Summary of Derivations and Equivalence between Bias Factor Methods and Adjustment Methods

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Action Agreed at June 2019 Meeting*

• K. Yokoyama to provide papers to G. Palmiotti which illustrate the equivalence of different bias factors methods and their equivalence to the extended adjustment method

• During the June 2019 meeting, eight papers listed in the last slide were delivered personally

• Through the mailing list of SG46, the list of the papers were distributed on September 19, 2019

• In this presentation, I would like to summarize the papers focusing on the equivalence between bias factor methods and adjustment methods

*: Summary Record, Meeting of WPEC SG46, 25-26 June 2019, NEA/NSC/WPEC/DOC(2019)4
Nomenclature in comparison with SG39’s

- \( P(A|B) \): conditional probability of \( A \) given \( B \)
- \( E(A) \): expectation of \( A \)
- \( V(A) \): variance of \( A \)
- \( T_0 \): unadjusted cross sections (= \( \sigma \) in the SG39’s common nomenclature)
- \( T_x \): adjusted cross sections by methodology \( x \) (= \( \sigma' \))
- \( R^{(1)}_e \): measured value of integral experiments (= \( E \))
- \( R^{(1)}_c(T) \): calculation value of integral experiments (= \( C \))
- \( R^{(2)}_x \): design value of the target system by methodology \( x \)
- \( G^{(1)} \): sensitivity matrix of integral experiments (= \( S \))
- \( G^{(2)} \): sensitivity matrix of integral parameters of the target system
- \( M \): covariance matrix of unadjusted cross sections (= \( M_\sigma \))
- \( V^{(1)}_e \): covariance matrix of experimental error for the integral experiments (= \( M_E \))
- \( V^{(1)}_m \): covariance matrix of analysis method error for the integral experiments (= \( M_C \))
- \( V^{(1)}_{e+m} = V^{(1)}_e + V^{(1)}_m \) (= \( M_{EC} \))
- \( V^{(2)}_m \): covariance matrix of analysis method error for the target system
- \( V^{(12)}_m \): cross-correlation between the integral experiments and the target system
- \( F_x \): combination factor matrix in the linear estimation by methodology \( x \)
CBCA (= Cross-section Adjustment, GLLS, etc.)

- Classical Bayesian Conventional XS Adjustment method

\[ T_{\text{CBCA}} \equiv \arg\max_{\hat{T}} P\left(\hat{T} \mid R_e^{(1)}\right) \]

\[ P \sim \mathcal{N}(\mu, \Sigma) \quad \text{Gaussian distribution} \]

\[ T_{\text{CBCA}} = T_0 + MG^{(1)T} D^{-1} \left( R_e^{(1)} - R_c^{(1)}(T_0) \right) \]

\[ D \equiv G^{(1)} MG^{(1)T} + V_e^{(1)} \]

- Well-known formula to members of SG46
EBPE & Matrix Form with Approximation

• **Extended Bias factor method (Product of Exponentiated values)**

\[
R_{EBPE}^{(2)} = R^{(2)}(T_0) \left( \frac{\prod_i (R_{e,i}^{(1)})^{F_{EBPE,i}}}{\prod_i (R_{c,i}^{(1)})^{F_{EBPE,i}}} \right)
\]

\[
\approx R^{(2)}(T_0) \left( 1 + F_{EBPE} \frac{R_e^{(1)} - R_c^{(1)}(T_0)}{R_c^{(1)}(T_0)} \right)
\]

\[
F_{EBPE} \equiv \arg\min_V \left( \frac{\hat{R}^{(2)}}{R_{true}^{(2)}} \right)
\]

1. Gaussian distribution is not assumed in EBPE
2. EBPE is equivalent to a linear estimation

NB: It is shown that a result of best representativity method by T. Umano et al. is completely equivalent to that of EBPE in Ref.2

\[
F_{EBPE} = \left( G^{(2)} M G^{(1)T} + V_m^{(12)T} \right) D^{-1}
\]

• This formula is similar to CBCA but different
CBEA

- Classical Bayesian Extended XS Adjustment method

\[ T_{\text{CBEA}} \equiv \arg\max_{\hat{T}} P \left( \hat{R}^{(2)} | R^{(1)}_e \right) \]

\[ P \sim \mathcal{N} (\mu, \Sigma) \quad \text{Gaussian distribution} \]

\[ T_{\text{CBEA}} = T_0 + \left( MG^{(1)T} + G^{(2)} + V_{m}^{(12)} \right) D^{-1} \left( R^{(1)}_e - R^{(1)}_c (T_0) \right) \]

\[ T_{\text{CBEA}} \neq T_{\text{CBCA}} \]

\[ R^{(2)}_c (T_{\text{CBEA}}) \approx R^{(2)}_{\text{EBPE}} \quad \text{K. Yokoyama et al.} \]


- Design values of CBEA and EBPE are approximately the same
Contrast of Bayesian Inference & Linear Estimation

**Bayesian inference**

\[ P \left( \hat{T} | R_e^{(1)} \right) \]

\[ P \left( \hat{R}^{(2)} | R_e^{(1)} \right) \]

\( \mathbf{X}_x \equiv \text{argmax} \ P \left( \hat{\mathbf{X}} | R_e^{(1)} \right) \)

\( P \sim \mathcal{N} (\mu, \Sigma) \)

(assumption of Gaussian distribution)

**Linear estimation**

\[ \hat{T} - T_0 = \mathbf{F} \left( R_e^{(1)} - R_c^{(1)}(T_0) \right) \]

\[ \hat{R}^{(2)} - R_c^{(2)}(T_0) = \mathbf{F} \left( R_e^{(1)} - R_c^{(1)}(T_0) \right) \]

\[ \mathbf{X}_x = \mathbf{X}_0 + \mathbf{F}_x \left( R_e^{(1)} - R_c^{(1)}(T_0) \right) \]

\( \mathbf{F}_x \equiv \text{argmin} \ \text{tr} \left( V(\hat{\mathbf{X}}) \right) \)

\[ E(T_0) = T_{\text{true}}, \ldots \]

(assumption of unbiased estimation)

where \( \mathbf{X}_x = T_x \) or \( R_x^{(2)} \)

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BFRS (Bias Factor Method by T. Endo et al.)

• Bias Factor Method Using Random Sampling Technique

\[ R_{BFRS}^{(2)} \equiv \text{argmax}_K P \left( \hat{R}^{(2)} | R_e^{(1)} \right) \]

\[ P \sim \mathcal{N}(\mu, \Sigma) \quad \text{Gaussian distribution} \]

\[ R_{BFRS}^{(2)} = R_c^{(2)}(T_0) + K \left( R_e^{(1)} - R_c^{(1)}(T_0) \right) \]

\[ K \equiv \left( \text{Cov} \left( R_c^{(2)}, R_c^{(1)} \right) + V_m^{(12)^T} \right) \left( \text{Cov} \left( R_c^{(1)}, R_c^{(1)} \right) + V_{e+m}^{(1)} \right)^{-1} \]

\[ \approx \begin{bmatrix} G^{(2)} & MG^{(1)T} \\ \end{bmatrix} \]

\[ R_{BFRS}^{(2)} \approx R_{EBPE}^{(2)} \]

• Design values of BFRS and EBPE are approximately the same
MLEA

- MVULE (Minimum Variance Unbiased Linear Estimation)-based Extended XS Adjustment method

\[ T_{\text{MLEA}} = T_0 + F_{\text{MLEA}} \left( R_{e}^{(1)} - R_{c}^{(1)}(T_0) \right) \]

\[ F_{\text{MLEA}} \equiv \arg \min_F \text{tr} \left( V \left( R_{c}^{(2)}(T) \right) \right) \cap \arg \min_F \text{tr} \left( V(T) \right) \]

\[ T_{\text{MLEA}} = T_{\text{CBEA}} \]

\[ R_{c}^{(2)}(T_{\text{MLEA}}) = R_{c}^{(2)}(T_{\text{CBEA}}) \approx R_{\text{EBPE}}^{(2)} \]

- EA (Extended Adjustment method) can be derived by a linear estimation

MLCA

• **MVULE (Minimum Variance Unbiased Linear Estimation)-based Conventional XS Adjustment method**

\[
T_{MLCA} = T_0 + F_{MLCA} \left( R_e^{(1)} - R_c^{(1)}(T_0) \right)
\]

\[
F_{MLCA} \equiv \arg\min_F \text{tr} \left( V(\hat{T}) \right)
\]

\[
T_{MLCA} = T_{CBCA}
\]

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• The conventional cross-section adjustment method can be derived by a linear estimation
• Gaussian distribution is not assumed in MLCA although the assumption of unbiased estimation is required
Concluding Remarks

• The extended bias factor method (EBPE) is approximately equivalent to the extended adjustment method (CBEA) although EBPE does not adjust cross sections explicitly
  • EBPE is derived by a linear estimation
  • CBEA is derived by a Bayesian inference

• The (classical Bayesian) conventional cross-section adjustment method (CBCA) is different from CBEA
  • CBCA maximizes the probability of cross sections
  • CBEA maximizes the probability of design values

• The conventional and the extended cross-section adjustment methods can be derived by a linear estimation (MLCA and MLEA)
  • MLCA minimizes the variance of cross sections
  • MLEA minimizes the variance of design values


In this presentation, summarization of Refs. 5-7 are omitted for simplification.