Sensitivity and Uncertainty Analysis Methodologies for Fast Reactor Physics and Design at JAEA

Kick off meeting of NEA Expert Group on Uncertainty Analysis for Criticality Safety Assessment
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Outline

- Introduction
- Overview of sensitivity analysis system at JAEA
- Nuclear-Data Covariance Processing Code
- Sensitivity Calculation
- A Standard Database of Fast Reactor Experiments
- Cross-Section Adjustment
- Evaluation and Improvement of Design Accuracy of FR Cores
- Concluding Remarks
The development of a fast reactor (FR) cycle is one of national projects in Japan.

Need to evaluate the design accuracy for FRs

Introduce sensitivity analysis system and adjusted cross section data for fast reactor design at JAEA.
Sensitivity Analysis System at JAEA

Code system for FBR analysis

FBR 70g multigroup data

JOINT- FR

Flux, Adjoint flux

Calculated value

C/E, V_m, V_e

Database of integral experiments

C/E value
Cal error
Exp error

SAGEP

SAGEP- BURN

Sensitivity G

Accept

M'

ABLE

Sensitivity Analysis System

M

Covariance matrix

ENDF Covariance File

ErrorJ

Cross section uncertainties

Design accuracy

• GMG'

• ΔGMΔG'

• GM'G'

Adjustment

• M' : adjusted covariance

• T' : adjusted cross section

ADJ

Adjusted cross section data

Cross section uncertainties

Design accuracy

• GMG'

• ΔGMΔG'

• GM'G'
Needs to evaluate the prediction accuracy of reactor core parameters with clear accountability.

One of the prediction uncertainties is the cross-section induced uncertainty.

Existing utility codes: ERRORRR module in NJOY, PUFF code developed by ORNL.

Necessity to process the covariance data of:
- the Reich-Moore resolved-resonance parameters,
- unresolved-resonance parameters,
- P1 components of elastic scattering for scattering average cosine,
- secondary neutron energy distributions of fission reaction.

ERRORJ code has been developed by JAEA.
Nuclear Data Covariance Processing Code (2/2)

Results by ERRORJ

Standard Deviations of U-235 Capture Cross-section processed by ERRORJ

ENERORJ was integrated into NJOY. (NJOY99.258)
Sensitivity Analysis System at JAEA

- **FBR 70g multigroup data**
  - Code system for FBR analysis
  - JOINT- FR
    - Flux, Adjoint flux
    - Calculated value

- **ENDF Covariance File**
  - SAGEP
    - SAGEP- BURN
  - ERRORJ
    - Covariance matrix

- **Database of integral experiments**
  - $C/E, V_m, V_e$
  - Design accuracy
    - $GMG'$
    - $\Delta GM\Delta G'$
    - $GM'G'$
  - Cross section uncertainties
  - Adjustment
    - $M'$: adjusted covariance
    - $T'$: adjusted cross section

- **Sensitivity Analysis System**
  - Adjusted cross section data
  - ADJ
Importance of Nuclear-Data Sensitivity in Reactor Analysis

1. To understand the physical mechanism quantitatively by breaking it down into components such as nuclides, reactions and energies.
2. To evaluate the prediction error of reactor core parameters, and to improve the design accuracy.

Needs to calculate sensitivity of various core parameters with respect to various nuclear data

1. Sensitivity of Doppler reactivity.
2. Sensitivity of burnup-related core parameters such as fuel component changes by power operation.
3. Average cosine of scattering angle, fission spectrum, and excited level-wise inelastic scattering matrix.
Sensitivity Calculation (2/3)

**Background**

- Conventional Perturbation Theory
  - Criticality
- Generalized Perturbation Theory
  - Reaction rate
  - Reactivity

- Usachev (1964): Formulation for reaction rate
- Gandini (1967): Extension to reactivity
- Hara and Takeda (1984): Developed the **SAGEP** code

- Improved SAGEP
- **SAGEP- BURN**

**Core parameters**
- Doppler reactivity
- Nuclear data
  - Average cosine of scattering angle
  - Fission spectrum
  - Excited level-wise inelastic scattering matrix, etc.

**Burnup-related core parameters**
- Atomic densities
- Burnup reactivity

Japan Atomic Energy Agency
Pu-239 fission and U-238 capture have large sensitivity but reactions of other nuclides cannot be neglected.

The sensitivity with respect to the fission spectrum has positive and negative components depending on energy.
Sensitivity Analysis System at JAEA

- Code system for FBR analysis
  - FBR 70g multigroup data
  - JOINT- FR
  - Flux, Adjoint flux
  - Calculated value
- Database of integral experiments
  - C/E value
  - Cal error
  - Exp error
- Code system for FBR analysis
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  - Design accuracy
    - GMG'
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- Cross section uncertainties
- Adjustment
  - M' : adjusted covariance
  - T' : adjusted cross section
- Adjusted cross section data
- ENDF Covariance File
- END
  - ERRORJ
- Sensitivity matrix
  - G
- Covariance matrix
  - M
- Sensitivity Analysis System
  - ACCEPT
  - ADJ
- Adjusted cross section data
  - ADJ
A Standard Database of FR Experiments (1/3)

Objectives and Status

- Importance of the integral experimental data to verify the quality of evaluated nuclear data libraries
  
  For last 15 years, JAEA has made efforts to collect the fast reactor experimental data from their original documents, and continued to evaluate those data by using the most detailed analytical methods with JENDL.

- To leave these precious data as a fortune of next generation.
  
  Some of these data such as ZPPR and JOYO are being prepared to provide the archiving activity of OECD/NEA, "International Reactor Physics Benchmark Experiments Project (IRPhE)".


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A Standard Database of FR Experiments (2/3)

**Experimental Data**

- **BFS-2 (IPPE):** 11 data
- **ZPPR (ANL):** 199 data
- **MASURCA (CEA):** 2 data
- **FCA (JAEA):** 9 data
- **Joyo (JAEA):** 7 data
  + 10 MA data

- Los Alamos Small Core Experiments
- ZEBRA (MOZART program, UK)
- SEFOR (General Electric, USA)
Summary of C/E values for Fast Reactor Core Parameters by using the Most Detailed Analytical Method with JENDL-3.2

C/E Changes by nuclear data from JENDL-3.2 to JENDL-3.3

<table>
<thead>
<tr>
<th>Core Parameter and Experimental Core</th>
<th>C/E Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criticality</td>
<td>+0.1 %</td>
</tr>
<tr>
<td>: ZPPR-9</td>
<td></td>
</tr>
<tr>
<td>: BFS-62-3A</td>
<td>-0.4%</td>
</tr>
<tr>
<td>Sodium Void Worth</td>
<td>-2%</td>
</tr>
<tr>
<td>: ZPPR-9 (Step 3)</td>
<td></td>
</tr>
<tr>
<td>: BFS-62-3A (LEZ region)</td>
<td>-27 %</td>
</tr>
<tr>
<td>Reaction Rate Distribution</td>
<td></td>
</tr>
<tr>
<td>: ZPPR(Pu-239 Fission, Core)</td>
<td>0.99-1.01</td>
</tr>
<tr>
<td>U-238 Capture/ Pu-239 Fission Ratio</td>
<td>1.02-1.04</td>
</tr>
<tr>
<td>: ZPPR</td>
<td></td>
</tr>
<tr>
<td>: BFS</td>
<td>0.99-1.03</td>
</tr>
<tr>
<td>Control Rod Worth</td>
<td></td>
</tr>
<tr>
<td>: ZPPR, ZEBRA, BFS</td>
<td>0.95-1.05</td>
</tr>
<tr>
<td>Sodium Void Worth</td>
<td></td>
</tr>
<tr>
<td>: ZPPR, FCA, BFS</td>
<td>0.90-1.05</td>
</tr>
<tr>
<td>Doppler Reactivity</td>
<td></td>
</tr>
<tr>
<td>: ZPPR, FCA, BFS, SEFOR</td>
<td>0.95-1.05</td>
</tr>
<tr>
<td>Burnup Reactivity</td>
<td></td>
</tr>
<tr>
<td>: JOYO</td>
<td>1.05</td>
</tr>
</tbody>
</table>

C/E Values

- 0.99-1.01
- 1.02-1.04
- 0.99-1.03
- 0.95-1.05
Sensitivity Analysis System at JAEA

Code system for FBR analysis

FBR 70g multigroup data

Joint- FR

Flux, Adjoint flux

C/E, \( V_m, V_e \)

Database of integral experiments

Design accuracy
- \( \Delta G M \Delta G' \)
- \( G M' G' \)

Cross section uncertainties

Adjustment
- \( \mathbf{M}' \) : adjusted covariance
- \( \mathbf{T}' \) : adjusted cross section

Sensitivity Analysis System

\( \mathbf{S} \)

\( \mathbf{G} \)

\( \mathbf{M} \)

\( \mathbf{M}' \)

\( \mathbf{G}' \)

\( \mathbf{T}' \)

\( \mathbf{C/E, V_m, V_e} \)

Errorj

Endf Covariance File

SAGEP

SAGEP- Burn

Accept

ABLE

Adjusted cross section data
Cross Section Adjustment (1/3)

Objectives and Status

- To improve the design accuracy of power reactor cores for assuring safety, reliability and economy.
  - All related information including C/E values, experimental and analytical errors, sensitivity coefficients of various experimental cores and parameters, and cross-section covariance, should be integrated.

- The cross-section adjustment is a powerful technique to improve the prediction accuracy.

- An adjusted cross-section set, ADJ2000R, based on JENDL-3.2 was released and utilized for fast reactor design study in Japan.
Based on the Bayes theorem, i.e., the conditional probability estimation method
To maximize the posterior probability that a cross-section set, $T$, is true, under the condition that the information of integral experiment, $R_e$, is obtained.

$$J(T) = (T-T_0)^t M^{-1} (T-T_0) + [R_e-R_c(T)]^t [V_e+V_m]^{-1} [R_e-R_c(T)]$$

Minimize the function $J(T)$. i.e. $dJ(T)/dT = 0$

The adjusted cross-section set $T'$, and its uncertainty (covariance), $M'$

$$T' = T_0 + MG'[GMG^t+V_e+V_m]^{-1} [R_e-R_c(T_0)]$$

$$M' = M - MG'[GMG^t+V_e+V_m]^{-1} GM$$

Prediction uncertainty induced by the cross-section uncertainties

Before adjustment : $GMG^t$  
After adjustment : $GM'G'$

Where, $T_0$ : Cross-section set before adjustment  
$V_e$ : Experimental errors of integral experiments  
$M$ : Covariance before adjustment  
$V_m$ : Analytical modeling errors of integral experiments  
$R_e$ : Measured values of integral experiments  
$G$ : Sensitivity coefficients,
Predict the criticality of various cores within 0.2%dk except for some cases.
Sensitivity Analysis System at JAEA

FBR 70g multigroup data

- JOINT- FR
  - Code system for FBR analysis
  - Flux, Adjoint flux
  - Calculated value

- SAGEP
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- Database of integral experiments
  - C/E value
  - Cal error
  - Exp error

- GMG'
- ΔGMΔG'
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- Design accuracy

- Cross section uncertainties

- Adjustment
  - M' : adjusted covariance
  - T' : adjusted cross section

- ADJ

Sensitivity Analysis System

- Sensitivity
  - G

- Covariance matrix
  - M

- M' : adjusted covariance

- T' : adjusted cross section

- ADJ

- Adjusted cross section data

- ACCEPT

- ABLE

- ACCEPT

- ABLE

- ABLE
Evaluation and Improvement of Design Accuracy of FR Cores (1/2)

- Based on formulae to evaluate the design accuracy
  - Takeda et al., NSE103, pp157-165 (1989)

- No-information from integral experiments (Before adjustment)
  - Design nominal values: \( R_c^{* (2)}(T_0) = R_c^{(2)}(T_0) \)
  - Design errors (variance): \( V[R_c^{* (2)}(T_0)] = G^{(2)} M G^{(2)t} + V_m^{(2)} \)

- Adjusted cross-section set based on integral experiments (After adjustment)
  - Design nominal values: \( R_c^{* (2)}(T_0) = R_c^{(2)}(T') = R_c^{(2)}(T_0) + G^{(2)}(T'-T_0) \)
  - Design errors (variance): \( V[R_c^{* (2)}(T_0)] = G^{(2)} M' G^{(2)t} + V_m^{(2)} - NV_m^{(12)} - V_m^{(12)t} N^t \)

where,
- \( T_0 \): Cross-section set before adjustment, \( T' \): Cross-section set after adjustment
- \( M \): Covariance before adjustment, \( M' \): Covariance after adjustment
- \( R_c \): Analytical values of core parameters, \( * \): Design nominal values (best-estimated values)
- \( G \): Sensitivity coefficients, \( (dR/R)/(d \sigma / \sigma) \), \( V_m \): Analytical modeling errors of core parameters
- \( (1) \): A set of integral experiments, \( (2) \): A design target core
- \( (12) \): Correlation between the integral experiments and the design target core
- \( N = G^{(2)} M G^{(1)t} [G^{(1)} M G^{(1)t} + V_e^{(1)} + V_m^{(1)}]^{-1} \), \( V_e \): Experimental error of integral experiments
### Accuracy of a Large FR Core Design

(Contribution from cross-section error before (B) and after (A) adjustment)

A 1500 MWe-class sodium-cooled MOX core (diameter: 4.9m, height: 0.8m)

<table>
<thead>
<tr>
<th>Error and Components</th>
<th>Criticality</th>
<th>Burnup Reactivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>Total Error</td>
<td>0.82</td>
<td>0.26</td>
</tr>
<tr>
<td>Pu-239 fission spectrum</td>
<td>0.37</td>
<td>0.01</td>
</tr>
<tr>
<td>Fe Inelastic</td>
<td>0.38</td>
<td>0.11</td>
</tr>
<tr>
<td>U-238 Capture</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>U-238 Inelastic</td>
<td>0.29</td>
<td>0.05</td>
</tr>
<tr>
<td>Pu-239 Capture</td>
<td>0.27</td>
<td>0.18</td>
</tr>
<tr>
<td>Pu-239 Fission</td>
<td>0.36</td>
<td>0.07</td>
</tr>
<tr>
<td>FPs of Pu-239 Capture</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>Beta of U-238</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Beta of Pu-239</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Others</td>
<td>0.30</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Unit: % (1 sigma)

Total errors are remarkably reduced by adjustment.

The contribution of each isotope is also clarified.
Concluding remarks

- The sensitivity analysis system for the FR design at JAEA has been introduced.
- The cross section adjustment and the evaluation method of design accuracy has been briefly reviewed.
- The methodology to estimate the cross-section induced uncertainties has been well established and would be available for criticality safety analysis.
- One of the remained issues is that the number of available covariance data is limited.
Thank you for your attention!
Generalized Perturbation Theory for Reactivity

Definition of Reactivity:

\[ R = \left[ \phi^* H_1 \phi \right] \left[ \phi^* H_2 \phi \right] = \int \int d\vec{r} dE \phi^* (\vec{r}, E) H_1(\vec{r}, E) \phi(\vec{r}, E) \]

Sensitivity:

\[ S = \frac{dR}{R} = \frac{d(\ln R)}{d\sigma} = \sigma \frac{d}{d\sigma} \left( \frac{d}{d\sigma} \right) = \begin{bmatrix} \phi^* \frac{dH_1}{d\sigma} \phi \\ \phi^* H_1 \phi \end{bmatrix} - \begin{bmatrix} \phi^* \frac{dH_2}{d\sigma} \phi \\ \phi^* H_2 \phi \end{bmatrix} - \begin{bmatrix} \Gamma^* \frac{dB}{d\sigma} \phi \\ \Gamma \frac{dB^*}{d\sigma} \phi^* \end{bmatrix} \]

Direct terms

\[ B^* \Gamma^* = \begin{bmatrix} H_1^* \phi^* \\ \phi^* H_1^* \end{bmatrix} - \begin{bmatrix} H_2^* \phi^* \\ \phi^* H_2^* \end{bmatrix} \]

\[ (A - \lambda F) \phi(\vec{r}, E) = B \phi = 0 \]

Flux terms

\[ B \Gamma = \begin{bmatrix} H_1 \phi \\ \phi^* H_1 \phi \end{bmatrix} - \begin{bmatrix} H_2 \phi \\ \phi^* H_2 \phi \end{bmatrix} \]

\[ (A^* - \lambda F^*) \phi^* (\vec{r}, E) = B^* \phi^* = 0 \]
Sensitivity Coefficient of a Sample Doppler Reactivity for ZPPR 9 Experiment

(* Sample material: Natural uranium dioxide, app. 1 kg)

The local sample Doppler reactivity is affected not only by shielding-factors, but by other infinite-diluted cross-sections through the effects to spectrum, importance distribution and the denominator.
**JUPITER Critical Experiment**

- Cooperative study of DOE and JAEA (former JNC) in 1978～1988, using ZPPR facility at ANL, USA.
- The largest FBR mockup experiment in history, 4,600 – 8,500 liters.
- Various core concepts, sizes, and structures:
  - 600～800MWe-class two-region homogeneous cores,
  - 650MWe-class radially-heterogeneous cores,
  - 650MWe-class axially-heterogeneous cores,
  - and, 1000MWe-class homogeneous cores with enriched uranium regions.
- Many kinds of measured parameters.

As-built experimental information is available for the public.

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ZPPR Critical Assembly (ANL)
A Standard Database of Fast Reactor Experiments in Japan (3/6)

- **FCA Critical Experiment**
  - Fast Critical Assembly at JAEA, Japan.
  - To simulate small FBR cores with plutonium and enriched uranium fuels.
    - FCA X-1 Core (1982) - 130 liters.

- **Experimental Reactor JOYO**
  - First Japanese FBR (1st Criticality in 1977)
    - Burnup, pin-wrapper structure.
      - Mixed one-region plutonium and enriched uranium core with 240 liter-size.
      - Criticality, fuel-blanket replacement reactivity, and burnup reactivity were adopted.

As-built experimental information is available for the public.
BFS-1, 2 Critical Experiment
✓ Fast Critical Assembly at IPPE, Obninsk, Russia.
  ◆ BFS-58-1-I1 Core (1996) — Uranium-free region in core center and enriched uranium region in periphery.

MASURCA Critical Experiment
✓ Fast Critical Assembly at CEA, Cadarache, France.
  ◆ ZONA2B Core (1996) — a 380 liter-size MOX fuel core with reflectors, which simulated Pu-burner.
Los Alamos Small Core Experiment

- Sphere-shaped cores of approx. ten centimeter in diameter with metallic fuel consisted ofPu-239, or degraded Pu, or U-235.
  - FLATTOP-Pu, FLATTOP-25, JEZEBEL, JEZEBEL-Pu, GODIVA

Benchmark models have already opened.

Other Experiments

- ZEBRA (MOZART program, UK) - a 550 liter-sized one-region MOX core as a clean benchmark.
- SEFOR (General Electric, USA) - a 20MWt fast power reactor core fueled with mixed PuO₂-UO₂ and cooled with sodium.