General hierarchical Bayesian procedures for calculating the bias and the a posteriori uncertainty of neutron multiplication factors

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In a Criticality Safety Analysis (CSA) uncertainties have to be treated on different hierarchical levels, where uncertainties on a lower level have an impact on uncertainties on a higher level. E. g., in a Burn-up Credit (BUC) CSA uncertainties in the parameters and the outcomes of post-irradiation experiments evaluated in order to estimate the isotopic biases in a predicted Spent Nuclear Fuel (SNF) isotopic inventory, which are due to the depletion code applied, result in uncertainties in the predicted SNF isotopic concentrations. These uncertainties affect the uncertainty in the neutron multiplication factor of any system containing the SNF. It is, therefore, preferable to use Bayesian hierarchical models making it possible to follow the impacts of the uncertainties through all the levels.

Uncertainties in a given set of parameters **x** are generally taken into account by treating **x** as a random vector defined by some probability distribution

$$F(\mathbf{x} \in R \mid \mathbf{\Theta}) = \int_R d\mathbf{x}\ p(\mathbf{x} \mid \mathbf{\Theta}).$$

$p(\mathbf{x}|\mathbf{\Theta})$ denotes the related probability density function, where $\mathbf{\Theta}$ represents the set of parameters characterizing the distribution model adequate to **x**. In general, the values of $\mathbf{\Theta}$ are unknown. They have therefore to be estimated from the evaluation of sampled data $\mathbf{D_x}$ of **x**. The model parameters $\mathbf{\Theta}$ can hence be regarded as random variables as it is done in Bayesian statistics. The Bayesian approach allows to perform random draws, by means of Monte Carlo techniques, of model parameters $\mathbf{\Theta}$ and parameters **x** on different hierarchical levels. The respective random draws can then be used as an input to the next higher level.

The hierarchical Bayesian procedures required for estimating an adequate upper tolerance limit of the sum ($k_{eff} + \Delta k_B$) of the neutron multiplication factor k_{eff} of an SNF system of interest and the bias Δk_B in k_{eff} related to this system due to the employed criticality calculation code will be described, and it is shown how these procedures consider all the uncertainties in the parameters characterizing the SNF system and the uncertainties in the nuclear data involved.