Modelling the widths of fission observables in GEF

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GEF (GEneral Fission model)

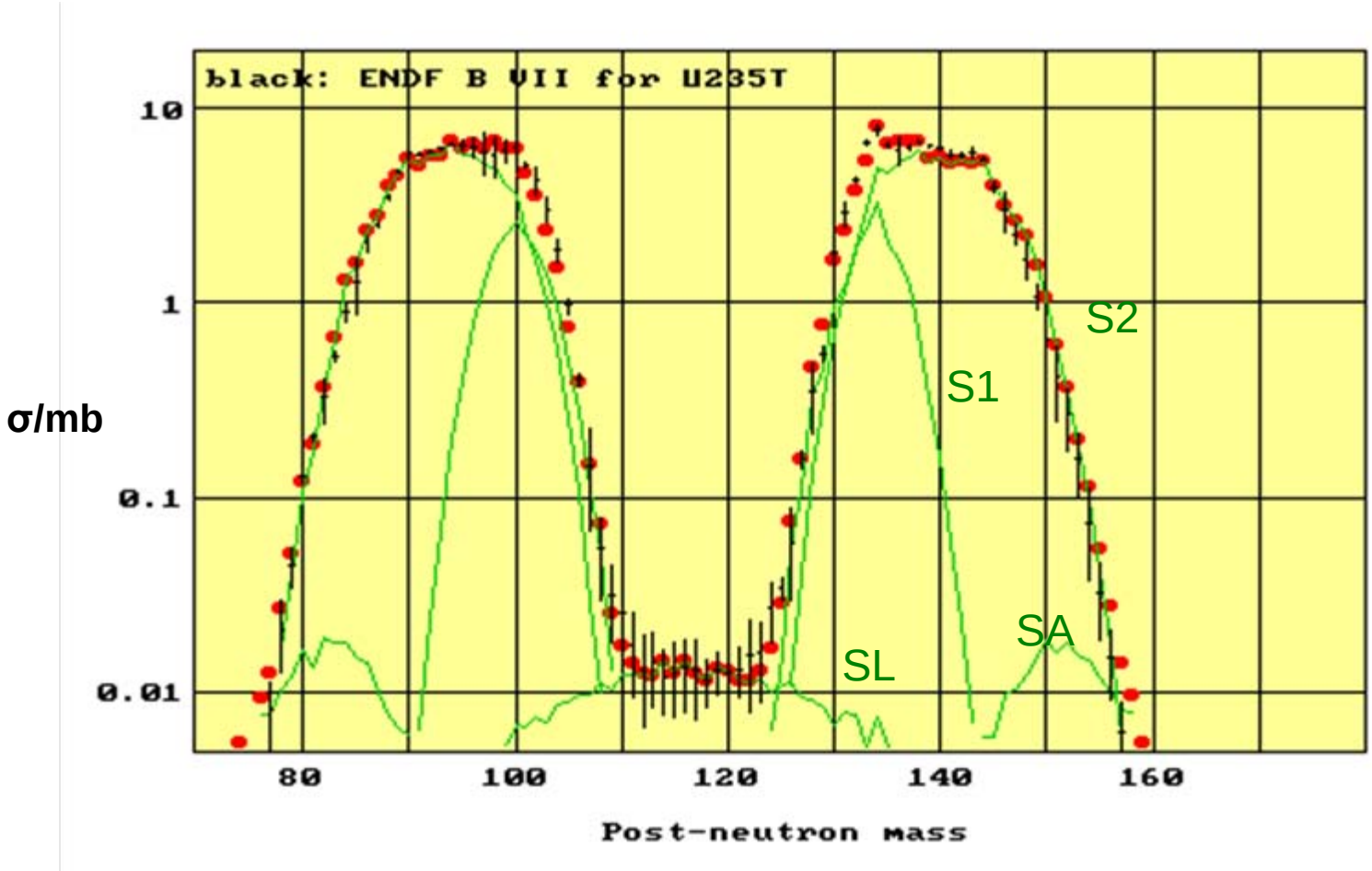
Reliable results for:
-->isotopic fission-fragment yields
-->energy and multiplicity distrib. of prompt neutrons and gammas

Predictions for nuclei where no data are available

Semi-empirical but based on solid physical concepts
Good predictive power!

www.khs-erzhausen.de
www.cenbg.in2p3.fr/GEF

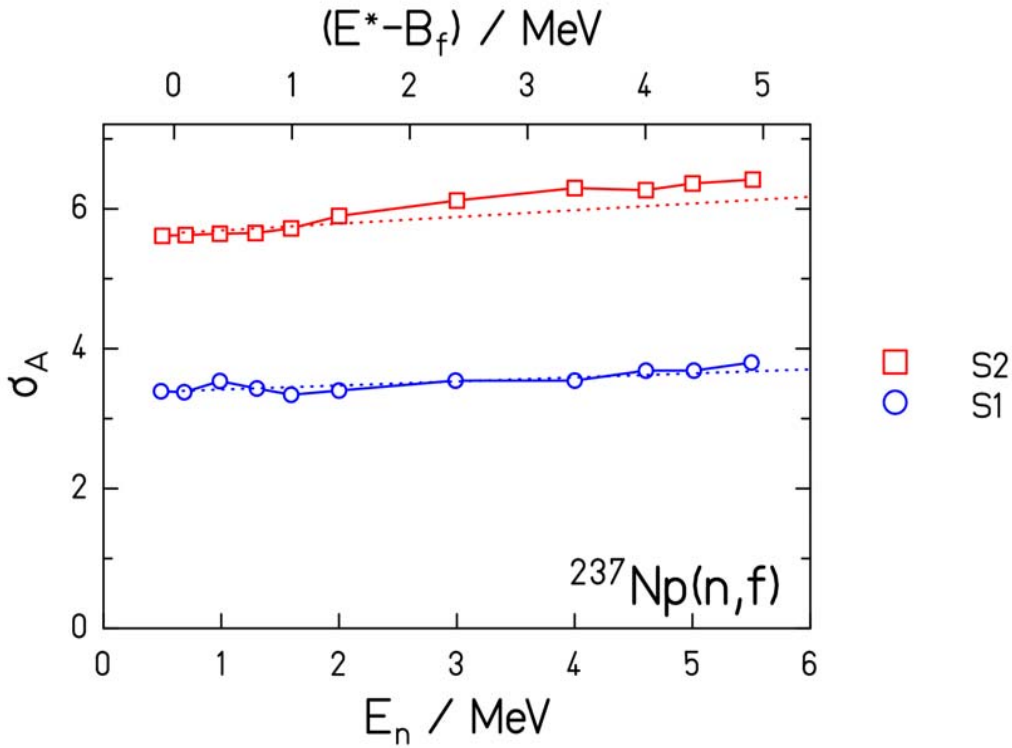
What determines the widths of fission observables?



Fission-fragment mass distribution of $^{235}U(n_{th},f)$.
GEF [JEF/DOC 1243] calculation with contribution of fission channels and data from ENDF B VII.

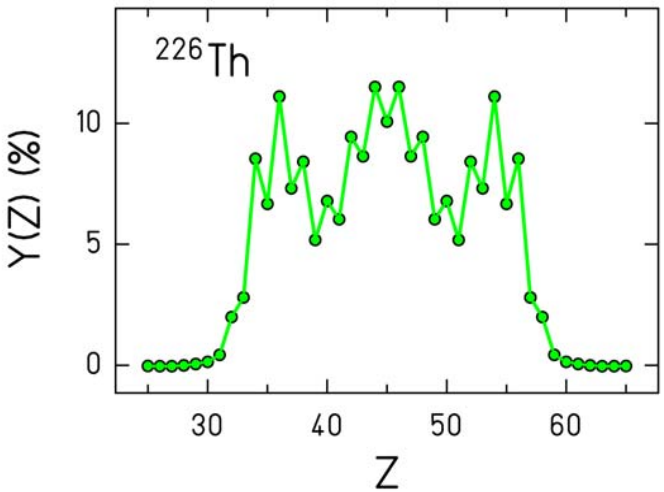
...and their dependence with energy?

Asymmetric modes



F.-J. Hambsch et al., Nucl. Phys. A 679 (2000) 3

Symmetric mode

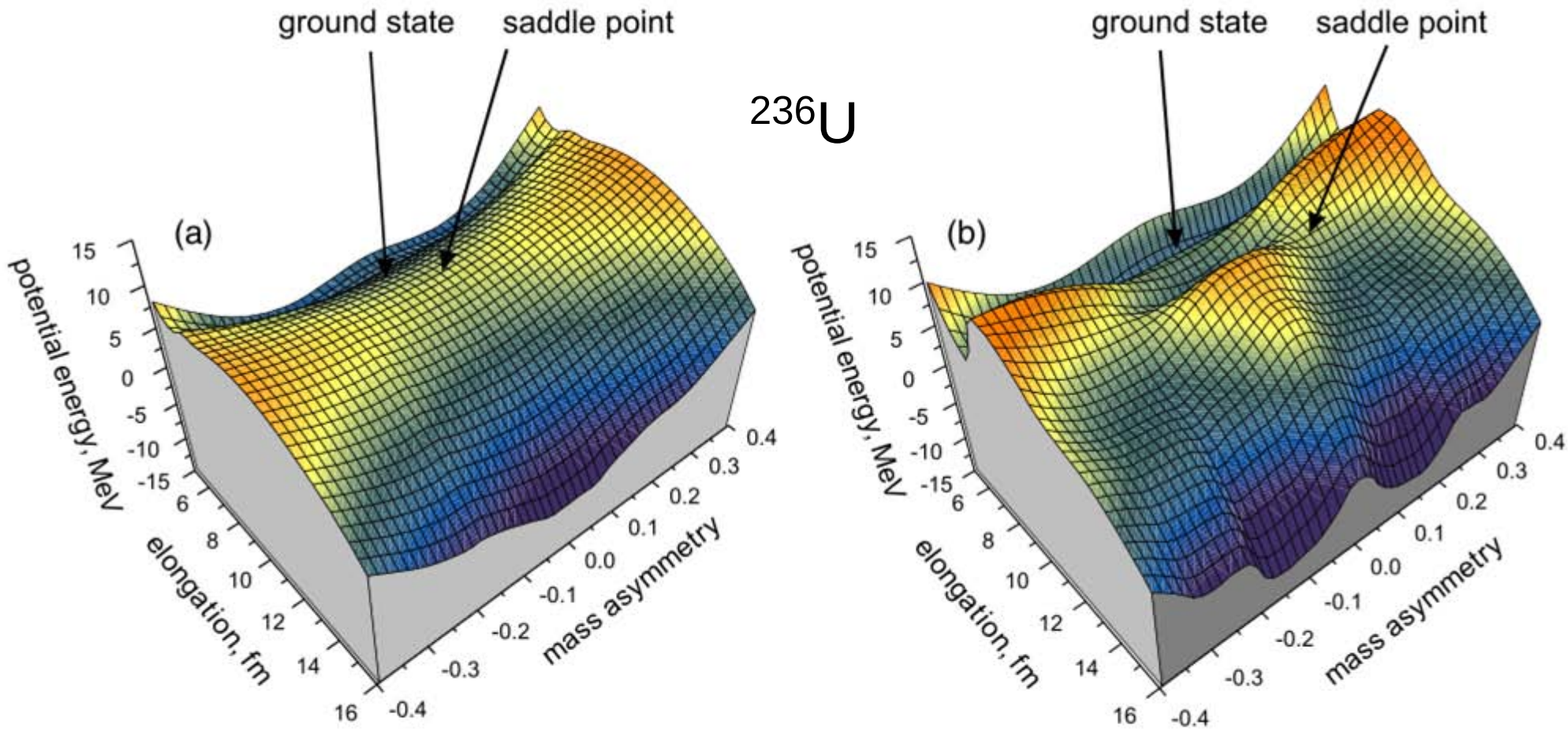


Corresponds to σ_A of 10 units

K.-H. Schmidt et al., Nucl. Phys. A 665 (2000) 221

No data for evolution with En of symmetric mode at low neutron energies

Potential-energy surface



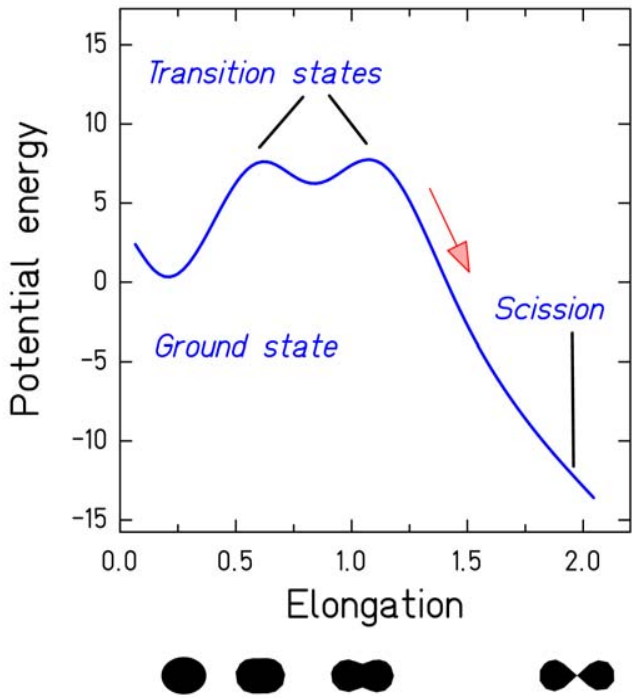
A. Karpov

Without shells
(symmetric mode and high energies)

With shells
(low energies)

Mass distribution results from dynamic evolution driven by the potential.

Different approaches



Time-dependent microscopic calculations based on the constrained HFB approach.
H. Goutte et al., PRC 71 (2005) 024316
+ Dynamical model
+ Fully quantum-mechanical
+ Self-consistent
- Very time consuming (limited degrees of freedom)
- Difficulties to handle dissipation

Stochastic approaches (Langevin-type)
J. Randrup et al., PRC 84 (2011) 034613
+ Dynamical model
- Not fully quantum mechanical
- Very time consuming (limited degrees of freedom)
- Smoluchowski equation assumes full dissipation

Statistical approach at scission
B. D. Wilkins et al, PRC 14 (1976) 1832
+ Simple calculation
- No dynamics
- Not fully quantum mechanical
- Macrocanonical

Critics on the statistical scission-point model from dynamical calculations[1)]

The **statistical scission-point models** are **unable** of explaining the widths of the mass and energy distributions.

During the descent from saddle to scission, the distribution keeps **memory on** the distribution at **former times**.

The width of the distribution of a specific normal mode is approximately given by the fluctuation of the corresponding **quantum oscillator** with an effective stiffness that is equal to the **stiffness** of the potential somewhere **between saddle and scission**.

→ **Dynamics can be considered by assuming an early freeze out of the distribution.**

1) G. D. Adeev, V. V. Pashkevich , Nucl. Phys. A 502 (1989) 405c

Statistical microcanonical model with dynamical and quantum-mechanical features

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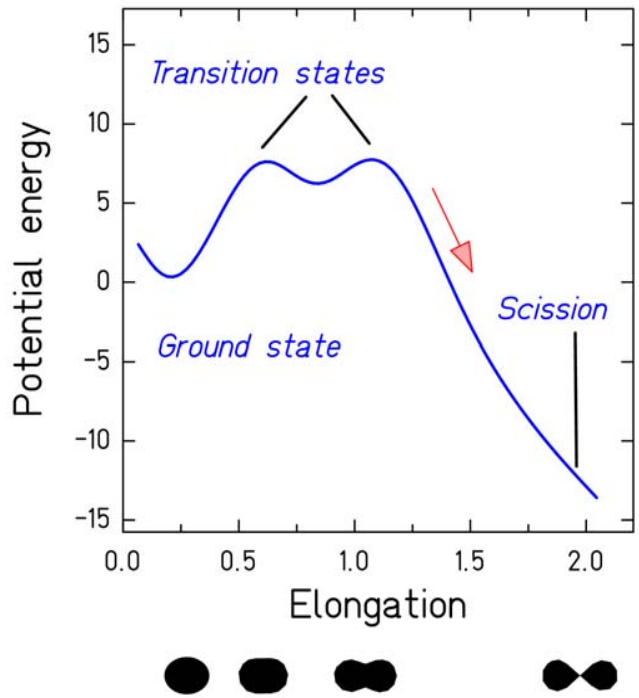
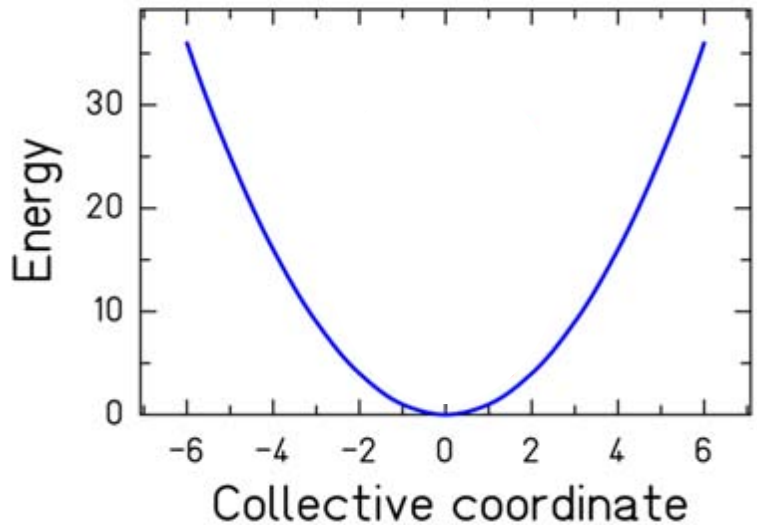
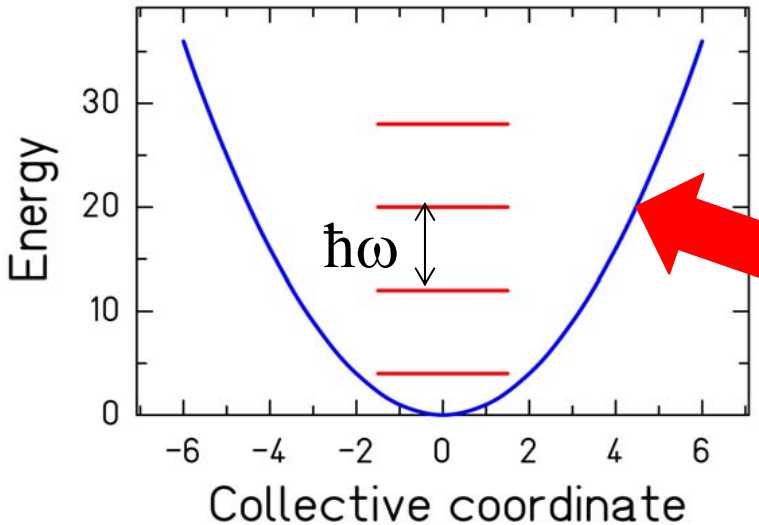




“at freeze out” (at the appropriate position between saddle and scission) we assume a parabolic potential as a function of mass asymmetry where m and stiffness are determined at the “freeze out” point.

Statistical microcanonical model with dynamical and quantum-mechanical features



Heat bath
Etot, T

Stifness $C \propto (\hbar\omega)^2$

Minimum E and width ≠ 0 (zero-point motion) $\sigma_q^2 = \frac{\hbar\omega}{2C}$

Population of the states given by the properties of the heat bath: Etot (not inifinite!) and T (the most probable configurations will be those of maximum entropy)

For nuclei at low E* $\rho \propto \exp(E^*/T)$ (constant-temperature)

If Etot>>T

$$\sigma_q^2 = \frac{\hbar\omega}{2C}\coth\left(\frac{\hbar\omega}{2T}\right)$$

If T<< ħω, zero-point motion $\sigma_q^2 = \frac{\hbar\omega}{2C}$

If T>> ħω, classical limit $\sigma_q^2 = \frac{T}{C}$

Quantitative formulation of the model

Symmetric fission channel

At higher energies:
Measured mass width σ_A[1)] and
temperature *T* of heat-bath from **Fermi-gas level density:**
$\rightarrow$ $C = T/\sigma_A^2$ (***C* = 0.0049 MeV** for ^{238}Np)
In agreement with theoretical value of *C* little beyond saddle [2)].

At lower energy (a few MeV above saddle):

$$\sigma_q^2 = \frac{\hbar\omega}{2C}\coth\left(\frac{\hbar\omega}{2T}\right)$$

ħω = 0.5 MeV
(Nix,1967: ħω = 1.2 MeV at saddle.)

C = 0.0049 MeV

***T* = 0.45** MeV (from systematics[3])

σ_A=10 units (experiment)

ħω ≈ *T*: Width is strongly influenced by the zero-point motion!

1) A.Ya. Rusanov, M.G. ltkis and V.N. Okolovich, Phys. At. Nucl. 60 (1997) 683.
2) E.G. Ryabov, A. V. Karpov, P. N. Nadtochy, G. D. Adeev, PRC 78 (2008) 044614
3) Till von Egidy et al., Phys. Rev. C 72 (2005) 044311

Quantitative formulation of the model

Asymmetric fission channels

Assuming that the mass asymmetry has the same inertia "m" as for the symmetric channel we obtain for $^{237}Np(n_{th},f)$:

$$\sigma_q^2 = \frac{\hbar\omega}{2C}\coth\left(\frac{\hbar\omega}{2T}\right)$$

S2:

$$\left\{\begin{array}{c} C=m(\hbar\omega)^2 \\ T = 0.45\ \text{MeV} \\ \sigma_A = 5.57 \end{array}\right\} \quad \hbar\omega = 3.3\ \text{MeV for S2}$$

S1:

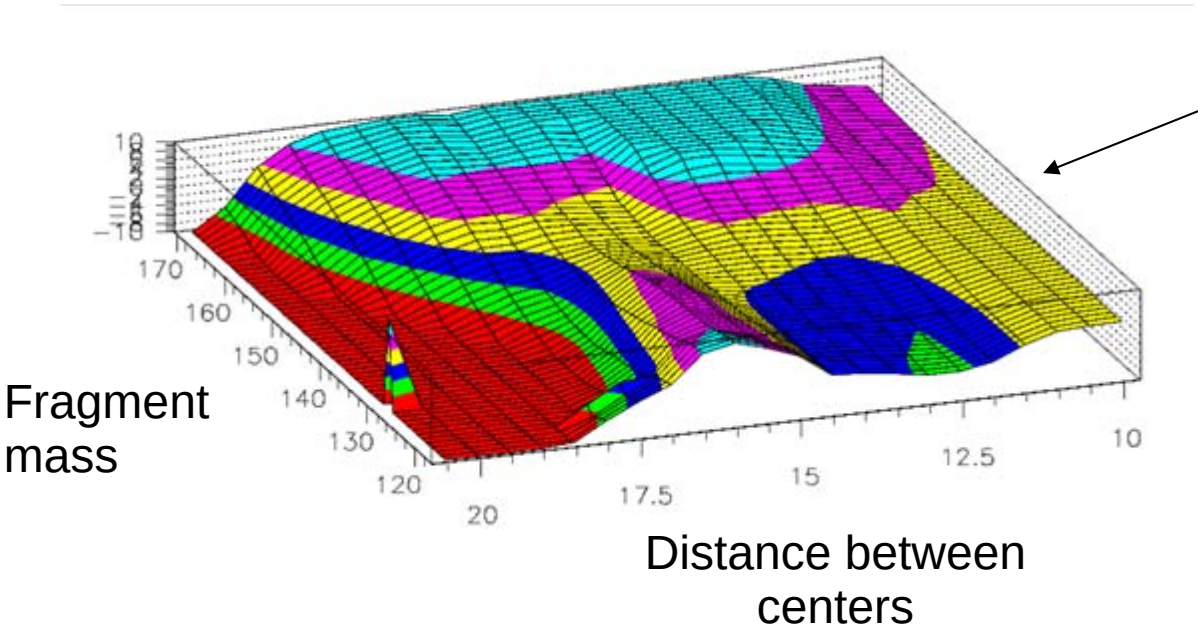
$$\left\{\begin{array}{c} C=m(\hbar\omega)^2 \\ T = 0.45\ \text{MeV} \\ \sigma_A = 3.37 \end{array}\right\} \quad \hbar\omega = 8.9\ \text{MeV for S1}$$

→ $\hbar\omega >> T$ (T = 0.45 MeV)

In thermal equilibrium:

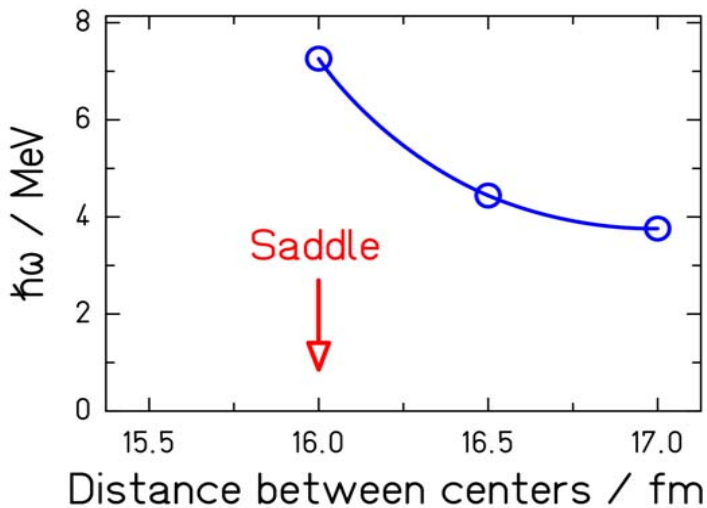
Width in mass asymmetry is totally determined by the zero-point motion!!!!

Quantum oscillator in mass asymmetry for asymmetric fission component (representative for S2)



Potential-energy landscape (M. Mirea)

Deduced ħω.



Like for symmetric fission: The deduced empirical value (ħω = 3.3 MeV) is about 1/2 the theoretical value at saddle.

Influence of shell effects

Constant-temperature level-density formula

$$\rho \sim 1/T \exp(E/T)$$

with

$$T = A^{-2/3} (17.45 - 0.51\, \delta U + 0.051\, \delta U^2)$$ [T.v. Egidy et al.]

and assuming:

$$\delta U\,(q) = \delta U_0 + C/2(q-q_0)^2 \;,\; \delta U_0 = -5 MeV$$

In an oscillator coupled to a heat bath, the restoring force F is given by

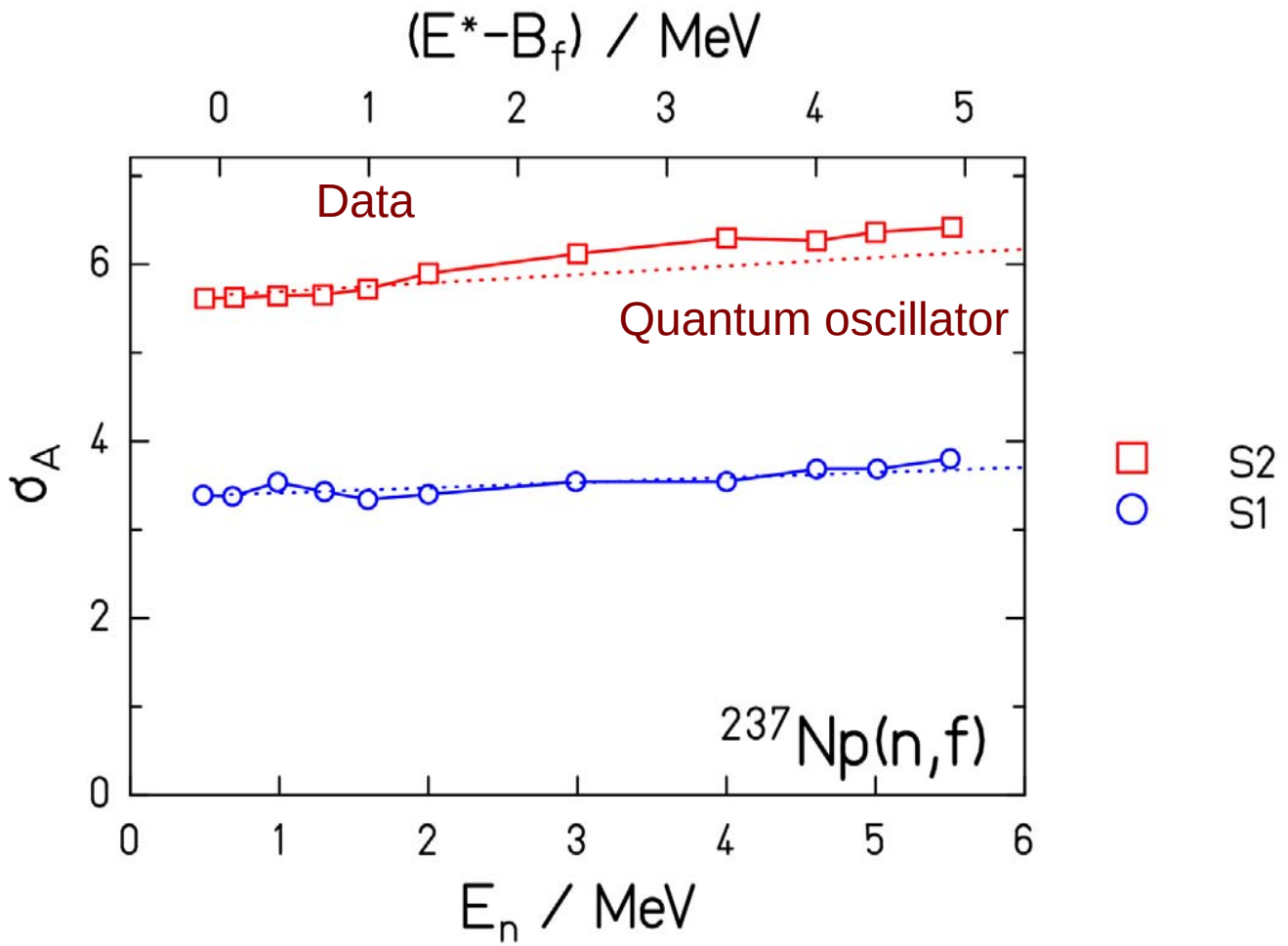
$$F = T\, dS/dq \;\; with \;\; S = \ln(\rho)$$

By integration one obtains the potential $U = \int F\, dq$.
The stiffness C is given by $C = d^2U/dx^2$.

--> We find a reduction of C due to shell effects
Since for the zero-point motion: $\sigma_A^{\,2} = \hbar\omega/(2C)$

The washing out of shell effects leads to an increase of σ_A with increasing E^*!

Overview asymmetric fission channels: energy dependence



Energy dependence of σ_A of the quantum oscillator fits rather well to the experimental data [F.-J. Hambsch et al., Nucl. Phys. A 679 (2000) 3].

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Conclusions

•The deduced properties of the quantum oscillators imply that **the widths of the asymmetric fission channels** (in low-energy fission) **are essentially given by the zero-point motion!**

•The width of the symmetric fission channel is strongly influenced by the zero-point motion

•The weak increase of the widths of asymmetric modes with En is due to the washing out of shell effects and not to the population of higher oscillator states

•**Models should include quantum-mechanical effects to give a realistic estimation of the widths of the mass distributions**