Evaluation of Spin Distributions in Fission Fragments using the Statistical Model

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- interpretation of measured kinetic energy distributions and spin distributions on the basis of the statistical model

- a new spectrometer to measure the prompt decay of fission products
**Statistical Model/Thermodynamic and Nuclear Fission**

2\textsuperscript{nd} law of thermodynamics:  \[ W \propto e^S \]

\[ S = \ln \rho \]

S=entropy, counts the number of nuclear levels in the potential

\[ W \propto \rho(A,Z,E^*) \]

\[ \rho = \frac{dN}{dE^*} \]

number of nuclear levels per energy interval

If Fermi-gas model:  \[ \rho(A,Z,E^*) = e^{2\sqrt{aE^*}} \]

\[ a(A,Z) \]

level density parameter

**Statistical model:** no selection rules, no barriers

\[ \rightarrow \]

All levels have the same weight, independent of quantum numbers
Thermodynamic model: following decay all levels in the residual nucleus (fission fragment) are occupied according to Boltzmann, which results in:

\[ W \propto \rho(a, E^*) \cdot e^{-\frac{E^*}{kT}} \]

\( kT \) depends on the Q-value of the reaction and on the type of interaction

\[ \frac{1}{kT} = \frac{d \ln \rho}{dE^*} \]

- for the nuclear fission process:

-if TXE (or TKE) distribution is known: derivative is taken at the maximum of the distribution
**Statistical ensemble in nuclear physics:**

- micro-canonical ensemble of nuclei (fission fragments) which are non interacting

(all fission products of one kind (e.g. 142Ba) from the same compound system e.g. from 235U(n,f))

- important: statistical decay is a one-step process from a initial to a final state

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**In contrast: antagonistic view of the fission process:**

Fission viewed as sequence of LD-shapes:

Here only the groundstate is followed in a moving barrier landscape, \( \rho = 1 \)
**statistical model approach for the fission process:**

**mechanism:** at the excitation energies for fragments (10 to 20 MeV) pairing can be neglected

**excitation energy:** independent particle model, nucleons are promoted to excited states according to Boltzmann

**spin:** the final fragment angular momentum is generated by the coupling of spin and orbital momenta of the individual nucleons to the fragment spin J

for the evaluation of energy and spin distributions of a statistical ensemble of fragments the nuclear temperature kT must be known. This is the only unknown parameter which enters the calculations
**We know:**

**thermodynamic model:** the calculation of fragment mass distribution leads to wrong results (unless the available phase space is strongly truncated by selection rules)

-in the following: only fragment excitation, fragment kinetic energy, spin distribution and alignment are addressed.
-questions on mass and nuclear charge distribution are not addressed

---it is assumed that we can decouple fragment mass/charge distribution and fragment excitation

\[
W(A_i, Z_i, E_i^*, J_i) = \Theta(A_i, Z_i) \cdot \Phi(E_i^*, J_i)
\]

----and further as a consequence of the statistical model----

\[
\Phi(E_i^*, J_i) = P(E_i^*) \cdot G(J_i)
\]

-separation for excitation energy and spin
The probability to excite a fragment at $E^*$ is

$$P(E^*)dE^* = \frac{1}{N} \rho(E^*) e^{-E^*/T} dE^*$$

$\rho(E^*)$  level density for excited states in the nucleus

$e^{-E^*/T}$  Boltzmann factor which populates the excited levels

$\rho(E^*) = \exp(2\sqrt{aE^*})$  Fermi gas expression
The probability to populate spin $J$ in the fragment is

$$G(J) \propto (2J + 1) \exp(-J(J + 1)/2\sigma^2)$$

$$\sigma^2 = 0.088 \cdot a \cdot kT \cdot A^{2/3}$$

$kT$ determines also the spin distribution for fragment $(A,Z)$ via the spin cutoff parameter

![Graph showing $G(J)$ vs $J$](image)

$$<J> = \sigma - 0.5$$
The dependence of the level density parameter on the excitation energy $\varepsilon$ is given by Ignatyuk et al.

The structure in the level density parameter imposes a structure on the mean excitation energies and mean spin values of the fragments as function of fragment mass.
Temperature of the statistical ensemble

The temperature parameter determines the distribution functions for the excitation and the spin of fragments of the same kind \((A,Z)\), e.g. 146Ba-statistical ensemble.

The knowledge of the temperature allows to know how the total excitation energy \(TXE\) in fission is shared between the light and the heavy fragment.

To determine the temperature we have 3 ways:

1) Find a decay law for nuclear fission, in analogy to gamma decay, beta decay, or neutron decay.

2) We have measured the total excitation energy \(TXE\), e.g. by the determination of the fragment kinetic energies and by application of conservation laws.

\[
\frac{1}{kT} = \frac{dS}{d(TXE)} = \frac{d(\ln \rho)}{d(TXE)}, \text{ for Fermi-gas: } kT = \sqrt{\frac{TXE}{a}}
\]

3) We find an empirical law which connects the temperature to the Q-value of the reaction.
Empirical relationship for the temperature $kT$ of a statistical ensemble of fragments with mass $A$ and nuclear charge $Z$:

$$kT = f \cdot Q$$

**Dependence of constant $f$ on the actinide system:** $f = aZ_c + b$

- $^{219}\text{Ac} [12\text{MeV}], \text{GSI}$
- $^{225}\text{Th} [12\text{MeV}], \text{GSI}$
- $^{234}\text{U} [12\text{MeV}], \text{GSI}; [6\text{MeV}], \text{LOHENGRIN}$
- $^{246}\text{Cm} [6\text{MeV}], \text{LOHENGRIN}$
If the temperature $kT$ of the system is known, all observables concerning the energy and spin distribution are determined

-for the excitation energy distributions of the fragments we have

$$P(E_1^*)dE_1^* = e^{-E_1^*}/kT \cdot \rho(E_1^*)dE_1^*$$

$$P(E_2^*)dE_2^* = e^{-E_2^*}/kT \cdot \rho(E_2^*)dE_2^*$$

with $\rho(E_{1,2})$ the level density parameter for fragment 1,2

-notice: independent excitation for fragment 1 and 2, not coupled to deformation

-for the spin distribution we have

$$G(J_1) = \exp(-J_1^2/2\sigma^2) - \exp(-J_1(J_1+1)/2\sigma^2)$$

$$G(J_2) = \exp(-J_2^2/2\sigma^2) - \exp(-J_2(J_2+1)/2\sigma^2)$$

and

$$\sigma^2 = 0.0888 \cdot a_{1,2} \cdot kT \cdot A_{1,2}^{2/3}$$
Start from excitation of fragments, not from Coulomb repulsion:

**Calculation of mean values for single fragment kinetic energies:**

\[ kT = fQ \]

\[
< E_1^* > = a_1 \cdot (fQ)^2 \\
< E_2^* > = a_2 \cdot (fQ)^2 \\
< TXE > = < E_1^* > + < E_2^* > \\
< TKE > = Q - < TXE >
\]

Nuclear fission is a binary reaction: from momentum and energy conservation law we get the distribution of the single fragment kinetic energies:

\[
< E_{kin1} > = \frac{< TKE >}{1 + \frac{A_1}{A_2}} \quad \text{for fragment 1}
\]

\[
< E_{kin2} > = \frac{< TKE >}{1 + \frac{A_2}{A_1}} \quad \text{for fragment 2}
\]
Calculated and experimental mean kinetic energies for 233U(n,f)

Calculations (open points) are done with the fermi gas approach for the nuclear level density. Experimental points are from LOHENGRIN experiments.

\[ f = 0.0045 \]
From the agreement of the calculated mean kinetic energies with the measured ones:

- we know the constant to calculate the temperature of the fragments for different systems

- we can address to the spin distributions for the fragments
**Mean fragment spin (1st moment)**

\[ \Phi_J = \frac{2J + 1}{2\sigma^2} \exp\left(-\frac{J(J+1)}{2\sigma^2}\right) \]

\[ \sigma^2 = 0.0888 \cdot a_{1,2} \cdot kT \cdot A_{1,2}^{2/3} \]
Knowing the excitation energy distribution and the spin distribution function we can construct the entry states:

Entry states for $A=94$ from $^{233}\text{U}(n,f)$

- In order to find experimentally the spin distribution of fragments we can:
  - Measure directly the population of the Yrast band
  - Deduce the mean spin values from the population of isomeric states

- A model is needed to extract spin values from the experiment. The model has to say how the entry states decay by gamma and neutron emission.
Decay pattern in fragment de-excitation

in general: statistical decay by neutrons and dipole gamma rays
assumed:
- neutrons do not take away angular momentum
- gamma rays take away 0 or +-1 units of angular momentum
- no transitions along rotational bands (beside gs-band)
**Madland/England-model for extraction of $\langle J \rangle$ values from population of isomeric states**

- separation line at $\frac{J_1 + J_2}{2}$
- not transitions along rotational bands included
235U(n,f)
kT=1.2MeV

<\textbf{J}> from isomer ratios

Huizenga/Vandenbosch approach
Compilation of data by Naik et al.

Calculated values for kT=1.2MeV (to guide the eye)

\textbf{note:} absolute values for <\textbf{J}> may be erroneous due to model dependence, but general trend should be OK

\textbf{note:} kT is different for different masses, but: complementary fragments have the same temperature

\textbf{result:} <\textbf{J}>\text{heavy}=<\textbf{J}>\text{light} +2

---the minimum for 132Sn seems to be confirmed (small level density parameter)---
Mean angular momentum in fragments as function of the fragment kinetic energy

Methodology

Calculation of $kT$ from TXE

$$kT = \sqrt{\frac{TXE}{a_1 + a_2}}$$

TXE from single fragment kinetic energy

$$TXE = Q - TKE$$

$$TKE = E_{kin1} + E_{kin2}$$

momentum + energy – conservation

$$kT = \sqrt{\frac{Q - E_{kin1}[1 + \frac{A_1}{A_2}]}{a_1 + a_2}}$$

Note: a discrete value of TXE leads to a temperature distribution of the fragments

No free parameters, $kT$ fixed by measurement of $E_{kin1}$
**Dependence of mean fragment spin on the fragment kinetic energy (on temperature)**

In general: mean fragment spin is increasing when the kinetic energy is decreasing (temperature goes up).
-distribution functions for fragment spin:

**Yrast-band model (deformed even-even fragments)**

- direct feeding of gs-band members $P(\mathcal{J})$ (simplified Huizenga/Vandenbosch)
- de-excitation along yrast band

$$P(\mathcal{J}) = \frac{2\mathcal{J} + 1}{2\sigma^2} \exp\left(-\frac{\mathcal{J}(\mathcal{J} + 1)}{2\sigma^2}\right)$$

![Diagram of energy versus spin](image)

assumption: higher bands do not play a big role due to $E_\gamma^3$ decay rule (beside from the feeding of the 0+ ground state)

-may be refined by taking into account feeding from odd spin members
simplified Huizenga/Vandenbosch: only one statistical gamma ray (in the mean)
Population of ground-state band members in 248Cm(sf)

Feeding of gs-band members:

\[ P(J) = I_\gamma^J (out) - I_\gamma^J (in) \]

Probable reason for deviation at high J: nearby additional bands take away intensity

\[ kT = fQ \quad , \quad f = 0.0058 \quad (248Cm(sf)) \]

data from Urban et al. Phys Rev C
Conclusions

-we have applied the well known statistical model to the fission process

-separation of the charge/masse distribution from energy and spin distribution

-Fermi gas description for level density and
-Bethe formulation for spin density (shell model state sequence)

-expression for the temperature from empirical law

-kinetic energy distributions are constructed from momentum and energy conservation

-this appears to lead to a full description of energetics in nuclear fission at low energies ((sf) and thermal neutron capture)
The gas filled magnet for the investigation of prompt gamma decay characteristics in nuclear fission

Collaboration: ILL, LPSC Grenoble, CEA Cadarache, CEA Saclay

aim:
- to provide a filter in mass and charge range for the recording of prompt gamma decay from fission fragments (selectivity)
- to provide focusing characteristics for ionic charge, velocity and solid angle (efficiency)

problems:
- due to collisions with the gas the ionic charge \(<q>\) of the incoming ions is strongly fluctuating
- the \(B\rho\) values in the magnetic field change along the path of the ion
- the velocity of the ions change along the path (electronic and nuclear stopping, \(dE/dx\))
the following difficulties arise:

A) concerning the mean ionic charge of the ions:
- shell effects at magic numbers for the ionic charge
- pressure dependence of the mean charge values

B) concerning the stopping of ions at energies of 1MeV/amu:
- effective charge of the ions
- validity of the Bethe Bloch formalism

\[ B \rho = \text{const} \cdot A \frac{<v>}{<q>} \]

\[ \frac{dE}{dx} = f\left(\frac{<q>^2}{<v>^2}, Z_{\text{target}}, \ldots\right) \]

\(<v>\) and \(<q>\) to be determined
Experimental set up:

Dipol-magnet RED filled with gas:
- Deflection radius: \( \rho = 60 \text{ cm} \)
- Deflection angle: 65 degrees
- Trajectory lengths: 68 cm
- Gas filling: He, N\(_2\), Ar, Kr, Xe
- Energy loss of ions: up to 70% in gas filled section
Spectrometer lay-out

Gamma-ray detection (Ge clovers, Ge planars)

Gas-filled magnet properties:
- Ionic charge focusing
- Determination of A, (Z)

Fragment detection (A,Z,E_{kin})

Rate $\sim 10^5$ fission/s
Possible targets: $^{229}$Th, $^{233,235}$U, $^{239,241}$Pu, $^{242}$Am, $^{243,245,247}$Cm, $^{249,251}$Cf

Intense cold neutron beam ($10^9$ n/cm$^2$.s at the exit of a bent neutron guide)

Gas-filled magnet properties:
- Ionic charge focusing
- Determination of A, (Z)

With the help of conservation laws:
**full picture of fission event**

left fragment: stopped in backing
Doppler free gamma detection, determination of A,Z,E$^*$,J,yield

right fragment:
TOF: velocity magnet: mass