

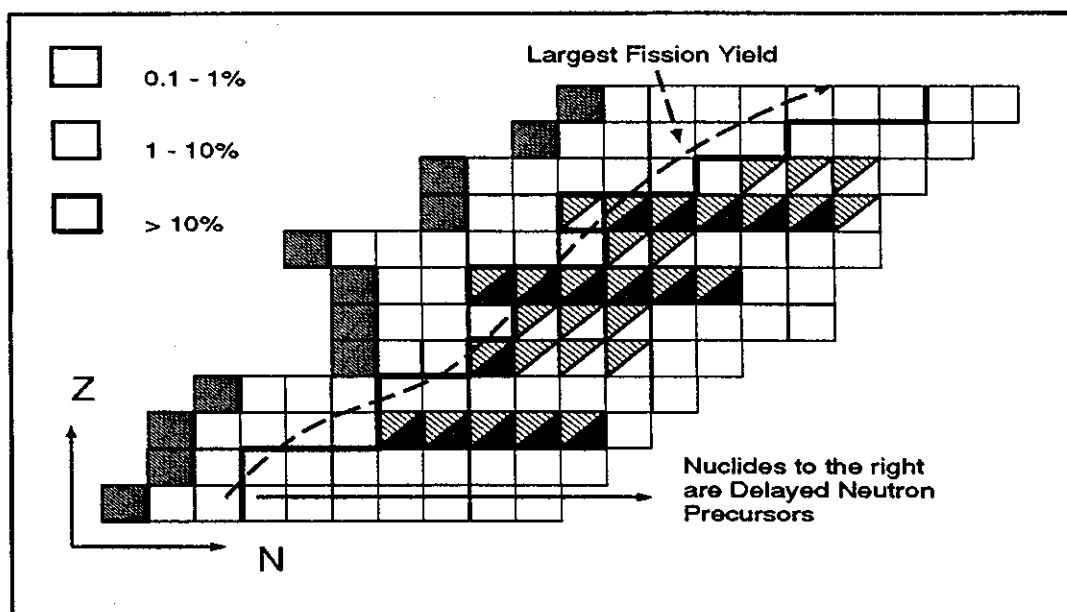
OECD NUCLEAR ENERGY AGENCY (NEA)

Committee on Reactor Physics (NEACRP) and  
Nuclear Data Committee (NEANDC)

## STATUS OF DELAYED NEUTRON DATA - 1990

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### - ABSTRACT -

Delayed neutron data play a key role in the reactor physics analysis of safety related parameters. This is the case for any type of reactor. For existing and operating reactors the interest is for an improvement of the basic data which are used to establish the reactivity scale and the reduction of the associated uncertainties. In this context delayed neutron data play a significant role. Moreover, there is at present a strong trend towards the study and development of new reactor types. For these advanced and innovative reactor concepts, there is a need to establish complete and sound data bases.

This paper carries out a review of the delayed neutron parameters and their uncertainties as available today. The review focuses on reactor technology and presents the data in a "consistent structure" having three levels of refinement. Conclusions on the quality of the data together with recommendations for improving them through modelling and measurements on each of the three levels are formulated.

Improvements in delayed neutron data make it possible to establish a more precise reactivity scale for existing reactors. Increased safety margins can be achieved by reducing, by about a factor of 2, the current uncertainty of 5 % (one standard deviation).

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## INTRODUCTION

# 1 INTRODUCTION

Trends in reactor technology reported at meetings such as PHYSOR'90 include the increased use of mixed-oxide fuels and the move to increased enrichment and burnup in current light water reactor designs as well as the development of fast reactor designs such as the actinide burning reactor. Accurate predictions of the kinetic response of these new reactor fuels require reliable data concerning delayed neutron production. These trends illustrate the necessity for improving the delayed neutron data available for transuranic nuclides and resolving the discrepancies existing in the current data, particularly those for  $^{238}\text{U}$ . The current delayed neutron data have been shown to contribute significantly to the large uncertainties<sup>1</sup> in the reactivity scale for fast reactors; improvements in the basic delayed neutron data and integral benchmarking are necessary to reduce these uncertainties.

This paper will briefly review the status of delayed neutron data, identify prominent discrepancies, and identify and prioritize areas for improvement in the data.

<sup>1</sup>uncertainties are given throughout as one standard deviation.

## BACKGROUND

### 2 BACKGROUND

The fundamental goals of every basic data library are precision and general applicability; the long term objective of the delayed neutron evaluation effort must be focused toward these same goals. A straightforward method for doing this is related to an effort to ensure consistency between 3 levels of treatment of delayed neutron parameters (ref. 1). These, as described in figure 1, are:

- Level 1. The individual precursor, or microscopic, level; this is the most extensive and physically based of the three levels.
- Level 2. The aggregate precursor, or macroscopic, level which is more synthetic but contains global parameters which are directly measurable and these can be used in reactor applications.
- Level 3. The integral level; this provides important and stringent tests related to applications such as reactor kinetics calculations and includes comparison of values for  $\beta_{\text{eff}}$  from measurements with values from calculations using lower level data.

Theoretically, a complete knowledge of the information at level 1 would be sufficient as the parameters at the other 2 levels can be deduced from this.

Practically and historically the 3 levels were unequally and nearly independently developed in terms of their Measurement, Modelling, and Evaluation (MM&E). The data initially produced in the first decades (1950-1970), were principally at level 2 with some at level 1, such as fission product yields. The basic work at this time was that performed by Keepin (ref. 2) which resulted in the now familiar six-group modelling of delayed neutron parameters as used in all later fission reactor design. The last two decades (1970-1990) have focused on the MM&E for level 1 data, primarily fission yields, the delayed neutron emission probabilities ( $P_n$ ) and spectra ( $\chi_d$ ). These advances in the level 1 data have been based upon work in nuclear physics investigating the detailed structure of nuclei. Some level 3 data have also been developed in this period, and small advances in level 2 parameters such as  $\bar{\nu}_d$  and spectra have also been made. With the evaluated microscopic data now available the macroscopic parameters of level 2, and consequently the integral parameters of level 3 (namely  $\beta_{\text{eff}}$ ), can or already have been calculated with satisfactory precision except for  $\bar{\nu}_d$  (see section 3.2.2).

In the next section the status of macroscopic delayed neutron data (level 2) is reviewed, focused on the fissile nuclides of principal importance for nuclear reactor technology,  $^{235}\text{U}$ ,  $^{238}\text{U}$  and  $^{239}\text{Pu}$ . Some suggestions for future experimental and theoretical work to improve the data are made.

### 3 MACROSCOPIC DELAYED NEUTRON DATA

These data are conventionally classified as time-dependent and time-independent (equilibrium) parameters.

#### 3.1 Time-Dependent Parameters

The time-dependent parameters such as delayed neutron activity,  $DN(t)$ , and the spectra  $\chi(t)$ , which result from the decays of individual precursors are currently obtained by "summation" from level 1 microscopic data and they have been shown to perform in general good agreement with level 2 data obtained experimentally. Specifically, this is done by starting with the fission yields of delayed neutron precursors, in either independent or cumulative form ( $Y_i$  or  $Y_c$ ), and then combining these results with decay data:  $P_n$  values and the spectra from individual precursors. See the scheme in figure 1. The  $DN(t)$  parameter is usually represented in a 6-temporal group form ( $\alpha$  and  $\lambda$ ) normalized to an absolute delayed neutron yield,  $\bar{\nu}_d$ . Spectra corresponding to the 6-temporal groups may also be derived from the level 1 data using indirect summation techniques (Brady, ref. 3).

##### 3.1.1 Six-temporal group constants

Current sets of six-group parameter data from the literature are:

- the (classical) Keepin (1965, ref. 2), which are essentially the data proposed by Cox (1974, ref. 4) for ENDF/B-IV (Evaluated Nuclear Data File) and retained for ENDF/B-V,
- the Waldo et al (1980, ref. 5) measured data.
- the Manevich et al (1988, ref. 6) evaluated summation data, and
- the Brady and England (1989, ref. 3), summation data.

The differences between these data sets are evident, but the repercussions of using a particular set on the calculated reactor kinetics parameters (a stringent test) are less noticeable (Stevenson, ref. 7), although some significant uncertainties in the reactor reactivity (inverse kinetic equation) were pointed out by Manevich and co-workers (1988, ref. 5). The most apparent differences are seen in the groups representing short decay times.

## MACROSCOPIC DELAYED NEUTRON DATA

More specific sensitivity studies are needed to determine the importance of these parameters relative to delayed neutron activity,  $DN(t)$ , and the kinetic response to reactivity changes.

### 3.1.2 Spectra associated with the six-temporal groups

Measured group spectra prior to 1986 were available only for groups 1-5 for  $^{238}\text{U}$  and only 1-4 for  $^{235}\text{U}$  and  $^{239}\text{Pu}$ . Measurements by Lowell (ref. 8) and Birmingham (ref. 9) have expanded the measured spectra to include all six delayed neutron groups for  $^{235}\text{U}$ ,  $^{238}\text{U}$ , and  $^{239}\text{Pu}$ .

The more recent and complete data sets for six-group spectra are:

1. the summation results of Brady et al. (1989, ref. 3) which utilized a great deal of level 1 individual precursor data (measured by Rudstam (ref. 10), Kratz (ref. 11), Shalev (ref. 12), Reeder (ref. 13), Greenwood (ref. 14) and many others). These were augmented for the unmeasured energy ranges by theoretical predictions (Mann, ref. 15) and rely on systematics models for precursor nuclides with no measured data (England et al. 1986, ref. 16).
2. directly measured level 2 macroscopic data. These include both time-dependent spectra and those arranged into six temporal groups. The results include earlier measurements (Batchelor (ref. 17), Feig (ref. 18), Shalev and Cuttler (ref. 19), and others) plus the more recent (1988-1989) measurements at the University of Lowell (ref. 8) and those of the University of Birmingham (1986, ref. 9).

The consistency of the two entirely independent methods (1) and (2), has been demonstrated by Brady et al. (1989, ref. 3), and the set of results they proposed are probably the most complete and accurate data available on aggregate (level 2) spectra.

The sensitivity of fast reactor kinetic behavior to variations in delayed neutron energy spectra has been studied by Das and Walker (1986, ref. 20) who concluded "no striking consequences of spectral changes have been observed". Further studies of  $\beta_{eff}$  sensitivity to delayed neutron spectra are in progress in Casaccia (D'Angelo, 1990, ref. 21).



## MACROSCOPIC DELAYED NEUTRON DATA

### 3.2 Time-Independent (Equilibrium) Delayed Neutron Data

#### 3.2.1 Equilibrium delayed neutron spectra

Equilibrium energy spectra are readily deduced from the time-dependent results noted in Sect. 3.1.2. The results of Los Alamos (ref. 3) are to be utilized consistently with the six-group spectra.

#### 3.2.2 Absolute delayed neutron yields from fission

This parameter is the most important from two perspectives:

1. as a stringent test of overall consistency of the two levels of data; e.g.,  $\bar{\nu}_d$  resulting from the summation calculations using the microscopic data of level 1 in comparison with experimental values for  $\nu_d$  on the macroscopic level (level 2), and
2. as a fundamental parameter in nuclear reactor kinetics and dynamics studies, via parameters derived at level 3. Specifically the integral  $\beta_{\text{eff}}$  is particularly sensitive to the  $\bar{\nu}_d$  parameter (D'Angelo, 1990, ref. 21).

In connection with this second perspective, it is useful to distinguish between three classes of fissile systems:

- Firstly, those concerning the short term goals associated with nuclear energy technologies, e.g. thermal and fast fission in  $^{235}\text{U}$ , thermal and fast fission in  $^{239}\text{Pu}$  and fast fission in  $^{238}\text{U}$ , (labeled for convenience  $^{235}\text{U}$  (T and F),  $^{239}\text{Pu}$  (T and F) and  $^{238}\text{U}$  (F)).
- secondly, fissile systems related to intermediate or long term goals (such as improved or new concepts for nuclear reactors); particularly  $^{232}\text{Th}$  (F),  $^{233}\text{U}$  (F and T),  $^{240}\text{Pu}$  (F),  $^{241}\text{Pu}$  (F and T) etc.
- finally, the higher transactinides and some exotic systems important for fundamental physics or particular applications (e.g. space reactors (Ronen, 1990, ref. 22)).

In this paper we focus our review on the first class of fissile systems.

Summation calculations using level 1 data have been performed by several groups. We have referred here to the most recent and/or complete results. There are three evaluation efforts effectively utilizing data on level 1:

## MACROSCOPIC DELAYED NEUTRON DATA

- ENDF/B-VI (ref. 3) based on preliminary data from ref. 23a for fission yields and on data from ref. 23b for  $P_n$ .
- JEF-2 (ref. 24) based on data from ref. 25 for fission yields and mainly on data from ref. 26 for  $P_n$ .  
(Joint Evaluated File)
- Soviet work based on data from ref. 5.

On the other hand, there are many proposed evaluated  $\bar{\nu}_d$  data resulting from MM&E on level 2. We will address these first.

A selection among the many proposed evaluated data sets is presented in table 1. In figure 2 are displayed the results of MM&E (evaluations by Tuttle 1979, ref. 27b) for all the measurements with uncertainties concerning the three most important fission systems for nuclear reactor technology - thermal and fast fission of  $^{235}\text{U}$  and  $^{239}\text{Pu}$ , and fast fission of  $^{238}\text{U}$ .

It is considered that the evaluations of Tuttle (1975 ref. 27a, 1979 ref. 27b) are the most careful and exhaustive and, therefore, recommendable provided that more recent measurements by Synetos, Waldo, Benedetti, and other (post 1979) results be included. Table 2 lists nuclides with measured  $\bar{\nu}_d$  data at one or more incident neutron energies.

However, some remarks are to be made:

1. The uncertainties appear to be too optimistic, as evident in figure 2. For a more quantitative illustration see table 3 where the uncertainties (associated with the mean of all measurements having uncertainties less than 15% in the evaluation by Tuttle) have been calculated in several classical ways.

It is apparent that the Tuttle uncertainties on the mean approach the standard deviation of the mean, relevant to the central limit theorem, which is not rigorously applicable since the measurements considered are not actually independent (they utilize some common techniques) and not always sufficiently numerous.

Consider the data for  $\bar{\nu}_d$  for fast fission in  $^{238}\text{U}$ . The inverse variance weighted mean calculated using the measured data reported in ref. 27b is 0.04354. The uncertainty in the value may be calculated as the internal error (1.96%) or the external error (4.01%). Note that the internal and external estimates of error are different methods of calculating the uncertainty on an average. The internal estimate is based on the determination of a mean of a set of values using the inverse of its variance as a weight for each value. The internal estimate of the variance of the mean is then the inverse of the sum of these weights, hence the internal estimate of the standard deviation of the mean is derived as the square root of this variance. The internal error thus represents the expected achievable standard deviation based upon the given uncertainty information on the input data.

## MACROSCOPIC DELAYED NEUTRON DATA

The external estimate of the error of the mean is derived from the weighted sum of squares of deviations of the data values from the weighted mean. It thus represents the standard deviation actually arising from the distribution of the data about the mean. If the original data is normally distributed, and the original estimates of errors on these values are reasonable, then the internal and external estimates of the standard deviation of the mean should be close to being equal. With respect to the data for  $^{238}\text{U}$  obtained from ref. 27b, the estimates of these errors are quite different indicating that either the data are not normally distributed or the estimates of the errors on the original data are not reasonable. In this case the external estimate, the more conservative error is the one to be recommended. The resultant  $\chi$ -squared test for these pre- 1979 values is 94% representing a good distribution about the mean. Inclusion of the Waldo 1981 (ref. 5) value only slightly changes the mean (.043577) and the internal and external errors are 1.95% and 4.04%, respectively. The  $\chi$ -squared test for these data is 96%. The Tuttle 1979 evaluation (ref. 27b) gives a value of 0.0439 for  $^{238}\text{U}$  (F) with an uncertainty of 2.3%.

It is considered that the real uncertainties lie between the standard deviation of the mean and the standard deviation of the individual typical measurement of  $\bar{\nu}_d$  that is:

$$\begin{aligned} &+/-3\% \text{ for } ^{235}\text{U} \text{ (T,F)} \\ &+/-4\% \text{ for } ^{239}\text{Pu} \text{ (T,F)} \end{aligned}$$

Based on the external error estimate and the dispersion of the data about the simple mean, we might expect the real uncertainty of  $\bar{\nu}_d$  to approach 4 to 6% for  $^{238}\text{U}$  (F).

2. The values of  $\bar{\nu}_d$  for these last three nuclides may be slightly different from the Tuttle (1975, ref. 27a and 1979, ref. 27b) evaluations. A selected set of MM&E values of  $\bar{\nu}_d$  for these three nuclides is reported in table 4. All the values starting from the Cox evaluation are equally probable except for  $^{238}\text{U}$ . The problem of  $^{238}\text{U}$  is an unresolved one at this time. It seems highly probable in the light of recent measurements (principally those of Birmingham (1986, ref. 28) that the recommended values of Tuttle may be too low. We believe the most probable value lies in the range 0.043 to 0.047.
3. The incident neutron energy dependence of  $\bar{\nu}_d$  (between 0 and 4 MeV) is not clear. Some increase in  $\bar{\nu}_d$  over this energy range is probable (3 to 4% for  $^{235}\text{U}$ ) but some structure is also possible in the intermediate energy ( $\leq 1$  MeV) region. Effort should be made to correlate the energy dependence of  $\bar{\nu}_d$  with threshold energies in the fission cross section.

Now to discuss the calculation of  $\bar{\nu}_d$  by summation using microscopic (level 1) data: among several examples of such work, we refer to the recent results of:

## MACROSCOPIC DELAYED NEUTRON DATA

- Los Alamos, USA (Brady and England, 1989, ref. 3),
- Winfrith and Birmingham, UK (James, Mills, and Weaver, 1990, ref. 29),
- Grenoble, France (Blachot and Benjelloun, 1990, ref. 30) and,
- Kurchatov, USSR (Manevich et al., 1987, ref. 5).

The sources of microscopic data ( $P_n$ 's and fission yields) in the first three cases are highly correlated. The fission yields are frequently generated by use of models (mostly Wahl's Zp model, ref. 31), fitted to the available data. As the evaluators attempt to collect all the published data, their data sets must therefore be very similar. For  $^{238}\text{U}$  (F) the summation methods have consistently low values compared with the Tuttle evaluation; the preliminary ENDF/B-VI and JEF-2 evaluations, use the Zp formalism but there is a suggestion (Blachot, ref. 30) that Wahl's Ap' model gives better agreement. One difference between the ENDF/B-VI and JEF-2 fission yield evaluations is that the latter uses a method of adjustment for the final results to provide agreement with physical constraints, but these adjustments are seldom greater than 1%. The measured  $P_n$ -values used come either from the Mann or Lund evaluations but as again these both attempt to include all measured results, their values are very similar. However,  $\bar{\nu}_d$  values computed from preliminary ENDF/B-VI (ref. 3b) are generally 2-4% higher than those obtained from JEF-2 (ref. 3c) for important fissile systems. Unmeasured  $P_n$ -values are predicted by the Kratz-Herrmann (ref. 32) or the Klapdor theory (ref. 33), although these represent generally less than 5% of the delayed neutron component of  $\bar{\nu}$ . So while the models used for the yields in the preliminary ENDF/B-VI and JEF-2 files are similar, it is interesting to note the differences in the calculated values. The Soviet work is quite different from the other three in its source data and especially in the selection for the summation.

Table 5 provides a comparison between these four sets of summation results for  $\bar{\nu}_d$  for  $^{235}\text{U}$  (T),  $^{238}\text{U}$  (F) and  $^{239}\text{Pu}$  (T) and also shows the values from the Tuttle 1979 evaluation. In the two last rows the weighted mean of the summation values derived from evaluated fission product yields is reported for comparison. The discrepancies generally fall within the uncertainty limits. However, it is noted that the summation uncertainties are, roughly, twice those of the direct measurements.

Decisive progress would be achieved if the summation uncertainties could be lowered to those of the direct (level-2) measurements (about 4%). Some insight might be gained through investigation of differences in fission yield prediction from use of the Ap' and Zp models. Equally, further study of even more representative models for the prediction of unknown fission yields would seem to be useful; Zp and Ap' are both based upon parameterization linked to empirical functions with little theoretical basis, but having mathematical convenience.

## MACROSCOPIC DELAYED NEUTRON DATA

The recent attempt by Soviet investigators (ref. 34) to explicitly link level-1, level-2 and MM&E data via a new elaborate model, accounting for, among other things, incident neutron energy dependence may be a source for further investigations.

On the other hand it has been shown that the major uncertainty in the summation  $\bar{\nu}_d$  comes from the uncertainty in the fission yield data rather than the  $P_n$  values (ref. 29 and 30). For cases like  $^{238}\text{U}$ , where independent yields are poorly known at present, while awaiting progress on the experimental side, one has to use results from models. However, this could introduce systematic errors which may be important when evaluating the uncertainty arising from the summation method. For example correlations between the errors in the contribution of different precursors could result in an increase in the overall uncertainty. This covariance problem is less important for  $^{235}\text{U}$  where much more uncorrelated experimental data are available. Thus the uncertainties quoted in table 4 for the summation calculation on  $^{238}\text{U}$  may be underestimates.

## RECOMMENDATIONS

# 4 RECOMMENDATIONS

### 4.1 Nubar-d ( $\bar{v}_d$ ) Absolute Data

The Tuttle (1979, ref. 27b) data are recommended, except for  $^{235}\text{U}$ ,  $^{233}\text{Pu}$ , and especially,  $^{238}\text{U}$  whose value must be some mean from table 4.

In view of the fact that the values in table 4 are equally probable, a simple mean with the dispersion for error may be proposed. Thus:

- $\bar{v}_d (^{235}\text{U})$  : 0.0166 +/- 3%
- $\bar{v}_d (^{239}\text{Pu})$  : 0.00654 +/- 4%
- $\bar{v}_d (^{238}\text{U})$  : 0.043 to 0.047

Due to the persistence of discrepancies in the reported values of  $\bar{v}_d$  of  $^{238}\text{U}$  (F) (see figure 2), it would be very *desirable to have a new and accurate measurement of this parameter.*

The summation calculations are a useful tool for deriving  $\bar{v}_d$  data from the microscopic data. However, the uncertainties (8-10%) in the  $\bar{v}_d$  values obtained from summation will have to be reduced to the level of the direct measurement (4%) before the method will be generally applicable. It has been shown (ref. 29 and 30) that the major uncertainty in the summation  $\bar{v}_d$  comes from the uncertainty in the fission yield data rather than the  $P_n$  value. This suggests that further effort in the level 1 data is necessary to *improve the fission yield data.*

Where there are no direct measurements of  $\bar{v}_d$  (level 2 data) but experimental fission yield data do exist, use of the summation technique to calculate  $\bar{v}_d$  is recommended.

For nuclides like  $^{243}\text{Am}$  where there are no direct measurements of  $\bar{v}_d$  (level 2 data) nor measured level 1 data (i.e., fission yields), it is recommended that a semi-empirical fit of  $\bar{v}_d$  to the parameter (3Z-A) as proposed by Tuttle (ref. 27b), Waldo (ref. 5) and Ronen (ref. 22) be used.

## RECOMMENDATIONS

### 4.2 Temporal Group Parameters and Spectra

Additional sensitivity studies should be performed to quantify the importance of the six-group parameters (abundances and half-lives) in calculations of integral  $\beta_{\text{eff}}$  and the kinetic responses to a change in reactivity. Experimental level 2 data is available only for a small number of fission nuclides.

The most recent and complete measured sets of six groups are those of:

- Waldo et al (ref. 5) concerning the  $(\alpha, \lambda)$  parameters
- University of Lowell (ref. 8) concerning the spectra.

However, due to the overall consistency of the data, the Brady summation group parameters and particularly the spectra are recommended for use in reactor applications.

### 4.3 Integral (level-3) Testing of Delayed Neutron Data.

Among the integral nuclear reactor kinetic parameters,  $\beta_{\text{eff}}$  is the best suited for testing the delayed neutron data as it is directly proportional to the  $\bar{\nu}_d$  parameter (see figure 1).

Two sets of  $\bar{\nu}_d$  values measured independently (level-2 and level-3) can be used to validate each other provided the two sets have comparable uncertainties, namely of about 4%. This precision is the one achieved today for level-2. The precision achieved at present in  $\beta_{\text{eff}}$  measurements is however still of the order of 10%.

Because of the importance this parameter has for economic and safety aspects of reactor technology such as the reactivity scale, an international  $\beta_{\text{eff}}$  benchmark experiment was proposed in the framework of NEACRP (ref. 35). The aim of this benchmark is to decrease substantially this uncertainty. It is desirable to use this opportunity to achieve also the validation of  $\bar{\nu}_d$  of  $^{235}\text{U}$ ,  $^{238}\text{U}$  and  $^{239}\text{Pu}$ . In this case the goal is *to reduce the uncertainty drastically but realistically* (ref. 36) to the following values:

- about 3.5% for the measurement (choice of technique, intercomparisons ...)
- about 2% for the interpretation (sensitivity studies, modelling ...)

The validation of the effect of the delayed neutron spectra on  $\beta_{\text{eff}}$  can also be achieved in the same work.

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## FIGURES

# 6 FIGURES

**Figure 1 Delayed Neutron Parameters**

**1. INDIVIDUAL LEVEL (summation of precursors)**

$$\bar{v}_d(E_f) = \sum_{A,Z} \lambda(A,Z) Y_c(E_f, A, Z) P_n(A, Z) \int_0^\infty \chi_d(E, A, Z) dE \int_0^\infty e^{-\lambda(A,Z)t} dt$$

**2. GLOBAL LEVEL (mathematical grouping of precursors)**

$$\bar{v}_d(E_f) = \sum_{k=1}^6 \lambda_k(E_f) \alpha_k(E_f) \int_0^\infty \chi_k(E) dE \int_0^\infty e^{-\lambda_k t} dt$$

**3. INTEGRAL LEVEL (reactors)**

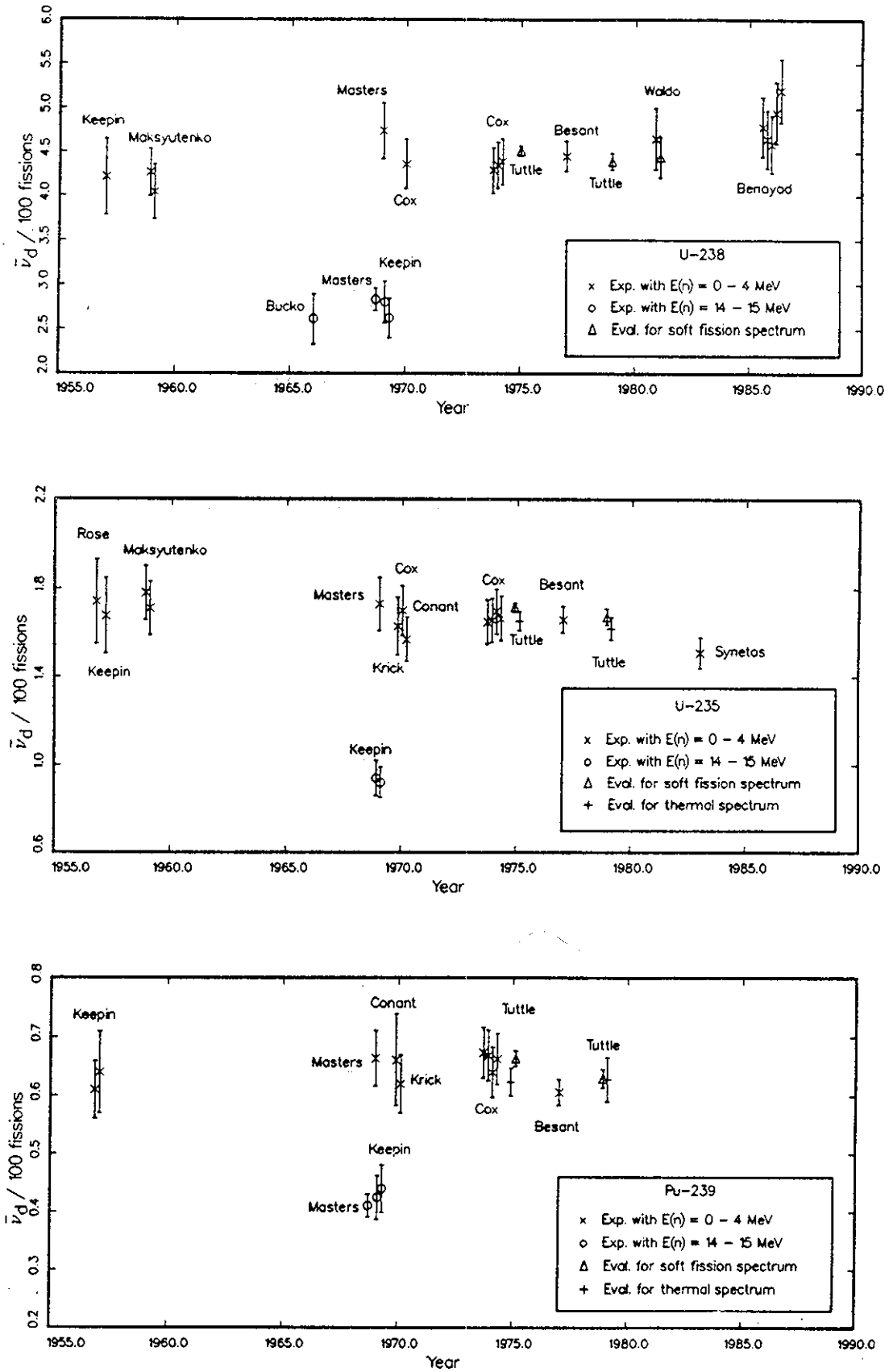
$$\frac{\bar{v}_d}{\bar{v}} \Rightarrow \bar{\beta}_{eff} = \sum_{k=1}^6 \kappa_k(\Delta\chi) \sum_i \beta_{ki} \frac{\bar{v}_i \Sigma_f \Phi}{\sum_i \gamma_i \Sigma_f \Phi}$$

**Notations:**

$\bar{v}_d$	Delayed neutron yield	$\bar{v}$	Total neutron yield
$E$	Delayed neutron energy	$E_f$	Incident neutron energy (inducing fission).
$A, Z$	Mass and charge of fission product precursor	$\lambda(A, Z)$	Decay constant of fission product $A, Z$ .
$Y_c(E_f, A, Z)$	Cumulative fission yield	$P_n$	Delayed neutron emission probability.
$\chi_d(E, A, Z)$	Energy spectrum of delayed neutrons for fission product $A, Z$	$\lambda_k$	Decay constant for group $k$
$\alpha_k$	Delayed neutron yield for group $k$	$\Sigma_f$	Macroscopic fission cross section
$\chi_k$	Energy spectrum of delayed neutrons for group $k$	$\kappa_k(\Delta\chi)$	Factor related to the importance weighted difference between prompt and delayed neutron spectrum
$\beta_{ki}$	$\bar{v}_{d,ki} / \bar{v}_i$	$\Phi$	Reactor spectrum

# FIGURES

Figure 2  $\bar{\nu}_d$  experiments after 1955 with uncertainty < 10 %.



TABLES

7 TABLES

Table 1 Evaluations of  $\bar{\nu}_d$  per 100 fissions

Nuclide	Keepin (1965)		Tomlinson	Cox
	Fast	Thermal	1972 (b) (c)	1974 (c)
<sup>232</sup> Th	4.96 (0.20)		5.2 (0.4)	5.27 (0.4)
<sup>233</sup> U	0.70 (0.04)	0.66 (0.03)	0.69 (0.02)	0.74 (0.04)
<sup>235</sup> U	1.65 (0.05)	1.58 (0.05)	1.65 (0.03)	1.668 (0.07)
<sup>238</sup> U	4.12 (0.17)		4.4 (0.21)	4.60 (0.25)
<sup>239</sup> Pu	0.63 (0.03)	0.61 (0.03)	0.64 (0.02)	0.645 (0.04)
<sup>240</sup> Pu	0.88 (0.06)		0.88 (0.09)	0.90 (0.09)
<sup>241</sup> Pu		1.54 (0.15)	1.59 (0.16)	1.57 (0.15)
<sup>242</sup> Pu			1.6 (0.5)	

Table 1 (continued)

Nuclide	Tuttle (1975)			Tuttle (1979)	
	Fast	Thermal	Combined	Fast (a)	Thermal
<sup>232</sup> Th	5.47 (0.12)		5.45 (0.11)	5.31 (0.23)	
<sup>233</sup> U	0.729 (0.02)	0.664 (0.02)	0.698 (0.013)	0.731 (0.036)	0.667 (0.029)
<sup>235</sup> U	1.714 (0.022)	1.654 (0.042)	1.697 (0.020)	1.673 (0.036)	1.621 (0.05)
<sup>238</sup> U	4.510 (0.061)		4.508 (0.060)	4.39 (0.10)	
<sup>239</sup> Pu	0.664 (0.013)	0.624 (0.024)	0.655 (0.012)	0.630 (0.016)	0.628 (0.038)
<sup>240</sup> Pu	0.96 (0.11)		0.96 (0.11)	0.95 (0.08)	
<sup>241</sup> Pu	1.63 (0.16)	1.56 (0.16)	1.60 (0.16)	1.52 (0.11)	1.52 (0.11)
<sup>242</sup> Pu	2.28 (0.25)		2.28 (0.25)	2.21 (0.26)	

(a) slightly softened fission spectrum (mean energy is nuclide dependent)

(b) deduced from sets of few measurements (2-6) for incident neutron energies between 0 and 4 MeV.

(c) Assembled by Tuttle (1975,ref. 27a) for the period 1958-1974.

TABLES

Table 2 Nuclides for which  $\bar{v}_d$  has been measured

$^{232}\text{Th}$	$^{240}\text{Pu}$
$^{232}\text{U}$	$^{241}\text{Pu}$
$^{233}\text{U}$	$^{242}\text{Pu}$
$^{235}\text{U}$	$^{241}\text{Am}$
$^{238}\text{U}$	$^{242\text{m}}\text{Am}$
$^{237}\text{Np}$	$^{245}\text{Cm}$
$^{238}\text{Pu}$	$^{249}\text{Cf}$
$^{239}\text{Pu}$	$^{252}\text{Cf}^*$

\*Value for spontaneous fission only.

Table 3 Uncertainties (in %) for the Evaluation of  $\bar{v}_d$ .

Nuclide		Number of Measurements (n)	D	BESD	SD	SDM	Evaluation Tuttle 1979
$^{235}\text{U}$	(T)	4	5.3	5.0	4.3	2.5	3.1
	(F)	8	3.5	2.5	2.4	0.9	2.1
$^{239}\text{Pu}$	(T)	5	12	10	8.9	4.5	6
	(F)	5	6.6	4.9	4.4	2.2	3.7
$^{238}\text{U}$	(F)	8	10	6.4	6.0	2.3	2.3

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$D(\text{Dispersion}) = \frac{x_i^{\text{max}} - x_i^{\text{min}}}{2\bar{x}}$$

$$\text{BESD}(\text{Best-Estimate-Standard-Deviation}) = \sigma \sqrt{\frac{n}{n-1}}$$

$$\text{SD}(\text{Standard-Deviation}) = \sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

$$\text{SDM}(\text{Standard-Deviation-of-the-Mean}) = \frac{\sigma}{\sqrt{n-1}}$$

TABLES

Table 4  $\bar{\nu}_d$  per 100 Fissions for  $^{235}\text{U}$ ,  $^{238}\text{U}$  and  $^{239}\text{Pu}$  (standard deviation in %).

Nuclide	Keepin (1965)		Cox (b)	Tuttle (1975)		Tuttle (1979)	
	F(a)	T(a)	1974	F	T	F	T
$^{235}\text{U}$	1.65(3)	1.58(3)	1.67(4)	1.71(1.3)	1.65(2.5)	1.67(2)	1.62(3)
$^{238}\text{U}$	4.45(4)	--	4.6(5.8)	4.51(1.3)	--	4.39(2.3)	--
$^{239}\text{Pu}$	0.63(4.8)	0.61(5)	0.645(6)	0.664(2)	0.624(4)	0.63(2.5)	0.628(6)

Table 4 (continued)

Nuclide	Waldo	Benayad	Kaneko	D'Angelo
	1981 (c)	1986 (d)	1988 (e)	1990 (f)
$^{235}\text{U}$	1.67	-- 1.65(1.2)	1.65(2)	
$^{238}\text{U}$	4.65(7.5)	4.84(8.3)	--	4.57(4)
$^{239}\text{Pu}$	0.68	--	--	0.66(3)

- (a) The neutron spectrum inducing fission is:  
 F : fast (softened fission spectrum)  
 T : thermal
- (b) The measurements/evaluations by Cox are those forming the basis for the ENDF/B-IV evaluation.
- (c) The yields relative to  $^{238}\text{U}$  are measured, those relative to  $^{235}\text{U}$  and  $^{239}\text{Pu}$  are measured and then adjusted using an ad-hoc systematics (ref. 5)
- (d) The  $\bar{\nu}_d(^{238}\text{U})$  is the mean over 5 measurements carried out at the University of Birmingham at the following energies: 1, 1.5, 2, 2.5, 3 MeV. (ref. 28)
- (e) The evaluation by Y. Kaneko et al. (ref. 37) is the result of a series of integral measurements ( $\beta_{\text{eff}}$ ) carried out in thermal critical assemblies at JAERI.
- (f) These values were obtained from a series of 10 integral measurements ( $\beta_{\text{eff}}$ ) on fast assemblies (ZPR and SNEAK) by using an adjustment procedure (ref. 21).

- Note (1) The number in parentheses next to the  $\bar{\nu}_d$  values represent the uncertainty of the measurement/evaluation in per cent.
- (2) The value of  $\bar{\nu}_d(^{238}\text{U})$  of Keepin has been rescaled because of a calibration error (factor 1.08 : see ref. 5)



TABLES

Table 5 Review of  $\bar{\nu}_d$  values per 100 fissions by summation

Reference	$^{235}\text{U}$ (T)	$^{238}\text{U}$ (F)	$^{239}\text{Pu}$ (T)
Tuttle 1979 (ref. 27b) (Evaluation)	1.621 (0.050)	4.39 (0.10)	0.628 (0.038)
Manevich et al. (ref. 6) 1988	1.712 (0.154)	4.25 (0.51)	0.64 (0.09)
Brady and England (ref. 3b) 1989	1.78 (0.10)	4.05 (0.29)	0.76 (0.04)
Blachot and Benjelloun(*) 1990 (ref. 30)	1.63 (0.11)	4.16 (0.24)	0.59 (0.06)
Zp model yields(**)	1.60	3.85	0.65
Ap' model yields(**)	1.63	4.59	0.69
James, Mills, Weaver(*) 1990 (ref. 29)	1.645 (0.114)	4.056 (0.199)	0.589 (0.054)
Weighted Mean	1.653	4.283	0.653
(The mean is only useful for comparison, and is not meant as a recommendation)			

(\*) The values from ref. 29 and 30 are both based on the same fission yield evaluation but have slightly different  $P_n$  evaluations.

(\*\*) These values result from the calculated yields by Wahl (ref. 31) and are given here only for comparison between models. They are not included in the weighted mean.