DECAY RATIO STUDIES IN BWR AND PWR USING WAVELET

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Abstract

The on-line stability of BWR and PWR is studied using neutron noise signals as the fluctuations reflect the dynamical characteristics of the reactor. Using appropriate signal modelling for time domain analysis of noise signals, the stability parameters can be obtained from the system impulse response directly. Here in particular for BWR, an important stability parameter is the decay ratio (DR) of the impulse response. The time series analysis involves the autoregressive modelling of the neutron detector signal and the DR determination is strongly effected by the low frequency behaviour since the transfer function characteristic tends to be a third order system rather than a second order system for a BWR and low frequency behaviour is modified by the Boron concentration in PWR. As a result, there are difficulties in consistent determination of the DR oscillations. Hence, the enhancement of the consistency of the DR estimation is aimed by wavelet transform using actual power plant data from BWR and PWR. A comparative study of the estimation with and without wavelets is presented.

Introduction

As a dynamic process system, stability is an important concern in control and operation of a nuclear reactor. From the design view point, such stable systems may be conceived so that they can exercise some incidental operational occurrences due to situations known as dynamic instabilities. Basically, this is due to time lag involved in a reactivity feedback. Nevertheless for PWRs, the stable regions established by the reactivity coefficients are rather well defined and the reactivity disturbances lead to limited power response. Some instability concerns might be due to low boron concentrations by the end of the fuel cycle. However, the issue is essentially important for boiling water reactors due basically to interaction between thermal hydraulics and neutronics, caused by the reactivity effect of steam bubbles and the time lag between a reactivity change and the corresponding change in steam void fraction due to power change. Added to this, mention may be made to the two-phase flow instabilities resulting in a peculiar thermohydraulic behaviour termed as density-wave oscillations.

Motivated by the stability considerations briefly mentioned above, there is a large amount of research reported in the literature especially for BWRs relative to PWRs. There are two aspects of these researches. On one hand is the early primary investigations to identify the underlying physics of the phenomena leading to instabilities. On the other hand, it is important to have the necessary information on the causes of instability and establish suitable means to monitor the system even in real-time. An important finding of these researches is that reactor neutron noise signals for the stability analysis and monitoring can be used so that the perturbation of the system in one way or other is circumvented. This means that continuous monitoring by these signals is possible. In this context, the conventional approach is the utilisation of the advanced signal modelling techniques. In the last two decades, with the aid of computer technology, such methods are intensively used in nuclear industry not only for reactor stability but also for general reactor analysis. The neutronic fluctuations reflect the dynamical characteristics of the reactor and in particular in BWRs they are additionally due to steam content change in the core exciting the system. The early researches with noise signals revealed that the BWR can reasonably be characterised as a second-order damped system with relevant characteristic quantities being defined.

The time series model is a causal representation of the dynamic behaviour of the system so that the dynamic character of the plant is thus effectively represented. Here, an important stability parameter is the decay ratio (DR) of the impulse response. Decay ratio is defined as the ratio of two consecutive maxima of the impulse response. For a second-order system, this ratio is constant during the course of the impulse response and it is also equal to two consecutive maxima of the autocorrelation function (ACF). Exploiting these features, stability determination for BWRs is endeavoured by several authors with a qualified success [1-3]. The same applies to PWRs [4]. In particular, the qualification stems from the fact that, reactor is not strictly a second-order system and even its global characteristic is more third-order rather than second-order for a BWR and from this viewpoint validity of a second-order system assumption for PWR is hardly justified. However, another important factor in such determinations is the method in use. Among the outstanding methods mention may be made of time-series analysis, spectral decomposition and neural networks. To pin down the problems a brief introduction is appropriate here.

The majority of time series analysis methods use autoregressive (AR) model for the impulse response acquisition where the signal is modelled for model order p and p+1 of the form

$$y_k = -\sum_{i=1}^{p} a_i y_{k-1} + \varepsilon_p(k) \tag{1}$$

$$y_{k+1} = -\sum_{i=1}^{p+1} a_i * y_{k-1} + \varepsilon_{p+1}(k) \tag{2}$$

where a_i and a_i^* are the model parameters which are known as AR parameters; $\varepsilon_p(k)$ and $\varepsilon_p^*(k)$ are white noise sequences. If we multiply Eq. (1) and Eq. (2) by y_k and take the average, we obtain after some arrangements

$$\sigma_{p^2} - \sigma_{p+1^2} = \sum_{i=1}^{p}(a * - a_i)R(i) - a *_{p+1} R(p+1) \tag{3}$$

The AR coefficients a^*_i can be expressed in terms of a_i by means of Levinson-Durbin recursive algorithm. The recursion is of the form

$$\begin{aligned} a_i * &= a_i - g_{p+1} a_{p+1-i} \qquad i = 1,\ldots,p \\ a_{p+1} * &= -g_{p+1}, \end{aligned} \tag{4}$$

where a_0 = 1 and g_{p+1} is known as reflection or *parcor* coefficient defined by

$$g_{p+1} = \frac{\sum_{i=0}^{p} a_i R(p+1-i)}{\sum_{i=0}^{p} a_i R_i} = \frac{\sum_{i=0}^{p} a_i R(p+1-i)}{\sigma_{p^2}} \tag{5}$$

so that the residual variances σ_{p^2} and σ_{p+1^2} are related to one another through the reflection coefficient of the form

$$\sigma_{p+1^2} = \sigma_{p^2}(1 - g_{p+1^2}) \qquad p = 0,1,2,... \tag{6}$$

Above, σ_{o^2} is the variance of the incoming raw data sequence. It is noteworthy to mention that, since both σ_{p+1^2} and σ_{p^2} are positive, it follows that the factor $(1 - g_{p+1^2})$ will be positive and less than unity. It represents the improvement in the prediction afforded by using a predictor of the order p+1 instead of order p. Another noteworthy point is that g_{p+1} has magnitude less than one and this is an indication of the stationarity of the signal.

The estimation of the impulse response function on the basis of this model i.e., univariate AR, is given by

$$hi = \sum_{k=1}^{j} a_{j,k} \cdot \tag{7}$$

The initial conditions being set to

$$h_m = 0 \text{ for } m < 0 \text{ and } h_o = 1 \tag{8}$$

For DR estimation, the problems encountered here are the excessive model orders (model order of 50 for a second-order system, for instance) due to low frequency effects at the measured spectrum. The same problem applies to the autocorrelation and the spectral decomposition analyses methods using multivariate AR or fast Fourier transform analysis. This is because of the model errors and the measurement errors and the systematic errors introduced during the signal processing (windowing in spectrum estimation, finite block size effects in correlation estimation, for instance). Concerning neural networks, the case is not fully under control due to the lack in the actual data needed for training. Added to this, the degree of the nonlinearity of the neural structure is dependent on the number of hidden layer nodes so that matching the nonlinearity of the neural structure to the nonlinearity of the functional relationship to be established is an important issue with regard to the performance of this approach. Some elaborations are imperative.

From the above discussion, one may note that the stability determination based on the decay ratio computation apparently is not totally satisfactory as the method may not always be conclusive due to both model errors and measurement errors, while it needs careful and lengthy measurements and signal processing. Therefore, in this research, studies on novel utilisation of neutron noise signals for stability monitoring is described. In this novel approach, a new signal analysis method called “wavelets” is used. “Wavelets” collectively indicates a new technology in the field of signal analysis and its use in nuclear technology is exemplified [5] and suggested [6].

The organisation of the paper is as follows. To start with, wavelet analysis is briefly outlined and we give basic definitions and properties related to wavelet analysis including discrete bases for signal decomposition and reconstruction which constitute the essential signal processing part of the work. Then, general wavelet approach for stability analysis is described. Afterward, the application of the method to BWR and PWR signalsis examined. At the end of the paper, a few concluding remarks are offered.

Wavelet analysis

In contrast to Fourier analysis in frequency domain and time series analysis in time domain, wavelet analysis is used to analyse the signal in both time and frequency domains. By doing so, a signal is decomposed to some components corresponding to different frequency ranges and each component is further considered with a resolution matched to its scale. The continuous wavelet-transform (CTWT) is defined as the inner product of f(t) with the basis functions

$$\psi_{a,b}(t) = a^{-1/2}\psi\left(\frac{t-b}{a}\right) \tag{9}$$

so that

$$CTWT_f(a,b) = a^{-1/2} \int_{-\infty}^{+\infty} f(t)\psi\left(\frac{t-b}{a}\right)dt \tag{10}$$

where ψ is referred to as *mother wavelet* and a and b are, respectively, scale and shift parameters. The Fourier coefficients of the wavelet transform are obtained from the equation

$$W(a,b) = a^{1/2} f(\omega)\psi(a\omega) \tag{11}$$

The computation of the continuous wavelet transform by the discretisation of the integral term is not a general approach due to high cost of computations and the large errors arising at small scales. For continuous time computation, Eq. (11) is generally used.

The basis functions $\psi_{a,b} \in L^2(R)$ are real and oscillating. They are called wavelets and can be viewed as contracted and shifted versions of the function $\psi_{a,b}(t)$. The function $\psi(t)$ has to satisfy the admissibility condition

$$C\psi = \int_{o}^{\infty} \frac{\left|\psi(\omega)^2\right|}{\omega} d\omega < \infty \tag{12}$$

in order to be able to reconstruct f(t) from its CTWT. $\psi(\omega)$ above is the Fourier transform of $\psi(t)$. The reconstruction formula is

$$f(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{o}^{\infty} CTWT_f(a,b) a^{-1/2} \psi[(t-b)/a] a^{-2} da db \tag{13}$$

In $CTWT_f(a,b)$ the parameters a,b vary continuously. It is possible to discretise the values for a and b, while still being able to reconstruct the signal from its transform. For this we substitute

$$a = a_o^i,\ b = jb_o a_o^i,\ ij \in Z,\ a_o > 1,\ b_o \neq 0 \tag{14}$$

The corresponding wavelets for the discretised a and b are

$$\psi_{i,j}(t) = a_o^{-i/2} \psi(a_o^{-i} t - j) \tag{15}$$

so that the wavelet transform becomes

$$(W_\psi f)_{i,j} = d_{i,j} = \int_{-\infty}^{\infty} a_o^{-i/2} \psi(a_o^{-i} t - j) f(t) dt \tag{16}$$

In order to clearly present what role wavelets play in the stability analysis, first we recall the mathematical foundations i.e., multiresolution analysis and orthogonal wavelet bases.

A multiresolution analysis of a function f consists of estimation a series of functions f_j corresponding different representations of that signal where j represents the detail index of size 2^{-j}. These estimates converge to f when j tends to infinity. This can be best described by the theory of function spaces. A multiresolution analysis is a description of $L^2(R)$ as a hierarchy of embedded subspaces V_m which have intersection {0} and for which the limit of their union is $L^2(R)$; namely

$$...\subset V_2 \subset V_1 \subset V_o \subset V_{-1} \subset V_{-2} \subset ...$$

verifying the following properties [7-9]:

(i) $\cap_{j\in Z} V_j = \{0\}$ $\cup_{j\in Z} V_j = L^2(R)$

(ii) $f\in V_j \Leftrightarrow f(2^{-j}) \in V_{j+1}, j\in Z$

(iii) $f\in V_o \Leftrightarrow f(2.-k) \in V_o, k\in Z$

(iv) There exists $\Phi\in V_o$ so that $\{\Phi(t-k)\}_{k\in Z}$ is an orthonormal base of V_o.

As the functions $\Phi_{o,j}(t)$ form an orthonormal basis for V_o, it follows that the functions

$$\Phi_{i,j}(t) = 2^{-i/2}\Phi(2^{-i}t - j) \tag{17}$$

constitutes an orthonormal basis for V_i. These basis functions are referred to as scaling functions since they build up scaled versions of the functions in $L^2(R)$. From the multiresolution analysis introduced one realises that a function f(t) in $L^2(R)$ can be seen as a successive approximation by functions $f_i(t)$ in V_i. Hence, the function f(t) is the limit of the approximations $f_i(t) \in V_i$ for i to $-\infty$, namely

$$f(t) = \lim_{i\to -\infty} f_i(t) \tag{18}$$

This creates the possibility to examine the function or signal at several resolutions or scales. The variable i indicates the scale and therefore called the "scale factor". If the scale factor is high, this means the function in V_i is coarse approximation of f(t), the details being neglected. On the contrary, if the scale factor is low, a detailed approximation of f(t) is achieved. All functions in V_i can be represented using linear combinations of the scaling functions. Hence one can see that $f_i(t)$ is an orthogonal projection of f(t) onto V_i, of the form

$$f_i(t) = \sum_j < \Phi_{i,j}(t), f(t) > \Phi_{i,j}(t) = \sum c_{i,j}\Phi_{i,j}(t) \tag{19}$$

Since

$$\Phi(t) = \Phi_{0,0}(t) \in V_0 V_{-1}$$

For a specific sequence h_j, we can write

$$\Phi_{0,0}(t) = 2^{1/2} \sum_j h_j \Phi_{-1,j}(t) = 2 \sum_j h_j \Phi(2t - j) \tag{20}$$

so that $\Phi_{0,0}(t)$ is a solution of a two-scale difference equation indicating the close relationship between the function $\Phi(t)$ and the sequence h_j.

In the above definition we can assume that the space $L^2(R)$ is built up the set of rings that are differences between two consecutive spaces. These difference spaces are denoted by W_i with respect to V_{i-1} so that

$$V_{i-1} = V_i \oplus W_i \tag{21}$$

$$\cap_{ieZ} W_i = \varnothing, \qquad \cup_{ieZ} = L^2(R) \tag{22}$$

where $\oplus$ indicates the summation of the orthogonal spaces. The W_j spaces verify the following properties.

(i) $f \in W_j \Leftrightarrow f(2^{-j}.) \in W_o,\ j \in Z$

(ii) $\psi \in W_o \Leftrightarrow \psi(.-k) \in W_o,\ k \in Z$

(iii) W_i is orthogonal to W_j for $i \neq j$

(iv) $\oplus_{j \in Z} W_j = L^2$

Let $\psi(t) = \psi_{o,o}(t)$ be a basis function of W_o. Since $\psi_{o,o}(t) \in W_o \subset V_{-1}$ we can write

$$\psi_{0,0}(f) = 2^{1/2} \sum g_j \Phi_{-1,j}(t) \tag{23}$$

for a certain sequence of g_j. The functions $\Phi_{i,j}(t)$ are shifted and dilated versions of each other. Therefore, we can also define functions $\psi_{i,j}(t)$ that are shifted and dilated versions of one prototype function $\psi(t)$, of the form

$$\psi_{i,j}(t) = 2^{-i/2} \psi(2^{-i} t - j) \tag{23}$$

The functions $\psi_{i,j}(t)$ are identical to the wavelets introduced earlier after the discretisation of Eq. (14). The parameter a_o in Eq.(15) is fixed and equal to 2 in this case. They form an orthonormal basis for $L^2(R)$.

The wavelet transform algorithm carries out the multiresolution decomposition as follows. Let Φ be the scaling function. At step j, we have the signal f_j which belongs to the space of approximations V_j and its coefficients $c_{j,k}$ on the bases of V_j. Then, using the equation $V_j = V_{j-1} \oplus W_{j-1}$ we compute its projection f_{j-1} on V_{j-1} where, in particular, $d_{j-1,k}$ are coefficients on the bases of W_{j-1} and $c_{j-1,k}$ are coefficients on the bases of V_{j-1}. The coefficients $c_{j-1,k}$ and $d_{j-1,k}$ are obtained by respectively applying a low-pass filter H and a high-pass filter G to the sequence $c_{j-1,k}$ [5].

Improved decay-ratio estimation by wavelet analysis

Utilisation of wavelet approach for stability analysis is due to low frequency effects in the power spectral density of the signal from the neutron detectors. Particularly for BWRs the DR determination is strongly effected by the low frequency behaviour since the transfer function characteristic tends to be a third-order system rather than a second-order system. The same effect to a lesser extent is for PWRs the effect being attributed to diminished boron concentration at the end of the fuel cycle. By means of wavelet transform the low frequency part of the spectrum is replaced with a flat spectrum, i.e. gaussian white, in order to eliminate the low frequency effects. To this end, initially, a discrete band-limited white noise signal is considered. This can easily be formed by means of a suitable algorithm and a built-in noise generator in a computer. This signal is decomposed by means of wavelet transform into several signal components matching to their individual multi-resolution scale of frequency. In the same way the detector signal is subjected to the same wavelet decomposition as well. Depending on the width of the low frequency part one would intend to replace, the scale of the signal's multi-resolution subject to this due replacement, is replaced with the counterpart of that from the white noise. The computation is rather straightforward due to one-to-one replacement. However, in order not to modify the original signal beyond the intention, perfect reconstruction from the wavelet analysis is required. Therefore here in the analysis, for orthogonal wavelets specifically, Daubechies's wavelets with a compact support length of 12 are used.

Application to BWR and PWR

Wavelet-based decay ratio estimation described above is implemented to the recorded data from two operating nuclear power plants of the Netherlands, namely, Dodewaard BWR (58 MWe) and Borssele PWR (450 MWe) introducing the data throughout the on-line data acquisition and processing systems at the Netherlands Energy Research Foundation (ECN) site.

Dodewaard BWR is a small-sized BWR with natural convection circulation and has been operating since 1968 by GKN (Gemeenschappelijk Kerncentrale Nederland). For the DR investigations using on-line DR measurements (on 8 November 1989) a demonstration experiment with ECN on-line data acquisition and data analyses system is carried out at Dodewaard reactor. Among the various reactor signals that were measured, signals of four ex-core neutron detectors of safety channels are used for real-time DR calculation. The monitored signals, spectra, and impulse responses derived by univariate autoregressive method and the DR in real-time displayed for the reactor supervisors and to the members of Dutch Nuclear Safety Authority [10], thereby, a stability monitoring based on real-time on-line decay-ratio computation is thus realised and launched for operation for actual use with endorsement. In this wavelet application, a small part of the recorded noise signals of the ex-core neutron detector (N-6) from that experiment is used. Sampling period of the data used in the experiment was selected as 16 (s/s). The analysis of the same data with wavelet is shown in Figures 1 (a-d) where respectively, impulse response, step response, power spectrum from AR modelling and DR estimation are shown. The model order in this case is as low as 6, data block length is 128 which is intentionally low for real-time and on-line DR estimations. Decay ratio resulting from this analysis through the wavelet application is verified with the on-line DR estimation [10].

The Dodewaard reactor is a very stable BWR at the complete fuel cycle of operation, with the DR between 0.10-0.35. The functionality of wavelet analysis and the improvement achieved by wavelet is clearly demonstrated.

The outcomes of the same studies for PWRs is presented in Figures 2 (a-f), where the plant is Borssele NPP in the Netherlands. The Borssele PWR is a two loop system built by KWU and operated by N.V. Electriciteits-Productiematschappij Zuid Nederland EPZ since 1973. On-line experiments have been carried out since 1982 for monitoring, surveillance and diagnostics research and implementation purposes [11]. For this wavelet investigation we used noise signals of the ex-core neutron detector (D621) during the nominal reactor power at the beginning of the operating fuel cycle (boron concentration at 910 ppm) and at the end of fuel cycle (20 ppm). Noise data of the sensory signals are sampled with 8 samples/second. The illustrations involved two different operational situations; namely operation data at the beginning of the fuel cycle and at the end of the fuel cycle. Wavelet conditions and AR signal modelling conditions are kept the same as those of BWR studies described. Here there is no obvious change by wavelet due to the intrinsically flat spectrum of PWR at the low frequencies. Decay ratio is found to be as relatively low as one would expect.

Conclusion

Real-time decay ratio estimation is important for monitoring the stability of a BWR. Among the decay ratio estimations by conventional means, i.e. autocorrelation (ACF) method, spectral decomposition and time-series signal modelling, only the latter is of interest while the others are especially suitable for off-line estimations. Neural network approach is quite suitable for real time estimations but the method is not mature enough for consideration here, or conclusive assessments. In the time-series signal modelling approach a block of data is considered at each time for modelling and thereafter DR estimation while statistical variations play an important role regarding the estimations. The situation is aggravated in the case in which the data block is short. In contrast with this, short length of data is preferable for real-time operations. Referring to these conflicting qualifications accurate estimates by signal modelling becomes an issue of optimal design of a measurement. The case is more hampered if the model errors are also an important factor on the parameter determination as this is the case in DR estimation due to second-order system approximation. Referring to these, the estimation is highly improved by the utilisation of wavelet transform for BWR case. For PWR, such improvement is found not to be obvious for the same conditions used during the investigations for BWR. However, for increased block length of data, the DR estimations are found to be improved and also in the case of PWRs, wavelets can still be of substantial help for accurate estimations.

Acknowledgement

The authors would like to thank the authorities of CKN and N.V.EPZ for according their permission with regard to the release of NPP data

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Figure 1: Wavelet approach for DR estimation of BWR

Broken lines indicate the outcomes of conventional analysis counterpart

(a) Impulse response, (b) Step response, (c) Power spectrum, (d) Decay ratio

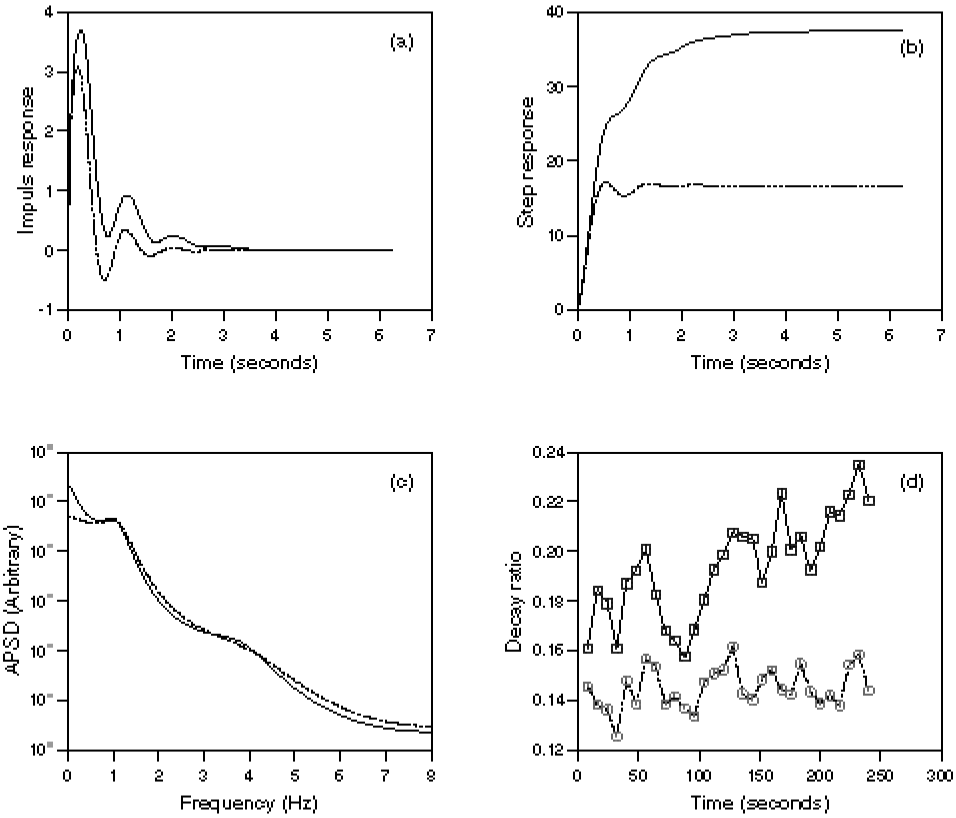

Figure 2: Wavelet approach for DR estimation of PWR

Broken lines indicate the outcomes of conventional analysis counterpart
(a) Impulse response, (b) Power spectrum, (c) Decay ratio abtained at the beginning of fuel cycle.
The Figures d,e,f are obtained at the end of the fuel cycle and
they are the counterpart of Figures a,b,c respectively