Does one shot Bayesian is equivalent to sucessive update ?
Bayesian inference : some matrix linear algebra

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General problem:

Multigroup cross section adjustment of N integral experiments $E_i$ with covariance matrix $M_E$.

$$M_E = \begin{pmatrix}
M_{E1} & C_{12} & \cdots & C_{1N} \\
C_{21} & M_{E2} & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
C_{N1} & \cdots & \cdots & M_{EN}
\end{pmatrix}$$

Two analyses could be made:

- case 1 $\rightarrow$ analysis of N experiments successively
- case 2 $\rightarrow$ once-through analysis: all experiments once as a meta-experiment

Are the posterior covariances on multigroup cross section equal for both analyses?
Analysis of N experiments successively

Step 1 : analyze 1st experiment

\[ M_1^{-1} = M_0^{-1} + S_1^T M_{E_1}^{-1} S_1 \]

where \( M_1 \) is the multigroup cross section covariances matrix and \( S_1 \) are the sensitivities of the calculated experiment (1st experiment).

Step n : analyze nth experiment with a priory coming from step (n-1) :

\[ M_n^{-1} = M_{n-1}^{-1} + S_n^T M_{E_n}^{-1} S_n \]

where \( M_n \) is the multigroup cross section covariances matrix at step n and \( S_n \) are the sensitivities of the calculated experiment (nth experiment). A simple recurrence demonstration gives :

\[ M_N^{-1} = M_0^{-1} + \sum_{i=1}^{N} S_i^T M_{E_i}^{-1} S_i \]
Once-throught adjustment: all experiments at the same time

Analysis of all experiments at the same time will give a posterior covariances matrices of the following form:

\[ M^{-1}_* = M^{-1}_0 + S^T M^{-1}_E S \]

Where:

\[ S = \begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_N \end{pmatrix} \]
No correlations between experiments ($C_{ij} = 0$)

If there are no correlations between experiments ($C_{ij} = 0$), we have:

$$M_{E} = \begin{pmatrix}
M_{E1} & 0 & \cdots & 0 \\
0 & M_{E2} & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & M_{EN}
\end{pmatrix}$$

$M_{E}$ being a block diagonal matrix, one can prove that:

$$M_{E}^{-1} = \begin{pmatrix}
M_{E1}^{-1} & 0 & \cdots & 0 \\
0 & M_{E2}^{-1} & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & M_{EN}^{-1}
\end{pmatrix}$$
No correlations between experiments\((C_{ij} = 0)\)

Which gives:

\[
M_*^{-1} = M_0^{-1} + (S_1^T \quad S_2^T \quad \cdots \quad S_N^T) \begin{pmatrix}
M_{E1}^{-1} & 0 & \cdots & 0 \\
0 & M_{E2}^{-1} & \ddots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & M_{EN}^{-1}
\end{pmatrix}
\begin{pmatrix}
S_1 \\
S_2 \\
\vdots \\
S_N
\end{pmatrix}
\]

Thus,

\[
M_*^{-1} = M_0^{-1} + \sum_i S_i^T M_{Ei}^{-1} S_i
\]
No correlations between experiments ($C_{ij} = 0$)

If the sensitivities calculated in case 1 are recalculated at each steps (potential effect of changes of cross section of step (n-1) to the calculation of sensitivities at step n) one may end up with slight differences on posterior covariance matrices of Case 1 and Case 2:

$$M^{-1}_* \sim M^{-1}_N$$

On the contrary, if the sensitivities are not recalculated in case 1 at each steps, once-trough calculation and successively analysis are giving the same results for posterior covariance matrices on cross sections:

$$M^{-1}_* = M^{-1}_N$$
Correlations between experiments

On the other hand, if there are correlations between experiments ($C_{ij} \neq 0$), then, we have:

$$M_{E}^{-1} \neq \begin{pmatrix}
M_{E1}^{-1} & 0 & \cdots & 0 \\
0 & M_{E2}^{-1} & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & M_{EN}^{-1}
\end{pmatrix}$$

There are no reasons why final covariances should be equal.

$$M_{*}^{-1} \neq M_{N}^{-1}$$