CSN / IAEA Workshop on
Advances in Applications of Burnup Credit for Spent Fuel Storage, Transport, Reprocessing, and Disposition
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Sensitivity/Uncertainty Analyses

- Chair Overview: Objectives, key concepts, and methods –

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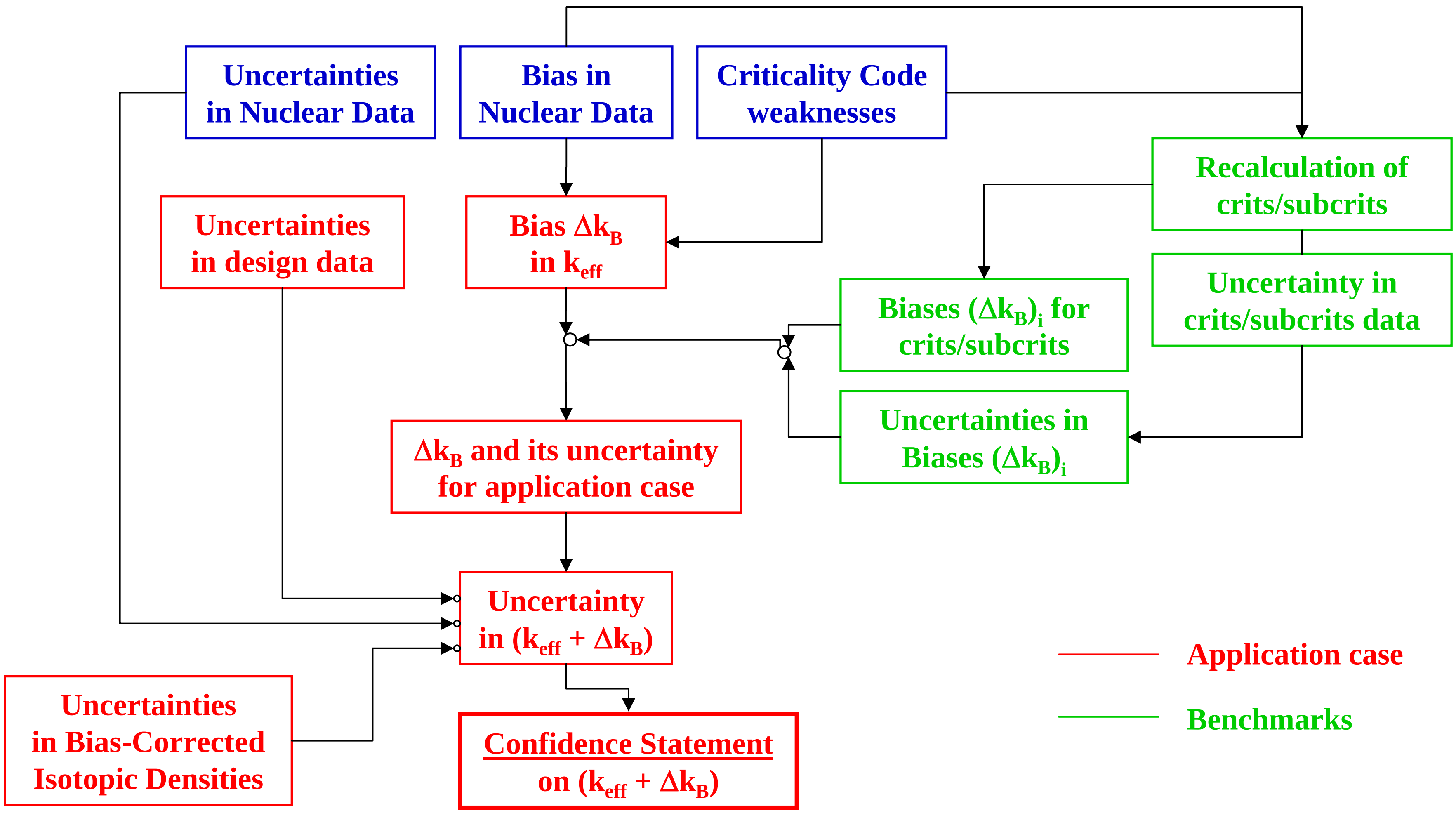
Objectives

Criticality validation → bias, uncertainty and confidence



Objectives and key concepts

Representativeness of benchmarks

Criticality validation

- **Critical experiments**
- **Reactivity worth measurements**
  (individual isotopes, spent fuel composition, REBUS Neckarwestheim II experiment)
- **Commercial reactor critical configurations**

Benchmark system

Application Case (AC)
(fuel system of interest)

Assessment of applicability of benchmark based on similarities

Traditional methods (based on expert judgment mainly)

- **comparisons of materials**
  (fuel compositions, neutron absorbing materials, …)
- **geometries**
- **comparison of gross integral parameters**
  (e.g., moderation ratio, lethargy of average neutron energy causing fission)

Modern methods: Neutron physics related methods

- **First step: Comparison of neutron spectra**
- **Methods developed during recent years: Sensitivity/uncertainty evaluation methods based on evaluation of sensitivity coefficients**

$$S_x = \frac{dk_{eff}/k_{eff}}{d\Sigma_x/\Sigma_x}, \; \Sigma_x := \text{macrosc. cross section of reaction } x$$

- **Representativeness:**

$$\mathrm{corr}(k_{AC}, k_{Bench}) \succ S\,\mathrm{cov}(\vec{\xi})\,S^T$$

Key concepts and methods

Traditional concepts and methods of bias and uncertainty estimation

Test on consistency of k_{eff} values of the benchmark systems analyzed:

- is a statistical test (χ^2-test) on equal expectation $E[k_{eff}]$ by using the weighted mean of all benchmark k_{eff} outcomes (special case of the weighted linear least squares method)
- statistical elimination of outliers (← physics arguments?)

Trending analysis of the k_{eff} values of the benchmark systems analyzed:

- k_{eff} as a function of traditional parameters
- Requires goodness-of-fit test
- Outliers elimination (← physics arguments?)

→ Neutron-physics-based benchmark selection arguments

Key concepts and methods

Representativeness of benchmarks

k(ξ):= set of neutron multiplication factors, ξ:= nuclear data

First order evaluation of k(ξ):

$$k(\tilde{\xi}) = k(E[\xi]) + Z \cdot \delta\xi \quad \text{with } Z \equiv \left(\ldots, \frac{\partial k}{\partial \xi_\nu}, \ldots \right)$$

$$E[k(\xi)] \approx k(E[\xi])$$

$$V[k] \approx Z\, V[\xi]\, Z^T \quad \text{with } V_{ij} = \text{cov}(k_i, k_j) = \sum_{\nu,\mu} \frac{\partial k_i}{\partial \xi_\nu} \text{cov}(\xi_\nu, \xi_\mu) \frac{\partial k_j}{\partial \xi_\mu}$$

$$C_{k_i k_j} = \frac{\text{cov}(k_i, k_j)}{k_i \cdot k_j} = \sum_{\nu,\mu} S_{i\nu} \frac{\text{cov}(\xi_\nu, \xi_\mu)}{\xi_\nu \cdot \xi_\mu} S_{j\mu} \quad \text{with sensitivity } S_{i\nu} = \frac{1}{k_i} \left(\frac{\partial k_i}{\partial \xi_\nu} \cdot \xi_\nu \right)$$

$$c_k := \text{correlation coefficient } \frac{C_{k_i k_j}}{\sqrt{C_{k_i k_i} \cdot C_{k_i k_i}}}$$

$$\frac{k(\tilde{\xi}) - k}{k} = \frac{\delta k}{k} = S \frac{\delta \xi}{\xi}$$

Key concepts and methods

Experimental information (m) → Increase of information on the nuclear data

Bayes' theorem

$$p(\xi\,|\,m) \succ L(m\,|\,\xi)\cdot\pi(\xi)$$

plus nuclear data adjustment based on the maximum likelihood (ML) procedure.

Due to the linearity assumption (← first order evaluation) (δk/k) = S (δξ/ξ)
→ ML estimator provides best estimates. For normal distributions π(ξ) and L(m|ξ) one gets:

$$\frac{d\xi}{\xi} = \tilde{V}_\xi\, S^T \left(C_{kk} + C_{mm}\right)^{-1} \frac{\delta k}{k} \;\text{with}\; \frac{\delta k}{k} = \frac{k-m}{k}$$

$$\tilde{V}_\xi^{posterior} = \tilde{V}_\xi - \tilde{V}_\xi\, S^T \left(C_{kk} + C_{mm}\right)^{-1} S\, \tilde{V}_\xi$$

→ **Computational bias of application case:** $\frac{\delta k_A}{k_A} = S_A \frac{\delta\xi}{\xi}\,,\; cov\left(\frac{\delta k_A}{k_A}\right) = S_A\, \tilde{V}_\xi^{posterior} S_A^T$

Key concepts and methods

Experimental information → Increase of information on the nuclear data

Generalized Linear Least Squares minimizes (GLLSM) quadratic form

$$Q^2(\psi,\zeta) = (\psi,\zeta)^T \begin{pmatrix} C_{mm} & 0 \\ 0 & \tilde{V}_\xi \end{pmatrix}^{-1} (\psi,\zeta) \text{ with } \psi = \frac{k(\xi^{adj}) - m}{k(\xi)} \text{ and } \zeta = \frac{d\xi}{\xi}$$

⟶

$$\zeta = \tilde{V}_\xi S^T (C_{kk} + C_{mm})^{-1} \frac{\delta k}{k}, \quad \psi = C_{mm} (C_{kk} + C_{mm})^{-1} \frac{\delta k}{k}$$

$$C_{\zeta\zeta} = \tilde{V}_\xi S^T (C_{kk} + C_{mm})^{-1} S \tilde{V}_\xi, \; C_{\psi\psi} = C_{mm} (C_{kk} + C_{mm})^{-1} C_{mm}$$

$$\tilde{V}_\xi^{\text{posterior}} = \tilde{V}_\xi - C_{\zeta\zeta}$$

⟶

Computational bias of application case: $\dfrac{\delta k_A}{k_A} = S_A \dfrac{\delta\xi}{\xi}, \; \text{cov}\left(\dfrac{\delta k_A}{k_A}\right) = S_A \tilde{V}_\xi^{\text{posterior}} S_A^T$

Key concepts and methods

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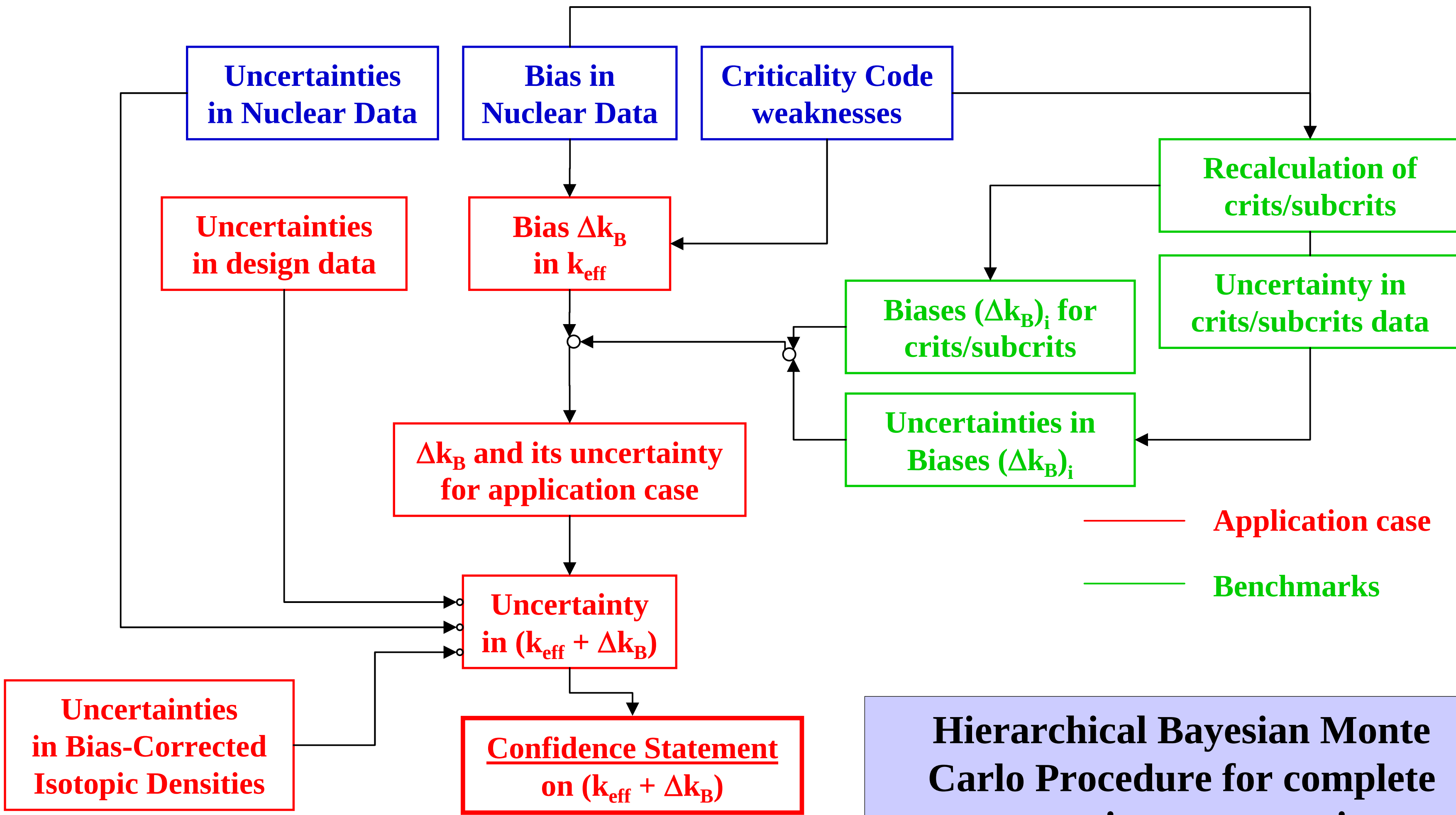

Hierarchical Bayesian Monte Carlo Procedure for complete uncertainty propagation

Some references

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