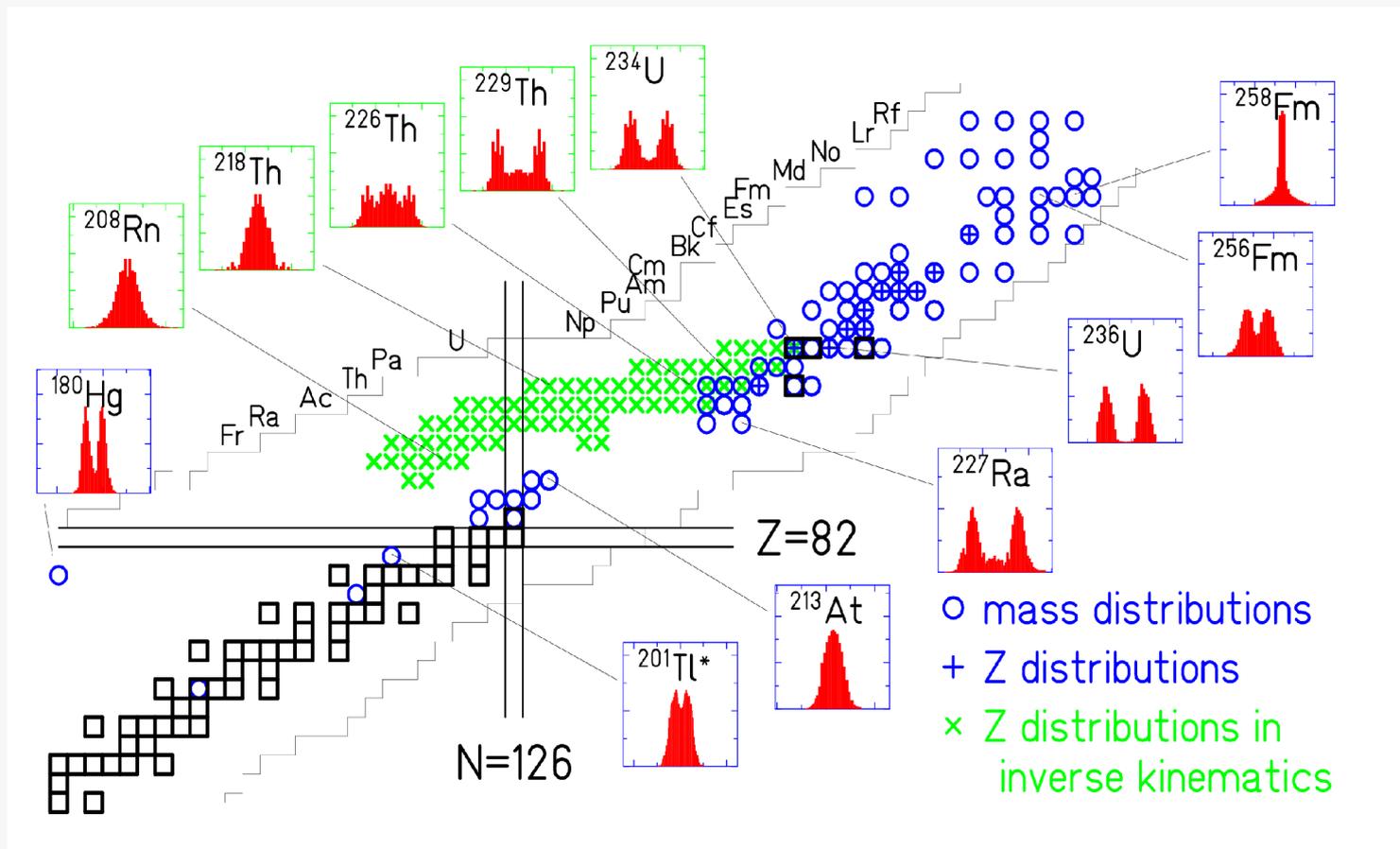


**A new theoretical approach
to low-energy fission
based on
general laws of quantum and
statistical mechanics**

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Beatriz Jurado

Contribution to the meeting of the WPEG subgroup
“Improved Fission product yield evaluation methodologies”
May 22, 2013
Issy-les-Moulineaux, France

Systematics of mass (and Z) distributions



$A \geq 258$: narrow single humped

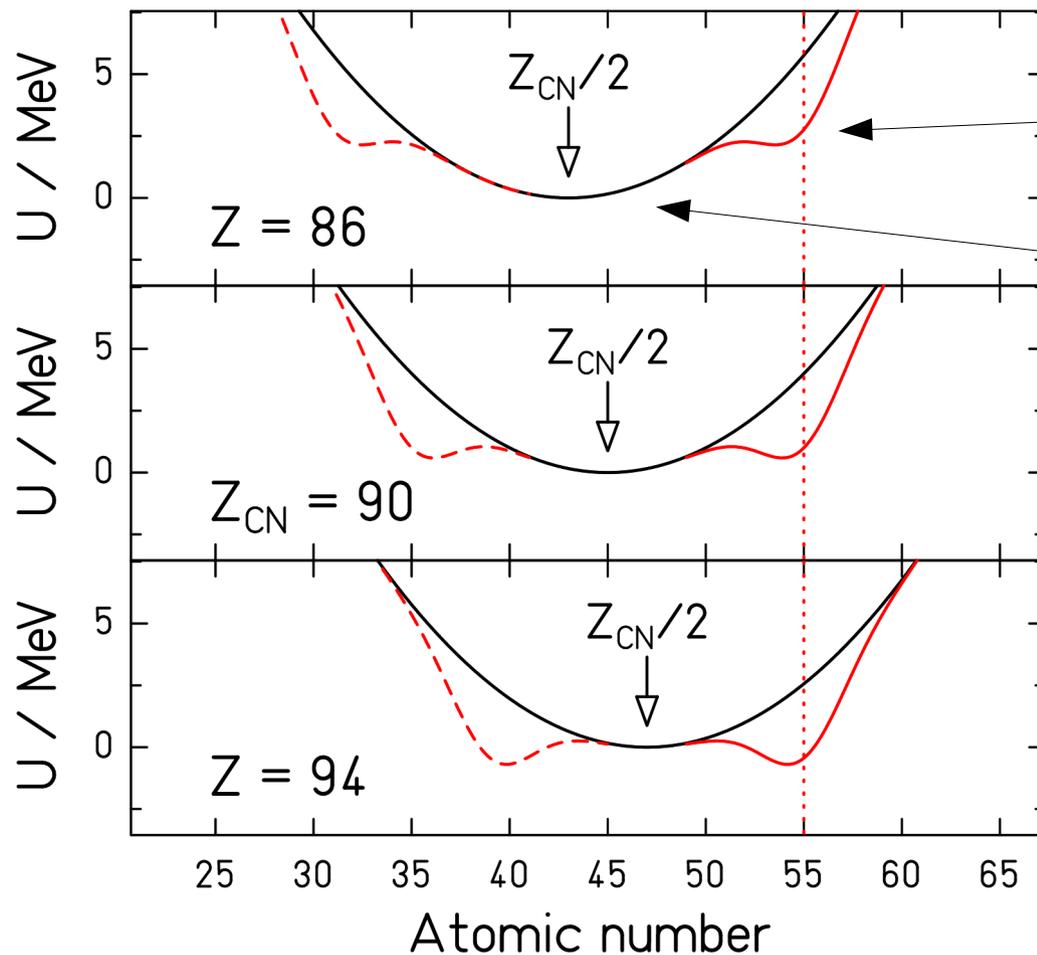
$A > 226$ to $A < 258$: double humped; $A(\text{heavy}) \approx 140$ or $Z(\text{heavy}) \approx 55$.

$Z = 82$ to $A < 226$: single-humped

$Z \approx 80$: narrow double humped

Complex behaviour, but some global trends.
 → Search for explanation on a deeper level.

An early idea



Shell

Liquid-drop potential

Potential “somewhere”
between saddle and
scission.

After M. Itkis et al.
Z. Phys. A 320
(1985) 433 :

Competition between liquid-drop potential and shells explains transition from symmetric to asymmetric fission around $A = 226$.

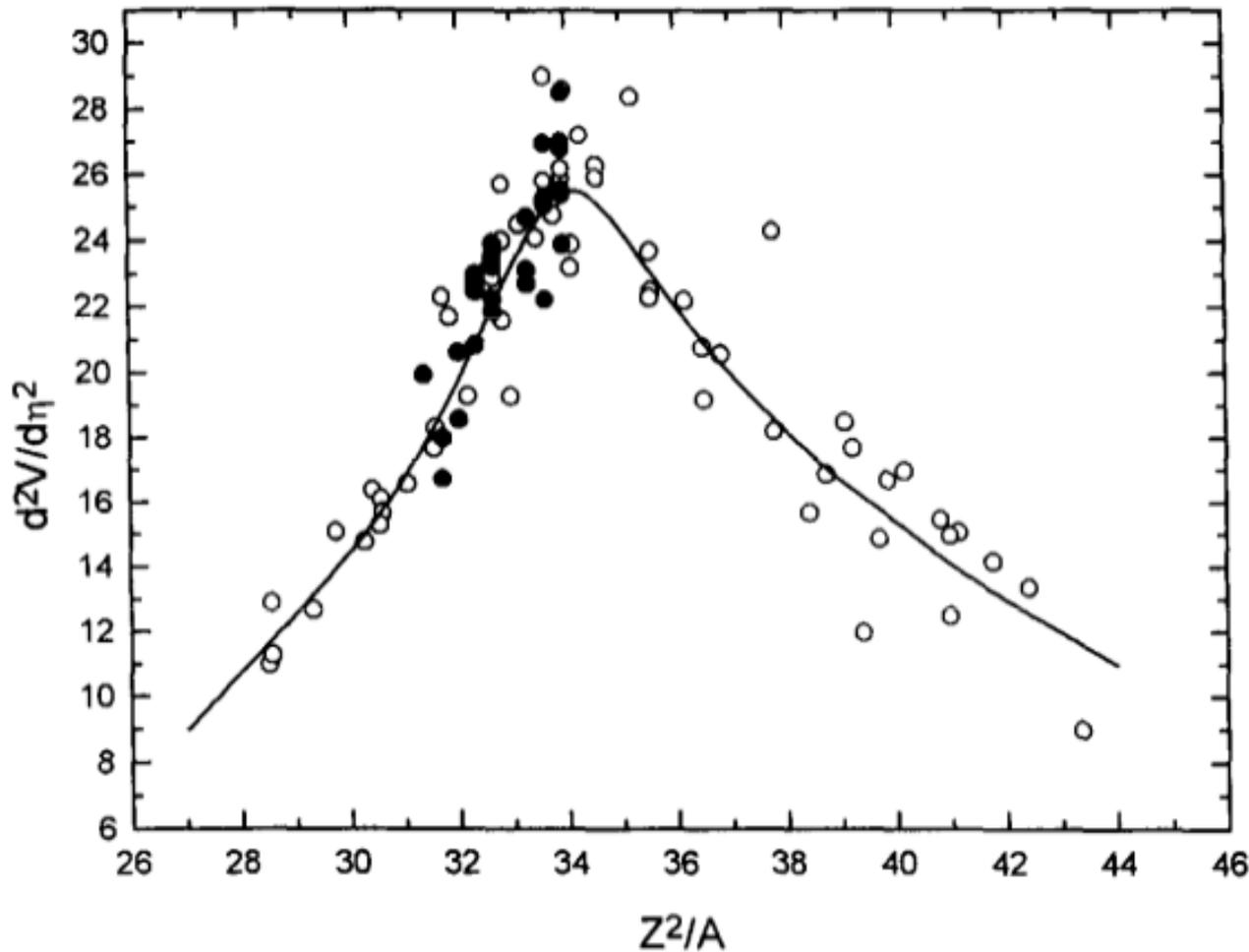
Macroscopic potential

From measured σ_A of symmetric fission mode.

$$\sigma^2 = \frac{\text{temperature}}{\text{stiffness}}$$

(Harmonic oscillator in a heat bath.)

Mulgin et al., NPA
640 (1998) 375

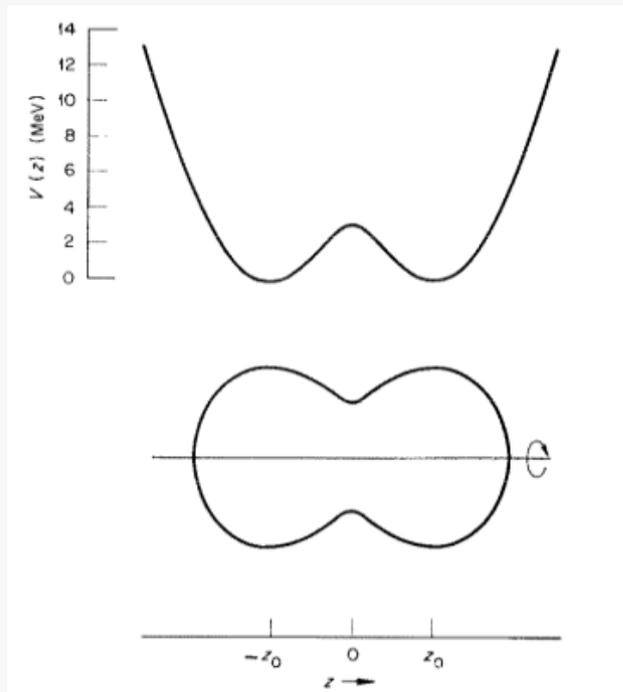


Stiffness vs. mass asymmetry of the macroscopic potential.
A unique function of fissility.

Shells

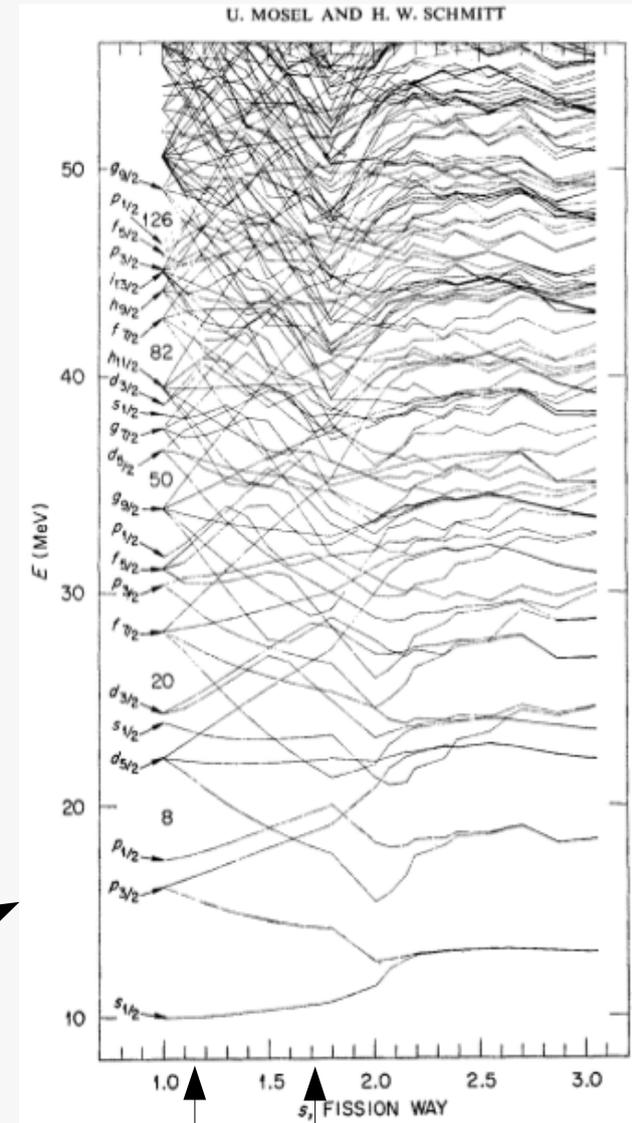
Two-centre shell model developed by:
Holzer, Mosel, Greiner (Nucl. Phys. A138 (1969) 241)

Allows continuous treatment from
ground state to separated fragments.



U. Mosel, H. W. Schmitt
Nucl. Phys. A
165 (1971) 73

Single-particle levels
in a di-nuclear potential



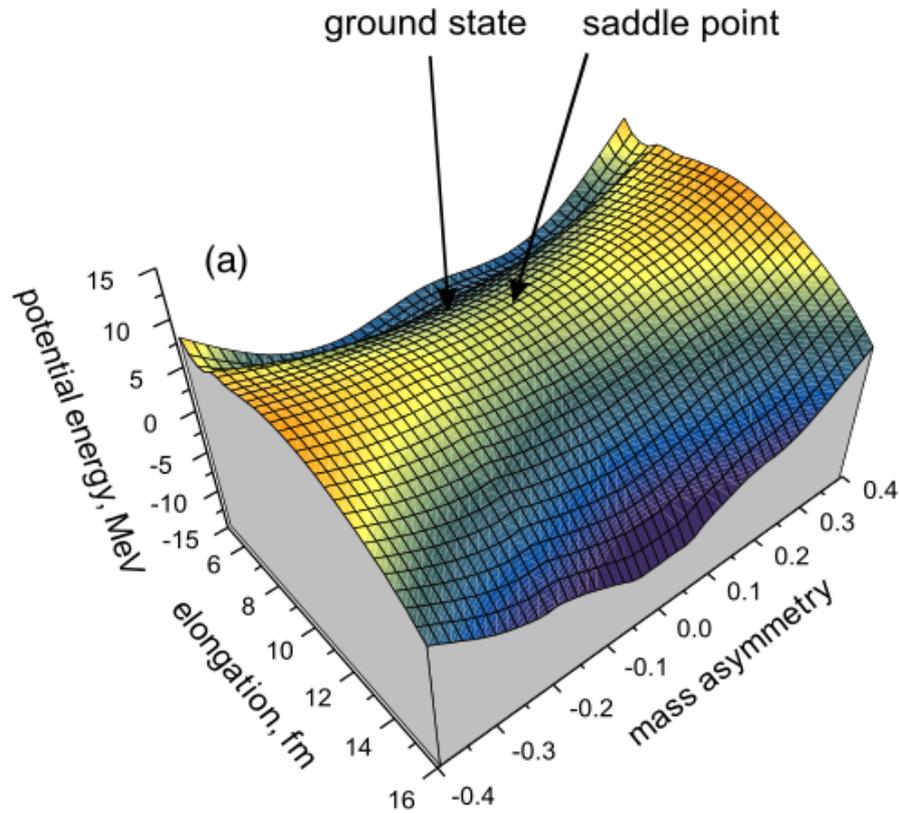
Ground state

Outer saddle

Early manifestation of fragment shells.

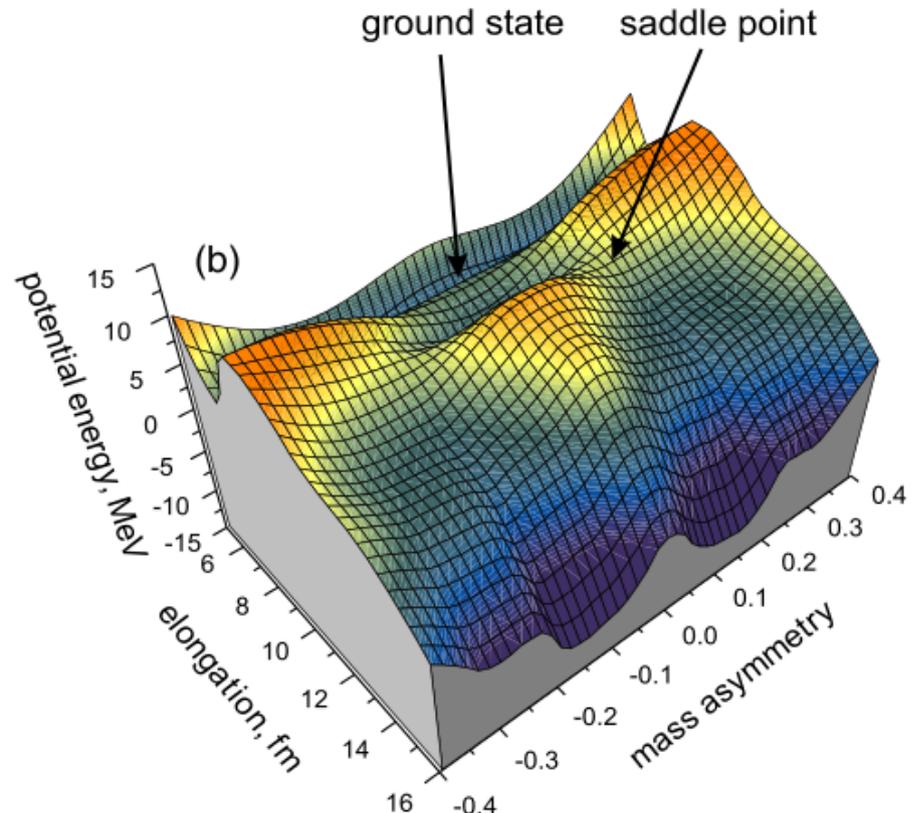
Potential energy landscape

2-dim. calculation by A. Karpov, 2008



liquid-drop potential

Property of the CN

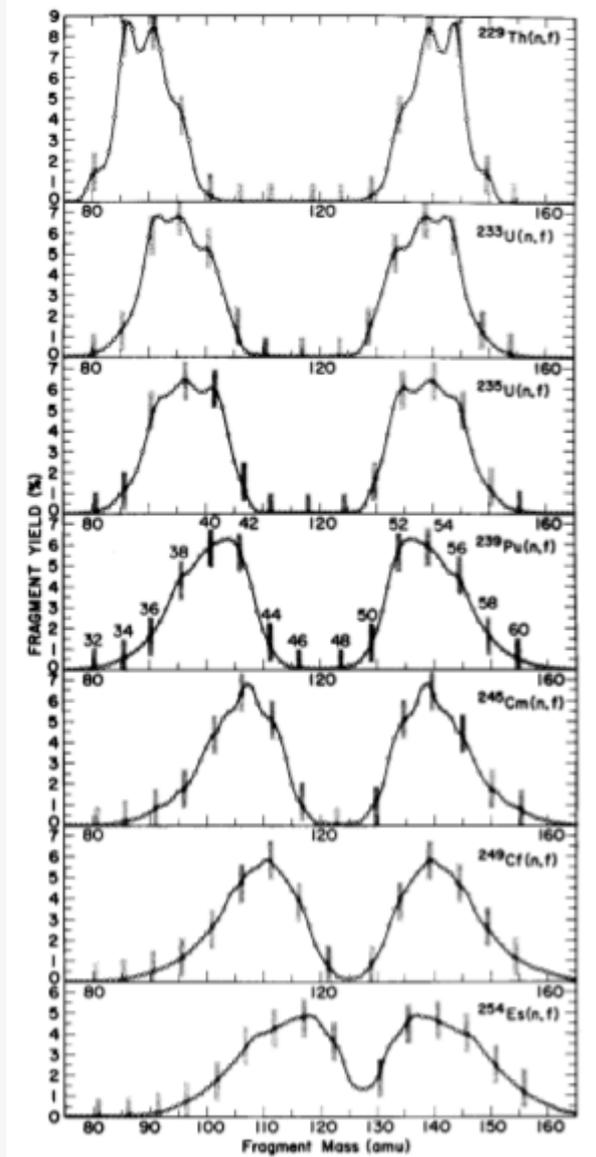


With shell effects

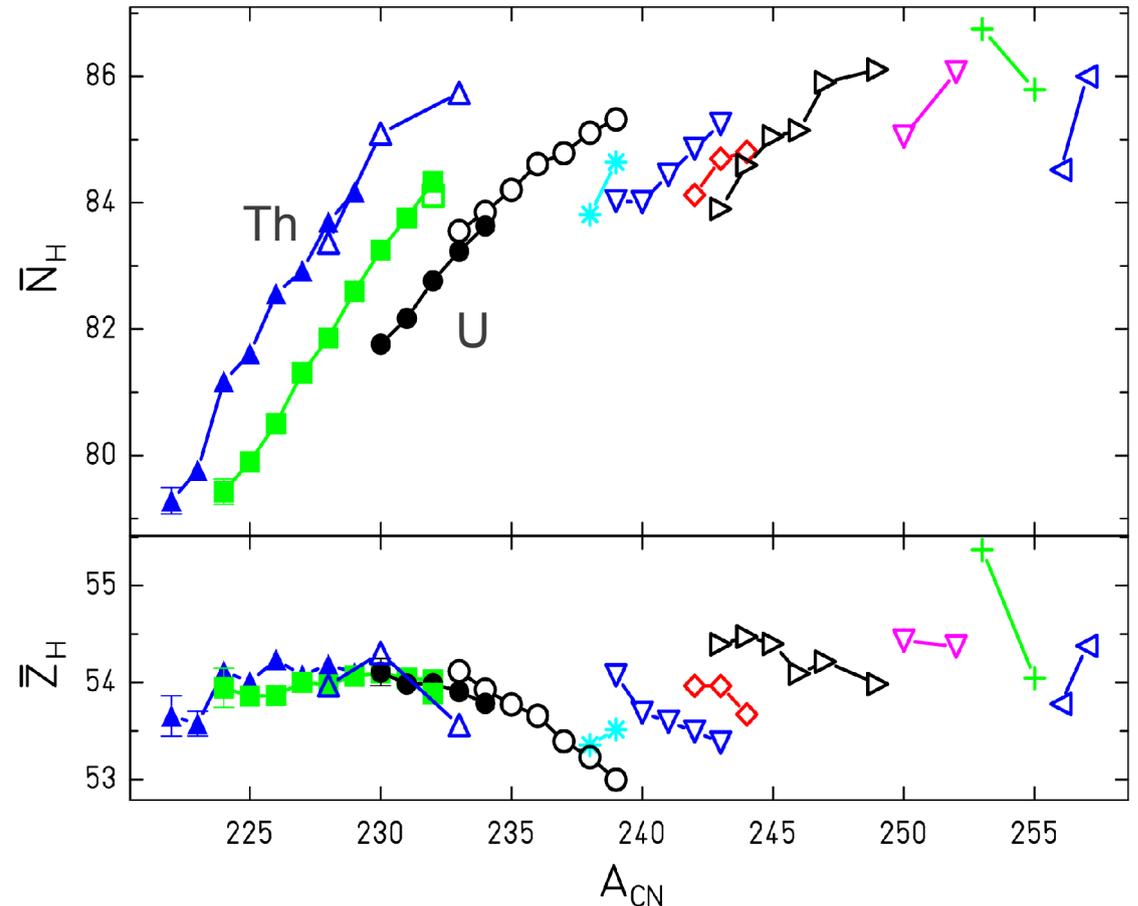
Shells behind outer saddle:
Property of the nascent fragments

→ **Separability principle**

Empirical information on the main shells



Unik et al., 1973
 $\langle A \rangle \approx 140$



Böckstiegel et al., NPA 802 (2008) 12

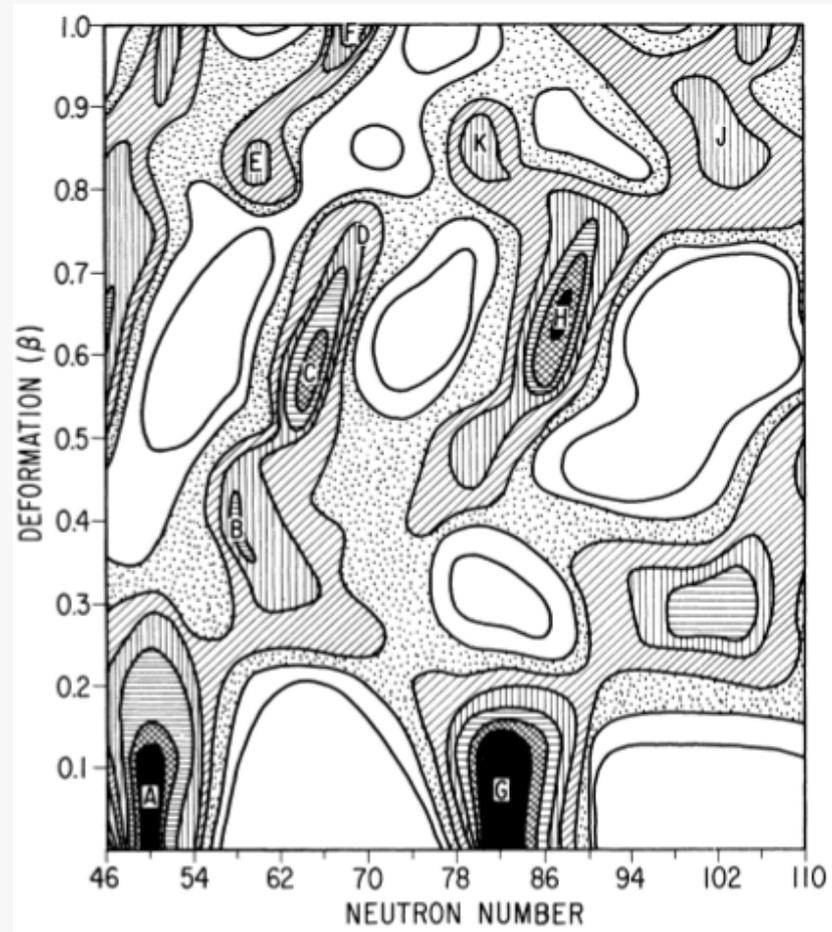
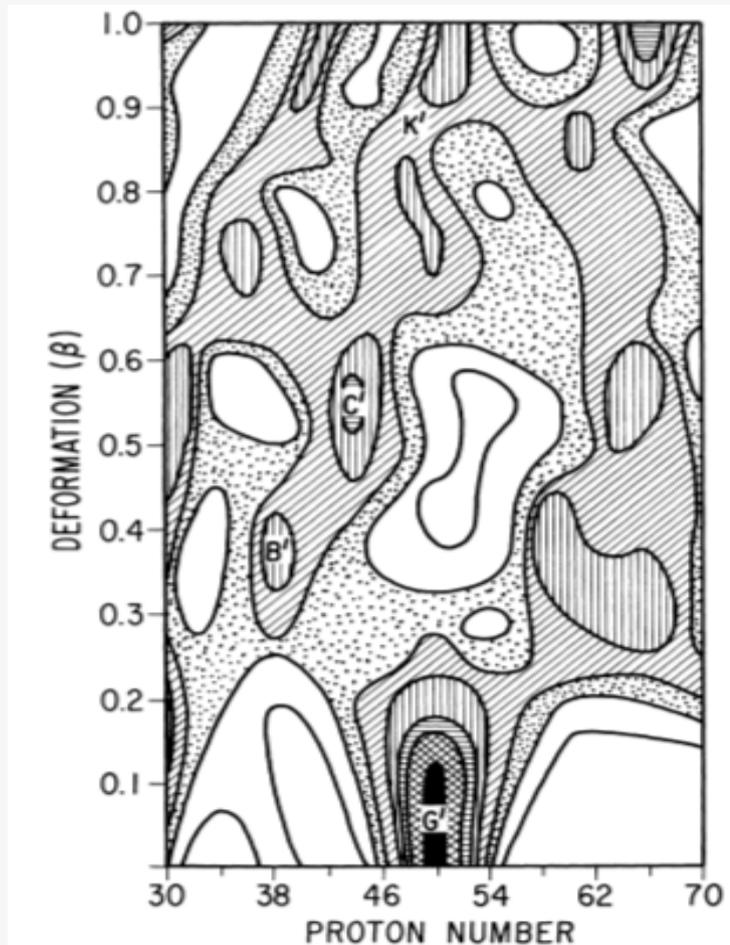
GSI data with long isotopic chains:

New empirical result: $\langle Z \rangle \approx 54$

Strong variation of $\langle A \rangle$!

Theory: Attributed to fragment shell!

Fragment shells



Wilkins et al., Phys. Rev. C 14 (1976) 1832
(Other calculations give similar results.)

No fragment shell at $Z = 54$ predicted \rightarrow puzzle for theory

Test of the separability principle

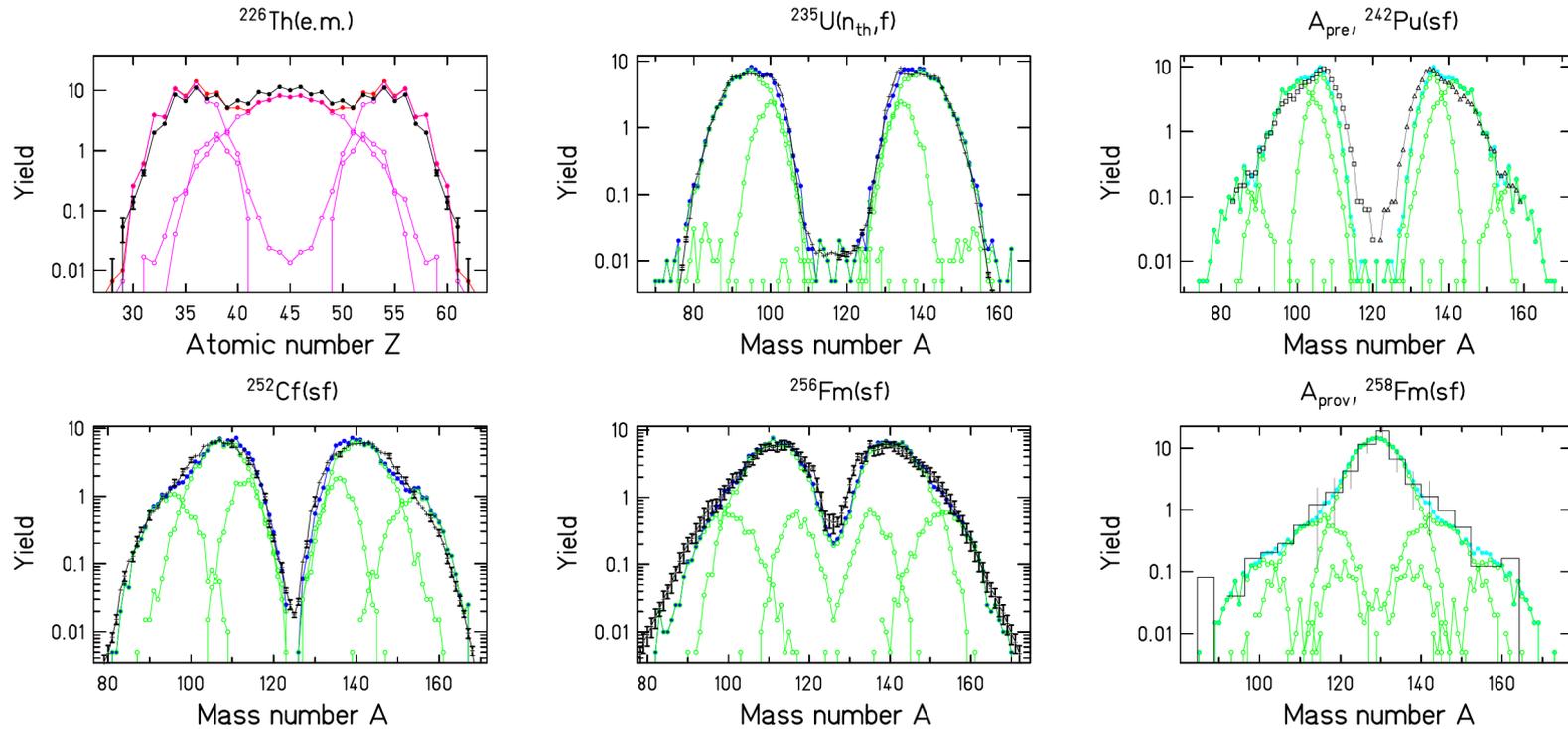
Deduce macroscopic potential from yield and shape of symmetric mode.

Deduce fragment shells (Z_{light} , Z_{heavy} , N_{light} , N_{heavy}) from yields and shapes of asymmetric modes (all systems).
(Shells depend only on the fragment, not on the CN!)

Idea: Mass distribution is given by quantum oscillator in mass asymmetry in a heat bath (nuclear temperature).
Superposition of the different fission valleys.

Dynamics: The deduced potential includes dynamical effects (friction and inertia).

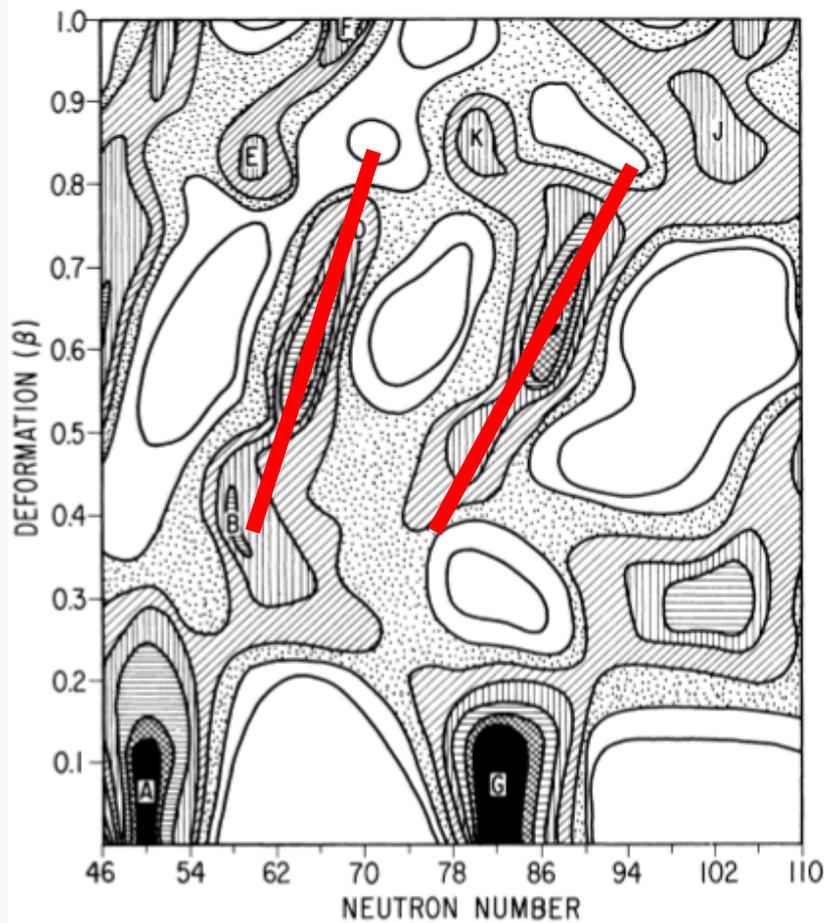
Hidden simplicity of fission !



Variety of mass (Z) distributions very well described with the same fragment shells ($Z \approx 51$, $Z \approx 55$, $Z \approx 59$, $Z \approx 42$)!
(All distributions obtained with the same parameter set: position, depth and width of shells.)

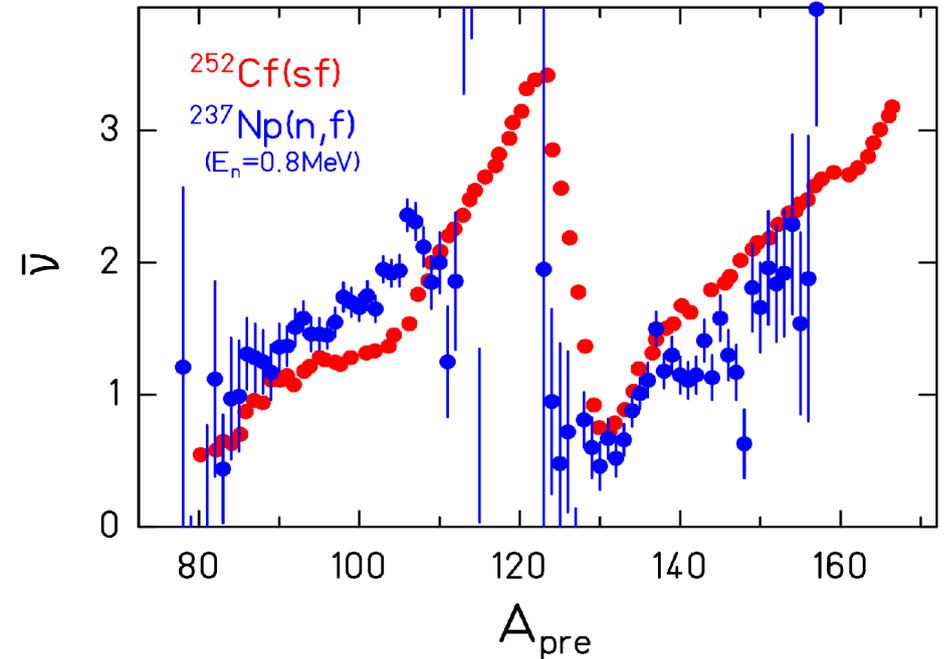
GEF code: www.khs-erzhausen.de

Fragment deformation \rightarrow prompt neutrons



Wilkins et al., Phys. Rev. C 14 (1976) 1832

General systematics of deformed shells:
Correlation particle number \leftrightarrow deformation



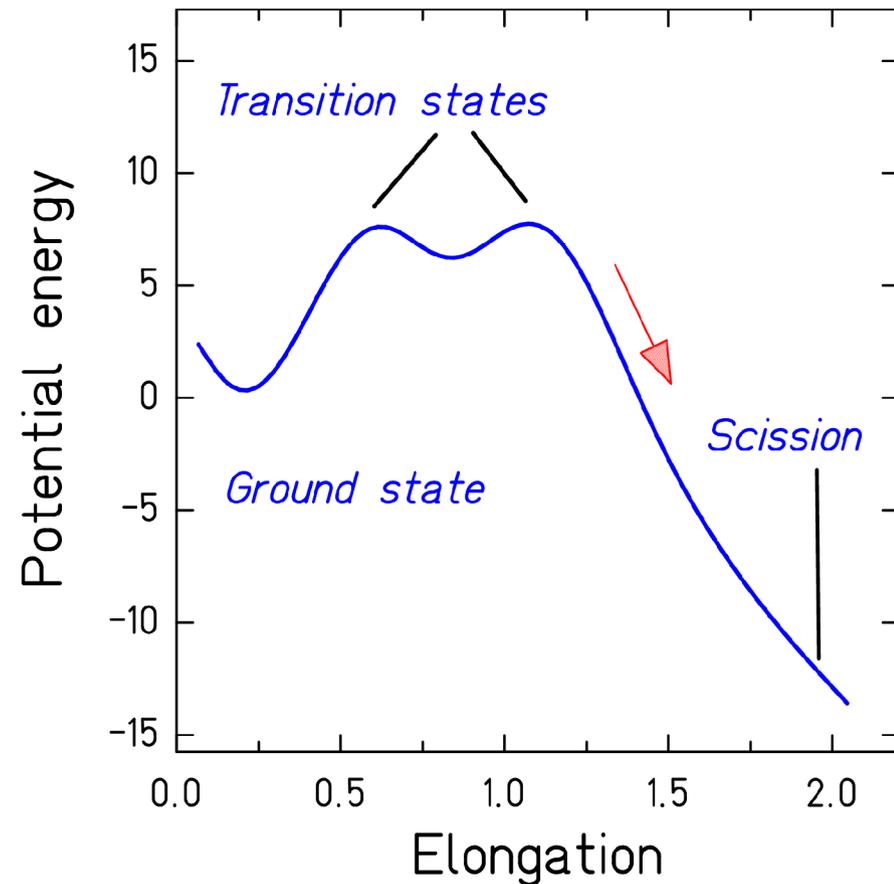
Naqvi et al, 1986 / Zeynalova et al., 2012

Deformation at scission
determined by fragment,
not by CN.
Confirms separability
principle.

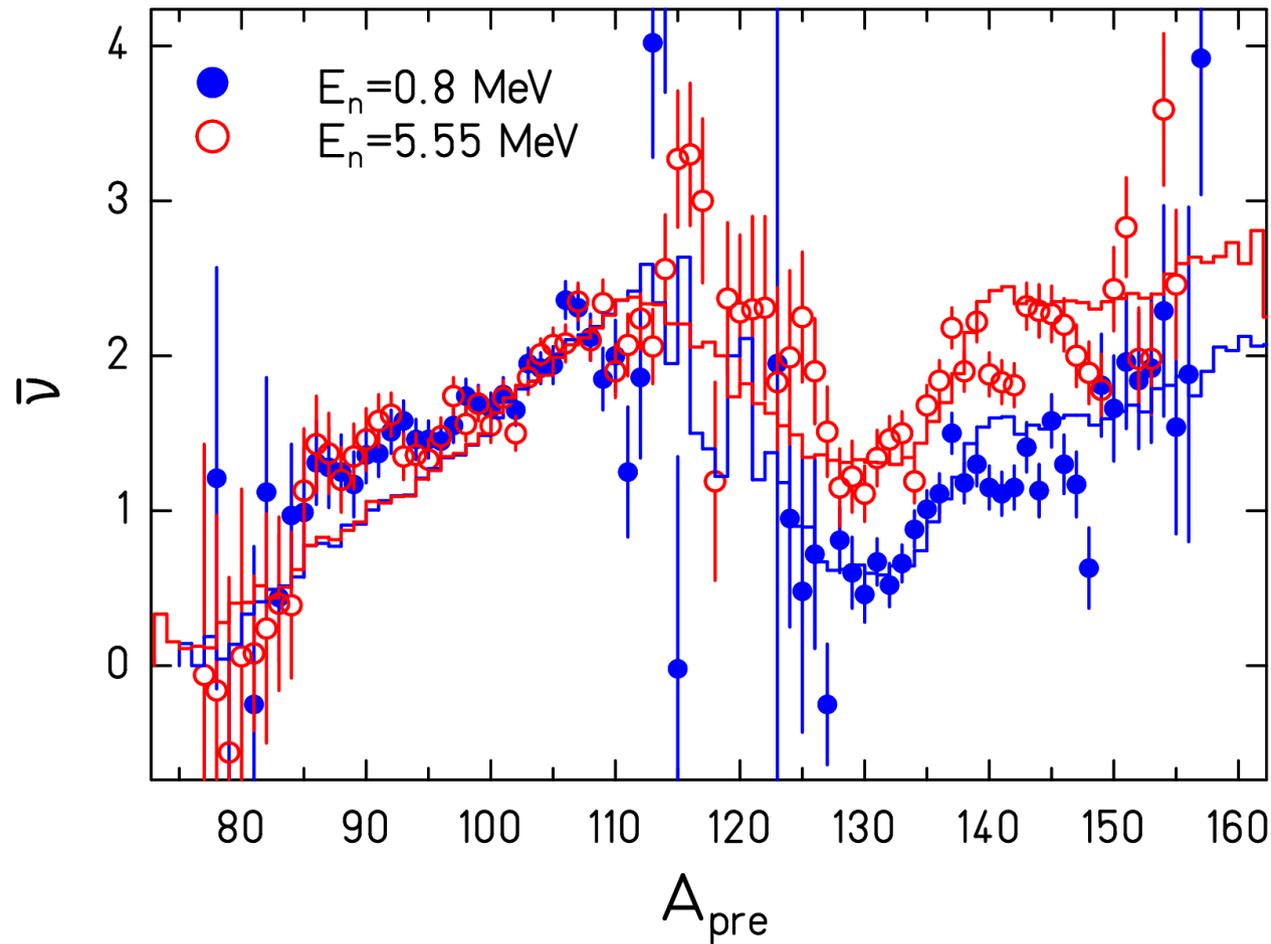
Thermodynamics: How does heat divide between fragments ?

Variation of initial E^* .

Not only nucleons,
but also thermal
energy can be
exchanged between
nascent fragments
until scission.



Heat moves to heavy fragment



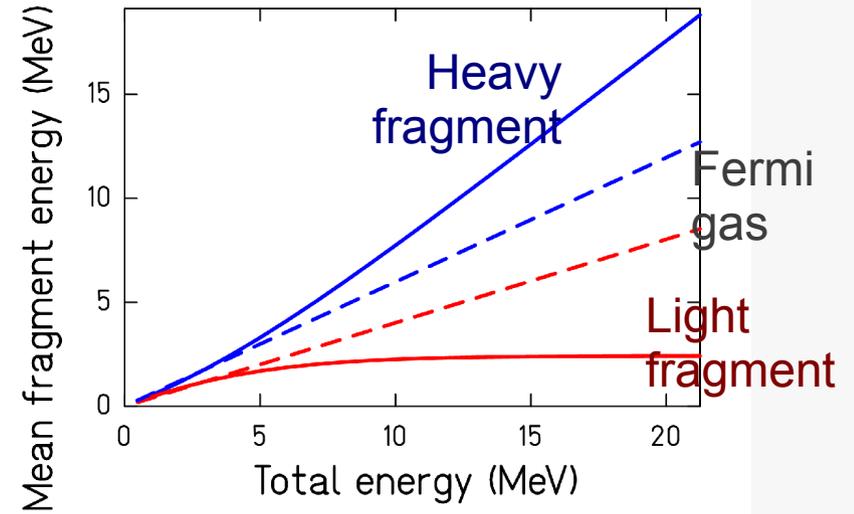
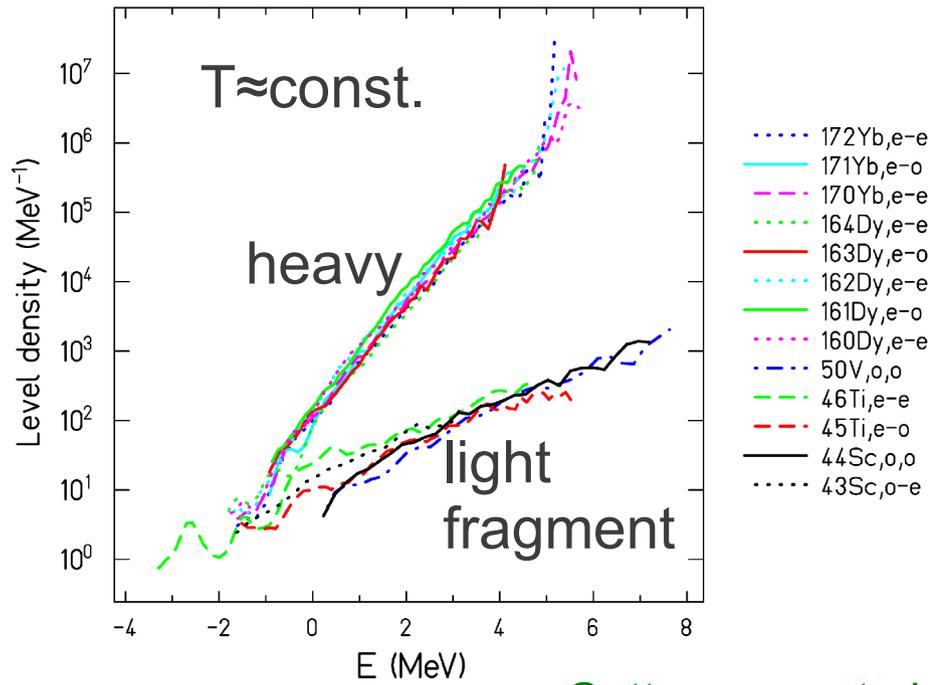
Naqvi et al., PRC 34 (1986) 21

Theory associates this result with new data on level densities:

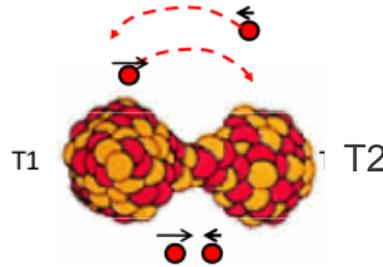
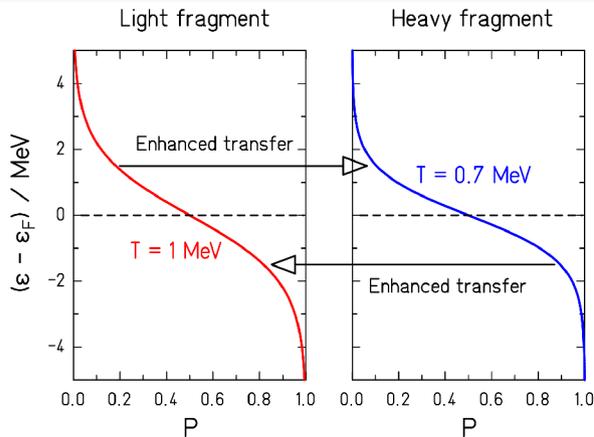
Statistical mechanics (energy sorting)

Nascent fragments: two thermostats!

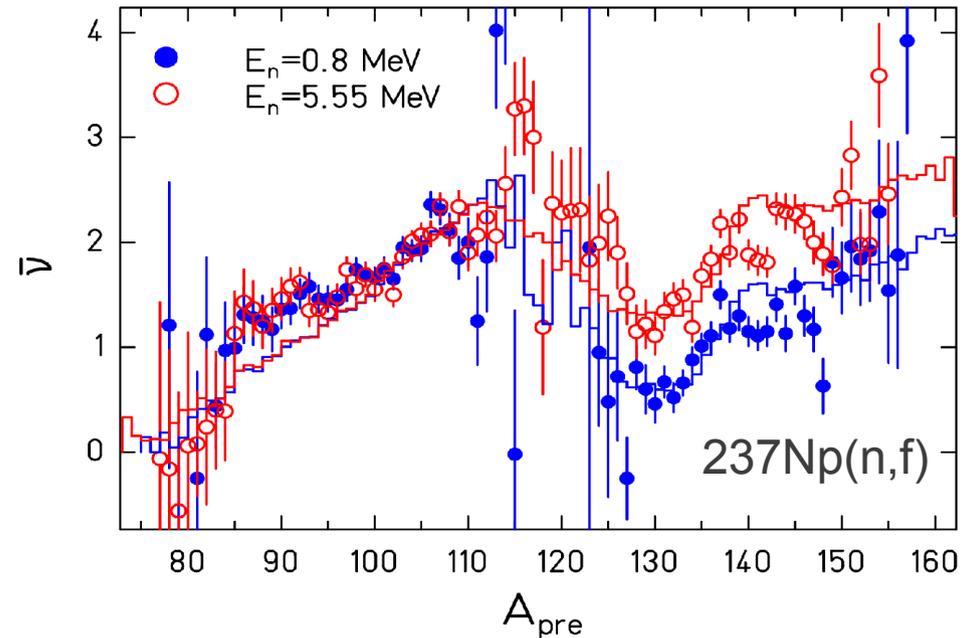
Schmidt, Jurado, PRC83 (2010) 061601



Guttormsen et al.



Naqvi et al., PRC 34 (1986) 21



Heat transport (Randrup NPA 327 (1979) 490)

Nuclear data:

GEFY - Library of fission-fragment yields

Tables of fission-fragment yields in ENDF format are provided for technical applications:

www.khs-erzhausen.de/GEFY.html or
www.cenbg.in2p3.fr/GEFY

Spontaneous and n-induced fission (E_n =thermal to 20 MeV)

About 40 specifically adjusted parameters *for all systems*.
(Compared to 30 to 60 parameters *for each system* in Wahl's empirical description.)

Quality is competitive with purely empirical descriptions:

1. Good reproduction of experimental data.
2. High predictive power due to validity of general laws.

Mass distributions vs. energy

En

225Th

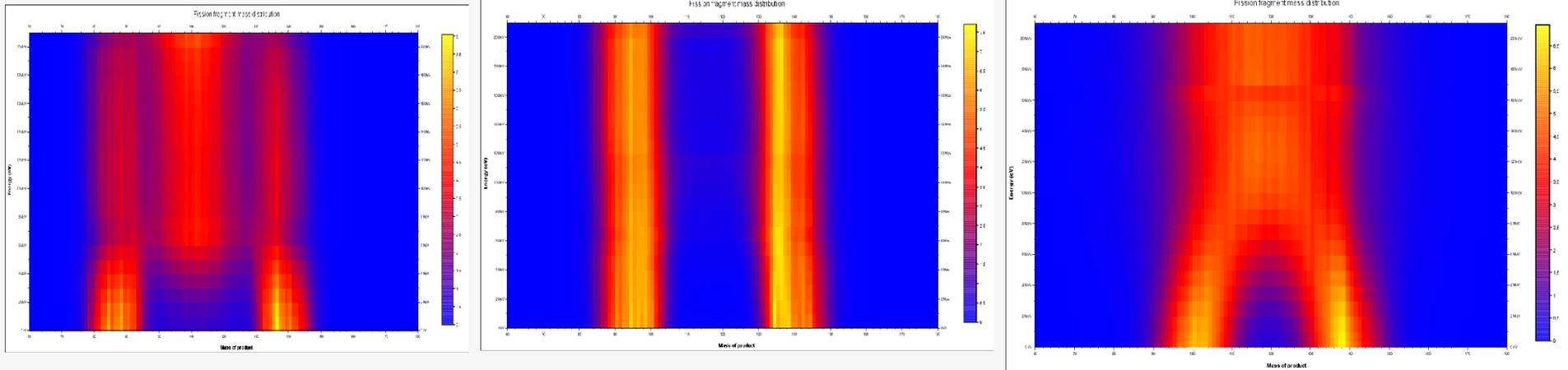
235U

241Cf

20
MeV



0
MeV



Fragment mass



Figure produced from GEFY tables
with the test version of JANIS.

Summary

General law of quantum mechanics: (→ mass division)

Localisation of wave functions in di-nucleus.

Early influence of fragment shells.

Application:

Separability principle (mac. and mic. properties).

Hidden simplicity of fission-channel properties.

General law of statistical mechanics: (→ heat division)

Dynamics driven by entropy.

Application:

Energy sorting. Also decisive for even-odd effect!

General conclusion:

Microscopic modelling of individual systems is blind to the power of general laws.

Application of general laws and careful analysis of firm empirical systematics puts the physics into a global context and leads to a better understanding.

GEFY: global set of fission-fragment yields.

Relevant publications

- [1] Experimental evidence for the separability of compound-nucleus and fragment properties in fission, K.-H. Schmidt, A. Kelic, M. V. Ricciardi, *Europh. Lett.* 83 (2008) 32001
- [2] Nuclear-fission studies with relativistic secondary beams: analysis of fission channels, C. Boeckstiegel et al., *Nucl. Phys. A* 802 (2008) 12
- [3] Shell effects in the symmetric-modal fission of pre-actinide nuclei, S. I. Mulgin, K.-H. Schmidt, A. Grewe, S. V. Zhdanov, *Nucl. Phys. A* 640 (1998) 375
- [4] Entropy-driven excitation-energy sorting in superfluid fission dynamics, K.-H. Schmidt, B. Jurado, *Phys. Rev. Lett.* 104 (2010) 212501
- [5] New insight into superfluid nuclear dynamics from the even-odd effect in fission, K.-H. Schmidt, B. Jurado, arXiv:1007.0741v1 [nucl-th]
- [6] Thermodynamics of nuclei in thermal contact, K.-H. Schmidt, B. Jurado, *Phys. Rev. C* 82 (2011) 014607
- [7] Final excitation energy of fission fragments, K.-H. Schmidt, B. Jurado, *Phys. Rev. C* 83 (2011) 061601(R)
- [8] Inconsistencies in the description of pairing effects in nuclear level densities, K.-H. Schmidt, B. Jurado, *Phys. Rev. C* 86 (2012) 044322