Application of Marginal Likelihood Optimization to Haicheng’s Stress Test

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Haicheng’s Stress Test

• In SG33 benchmark exercise, adjusted results for ZPR6-7 F49/F25, ZPPR9 F28/F25 and F49/F25 reaction ratios are getting worse (decreased) due to missing essential constraints on $^{235}U$ (n,f) in the energy region between 1 to 10keV.

• A stress test on $^{235}U$ (n,f) to identify compensation errors was suggested with critical benchmarks sensitive to $^{235}U$ (n,f) between 1 to 10keV.
  - HEU-COMP-INTER-004-001 (HCI-004) $k_{eff}$ (infinity HEU moderated with graphite)
  - HEU-MET-INTER-006-002 (HMI-006) $k_{eff}$ (intermediate-spectrum critical assembly with a graphite-HEU core surrounded by a copper reflector)

• Lead to different, even contrary adjustments for both integral and differential data
Marginalized Likelihood Optimization (MLO)

- **Idea:** Account for biases or underestimated uncertainties with an extra uncertainty term, $M_{\text{extra}}$
- Minimize the negative of the log-likelihood to estimate $M_{\text{extra}}$

\[
\chi^2 = (E - C)^T (M_E + M_C + M_{\text{extra}})^{-1} (E - C)
\]  

\[
L = \frac{e^{-\chi^2/2}}{\sqrt{(2\pi)^N \det(M_E + M_C + M_{\text{extra}})}}
\]  

\[
\min \left[ \frac{1}{2} (N \times \log(2\pi) + \log(\det(M_E + M_C + M_{\text{extra}})) + \chi^2) \right]
\]  

- Objective of the present study is to analyze the effect of MLO on Haicheng’s stress test
Extra Uncertainty produced by MLO

- $M_{\text{extra}}$ is small for SG33 benchmarks
- No surprise as they have been chosen to be consistent
MLO Outcomes

Zoom on the extra benchmarks

- MLO produces large extra uncertainty for HCI-004
- Most likely due to inconsistencies between C & E

<table>
<thead>
<tr>
<th></th>
<th>C/E</th>
<th>$\sigma_E$</th>
<th>$\sigma_C$</th>
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<tbody>
<tr>
<td>HCI-004</td>
<td>1.015</td>
<td>0.4%</td>
<td>0.5%</td>
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<tr>
<td>HMI-006</td>
<td>0.998</td>
<td>0.1%</td>
<td>0.6%</td>
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</tbody>
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C/E adjustments

Without MLO, adding HMI-004 constraint leads to different direction for the posterior with respect to prior:
- ZPR6/7 F49/F25 and C28/F25
- ZPPR9 F49/F25 and C28/F25

With MLO, all adjustments are going in one direction! Even though not always towards C/E=1
\( ^{235}U \) (n,f) adjustments

- All adjustments go in the same direction with MLO although their magnitude changes as constraints are added.
- Without MLO, adjustment does not recover from HCI-004....
$^{235}U \ (n,c)$ adjustments

- All adjustments go in the same direction with MLO although their magnitude changes as constraints are added.
$^{10}B \ (n,c)$ adjustments

- All adjustments go in the same direction with MLO
- magnitude of adjustment is reduced.
$^{23}Na$ (n,el) adjustments

- All adjustments go in the same direction with MLO
- magnitude of adjustment is reduced.
MLO effect on adjustment magnitude

- Magnitude of adjustments is damped with MLO
Conclusions

• Outcomes of stress test (H. Wu presentation SG39 / Nov. 2015)
  ▪ Different constraints can lead to different, even contrary adjustments for both integral and differential data.
  ▪ To avoid compensation error and make adjusted nuclear data for general purpose, we need complete constraints.

• MLO helps resolving those issues
  ▪ Contrary adjustments are avoided
  ▪ Reduce the weight of discrepant benchmarks on adjustments

• Future work
  ▪ Consider a larger experimental data set (towards complete constraints)
  ▪ Maximizing the likelihood function becomes difficult