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A NODAL METHOD FOR FAST REACTOR ANALYSIS

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I. INTRODUCTION

In the past few years, coarse-mesh or "nodal" methods have had considerable success in solving fewgroup diffusion equations for light-water reactors (LWR's).[1,2,3] However, as Fröhlich has pointed out,[4] not much work has been done to apply coarse-mesh methods to the solution of realistic fast breeder reactor problems. This paper describes a method currently under development at the Argonne National Laboratory for multidimensional, multigroup fast reactor analysis using diffusion theory. The method is an extension of a highly successful method[5,6,7] developed for LWR static and transient analysis. The restriction made in the previous method to one or two neutron energy groups has been completely removed.

II. DEVELOPMENT OF THE BASIC METHOD

Let us write the time-independent multigroup diffusion equations in two-dimensional x-y geometry:

$$-\frac{\partial}{\partial x} D_g(x,y) \frac{\partial}{\partial x} \phi_g(x,y) - \frac{\partial}{\partial y} D_g(x,y) \frac{\partial}{\partial y} \phi_g(x,y) + \Sigma_{R_g}(x,y)\, \phi_g(x,y)$$

$$= \frac{\chi_g}{\lambda} \sum_{g'=1}^{G} \nu\Sigma_{f_{g'}}(x,y)\, \phi_{g'}(x,y) + \sum_{\substack{g'=1 \\ g\neq g'}}^{G} \Sigma_{s_{gg'}}(x,y)\, \phi_{g'}(x,y) \qquad (1)$$

We define $R_{i,j}$ to be a homogeneous region such that

$$R_{i,j} \;\varepsilon\; \begin{cases} (x_i, x_{i+1}) \\ (y_j, y_{j+1}) \end{cases}$$

We obtain the neutron balance equation by integrating Eq. (1) over $R_{i,j}$:

$$h_j \left(J_{gx_{i+1,j}} - J_{gx_{i,j}} \right)$$

$$+ h_i \left(J_{gy_{i,j+1}} - J_{gy_{i,j}} \right) + h_i h_j \Sigma_{R_{g\ i,j}} \bar{\phi}_{g\ i,j} =$$

$$\frac{\chi_g}{\lambda} \sum_{g'=1}^{G} h_i h_j \nu\Sigma_{f_{g'\ i,j}} \bar{\phi}_{g'\ i,j} + \sum_{\substack{g'=1 \\ g \neq g'}}^{G} h_i h_j \Sigma_{s_{gg'\ i,j}} \bar{\phi}_{g'\ i,j} \qquad (2)$$

where the above terms have their conventional meanings.[6,7]

To obtain a solution of Eq. (2), relationships must be found between the total currents and the adjacent average fluxes. This may be accomplished by solving analytically the partially integrated diffusion equation.[3,5,6,7] Thus, a relationship between the x-directed total current and the adjacent average fluxes is obtained by solving the equation resulting from integrating Eq. (1) over (y_j, y_{j+1}):

$$-D_{g\ i,j} \frac{\partial^2}{\partial x^2} \phi_{gj}(x) + \Sigma_{R_{g\ i,j}} \phi_{gj}(x) = \sum_{\substack{g'=1 \\ g \neq g'}}^{G} \Sigma_{s_{gg'\ i,j}} \phi_{g'j}(x)$$

$$+ \frac{\chi_g}{\lambda} \sum_{g'=1}^{G} \nu\Sigma_{f_{g'\ i,j}} \phi_{g'j}(x)$$

$$+ D_{g\ i,j} \int_{y_j}^{y_{j+1}} \frac{1}{h_j} \frac{\partial^2}{\partial y^2} \phi_g(x,y)dy \qquad (3)$$

where $\phi_{gj}(x)$ is the partially integrated flux.[6,7]

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Equation (3) has the same form as the one-dimensional diffusion equation, with the exception of the extra integral on the right hand side representing the y-directed leakage as a function of x. The equation may be solved analytically if the functional form of the right hand side is known. Let us assume at present the right hand side may be approximated as a quadratic polynomial. The transverse leakage term may be approximated as a quadratic as done in Refs. 1 and 3. The scattering and fissioning term may be represented as quadratics by assuming the following expansion of the partially integrated flux:

$$\phi_{gj}(W) = A_0^{gj,i} + A_1^{gj,i} \frac{W}{h_i} + A_2^{gj,i} \frac{W^2}{h_i^2} \tag{4}$$

where $0 \leq W \leq h_i$. With these approximations, Eq. (3) may be solved analytically, and relationships between the currents and average fluxes found.

Lastly, the flux expansion coefficients $A_k^{gj,i}$, k=0, 1, 2 (and their counterparts in the y direction) must be calculated. This is accomplished by utilizing an equation derived within the analytic solution procedure. The following equation for the detailed flux is found

$$\begin{aligned} \phi_{gj}(W) = {} & H_1^{gj}(W)\, \phi_{gj}(0) + H_2^{gj}(W)\, J_{xg\,j}(0) \\ & + H_3^{gj}(W)\, S_0^{gj,i} + H_4^{gj}(W)\, S_1^{gj,i} \\ & + H_5^{gj}(W)\, S_2^{gj,i} \quad (0 \leq W \leq h_i) \end{aligned} \tag{5}$$

where $S_k^{gj,i}$, k = 0, 1, 2 are the total components of the x-directed quadratic expansion; and $H_m^{gj}(W)$, m = 1,5 are trigonometric functions of the mesh spacings and cross sections within $R_{i,j}$. To obtain a quadratic function from the detailed solution in Eq. (5), several procedures could be used. In this paper, various collocation techniques have been tested. The collocation points are chosen to be the zeroes of the appropriate order Legendre polynomials.

III. FLUX AND LEAKAGE APPROXIMATIONS

Results are presented in this paper for three combinations of flux and leakage approximations:

1) Linear Flux - Flat Leakage

2) Quadratic Flux - Quadratic Leakage

3) Quadratic → Linear Flux and Leakage

In 1) and 2), the flux is found from a two and three point collocation, respectively. The leakage is taken as flat[3] in 1), and quadratic[1,3] in 2). In 3), the flux and leakage are first calculated as in 2); then a least squares technique is used to reduce the quadratic functions to linear. It was hoped this procedure would be an acceptably accurate approximation to that of 2).

IV. RESULTS

A static two-dimensional x-y test problem was devised which would be as similar as possible to a full-size heterogeneous core LMFBR. Eight-group cross sections were used, and the hexagonal geometry is represented by an equivalent x-y model. The area of the fuel assembly was chosen to match that of the original hexagonal fuel assembly. The problem is solved on a 15 × 15 grid (which includes two assembly thicknesses for reflector). Each assembly is 15 cm wide.

Solutions of this problem have been calculated using the AN2D (nodal) code for the three options given above, as well as finite difference (DIF3D[8]) results using a variety of mesh spacings. DIF3D runs using 1 × 1, 2 × 2, 3 × 3, 4 × 4, and 8 × 8 mesh cells per fuel assembly were made. The reference solution is taken as an extrapolation of the 4 × 4 and 8 × 8 DIF3D results. The nodal solutions used a 1 × 1 mesh grid. Table I shows the results of these runs.

The AN2D solution using linear flux and flat leakage is not acceptably accurate in power distribution, however the other nodal solutions are very accurate. To obtain an equivalently accurate DIF3D, a 4 × 4 mesh per fuel assembly must be used; however, in the true hexagonal geometry it is likely that the equivalent of the 3 × 3 DIF3D would be taken as acceptably accurate. The quadratic → linear nodal solution has an execution speed advantage of a factor of 3.6 and 8.2 over the 3 × 3 and 4 × 4 DIF3D runs, respectively.

V. SUMMARY

A nodal method has been developed which gives acceptably accurate results for realistic LMFBR problems at a considerable savings over finite difference methods. This method can also produce accurate and computationally efficient solutions for LWR configurations such as the IAEA benchmark problem. Nodal schemes of this kind hold significant promise for LMFBR analysis.

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TABLE I. Heterogeneous Core Problem Summary

Method	Eigenvalue λ	Execution Time (sec on 370/165)	Maximum Error In Assembly Power (%)
Reference	1.113273		
DIF3D			
1 × 1	1.148243	5.8	25.9
2 × 2	1.121355	32.4	8.6
3 × 3	1.116699	93.3	4.0
4 × 4	1.115129	213.4	2.3
8 × 8	1.113644	1282.8	0.5
AN2D			
Linear-Flat	1.113253	24.4	6.5
Quadratic-Quadratic	1.113347	38.9	1.4
Quadratic → Linear	1.112156	26.1	1.6

TABLE VI. Comparison of Calculated Components of Na Voiding

Zone Description, mm	Leakage			Non Leakage		
	8A	8C	8A/8C	8A	8C	8A/8C
Fuel Ring 1 ± 306	- 9.84	-10.87	0.91	34.86	32.32	1.08
Fuel Ring 1 ± 457	-16.01	-15.15	1.06	8.01	7.60	1.05
Fuel Ring 1 ± 814	-15.24	-14.40	1.06	4.99	5.03	0.99
Central Blanket ± 306	- 4.12	- 5.88	0.70	14.22	13.91	1.02
Central Blanket ± 814	- 3.39	- 4.36	0.78	3.16	3.23	0.98