Application of a Bayesian/Generalised Least-Squares method to generate correlations between independent neutron fission yield data

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Outline

- Introduction to fission yields
- Discrepancy in the current libraries
- Correlation matrix generation
- Practical applications
- Conclusions

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Fission Yields

- Independent Fission Yields (IFY) $y(A,Z,I)$

$$y(A,Z,I) = Y(A) \times f(A,Z) \times r(A,Z,I)$$

- Cumulative Fission Yields (CFY) $C(A,Z,I)$
- Chain Fission Yields (ChFY/MFY) $Y(A)$

Recommended FY data + uncertainties (std.dev.)

JEFF-3.1.2

ENDF/B-VII.1

JENDL-4

Constraints

$$\sum_{Z} f(A,Z) = 1 \quad \forall A$$

$$\sum_{I} r(A,Z,I) = 1 \quad \forall A,Z$$

$$Y(A) = \sum_{Z,I} y(A,Z,I) \quad \forall A$$

$$\sum_{A,Z,I} y(A,Z,I) = 2$$

$$\sum_{A,Z,I} Ay(A,Z,I) = A_f - \nu(E)$$

$$\sum_{A,Z,I} Zy(A,Z,I) = Z_f$$

$$C(A,Z,I) = Qy(A,Z,I)$$

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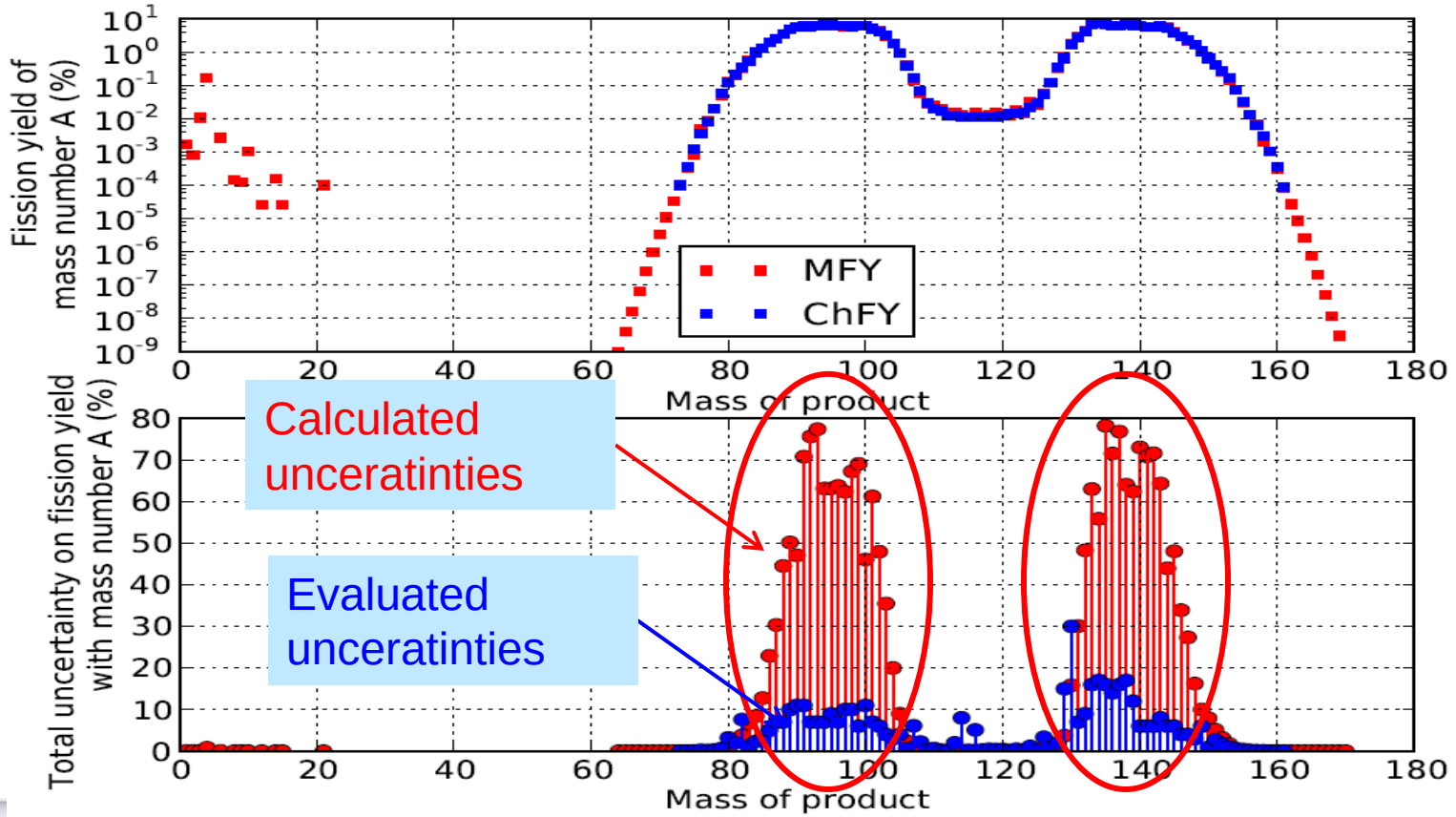
Need for correlations

- Current general-purpose libraries do not provide FY correlations

$$Y(A) = \sum_{Z,I} y(A,Z,I) \qquad Y(A) = \sqrt{\left(\sum_i \Delta y_i\right)^2}$$

NO COV



Need for correlations

Uncertainty on ^{148}Nd cumulative fission yield

$$C_j = \sum_i \left[N_{Nd148}(t) \approx \frac{\Sigma_f \phi y_c^{Nd148}}{\sigma_c^{Nd148}\phi}\left(1 - e^{-\sigma_c^{Nd148}\phi t}\right) \right] Q_{j,i}^2 var(y_j) \sum_j \sum_k Q_{j,i} cova(y_j, y_k) Q_{k,i}$$

JEFF-3.1.2	Uncertainty (%)
Evaluated ΔC_{Nd148}	0.70
Calculated ΔC_{Nd148} (no corr.)	9.67

Nd148 is a burnup tracker

ENDF/B-VII.1	Uncertainty (%)
Evaluated ΔC_{Nd148}	0.35
Calculated ΔC_{Nd148} (no corr.)	21.42

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Bayesian/General Least-Squares (GLS) method

$$\chi^2 = \begin{bmatrix} \vartheta - \vartheta_a \\ \eta - y \end{bmatrix}^+ \begin{bmatrix} V_a & H \\ H^+ & V \end{bmatrix}^+ \begin{bmatrix} \vartheta - \vartheta_a \\ \eta - y \end{bmatrix} = minimum$$

Covariance matrix generation

Constraint:

$$y - y_a = S(\vartheta - \vartheta_a)$$

ENDF/B-VII.1
JEFF-3.1.2
JENDL-4
…

Least-squares adjustment of the IFY variance

$$\vartheta - \vartheta_a = V_a S^t \left(S V_a S^t + V\right)^{-1} (\eta - y_a)$$

$$V_s = V_a - V_a S^t \left(S V_a S^t + V\right)^{-1} S V_a$$

IFYs (prior)

IFY variance matrix (no correlation)

Calculated CFYs / MFYs (observable)

Recommended CFYs / MFYs (new data)

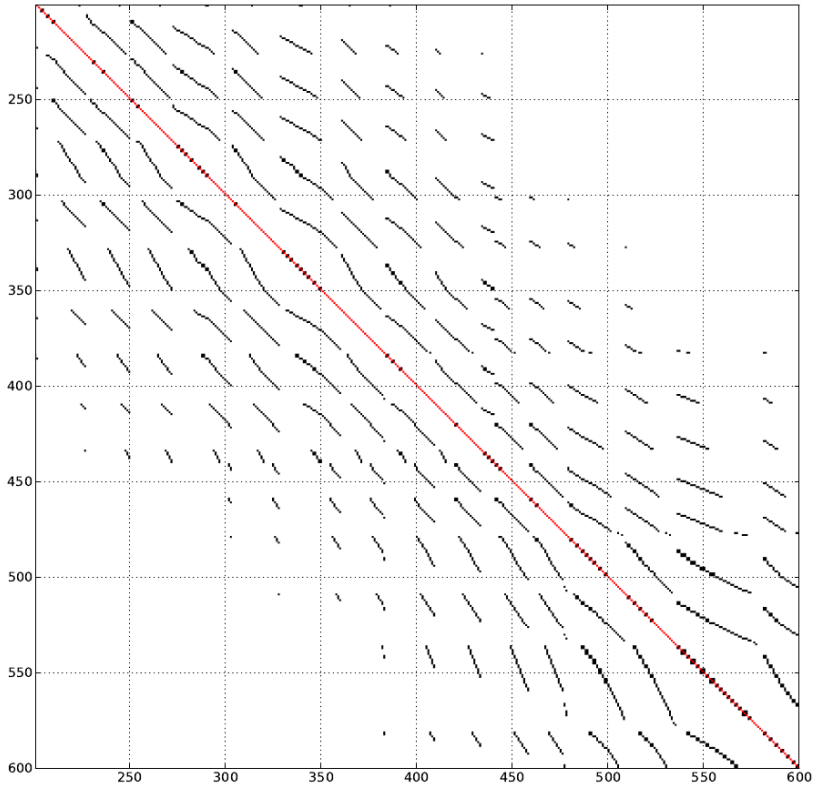
Recommended CFY / MFY variance matrix (no correl.)

Variance matrix of observables

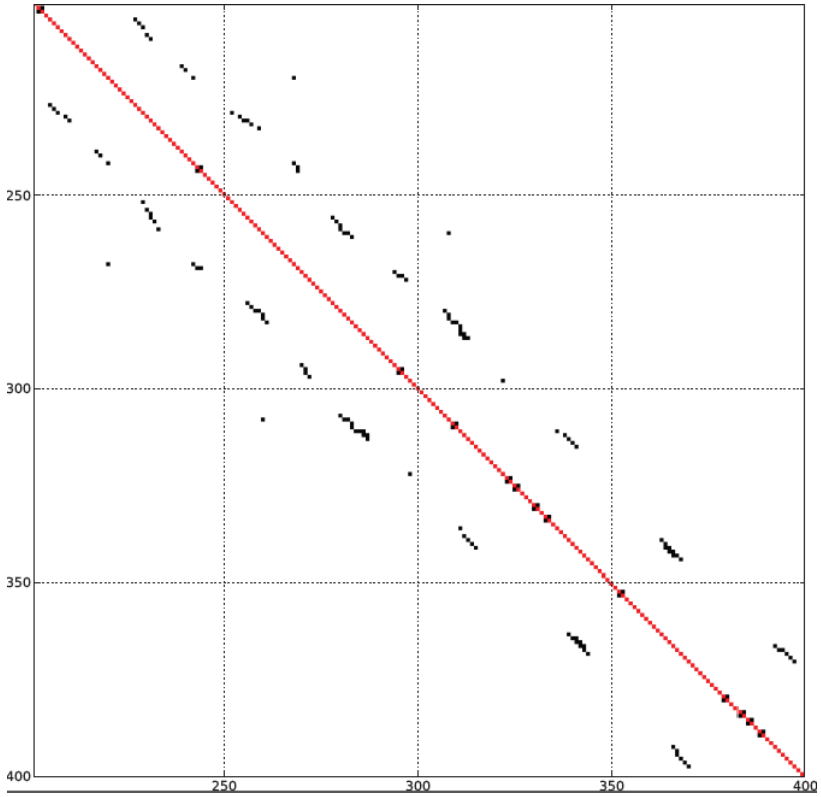
IFY updated variance matrix (posterior)

Covariance matrix generation

Updating with MFY *(JAEA, 2012)*



Updating with CFY



Negative correlations

$$\vartheta - \vartheta_a = V_a S^t \left(S V_a S^t + V\right)^{-1} \left(\eta - y_a\right)$$

$$V_s = V_a - V_a S^t \left(S V_a S^t + V\right)^{-1} S V_a$$

Need for correlations

Uncertainty on ^{148}Nd cumulative fission yield

$$C_j = \sum_i Q_{i,j} y_i$$

$$Var(C_i) = \sum_j Q_{j,i}^2 var(y_j) + \sum_j \sum_k Q_{j,i} cova(y_j, y_k) Q_{k,i}$$

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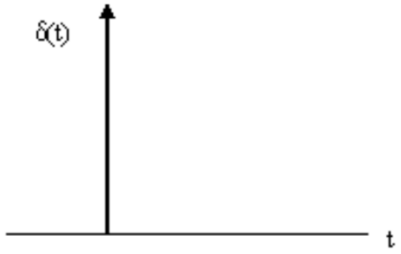
Application to FPDH

Impact of FY covariance matrix on Fission Pulse Decay Heat (FPDH)

FISSION



DECAY

$$\frac{dN_i}{dt} = -\lambda_i N_i + \sum_j \lambda_j \beta_{ji} N_j$$

$$DH = \sum_i DH_i = \sum_i \lambda_i N_i \left(\sum_j \beta_{ij} E_j \right)$$

2 burnup codes:

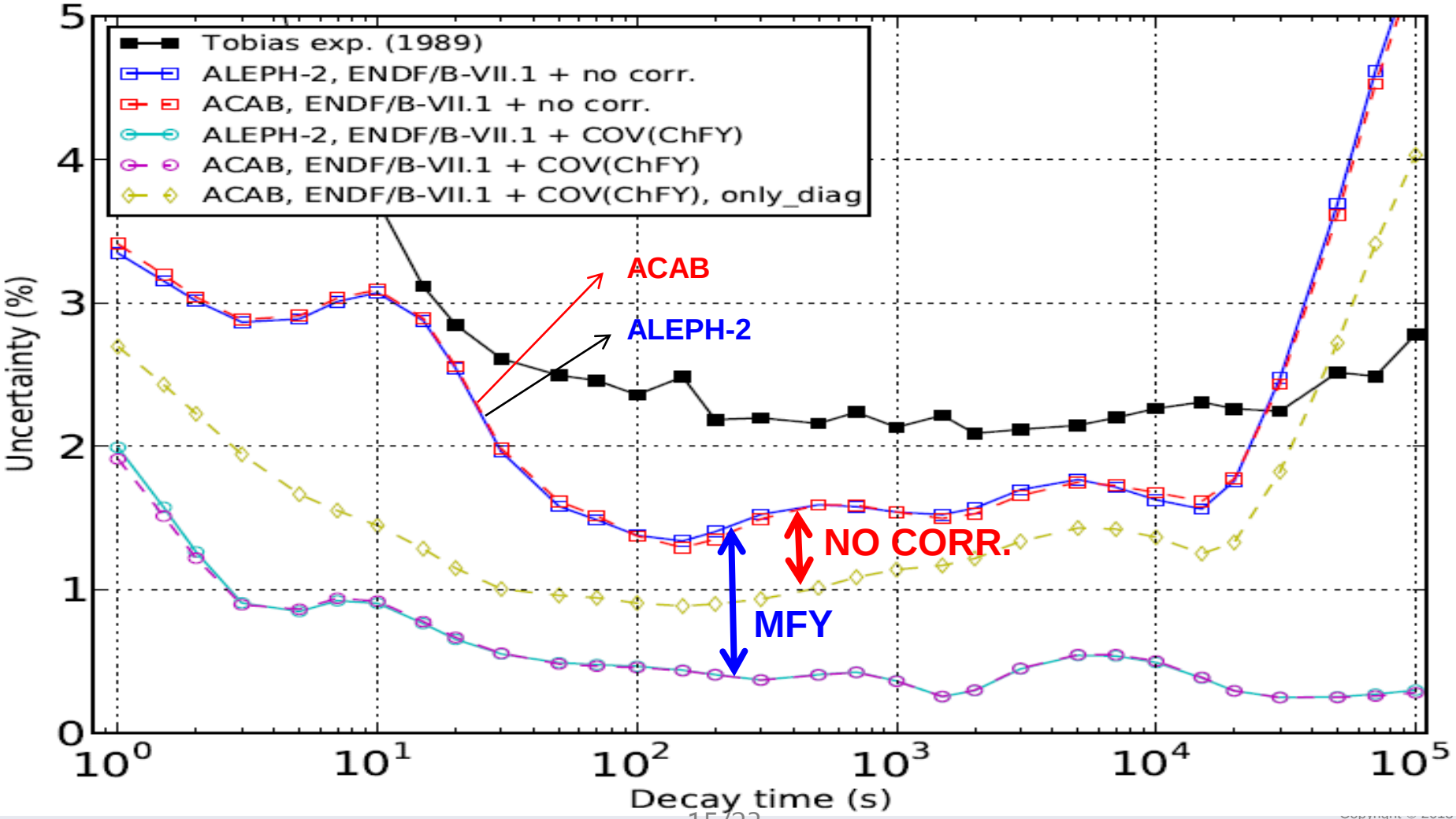
- ACAB (UPM)
- ALEPH-2 (SCK• CEN)

Monte Carlo sampling vs. linear perturbation

- 1000 independent $N(0,1)$ samples for each IFY
- Cholesky decomposition

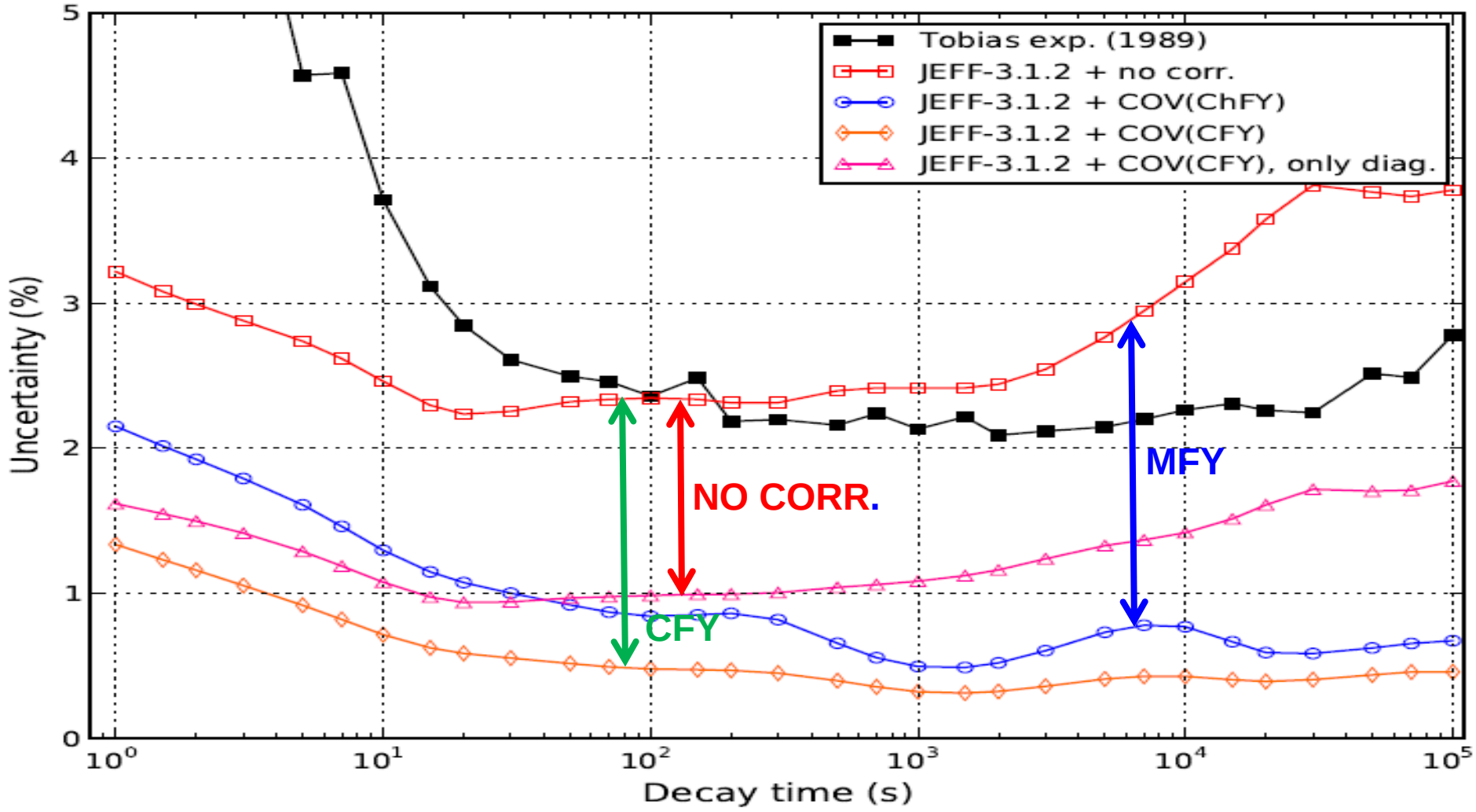
Reduced FPDH uncertainty

Calculation using **ENDF/B-VII.1**



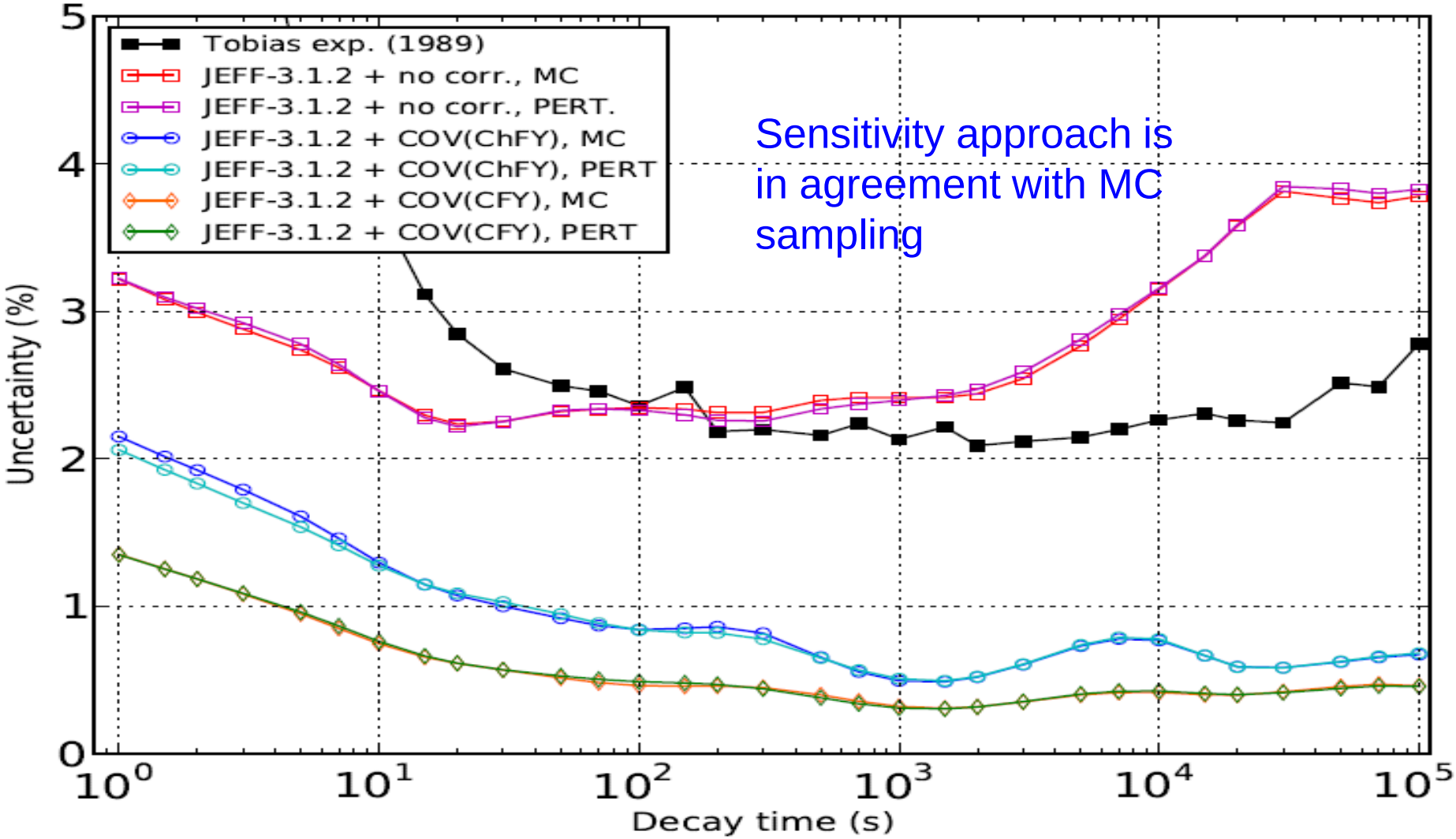
Reduced FPDH uncertainty

Calculation using **JEFF-3.1.2**



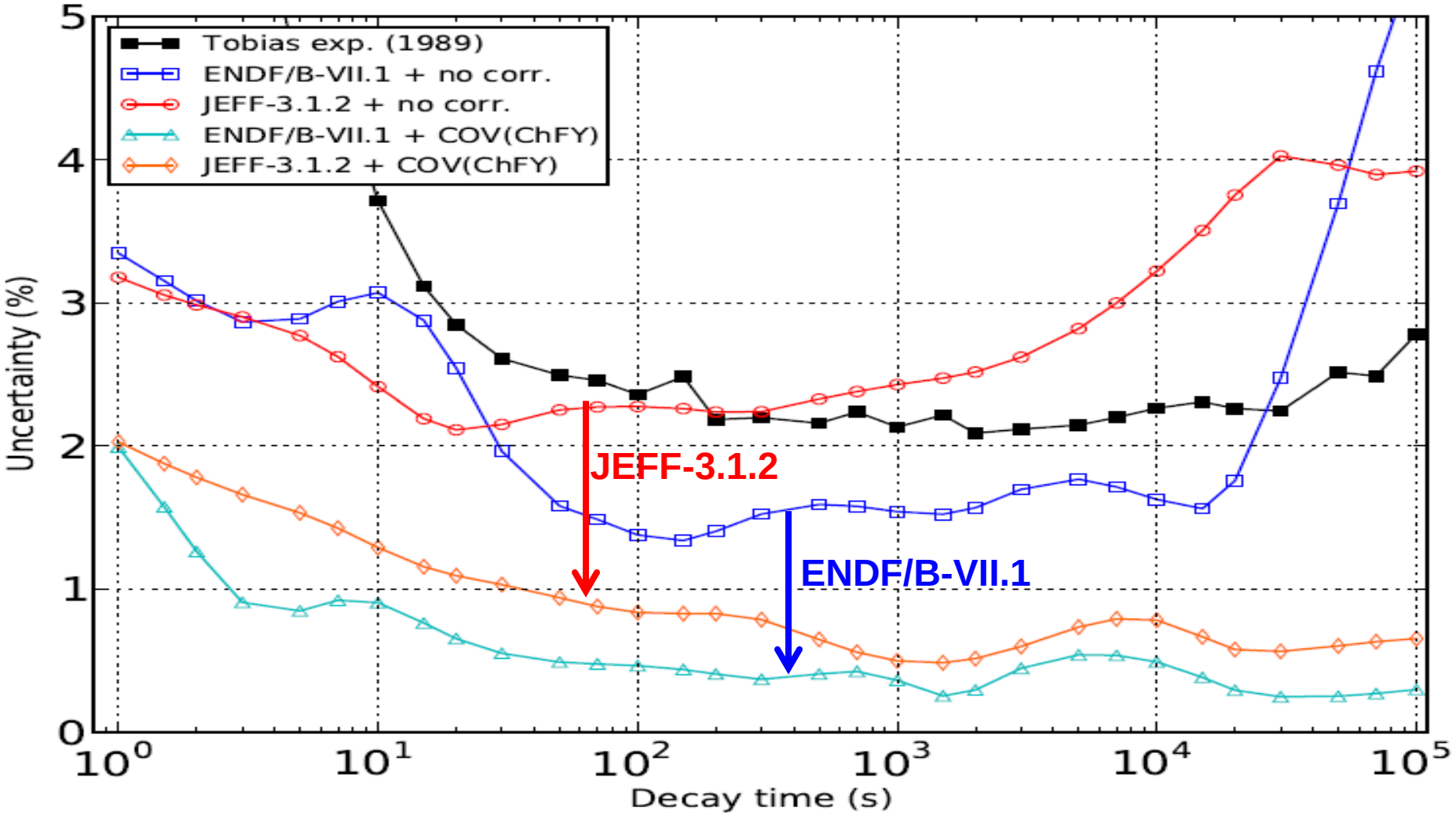
Reduced FPDH uncertainty

Comparison between **MC** and **linear pert.**



Reduced FPDH uncertainty

Comparison between **ENDF/B-VII.1** and **JEFF-3.1.2**



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Conclusions

- Discrepancies in uncertainties between libraries
- Covariance generation through Bayesian/GLS method
  - Update with MFY data
  - Update with CFY data
- Correlations “solve” discrepancies
- Application to FPDH scales down the overestimated uncertainties

THANK YOU FOR YOUR ATTENTION!

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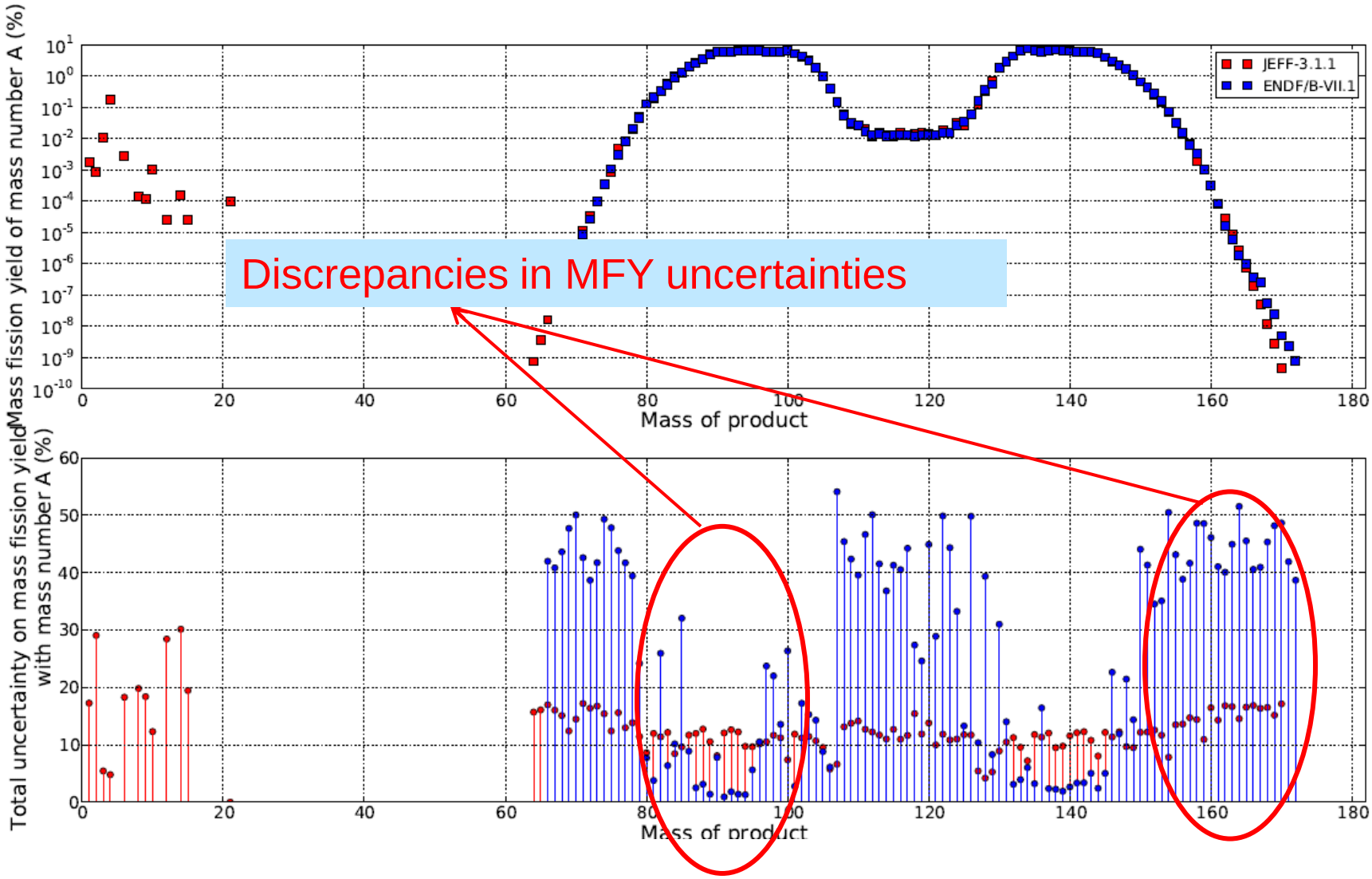
Mass Fission Yields



Cumulative Fission Yields