OECD/NEA WPNCS

Expert Group on Monte Carlo Source Convergence in Criticality Safety Analysis

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Chronological Fission Source Convergence Bibliographie


• “Convergence of probability measures”, by P. Billingsley, John Wiley & Sons (1968). Theory of the Brownian Bridge. Billingsley (University of Chicago) presents weak-convergence methods in metric spaces, with applications that illustrate the power and utility of the probability limit theory in such disciplines as number theory, economics, and population biology. It will also cover an introduction to Brownian motion.


• “Detecting Initialization Bias in Simulation Output”, by L. Schruben, Operations Research, vol. 30, May-June 1982 (1982). The output is transformed into a standardized test sequence that can be contrasted with a known limiting stochastic process. This transformation requires very little computation and the asymptotic theory is applicable to a wide variety of simulations.


• “Simulation Run Length Control in the Presence of an Initial Transient”, by P. Heidelberger and P. Welch, Operations Research, vol. 31, Nov-Dec 1983 (1983). This paper explores a procedure based on Schruben's Brownian bridge model for the detection of nonstationarity and a spectral method for estimating the variance of the sample mean. The procedure is evaluated empirically for a variety of output sequences. The performance measures considered are bias, confidence interval coverage, mean confidence interval width, mean run length, and mean amount of deleted data.


• “Testing for Initialization Bias in Simulation Output”, by G. Vassilacopoulos, Simulation Councils (1989). A test is presented for detecting the presence of initialization bias in the mean of a simulation output process. The test is based on a standardized sequence of linear combinations of the rank of the simulation observations.


• “Computation of Standard Deviations in Eigenvalue Calculations,”, by E. M. Gelbard and R. E. Prael, Proceedings of the International Monte Carlo Methods for Neutron and Photon Transport Calculations Conference, September 25-28, 1990, Budapest, Hungary. ). The batching method for correcting uncertainty estimates is evaluated. The tradeoff between fewer large batches (reduced autocorrelation at the expense of a smaller sample size) and more small batches (a larger sample size with greater autocorrelation) is identified.
• “Acceleration of Fission Distribution Convergence Using Eigenvectors from Matrix K Calculations in the KENO Code”, by H. Kadotani, Y. Hariyama, M. Shiota, and T. Takeda, *Proc. ICNC ’91*, paper II-1, Oxford, UK, September 9-13, 1991. In their article, Kadotani et al proposed using the eigenvector of the fission matrix at each iteration to recalibrate the sources. These eigenvector components do not correspond to the true fission distribution of each single unit, but since they are the region-integrated fission distribution, they are supposed to be more stable than the individual fission events calculated by the Monte Carlo method. Kadotani et al. proposed calculating the elements of the matrix only by considering the neutrons of the last generation. Success limited by noise.


• “Iterative Acceleration Methods for Monte Carlo and Deterministic Criticality Calculations”, by Todd Urbatsch, *LA-13052-T*, November, 1995. Develops three methods for improving fission source convergence in problems with high dominance ratios: the Fission Matrix Acceleration Method, the Fission Diffusion Synthetic Acceleration Method, and a hybrid Monte Carlo method. Showed that Fission Matrix Acceleration with filtering can work. Fission Diffusion Synthetic Acceleration is successful in grid-based geometries, but is sensitive to acceleration parameters that are not automatically set. The Hybrid Monte Carlo Method, which requires filtering the statistical noise, was effective and efficient, but has a truncation error.

• “Accelerating the convergence of Monte Carlo Calculations of k-eff in Systems with Weakly Coupled Fissile Units”, by I. De la Rive Box, *Graduation report for the faculty of Applied Physics of the Delft University of Technology, RF-96*, November, 1996. The goal of this study is to assess the developments made recently to improve the Monte Carlo method for the calculation of k-eff in geometries containing different weakly coupled fissile units. Four methods presented by different teams have been compared, which have a different approach to the problem. The four different methods are: Superhistory powering, fission matrix method, importance sampling and stratified sampling.

• “Error Estimations and their Biases in Monte Carlo Calculations”, by T. Ueki, T. Mori, and M. Nakagawa, *Nucl. Sci. Eng.*, 125, 1 (1997). An iterative method applied to real variance estimation. Biases in the estimators of the variance and intercycle covariances in Monte Carlo eigenvalues calculations are analyzed. The relations among the "real" and "apparent" values of variances and intercycle (correlation?) are derived, where “real” refers to a true value that is calculated from independently repeated Monte Carlo runs and “apparent” refers to the expected value of estimates from a single Monte Carlo run. Next, iterative methods based on the foregoing relations are proposed to estimate the standard deviation of the eigenvalues. The methods work well for the cases in which the ratios of the real to apparent values of variance are between 1.4 and 3.1, even in the case where the foregoing ratio is > 5, >70% of the standard deviation estimates fall within 40% from the true value.


• “Assessment of MCNP Statistical Analysis of keff Eigenvalue Convergence with an Analytical Criticality Verification Test Set,”, by A. Sood, R. A. Forster, and D.
K. Parsons, *Trans. Am. Nucl. Soc.*, **84**, 171 (2000). MCNP-4B was run on 75 multigroup problems from the analytic problem test set. 10 were also run incorrectly to check the statistical tests when applied to unconverged source distributions. The tests succeeded reliably only if enough cycles were run.


- “Development and Comparison of Monte Carlo Techniques for the Calculation of Loosely Coupled Systems”, by J. Miss, A. Nouri and O. Jacquet, *Advanced Monte Carlo for Radiation Physics, Particle Transport Simulation and Applications, Proceedings of the Monte Carlo 2000 Conference, Lisbon, 23-26 October 2000*, A. Kling et.al., Eds. Springer-Verlag, Berlin Heidelberg (2001). An assessment of recent developments made during the last decade to improve the Monte Carlo method for the calculation of keff in geometries containing different weakly coupled fissile units. Four methods, approaching the problem from various angles, have been proposed by different teams: Superhistory powering, Fission matrix method, Importance sampling and Stratified sampling. The paper will present results found with the Monte Carlo code MORET4, in which the different methods have been introduced, for various weakly coupled geometries.

- “An Alternative Monte Carlo Method for the k-Eigenfunction?”, by T. Booth, *Advanced Monte Carlo for Radiation Physics, Particle Transport Simulation and

- “Eigenvalue Uncertainty Evaluation in MC Calculations, Using Times Series Methodologies,” by O. Jacquet, et.al., Advanced Monte Carlo for Radiation Physics, Particle Transport Simulation and Applications, Proceedings of the Monte Carlo 2000 Conference, Lisbon, 23-26 October 2000, A. Kling et.al., Eds. Springer-Verlag, Berlin Heidelberg (2001). This paper deals with the accurate determination of confidence intervals of the standard Monte Carlo algorithm used for solving the homogeneous neutrons transport equation. It is crucial to control the convergence of the source distribution before starting the estimation of k-effective and its statistical error. Moreover, the correlations between generations should not be neglected even in the stationary phase, otherwise the variance can be underestimated. The paper introduces some statistical methods, based on the Brownian Bridge theory, which may be used for the elimination of the initial transient. Then it compares on a real case three methods aiming at improving the variance estimation, all three assuming the stationarity of the k-effective series: the batching method, the method by Ueki et al. and the fitting of the k-effective series to an autoregressive model.


• “Automated Suppression of the Initial Transient in Monte Carlo Calculations based on Stationarity Detection using the Brownian Bridge Theory”, by Y. Richet, O. Jacquet and X. Bay, Proceedings of the Seventh International Conference on Nuclear Criticality Safety, Vol. II, 578-583, Tokai, Ibaraki, Japan, October 20-24, 2003. A post-processing method to determine how many early generations to ignore. Works even when no trend is visually detectable. Vassilacopoulos (based on the ranks bridge) and Schruben (based on the series bridge) statistics. The accuracy of a criticality Monte Carlo calculation requires the convergence of the keffective series. Once the convergence is reached, the estimation of the keffective eigenvalue must exclude the initial transient of the k-effective series. The present paper deals with a postprocessing algorithm to suppress the initial transient of a criticality MC calculation, using the Brownian Bridge theory. This study on bridges based statistical tests shows good results in terms of transient bias gains, and even when transients are not visible to the naked eye. This preliminary study on automated transient suppression allowed to validate the iterative truncation methodology and some stationarity tests.


convergence diagnostics of the first OECD source convergence benchmark problem (checkerboard array of fresh fuel and water lattices).


- "The Sandwich Method for Determining Source Convergence in Monte Carlo Calculation", by Y. NAITO and J. YANG. *Journal of NUCLEAR SCIENCE and TECHNOLOGY*, VOL.41,No.5, (May 2004) The essence of the method is that a finally converged eigenvalue keff is approached starting from two kinds of initial source guesses which give higher and lower neutron multiplication factors. By this method, a range of a finally converged keff is estimated.

- "Entropy of the Fission Source Distribution & Stationarity Diagnostics", F. B. Brown and R. D. Mosteller. 9 very informative slides from Advances in Monte Carlo Criticality Calculations Workshop, mentioned in I above. Examples with some theory, examples and results including OECD/NEA source convergence problems.

- "Monte Carlo Criticality Calculations - Power Iteration, Convergence & Wielandt's Method", F. B. Brown and R. D. Mosteller. 27 very informative slides from Advances in Monte Carlo Criticality Calculations Workshop, mentioned in I above. Theoretical description of power iteration, Wielandt and Superhistory methods. Several references, four of which are not on our list (the first three are k-eff Monte Carlo, maybe not so much source convergence): Mendelson NSE 1968, Rief and Kschwendt NSE 1967, Goad and Johnston NSE 1959 and Nakamura 1986 (book, under Wielandt Method).


• "Vacation Matrix Method for Correct Source Distribution in Monte Carlo Criticality Calculations", J. P. Finch, J. S. Hendricks, C. K. Choi, From 2005 MC meeting in Chattanooga, see II above.

• "Asymptotic Equipartition Property and Undersampling Diagnostics in Monte Carlo Criticality Calculation", T. Ueki, From 2005 MC meeting in Chattanooga, see II above.

