# The Inverse Problem: would it be possible?

P. Romain, B. Morillon, H. Duarte

*14th october 2014*

# Surrogate Reactions ? ? ?

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Can we bet on surrogate reactions?

1) In a first time we will show that surrogate seems unable to give back an "equivalent neutron induced cross section"

2) But in a second time we will present the interest of surrogate method in fission studies to reconstruct a one-dimensional fission barrier

**Surrogate will survive**

For energies $E < E^{seuil}_{(n,2n)}$ when considering **compound emission processes**, we get :

$$\sigma_{CN} = \sigma_R - \sigma_{DI} \quad = \sigma_{CE} + \sigma_{n,n'} + \sigma_{n,\gamma} + \sigma_{n,f}$$

By defining compound neutron emissions as :

$$\sigma_{n,n} \quad = \sigma_{CE} + \sigma_{n,n'}$$

we get :

$$\begin{aligned} \sigma_{CN} &= \sigma_{n,n} + \sigma_{n,\gamma} + \sigma_{n,f} \\ \iff \quad 1 &= \frac{\sigma_{n,n}}{\sigma_{CN}} + \frac{\sigma_{n,\gamma}}{\sigma_{CN}} + \frac{\sigma_{n,f}}{\sigma_{CN}} \end{aligned}$$

# compound emission probabilities

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Then defining GLOBAL compound emission probability for each process occuring in this energy range :

$$1 \quad = P_{n,n} + P_{n,\gamma} + P_{n,f}$$

Now, for a surrogate reaction defined by an entrance channel $EC$, we have :

$$1 \quad = P_{EC,n} + P_{EC,\gamma} + P_{EC,f}$$

# Use of Shannon theorem towards surrogate reactions

From another point of view, we can also define the lack of information on the compound emission processes for an emitting system (CN). When using the same notations as previously defined, according to the Shannon theorem [1], the lack of information (also called entropy) on the compound emission processes for an emitting CN is defined as :

$$\begin{aligned} H_{EC}(CN,E) = \quad & -P_{EC,\gamma}(E)\log_2 P_{EC,\gamma}(E) \\ & -P_{EC,n}(E)\log_2 P_{EC,n}(E) \\ & -P_{EC,f}(E)\log_2 P_{EC,f}(E). \end{aligned}$$

Instead of plotting each probabibality of all exit channels for each studied entrance channel versus energy, this representation is more compact, and so, more clearly to read.
**In addition, using these functions, Bohr independence hypothesis implies that for two different entrance channels $EC_1$ and $EC_2$ leading to the same CN at the same excitation energies in the same spin-parity states, the lack of information on these compound nuclei emission processes or the uncertainties on their emission processes should be identical :**

$$H_{EC_1}(CN,E) = H_{EC_2}(CN,E).$$


[1] C.E Shannon, Bell System Technical Journal, **27** , 379 and 623, (1948).

# Shannon theorem - non fissile nuclei : 2 exit channels

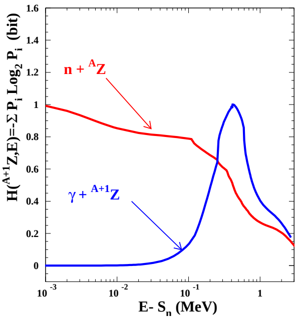



Figure : *Lack of information on the compound emission processes,* $H_\gamma(^{A+1}Z, E)$ *and* $H_n(^{A+1}Z, E)$ *related to a given entrance channel* $EC = \gamma$ *or* $EC = n$*, plotted versus* $E - S_n$.

$$\begin{aligned} H_{EC}(CN, E) = & -P_{EC,\gamma}(E)\log_2 P_{EC,\gamma}(E) \\ & -P_{EC,n}(E)\log_2 P_{EC,n}(E) \\ = & -P_{EC,\gamma}(E)\log_2 P_{EC,\gamma}(E) \\ & -(1 - P_{EC,\gamma}(E))\times \\ & \quad \log_2(1 - P_{EC,\gamma}(E)). \end{aligned}$$

**When, (for the $EC_1 = \gamma$), $\gamma$ emission is the most important emission process then :**

$$P_{EC_1,\gamma}(E) \approx 1, \quad P_{EC_1,n}(E) = 1 - P_{EC_1,\gamma}(E) \approx 0$$

**and**

$$H_{EC_1}(CN, E) = 0$$

**For the $EC_1 = \gamma$ and $EC_2 = n$ entrance channels, the lack of information on the compound emission processes is maximum when :**

$$P_{EC,\gamma}(E) = P_{EC,n}(E) = \frac{1}{2}$$

[since $h = -p\log_2 p - (1 - p)\log_2(1 - p)$ is maximum for $p = \frac{1}{2}$].

# Shannon theorem - non fissile nuclei : 2 exit channels

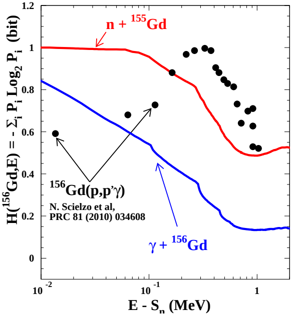



Figure : *Lack of information on the compound emission processes, $H_\gamma(^{156}Gd, E)$ and $H_n(^{156}Gd, E)$ and, $H_{(p,p'\gamma)}(^{156}Gd, E)$ related to a given entrance channel, plotted versus $E - S_n$ ($S_n = 8.536$ MeV).*

$$H_{EC}(CN, E) = -P_{EC,\gamma}(E)\log_2 P_{EC,\gamma}(E) - P_{EC,n}(E)\log_2 P_{EC,n}(E)$$

becomes here :

$$H_{EC}(CN, E) = -P_{EC,\gamma}(E)\log_2 P_{EC,\gamma}(E) - (1 - P_{EC,\gamma}(E))\log_2(1 - P_{EC,\gamma}(E))$$

**Shannon Theorem translation of Bohr independence hypothesis :**

$$H_{EC_1}(CN, E) = H_{EC_2}(CN, E).$$

BUT HERE :

$$H_n(^{156}Gd, E) \neq H_\gamma(^{156}Gd, E)$$

**AND HERE ALSO FOR $SR = (p, p'\gamma)$ :**

$$H_n(^{156}Gd, E) \neq H_{(p,p'\gamma)}(^{156}Gd, E)$$

$$\Updownarrow$$

$$J_n, \pi_n \neq J_{SR}, \pi_{SR}$$

*or at least*

Bohr independence hypothesis failed for $E \sim S_n$ with incident n since $W \neq 1$

# Interest of Shannon information for the 2 exit channels case

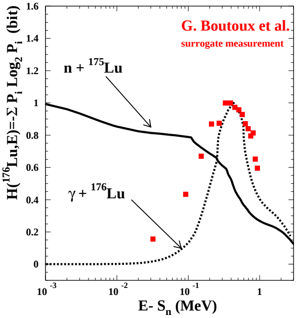



**Figure :** *Lack of information on the compound emission processes,* $H_\gamma(^{176}Lu, E)$ *and* $H_n(^{176}Lu, E)$ *and,* $H_{SR}(^{176}Lu, E)$ *related to a given entrance channel, plotted versus* $E - S_n$.

$$H_{EC}(CN, E) = \begin{array}{l} -P_{EC,\gamma}(E)\log_2 P_{EC,\gamma}(E) \\ -P_{EC,n}(E)\log_2 P_{EC,n}(E) \end{array}$$

becomes here :

$$H_{EC}(CN, E) = \begin{array}{l} -P_{EC,\gamma}(E)\log_2 P_{EC,\gamma}(E) \\ -(1 - P_{EC,\gamma}(E))\log_2(1 - P_{EC,\gamma}(E) \end{array}$$

**Shannon Theorem translation of Bohr independence hypothesis :**

$$H_{EC_1}(CN, E) = H_{EC_2}(CN, E).$$

BUT HERE :

$$H_n(^{176}LU, E) \quad \neq \quad H_\gamma(^{176}Lu, E)$$

**AND HERE ALSO FOR** $SR = {}^{174}\mathbf{Yb}(^{3}\mathbf{He}, p)^{176}\mathbf{Lu}$ **:**

$$\begin{array}{ccc} H_n(^{176}Lu, E) & \neq & H_{SR}(^{176}Lu, E) \\ & \Updownarrow & \\ J_n, \pi_n & \neq & J_{SR}, \pi_{SR} \\ & \textit{or at least} & \end{array}$$

Bohr independence hypothesis failed
for $E \sim S_n$ with incident n since $W \neq 1$

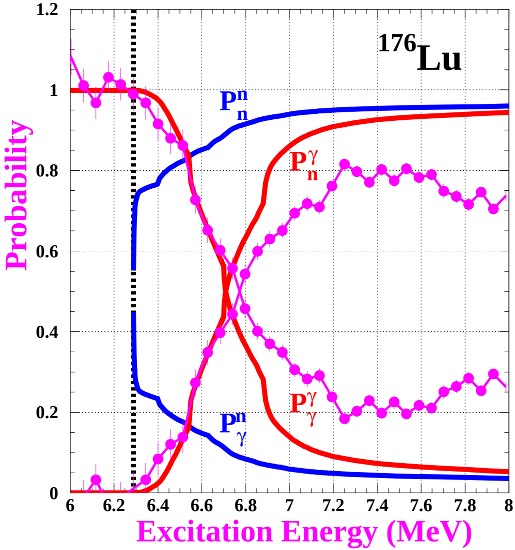
# E.D.P.S : the only 2 exit channels case

$$P_{EC,\gamma} + P_{EC,n} = 1$$

$$\Updownarrow$$

$$P_{EC,n} = 1 - P_{EC,\gamma}$$

# E.D.P.S : the only 2 exit channels case



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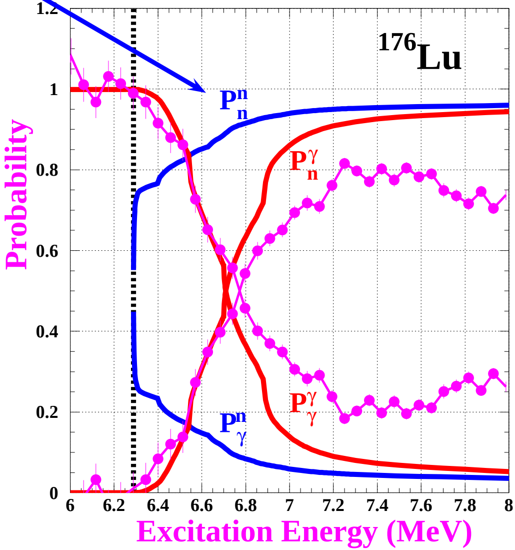
calc. $^{175}$Lu$(n,n)^{175}$Lu



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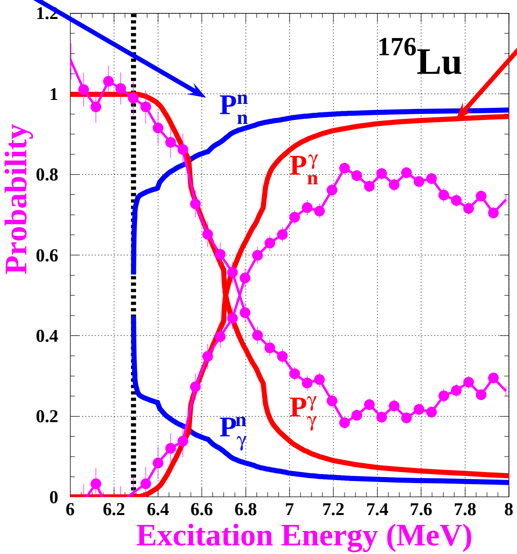

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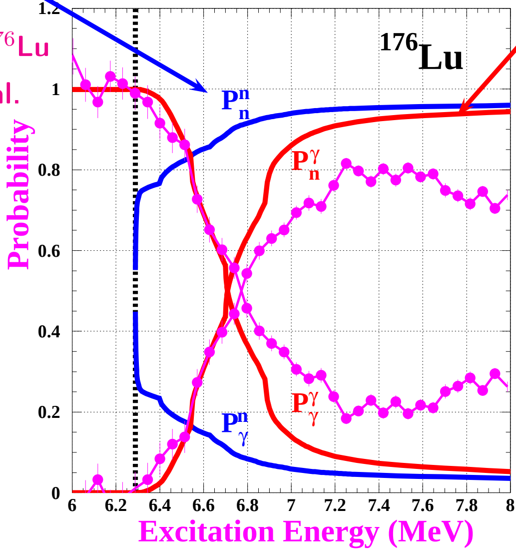
G. Boutoux et al.

CENBG



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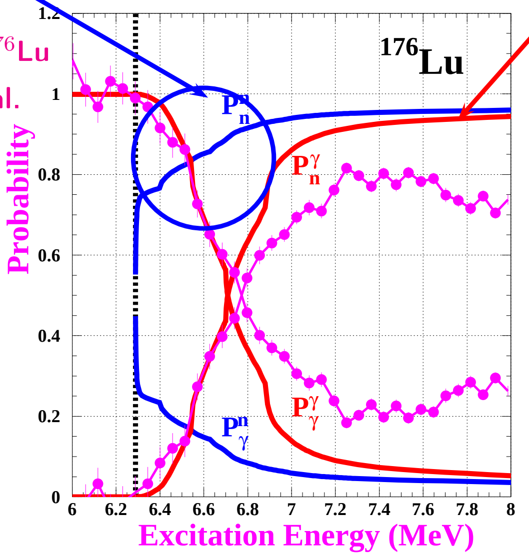


structures = opening channels

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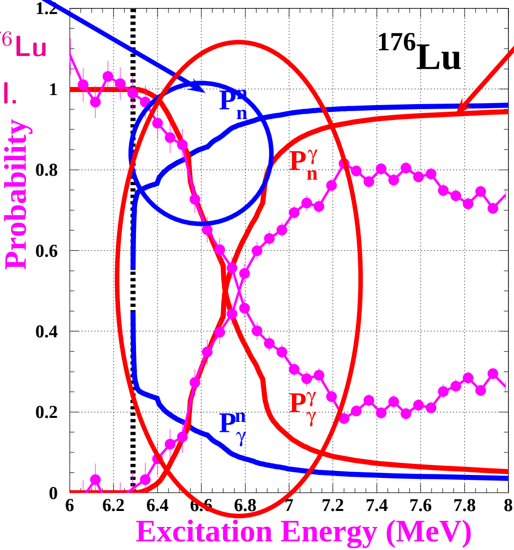


structures = opening channels

id. for $P_n^\gamma$ et $P_n^{SR}$

# E.D.P.S : the only 2 exit channels case

$$P_{EC,\gamma} + P_{EC,n} = 1$$

$$\Updownarrow$$

$$P_{EC,n} = 1 - P_{EC,\gamma}$$

But neutron emissions on discret states = threshold reactions

$\Longrightarrow$ interest of energy derivative study :

$$\frac{dP_{EC,n}}{dE} = -\frac{dP_{EC,\gamma}}{dE}$$

# Inelastic neutron emission probability

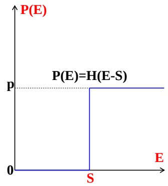


Figure : *Inelastic neutron emission probability for populating an $E = S$ energy state (Heaviside function)= threshold reaction.*

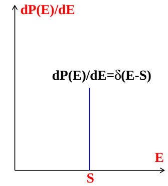


Figure : *Energy derivative of the inelastic neutron emission probability for populating an $E = S$ energy state (Dirac distribution =* energy distribution of populated states*).*

# Energy distribution of populated states

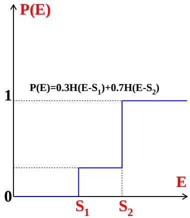


Figure : *Inelastic neutron emission probability for populating two energy states $E = S_1$ and $E = S_2$ (Heaviside functions)= threshold reactions.*

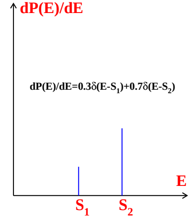


Figure : *Energy derivative of the Inelastic neutron emission probability for populating two energy states $E = S_1$ and $E = S_2$ (Dirac distributions =* **energy distribution of populated states***).*

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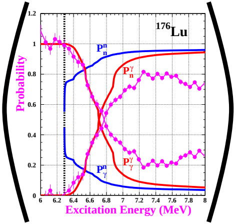
$\frac{d}{dE}$ 

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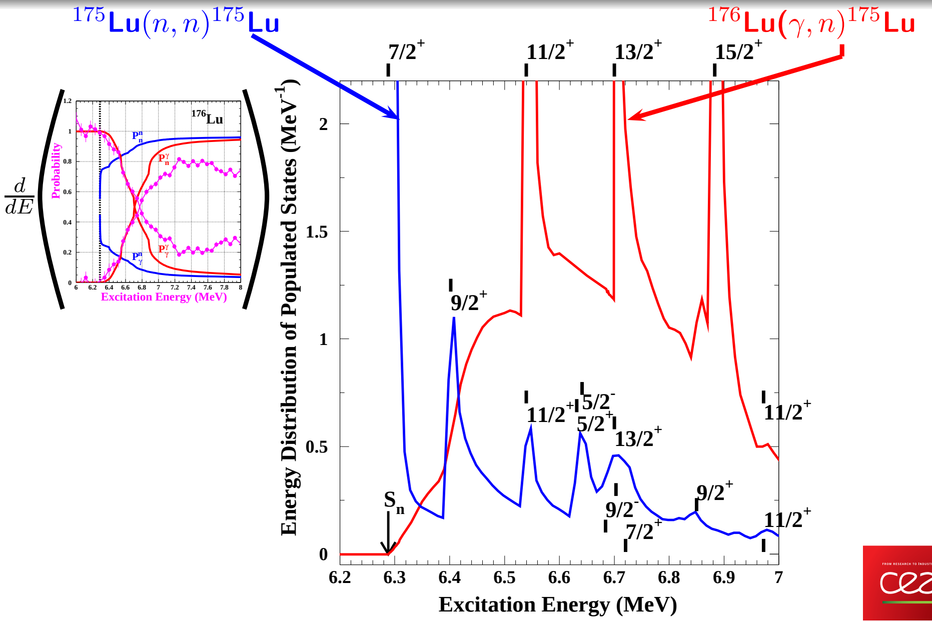
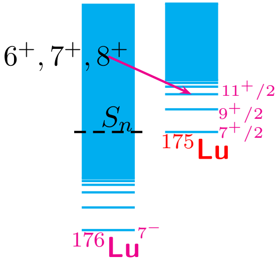


# Spin/Parity states reached

| absorption : | composition : | states : | states pop. after n-emission : |
|---|---|---|---|
| $\gamma +^{176} Lu$ : | $1(-) + 7^-$ | $6^+, 7^+, 8^+$ | $\frac{11}{2}^+, \frac{13}{2}^+$<br>$\frac{15}{2}^+, \frac{17}{2}^+$ |



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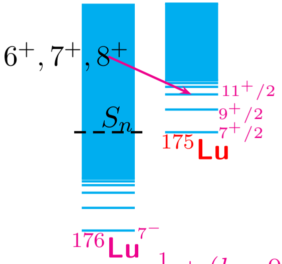
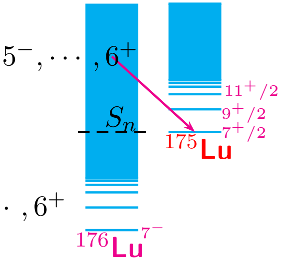
| $n + ^{175}Lu$ : | $\frac{1}{2} + (l=0) + \frac{7}{2}^+$ <br> $\frac{1}{2} + (l=1) + \frac{7}{2}^+$ <br> $\frac{1}{2} + (l=2) + \frac{7}{2}^+$ | $(l=0,2) : 1^+, \cdots, 6^+$ <br> $(l=1) : 2^-, \cdots, 5^-$ | $\frac{1}{2}^+, \cdots, \frac{13}{2}^+$ <br> $\frac{1}{2}^-, \cdots, \frac{13}{2}^-$ |



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$^{175}\mathbf{Lu}(n,n)^{175}\mathbf{Lu}$

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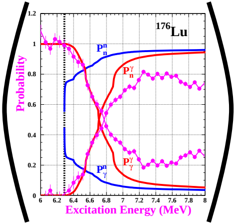
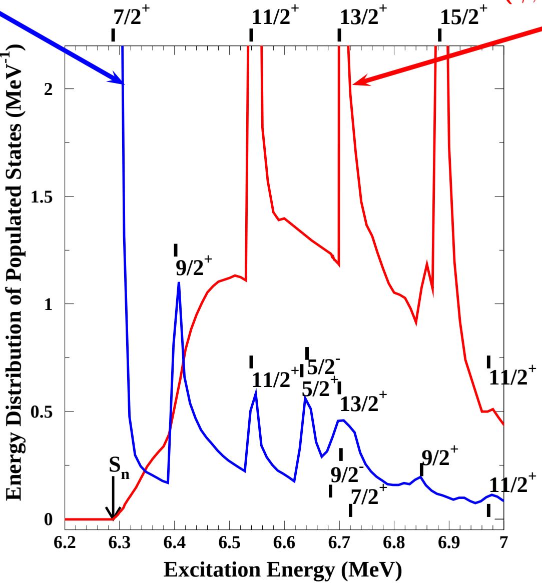
$\frac{d}{dE}$



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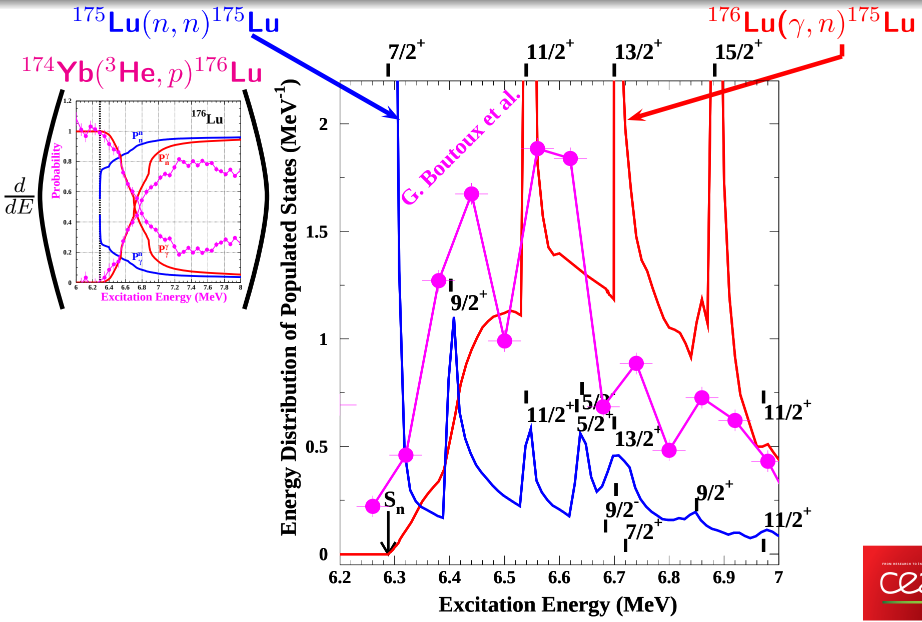

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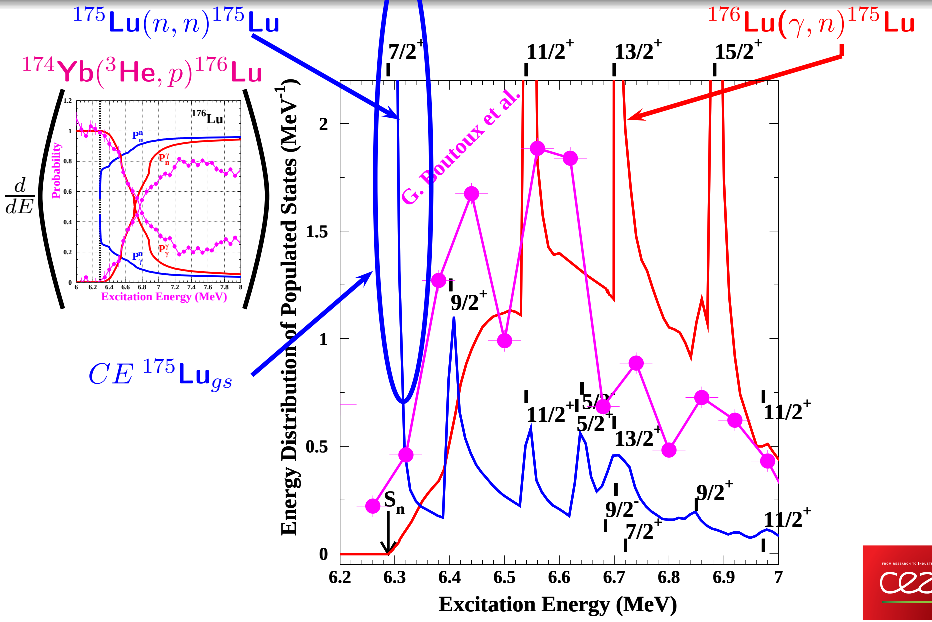

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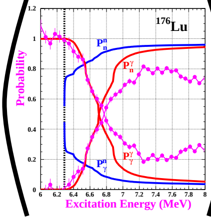
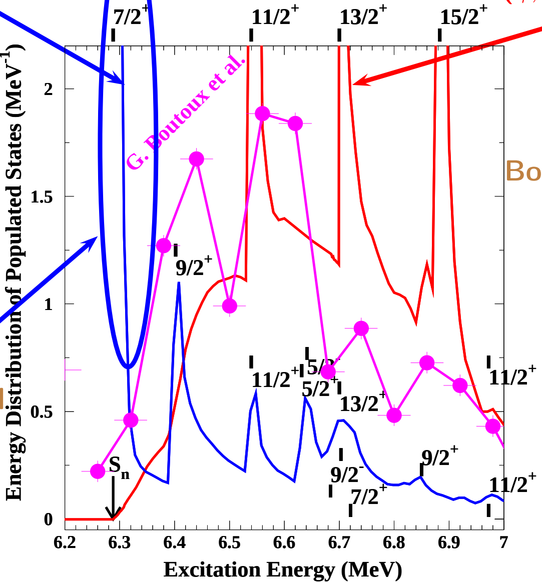
$\frac{d}{dE}$




$CE\ ^{175}$**Lu**$_{gs}$

$\frac{dP_n}{dE} \to J\pi$ **populated**




Bohr-ography

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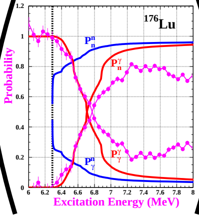
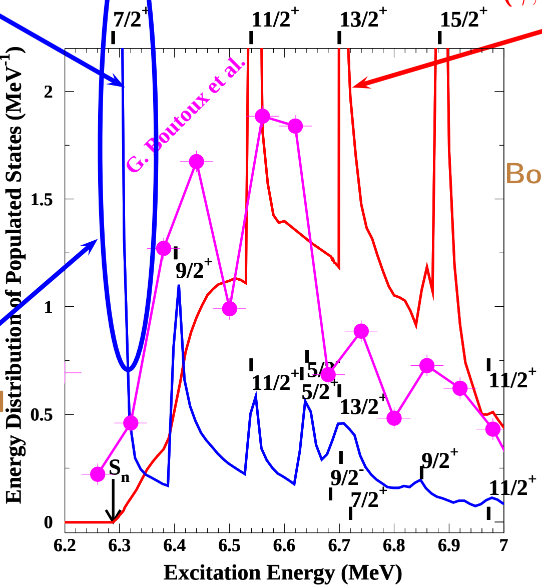



$CE\ ^{175}\mathbf{Lu}_{gs}$

$\frac{dP_n}{dE} \to J\pi$ **populated**

**surrogate**

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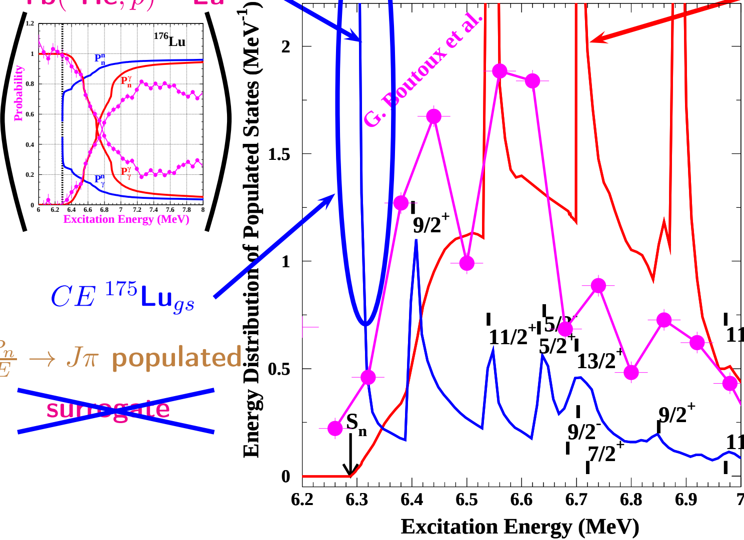


Bohr-ography

$CE\ ^{175}$Lu$_{gs}$

$\frac{dP_n}{dE} \rightarrow J\pi$ populated

~~surrogate~~

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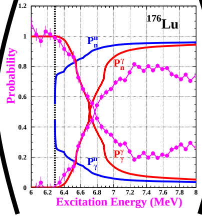
$\frac{d}{dE}$ (  )



$CE\ ^{175}$**Lu**$_{gs}$

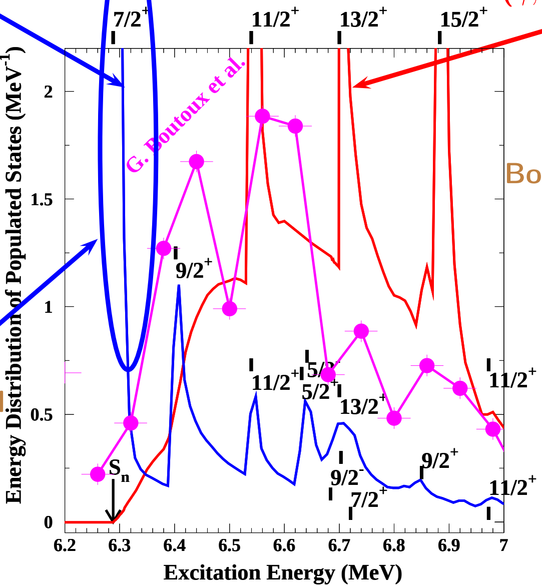
$\frac{dP_n}{dE} \rightarrow J\pi$ **populated**

~~surrogate~~

forget it

even for fission !

$^{176}$**Lu**$(\gamma,n)^{175}$**Lu**





Bohr-ography

# Fission barriers distributions

2376 GAVRON, BRITT, KONECNY, WEBER, AND WILHELMY 13

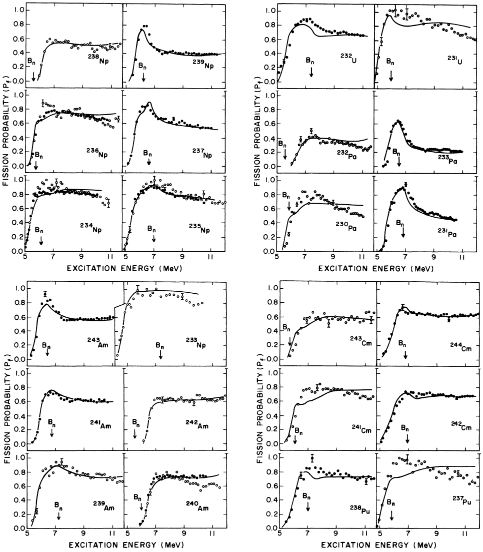



FIG. 2. Measured fission probabilities: open circles, from ($^3$He,$tf$) reactions; closed circles, from ($^3$He,$df$) reactions; full lines, model fits as discussed in the text.

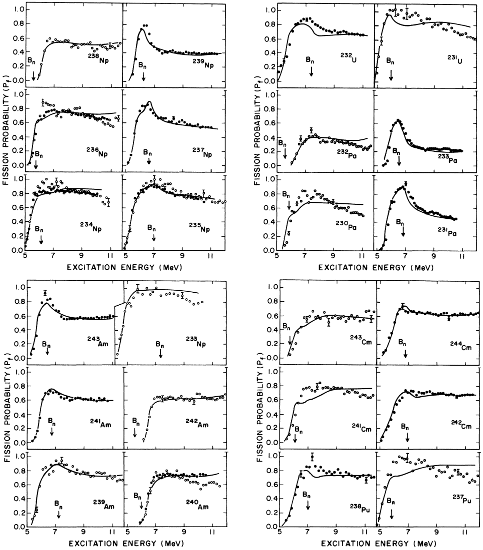
**fission Probabilities from**

**Gavron et al. : Phys. Rev. C 13 p. 2374 (1976)**

# Fission barriers distributions

$P_f \rightarrow 1$ ! ! !

2376 GAVRON, BRITT, KONECNY, WEBER, AND WILHELMY 13

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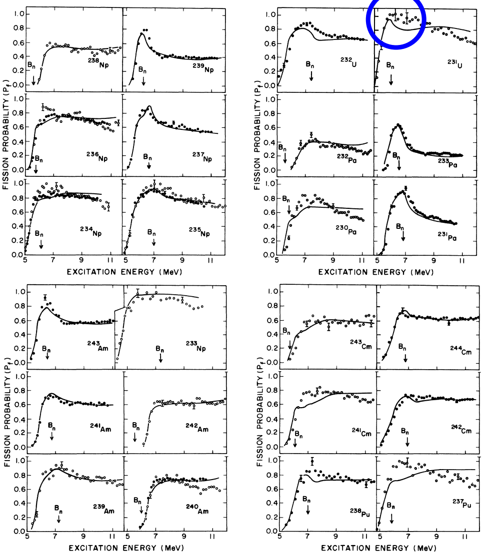
$^{231}$U

2376 GAVRON, BRITT, KONECNY, WEBER, AND WILHELMY 13

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$^{233,234,235}$Np

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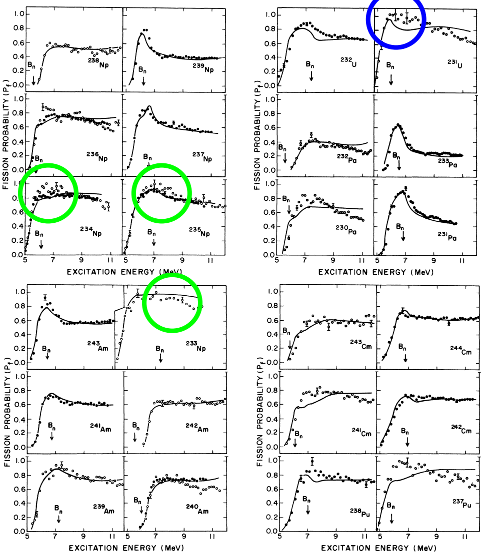


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$^{231}$**U**

$^{233,234,235}$**Np**

$^{237}$**Pu**



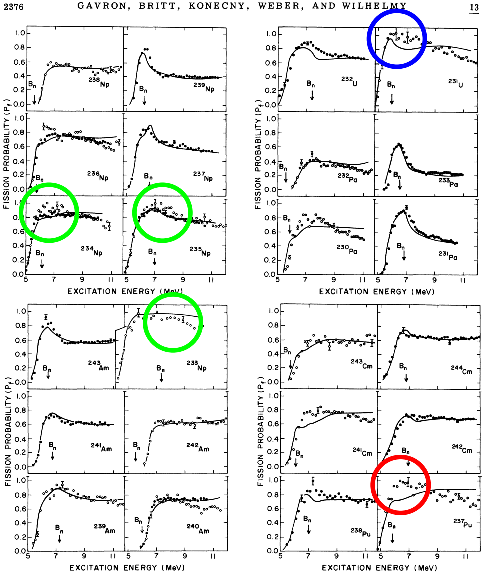


FIG. 2. Measured fission probabilities: open circles, from ($^3$He,$tf$) reactions; closed circles, from ($^3$He,$df$) reactions; full lines, model fits as discussed in the text.

fission Probabilities from

Gavron et al. : Phys. Rev. C 13 p. 2374 (1976)

# Fission barriers distributions

$P_f \to 1$ ! ! !

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**no neutron emissions ! ! !**

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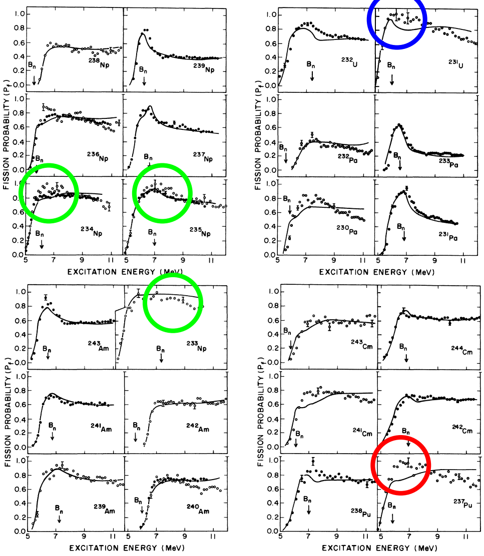
2376 GAVRON, BRITT, KONECNY, WEBER, AND WILHELMY 13

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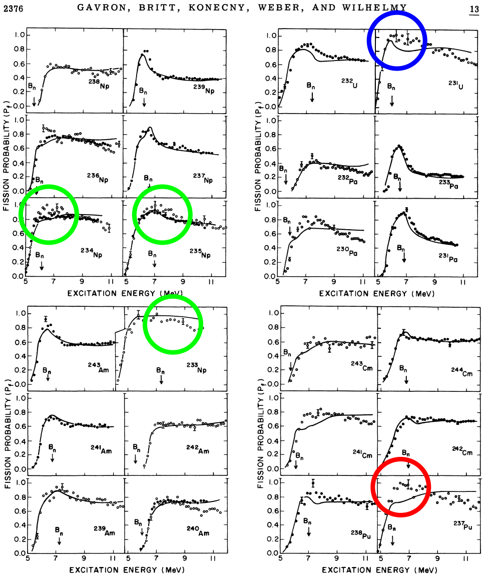


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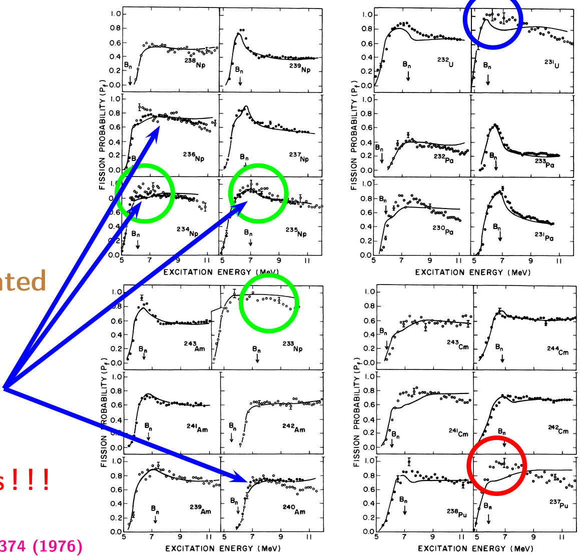


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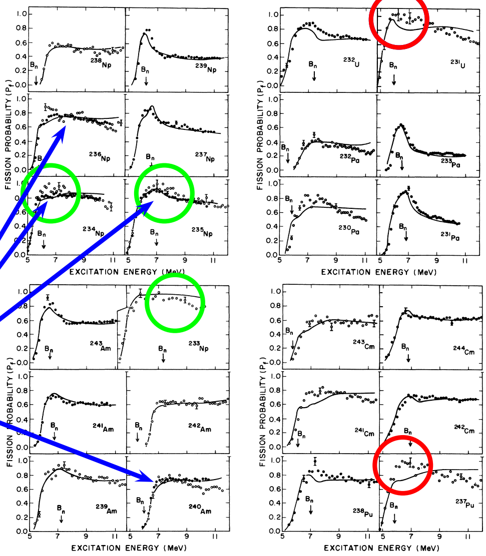


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# Fission barriers distributions = interest of surrogate reactions

Like in heavy ions fusion reactions

it is very interesting to study the energy derivative :

$$\Downarrow$$

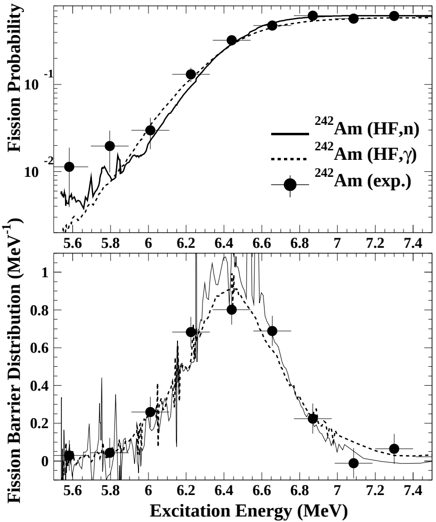
$$\frac{dP_{EC,f}}{dE} = D_f(E)$$

$$\Downarrow$$

$D_f(E)$ defines fission barriers distributions

# Fission barriers distributions

$$D_f(E) = \frac{dP_f(E)}{dE}$$




$$SR = ({}^{3}\mathbf{He}, \alpha f)$$

(P. Romain, H. Duarte, B. Morillon, PRC 85 044603)

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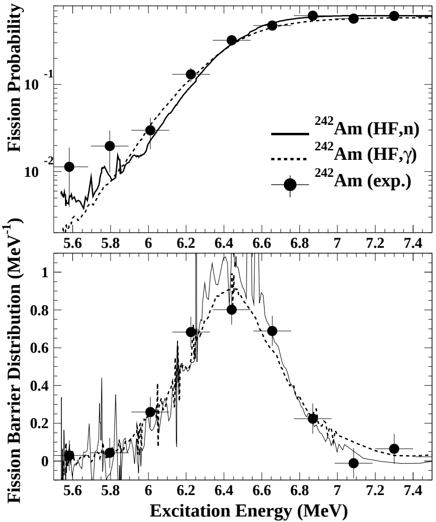


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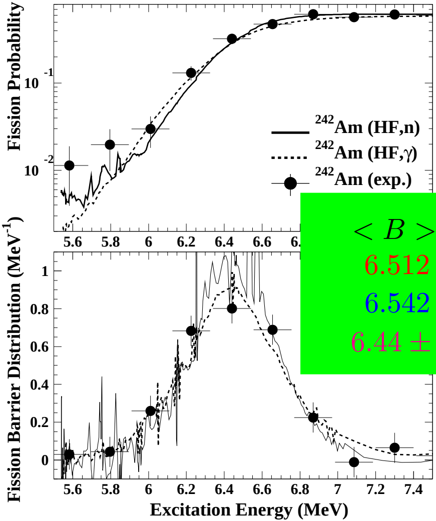


$SR = (^3\text{He}, \alpha f)$

| $< B >$ | reaction |
|---|---|
| 6.512 | $n, f$ |
| 6.542 | $\gamma, f$ |
| $6.44 \pm 0.11$ | $SR$ |

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consistency

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HOPE

surrogate for

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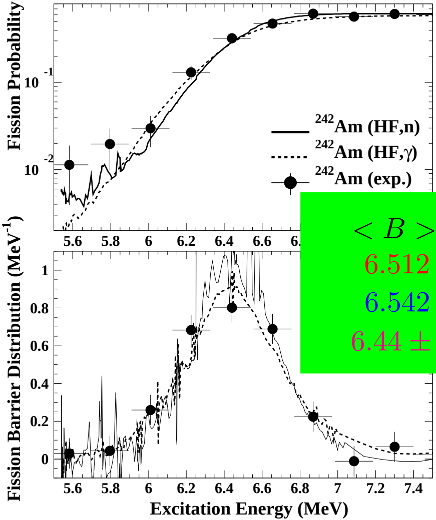
$< B^{exp} >$

completly exp.

using no model

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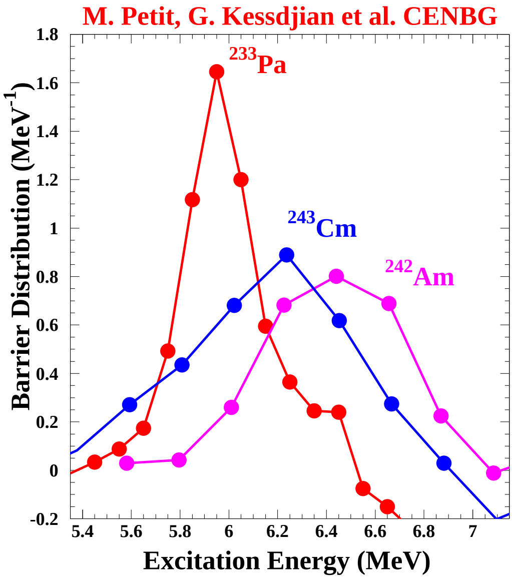

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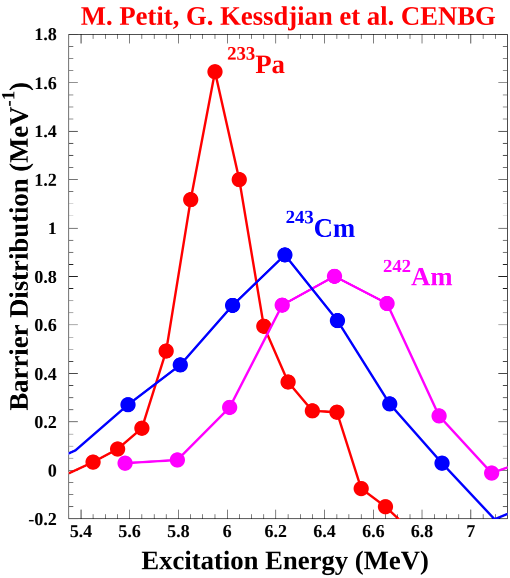

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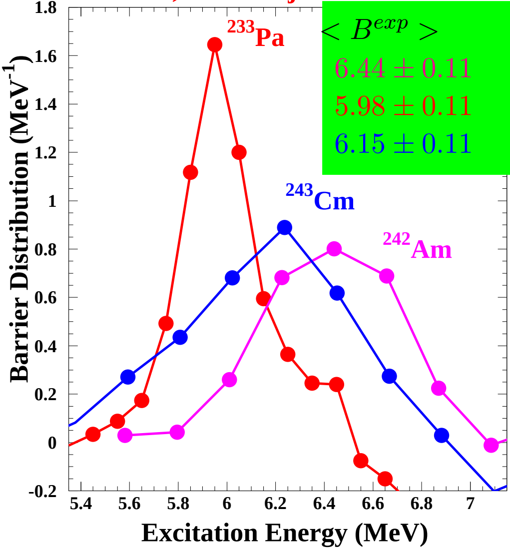
M. Petit, G. Kessdjian et al. CENBG




| $< B^{exp} >$ | Bjorn-Lynn |
|---|---|
| $6.44 \pm 0.11$ | $6.5 \pm 0.2$ |
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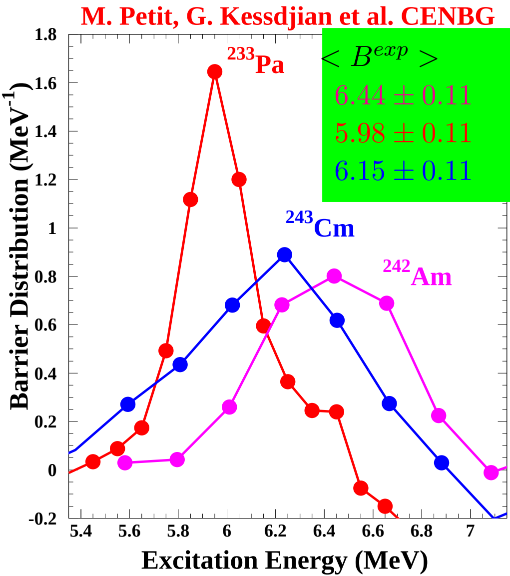


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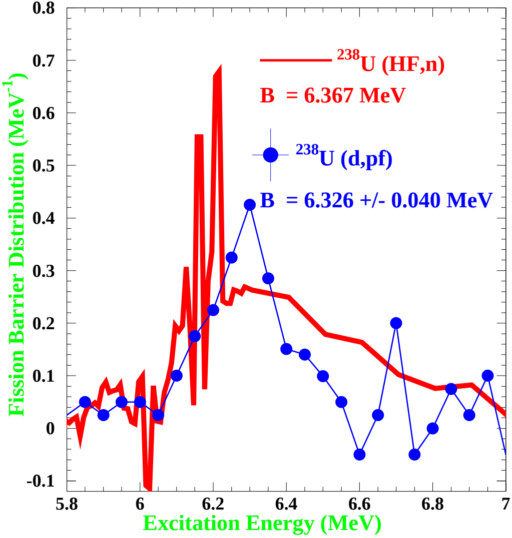


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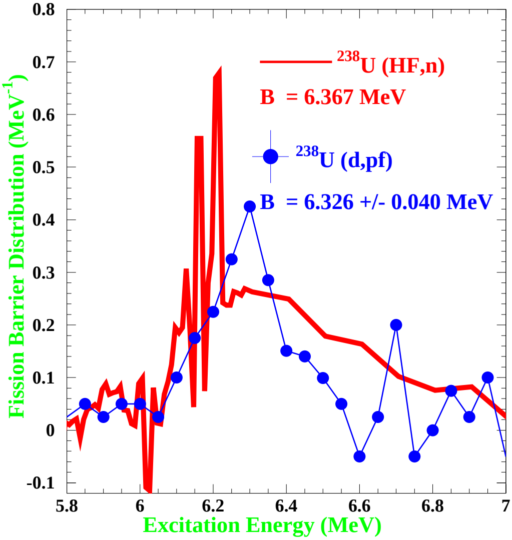


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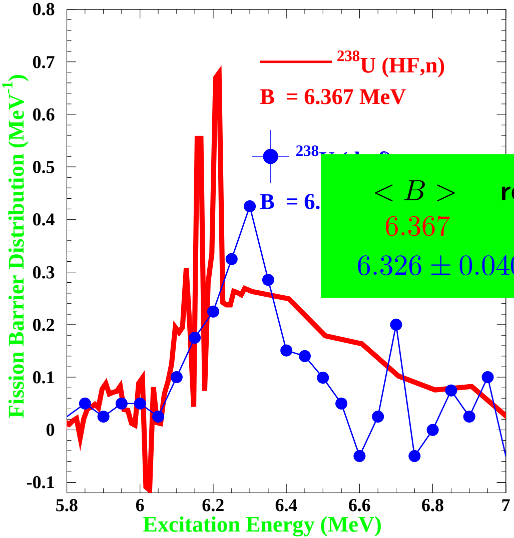


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| 6.367 | $n, f$ |
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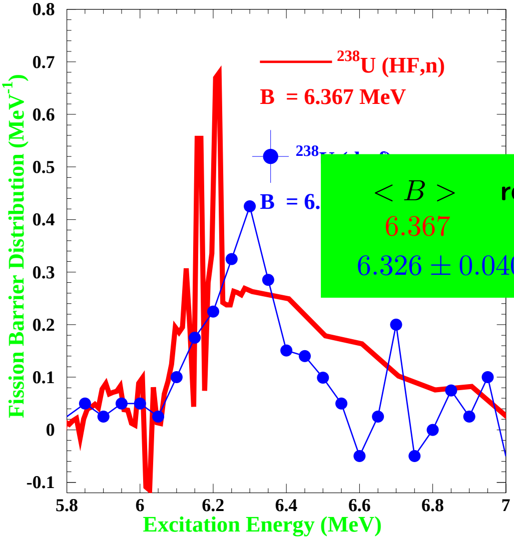


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# Fission barriers distributions (interest of surrogate)

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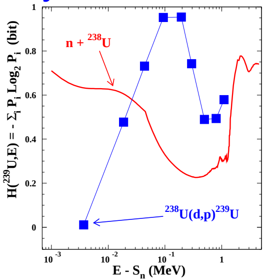
**Since $P_f$ and $P_\gamma$ simul. measured by Ducasse et al.**

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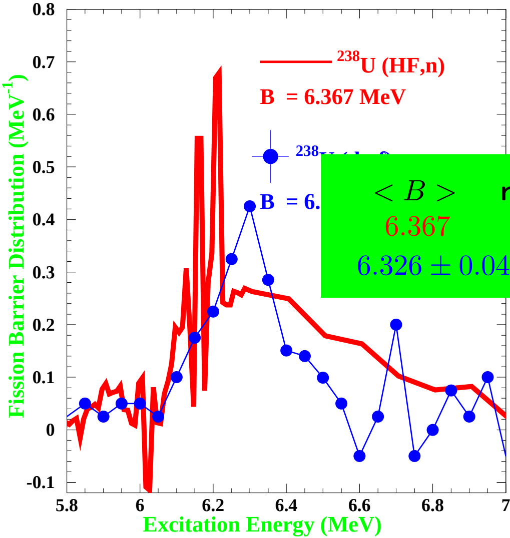


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# Surrogate Reactions for fission barriers ?

$$D_f(E) = \frac{dP_f(E)}{dE} \rightarrow < B > = \frac{\int E D_f(E) dE}{\int D_f(E) dE}$$

Consistency of $< B >$ values relatively to different Entrance Channels

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Can we go a step further, and reconstruct a barrier shape ?

**this means solving the Inverse Problem**

# Inverse Problem : Why and How ?

Statistical Model, Hauser-Feshbach formula :

$$\frac{T_a \times T_b}{\sum T_c} \longrightarrow T_n, T_p, \cdots T_\alpha \quad \approx OK \;\; OMP$$

$$\longrightarrow T_\gamma \quad \approx OK \;\; B.A., \;\; K.U.$$

$$\longrightarrow T_{fission} \;\; ??????$$

$$T_{fission} \longleftarrow \text{V microscopic (HFB) ????}$$

$$\longleftarrow \text{adjustment and fitting } \; (\text{Hill} - \text{Wheeler})$$

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**Another solution : Inverse Problem ?**

Let's start by discussing the problem of classical mechanics posed and solved in 1826 by **Niels Henrik Abel (1802-1829)**, namely :

**How to reconstruct the shape of a toboggan, knowing the total time of descent (frictionless) for a given starting height (without intial velocity) ?**

Energy conservation (assuming $m = 2$)

$$\left(\frac{dx}{dt}\right)^2 + V(x) = E.$$

From this equation, the time of descent can be deduced :

$$\tau(E) = \int_{x(0)}^{0} \frac{dx}{\sqrt{E - V(x)}}.$$

Setting $u = V(x)$ and defining the inverse function of $V$ as $x = W(u)$, we obtain then with the change of variables :

$$\tau(E) = -\int_{0}^{E} \frac{W'(u)du}{\sqrt{E-u}}.$$

# Inverse Problem in Classical Mechanics : Abel Transform formal

$$\tau(E) = -\int_0^E \frac{W'(u)du}{\sqrt{E-u}}.$$

In this equation appears what is named now the **Abel Transform** :
*Let $\mathcal{A}$ the linear operator defined for every continuous real function $f$ on [0,b], by :*

$$\forall y \in ]0,b] \quad : \quad \mathcal{A}f(y) = \int_0^y \frac{f(x)dx}{\sqrt{y-x}} \qquad \text{et} \quad \mathcal{A}f(0) = 0.$$

(This can be generalized to the fractionnal integration cases more precisely here : semi-integration $\mathcal{A} \equiv I_E^{\frac{1}{2}}$ ).

$$I_E^\alpha f(E) = \frac{1}{\Gamma(\alpha)} \int_{E_0}^E (E-E')^{\alpha-1} f(E')dE'$$

# Inverse Problem in Classical Mechanics : Abel Transform formal

One of the Abel Transform property is the following :

$$\forall y \in ]0, b] \quad : \quad \mathcal{A}(\mathcal{A}f)(y) = \pi \int_0^y f(x)dx$$

Indeed :

$$\mathcal{A}(\mathcal{A}f)(y) = \int_0^y \frac{1}{\sqrt{y-z}} \Big( \int_0^z \frac{f(x)dx}{\sqrt{z-x}} \Big) dz$$

which gives using Fubini Theorem :

$$\mathcal{A}(\mathcal{A}f)(y) = \int_0^y \Big( \int_x^y \frac{dz}{\sqrt{(y-z)(z-x)}} \Big) f(x)dx$$

and using the idententity : $\int_x^y \frac{dz}{\sqrt{(y-z)(z-x)}} = \pi$

we obtain then : $\mathcal{A}(\mathcal{A}f)(y) = \pi \int_0^y f(x)dx$

# Inverse Problem in Classical Mechanics : Abel Transform formal

Now coming back to the Classical Mechanics problem posed by Abel we get :

$$\tau(E) = -\int_0^E \frac{W'(u)du}{\sqrt{E-u}} = -\mathcal{A}W'(E).$$

Applying a second Abel transform we get :

$$\mathcal{A}\tau(E) = -\mathcal{A}^2 W'(E) = -\pi \int_0^E W'(u)du = -\pi W(E).$$

from which :

$$W(E) = -\frac{1}{\pi}\mathcal{A}\tau(E)$$

**We are now able to calculate $W$, in fact the potential $V$ from the $\tau$ function.**

Initially O. Klein and R. Rydberg (1931,1932)
defined a method for the construction of potential energy curves
for diatomic molecules

Later J.A. Wheeler (1976)

If we consider the action integral :

$$S(E) = \int_{x_1}^{x_2} \sqrt{\frac{2m}{\hbar^2}[V(x) - E]}dx$$

used in the WKB approximation for the calculation of the potential barrier penetration coefficient :

$$T(E) = \frac{1}{1+e^{2S(E)}} \quad \Longleftrightarrow \quad S(E) = \frac{1}{2}Log\Big(\frac{1}{T(E)} - 1\Big).$$

**Applying the Abel transform to** $-\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\frac{dS(E)}{dE}$ **:**

$$\mathcal{A}\Big(-\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\frac{dS(E)}{dE}\Big) = -\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\int_E^B \frac{dS(E')}{dE'}\frac{dE'}{\sqrt{E'-E}}$$

$$\begin{aligned}
\mathcal{A}\left(-\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\frac{dS(E)}{dE}\right) &= -\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\int_E^B \frac{dS(E')}{dE'}\frac{dE'}{\sqrt{E'-E}} \\
&= -\frac{2}{\pi}\int_E^B \int_{x_1}^{x_2} -\frac{1}{2}\frac{dx}{\sqrt{V(x)-E'}}\frac{dE'}{\sqrt{E'-E}} \\
&= \frac{1}{\pi}\int_{x_1}^{x_2}\left(\int_E^B \frac{1}{\sqrt{V(x)-E'}}\frac{dE'}{\sqrt{E'-E}}\right)dx \\
&= \int_{x_1}^{x_2} dx \\
&= x_2(E) - x_1(E) = \Phi(E)
\end{aligned}$$

$$\mathcal{A}\left(-\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\frac{dS(E)}{dE}\right) = x_2(E) - x_1(E) = \Phi(E) \qquad ???$$

$$T(E) = \frac{1}{1+e^{2S(E)}} \iff S(E) = \frac{1}{2}Log\left(\frac{1}{T(E)} - 1\right)$$

$$\begin{aligned} \frac{dS}{dE} &= \frac{d}{dE}\left(\frac{1}{2}Log\left(\frac{1}{T(E)} - 1\right)\right) \\ &= \frac{1}{2} \times \frac{-\left(\frac{dT(E)/dE}{T^2(E)}\right)}{\left(\frac{1}{T(E)} - 1\right)} \\ &= -\frac{1}{2} \times \frac{D(E)}{T(E)[1-T(E)]} \end{aligned}$$

where we used : $\frac{dT(E)}{dE} = D(E)$ and defined
$B = \langle B \rangle = \frac{\int ED(E)dE}{\int D(E)dE}$, which finally gives :

$$x_2(E) - x_1(E) = \frac{1}{\pi}\sqrt{\frac{\hbar^2}{2m}}\int_E^B \frac{D(E')}{T(E')[1-T(E')]}\frac{dE'}{\sqrt{E'-E}}$$

$$x_2(E) - x_1(E) = \frac{1}{\pi}\sqrt{\frac{\hbar^2}{2m}} \int_E^B \frac{D(E')}{T(E')[1-T(E')]} \frac{dE'}{\sqrt{E'-E}}$$

There the advantage is that we know the barrier height

$B =< B >= \frac{\int ED(E)dE}{\int D(E)dE}$.

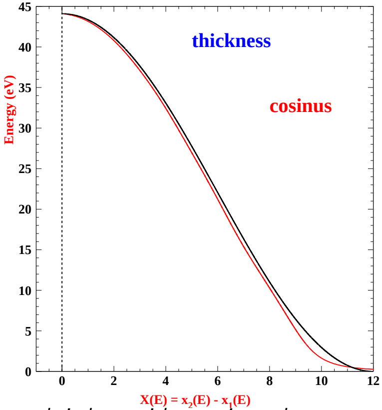



Figure : *Black curve from analytical potential expression, red curve according to Inverse Problem treatment.*

Thickness :

$$\Phi(E) \quad = \quad x_2(E) - x_1(E)$$

OK, but not sufficient to define completely a potential barrier shape, how to go further ?
Need to use a second equation :

$$\Psi(E) = \psi\big(x_1(E), x_2(E)\big)$$

Applying the same methodolgy, if $V(x) = V_0(x) - \lambda\phi(x)$

$$S(E,\lambda) = \int_{x_1}^{x_2} \sqrt{\frac{2m}{\hbar^2}[V(x) - E]}dx$$

$$= \int_{x_1}^{x_2} \sqrt{\frac{2m}{\hbar^2}[V_0(x) - \lambda\phi(x) - E]}dx$$

firstly with :

$$\frac{\partial S(E,\lambda)}{\partial\lambda} = \sqrt{\frac{2m}{\hbar^2}} \int_{x_1}^{x_2} \frac{\partial}{\partial\lambda}[\sqrt{V_0(x) - \lambda\phi(x) - E}]dx$$

$$= \sqrt{\frac{2m}{\hbar^2}} \int_{x_1}^{x_2} \frac{1}{2} \frac{-\phi(x)dx}{\sqrt{V_0(x)-\lambda\phi(x)-E}}$$

$$= \sqrt{\frac{2m}{\hbar^2}} \int_{x_1}^{x_2} \frac{1}{2} \frac{-\phi(x)dx}{\sqrt{V(x)-E}}$$

When considering Abel Transform $\mathcal{A}\left(-\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\frac{\partial S}{\partial \lambda}\right)$ of $\frac{\partial S}{\partial \lambda}$, and using always the same trick $\int_E^B \frac{dE'}{\sqrt{(V(x)-E')(E'-E)}} = \pi$, it gives :

$$\begin{aligned}
\mathcal{A}\left(-\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\frac{\partial S}{\partial \lambda}\right) &= -\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\int_E^B \frac{\partial S(E')}{\partial \lambda}\frac{dE'}{\sqrt{E'-E}} \\
&= -\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\int_E^B \sqrt{\frac{2m}{\hbar^2}}\int_{x_1}^{x_2}\frac{1}{2}\frac{-\phi(x)dx}{\sqrt{V(x)-E'}}\frac{dE'}{\sqrt{E'-E}} \\
&= \frac{1}{\pi}\int_{x_1}^{x_2}\phi(x)\left[\int_E^B \frac{1}{\sqrt{V(x)-E'}}\frac{dE'}{\sqrt{E'-E}}\right]dx \\
&= \int_{x_1}^{x_2}\phi(x)dx
\end{aligned}$$

and secondly, as we know $\frac{\partial S(E)}{\partial \lambda}$ :

$$T(E,\lambda) = \frac{1}{1+e^{2S(E,\lambda)}} \iff S(E,\lambda) = \frac{1}{2} Log\Big(\frac{1}{T(E,\lambda)} - 1\Big) :$$

$$\begin{aligned}
\frac{\partial S(E,\lambda)}{\partial \lambda} &= \frac{\partial}{\partial \lambda}\Big(\frac{1}{2} Log\Big(\frac{1}{T(E,\lambda)} - 1\Big)\Big) \\
&= \frac{1}{2} \times \frac{-\left(\frac{\partial T(E,\lambda)/\partial \lambda}{T^2(E,\lambda)}\right)}{\left(\frac{1}{T(E,\lambda)} - 1\right)} \\
&= -\frac{1}{2} \times \frac{\partial T(E,\lambda)/\partial \lambda}{T(E,\lambda)[1-T(E,\lambda)]}
\end{aligned}$$

and finally :

$$\begin{aligned}
\mathcal{A}\Big(-\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\frac{\partial S(E)}{\partial \lambda}\Big) &= \frac{1}{\pi}\sqrt{\frac{\hbar^2}{2m}} \int_E^B \frac{\partial T(E',\lambda)/\partial \lambda}{T(E',\lambda)[1-T(E',\lambda)]} \frac{dE'}{\sqrt{E'-E}} \\
&= \Psi(E)
\end{aligned}$$

$$\int_{x_1}^{x_2} \phi(x)dx = \mathcal{A}\Big( -\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\frac{\partial S(E)}{\partial \lambda}\Big) = \Psi(E)$$

But this is interesting just if we know $\phi(x)$, since it gives a new relation between $x_1$ and $x_2$, as exemple :
if $\lambda\phi(x) = \frac{\lambda}{x^2}$ (centrifugal term) then :

$$\int_{x_1}^{x_2} \phi(x)dx = \frac{1}{x_1} - \frac{1}{x_2} = \Psi(E)$$

In the same way, if $\lambda\phi(x) = V_0\frac{(x-x_0)}{s-x_0} = \lambda(x-x_0)$ (field emission from a metal) then :

$$\int_{x_1}^{x_2} \phi(x)dx = \frac{1}{2}\Big[(x-x_0)^2\Big]_{x_1}^{x_2} = \frac{1}{2}\Big[(x_2-x_0)^2-(x_1-x_0)^2\Big] = \Psi(E)$$

if in addition we always define $x_2(E) - x_1(E) = \Phi(E)$ it gives :

$$
\begin{aligned}
(x_2 - x_0)^2 - (x_1 - x_0)^2 &= (x_2 - x_0 + x_1 - x_0)(x_2 - x_0 - [x_1 - x_0]) \\
&= (x_2 + x_1 - 2x_0) \times \Phi(E) = 2\Psi(E)
\end{aligned}
$$

and finally :

$$x_2 + x_1 = \frac{2\Psi(E)}{\Phi(E)} + 2x_0$$

and

$$x_2 - x_1 = \Phi(E)$$

which leads to :

$$
\begin{aligned}
x_1(E) &= \frac{1}{2}\left[\frac{2\Psi(E)}{\Phi(E)} + 2x_0 - \Phi(E)\right] \\
x_2(E) &= \frac{1}{2}\left[\frac{2\Psi(E)}{\Phi(E)} + 2x_0 + \Phi(E)\right]
\end{aligned}
$$

The potential is **known close to a** $2x_0$ **translation.**

# Inverse Problem : towards a potential barrier shape

## concrete

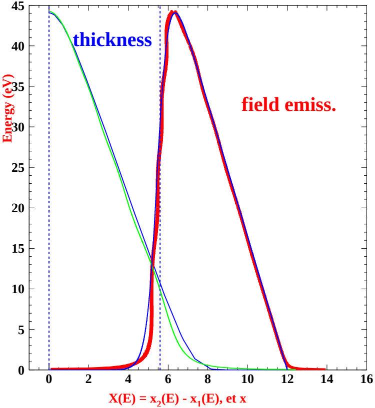



Figure : *Blue curves from analytical potential expression, green curve and red curve according to Inverse*

Field emission from a metal, for which $\lambda$ is proportional to the electric field at the surface

$$\Phi(E) = \frac{1}{\pi}\sqrt{\frac{\hbar^2}{2m}}\int_E^B \frac{D(E')}{T(E')[1-T(E')]}\frac{dE'}{\sqrt{E'-E}}$$

$$\Psi(E) = \frac{1}{\pi}\sqrt{\frac{\hbar^2}{2m}}\int_E^B \frac{\partial T(E',\lambda)/\partial\lambda}{T(E',\lambda)[1-T(E',\lambda)]}\frac{dE'}{\sqrt{E'-E}}$$

with :

$$B = \langle B \rangle = \frac{\int E D(E) dE}{\int D(E) dE}$$

$$D(E) = \frac{d}{dE}T(E)$$

$$x_1(E) = \frac{1}{2}\left[\frac{2\Psi(E)}{\Phi(E)} + 2x_0 - \Phi(E)\right]$$

$$x_2(E) = \frac{1}{2}\left[\frac{2\Psi(E)}{\Phi(E)} + 2x_0 + \Phi(E)\right]$$

In the same way, using the same tricks, potential well can be reconstruct with :

$$N(E) = \int_{x_1}^{x_2} \sqrt{\frac{2m}{\hbar^2}[E - V(x)]}dx = \int pdx$$

$$= Bohr - Sommerfeld = Weyl = WKB$$

$$= (n(E) + 1/2)\pi$$

$$x_2(E) - x_1(E) = \mathcal{A}\left(\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\frac{dN(E)}{dE}\right)$$

$$= \frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}}\int_{V_{min}}^{E} \frac{dN(E')}{dE'}(E - E')^{-1/2}dE'$$

Now If we set $E \approx (n(E) + 1/2)\hbar\omega$ then we get :
$n(E) + 1/2 \approx \frac{E}{\hbar\omega} \approx \frac{N(E)}{\pi}$ from which :

$$\frac{dN(E)}{dE} \approx \frac{\pi}{\hbar\omega}$$

here $V_{min}$ and $\hbar\omega$ were obtained using $D(E) = \frac{dT(E)}{dE}$ which allows to access at the peaks energy position and to get part of the spectrum ($E_n$ energies) inside the potential well and finally :

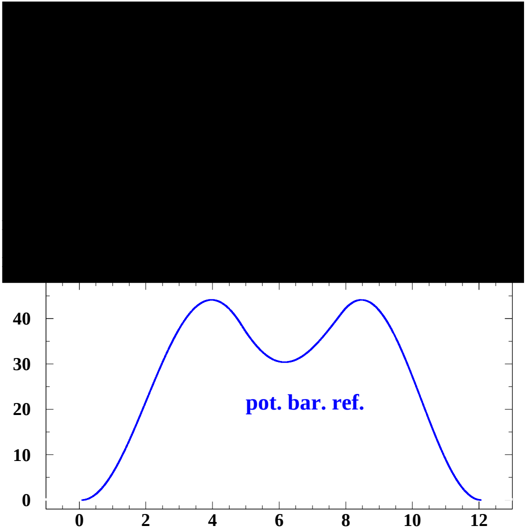
$$\hbar\omega = E_1 - E_0 \text{ and } V_{min} = E_0 - \frac{\hbar\omega}{2}.$$

$$
\begin{aligned}
x_2(E) - x_1(E) &= \frac{2}{\pi}\sqrt{\frac{\hbar^2}{2m}} \int_{V_{min}}^{E} \frac{dN(E')}{dE'}(E - E')^{-1/2} dE' \\
&= 2\sqrt{\frac{\hbar^2}{2m}} \int_{V_{min}}^{E} \frac{1}{\hbar\omega} \frac{dE'}{\sqrt{E-E'}} \\
&= \Phi(E)
\end{aligned}
$$

In the Semiclassical Quantum theory the inverse of the potential is proportional to the half-derivative of the eigenvalues counting function $N(E)$

# Status of double humped Potential Barrier Reconstruction

## concrete



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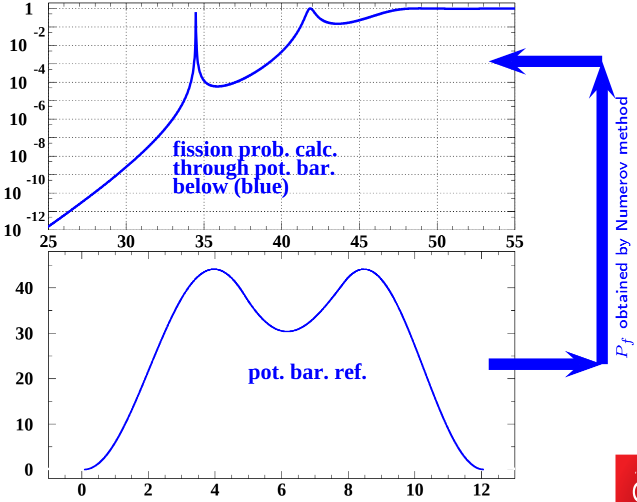
concrete



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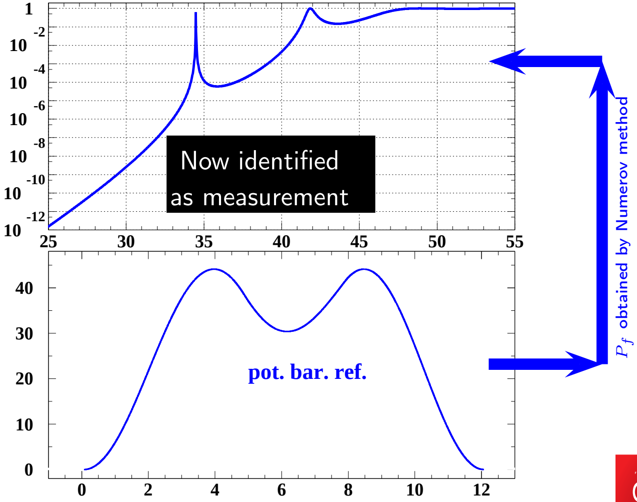

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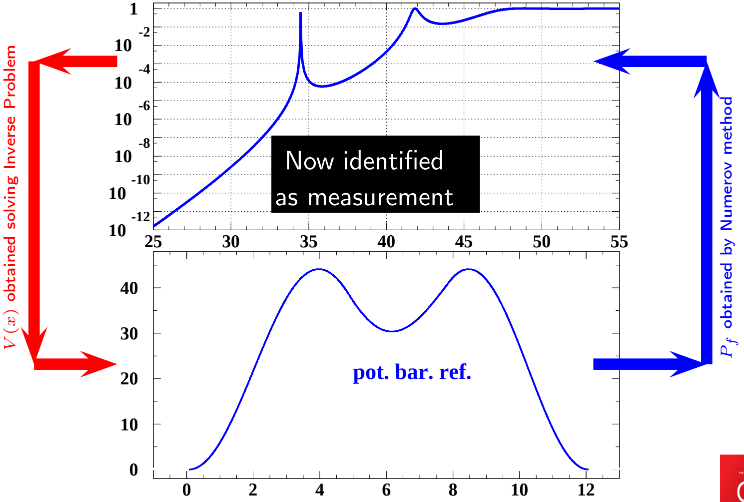

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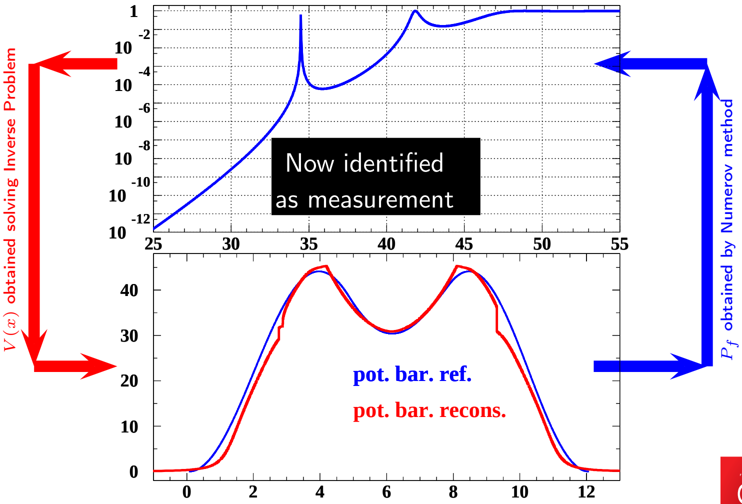

# HOW to reconstruct "true" fission barriers ? ? ?

$$\mathbf{\Phi(E) = x_2(E) - x_1(E)}$$

hump

$$\mathbf{\Phi(E)} = \frac{1}{\pi}\sqrt{\frac{\hbar^2}{2m}}\int_E^{\mathbf{B}} \frac{D(E')}{T(E')[1-T(E')]}\frac{dE'}{\sqrt{E'-E}}$$

$$\mathbf{B} = <\mathbf{B}> = \frac{\int E D(E) dE}{\int D(E) dE} \Longleftarrow \quad \mathbf{D(E)} = \frac{\mathbf{dT(E)}}{\mathbf{dE}}$$

well

$$\mathbf{\Phi(E)} = 2\sqrt{\frac{\hbar^2}{2m}}\int_{\mathbf{V_{min}}}^{E} \frac{\mathbf{1}}{\hbar\omega}\frac{dE'}{\sqrt{E-E'}}$$

$$\Longrightarrow \hbar\omega = E_1 - E_0$$

$$\Longrightarrow \mathbf{V_{min}} = E_0 - \frac{\hbar\omega}{2}$$

1) Here was assumed $\Psi(E) = x_1(E) + x_2(E) = cte$ only for symmetrical barriers $\Longrightarrow$ second equation needed :

$$\Psi(E) = \psi(x_1(E), x_2(E))$$

$$\lambda\Phi(E) \bigotimes \mu\Psi(E) \Longrightarrow x_1(E), x_2(E)$$

2) Kac's problem [(1966) Mark Kac, Lipman Bers, Hermann Weyl] : *"Can one hear the shape of a drum ?"* since 1992 (Gordon-Webb-Wolpert) we know the answer : NO !

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## Would it be possible to hear a fissionning nucleus ?