Primer on Input-Output Modelling

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W.W. Leontief, *The Structure of American Industry 1919-1939*, 2nd ed., Oxford Univ. Press, 1951

Leontief's question: "What level of output should each of the *n* industries in an economy produce such that it will just be sufficient to satisfy the total demand for that product?"

Our question: "What level of employment is required in each of *n* industries to satisfy the demand for 8 TWh, 80 TWh, or 800 TWh of nuclear–generated electricity per year?"

where 1,000 MW(e) x 91.32% capacity factor yields 8,000 gigawatt-hours or 8 terawatt-hours (TWh), so for 1 GWe, 10 GWe, or 100 GWe

Production functions: General and Input-Output

from presentations by Prof. J.M. Guilhoto, Univ of San Paulo, Brazil

http://guilhotojjmg.wordpress.com/

$$x_j = f\left(z_{1j},\ldots,z_{nj},W_j,M_j\right) \qquad x_j = \min\left(\frac{z_{1j}}{a_{1j}},\ldots,\frac{z_{nj}}{a_{nj}}\right)$$

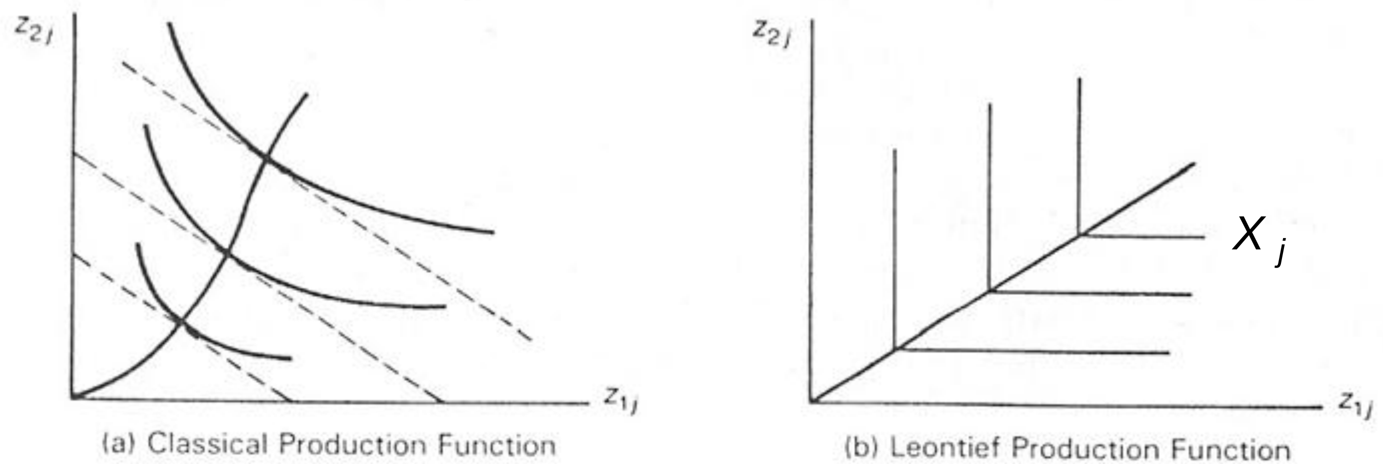


FIGURE 2-1 Production functions in input space.

Input-Output Modelling Assumptions:

(1) Each industry produces only one homogeneous product, x_i, e.g., MWh of electricity, tonnes of uranium, reactor vessels, etc., valued in currency units (e.g., euros). If there is more than one product, there should be more than one industry.

(2) Each industry uses a fixed input ratio, a_{ij} (or factor combination) for the production of its output, i.e., the capital to labour ratio is constant.

(3) Production in every industry is subject to constant returns to scale, i.e., a k% change in every input yields a k% change in output.

Industries related to the nuclear power sector according to the North American Industry Classification System (NAICS)

http://www.census.gov/cgi-bin/sssd/naics/naicsrch?chart=2012

21	Mining
212291	Uranium-Radium-Vanadium Ore Mining
22	Utilities
221113	Nuclear Electric Power Generation
23	Construction
237130	Electric light and power plant (except hydroelectric) construction
238220	Cooling tower installation
31-33	Manufacturing
325180	Radioactive isotopes manufacturing
332410	Reactors, nuclear, manufacturing
333611	Turbine and Turbine Generator Set Units Manufacturing
335310	Electrical Equipment Manufacturing
335312	Motor and Generator Manufacturing
92	Public Administration
926130	Regulation of Communications, Electric, Gas, and Other Utilities

South Korean Sectoral Outputs, 1995 and 2000

http://www.iaea.org/Publications/Booklets/ROK/rok0809.pdf

TABLE A-3.1. SECTORAL OUTPUTS AT THE NATIONAL LEVEL, 1995 AND 2000 (BILLION WON)

	Sector	1995	2000
1.	Agriculture and Forestry and Fishing	31 942	38 287
2.	Mining and Quarrying	3 256	2 648
3.	Manufacturing	400 873	647 344
4.	Nuclear Generation	3 934	7 927
5.	Construction	82 508	99 269
6.	Wholesale and Retail	49 599	69 844
7.	Food and Hotels	7 008	41 144
8.	Transport and warehousing	33 320	51 161
9.	Communication	11 869	33 891
10.	Finance and Insurance	32 282	63 435
11.	Real estate, Renting	72 498	137 433
12.	Public Administration	25 702	43 601
13.	Education/Research	26 421	41 762
14.	Health and Welfare Services	13 601	31 045
15.	Other Services	46 704	84 136
16.	Household	n/a	n/a
Total		841 519	1 392 928

The Open Model Input-Output Table:

(1) To produce each (one) unit of the *j*th output commodity, the amount needed of the *i*th input commodity must be a fixed input coefficient: a_{ij}, measured in currency units, e.g., 2014 euros (€).

(2) For an *n*-industry economy, the input coefficients can be arranged into a matrix $A = [a_{ij}]$.

(3) Consider a 3-industry economy: uranium, nuclear electricity generation (net), and all other, where "all other" does not include the household sector that supplies labour. This is an "open economy model." Households determine a final demand and supply a primary input, i.e., labor. (Later, we will close the model by including labor as an input.)

Input-Output Table for a 3-industry economy

	Output			
	Uran	MWh	Other	
Input	$j = 1$	$j = 2$	$j = 3$	
$i = 1$	$€a_{11}$	$€a_{12}$	$€a_{13}$	Uranium
$i = 2$	$€a_{21}$	$€a_{22}$	$€a_{23}$	Nuclear Generation (net)
$i = 3$	$€a_{31}$	$€a_{32}$	$€a_{33}$	All other
	$= \sum €a_{i1}$	$= \sum €a_{i2}$	$= \sum €a_{i3}$	
Uranium	0	$€a_{12}$	0	
Nuclear Generation	$€a_{21}$	0	$€a_{23}$	$= A$
All other	0	$€a_{32}$	$€a_{33}$	
	$= €a_{21} + €a_{31}$	$= €a_{12} + €a_{32}$	$= €a_{23} + €a_{33}$	

Payments to Labour and Final Demand:

(1) Given that the input coefficients, a_{ij} (measured in currency units), the sum of those coefficients in a particular column, e.g., $\sum a_{1j}$, $\sum a_{2j}$, and $\sum a_{3j}$, are less than 1 (euro) and the difference between 1 and the sum is equal to the payment to labour to produce 1 unit of output *j*.

(2) If industry *1* is to produce an output just sufficient to meet the input requirements of the *n* industries and the final demand of the households, d_1, its output level x_1 must satisfy:

$$x_1 = a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + d_1 \text{ or}$$

$$(1 - a_{11})x_1 - a_{12}x_2 - \ldots - a_{1n}x_n = d_1$$

The Open Input-Output Model:

(1) For all industries, a set of equations must be satisfied:

$$(1 - a_{11})\, x_1 - a_{12}\, x_2 - \ldots - a_{1n}\, x_n = d_1$$
$$- a_{21}\, x_1 + (1 - a_{22})\, x_2 - \ldots - a_{2n}\, x_n = d_2$$
$$\ldots \qquad\qquad\qquad \ldots$$
$$- a_{n1}\, x_1 - a_{n2}\, x_2 - \ldots + (1 - a_{nn})\, x_n = d_n$$

For example, our set of three equations must be satisfied:

$$(1 - 0)\, x_1 - a_{12}\, x_2 - (0)\, x_3 = d_1 = 0$$
$$- a_{21}\, x_1 + (1 - 0)\, x_2 - a_{23}\, x_3 = d_2$$
$$- (0)\, x_1 - a_{32}\, x_2 + (1 - a_{33})\, x_3 = d_3$$

(2) This system can be expressed in matrix notation:

$$(I - A)\, \underline{x} = \underline{d} \quad or \quad \underline{x}^* = (I - A)^{-1}\, \underline{d}$$

where I is the identity matrix with 1s on the diagonal and 0s everywhere else, $(I - A)$ is the "technology matrix," and $\underline{x}^*$ is the solution to the system

An Open 3 Input-Output Economy:

For our three equation economy, assume the input coefficients are

0.10 units of uranium used in producing electricity, $a_{12} = 0.10$
0.05 units of electricity used in producing uranium, $a_{21} = 0.05$
0.10 units of electricity used in producing all other, $a_{23} = 0.10$
0.80 units of all other used in producing electricity, $a_{32} = 0.80$
0.70 units of all other used in producing all other, $a_{33} = 0.70$

For our three equation economy, assume the final demands are

Final demand for uranium from households is 0 = d_1
Final demand for electricity from households is €1B = d_2
Final demand for all other from households is €20B = d_3

An Open 3 Input-Output Economy :

(1) Our set of three equations must be such that the levels of $\underline{x}$ solve the following three equations:

$$x_1 - 0.10\, x_2 = 0 \quad (1)$$

$$-0.05\, x_1 + x_2 - 0.1\, x_3 = €1B \quad (2)$$

$$-0.8\, x_2 + 0.3\, x_3 = €20B \quad (3)$$

(2) This system can be solved with matrix algebra or by solving for x_1 in Equation (1) and substituting into Equation (2), etc.:

$$\underline{x}^* = [\ 1.05,\ 10.5,\ 94.75\ ]^T$$

(3) The economy must produce about €1.05B in uranium units, €10.5B in electricity units, and €94.75B in all other units to pay the household sector (supplying labor) €21B.

Dynamic Input-Output Models:

(1) In the preceding discussion, we assumed an equilibrium, static economy. When considering changes over time, the linear equations become difference or differential equations. For example, if we were to consider the construction of new plant.

(2) Assume that the level of inputs changes over time so that x_i become x_{it}, where x_{it} is the level in period t. For example, the level of electricity changes so $x_{2t+1} > x_{2t}$. The system becomes

$$x_{1t+1} - a_{11} x_{1t} - a_{12} x_{2t} - \dots - a_{1n} x_{nt} = d_{1t}$$

$$- a_{21} x_{1t} + x_{2t+1} - a_{22} x_{2t} - \dots - a_{2n} x_{nt} = d_{2t}$$

$$\dots \qquad \dots$$

$$- a_{n1} x_{1t} - a_{n2} x_{2t} - \dots + x_{nt+1} - a_{nn} x_{nt} = d_{nt}$$

The x_{it} can be solved with particular integrals while the complementary functions remain unaffected.

The Closed Input-Output Model:

(1) In the general I-O model (as practiced, see for example, IAEA 2009) the household sector sells labor like any other input and consumes what is required to reproduce the labour. Final demand, d_j, includes savings, exports, etc.

(2) For all industries, the set of equations becomes:

$$(1 - a_{11})\, x_1 - a_{12}\, x_2 - \ldots - a_{1n}\, x_n - a_{1,n+1}\, x_{n+1} = d_1$$

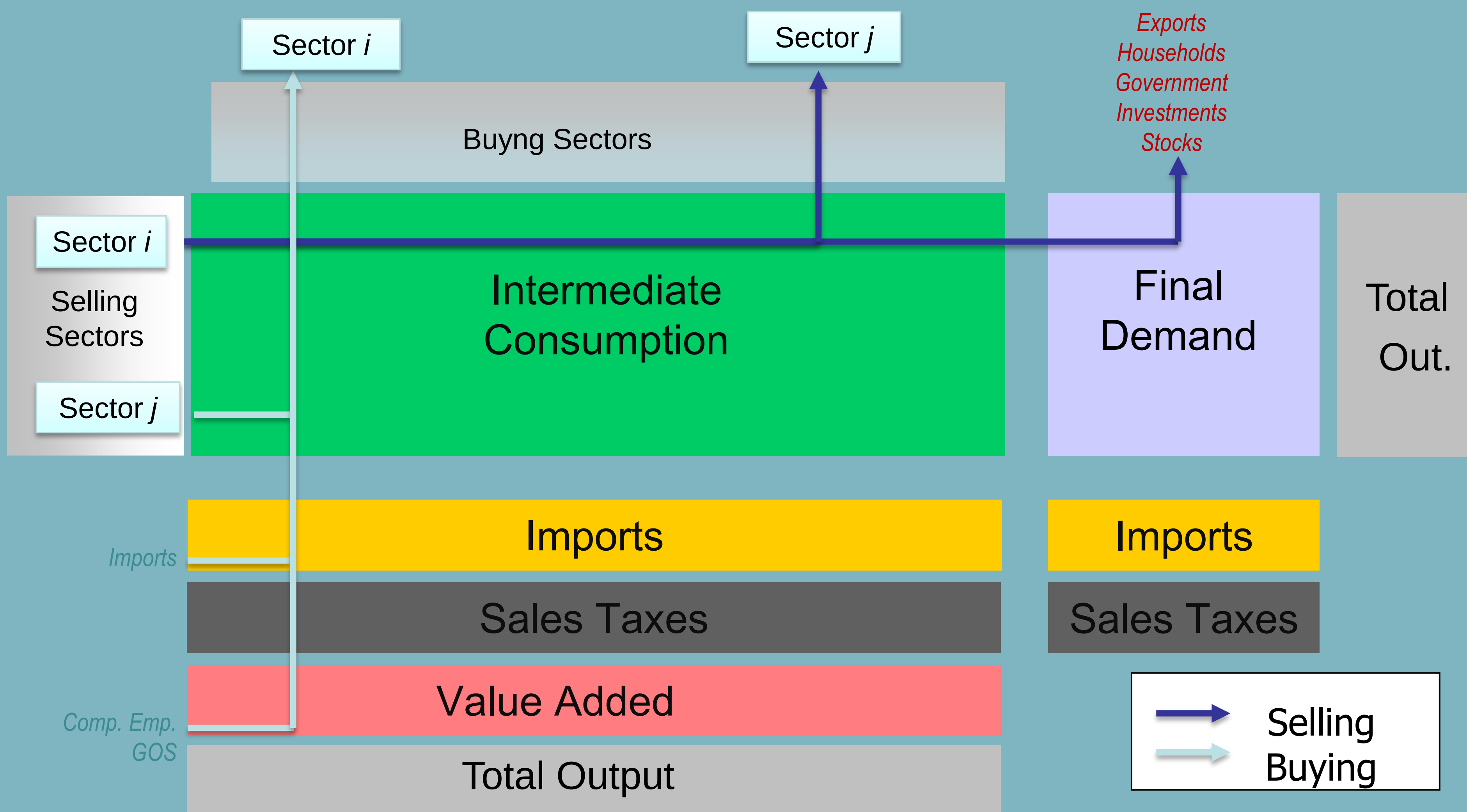
$$- a_{21}\, x_1 + (1 - a_{22})\, x_2 - \ldots - a_{2n}\, x_n - a_{2,n+1}\, x_{n+1} = d_2$$

$$\ldots \qquad \ldots$$

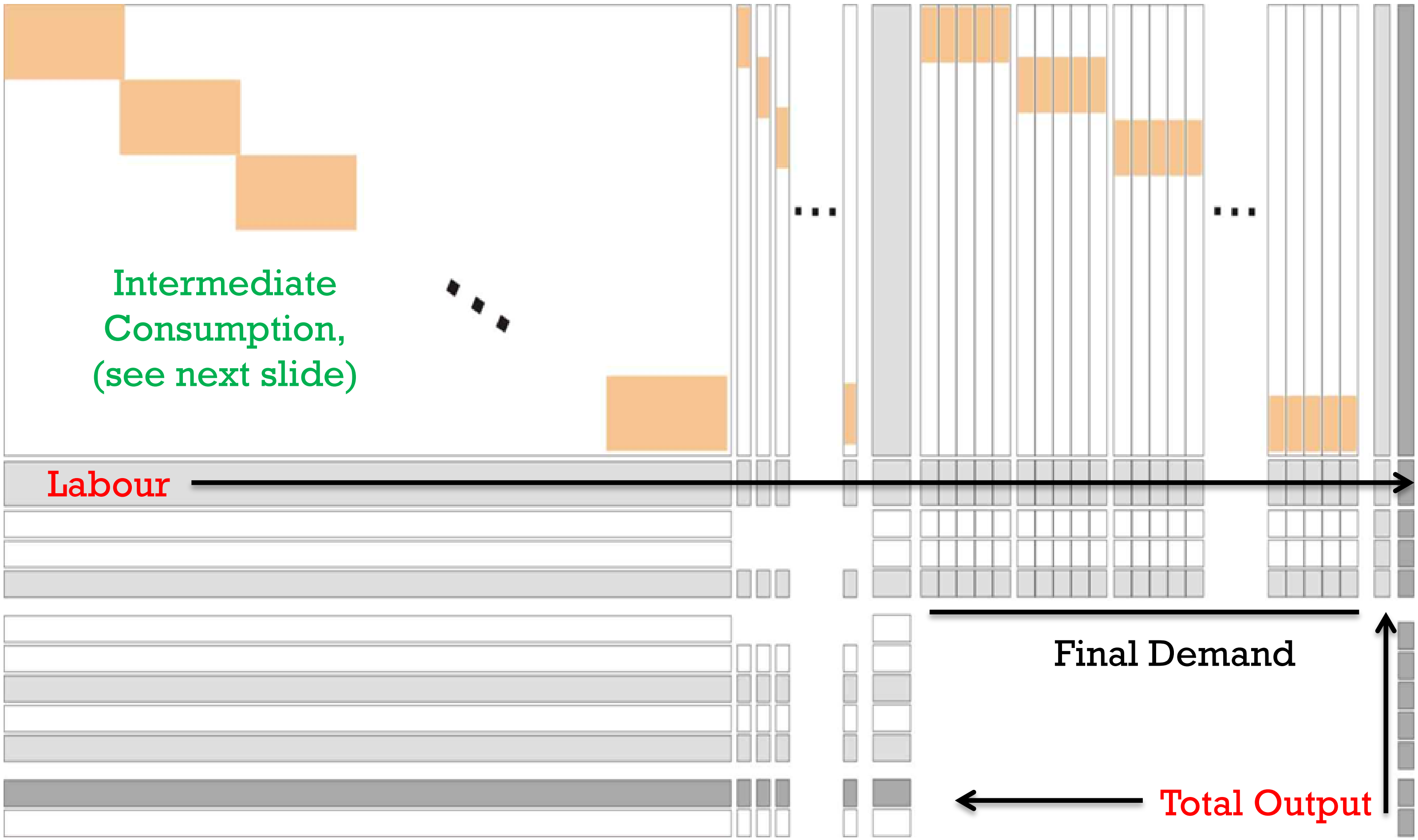
$$- a_{n1}\, x_1 - a_{n2}\, x_2 - \ldots + (1 - a_{nn})\, x_n - a_{n,n+1}\, x_{n+1} = d_n$$

$$- a_{n+1,1}\, x_1 - a_{n+1,2}\, x_2 - \ldots - a_{n+1,n}\, x_n + (1 - a_{n+1,n+1})\, x_n = d_{n+1}$$

Generalized Input-Output Table (from Guilhoto)



Generalized Input-Output Table (20 x 20) Lots of Data!



South Korean Input-Output Table, 2000

http://www.iaea.org/Publications/Booklets/ROK/rok0809.pdf

Labor is the 16th Sector

TABLE A-3.6. NATIONAL INPUT COEFFICIENTS BY SECTOR, 2000

Sector	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.0481	0.0019	0.0393	0	0.0024	0	0.0553	0	0	0	0.0001	0.0004	0.0001	0.0063	0.0054
2	0	0	0.0549	0	0.0038	0	0.0002	0	0	0	0	0.0001	0	0	0.0773
3	0.2226	0.1215	0.5005	0.0913	0.3729	0.0407	0.3580	0.2399	0.0405	0.0197	0.0397	0.1280	0.0620	0.2717	0.1836
4	0.0010	0.0157	0.0051	0.0127	0.0008	0.0058	0.0055	0.0017	0.0038	0.0018	0.0048	0.0062	0.0042	0.0044	0.0045
5	0.0007	0.0020	0.0005	0.0539	0.0003	0.0019	0.0039	0.0005	0.0030	0.0007	0.0481	0.0066	0.0034	0.0026	0.0078
6	0.0117	0.0070	0.0260	0.0036	0.0249	0.0352	0.0440	0.0103	0.0044	0.0022	0.0027	0.0099	0.0079	0.0253	0.0234
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1961
8	0.0081	0.0134	0.0097	0.0017	0.0095	0.0193	0.0081	0.1896	0.0057	0.0106	0.0053	0.0141	0.0042	0.0056	0.014
9	0.0030	0.0046	0.004	0.0029	0.0039	0.0623	0.0063	0.0076	0.1670	0.0214	0.0261	0.0117	0.0064	0.007	0.0138
10	0.0234	0.0615	0.0187	0.0289	0.0201	0.0322	0.0114	0.0246	0.0197	0.1368	0.0540	0.0142	0.0160	0.0223	0.0187
11	0.0378	0.0771	0.0304	0.0208	0.0960	0.1290	0.0719	0.0754	0.0696	0.0759	0.0658	0.0459	0.0264	0.0783	0.0572
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0.0004	0.0013	0.0114	0.0188	0.0068	0.0039	0.0005	0.0033	0.0076	0.0025	0.0050	0.0026	0.0200	0.0013	0.0023
14	0.0022	0.0006	0.0014	0.0032	0.0018	0.0018	0.0042	0.0024	0.0009	0.0015	0.0021	0.0037	0.0018	0.0102	0.0014
15	0.0168	0.0593	0.0248	0.0184	0.0173	0.0479	0.0259	0.0275	0.0932	0.0393	0.0336	0.0748	0.0606	0.0519	0.0915

There is no Mining, Food-Hotels, or Public Administration entering Agriculture

Nuclear's Employment Impacts:

To account for the different manner in which they arise, employment effects are divided into three categories:

direct employment effects **at nuclear power facilities;**

indirect employment effects **in facilities' supply chains; and**

induced employment effects **in all other sectors of the economy, resulting from firm and household spending accrued from direct and indirect effects.**

How are these calculated? The final demand for nuclear power is perturbed, e.g., increased by 8 TWh, and the changes in employment in (1) the nuclear power sector, (2) the nuclear power supply chain, and (3) the economy as a whole.

Employment Multipliers:

http://faculty.washington.edu/krumme/systems/multp.html

(1) Direct Employment Effects can be calculated by looking at changes in labour input in the nuclear power sector, e.g., line 4 in the Korean I-O table above: $e_4^* \, a_{4\,n+1}$, *where* e_4^* is the new equilibrium employment per € in the nuclear power sector.

(2) e^*_i =new equilibrium employment per € of total inputs in industry *i* (supplier)

e^*_j =new equilibrium employment per € of total output in industry *j* (receiver)

(3) Direct and Indirect Effects: Type I Multiplier

$$M_I = \sum e_i^* a_{ij} / e^*_j \quad \text{for } i = 1, \ldots, n$$

Direct, Indirect, and Induced Effects: Type II Multiplier

$$M_{II} = \sum e_i^* a_{ij} / e^*_j \quad \text{for } i = 1, \ldots, n+1$$

Nuclear Employment Modelling Questions:

(1) What constitutes the "nuclear power sector"?

(2) How should employment be determined in this sector?

(3) What model/methodology is best able to make this calculation?

(4) When adding the nuclear power sector to a model that does not specifically have one, how should production coefficients, e.g., a_{ij}, be determined?

(5) When constructing, operating, and decommissioning nuclear power plants, how should changes over time be accounted?

(6) When perturbing the model, gross changes are observed. How can net changes be calculated?

(7) Are multipliers appropriate for short-term or long-term?

(8) Are multipliers stable over time (e.g., with new technology)?

(9) What are appropriate error terms associated with multipliers?

(10) What modifications would be required for other electricity sources?