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Modelling odd-even staggering in fission fragment yields in the Brownian shape-motion approach: Current results and prospects

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More details about fission (PRC 79 064304, PRC 84 034613, PRC 88 064606), other projects (beta-decay,masses), associated ASCII data files, interactive access to data (type in Z, A and get specific data, contour maps) and figures are at

http://t2.lanl.gov/nis/molleretal/

Potential Energy of Deformation

We use the macroscopic-microscopic method introduced by Swiatecki and Strutinsky:

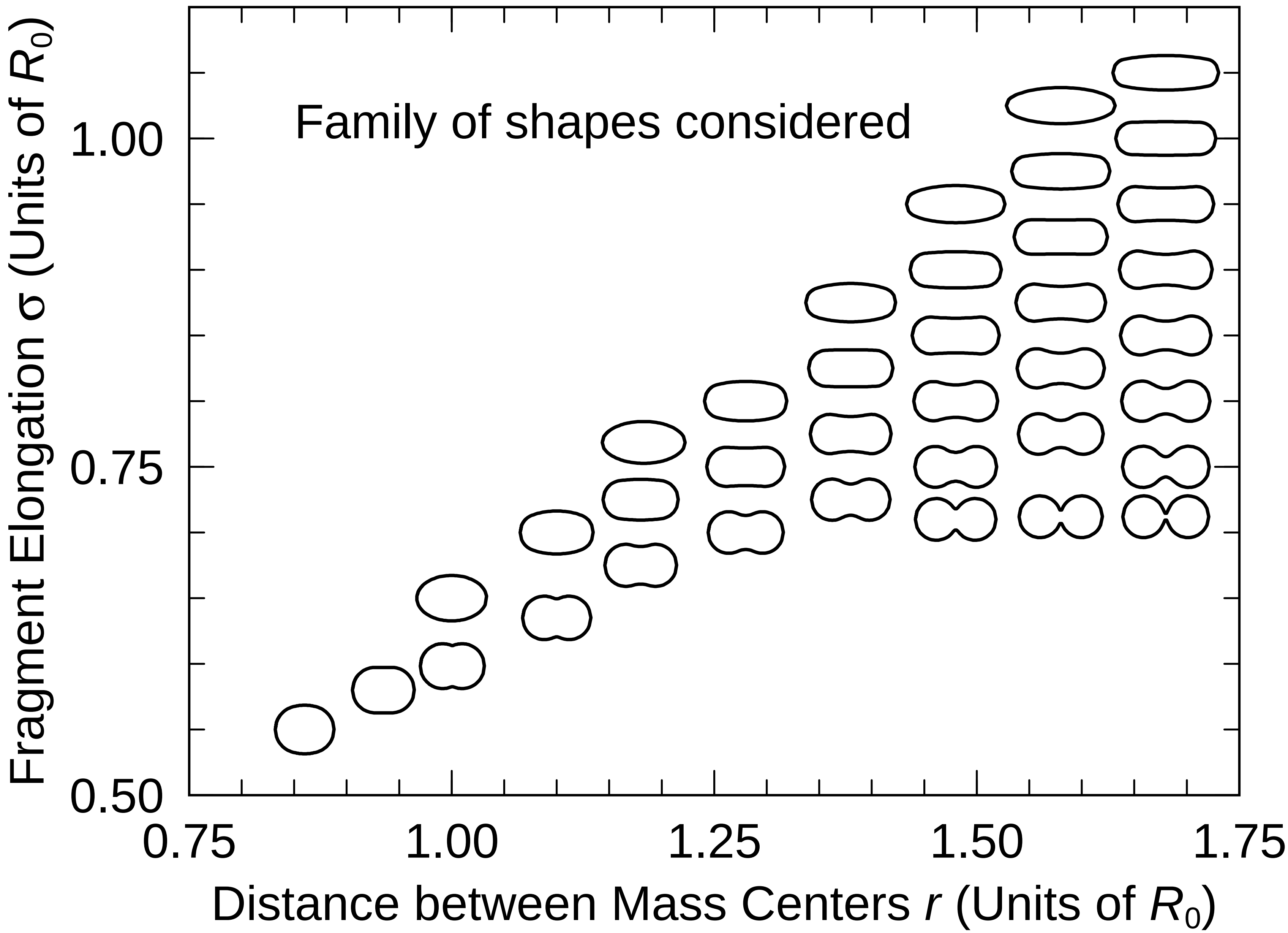
$$E_{\rm pot}({\rm shape}) = E_{\rm macr}({\rm shape}) + E_{\rm micr}({\rm shape}) \quad (1)$$

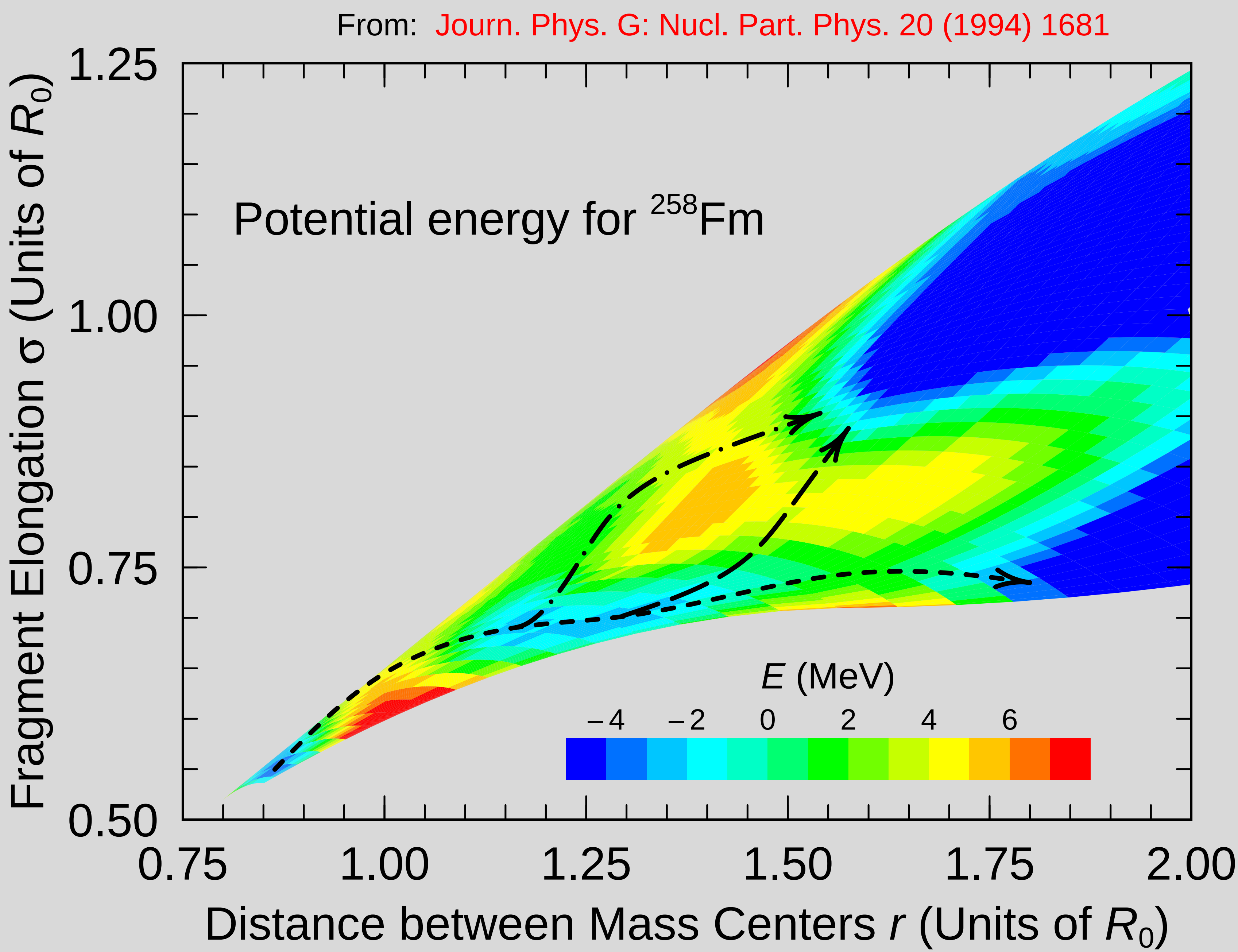
The macroscopic term is calculated in a liquid-drop type model (for a specific deformed shape).

The microscopic correction is determined in the following steps

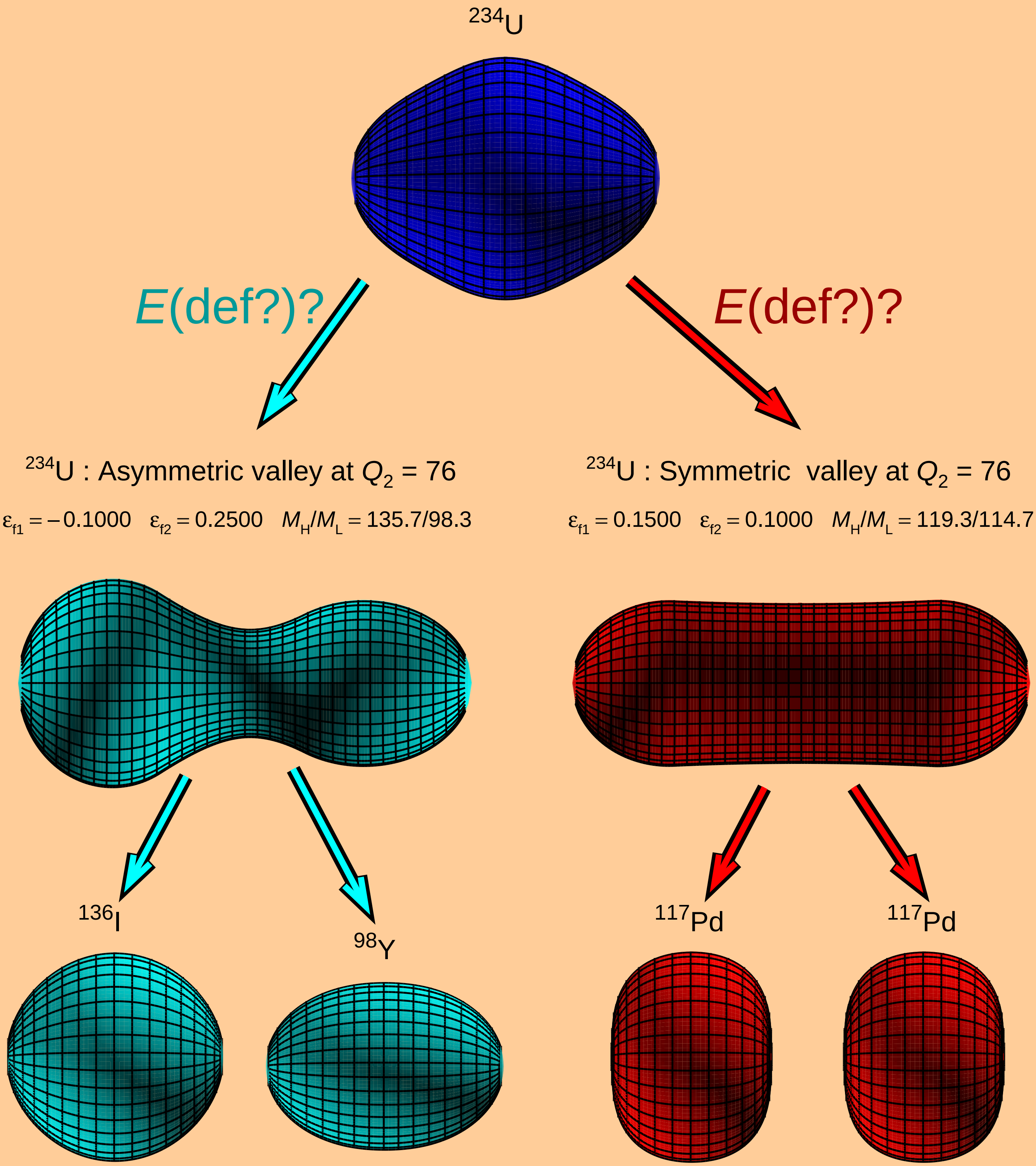
1. A shape is prescribed
2. A single-particle potential with this shape is generated. A spin-orbit term is included.
3. The Schrödinger equation is solved for this deformed potential and single-particle levels and wave-functions are obtained
4. The shell correction is calculated by use of Strutinsky's method.
5. The pairing correction is calculated in the BCS or Lipkin-Nogami method.

From: Nucl. Phys A469 (1987) 1

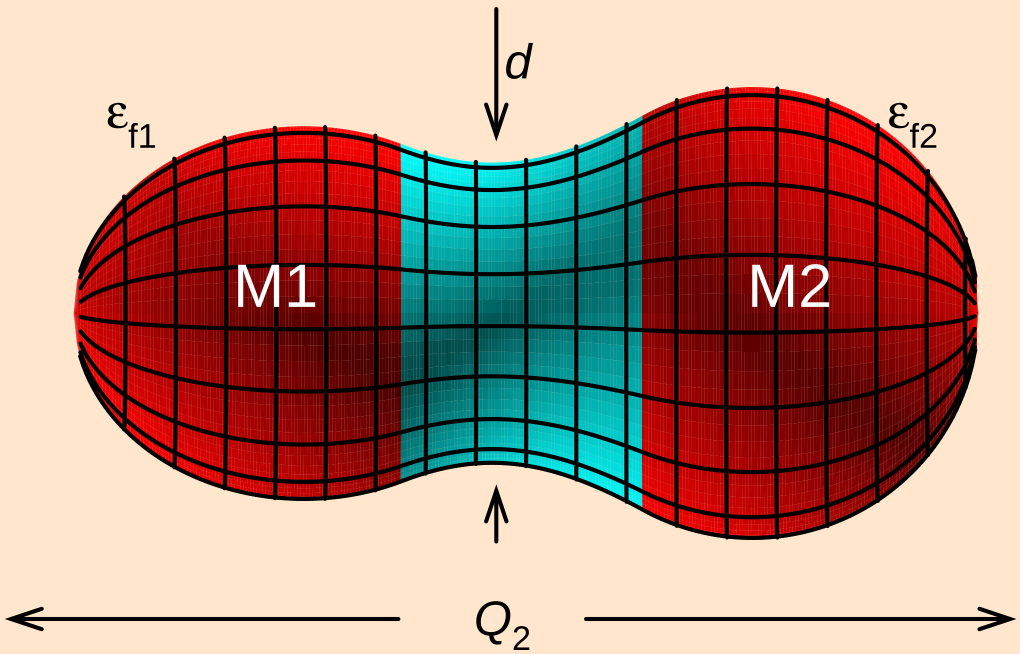



From: Journ. Phys. G: Nucl. Part. Phys. 20 (1994) 1681
Potential energy for 258Fm
Fragment Elongation σ (Units of R0)
1.25
1.00
0.75
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Distance between Mass Centers r (Units of R0)
E (MeV)
−4
−2
0
2
4
6

In fission, what are the shapes and related energies involved in the transition from a single ground-state shape to two separated fission fragments?



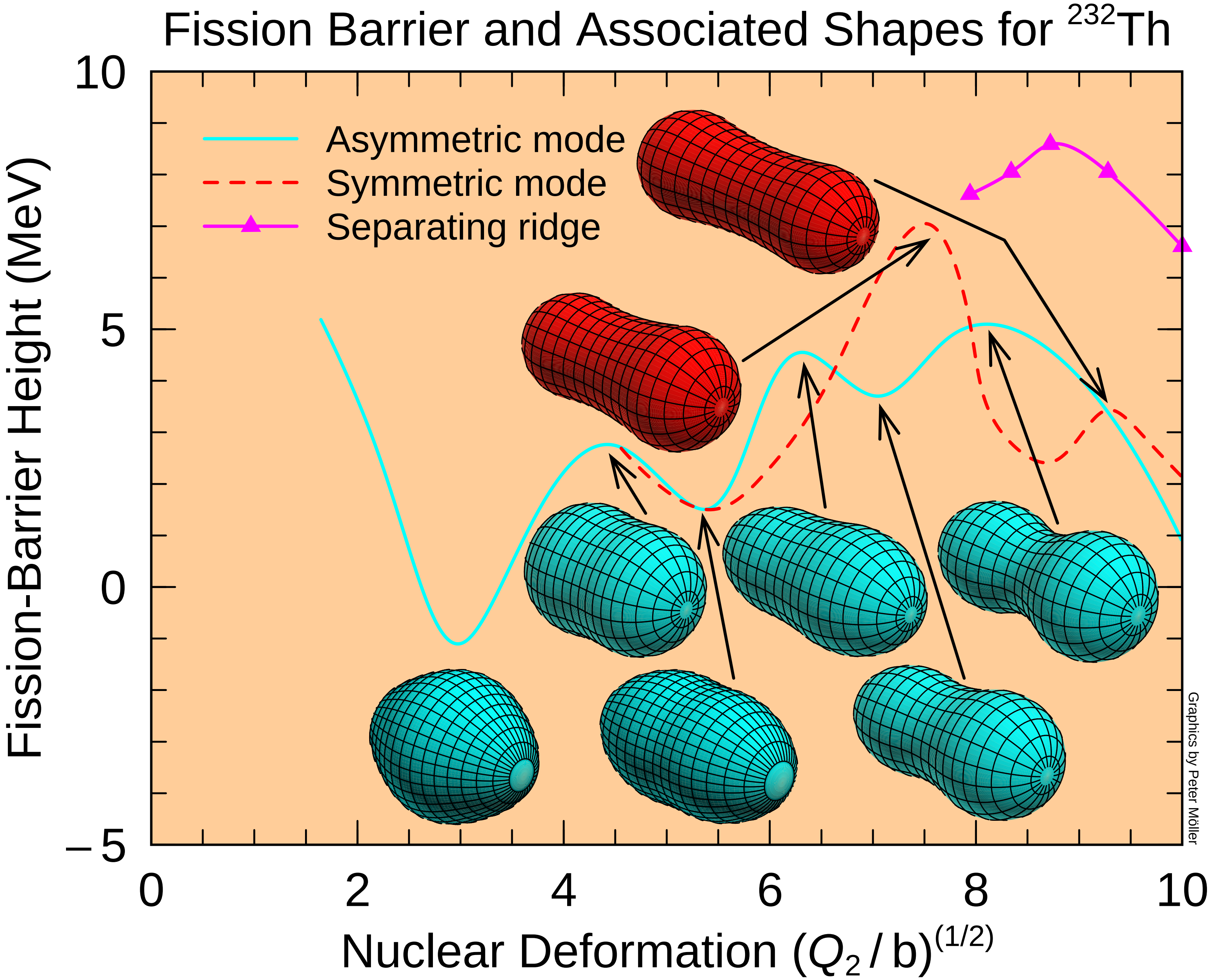
Five Essential Fission Shape Coordinates

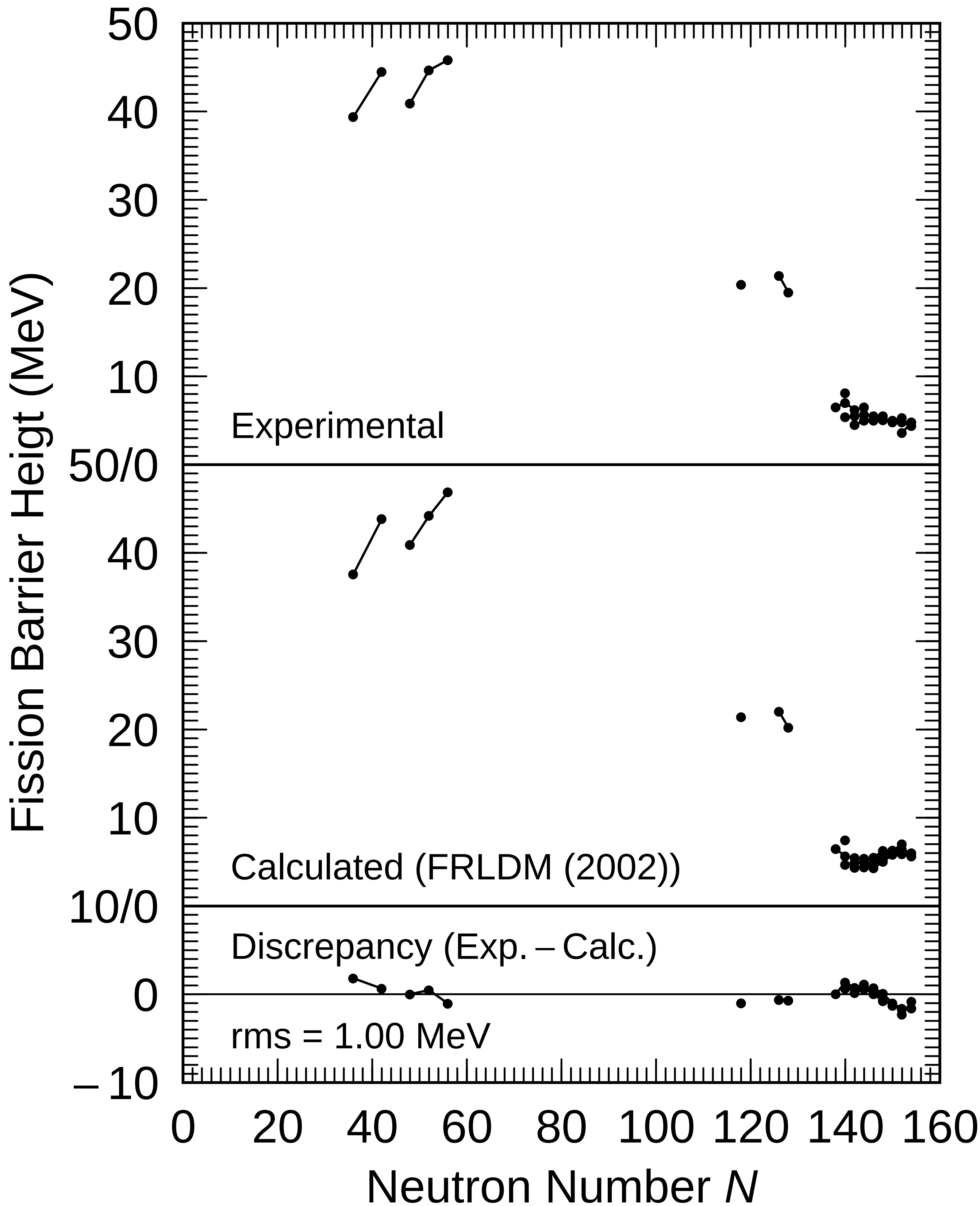


45	Q_2 ~	Elongation (fission direction)
⊗		
15	d ~	Neck
⊗		
15	ε_{f1} ~	Left fragment deformation
⊗		
15	ε_{f2} ~	Right fragment deformation
⊗		
35	α_g ~	(M1-M2)/(M1+M2) Mass asymmetry

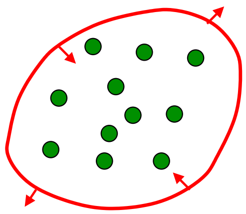
⇒ 5 315 625 grid points – 306 300 unphysical points

⇒ 5 009 325 physical grid points


Fission Barrier and Associated Shapes for ^{232}Th
Asymmetric mode
Symmetric mode
Separating ridge
Fission-Barrier Height (MeV)
10
5
0
−5
0
2
4
6
8
10
Nuclear Deformation $(Q_2 / b)^{(1/2)}$
Graphics by Peter Möller



Fission Barrier Heigt (MeV)
50
40
30
20
10
50/0
40
30
20
10
10/0
0
– 10
Experimental
Calculated (FRLDM (2002))
Discrepancy (Exp. – Calc.)
rms = 1.00 MeV
0
20
40
60
80
100
120
140
160
Neutron Number *N*



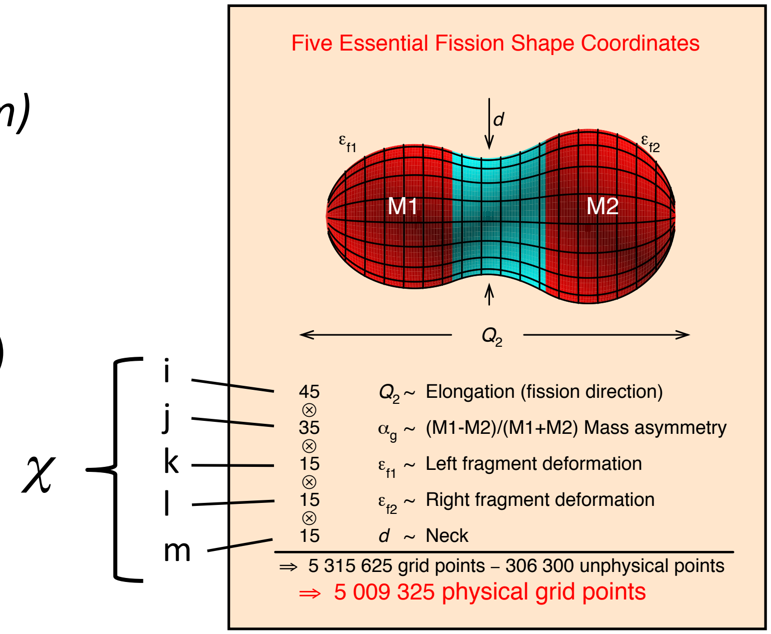
Brownian shape motion

Nuclear deformation energy: $E_{def}(i,j,k,l,m)$

Bias potential: $V_{bias}(i) = V_0\,(Q_0/Q_2)^2$

Level density parameter: $a_A = A/(8\text{ MeV})$

Temperature T: $E^* - E_{def} = a_A T^2$



$$\Rightarrow\; V(\chi) = E_{def} + V_{bias}$$

P. Möller *et al*, Nature 409 (2001) 785

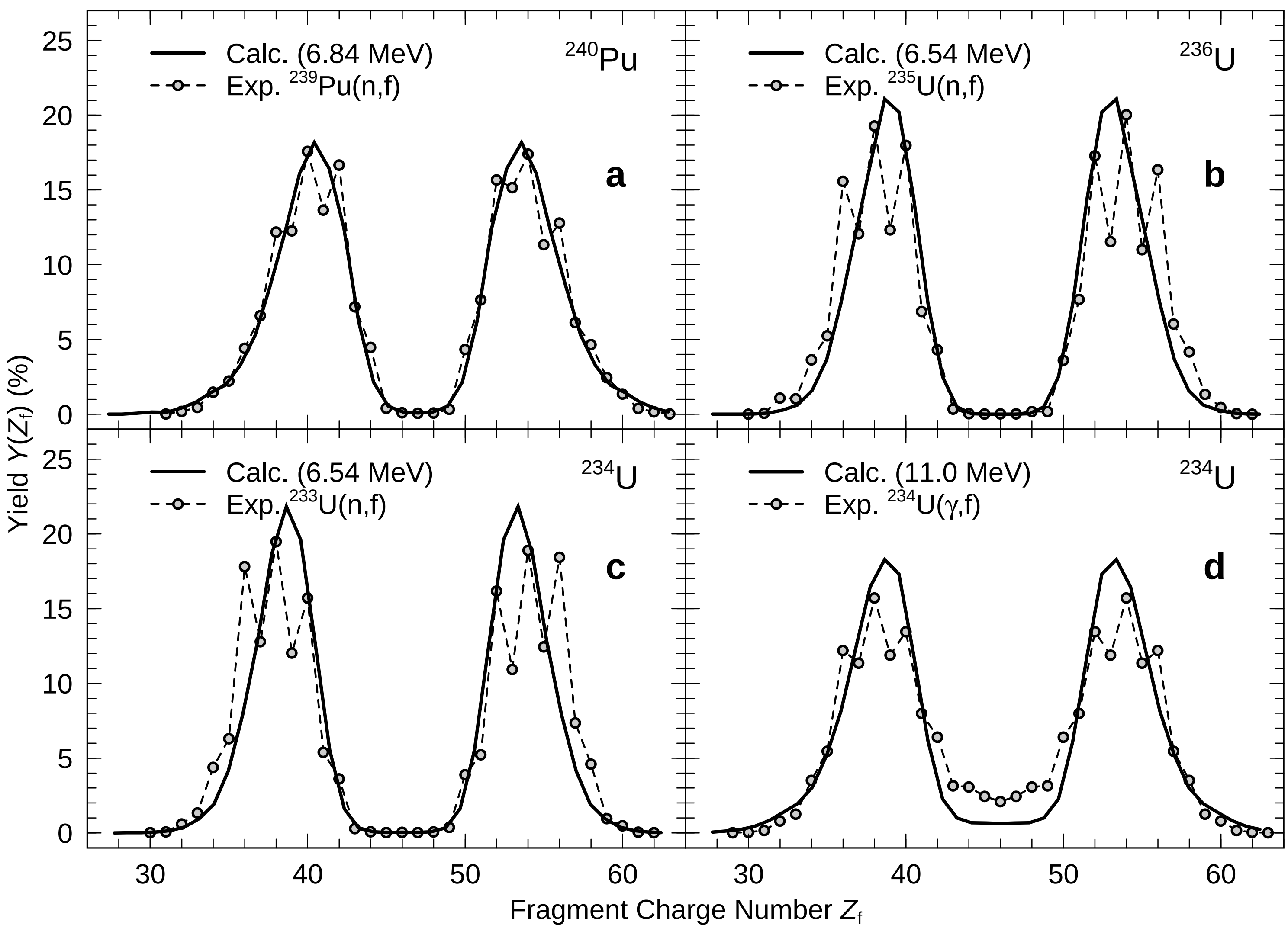
Metropolis walk:

N. Metropolis *et al*, J Chem Phys 26 (1953) 1087

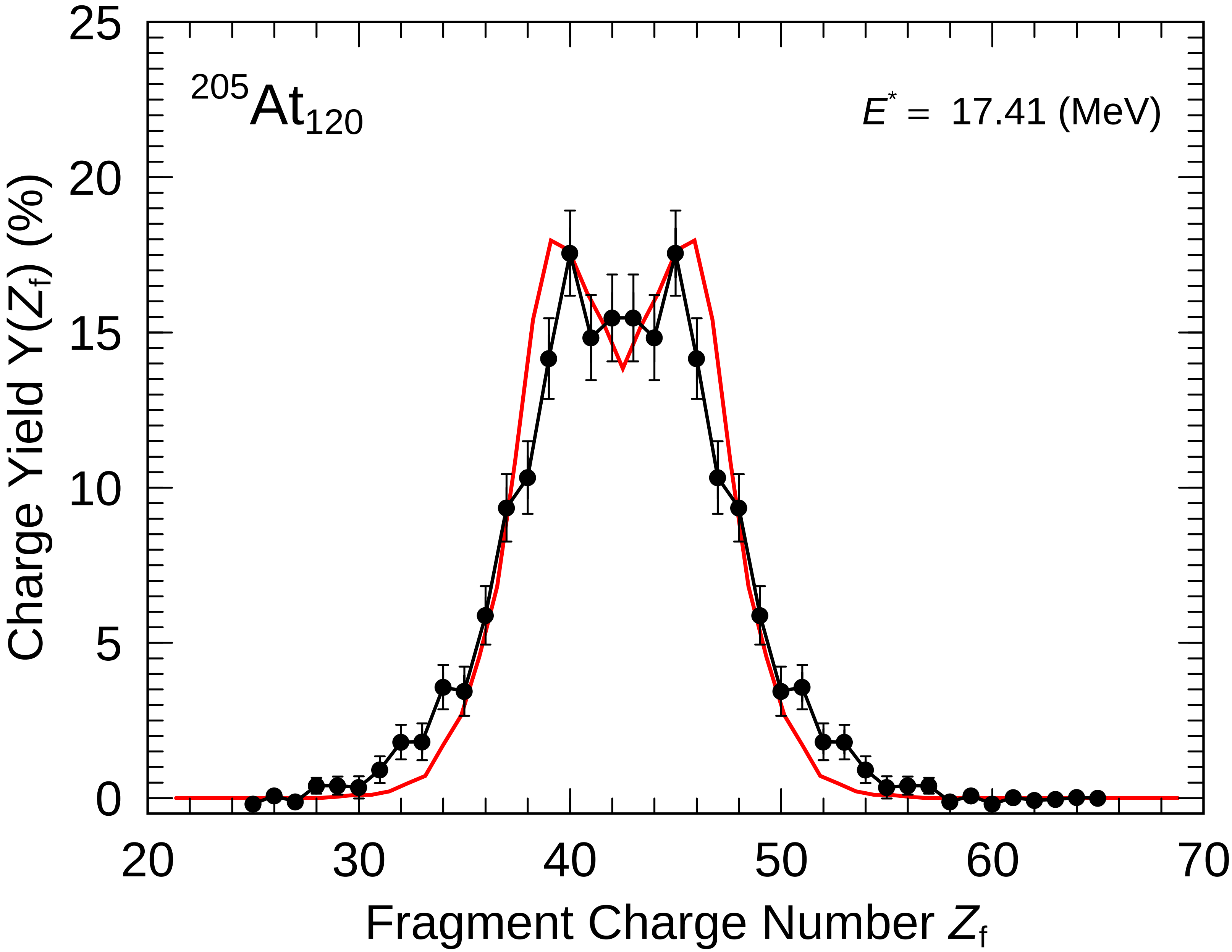
Change shape: $\chi \rightarrow \chi'$?

- $V(\chi') < V(\chi)$: *move with* $P = 1$
- $V(\chi') > V(\chi)$: *move with* $P = \exp(-\Delta V/T)$

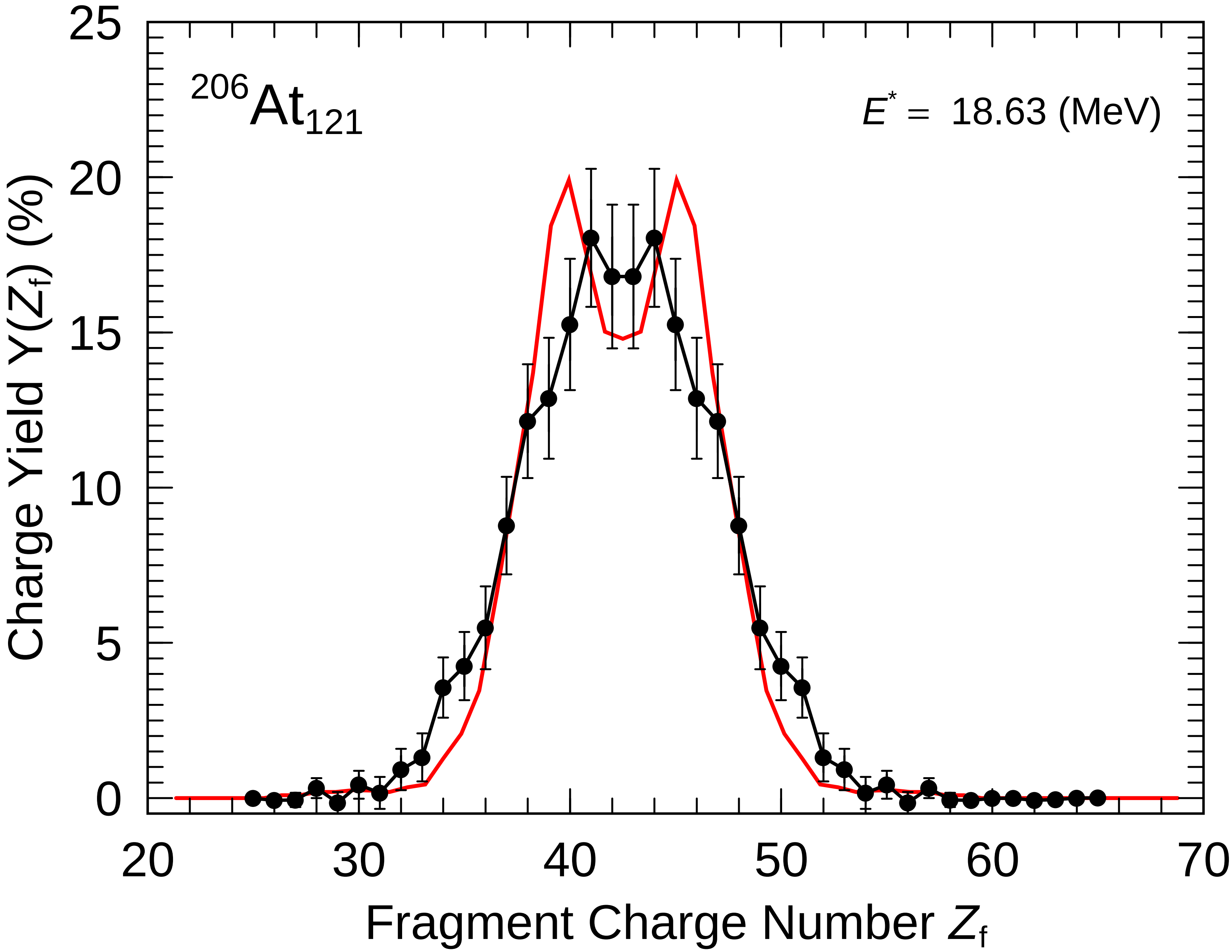
Scission: Critical neck radius $c_0 \approx 2.5$ fm



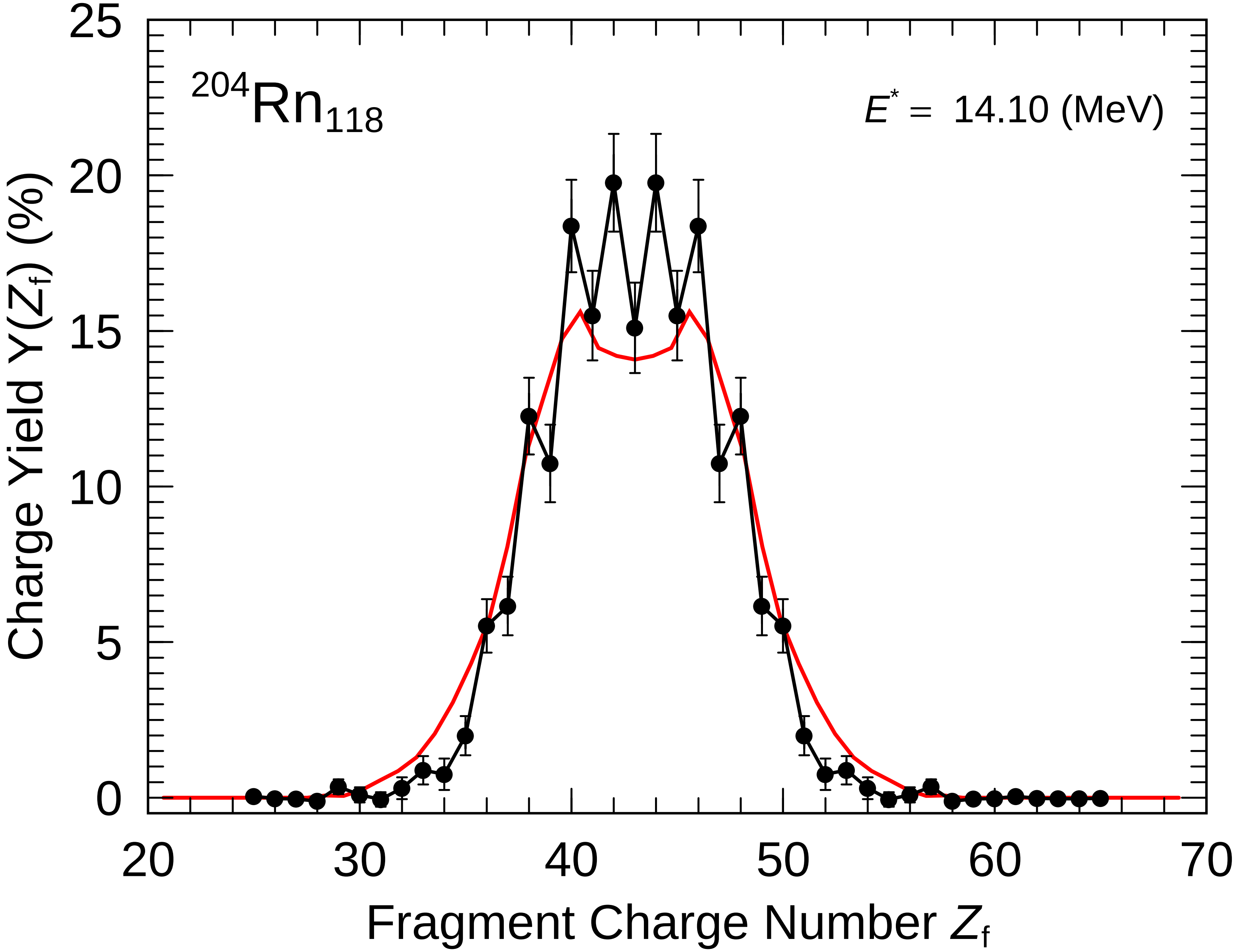
Calc. (6.84 MeV)
Exp. ^{239}Pu(n,f)
^{240}Pu
a
Calc. (6.54 MeV)
Exp. ^{235}U(n,f)
^{236}U
b
Calc. (6.54 MeV)
Exp. ^{233}U(n,f)
^{234}U
c
Calc. (11.0 MeV)
Exp. ^{234}U(γ,f)
^{234}U
d
Yield $Y(Z_f)$ (%)
Fragment Charge Number Z_f



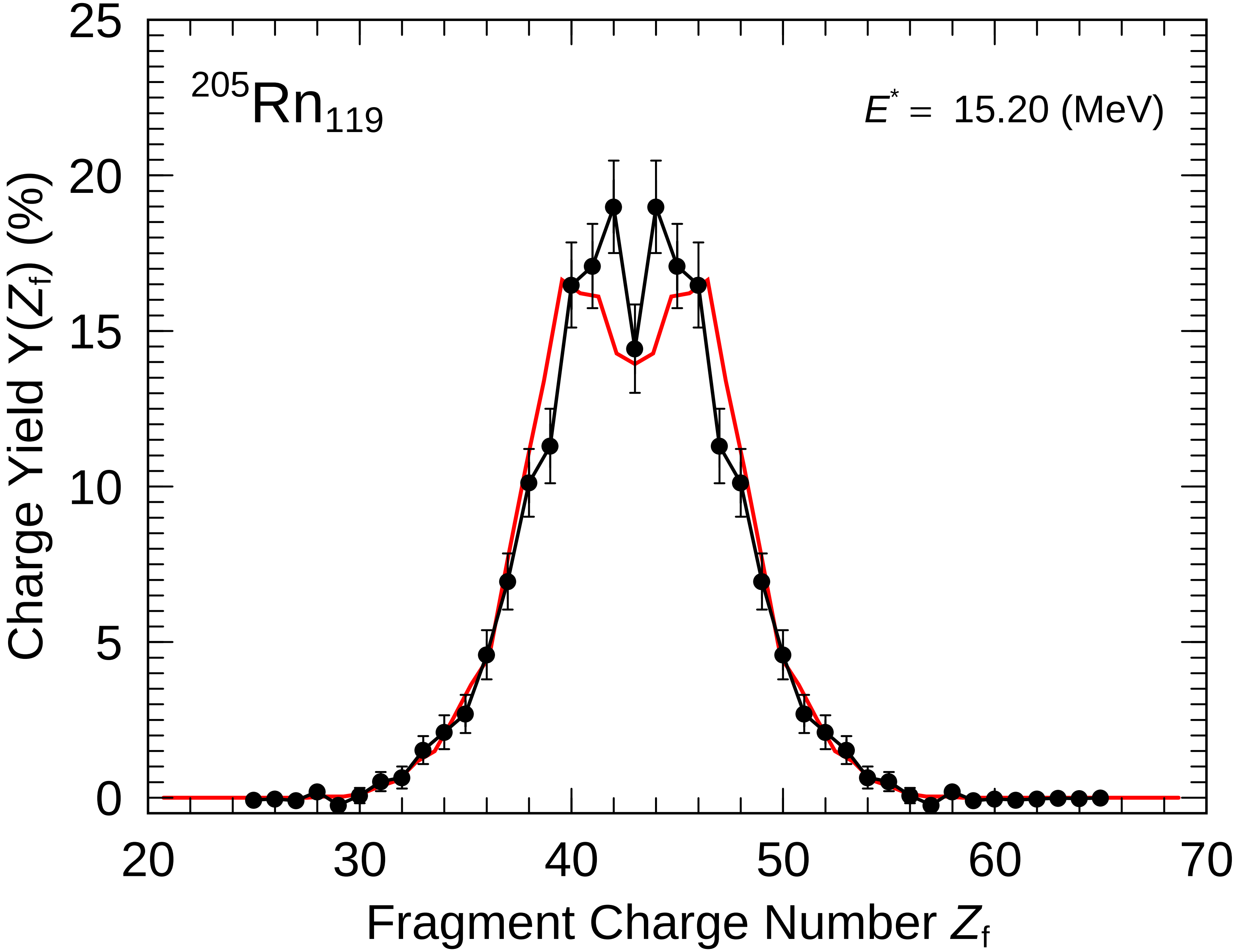
25
20
15
10
5
0
20
30
40
50
60
70
$^{205}At_{120}$
E^{*} = 17.41 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f



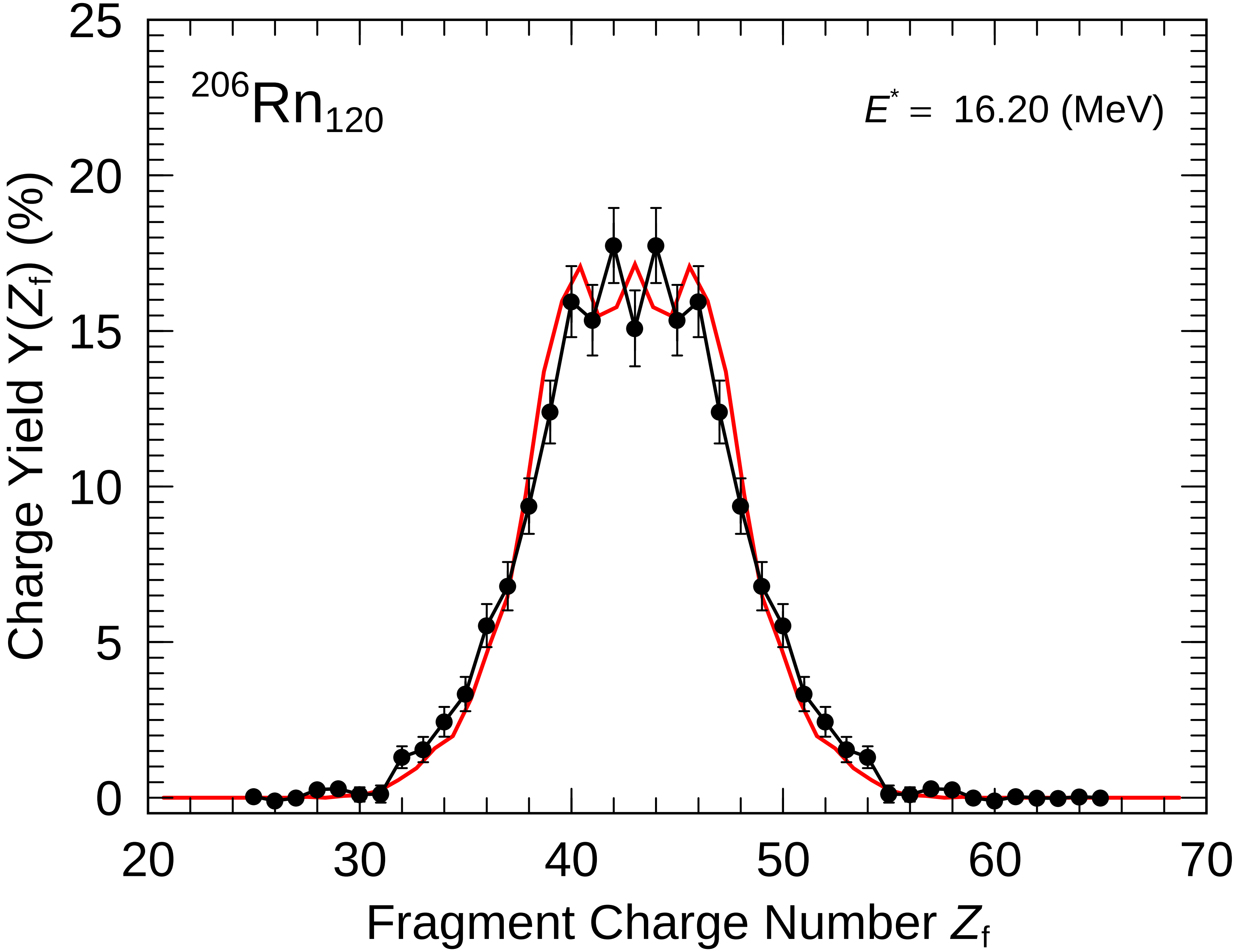
$^{206}At_{121}$
$E^* =$ 18.63 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



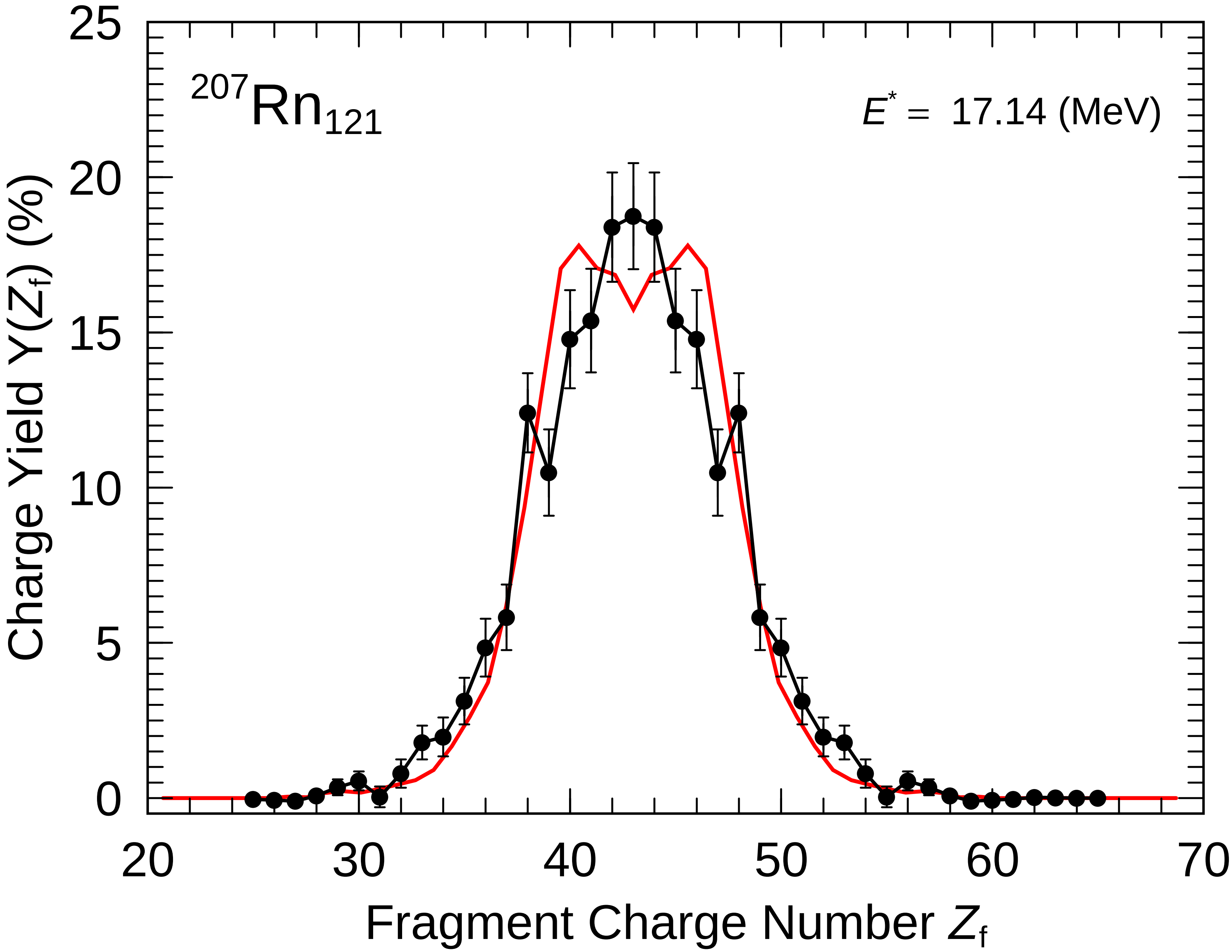
$^{204}Rn_{118}$
$E^* =$ 14.10 (MeV)
Charge Yield $Y(Z_f)$ (%)
Fragment Charge Number Z_f
25
20
15
10
5
0
20
30
40
50
60
70



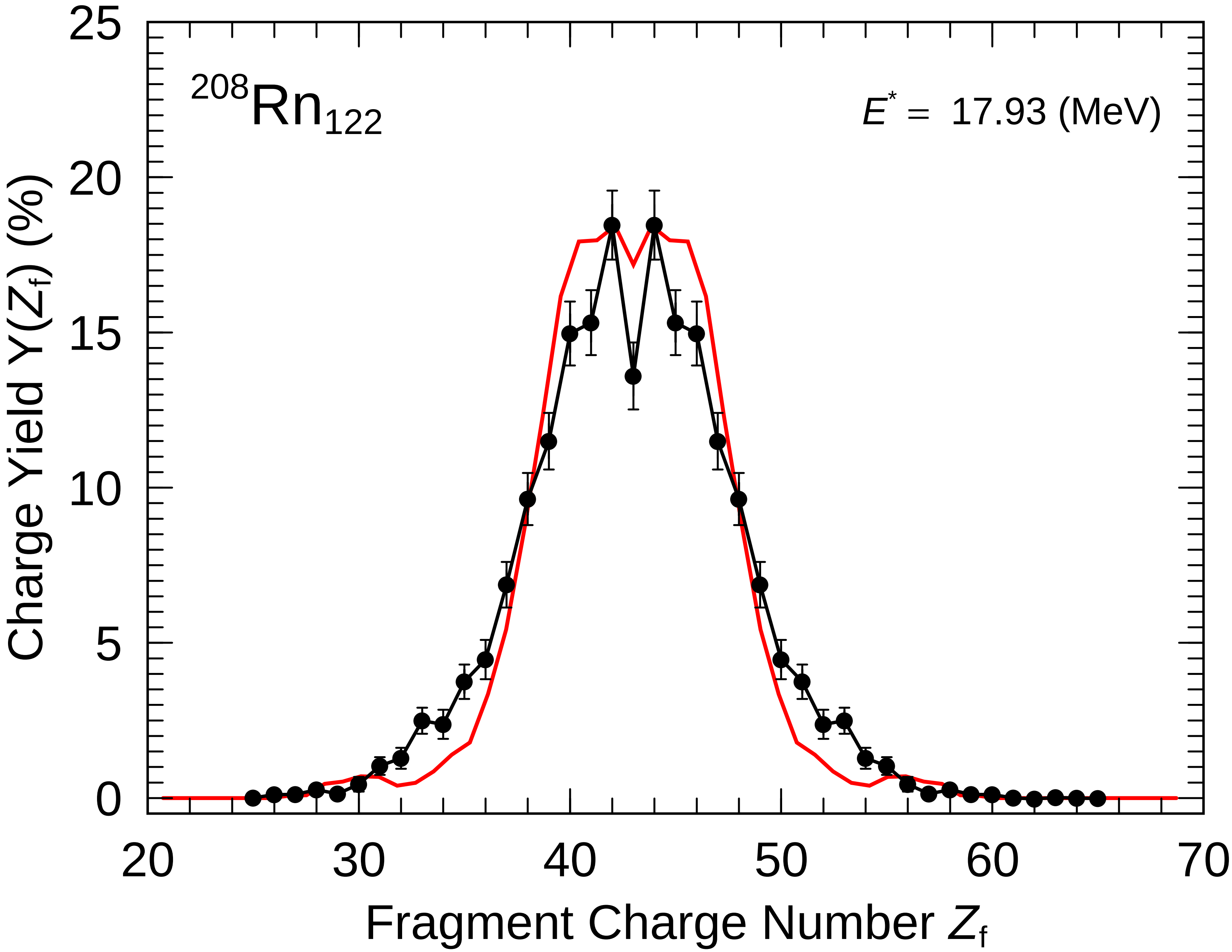
$^{205}Rn_{119}$
$E^* =$ 15.20 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



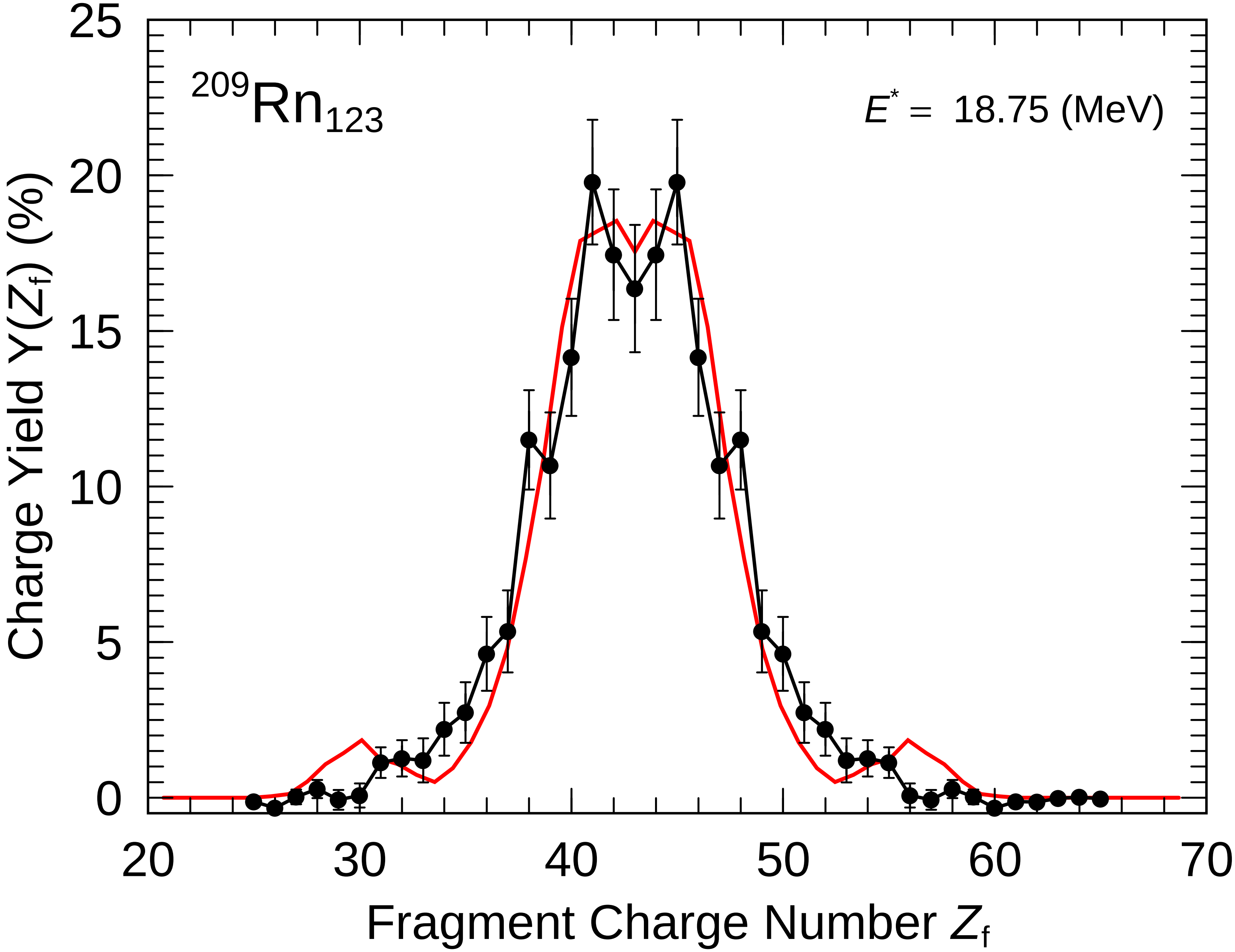
25
20
15
10
5
0
20
30
40
50
60
70
$^{206}Rn_{120}$
E^{*} = 16.20 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f



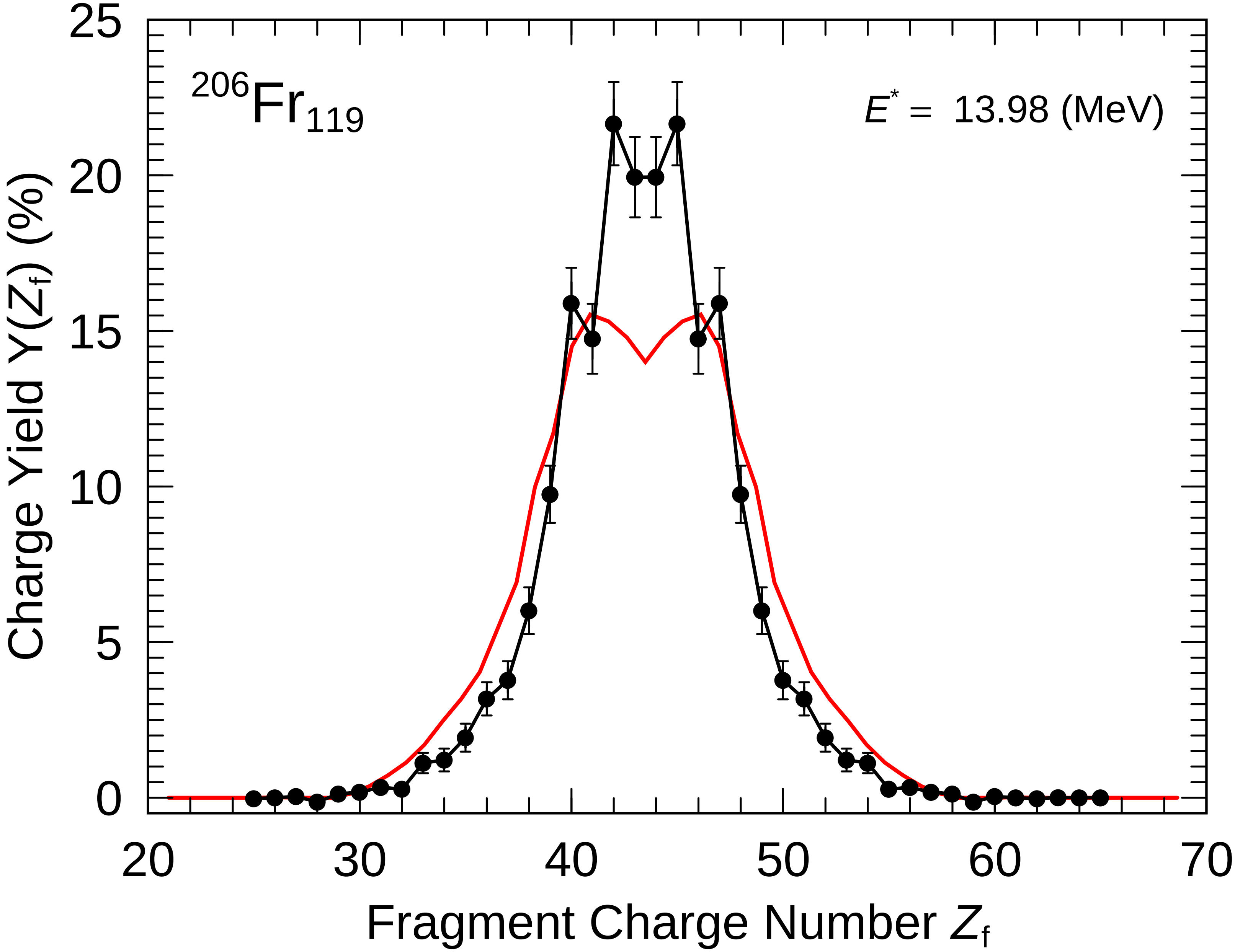
$^{207}Rn_{121}$
$E^* =$ 17.14 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



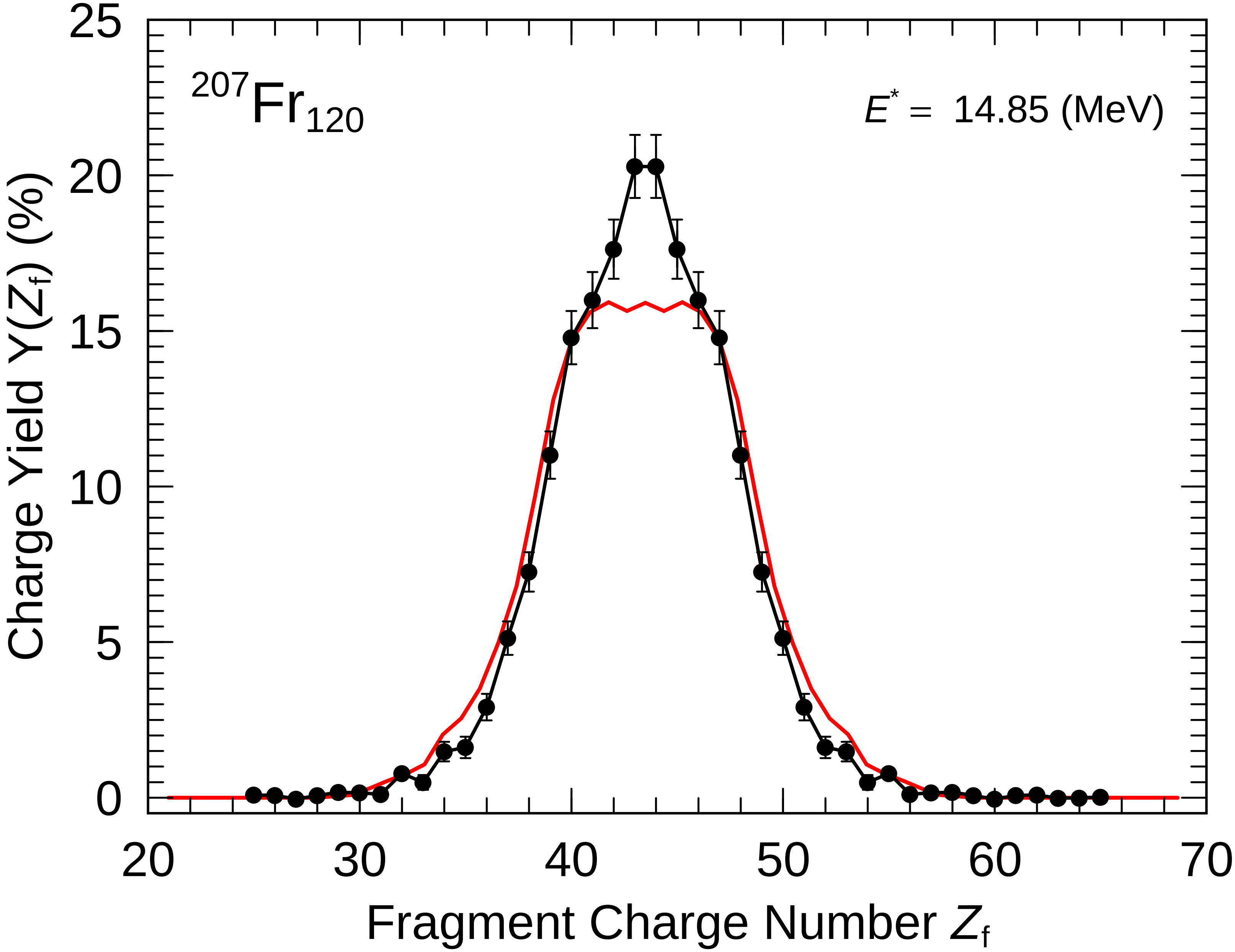
$^{208}Rn_{122}$
$E^* =$ 17.93 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



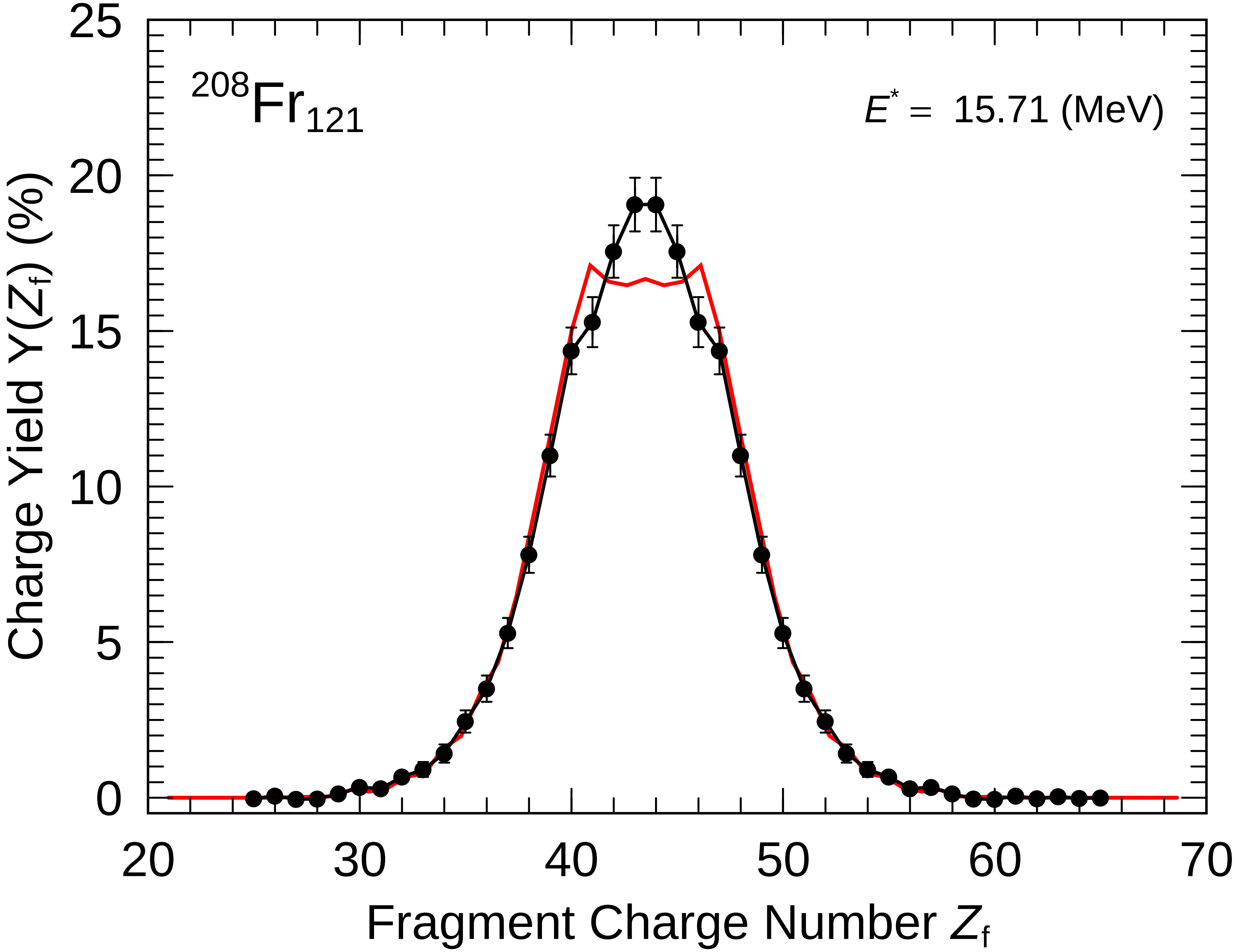
$^{209}Rn_{123}$
$E^* =$ 18.75 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



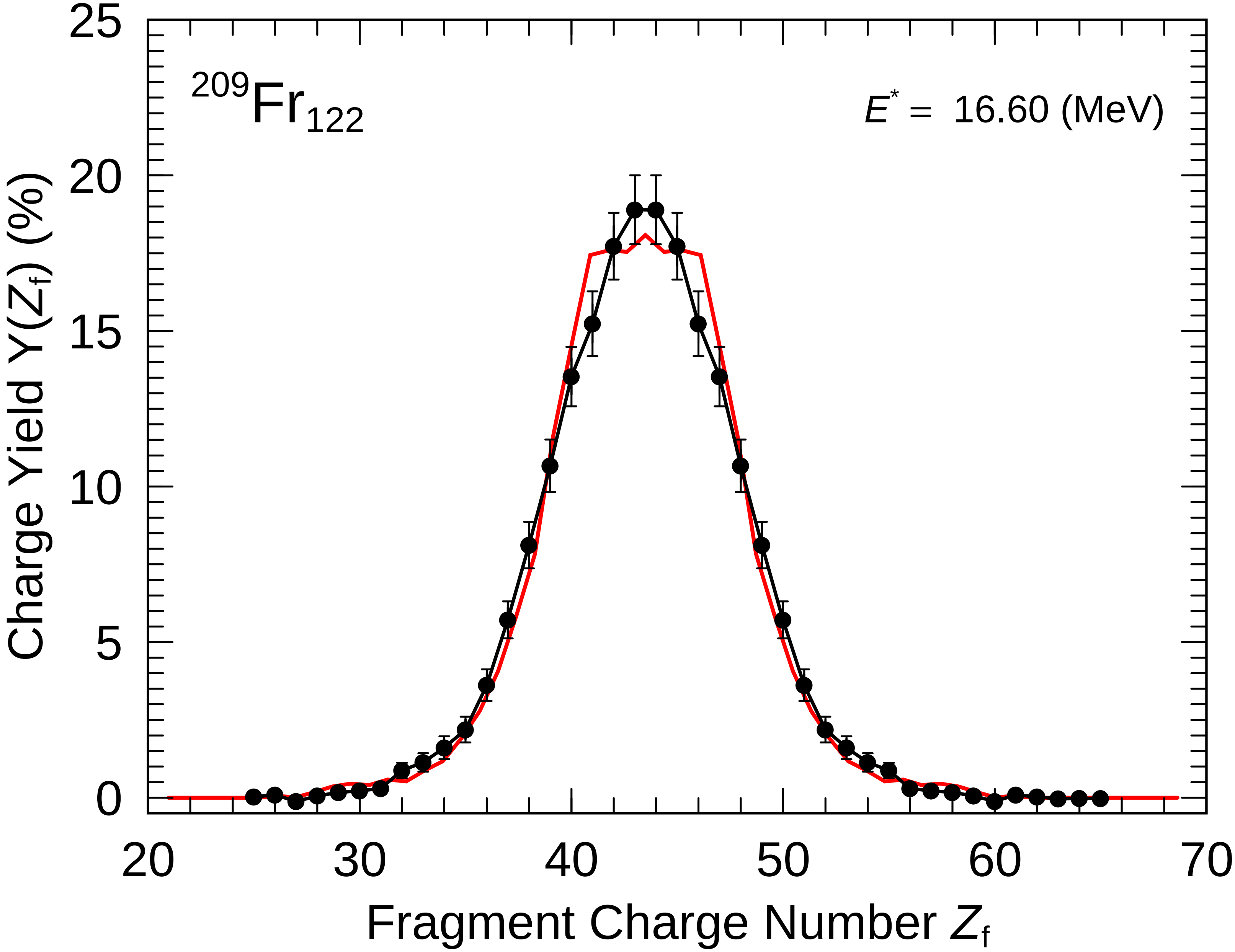
25
20
15
10
5
0
Charge Yield Y(Zf) (%)
206Fr119
E* = 13.98 (MeV)
20
30
40
50
60
70
Fragment Charge Number Zf



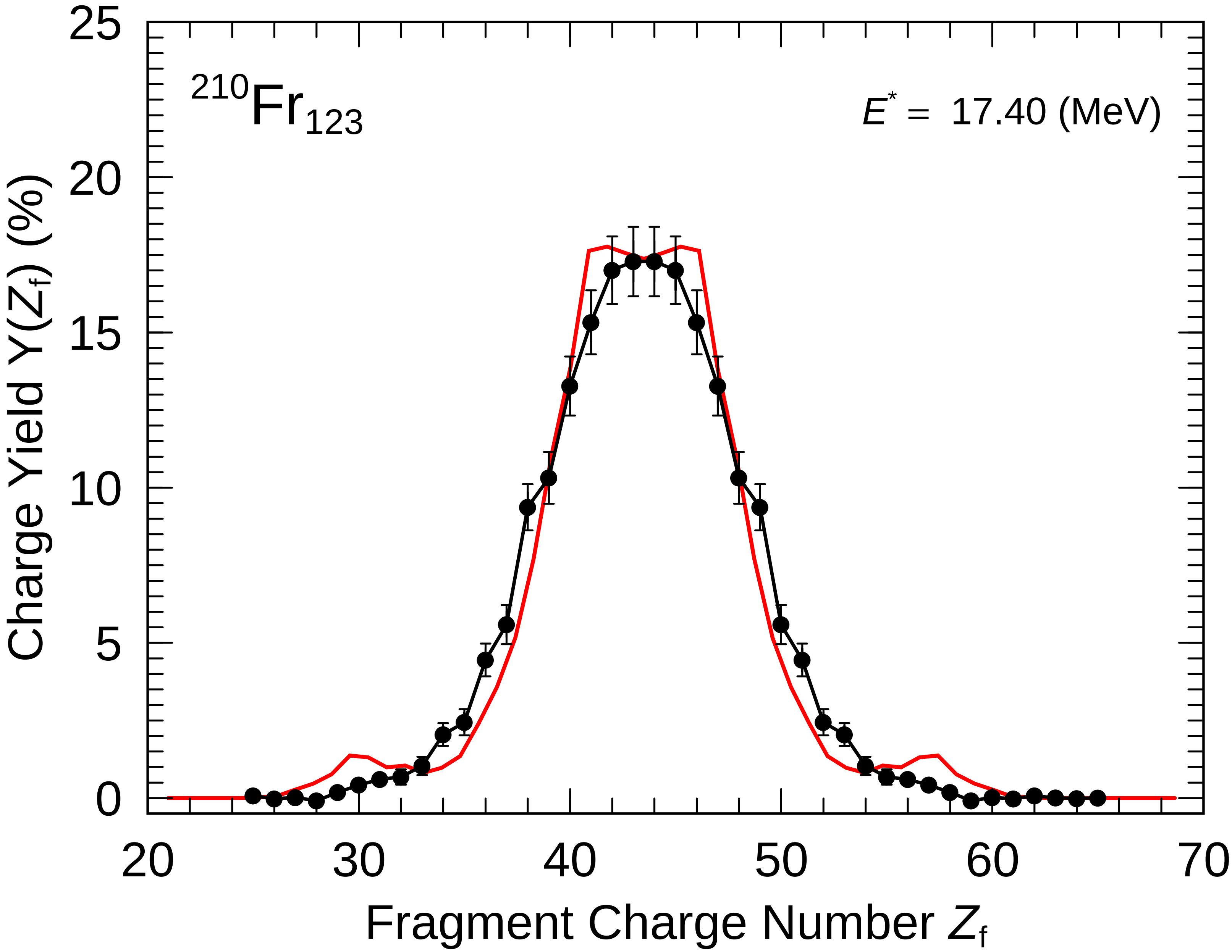
207Fr120
E* = 14.85 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
20
30
40
50
60
70



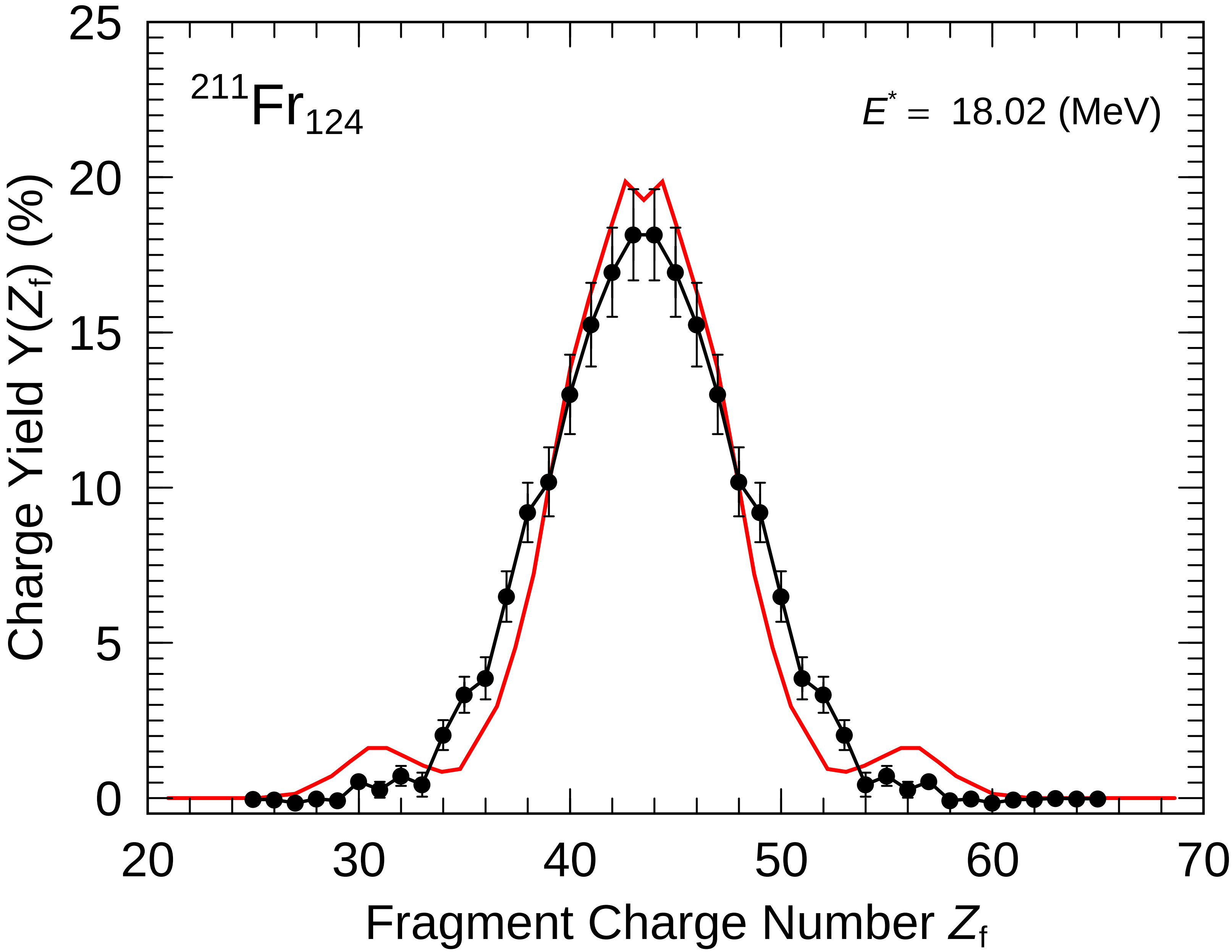
$^{208}Fr_{121}$
$E^* =$ 15.71 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



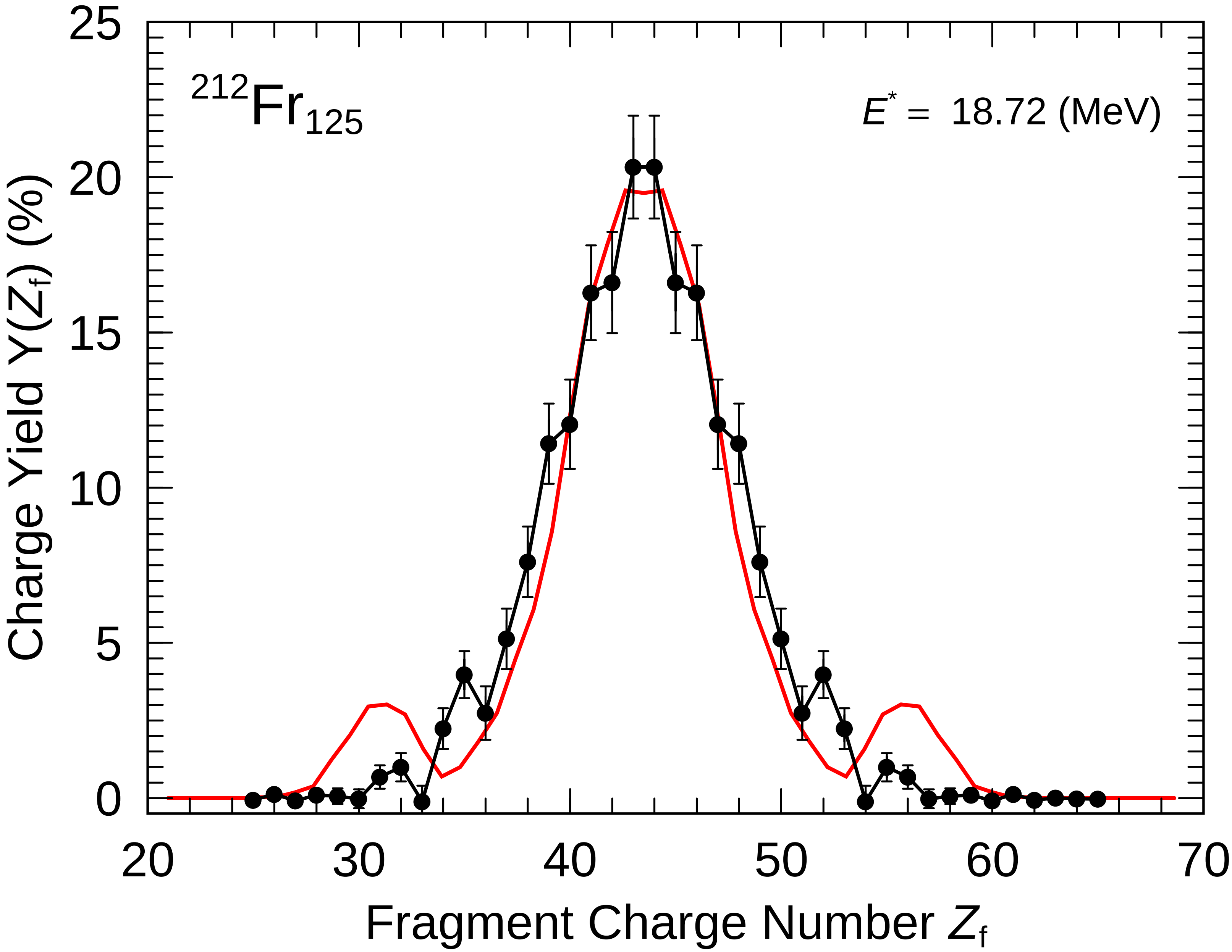
$^{209}Fr_{122}$
E^{*} = 16.60 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



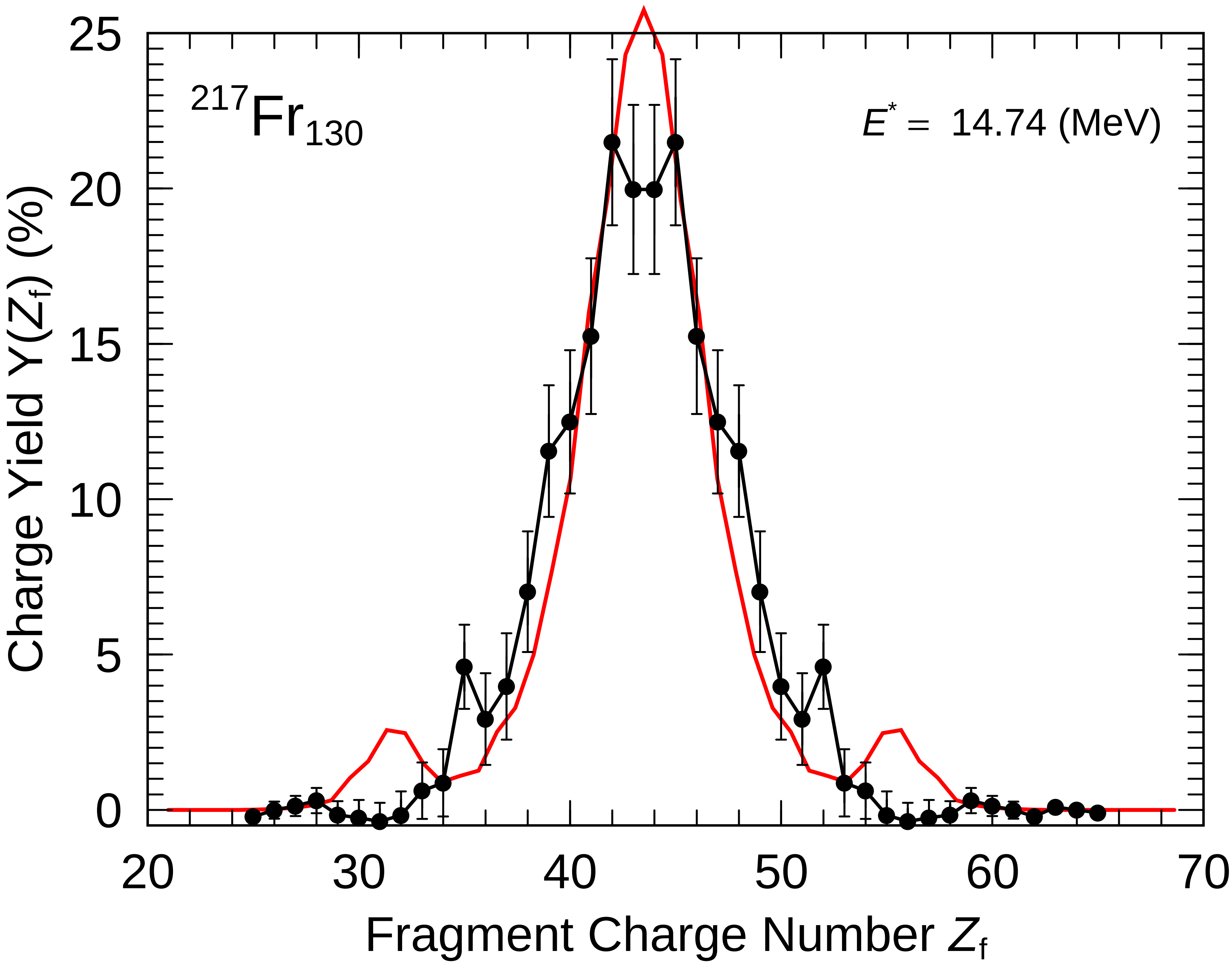
25
20
15
10
5
0
20
30
40
50
60
70
$^{210}Fr_{123}$
E^{*} = 17.40 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f



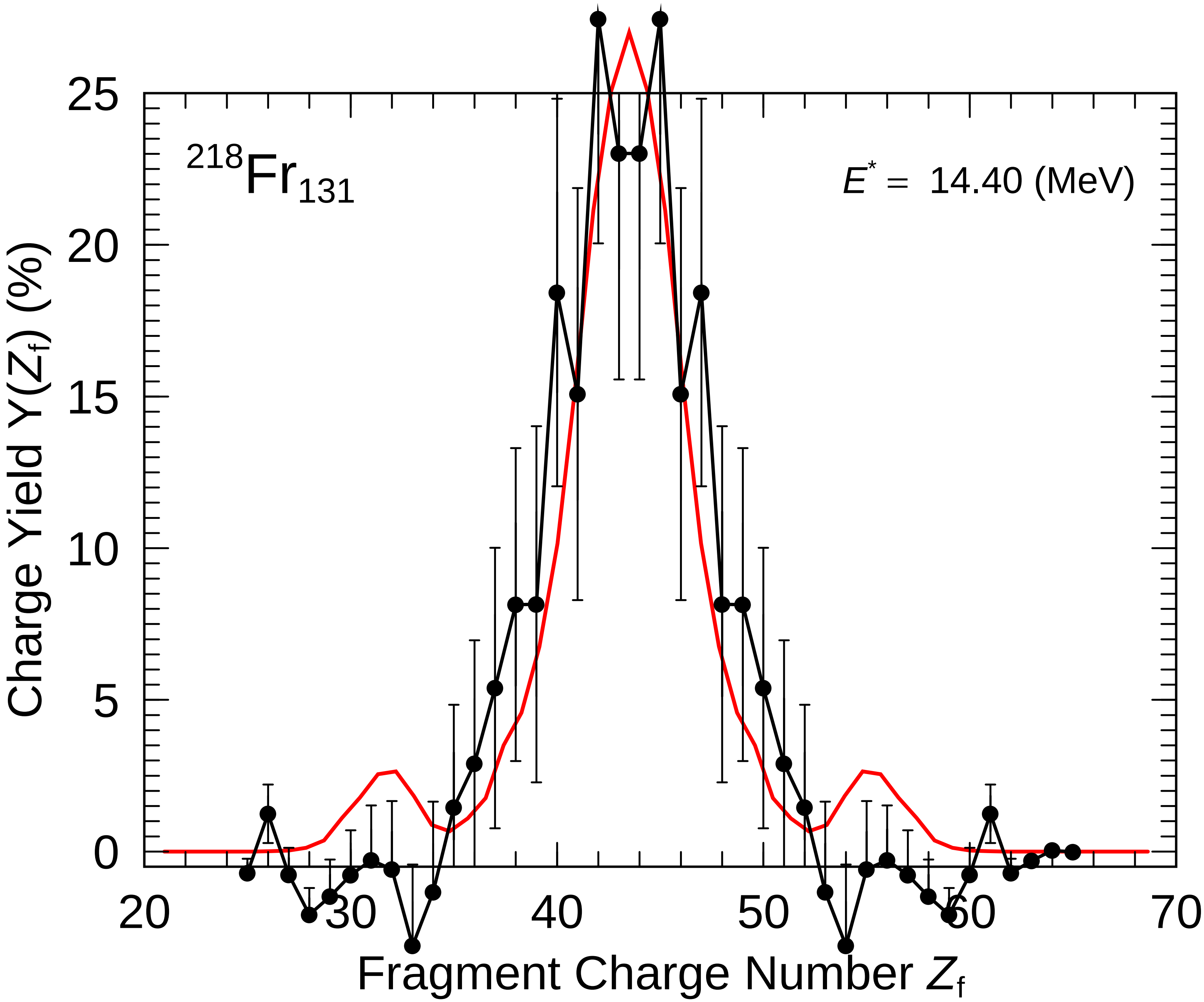
25
20
15
10
5
0
Charge Yield Y(Zf) (%)
211Fr124
E* = 18.02 (MeV)
20
30
40
50
60
70
Fragment Charge Number Zf



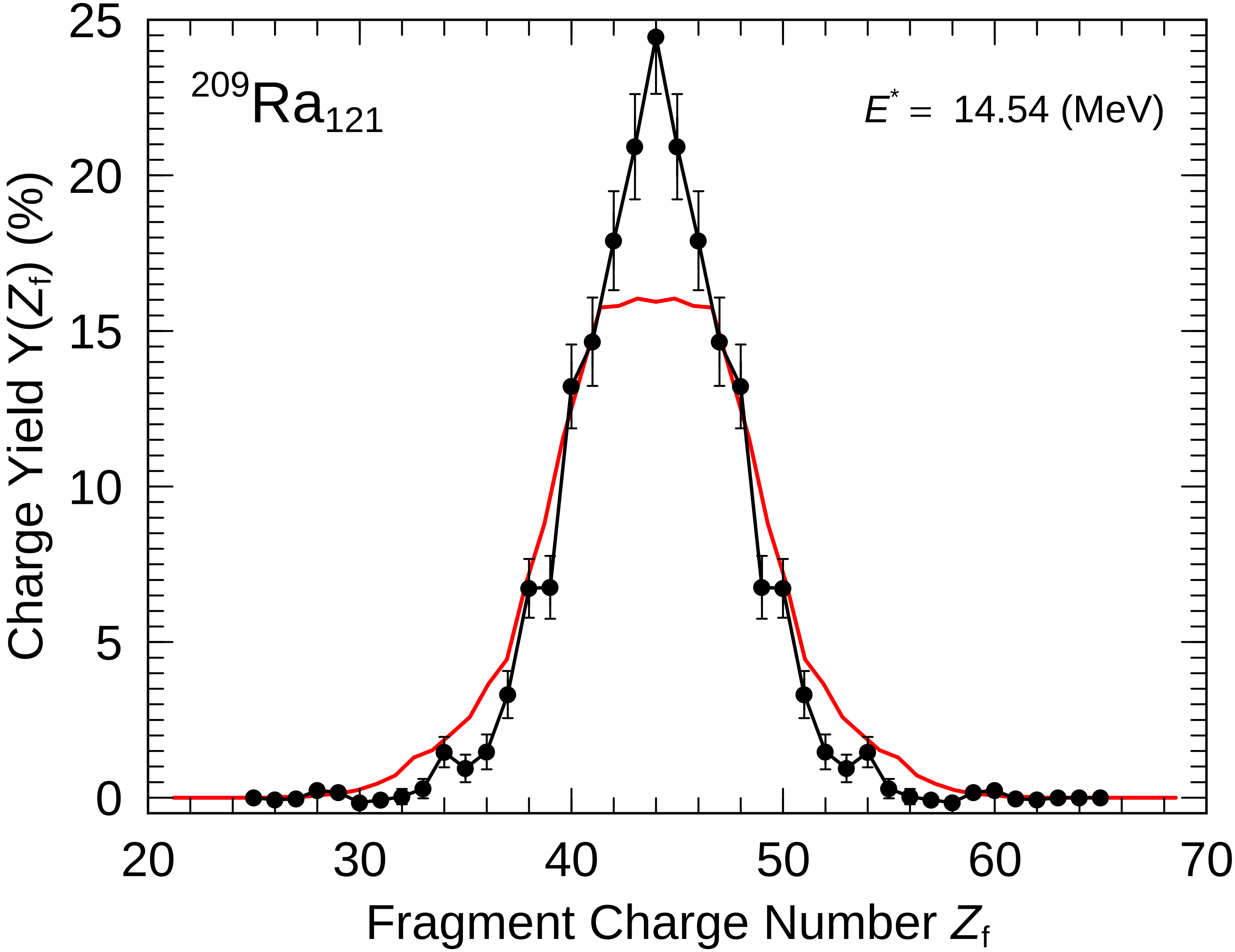
212Fr125
E* = 18.72 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
20
30
40
50
60
70



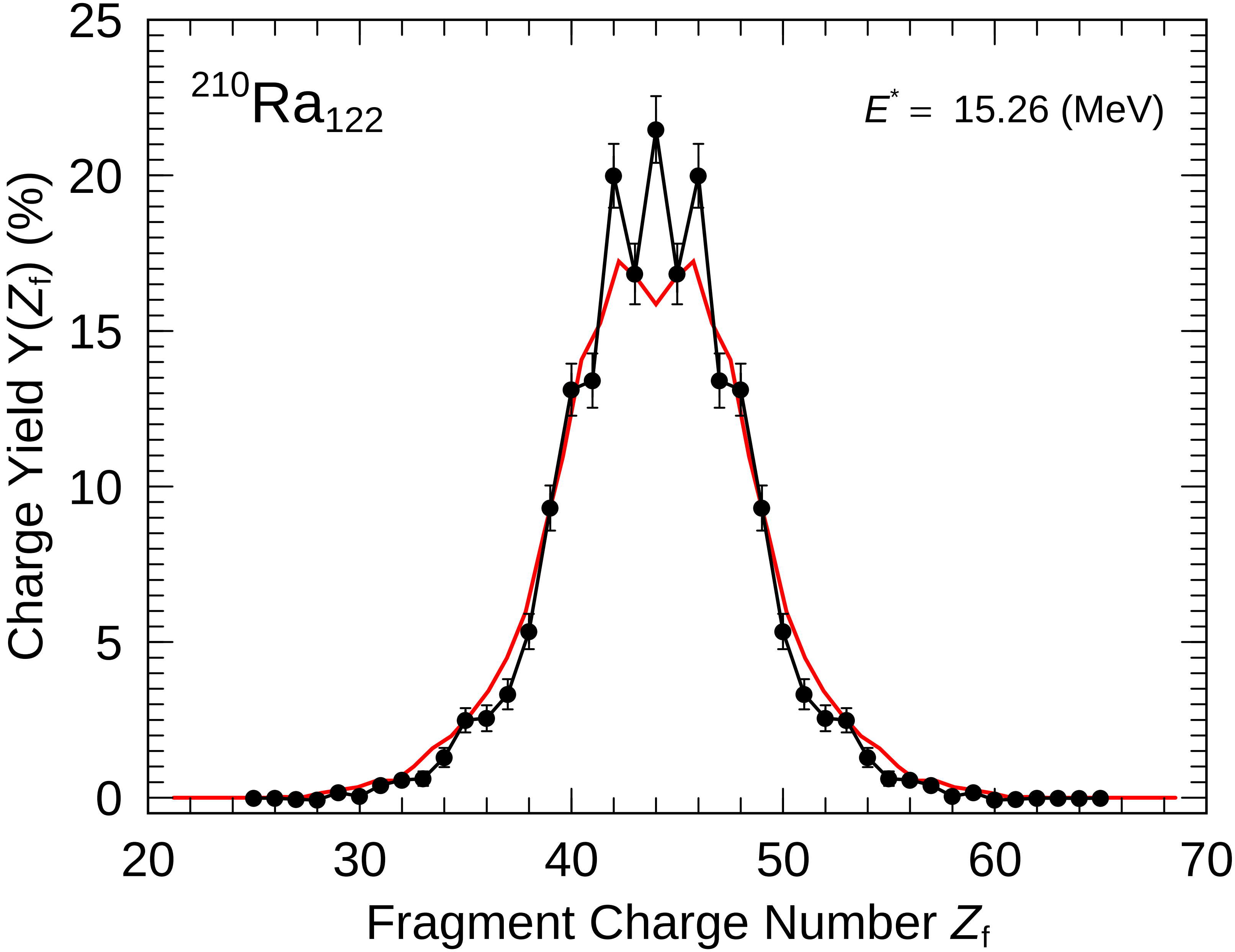
217Fr130
E* = 14.74 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
25
20
15
10
5
0
20
30
40
50
60
70



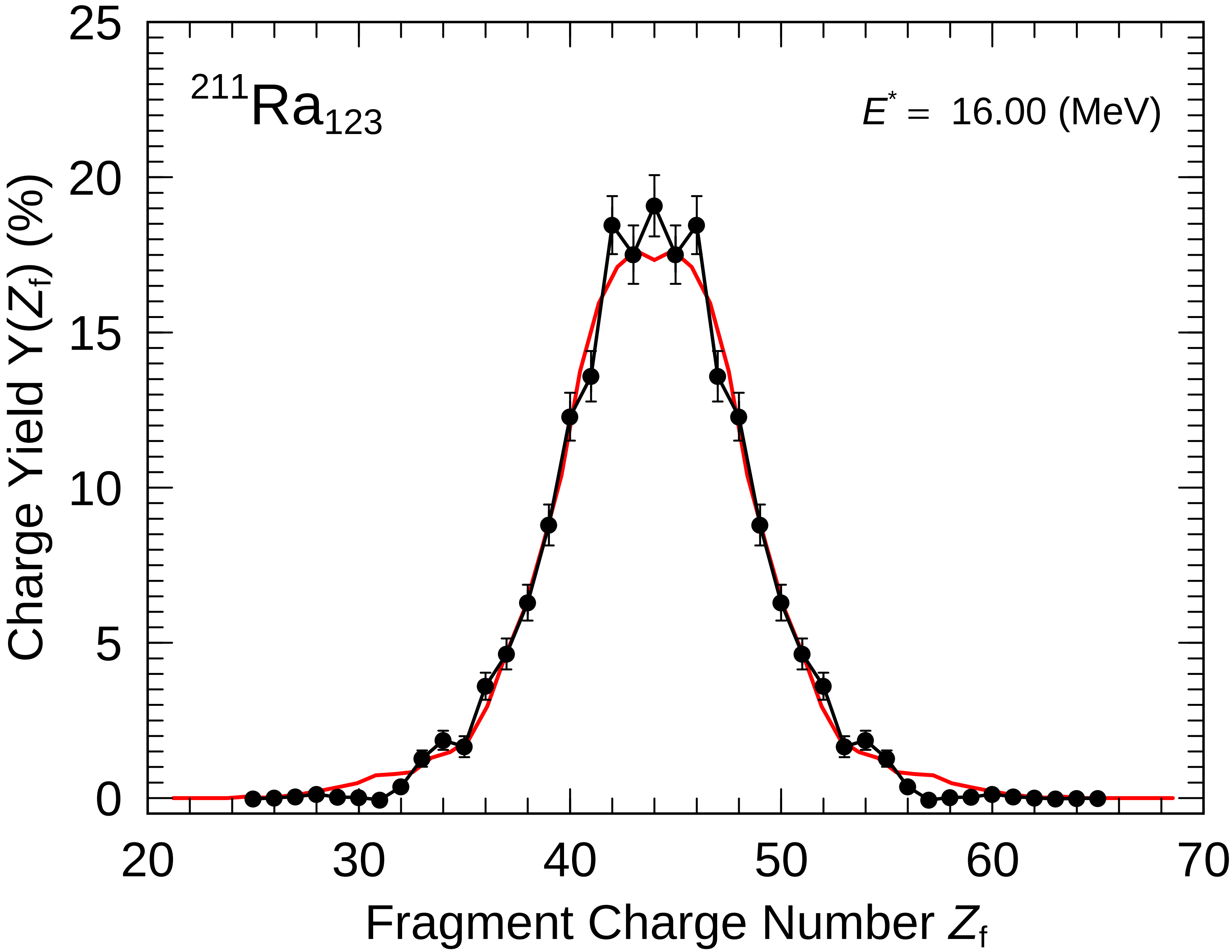
$^{218}Fr_{131}$
E^{*} = 14.40 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



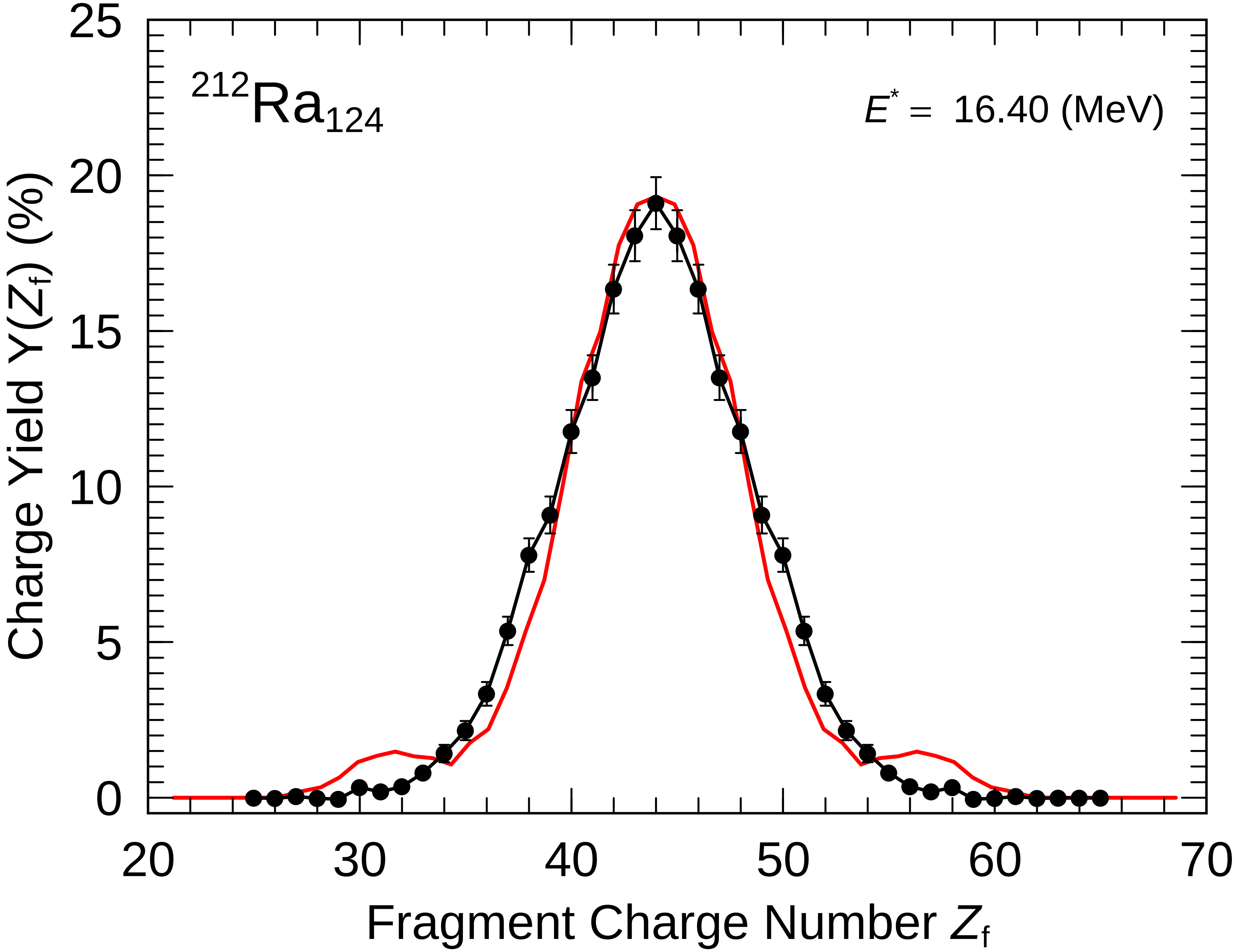
209Ra121
E* = 14.54 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
20
30
40
50
60
70



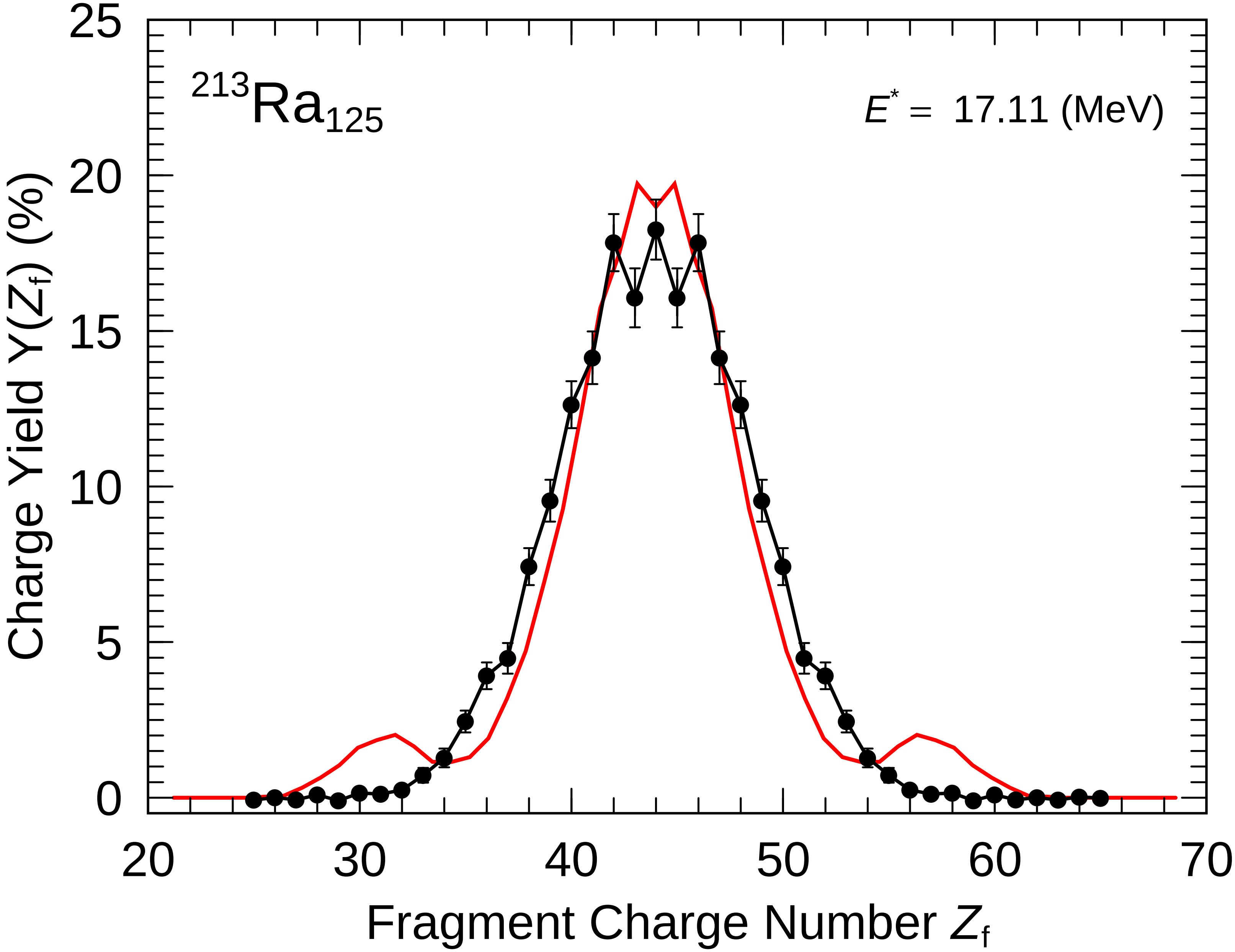
$^{210}Ra_{122}$
$E^* =$ 15.26 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



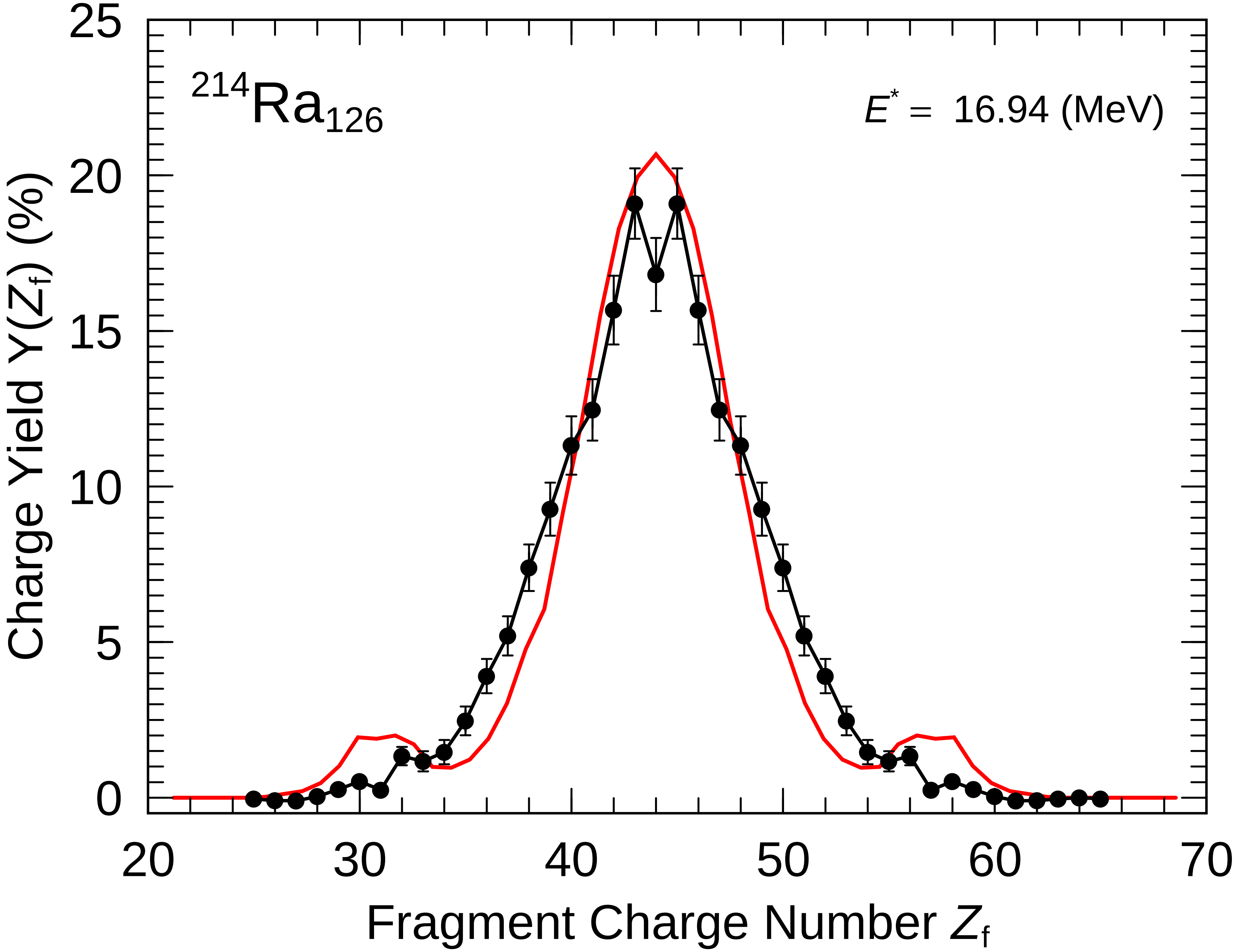
25
20
15
10
5
0
20
30
40
50
60
70
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
211Ra123
E* = 16.00 (MeV)



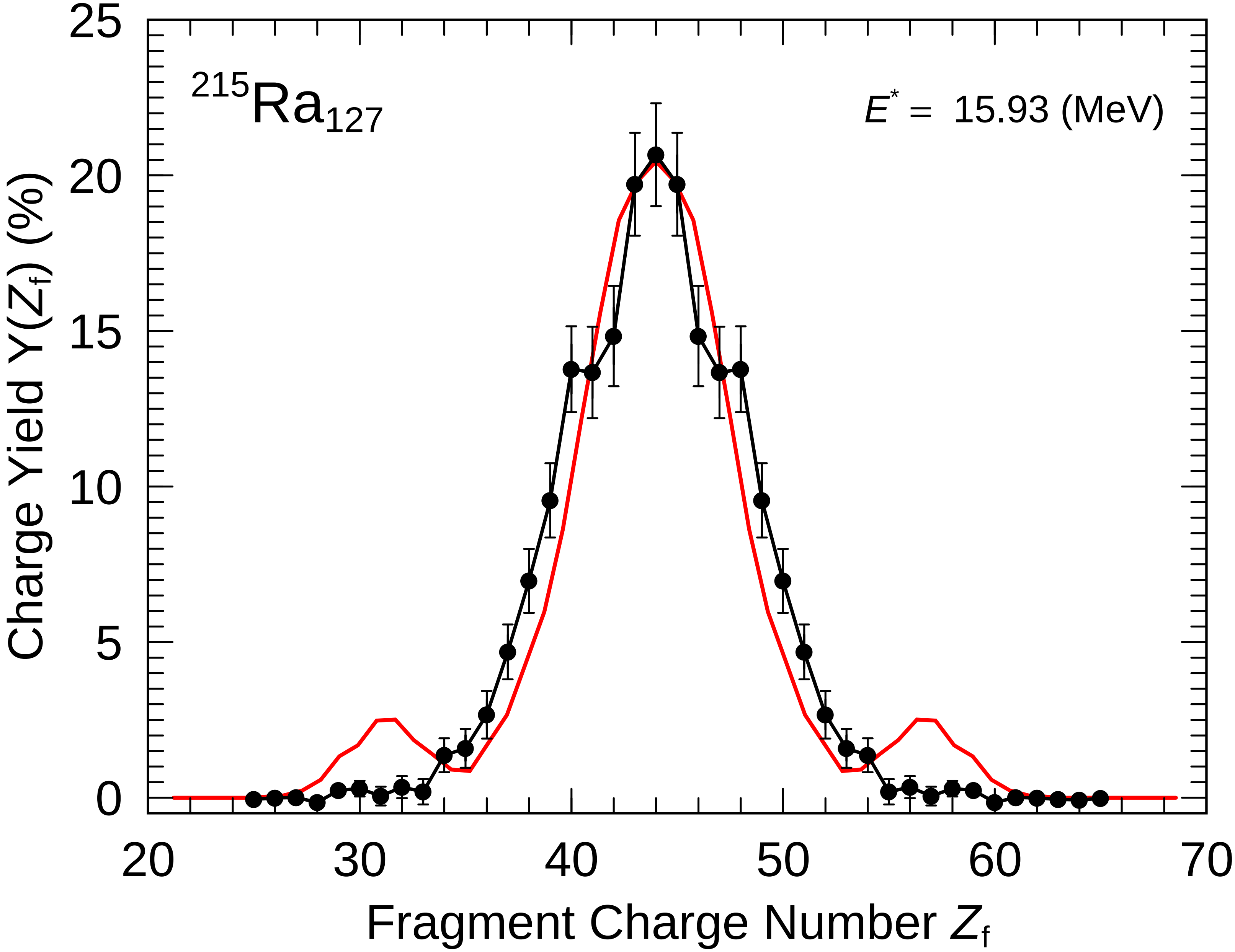
25
20
15
10
5
0
Charge Yield Y(Zf) (%)
212Ra124
E* = 16.40 (MeV)
20
30
40
50
60
70
Fragment Charge Number Zf



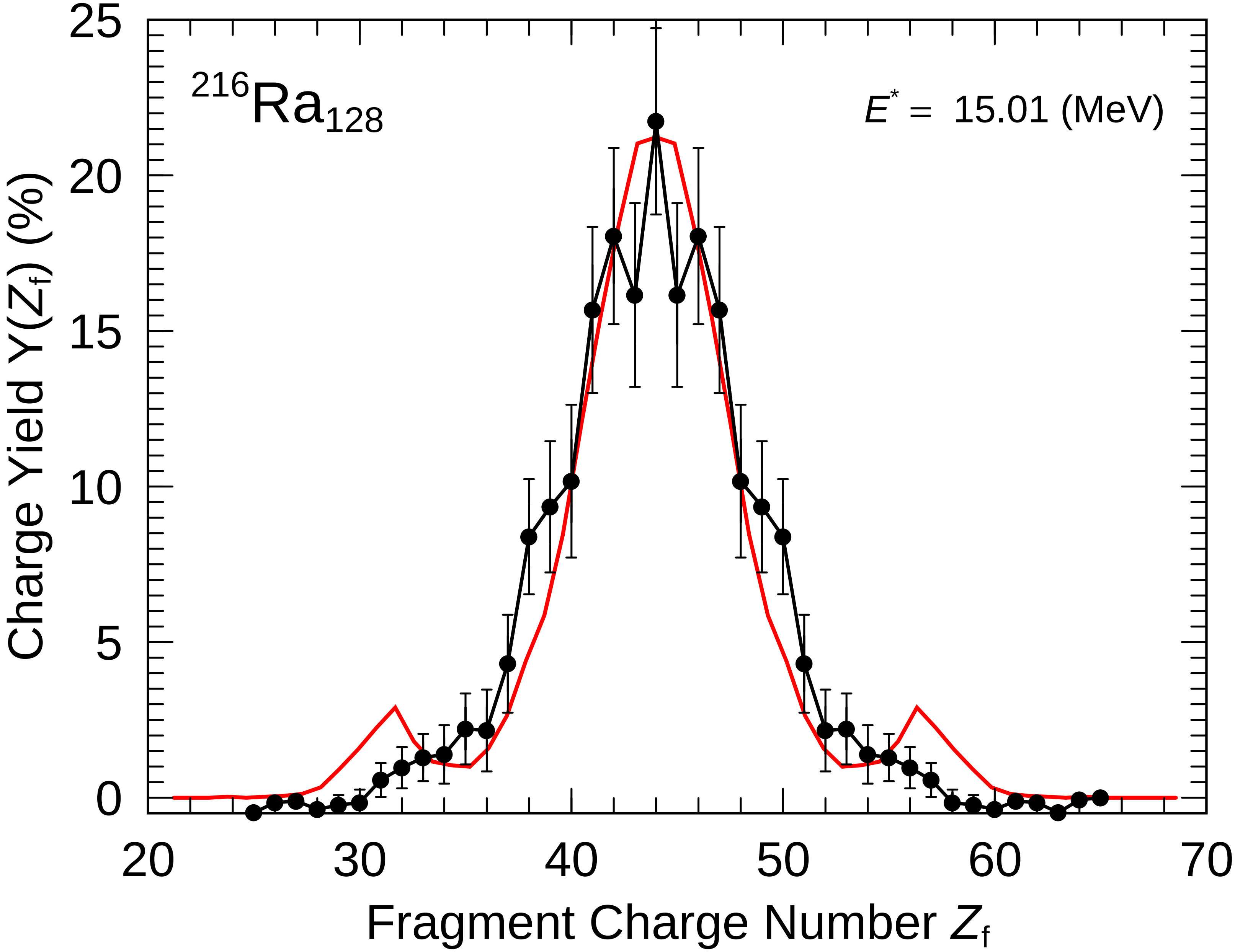
213Ra125
E* = 17.11 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
20
30
40
50
60
70



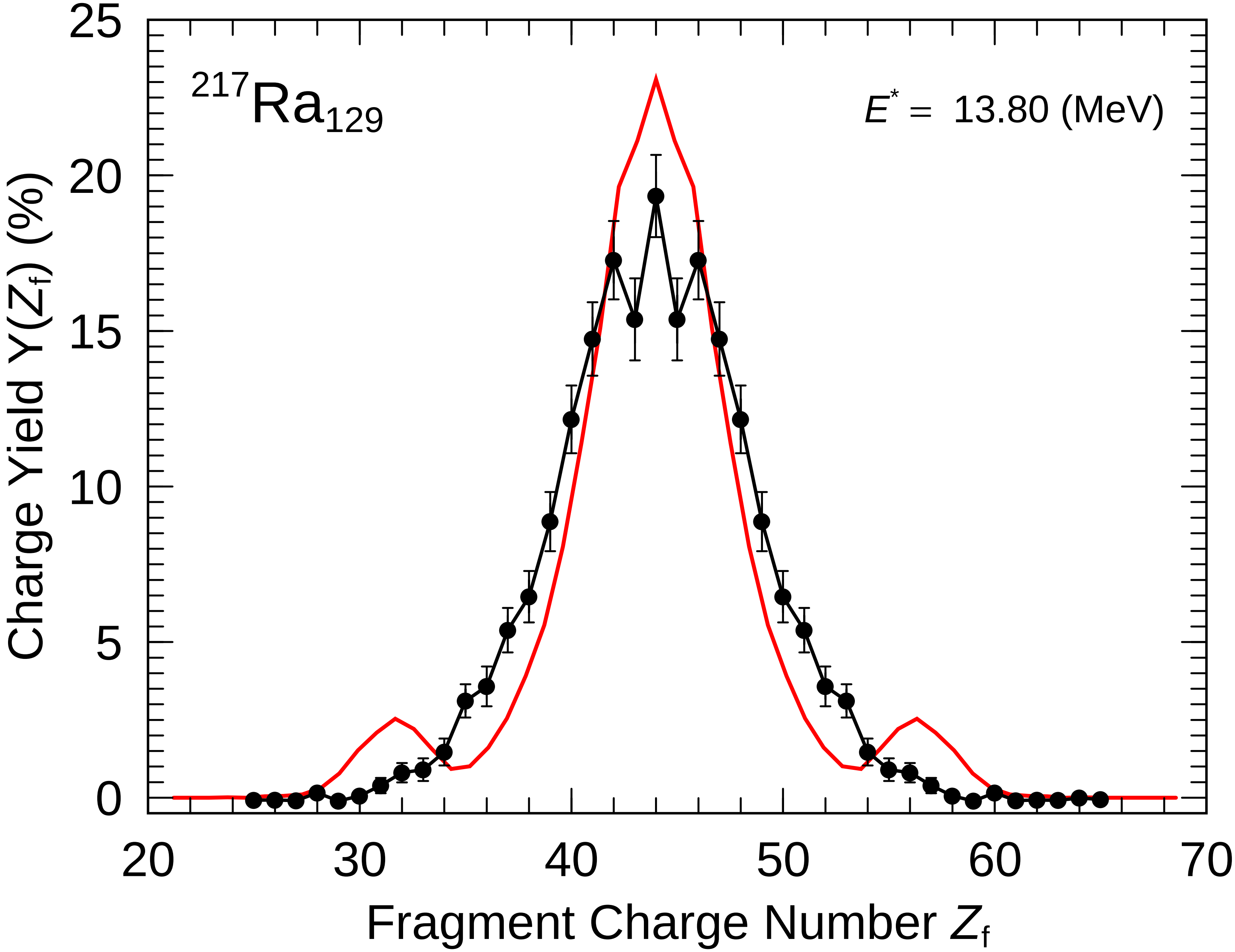
214Ra126
E* = 16.94 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
20
30
40
50
60
70



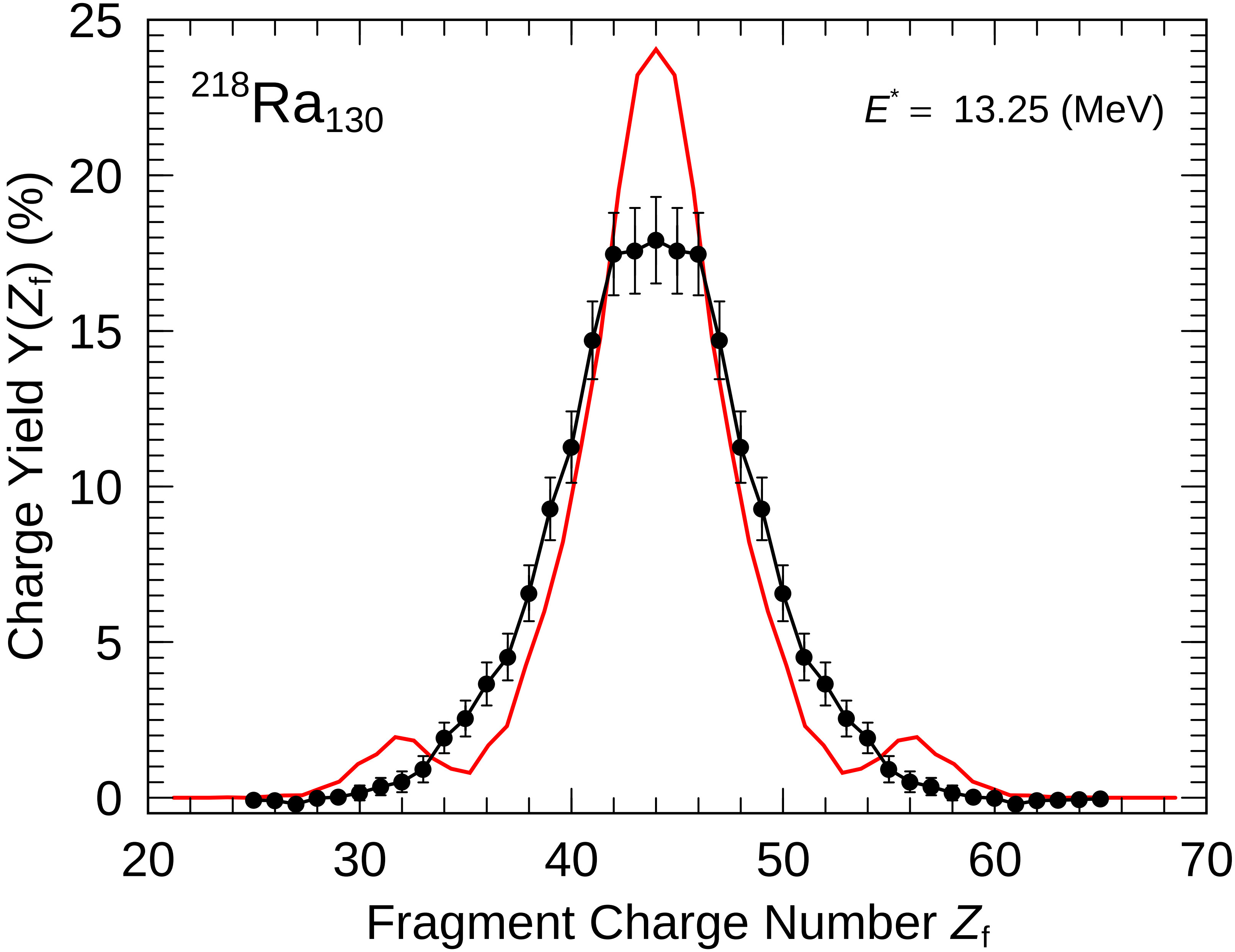
$^{215}Ra_{127}$
$E^* =$ 15.93 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



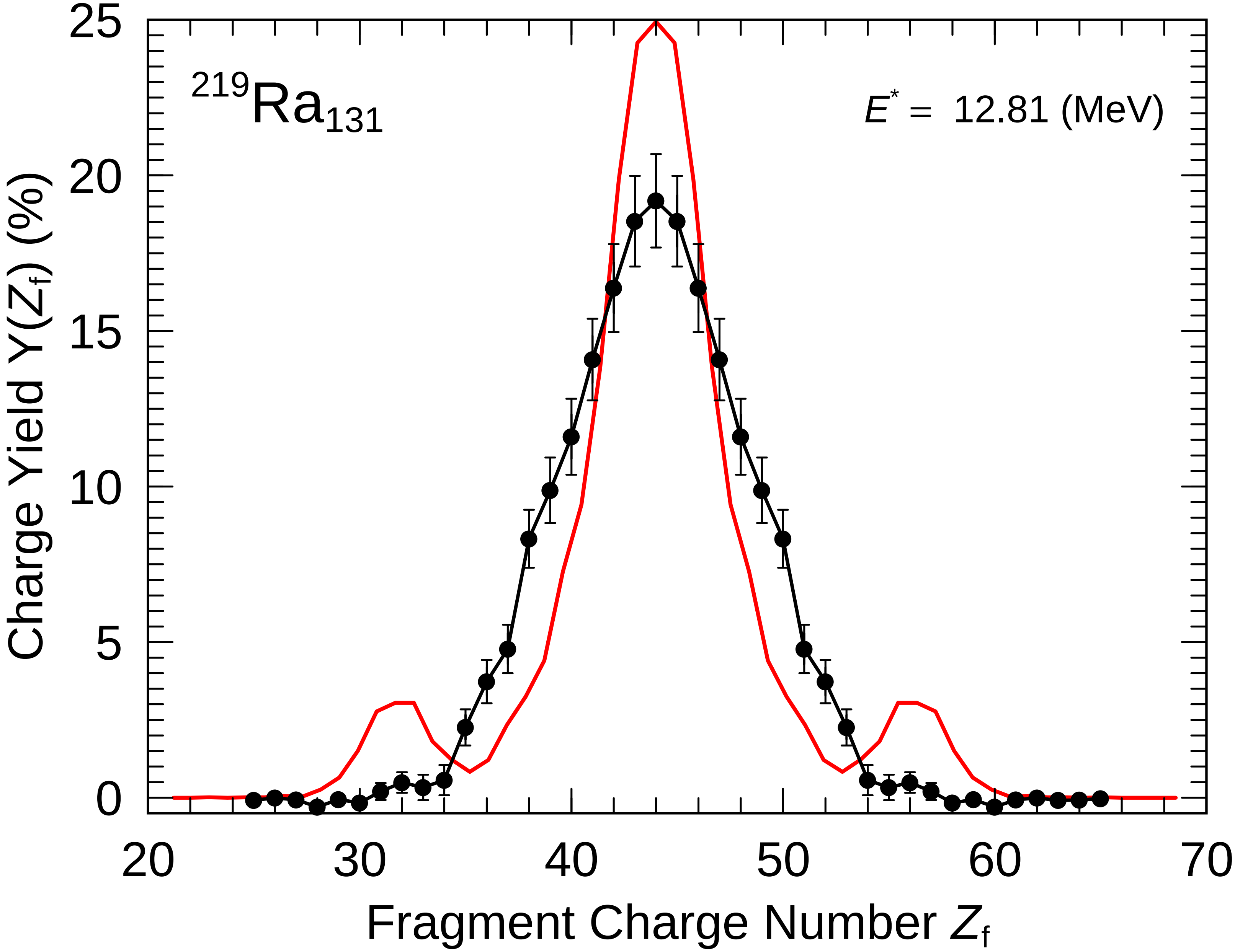
216Ra128
E* = 15.01 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
20
30
40
50
60
70



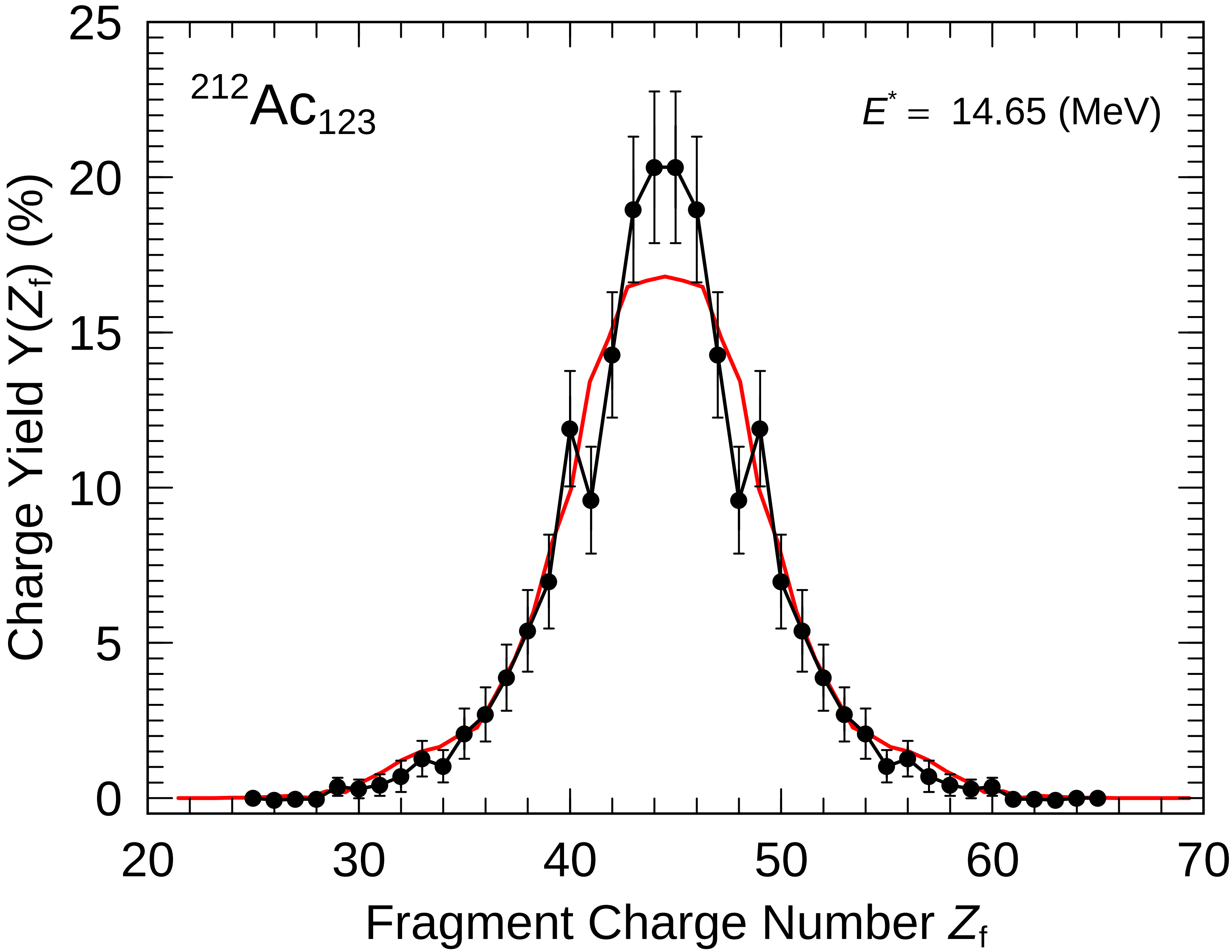
$^{217}Ra_{129}$
$E^* =$ 13.80 (MeV)
Charge Yield $Y(Z_f)$ (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



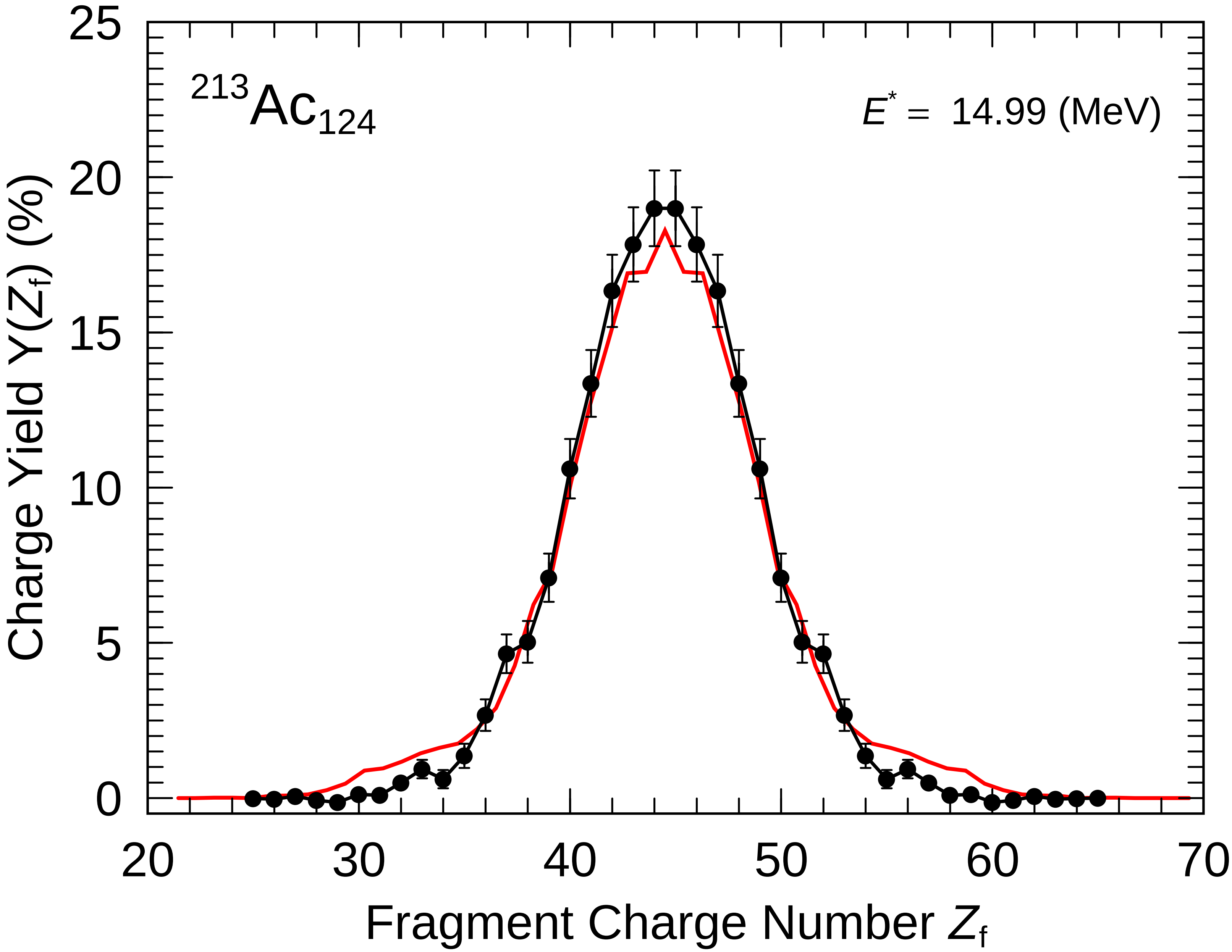
$^{218}Ra_{130}$
E^{*} = 13.25 (MeV)
Charge Yield $Y(Z_f)$ (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



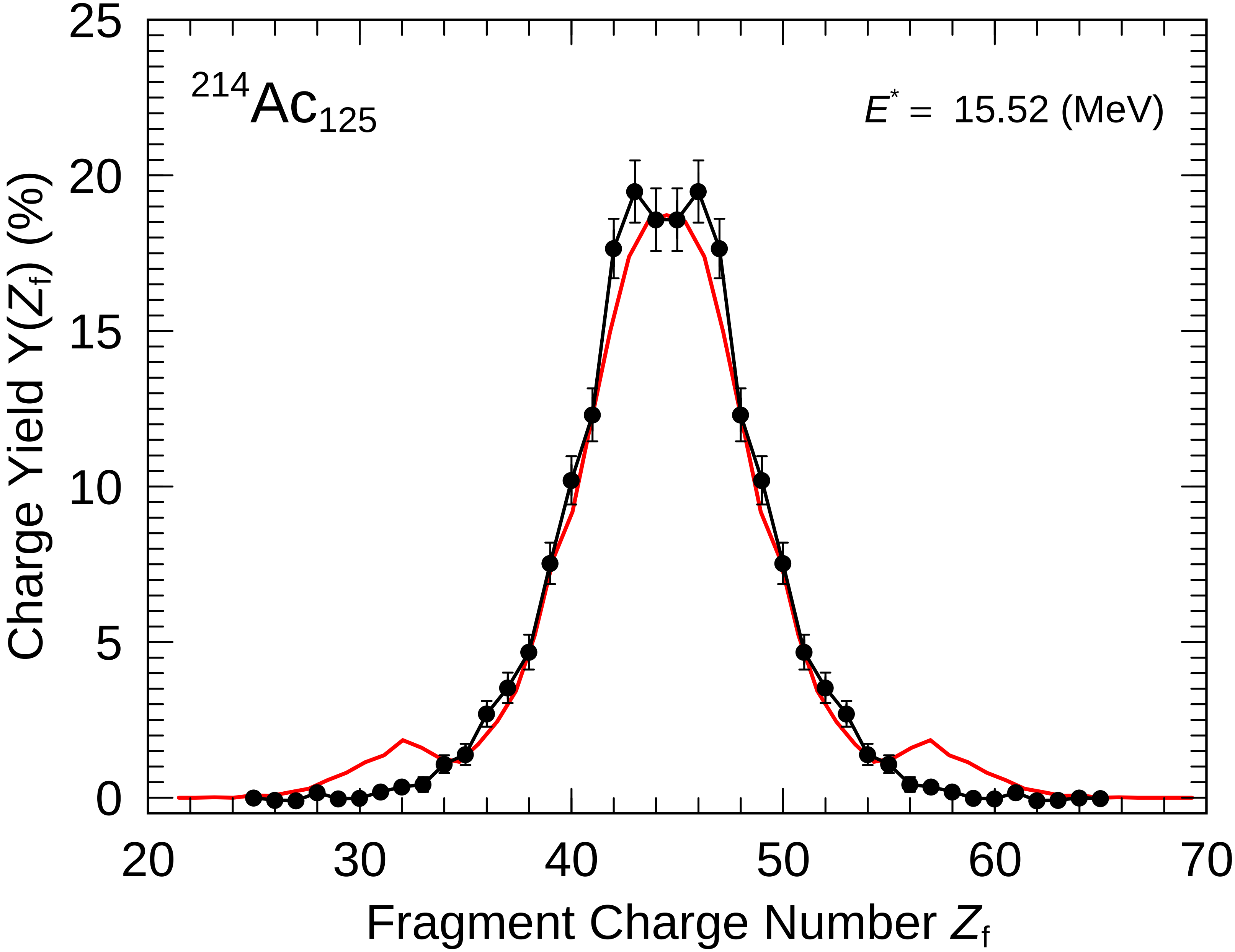
25
20
15
10
5
0
20
30
40
50
60
70
219Ra131
E* = 12.81 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf



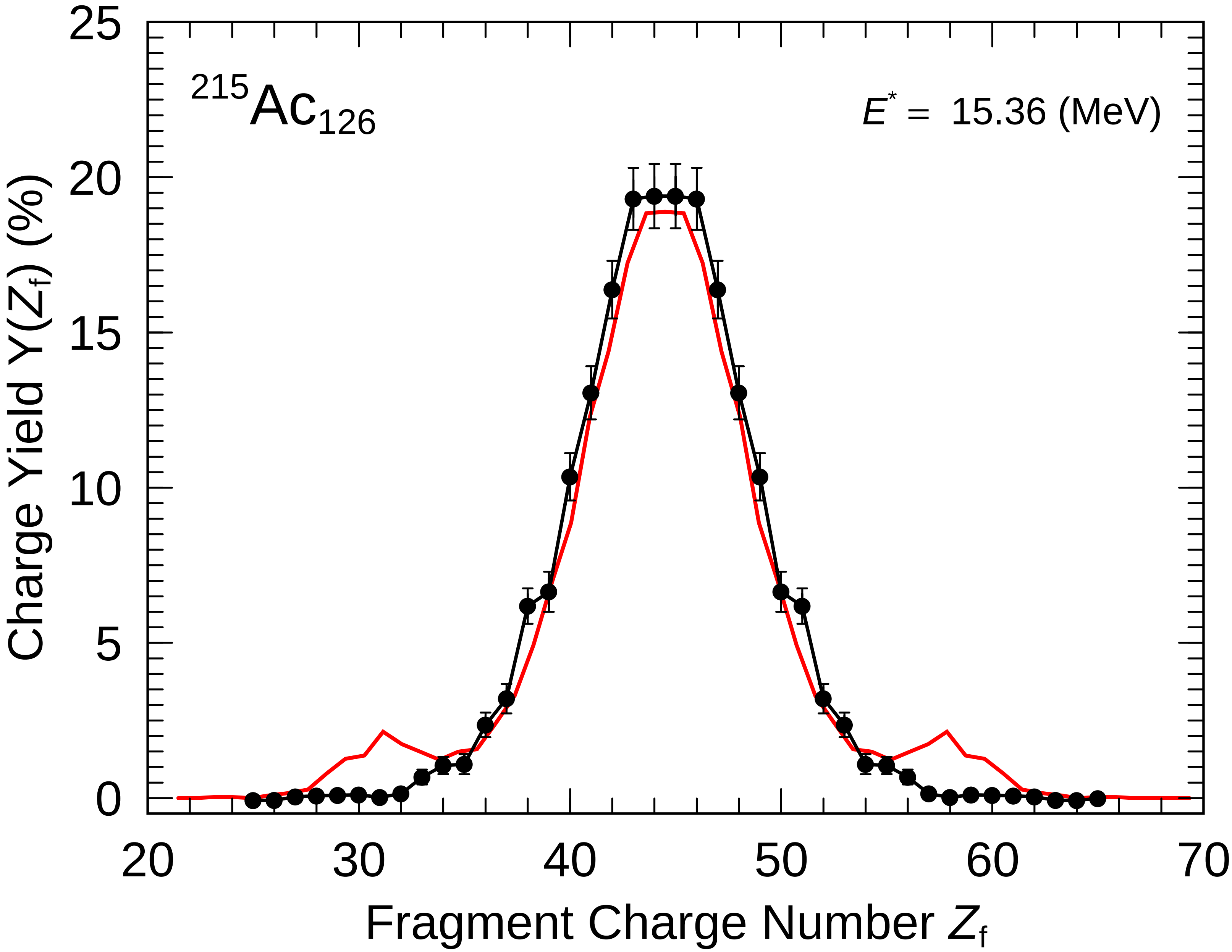
$^{212}Ac_{123}$
$E^* =$ 14.65 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



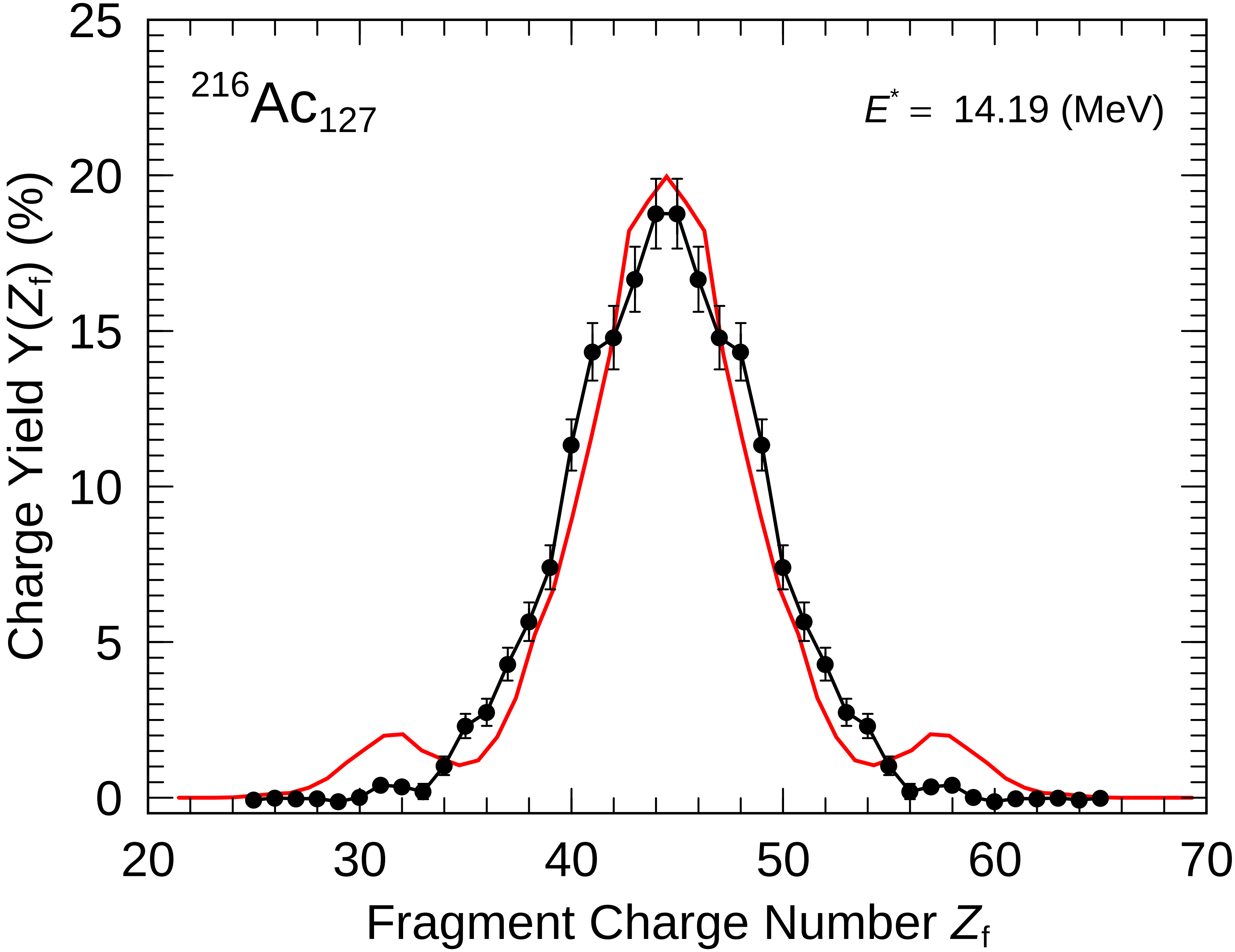
$^{213}Ac_{124}$
$E^* =$ 14.99 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



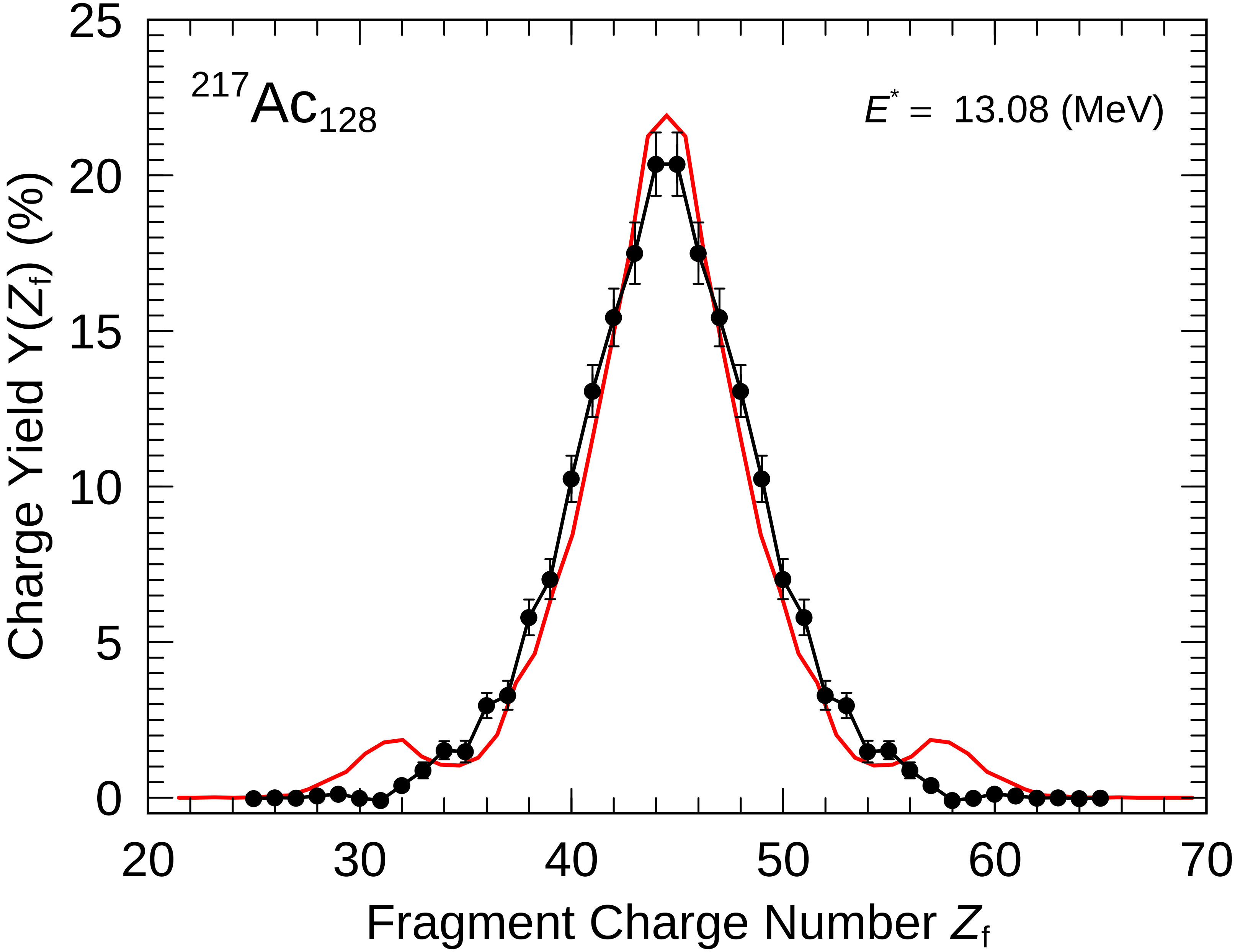
25
20
15
10
5
0
$^{214}Ac_{125}$
$E^* =$ 15.52 (MeV)
Charge Yield Y(Z_f) (%)
20
30
40
50
60
70
Fragment Charge Number Z_f



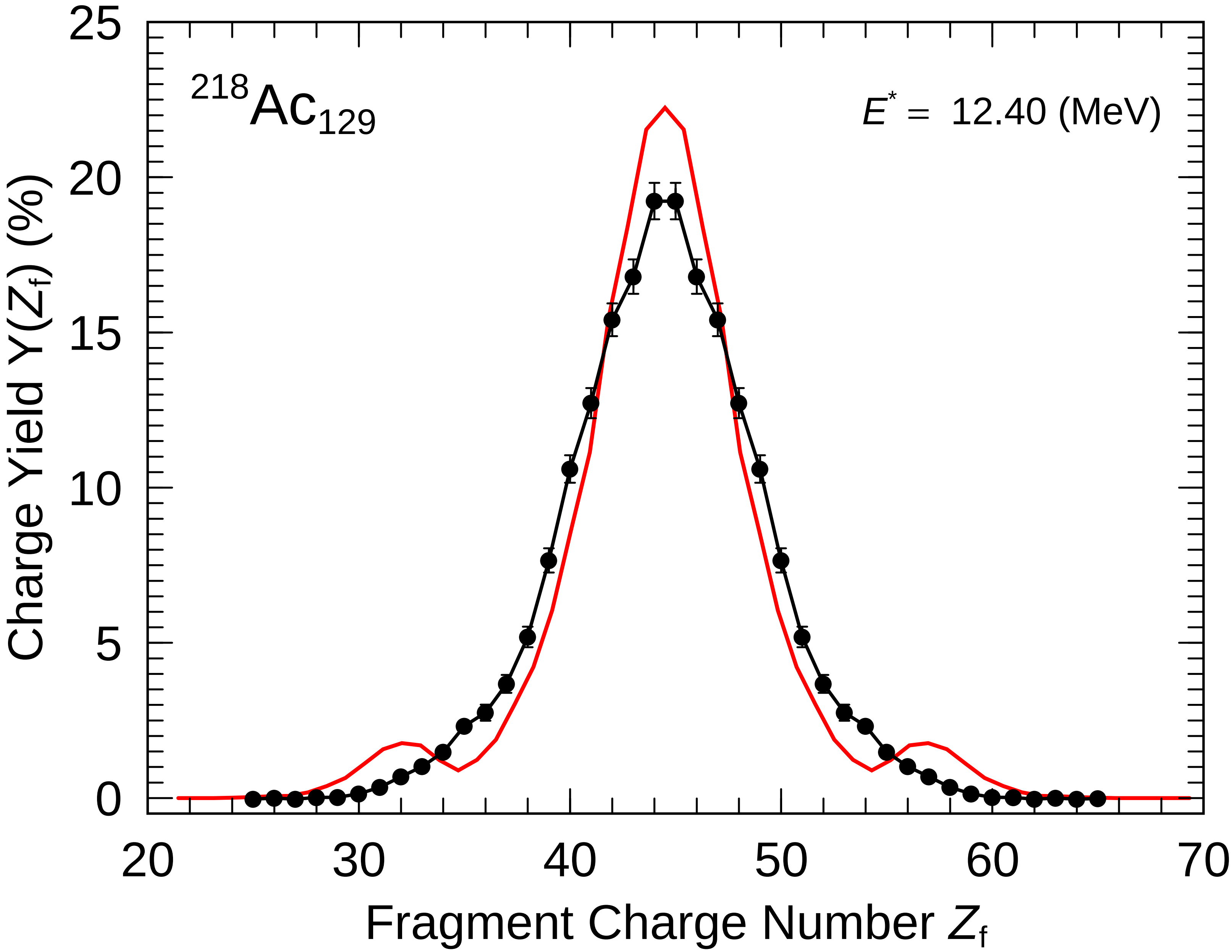
$^{215}Ac_{126}$
$E^* =$ 15.36 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



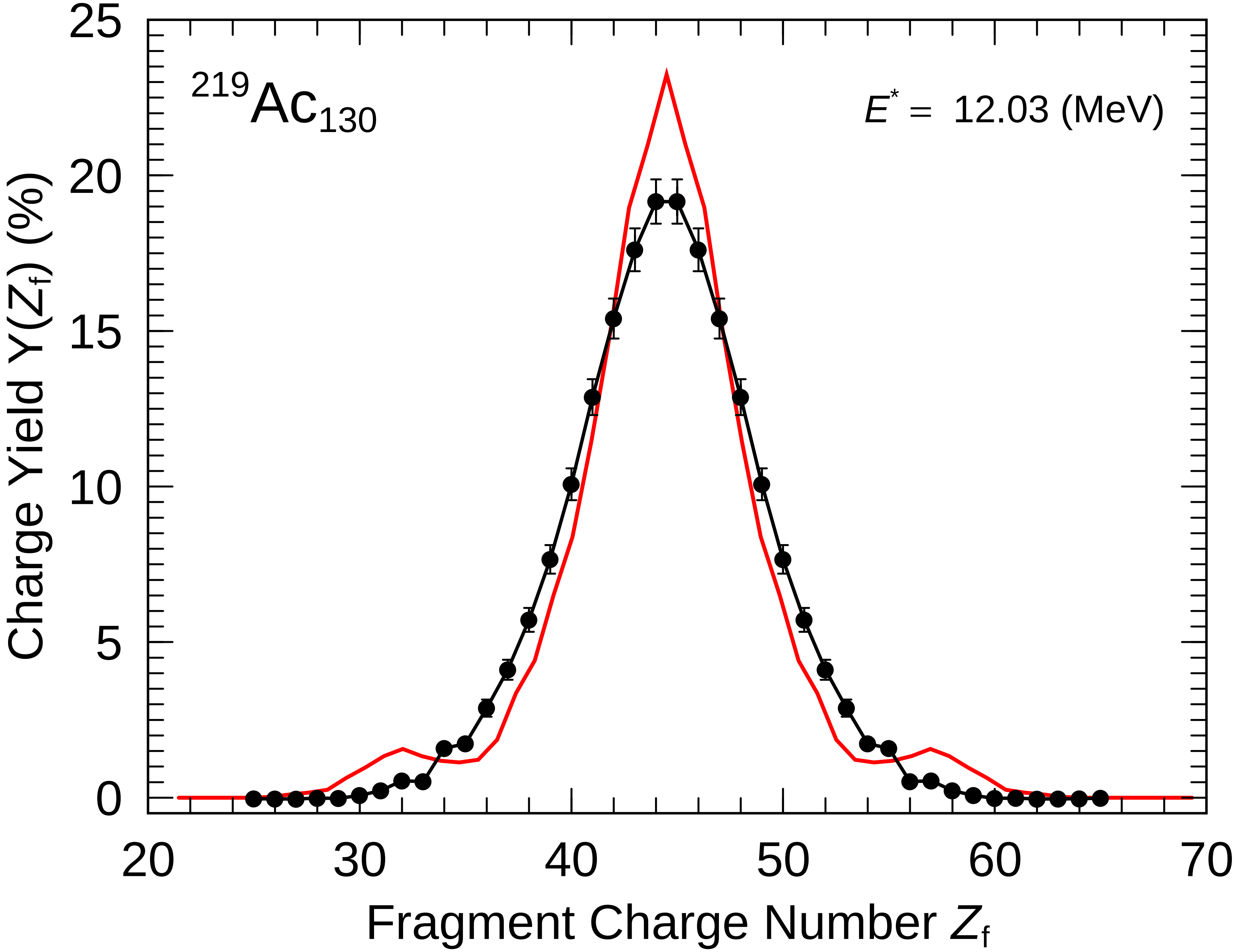
25
20
15
10
5
0
Charge Yield Y(Z_f) (%)
216Ac127
E* = 14.19 (MeV)
20
30
40
50
60
70
Fragment Charge Number Z_f



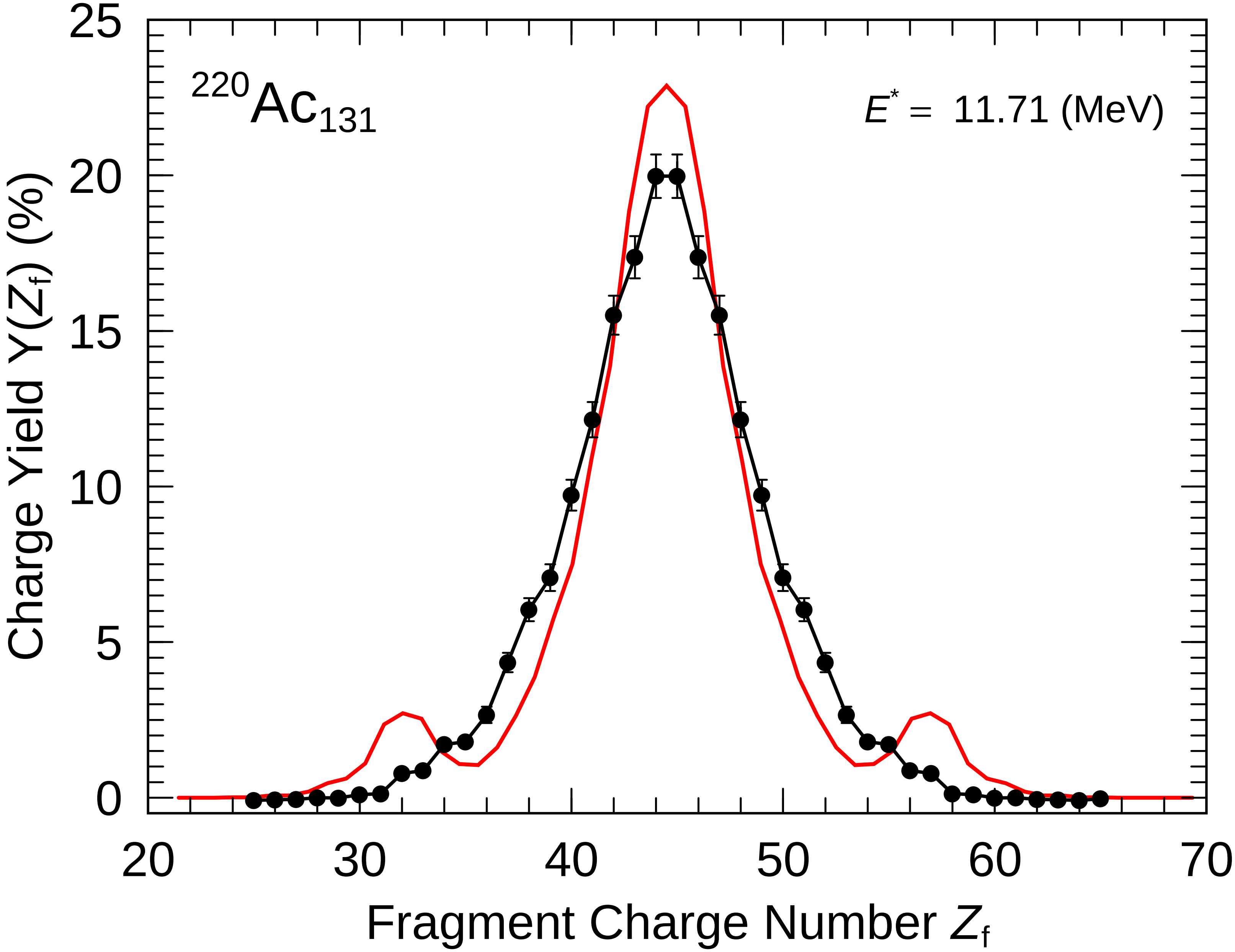
25
20
15
10
5
0
20
30
40
50
60
70
217Ac128
E* = 13.08 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf



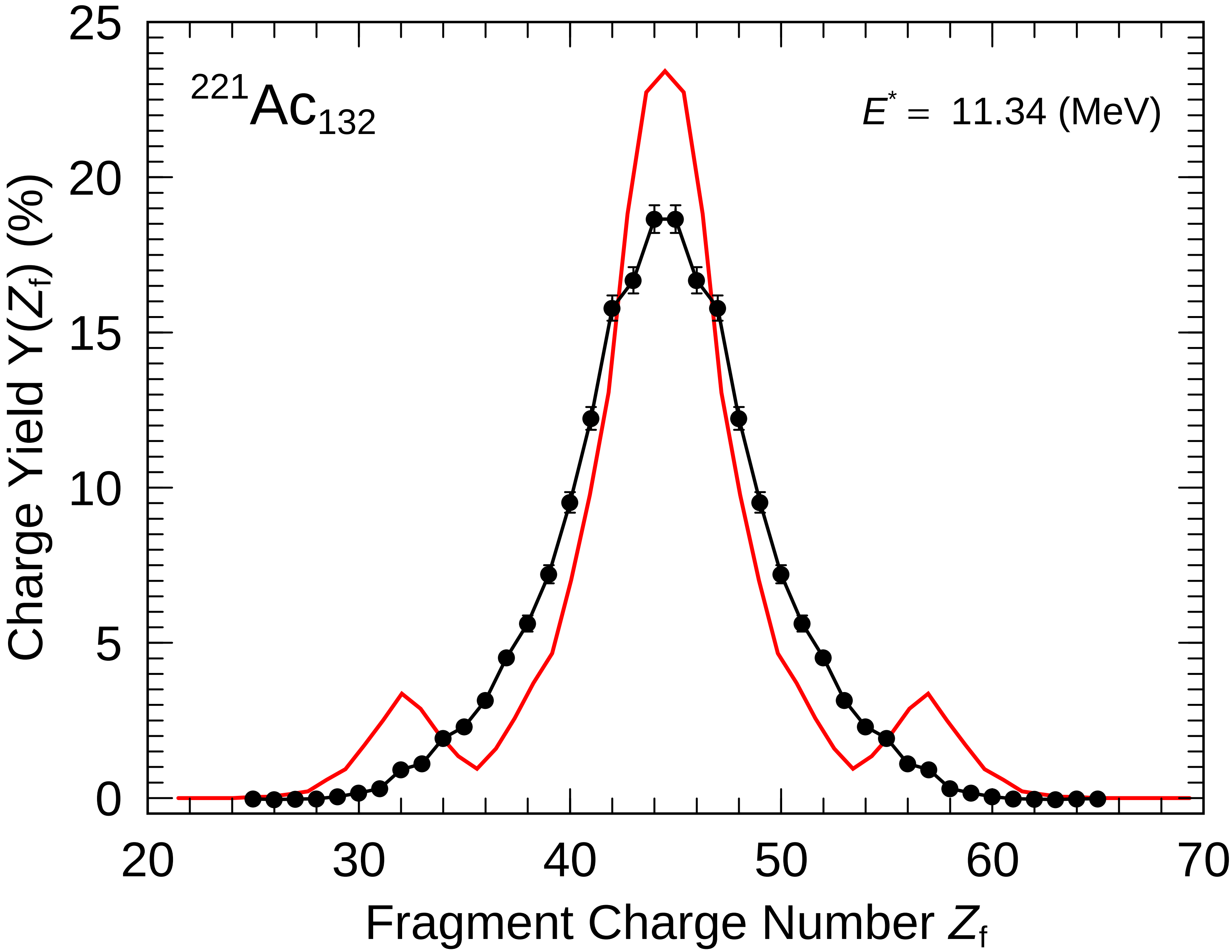
$^{218}Ac_{129}$
$E^* =$ 12.40 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



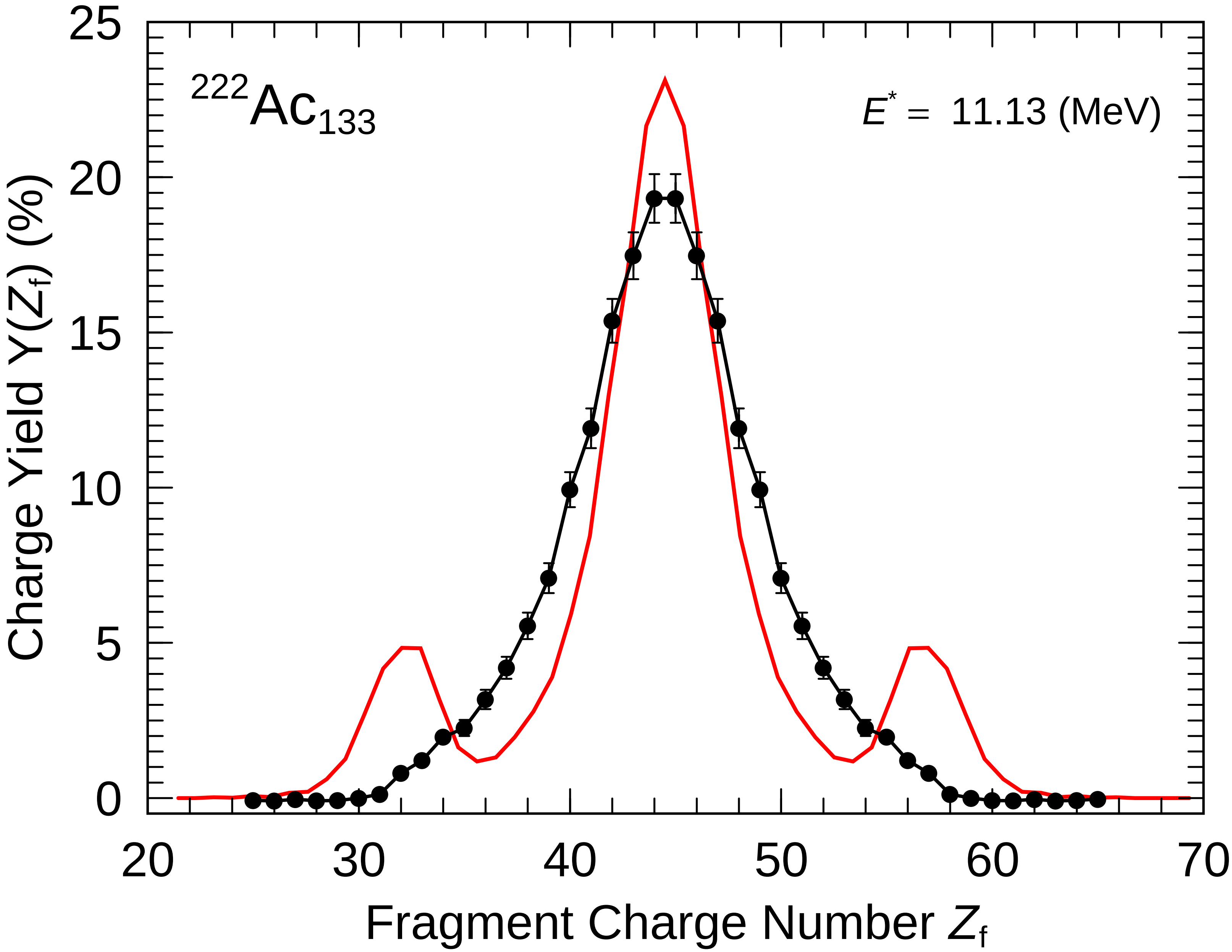
219Ac130
E* = 12.03 (MeV)
Charge Yield Y(Zf) (%)
0
5
10
15
20
25
20
30
40
50
60
70
Fragment Charge Number Zf



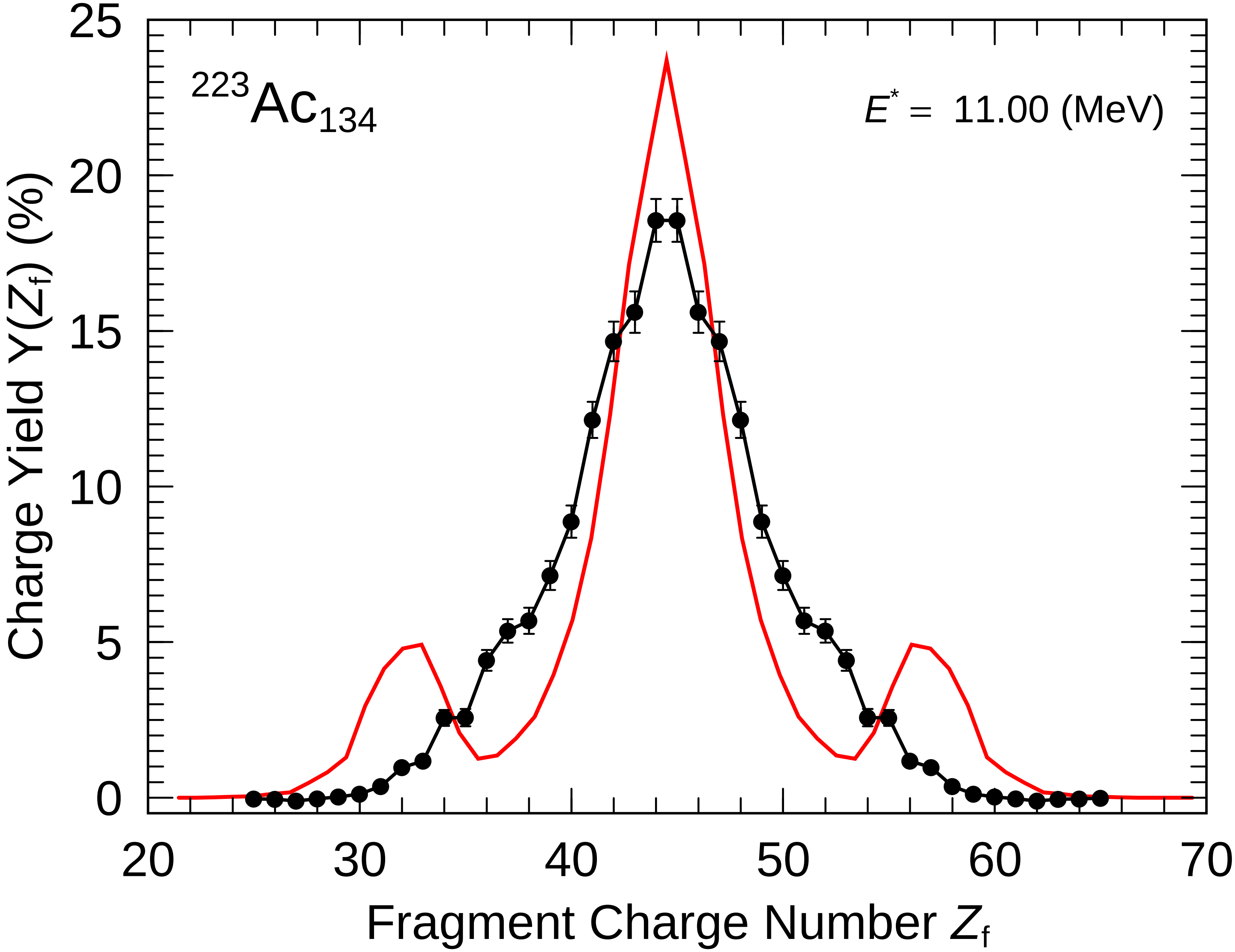
$^{220}Ac_{131}$
E^{*} = 11.71 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



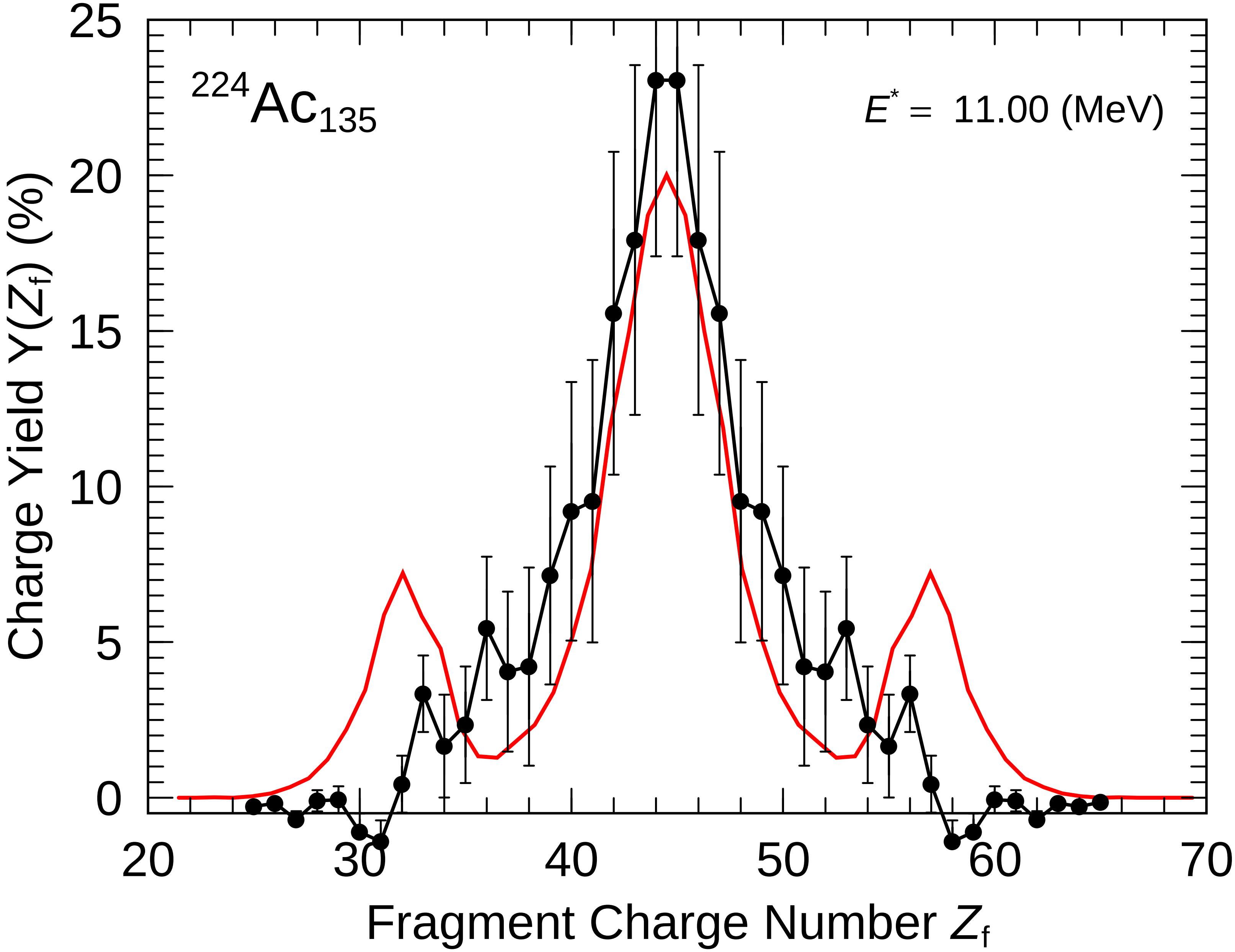
25
20
15
10
5
0
Charge Yield Y(Zf) (%)
221Ac132
E* = 11.34 (MeV)
20
30
40
50
60
70
Fragment Charge Number Zf



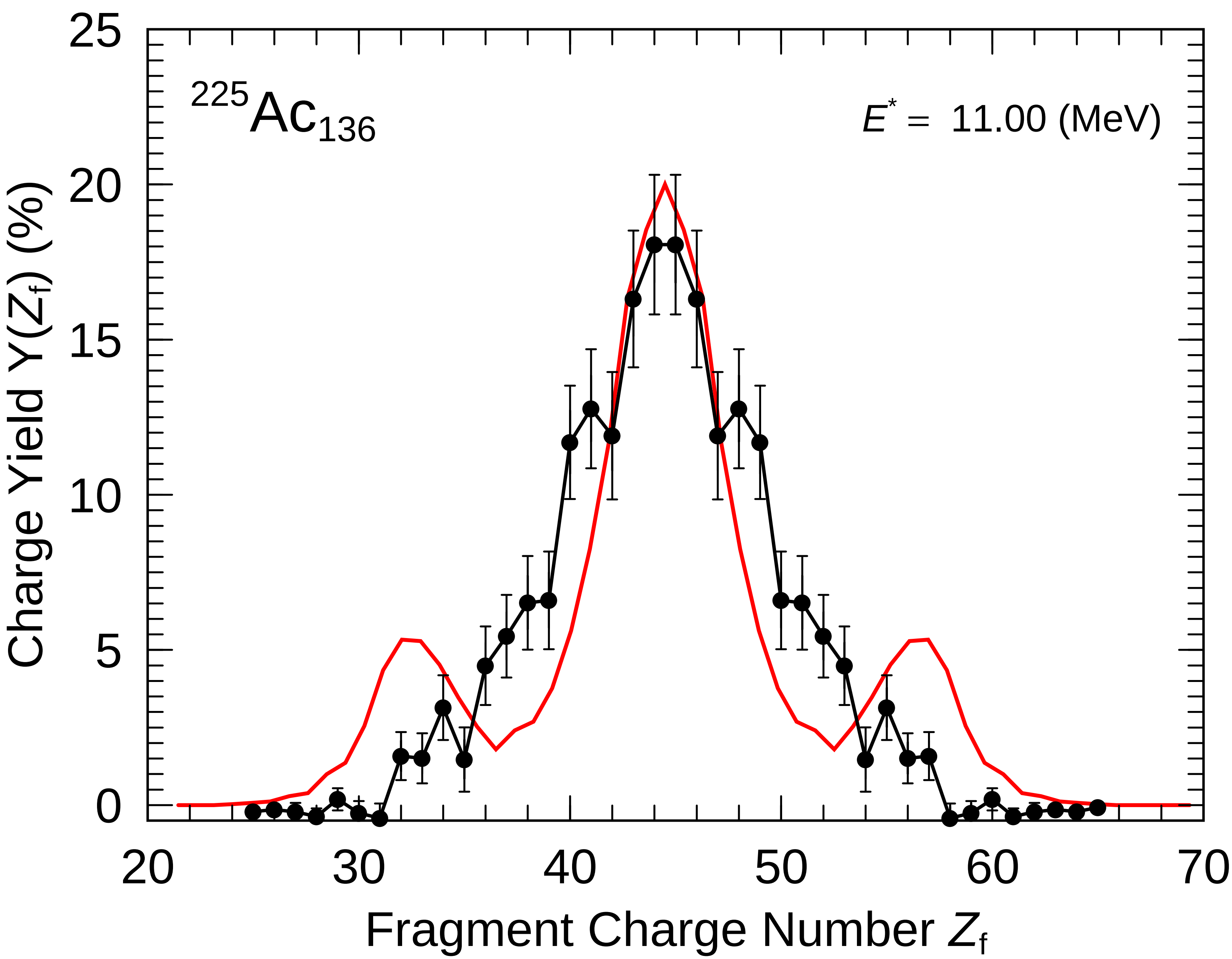
$^{222}Ac_{133}$
E^{*} = 11.13 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



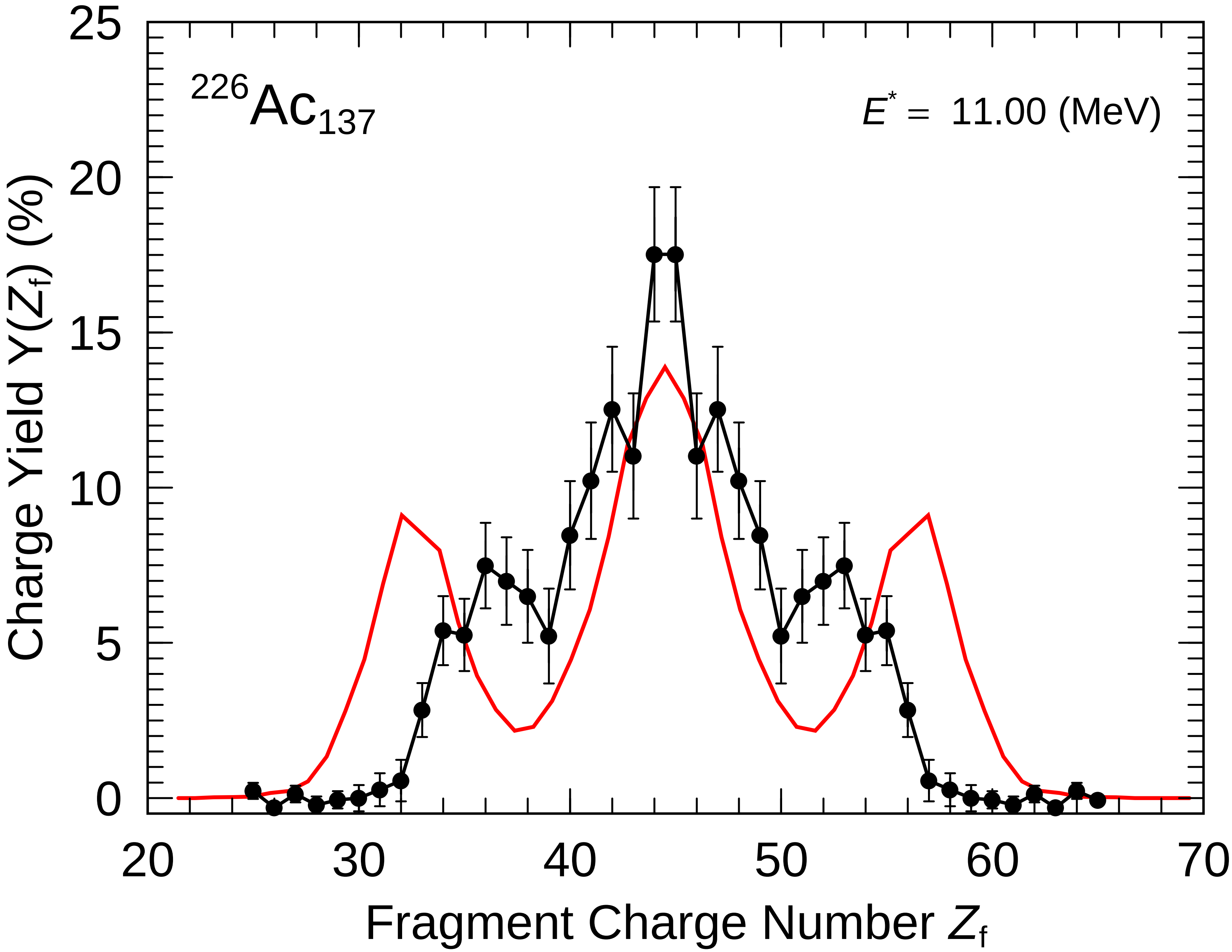
25
20
15
10
5
0
20
30
40
50
60
70
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
223Ac134
E* = 11.00 (MeV)



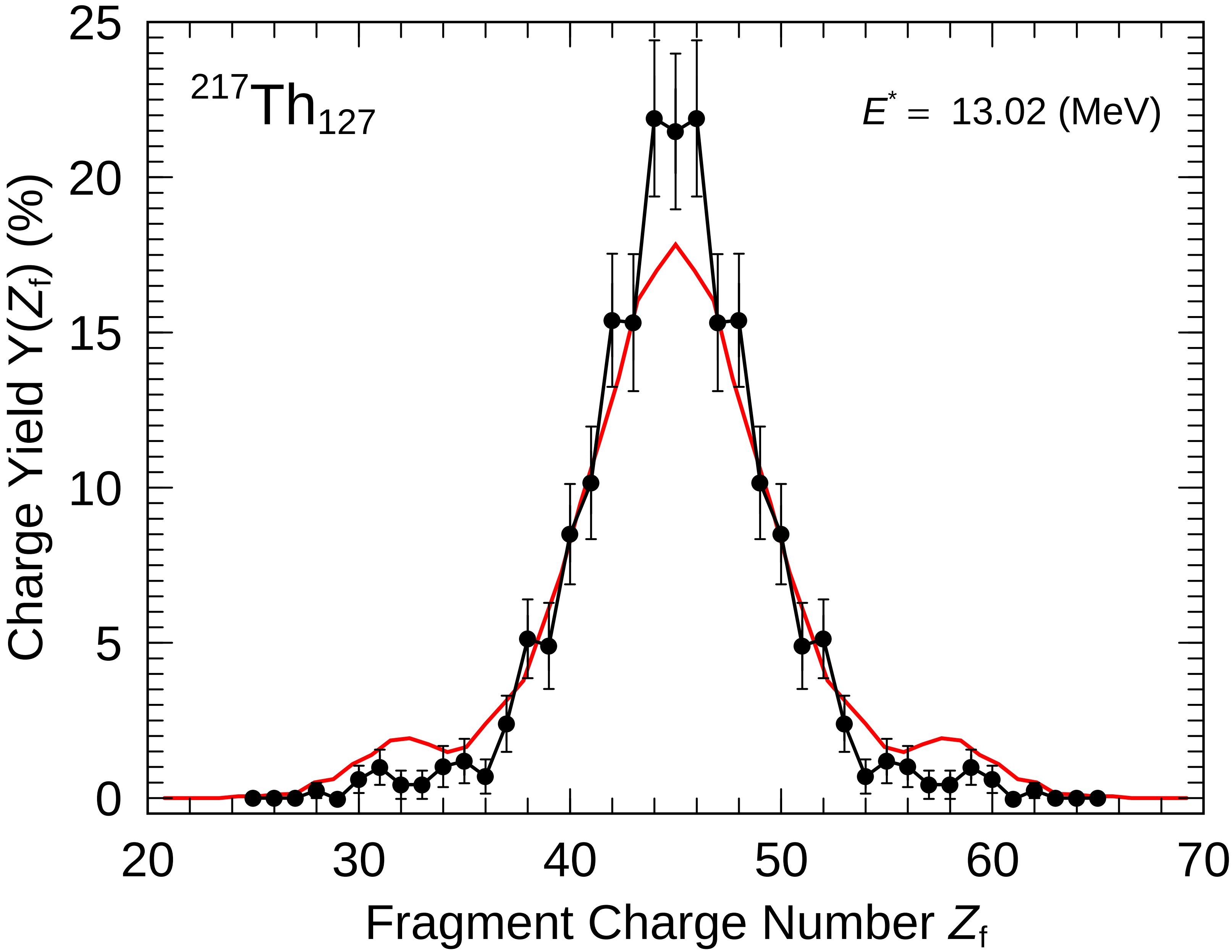
$^{224}Ac_{135}$
$E^* =$ 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



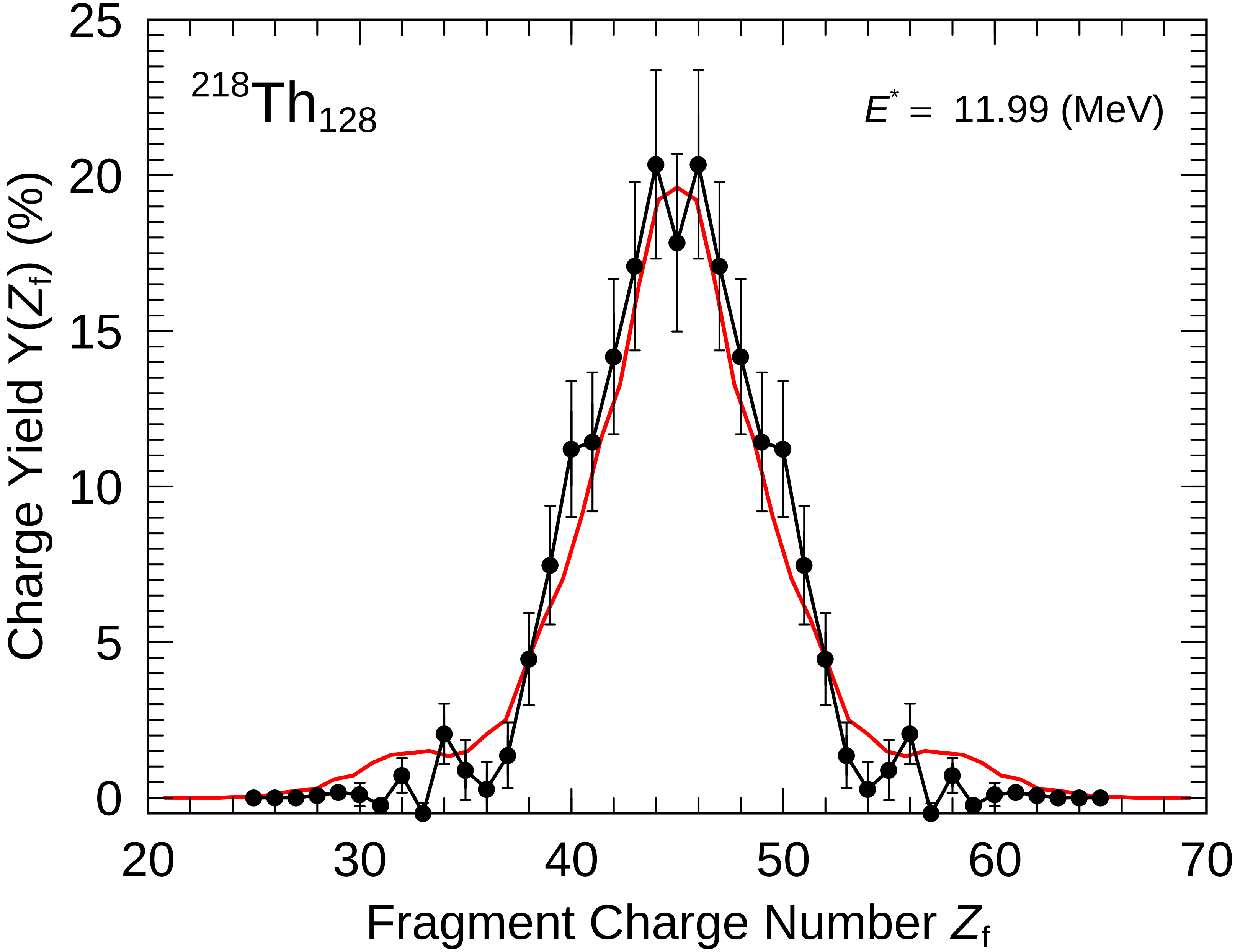
$^{225}Ac_{136}$
E^{*} = 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



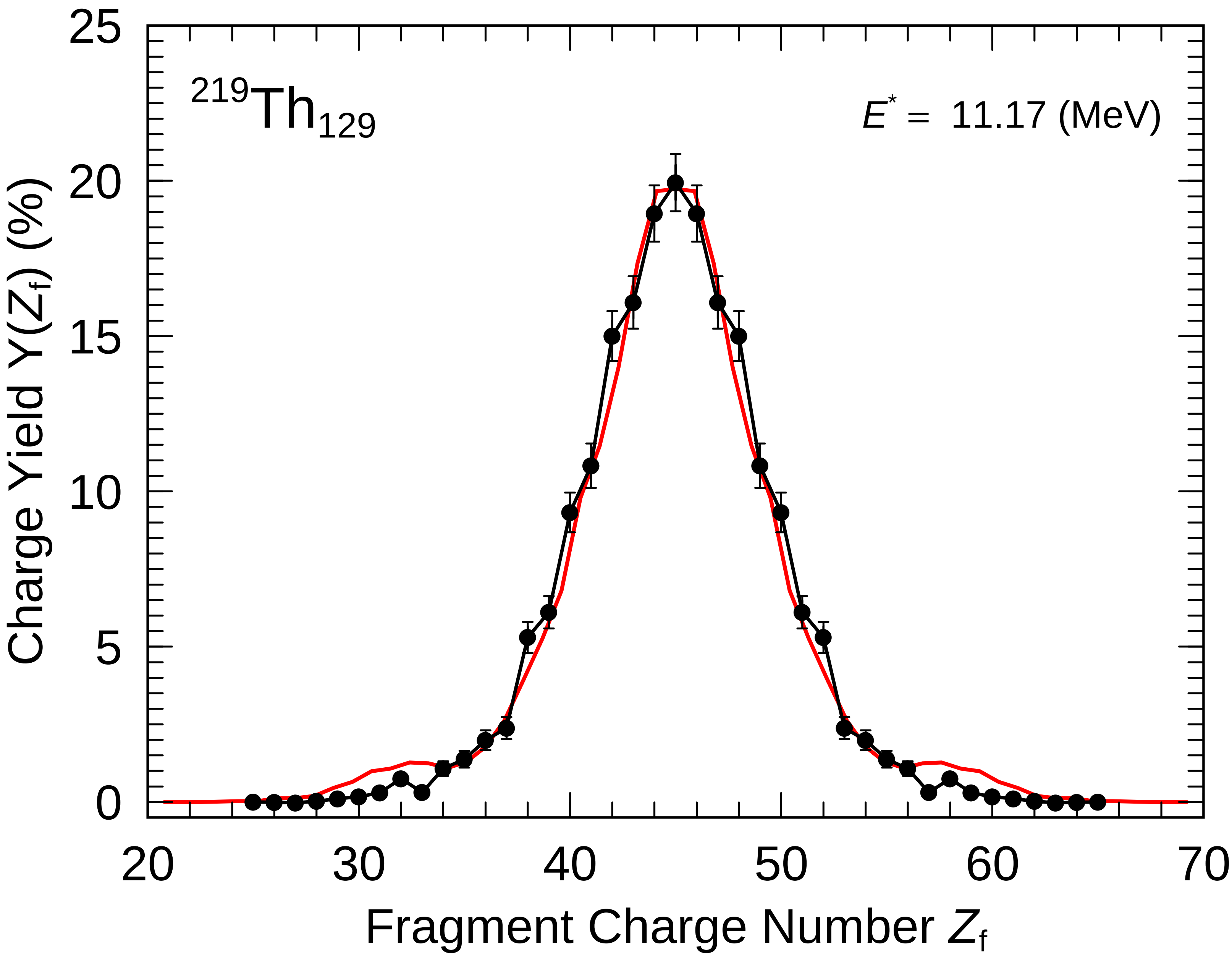
226Ac137
E* = 11.00 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
20
30
40
50
60
70



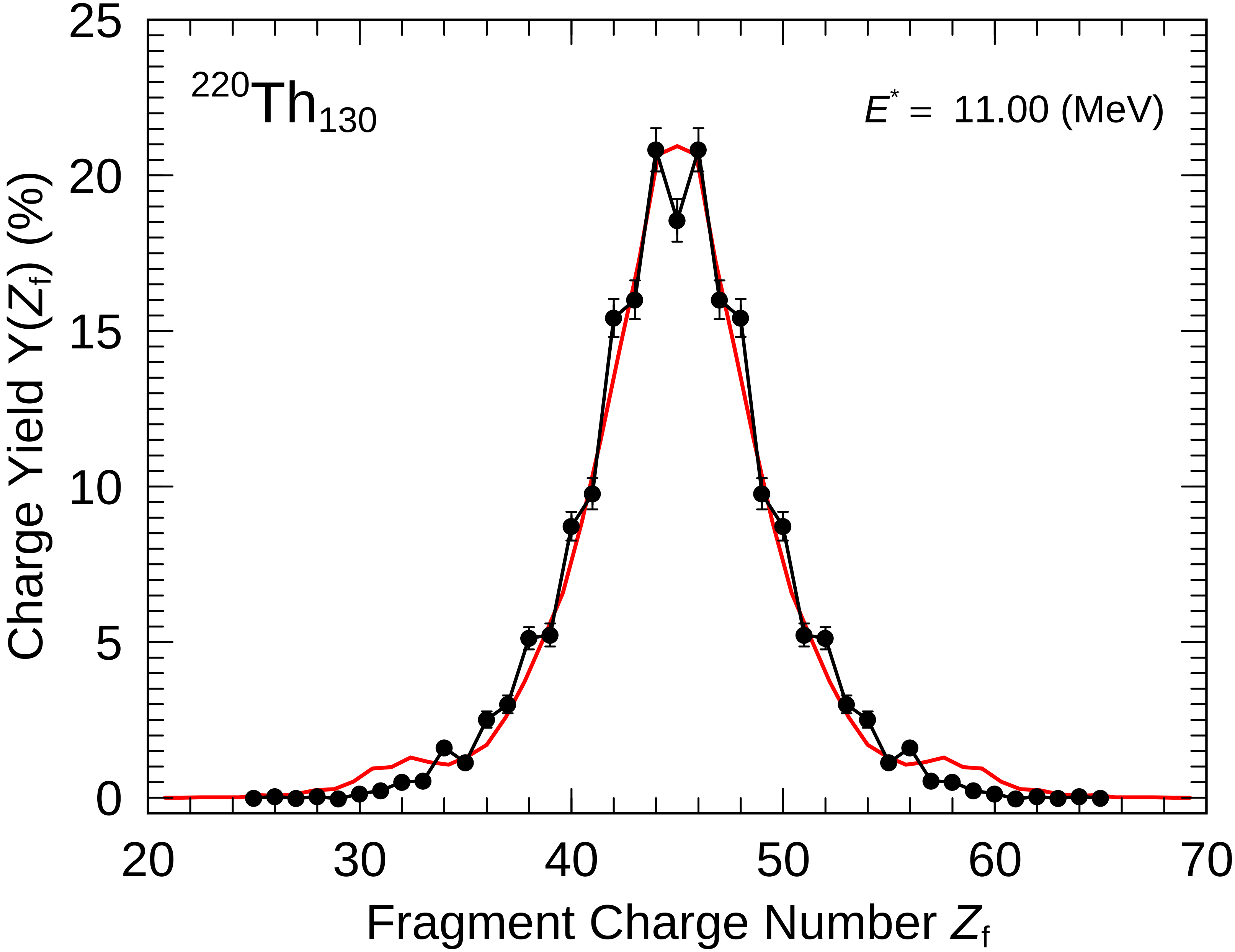
^{217}Th$_{127}$
E^{*} = 13.02 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



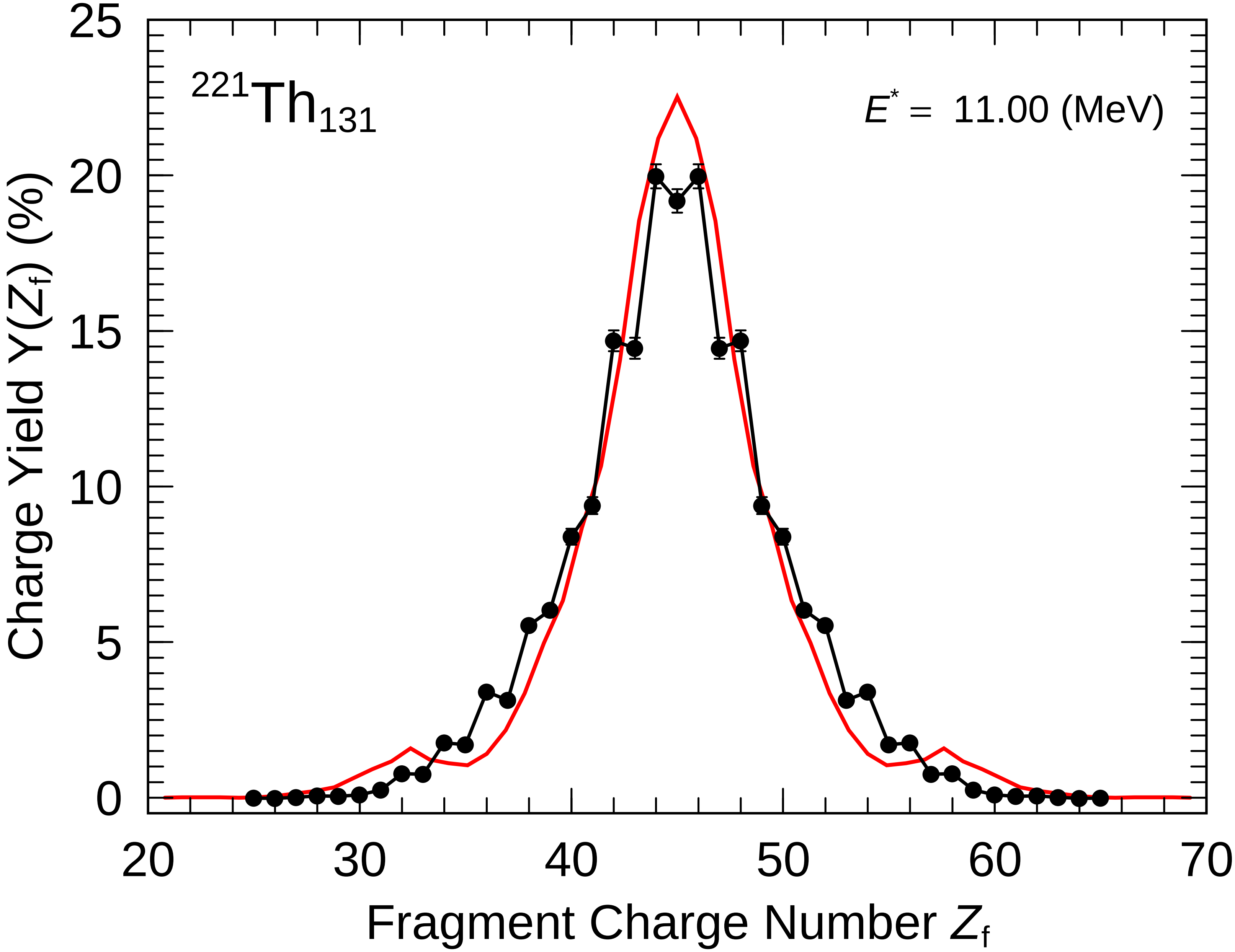
$^{218}Th_{128}$
E^{*} = 11.99 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



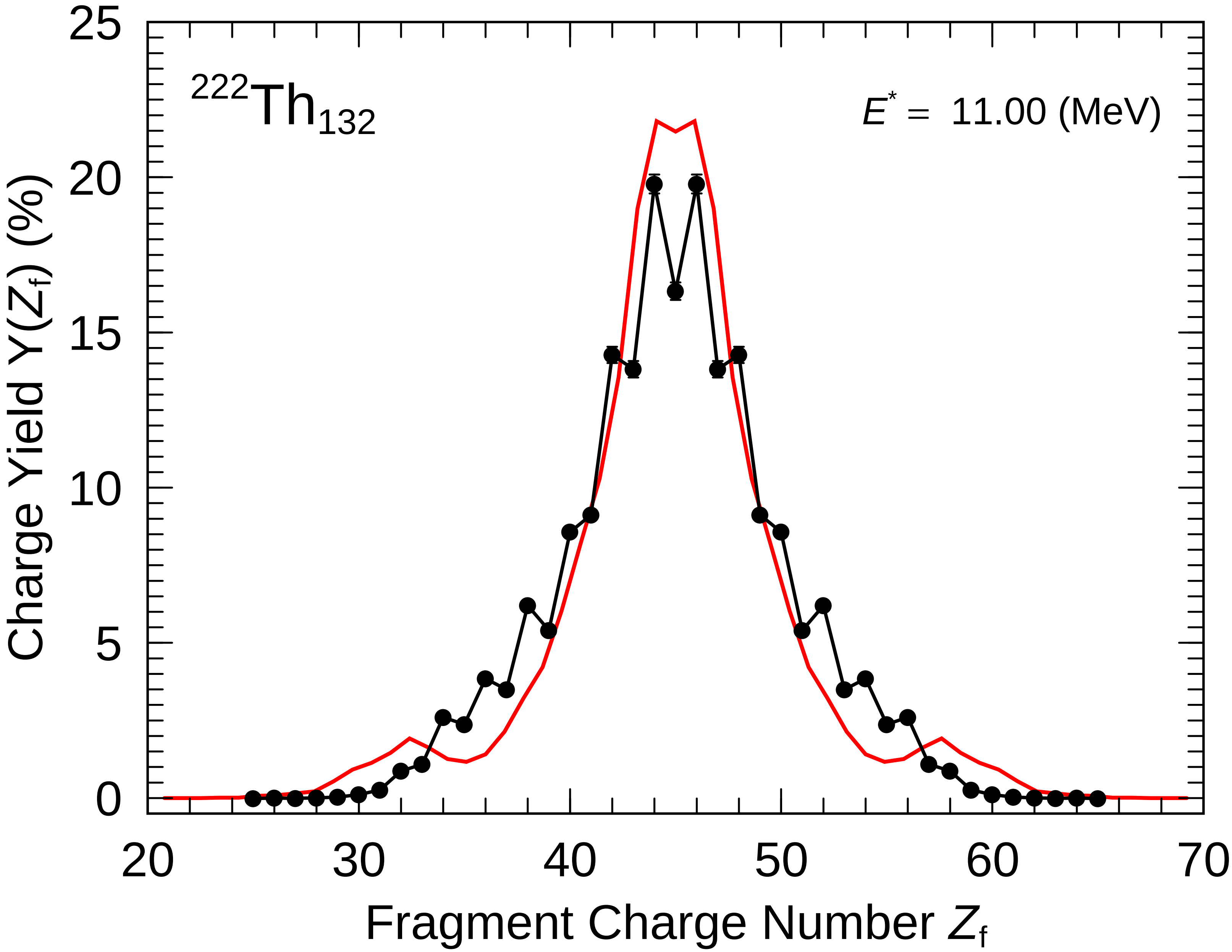
25
20
15
10
5
0
$^{219}Th_{129}$
E^{*} = 11.17 (MeV)
Charge Yield Y(Z_f) (%)
20
30
40
50
60
70
Fragment Charge Number Z_f



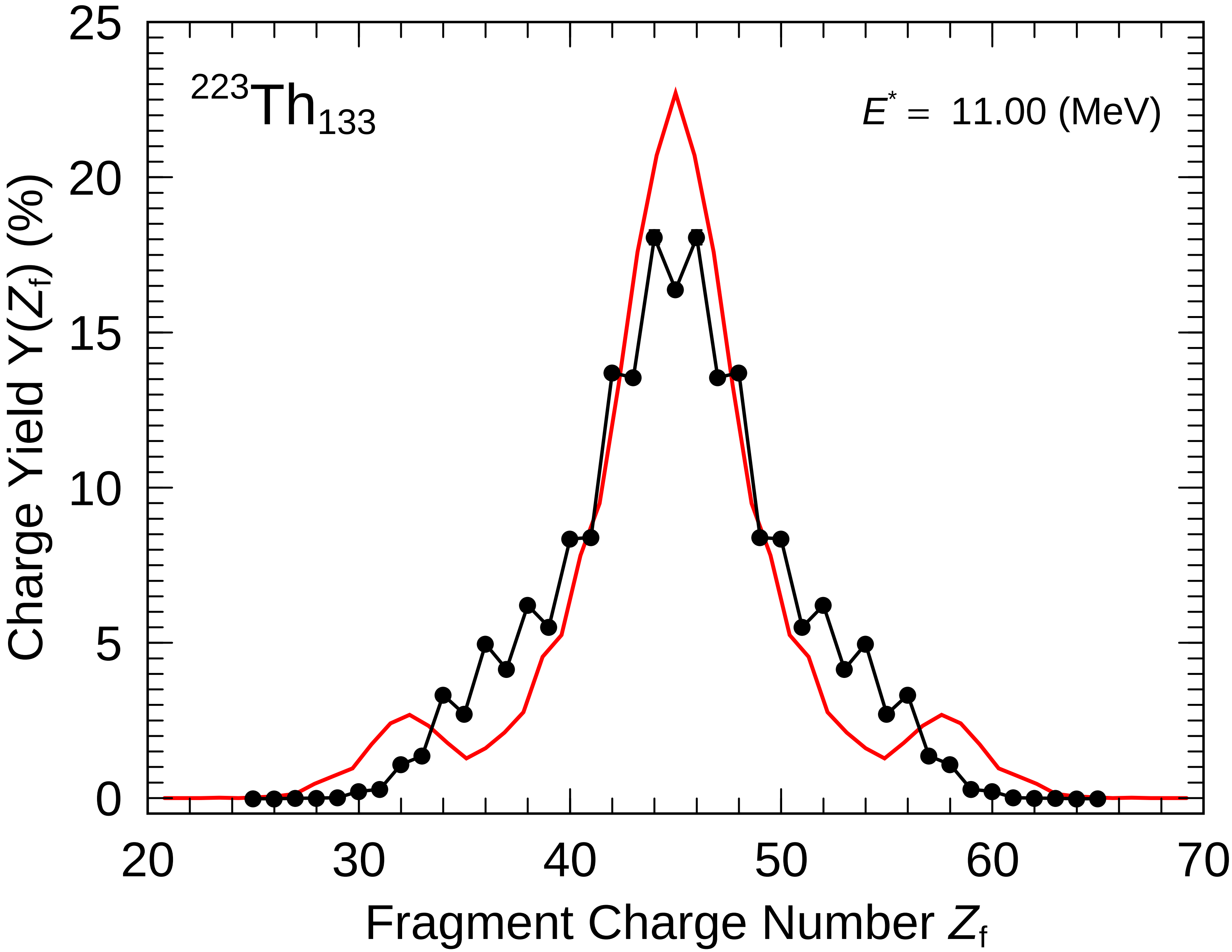
220Th130
E* = 11.00 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
20
30
40
50
60
70



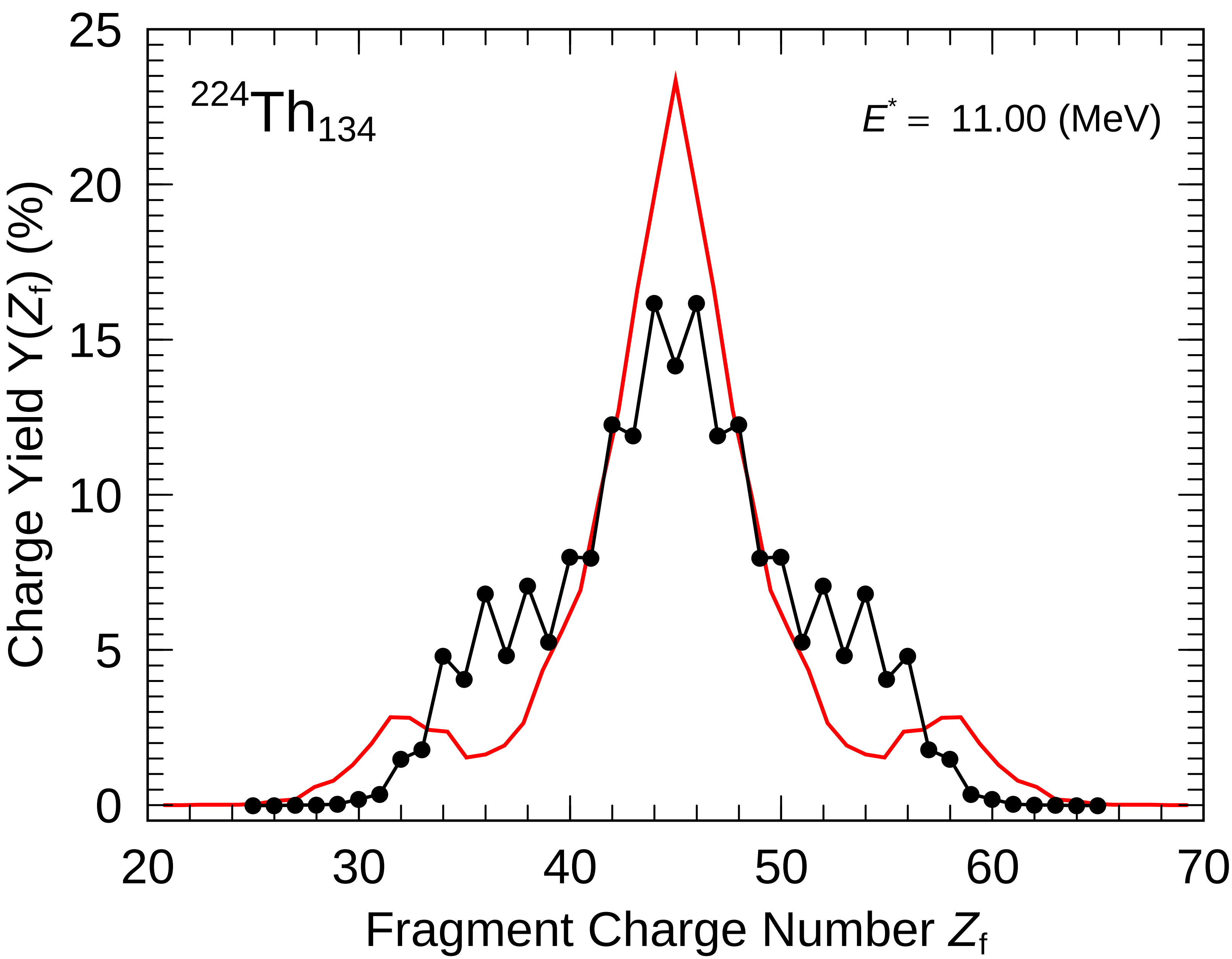
221Th131
E* = 11.00 (MeV)
Charge Yield Y(Zf) (%)
0
5
10
15
20
25
20
30
40
50
60
70
Fragment Charge Number Zf



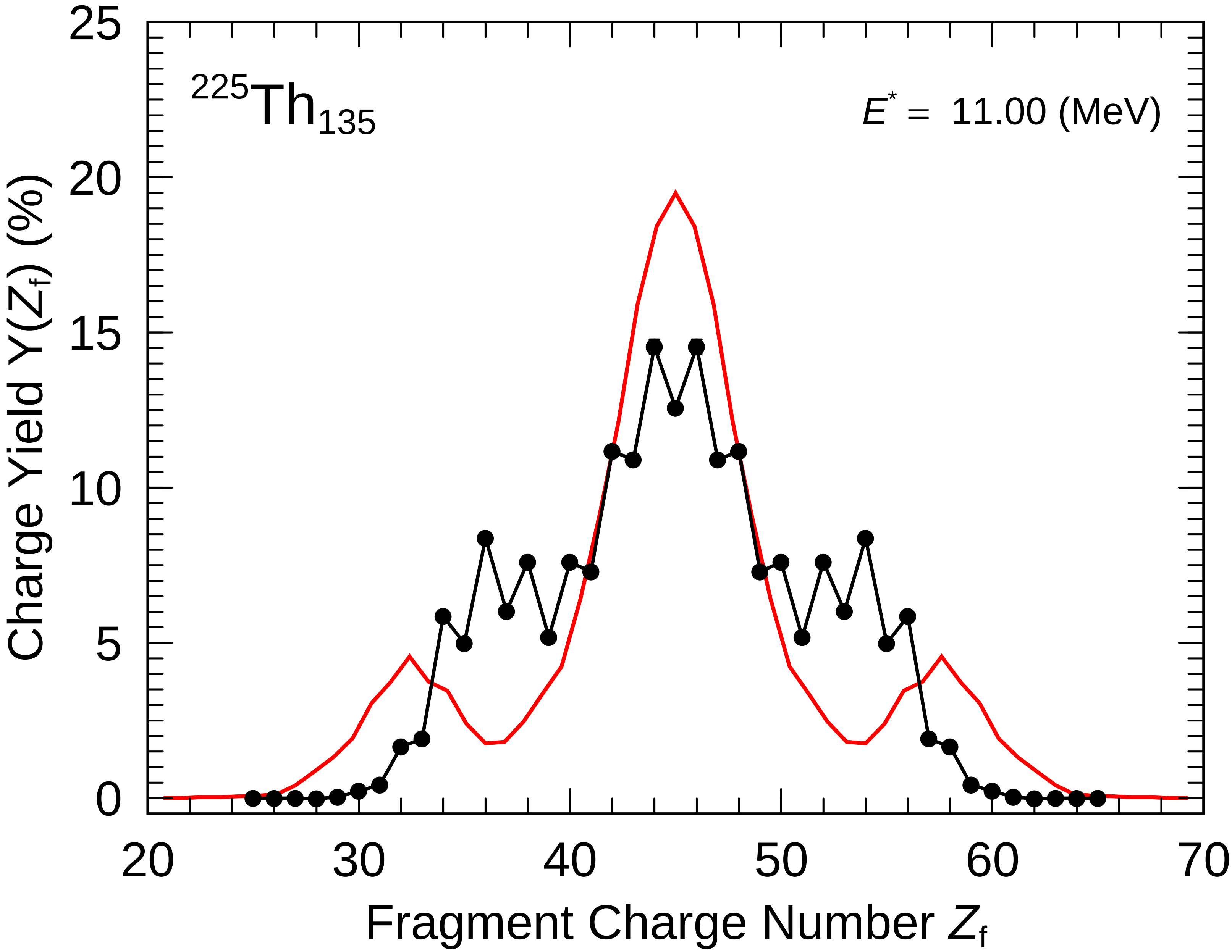
25
20
15
10
5
0
20
30
40
50
60
70
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
222Th132
E* = 11.00 (MeV)



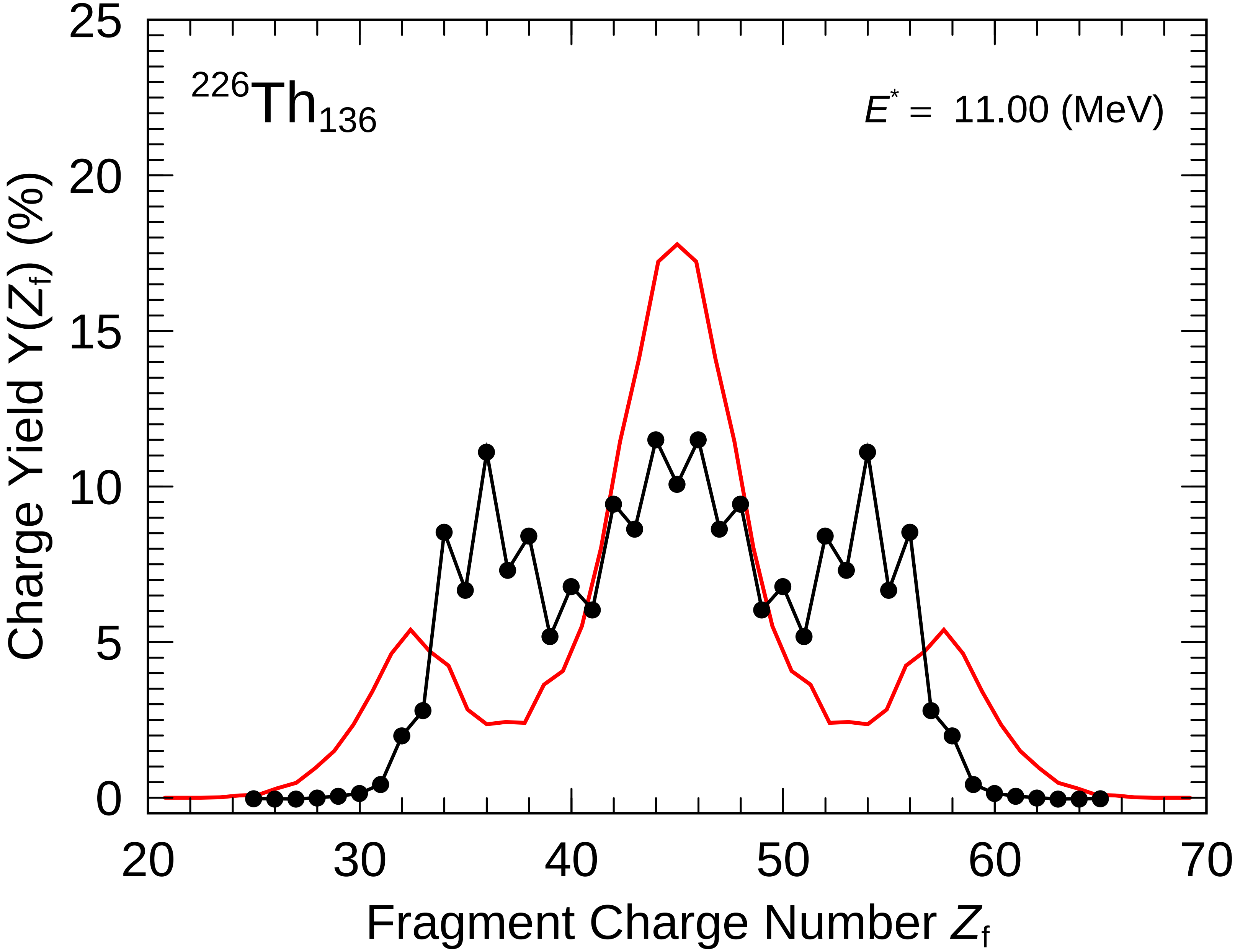
25
20
15
10
5
0
20
30
40
50
60
70
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
223Th133
E* = 11.00 (MeV)



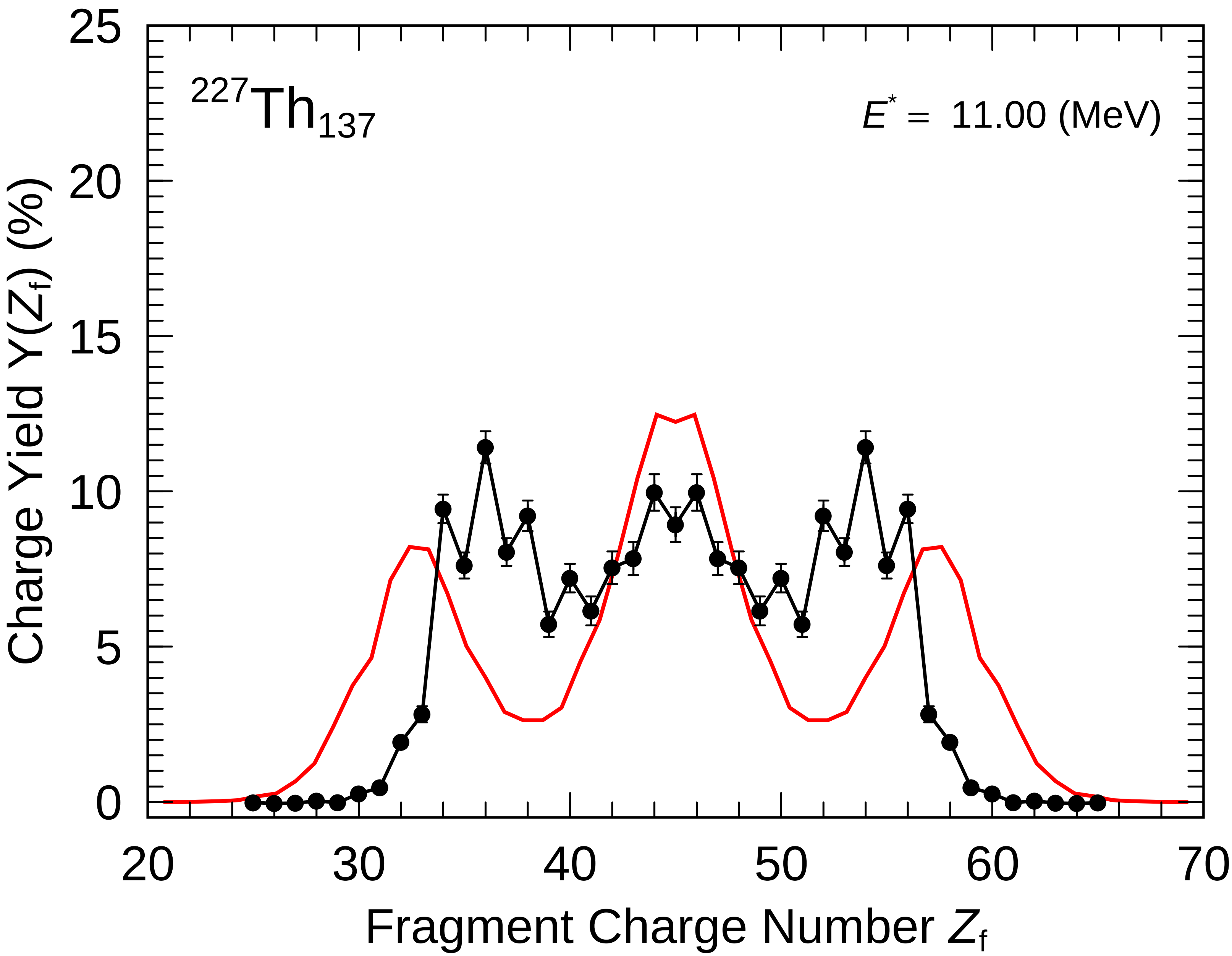
$^{224}Th_{134}$
E^{*} = 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



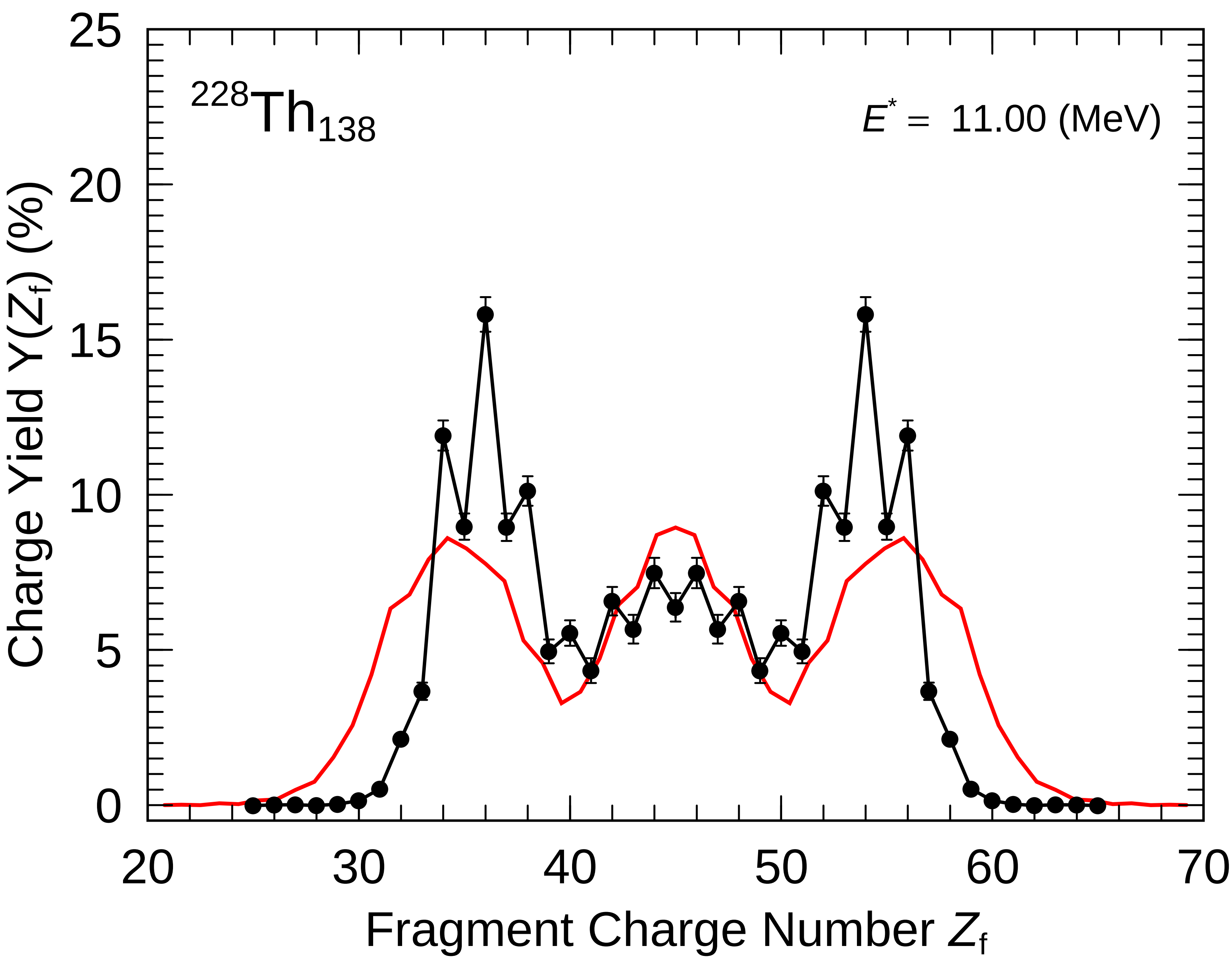
25
20
15
10
5
0
Charge Yield Y(Zf) (%)
225Th135
E* = 11.00 (MeV)
20
30
40
50
60
70
Fragment Charge Number Zf



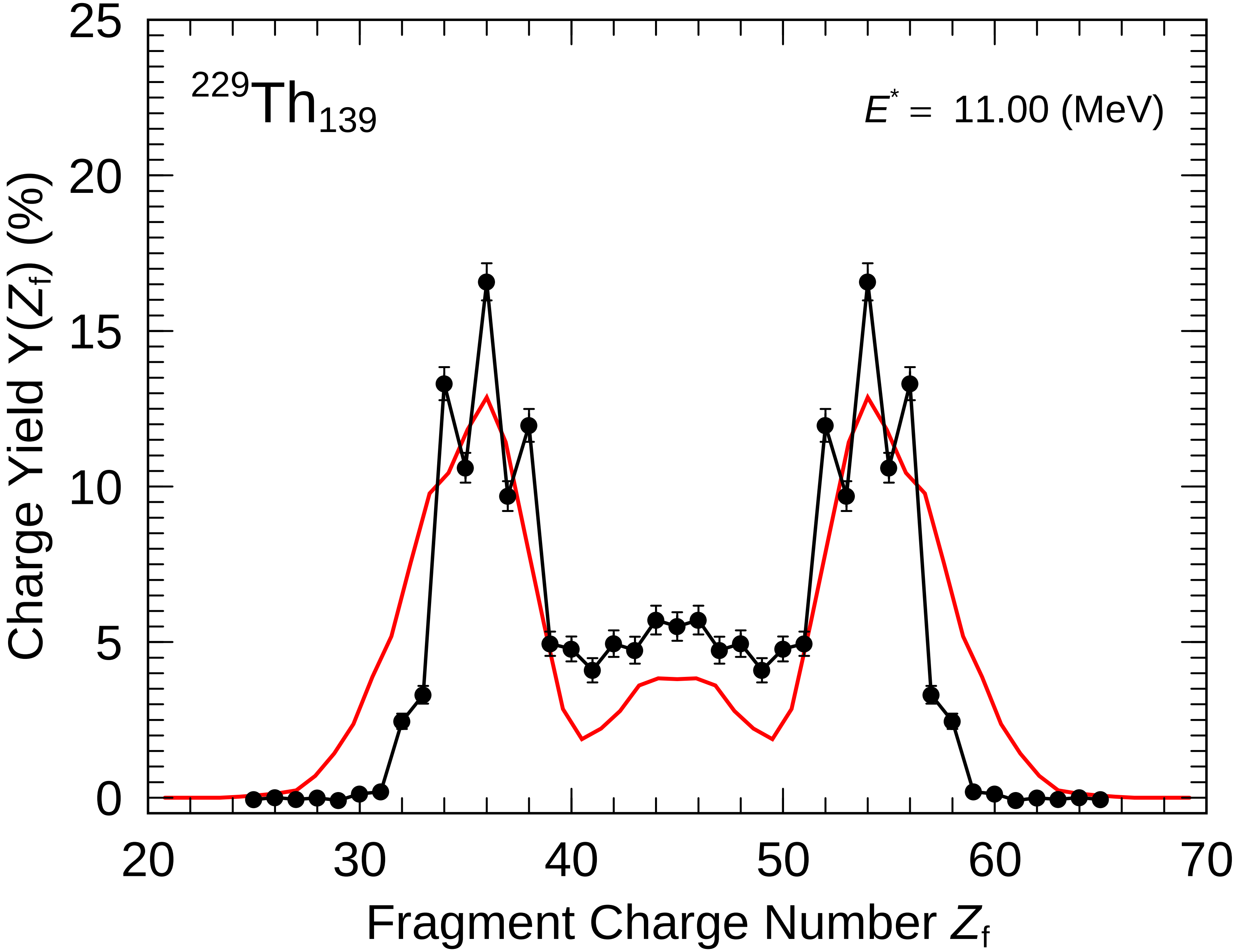
226Th136
E* = 11.00 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
20
30
40
50
60
70



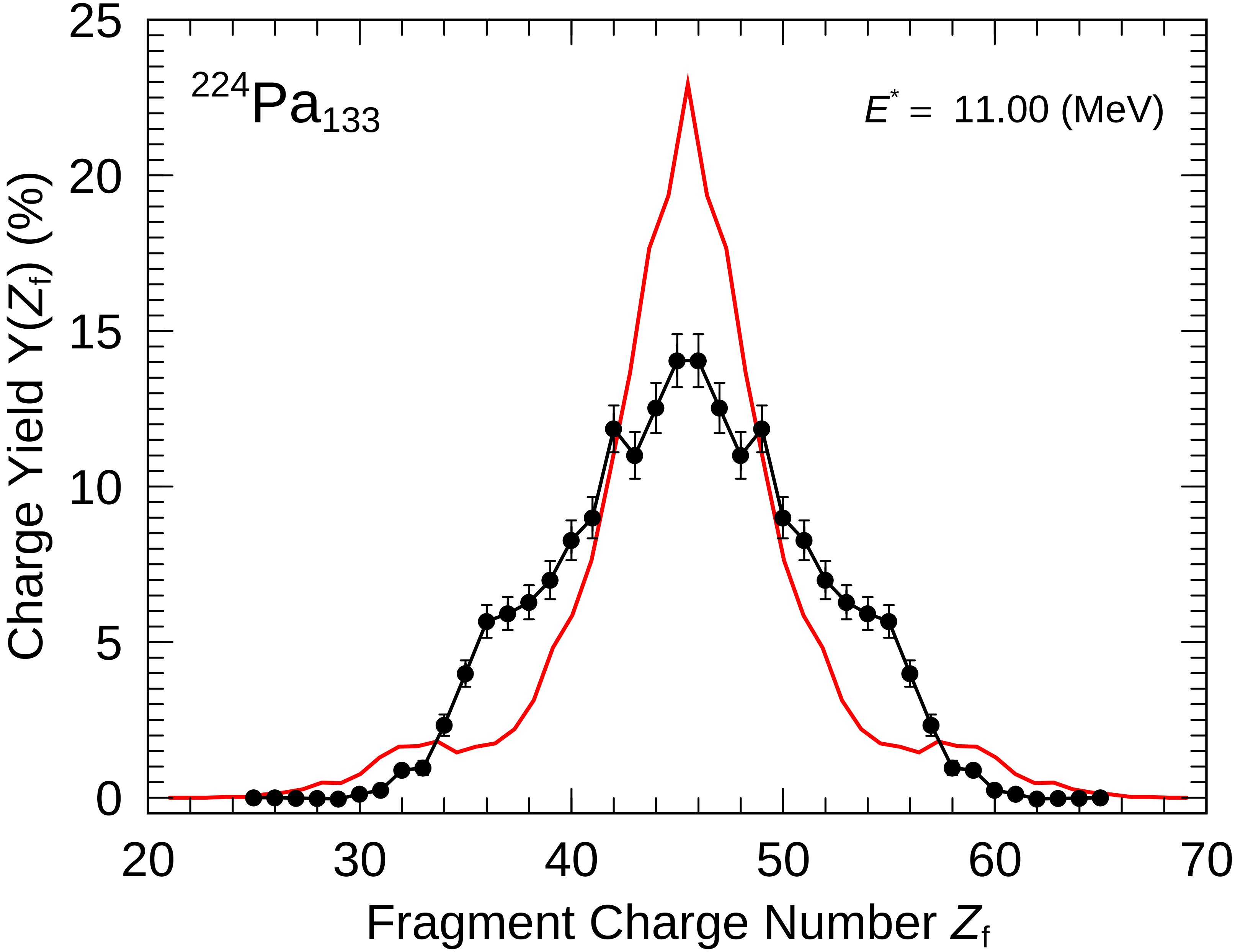
$^{227}Th_{137}$
E^{*} = 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
25
20
15
10
5
0
20
30
40
50
60
70



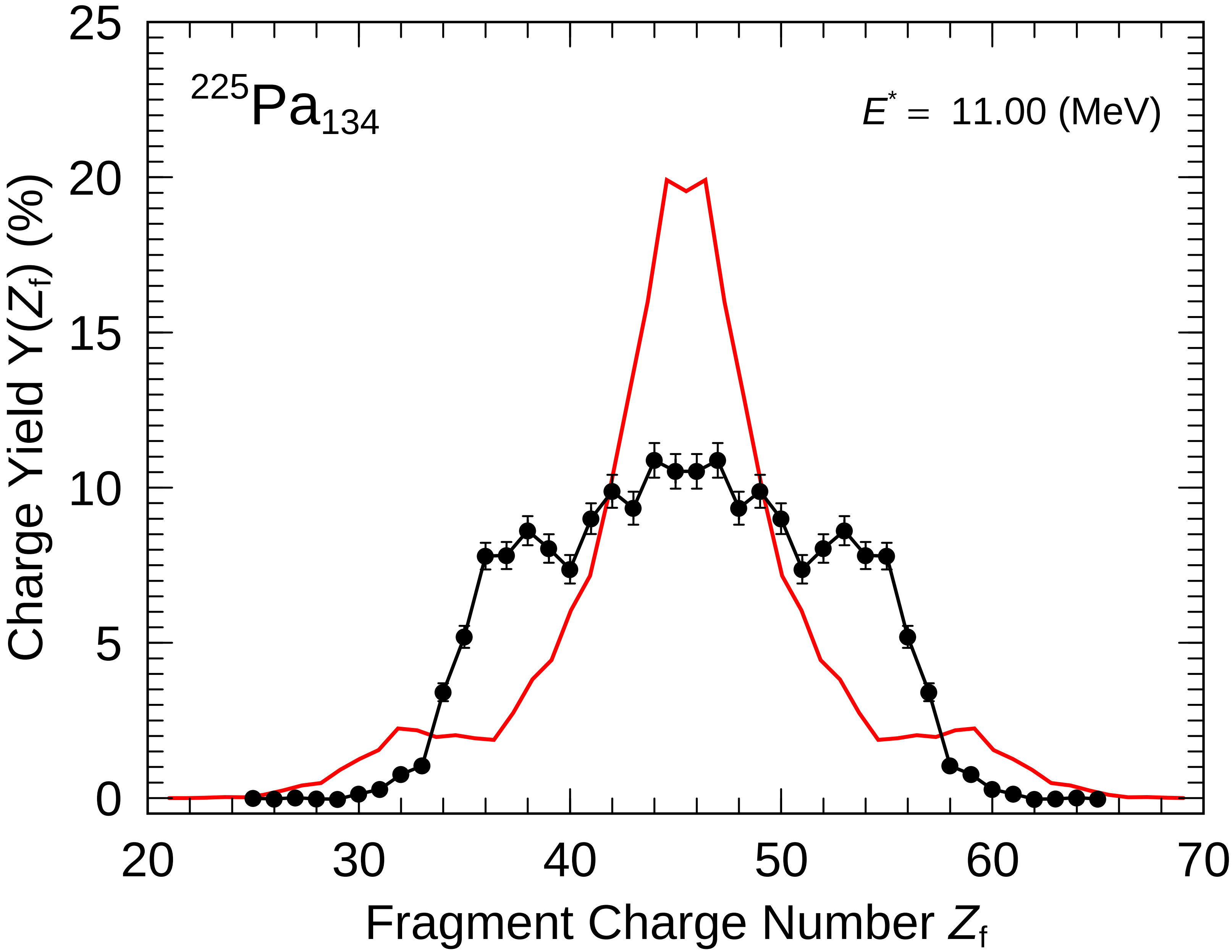
^{228}Th$_{138}$
$E^* =$ 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



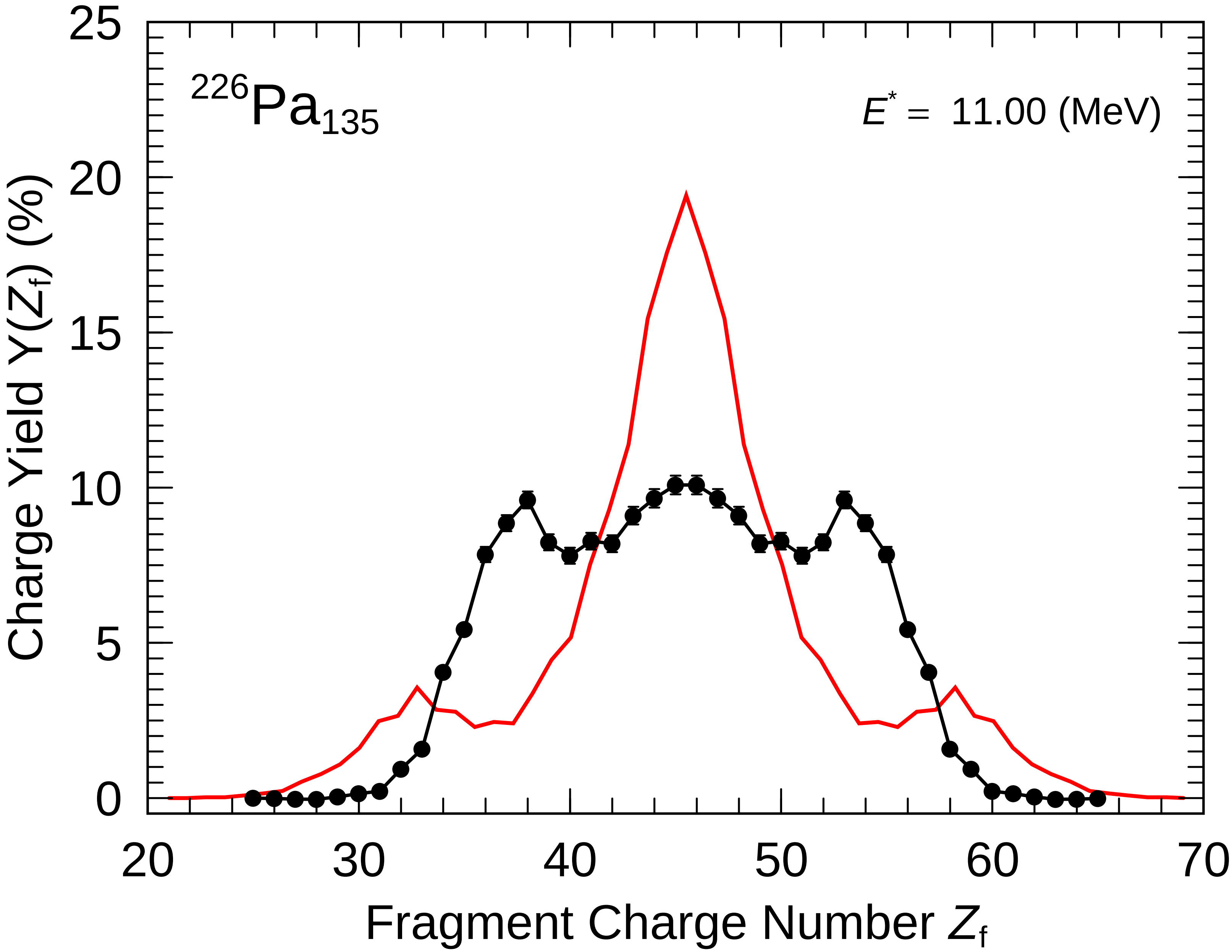
25
20
15
10
5
0
Charge Yield Y(Z_f) (%)
229Th139
E* = 11.00 (MeV)
20
30
40
50
60
70
Fragment Charge Number Z_f



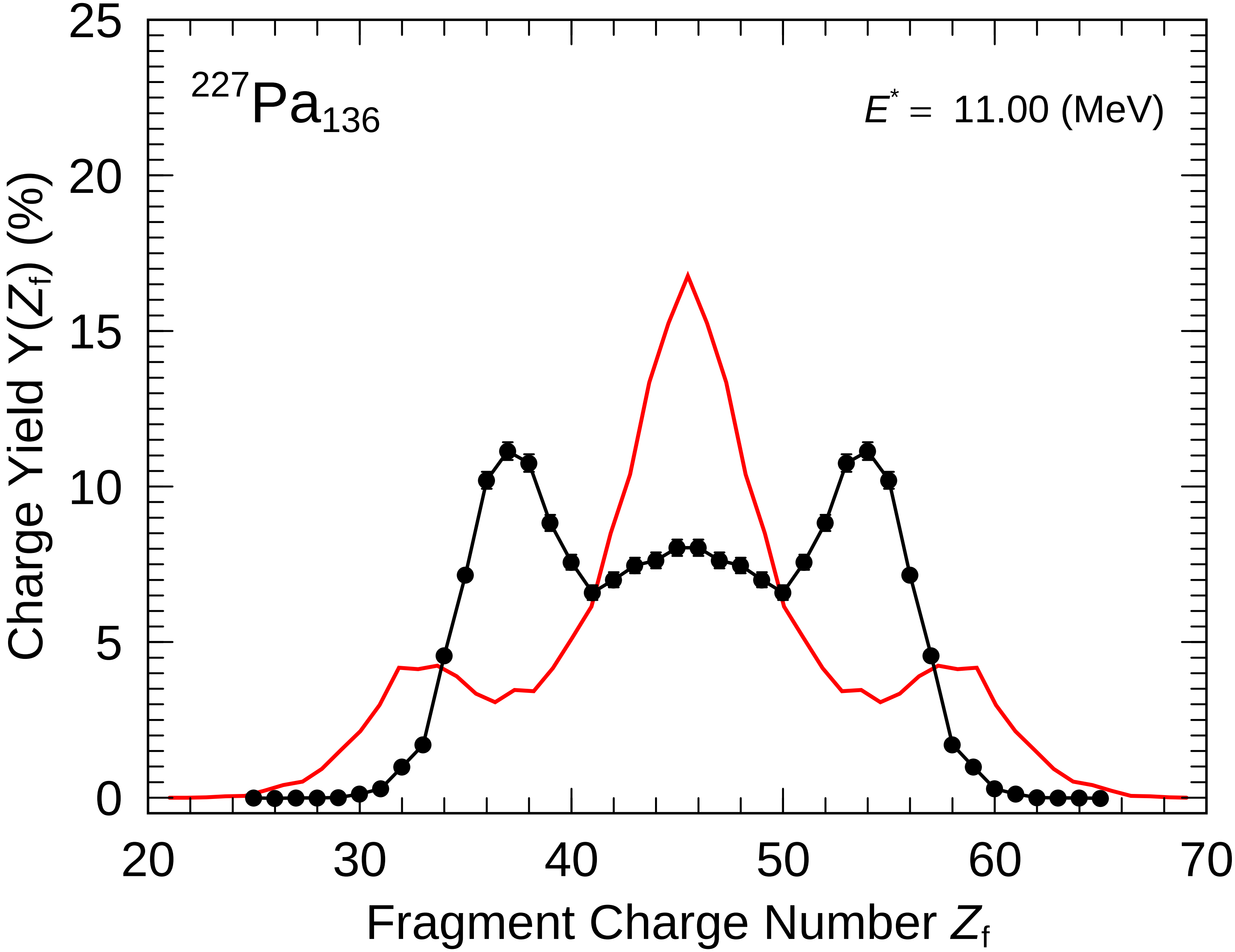
$^{224}Pa_{133}$
$E^* =$ 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



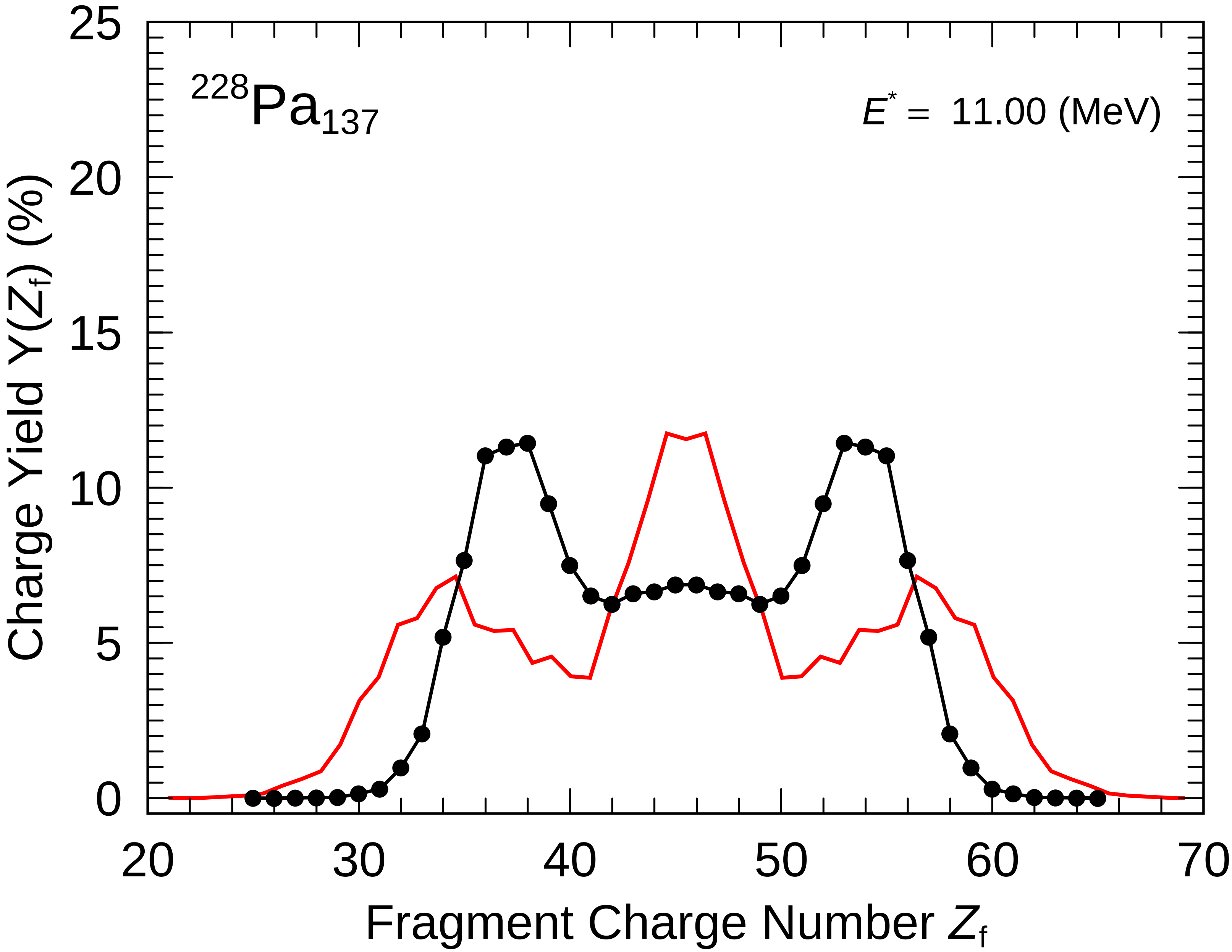
225Pa134
E* = 11.00 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
20
30
40
50
60
70



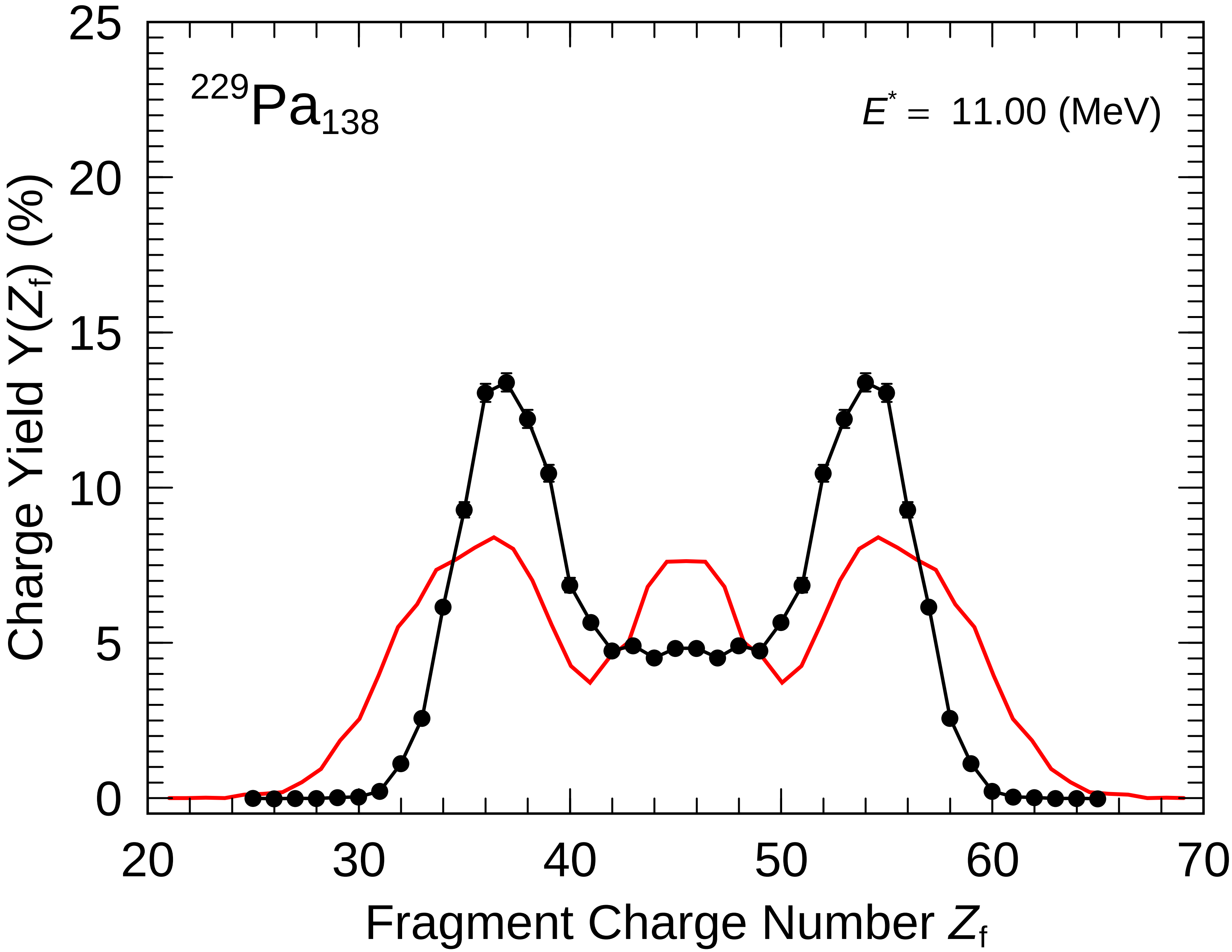
$^{226}Pa_{135}$
$E^* =$ 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



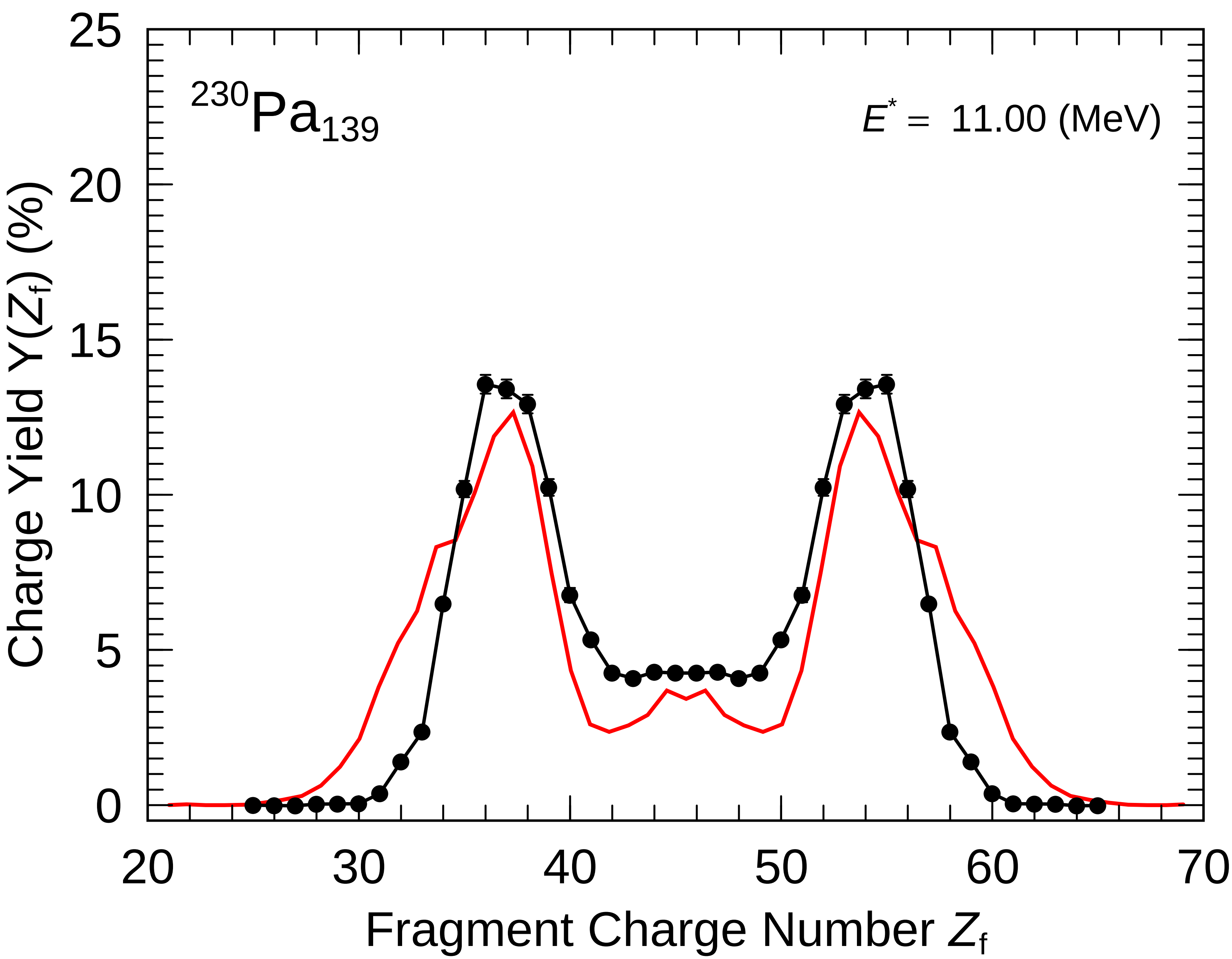
227Pa136
E* = 11.00 (MeV)
Charge Yield Y(Zf) (%)
0
5
10
15
20
25
20
30
40
50
60
70
Fragment Charge Number Zf



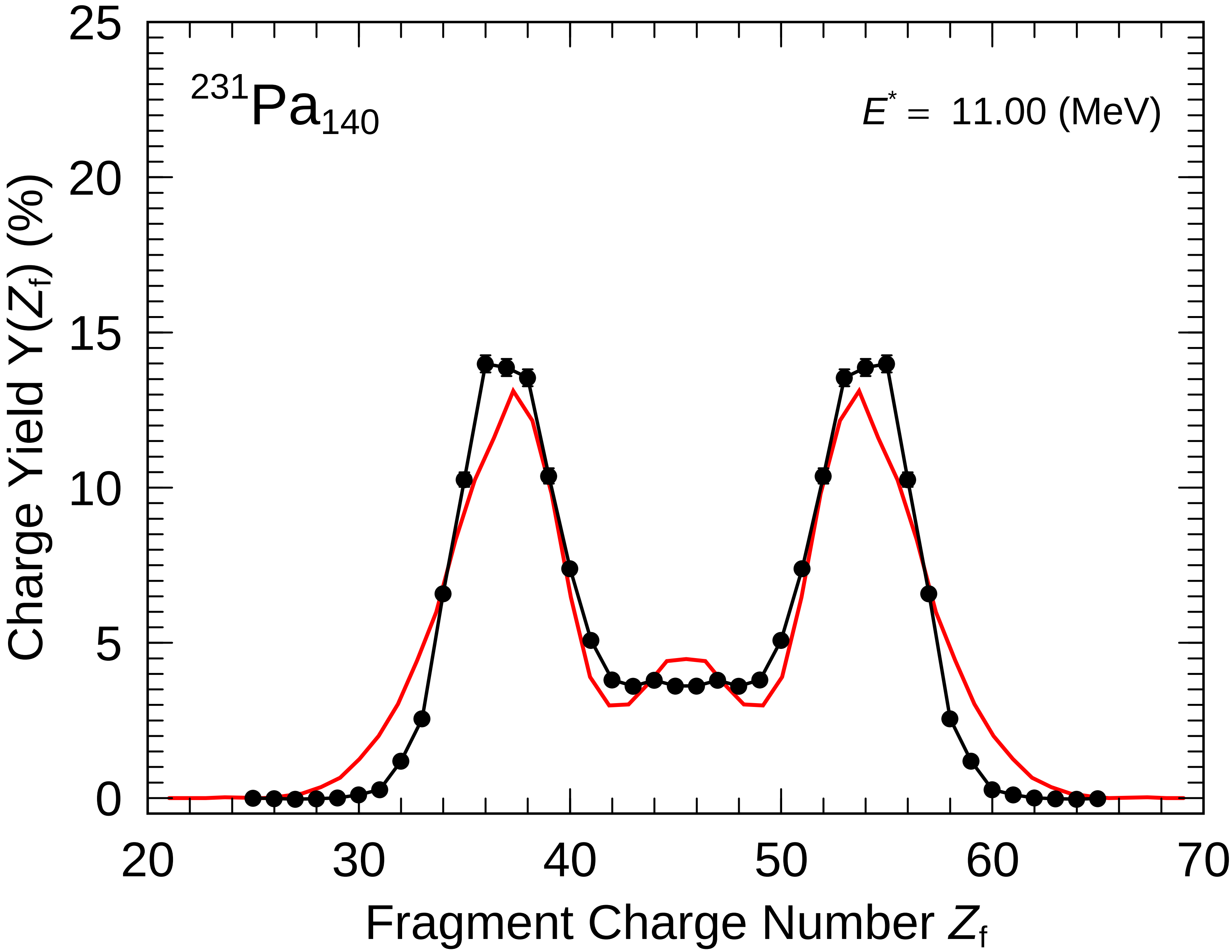
25
20
15
10
5
0
20
30
40
50
60
70
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
228Pa137
E* = 11.00 (MeV)



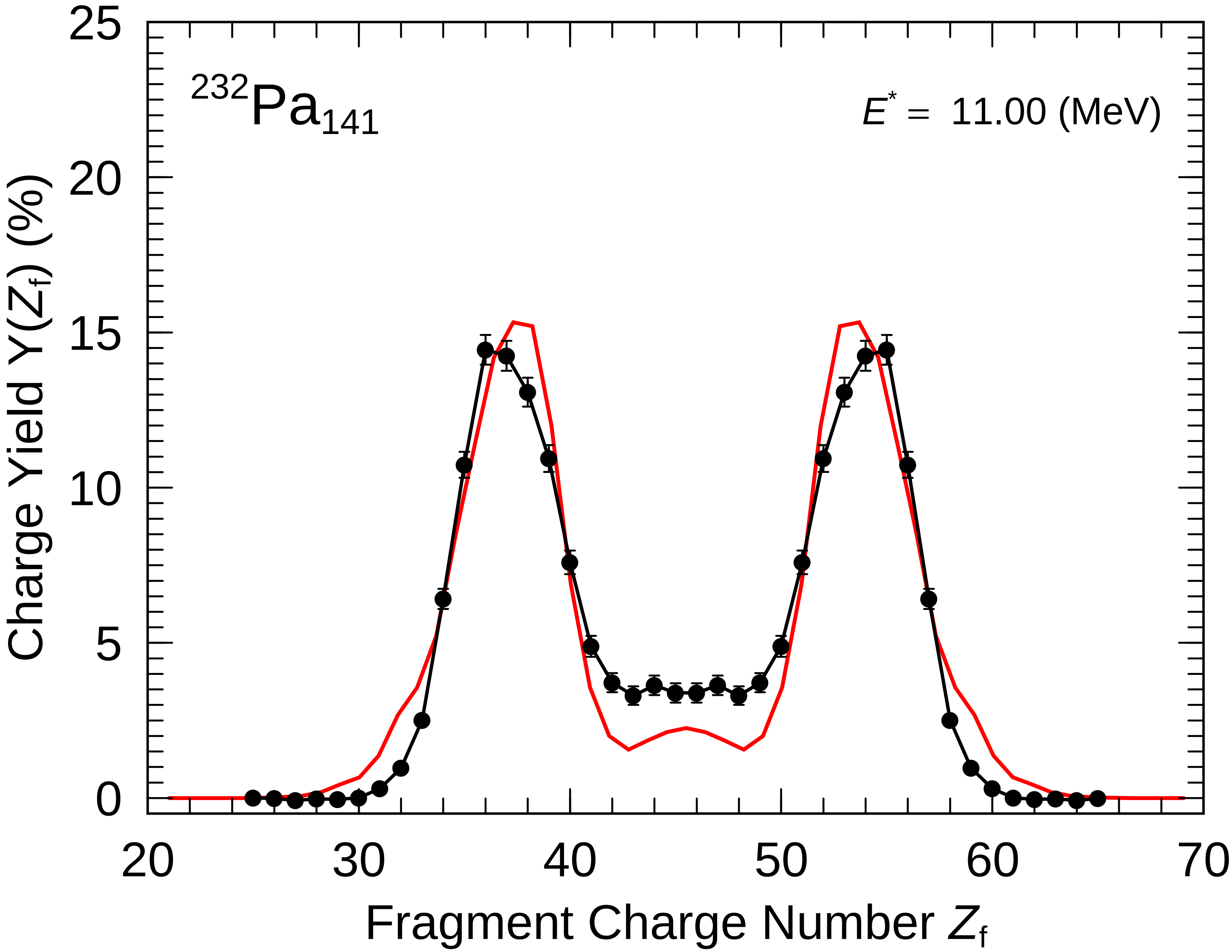
$^{229}Pa_{138}$
E^{*} = 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



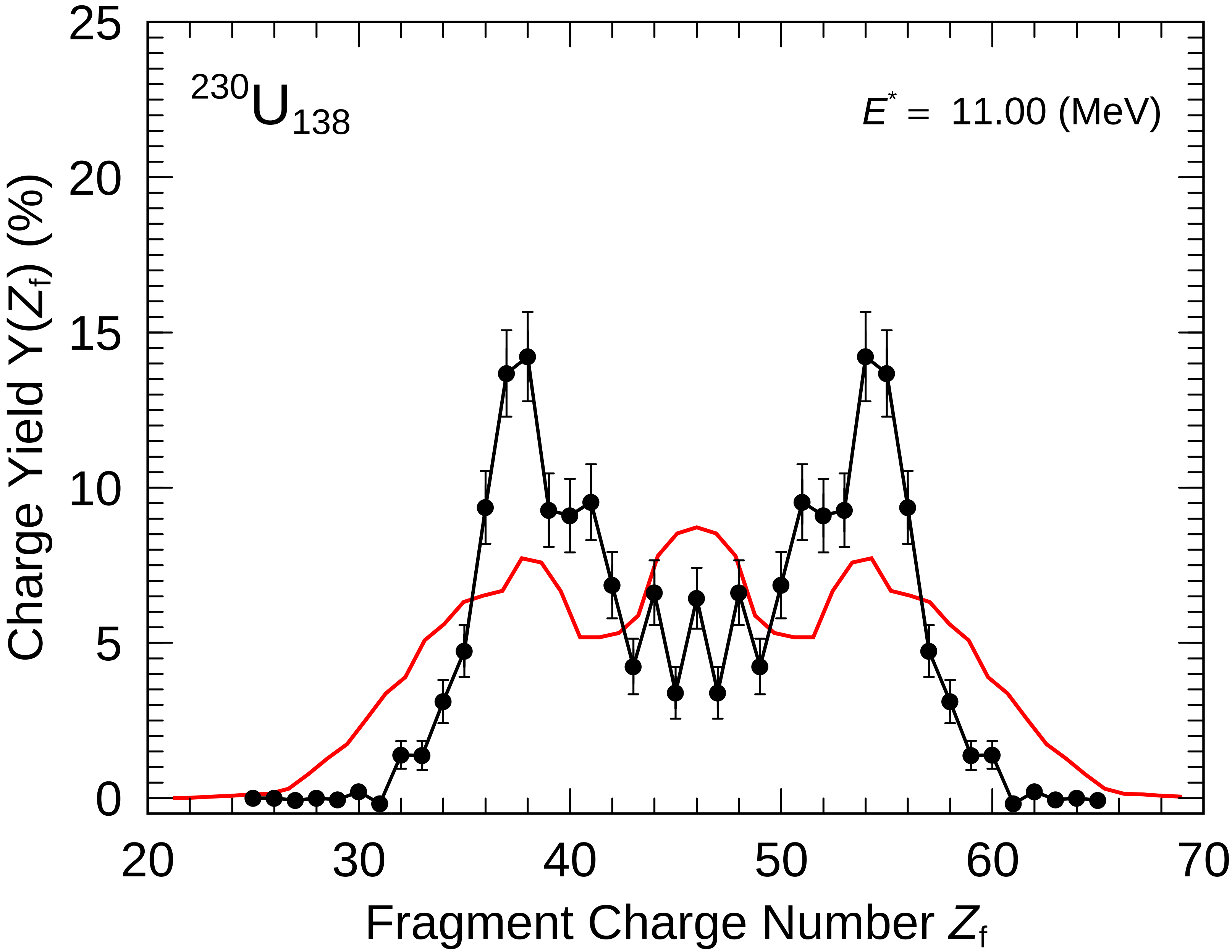
$^{230}Pa_{139}$
E^{*} = 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



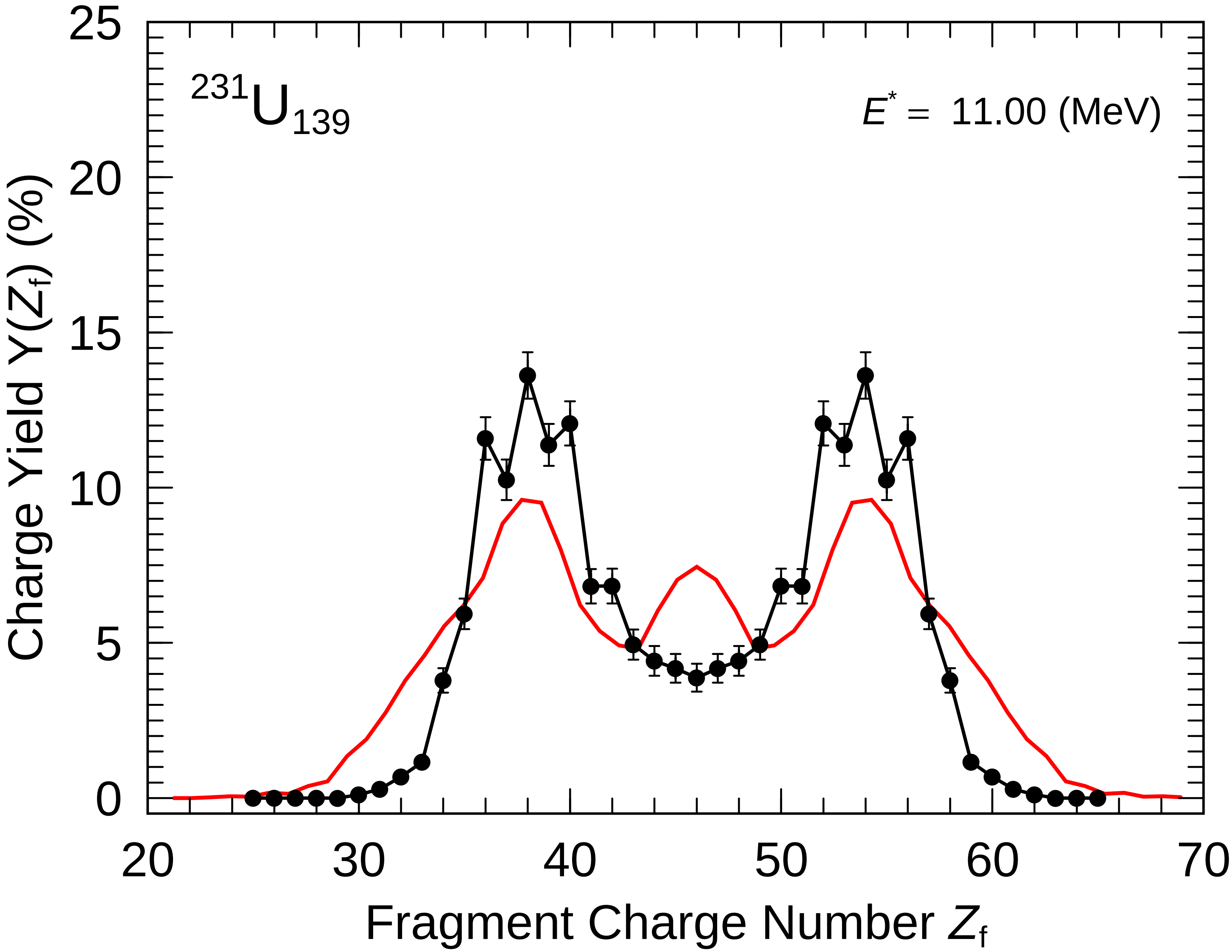
$^{231}Pa_{140}$
E^{*} = 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
25
20
15
10
5
0
20
30
40
50
60
70



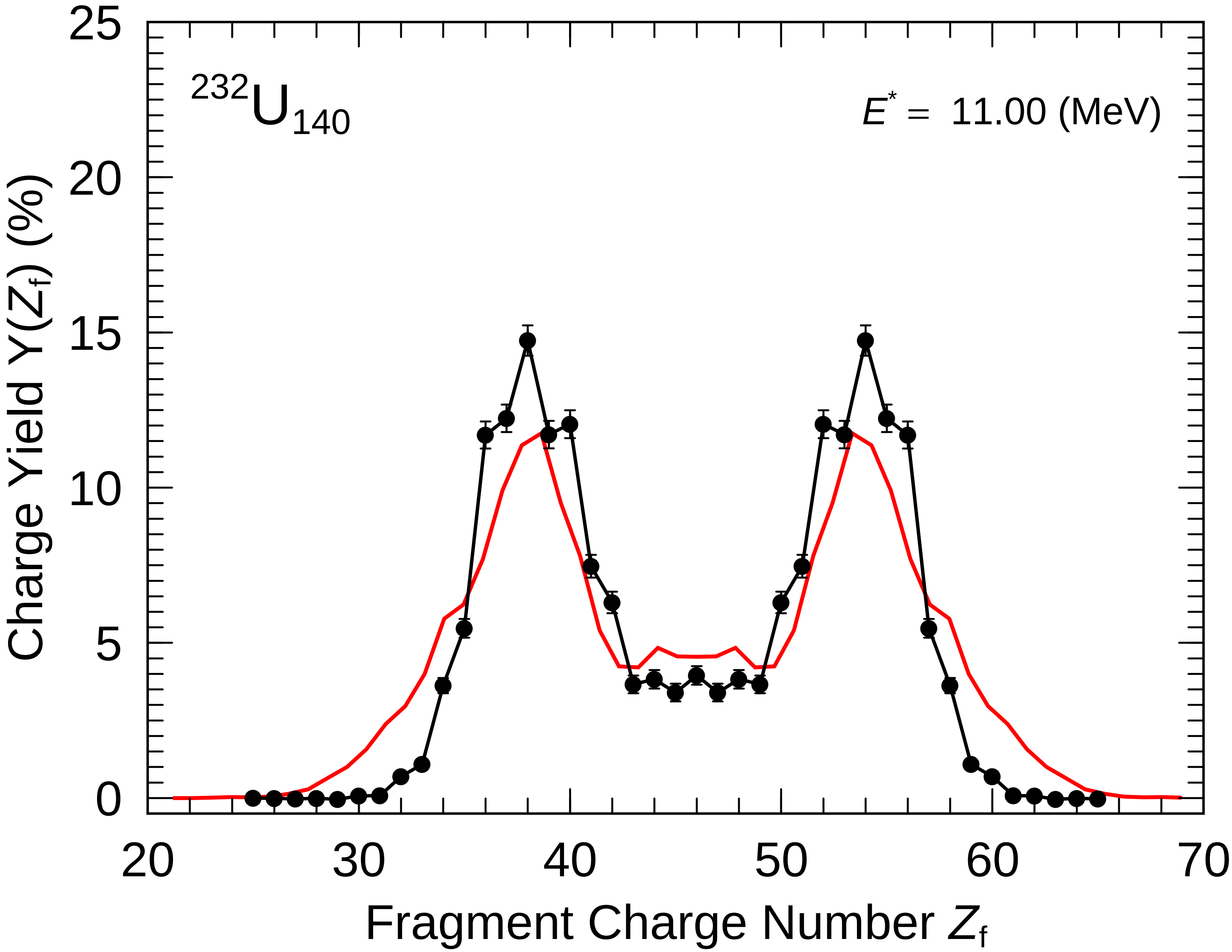
232Pa141
E* = 11.00 (MeV)
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
20
30
40
50
60
70



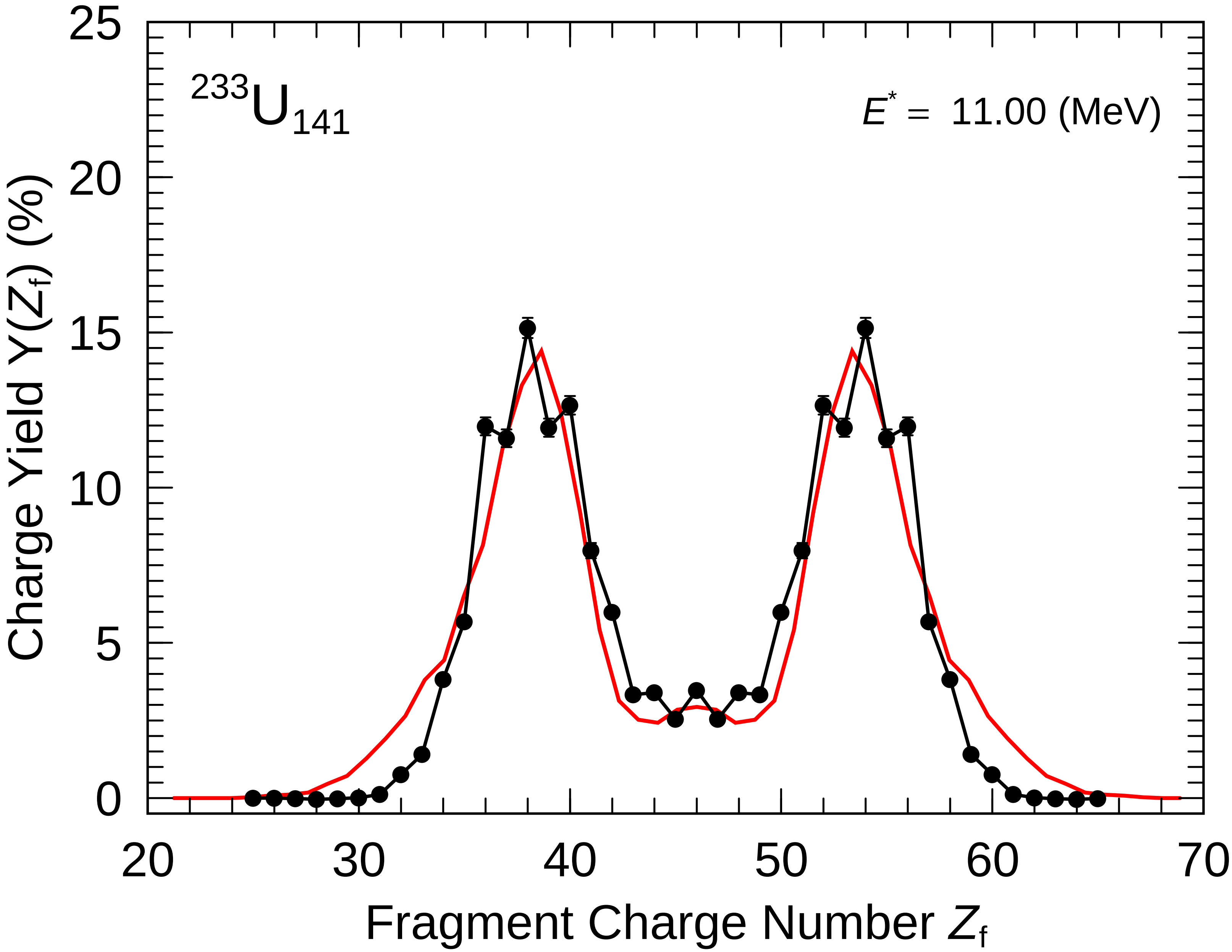
$^{230}U_{138}$
$E^* =$ 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



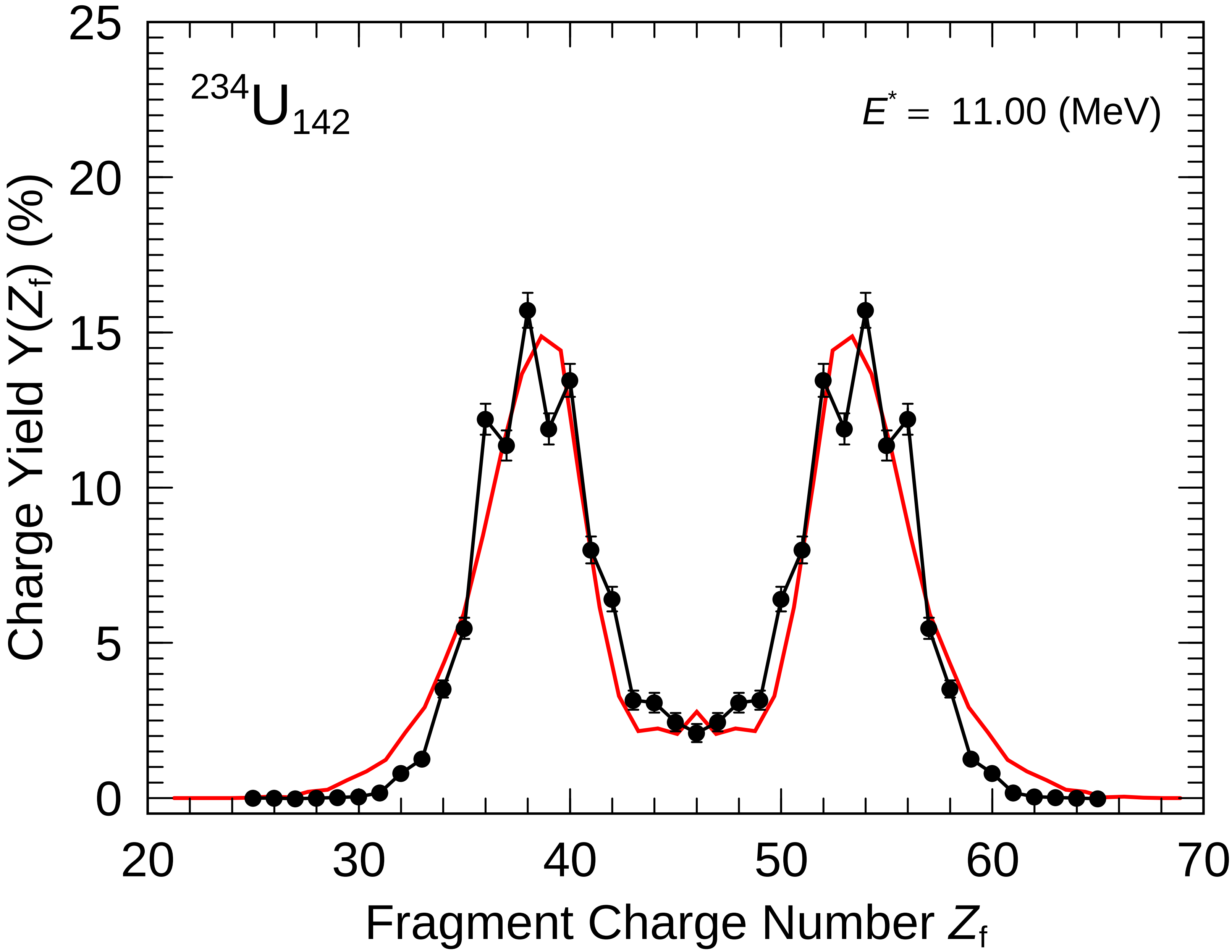
$^{231}U_{139}$
$E^* =$ 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
20
30
40
50
60
70



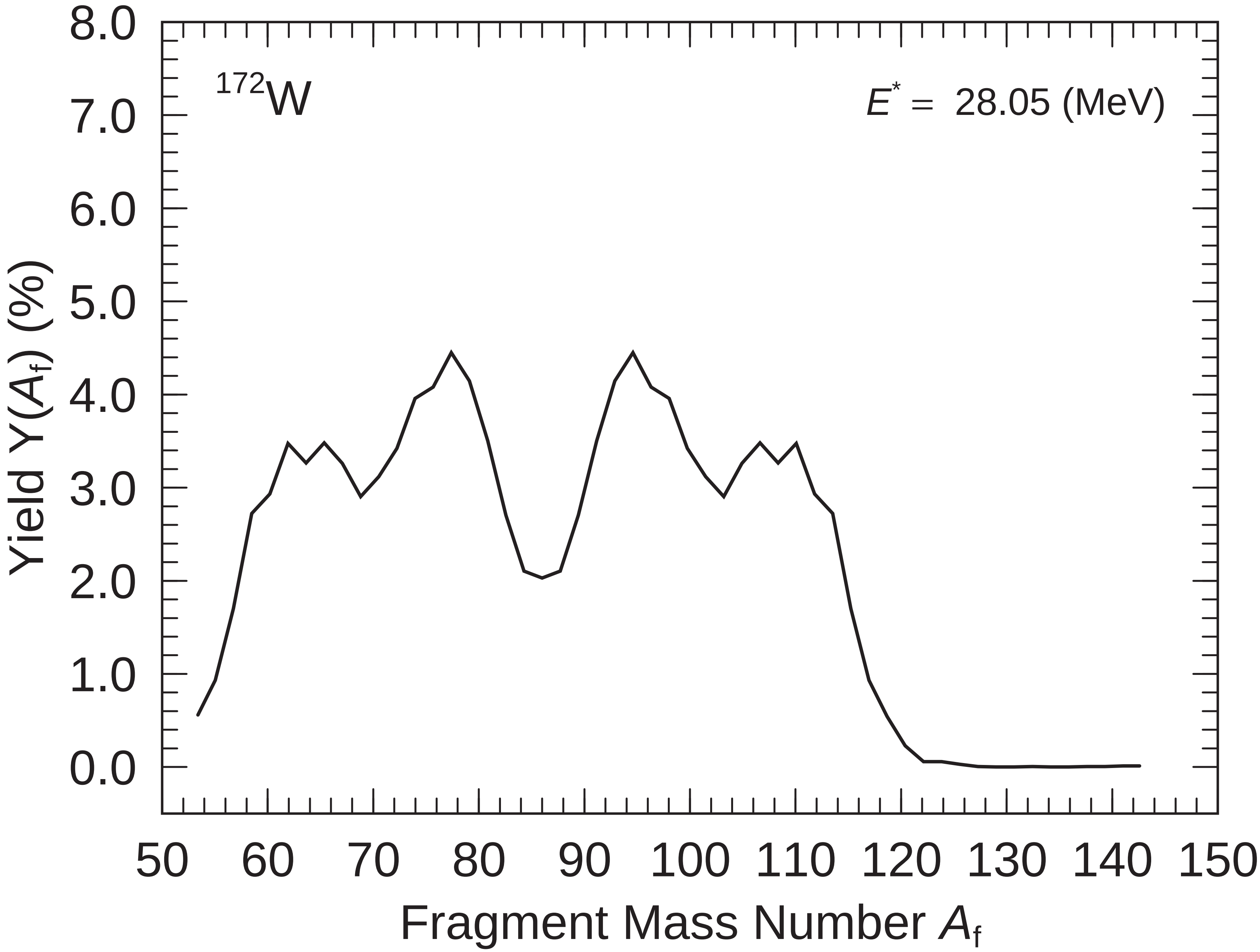
$^{232}U_{140}$
E^{*} = 11.00 (MeV)
Charge Yield Y(Z_f) (%)
Fragment Charge Number Z_f
25
20
15
10
5
0
20
30
40
50
60
70



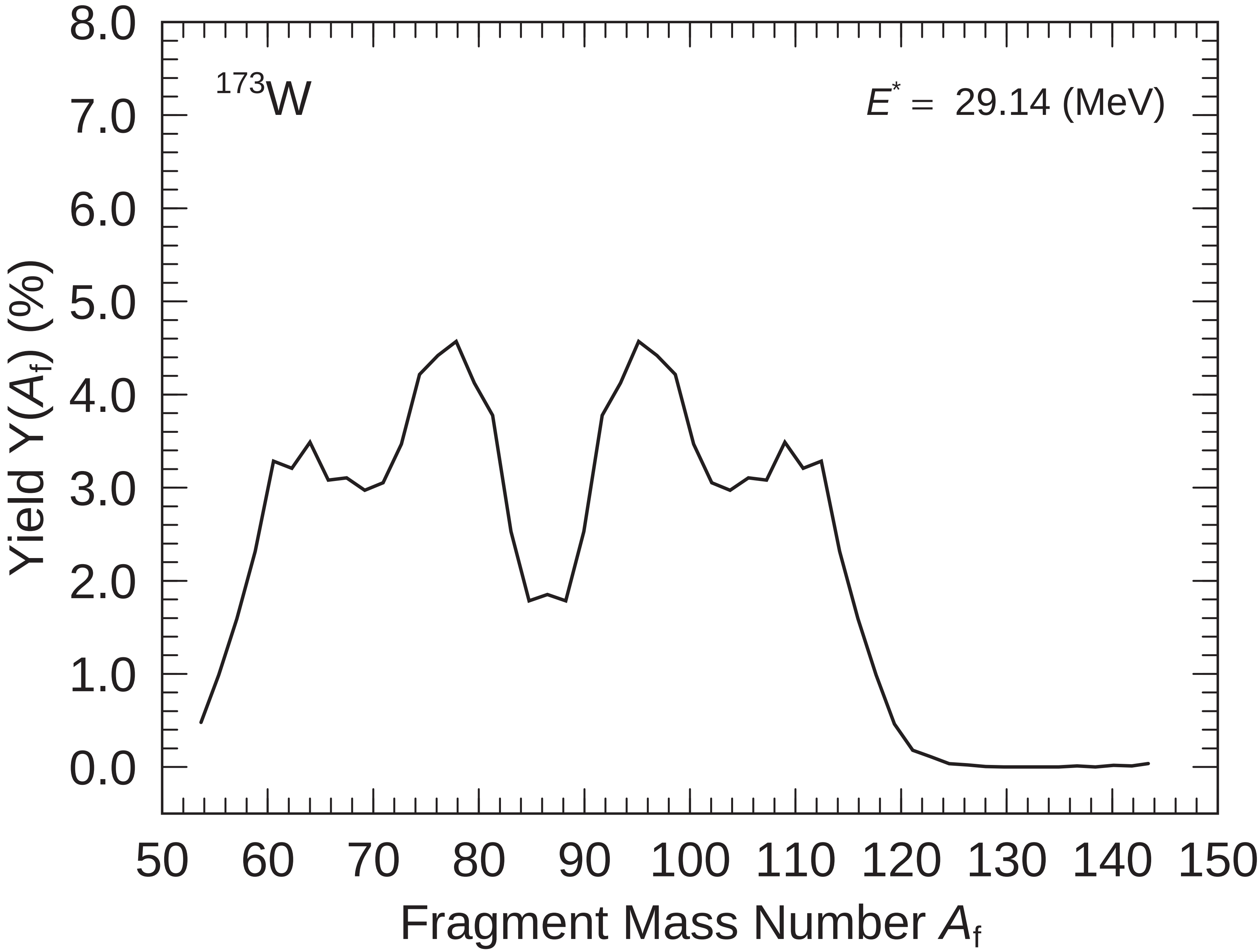
25
20
15
10
5
0
233U141
E* = 11.00 (MeV)
Charge Yield Y(Zf) (%)
20
30
40
50
60
70
Fragment Charge Number Zf



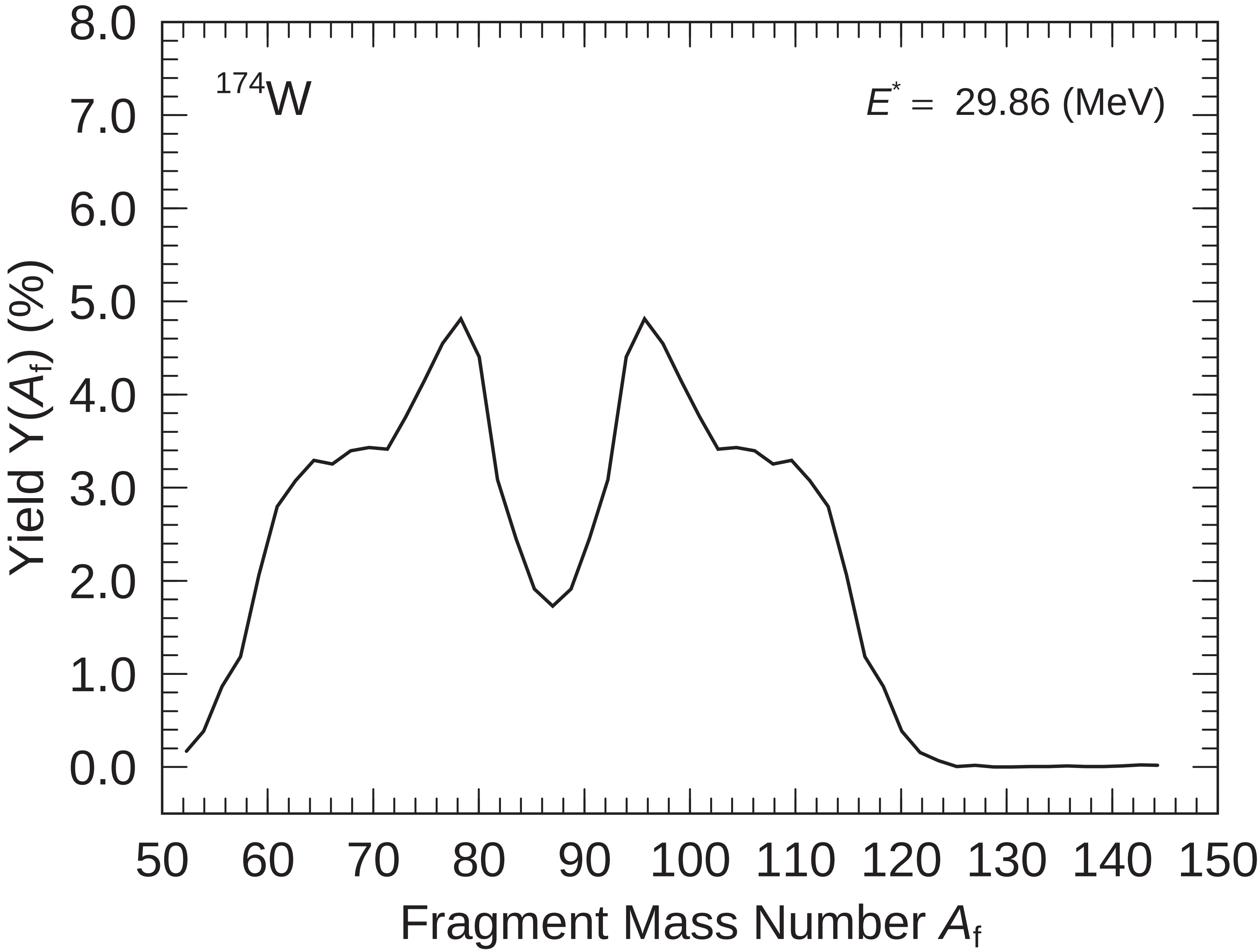
25
20
15
10
5
0
Charge Yield Y(Zf) (%)
234U142
E* = 11.00 (MeV)
20
30
40
50
60
70
Fragment Charge Number Zf



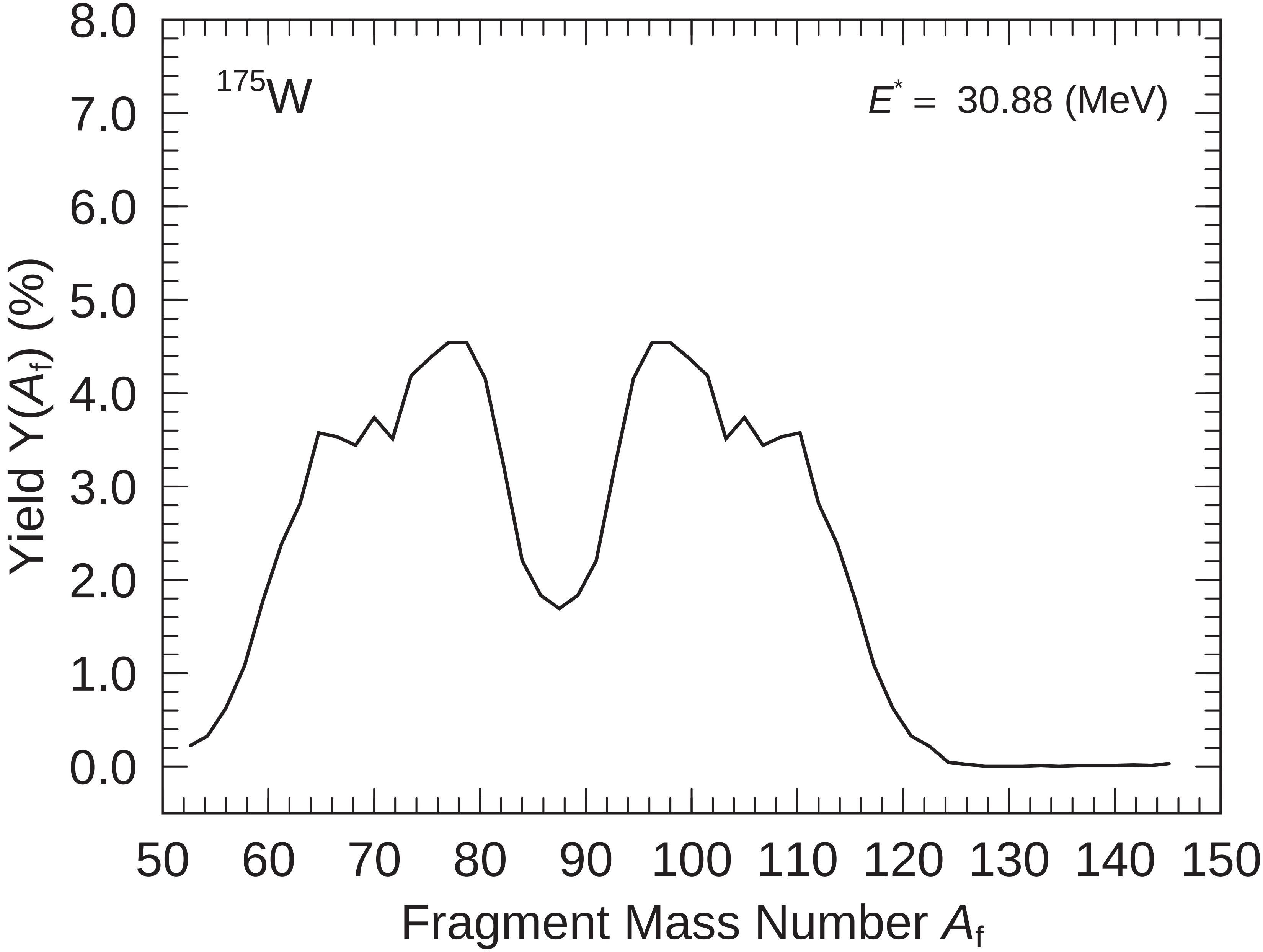
^{172}W
E^{*} = 28.05 (MeV)
Yield Y(A_f) (%)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number A_f



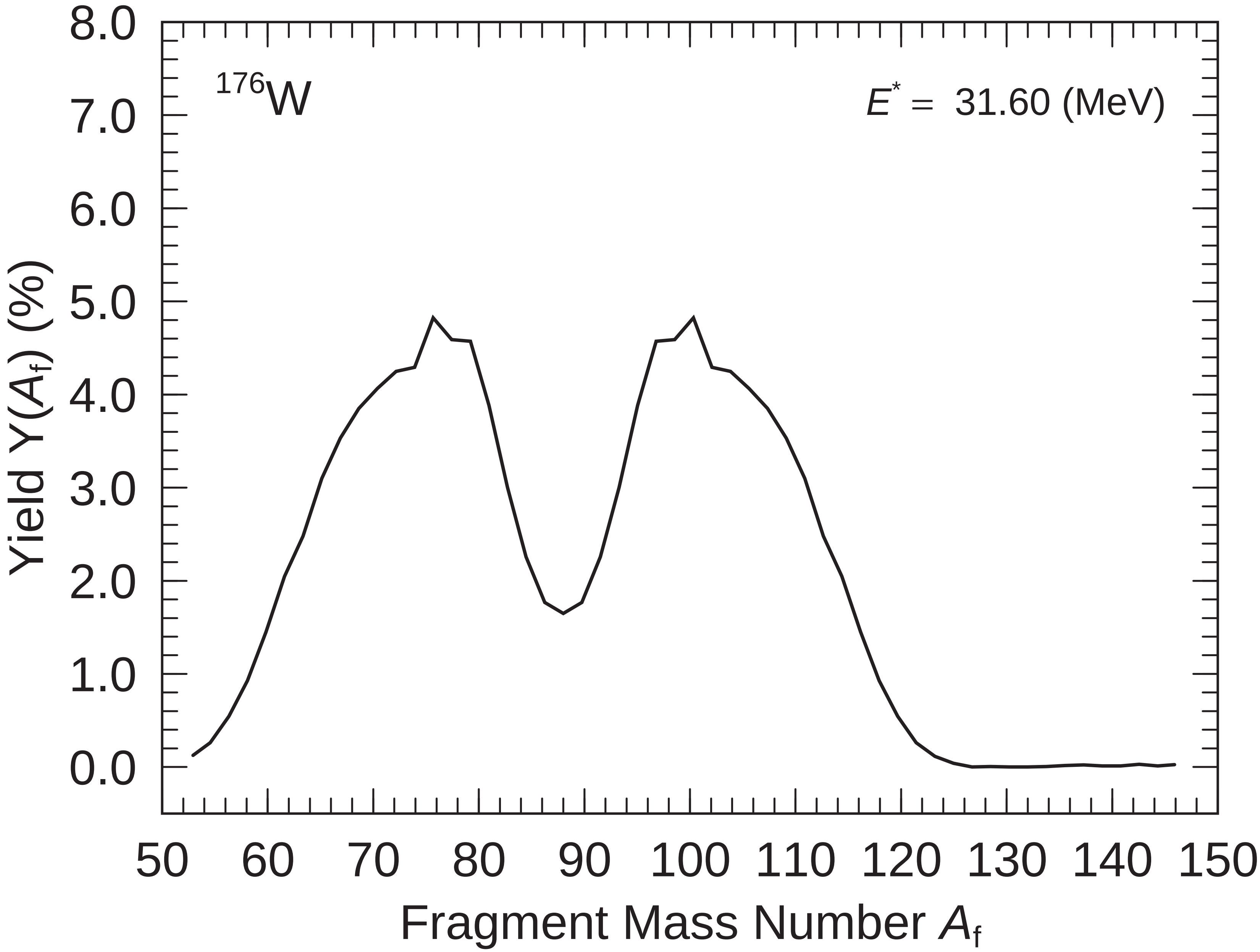
173W
E* = 29.14 (MeV)
Yield Y(Af) (%)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number Af



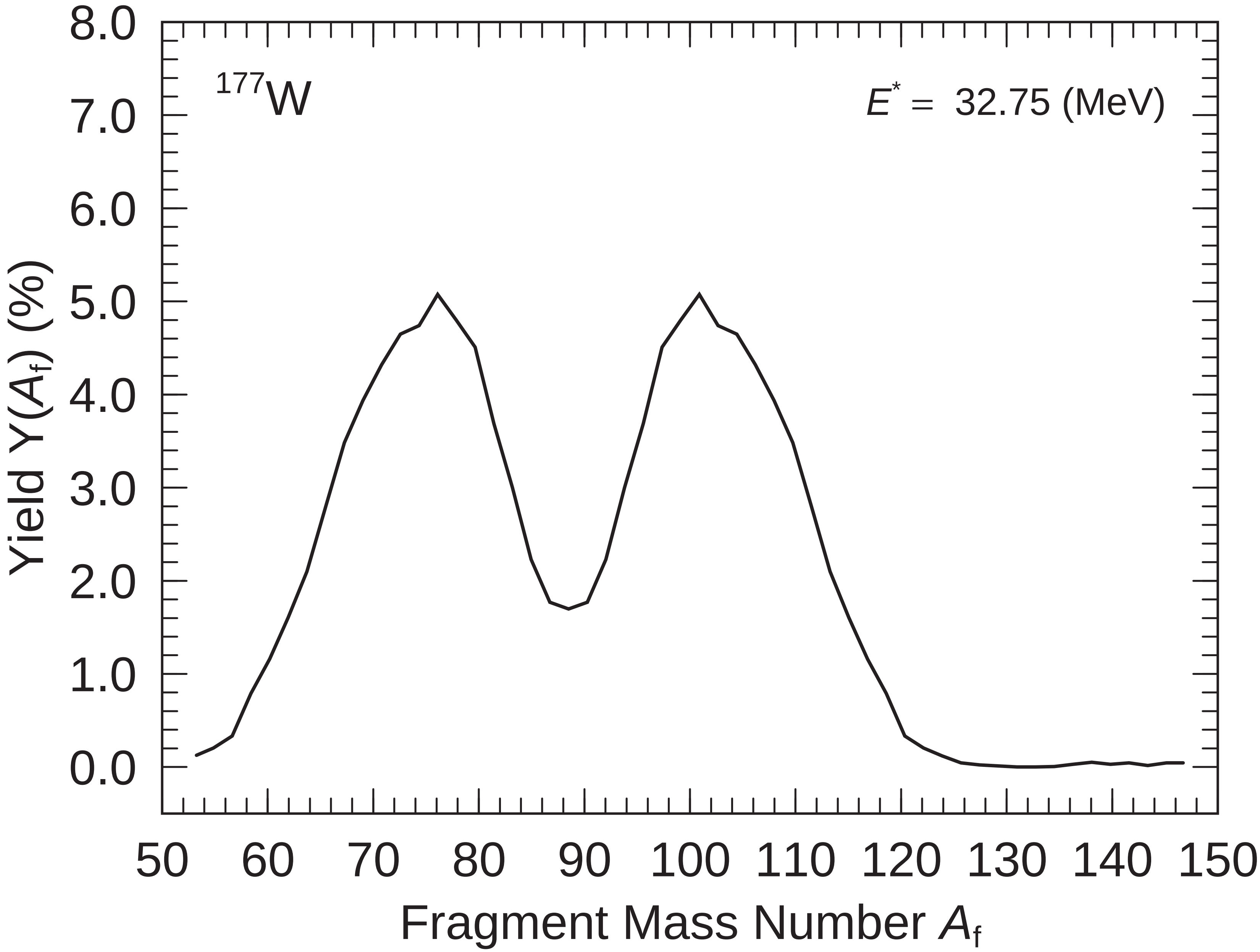
174W
E* = 29.86 (MeV)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Yield Y(Af) (%)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number Af



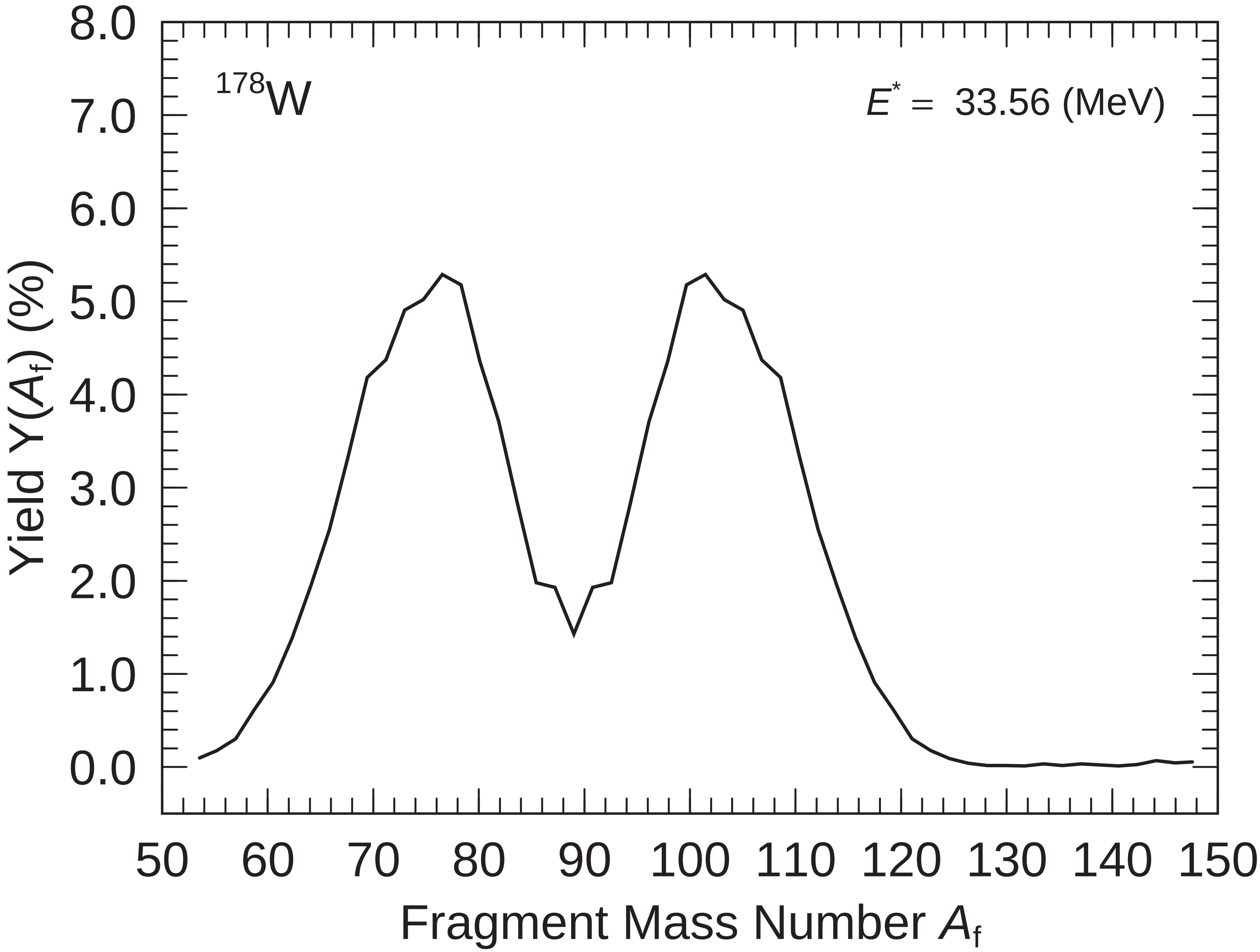
^{175}W
$E^* =$ 30.88 (MeV)
Yield Y(A_f) (%)
Fragment Mass Number A_f
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150



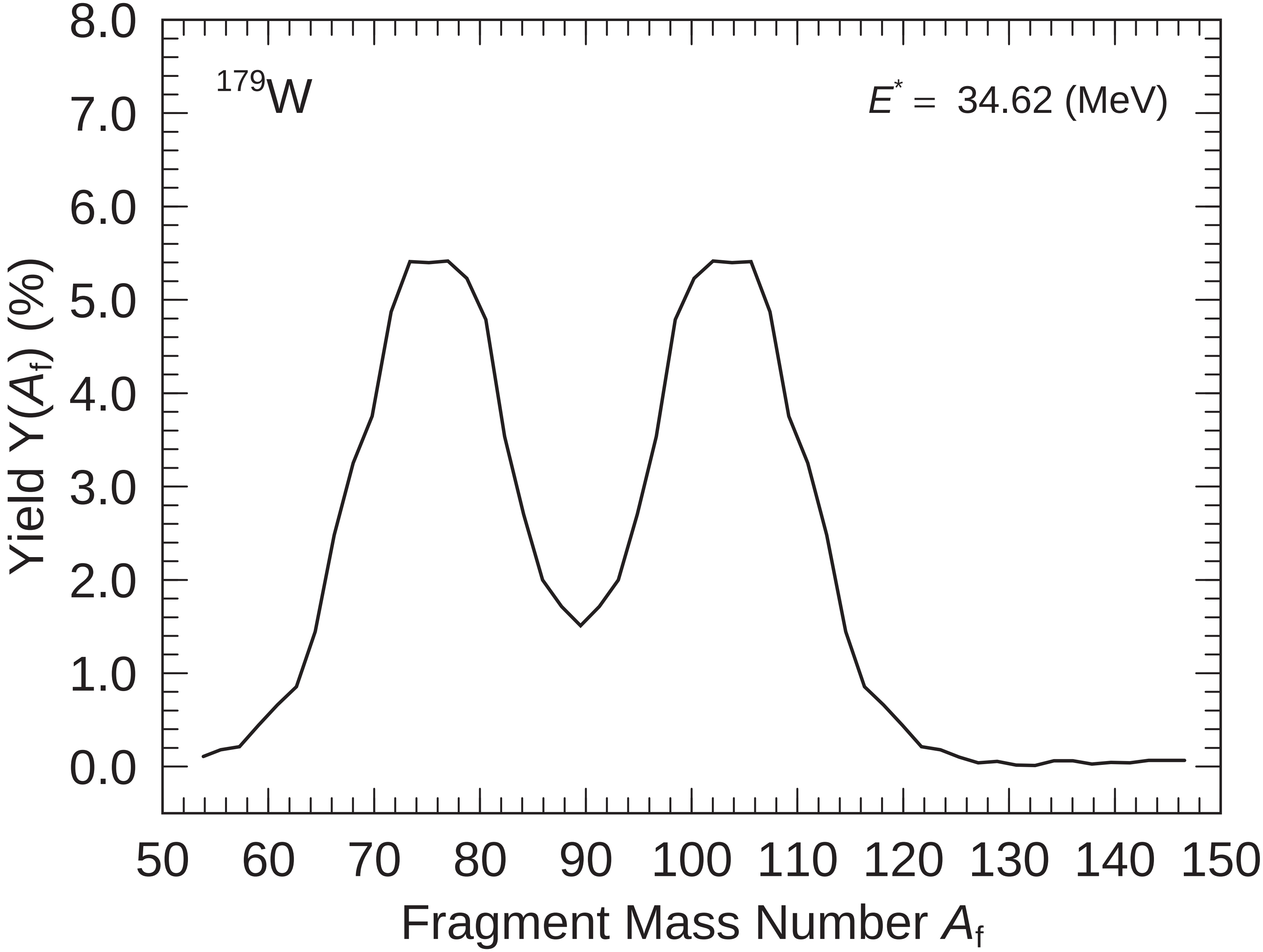
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Yield Y(A_f) (%)
176W
E* = 31.60 (MeV)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number A_f



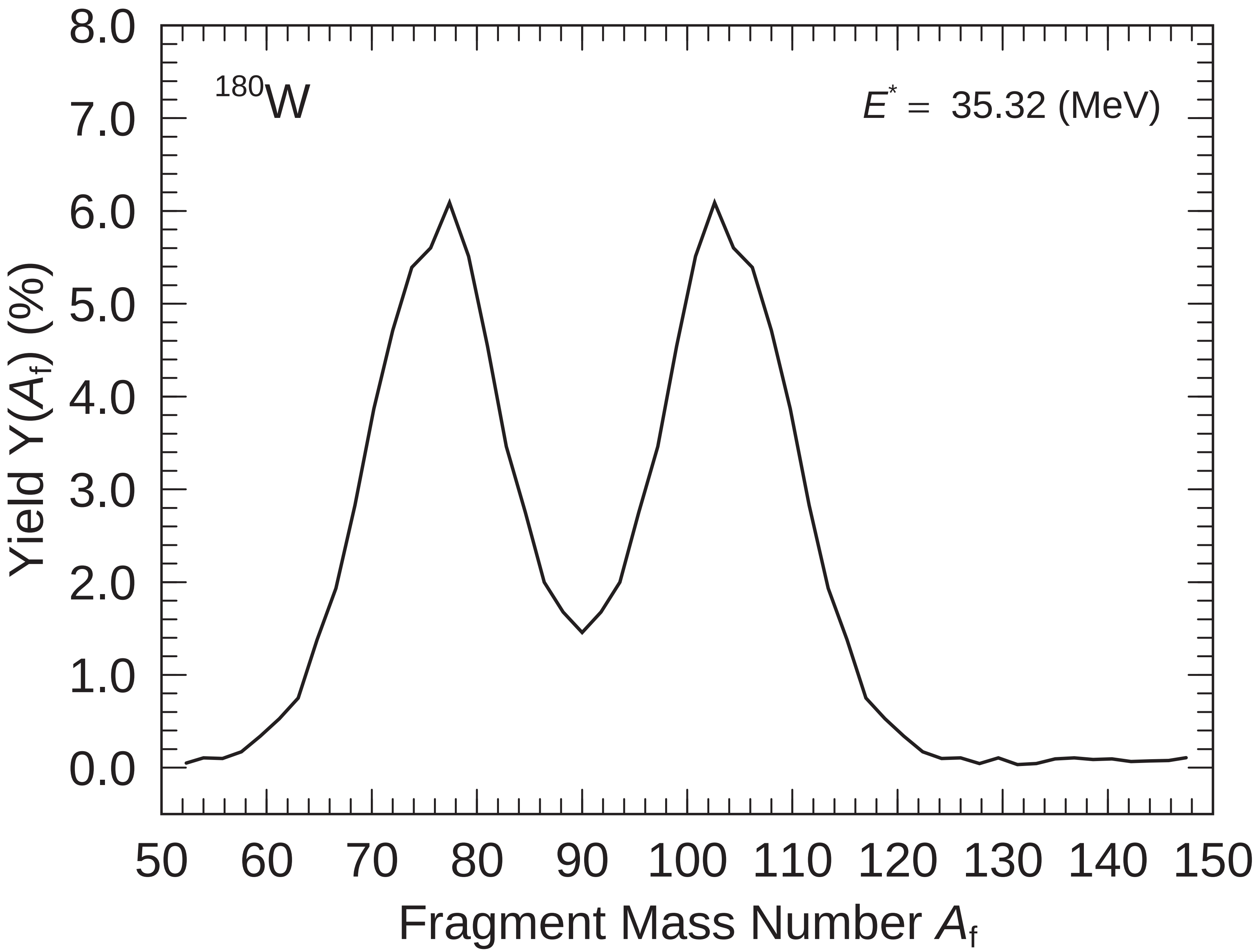
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Yield Y(Af) (%)
177W
E* = 32.75 (MeV)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number Af



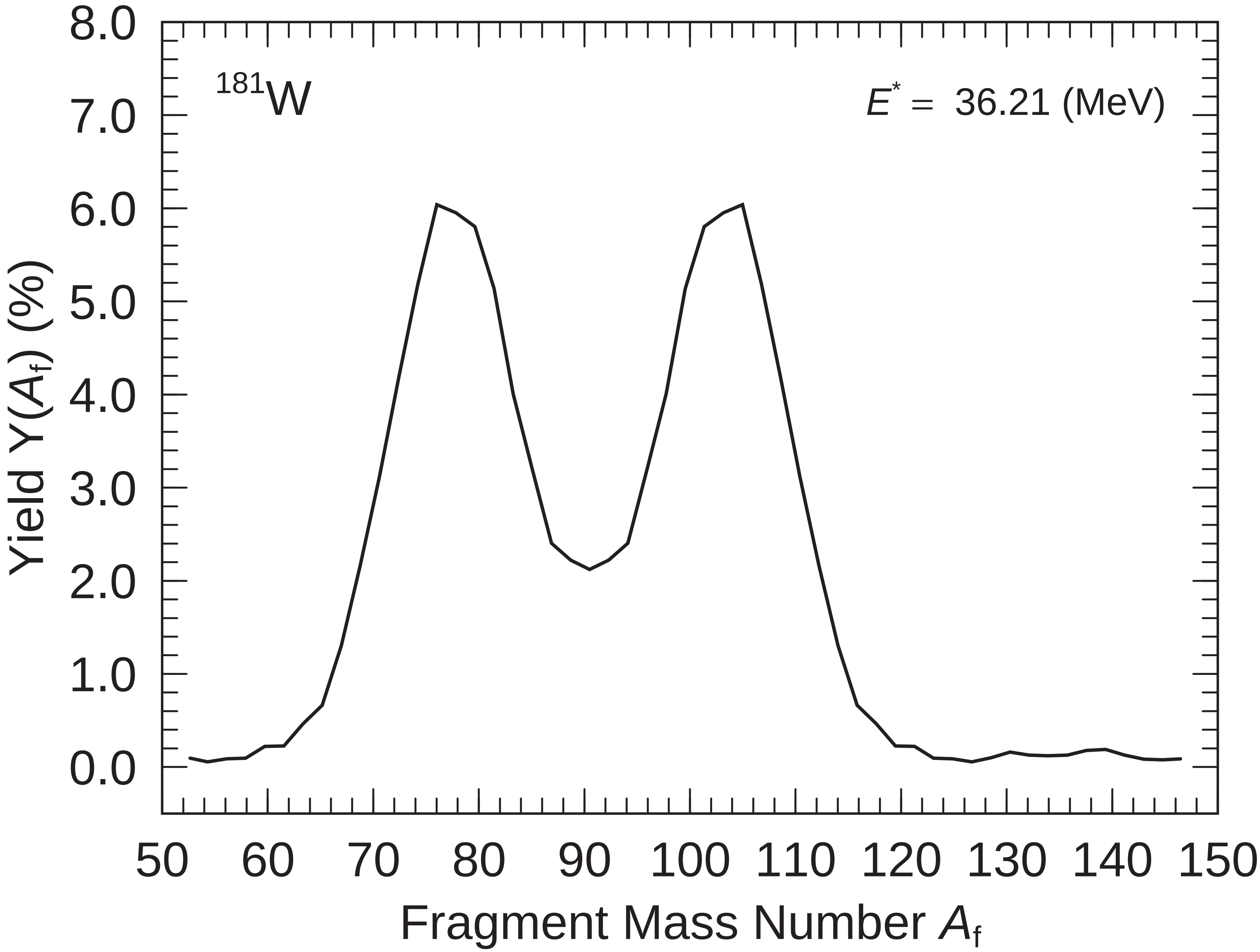
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Yield Y(A_f) (%)
^{178}W
E^* = 33.56 (MeV)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number A_f



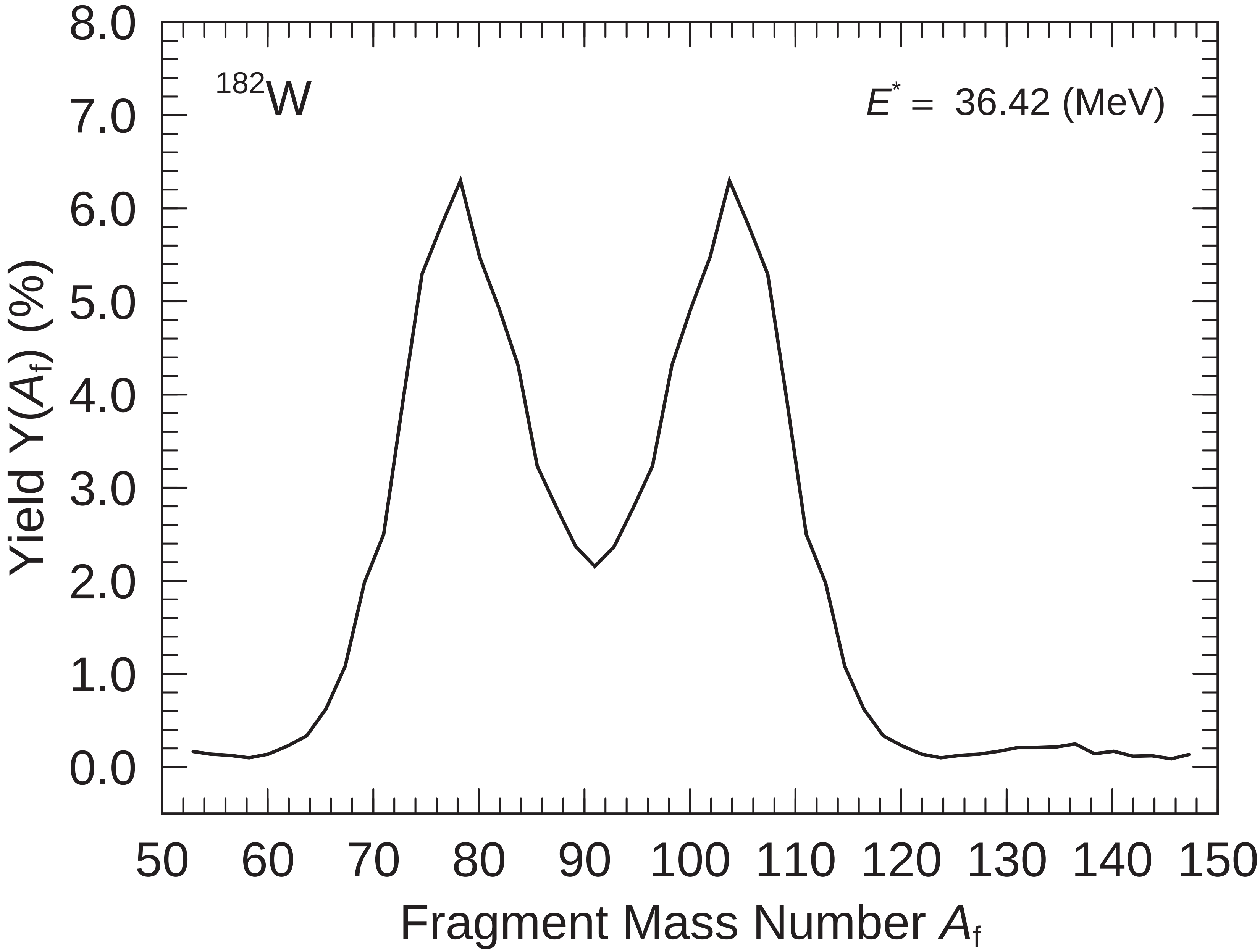
179W
E* = 34.62 (MeV)
Yield Y(Af) (%)
Fragment Mass Number Af
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
50
60
70
80
90
100
110
120
130
140
150



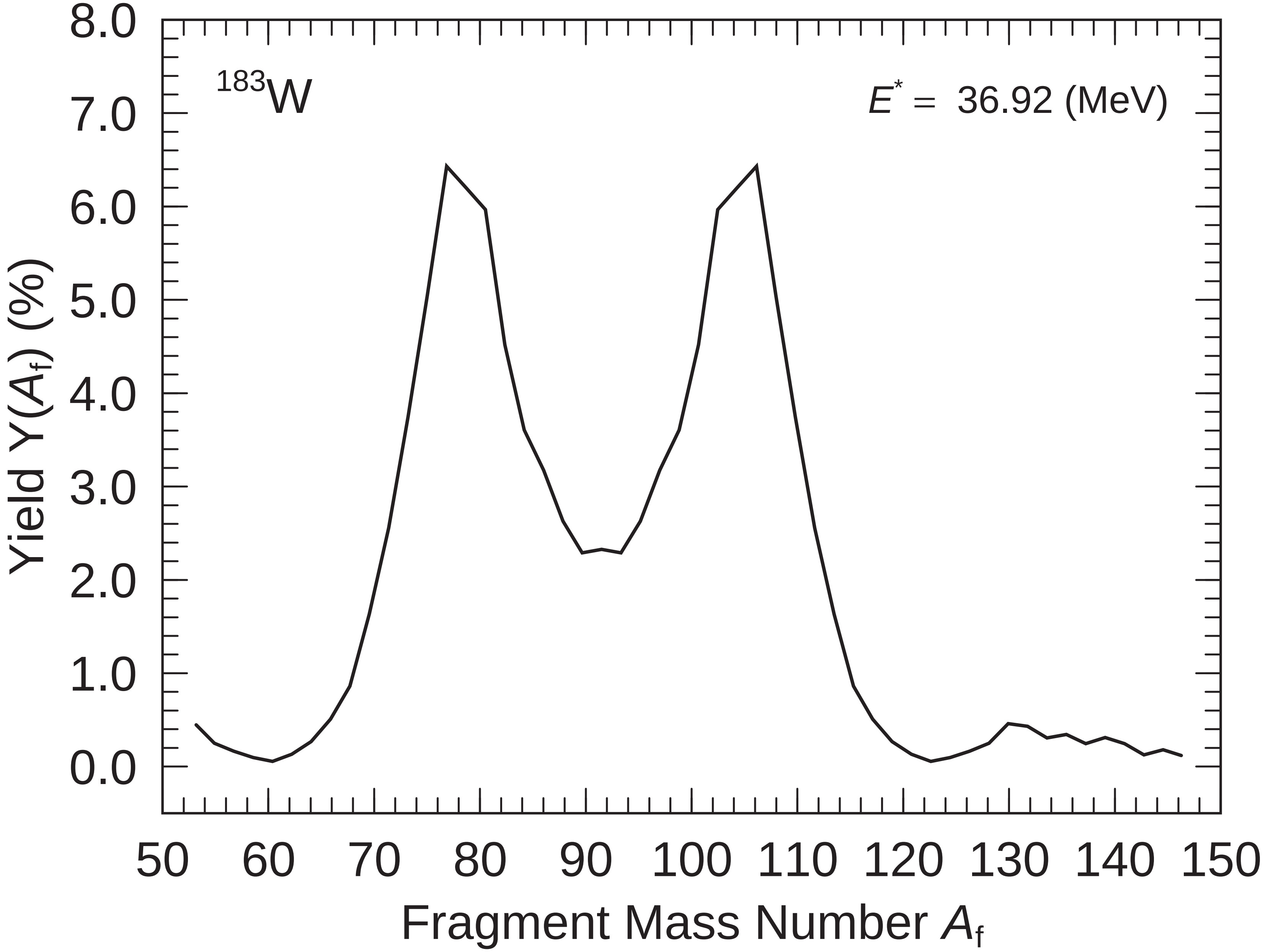
180W
E* = 35.32 (MeV)
Yield Y(Af) (%)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number Af



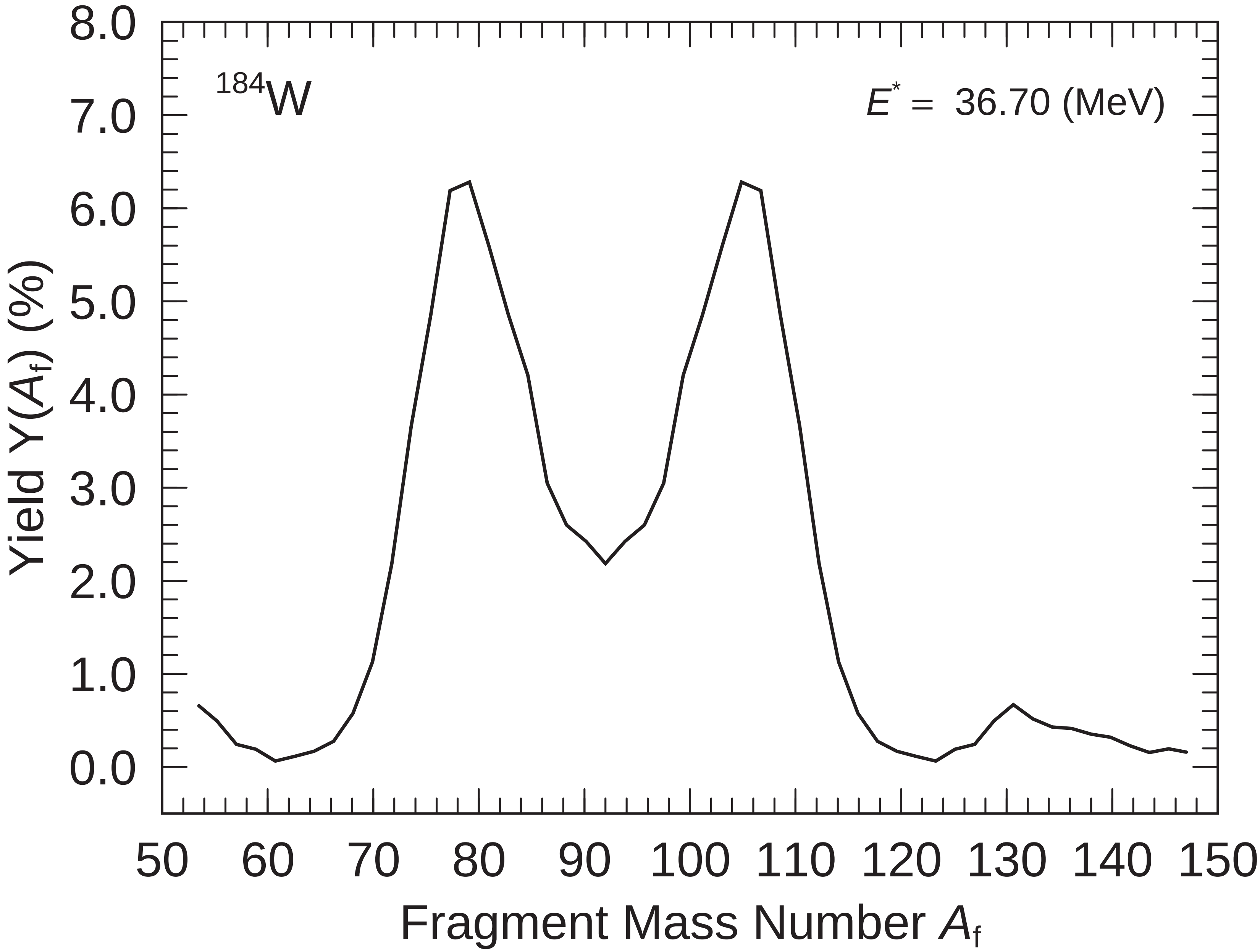
181W
E* = 36.21 (MeV)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Yield Y(Af) (%)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number Af



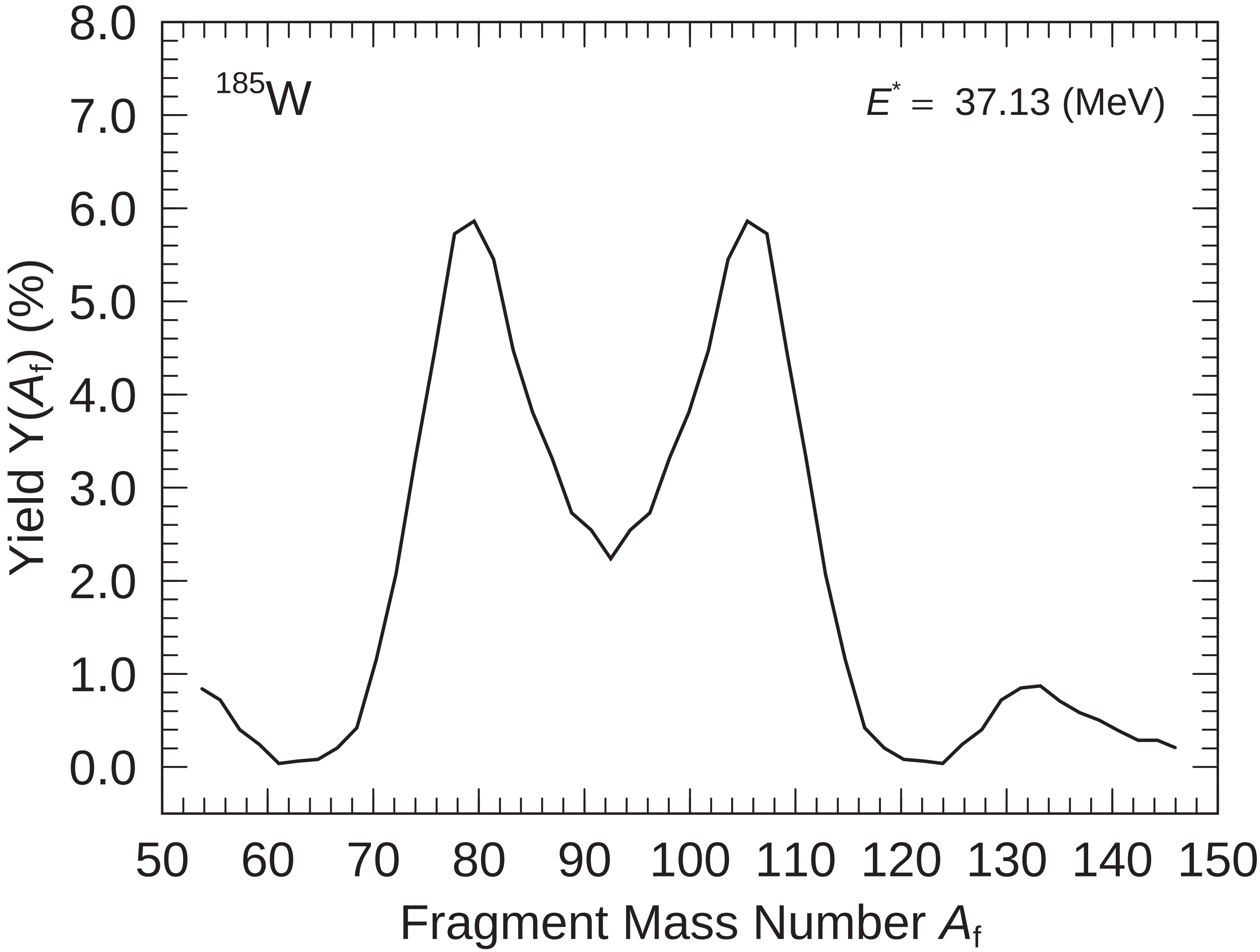
^{182}W
$E^* =$ 36.42 (MeV)
Yield Y(A_f) (%)
Fragment Mass Number A_f
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
50
60
70
80
90
100
110
120
130
140
150



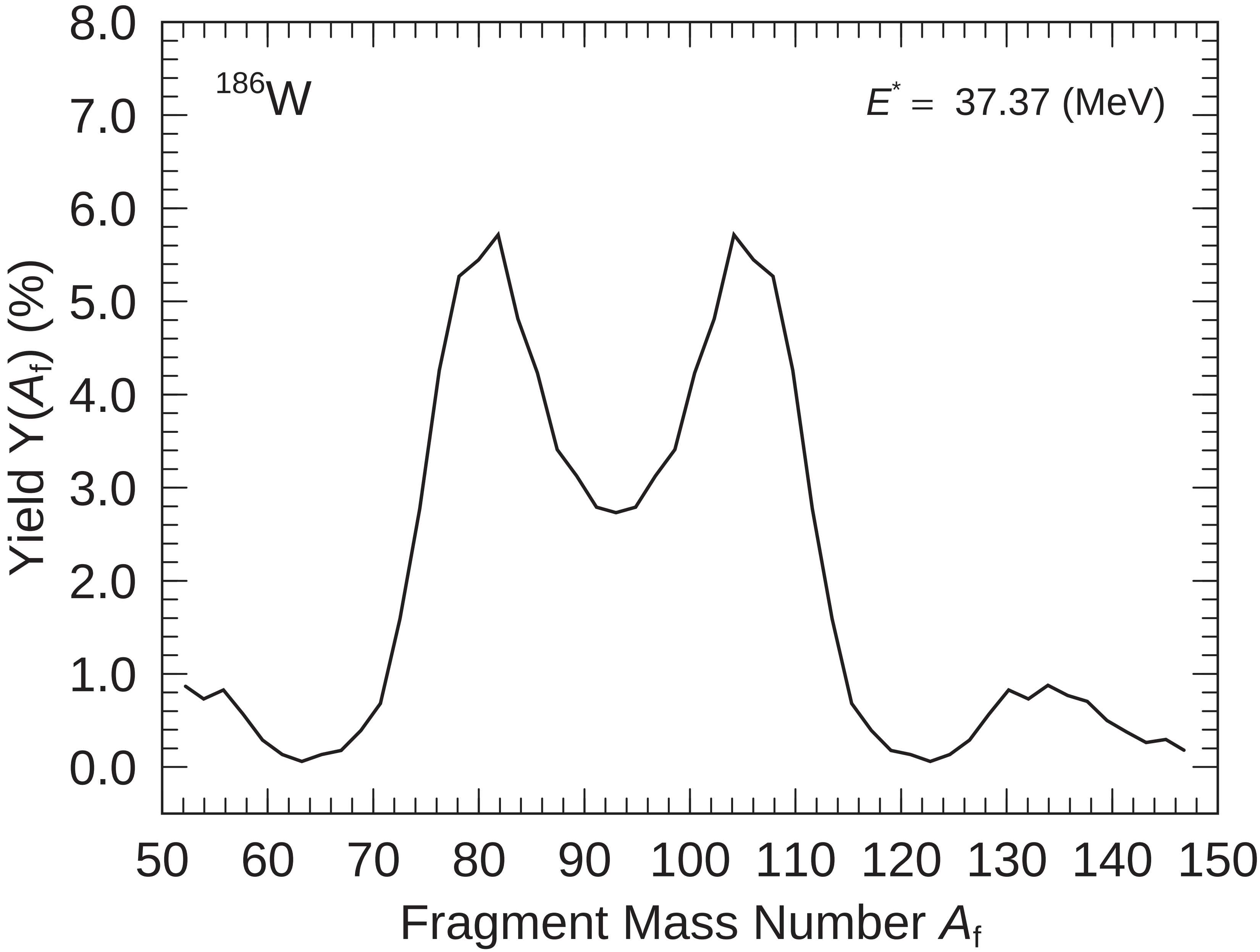
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Yield Y(A_f) (%)
^{183}W
E^* = 36.92 (MeV)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number A_f



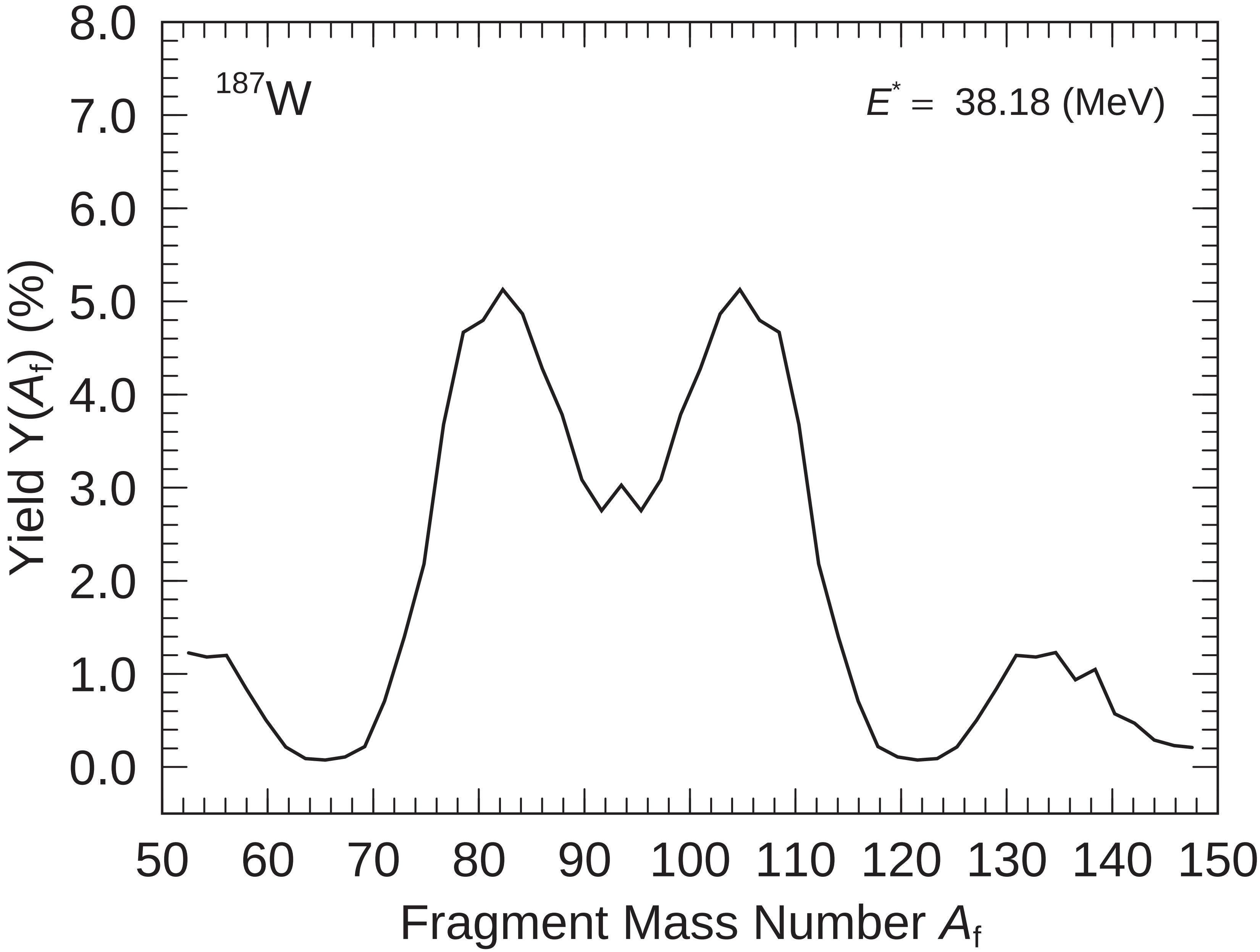
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Yield Y(Af) (%)
184W
E* = 36.70 (MeV)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number Af



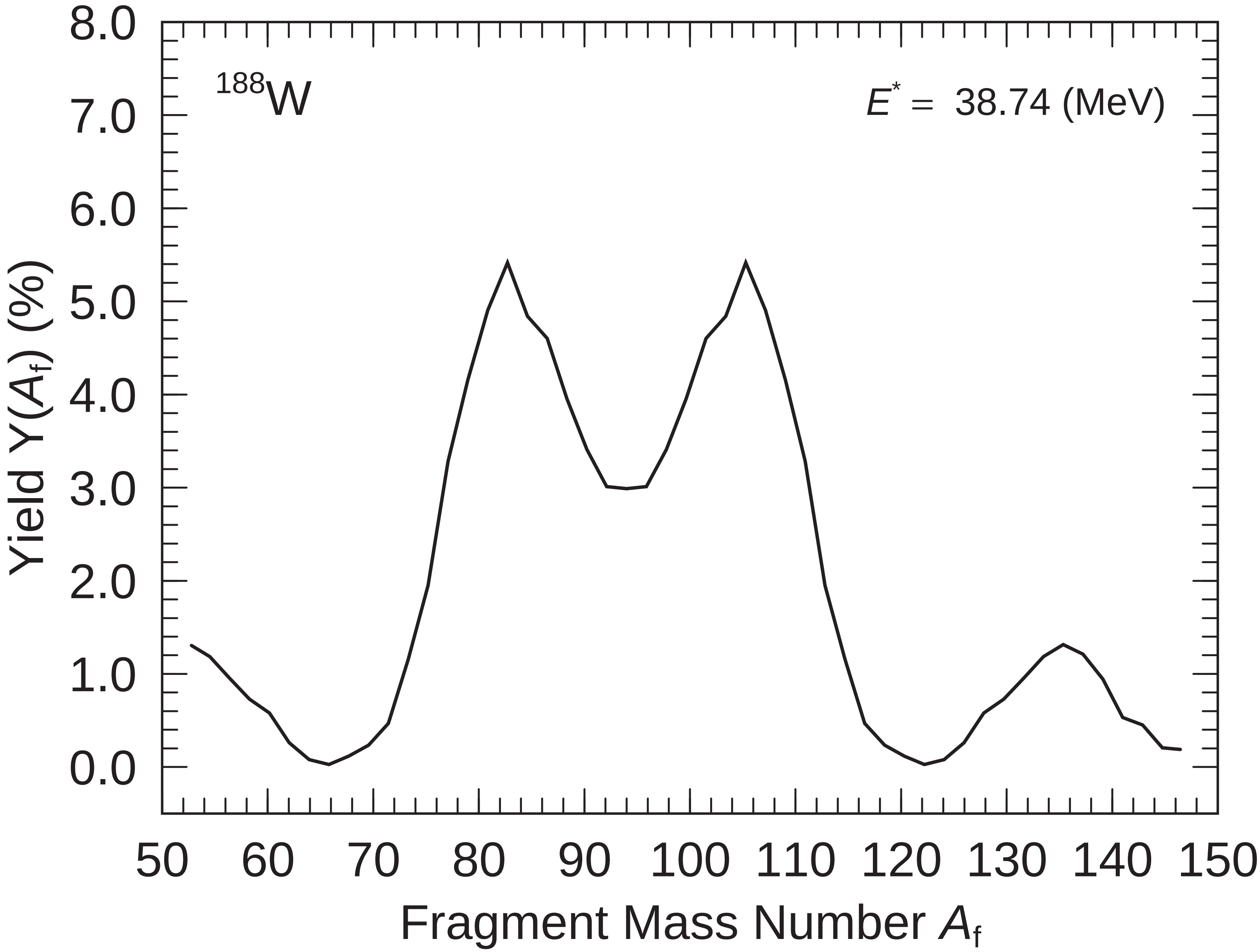
185W
E* = 37.13 (MeV)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Yield Y(Af) (%)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number Af



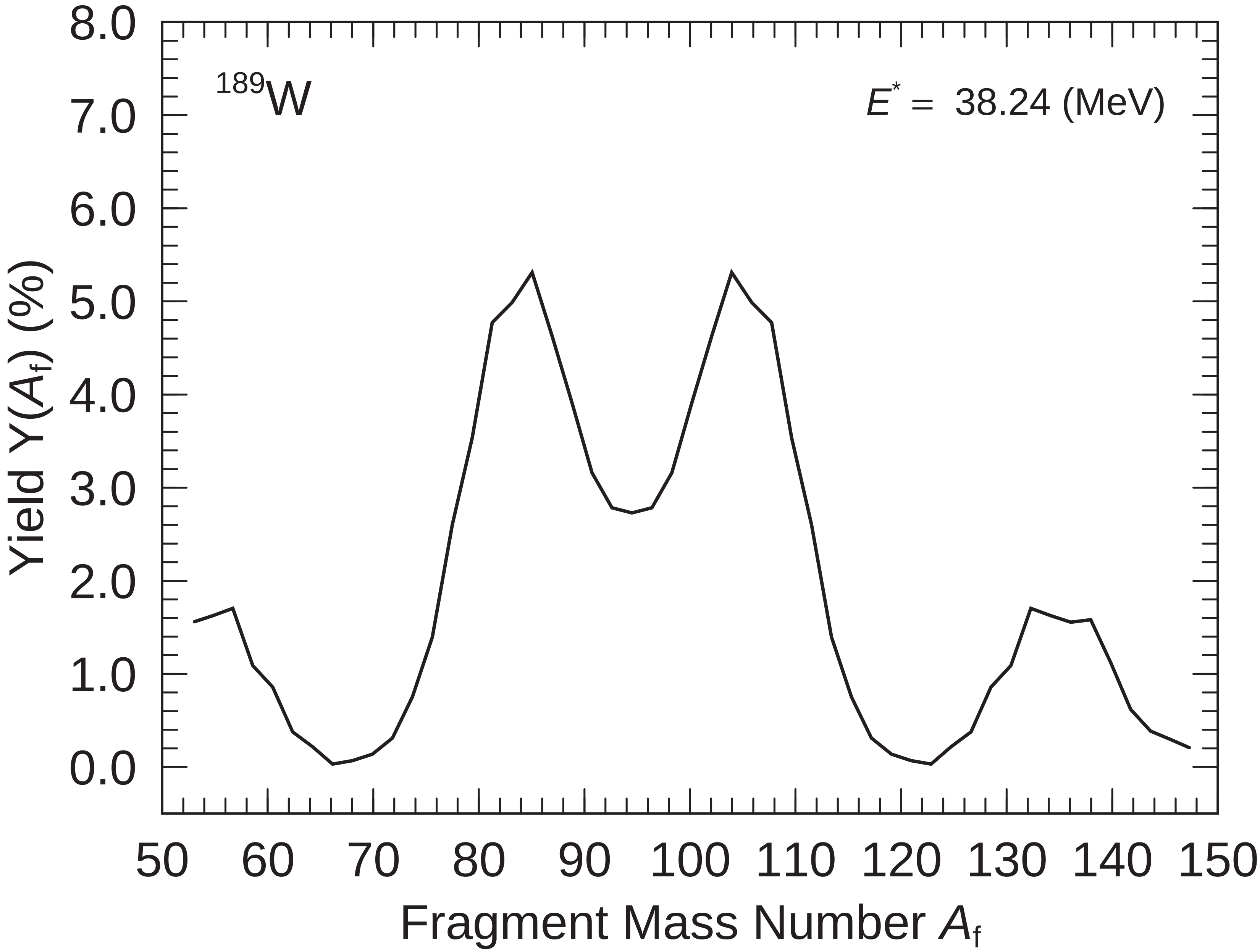
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Yield Y(A_f) (%)
186W
E* = 37.37 (MeV)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number A_f



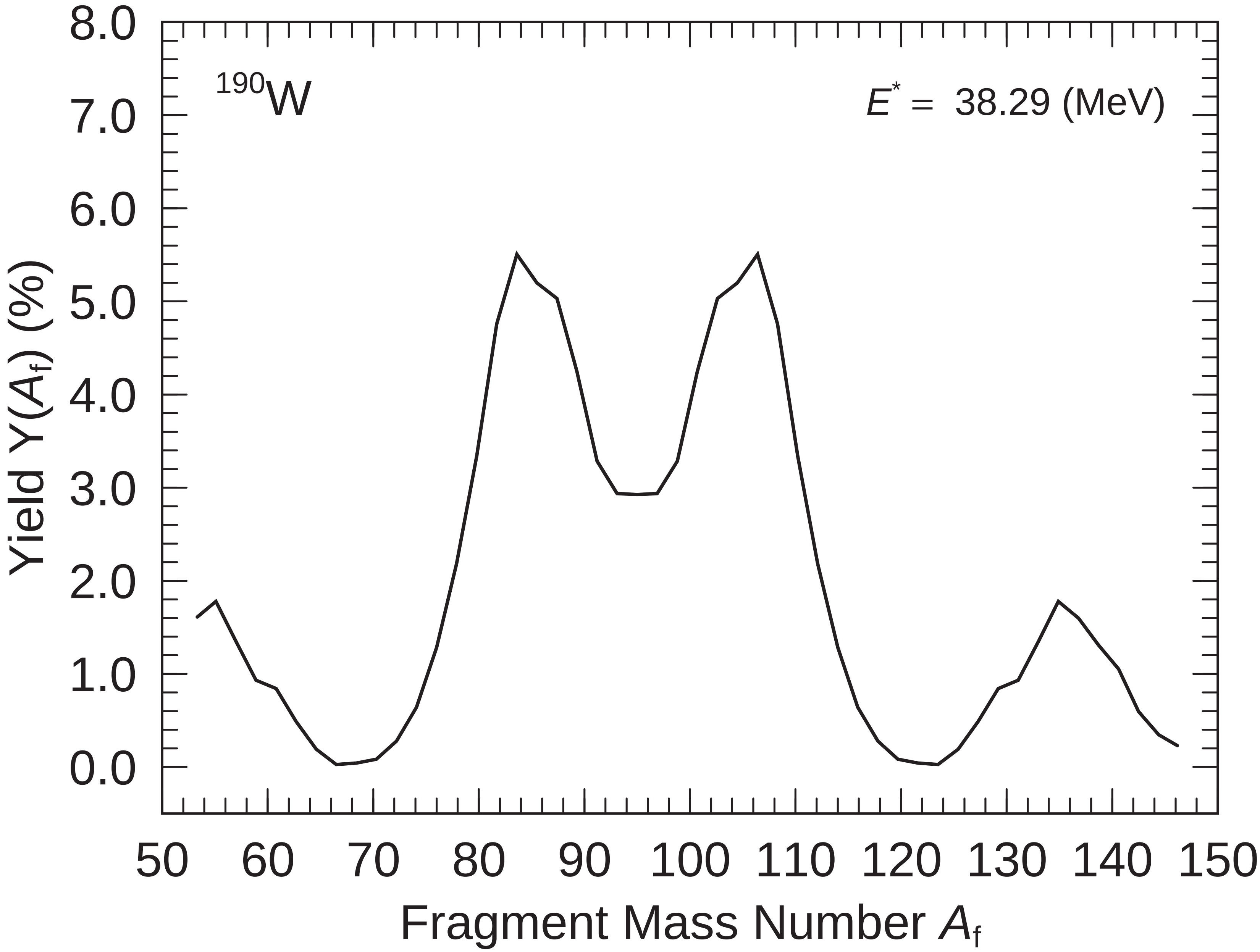
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Yield Y(Af) (%)
187W
E* = 38.18 (MeV)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number Af



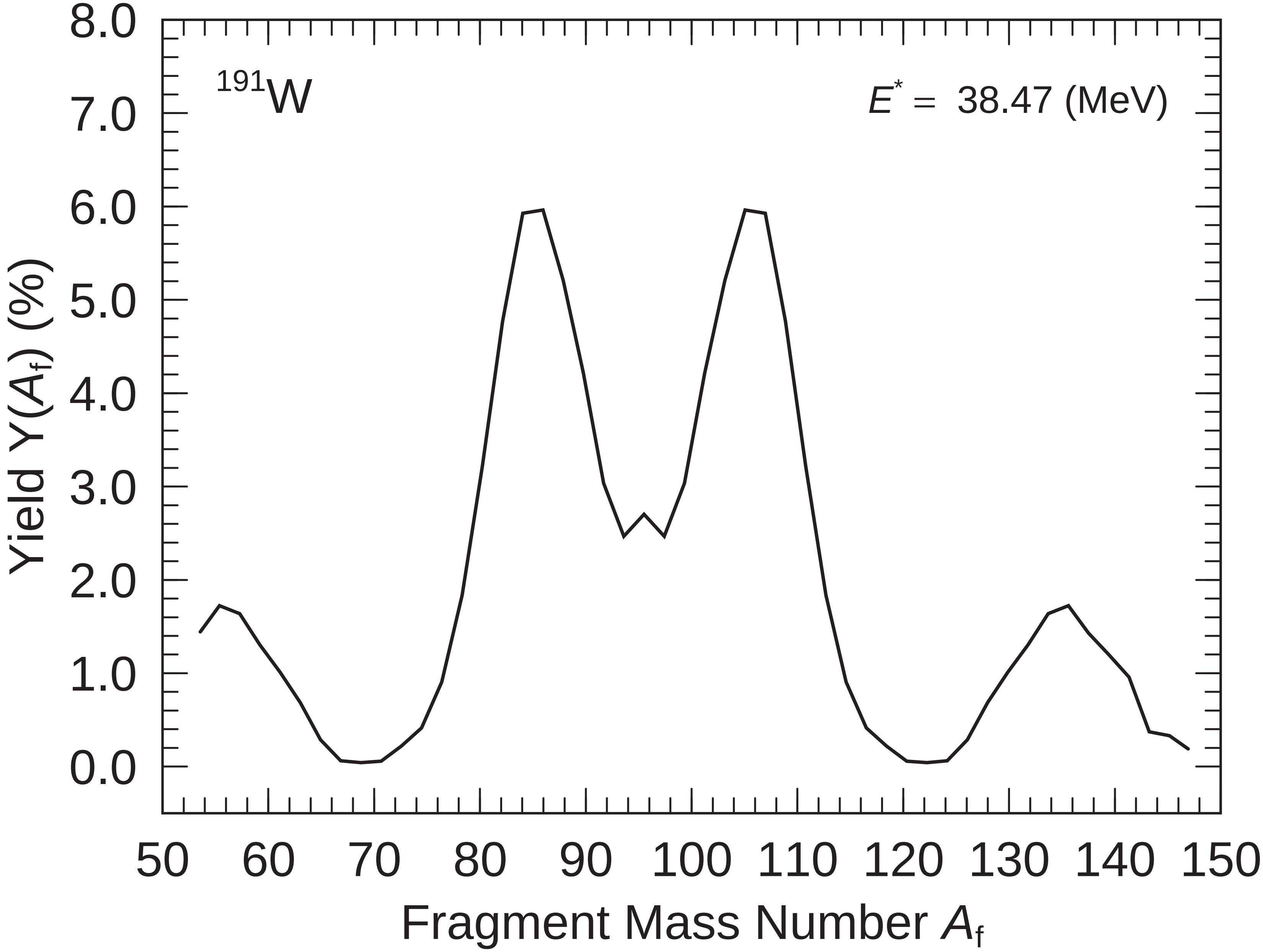
^{188}W
E^{*} = 38.74 (MeV)
Yield Y(A_f) (%)
Fragment Mass Number A_f
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150



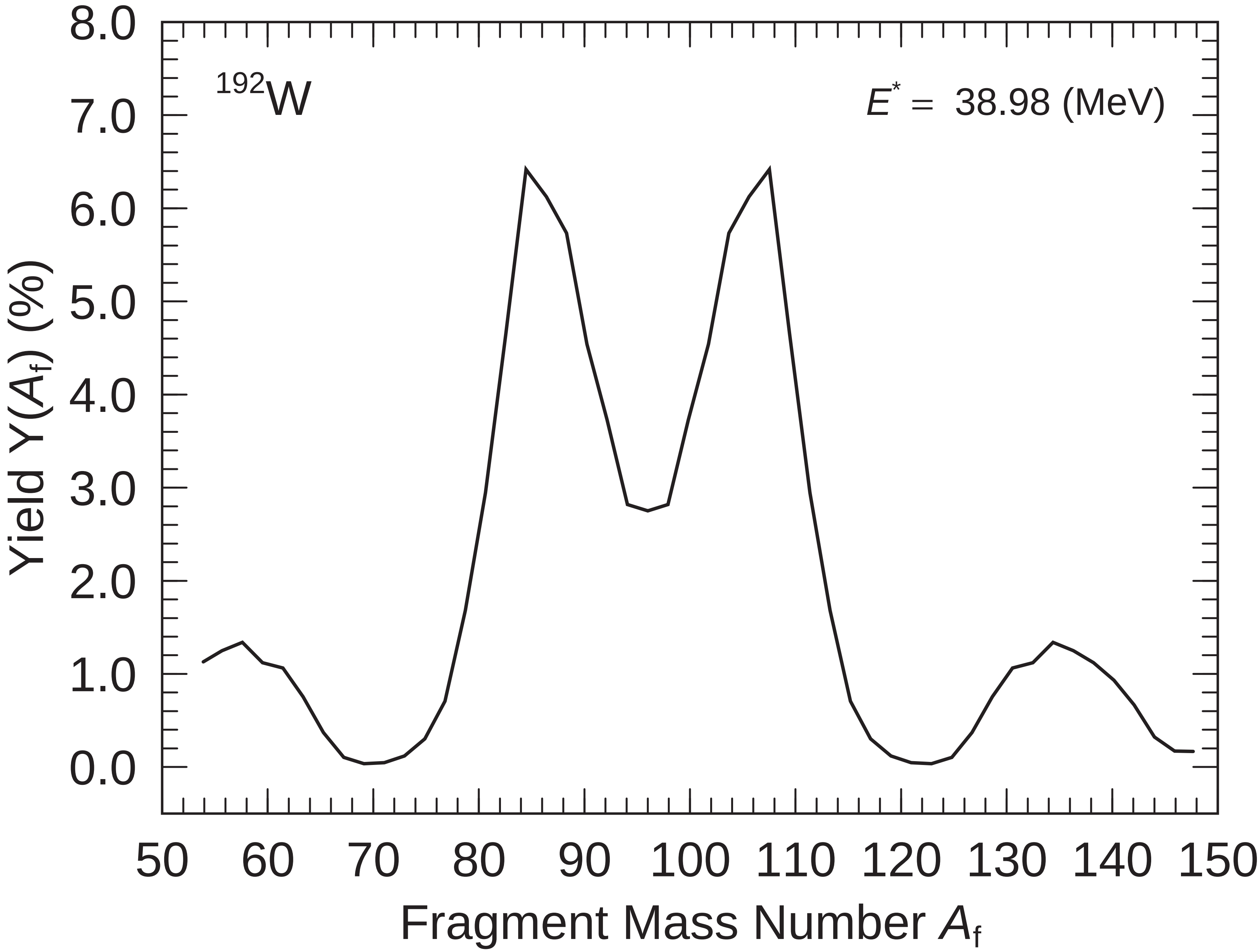
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
189W
E* = 38.24 (MeV)
Yield Y(Af) (%)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number Af



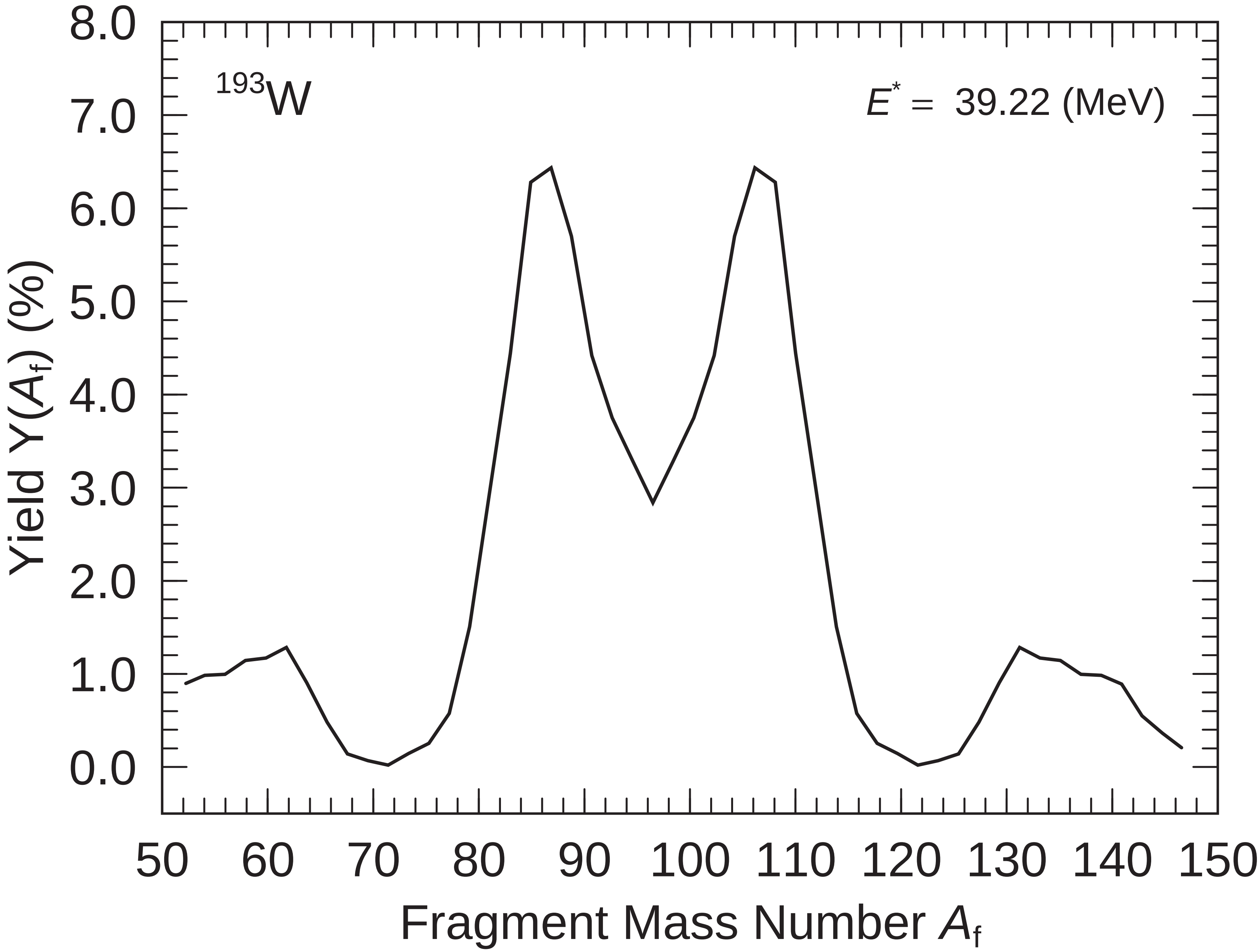
^{190}W
E^{*} = 38.29 (MeV)
Yield Y(A_f) (%)
Fragment Mass Number A_f
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150



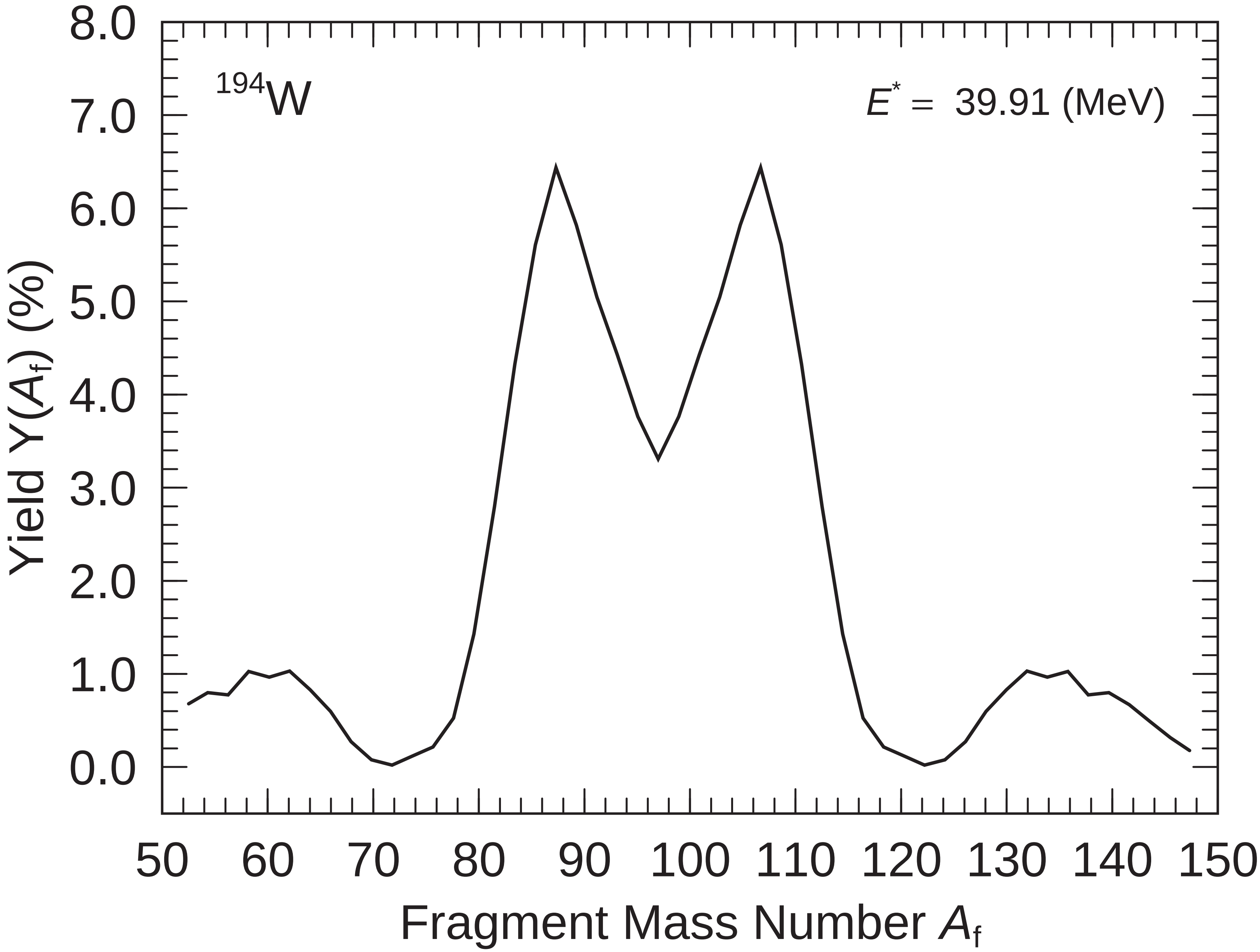
^{191}W
E^{*} = 38.47 (MeV)
Yield Y(A_f) (%)
Fragment Mass Number A_f
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150



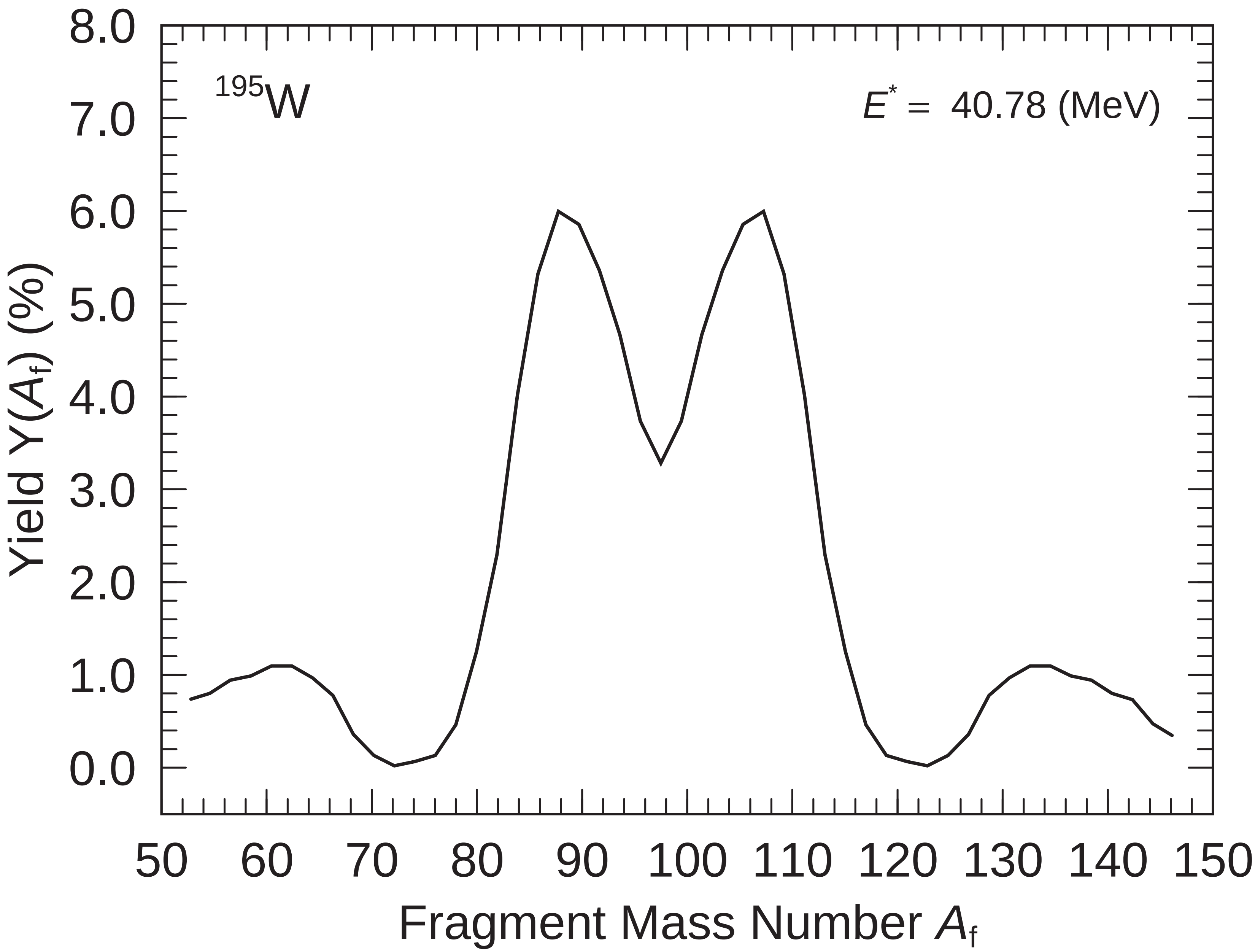
^{192}W
E^{*} = 38.98 (MeV)
Yield Y(A_f) (%)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number A_f



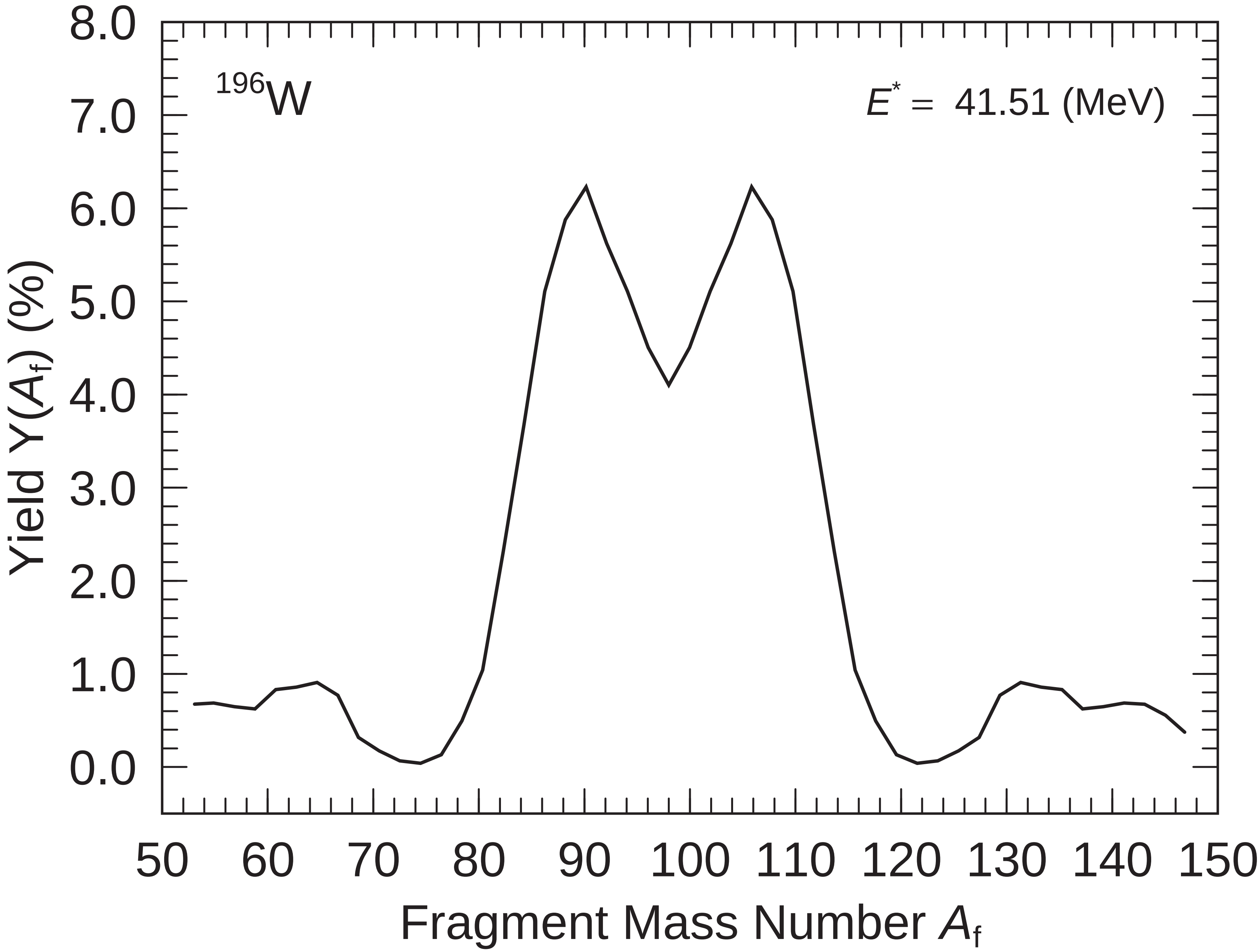
^{193}W
E^{*} = 39.22 (MeV)
Yield Y(A_f) (%)
Fragment Mass Number A_f
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
50
60
70
80
90
100
110
120
130
140
150



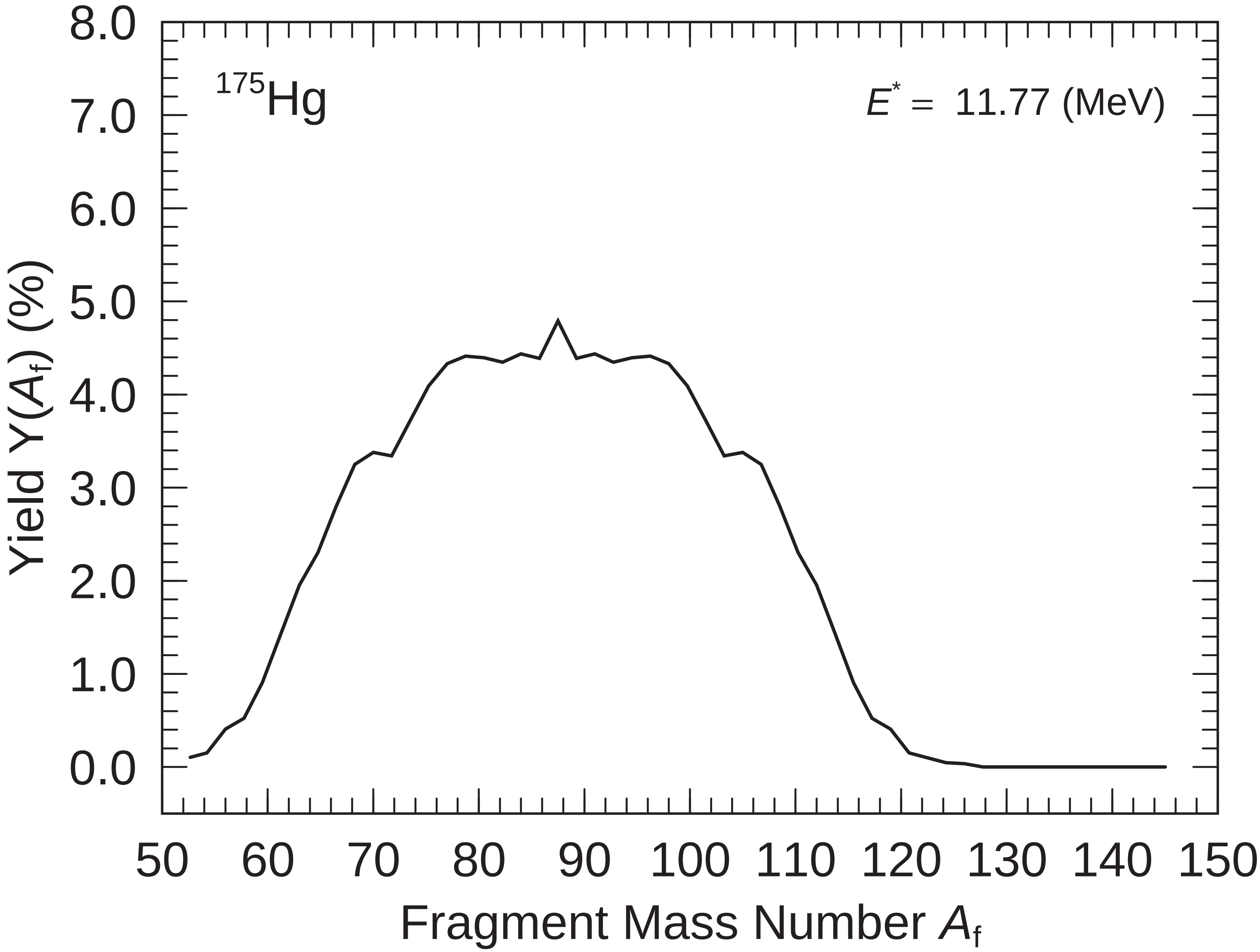
194W
E* = 39.91 (MeV)
Yield Y(Af) (%)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number Af



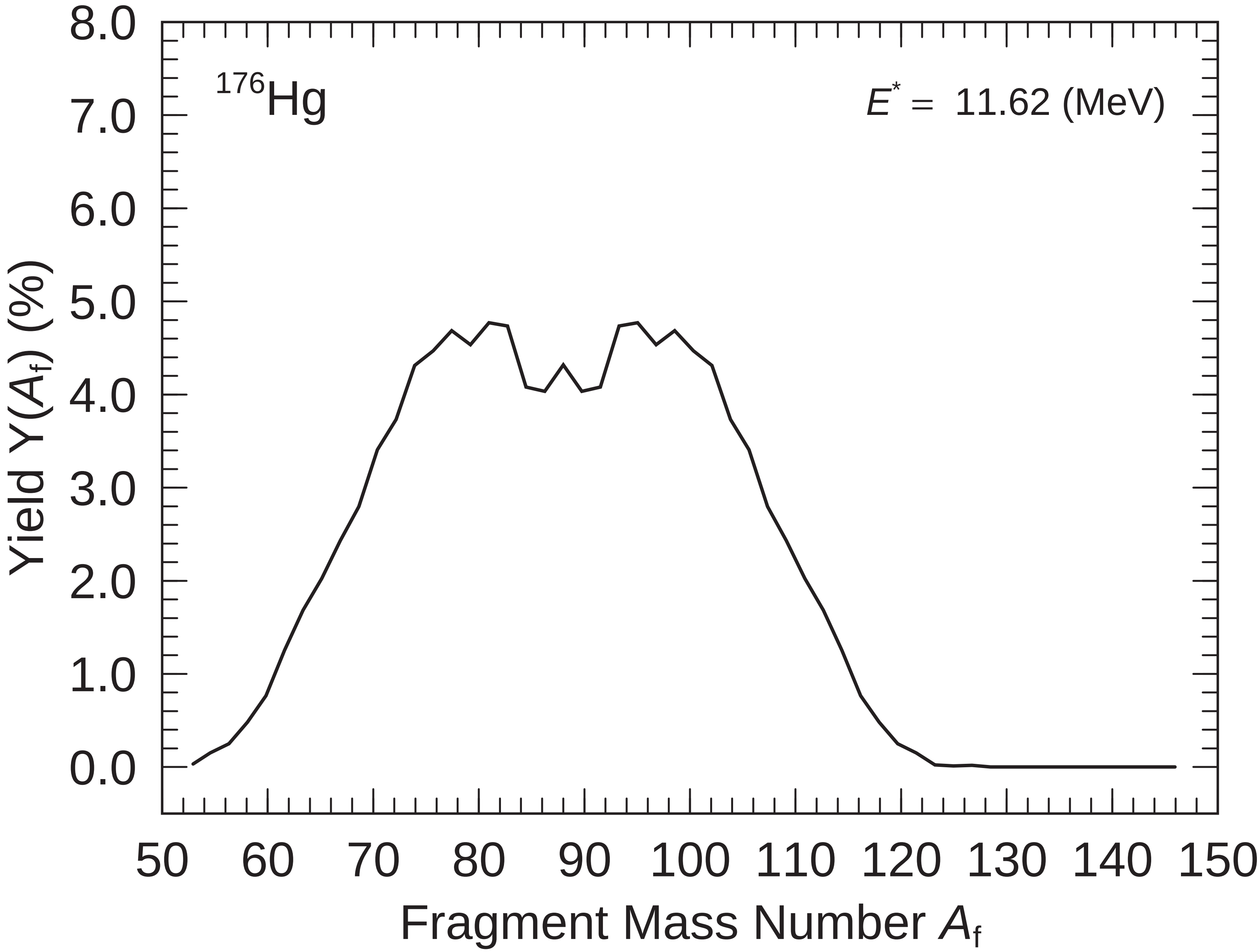
^{195}W
E^{*} = 40.78 (MeV)
Yield Y(A_f) (%)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number A_f



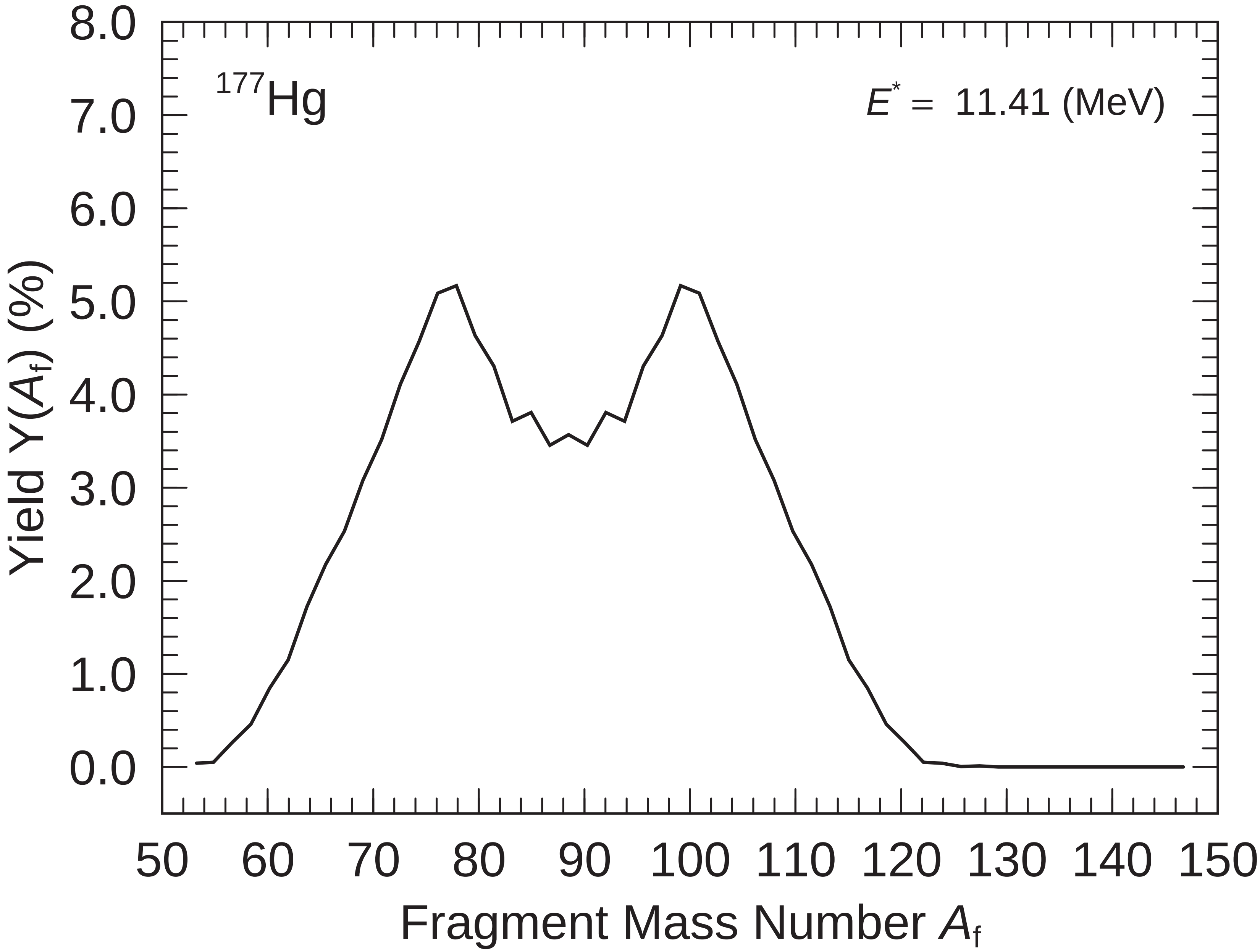
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Yield Y(A_f) (%)
196W
E* = 41.51 (MeV)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number A_f



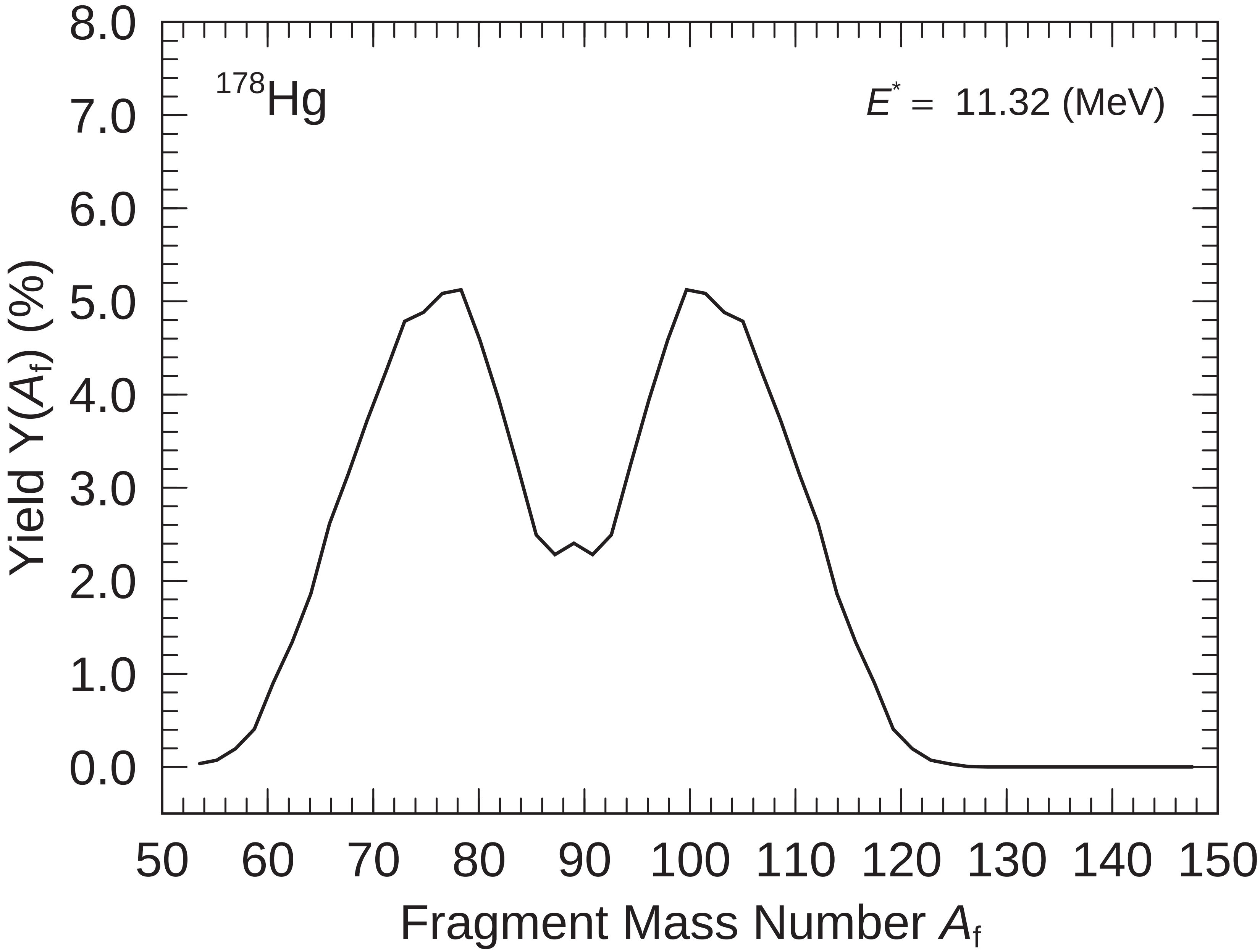
^{175}Hg
E^{*} = 11.77 (MeV)
Yield Y(A_f) (%)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number A_f



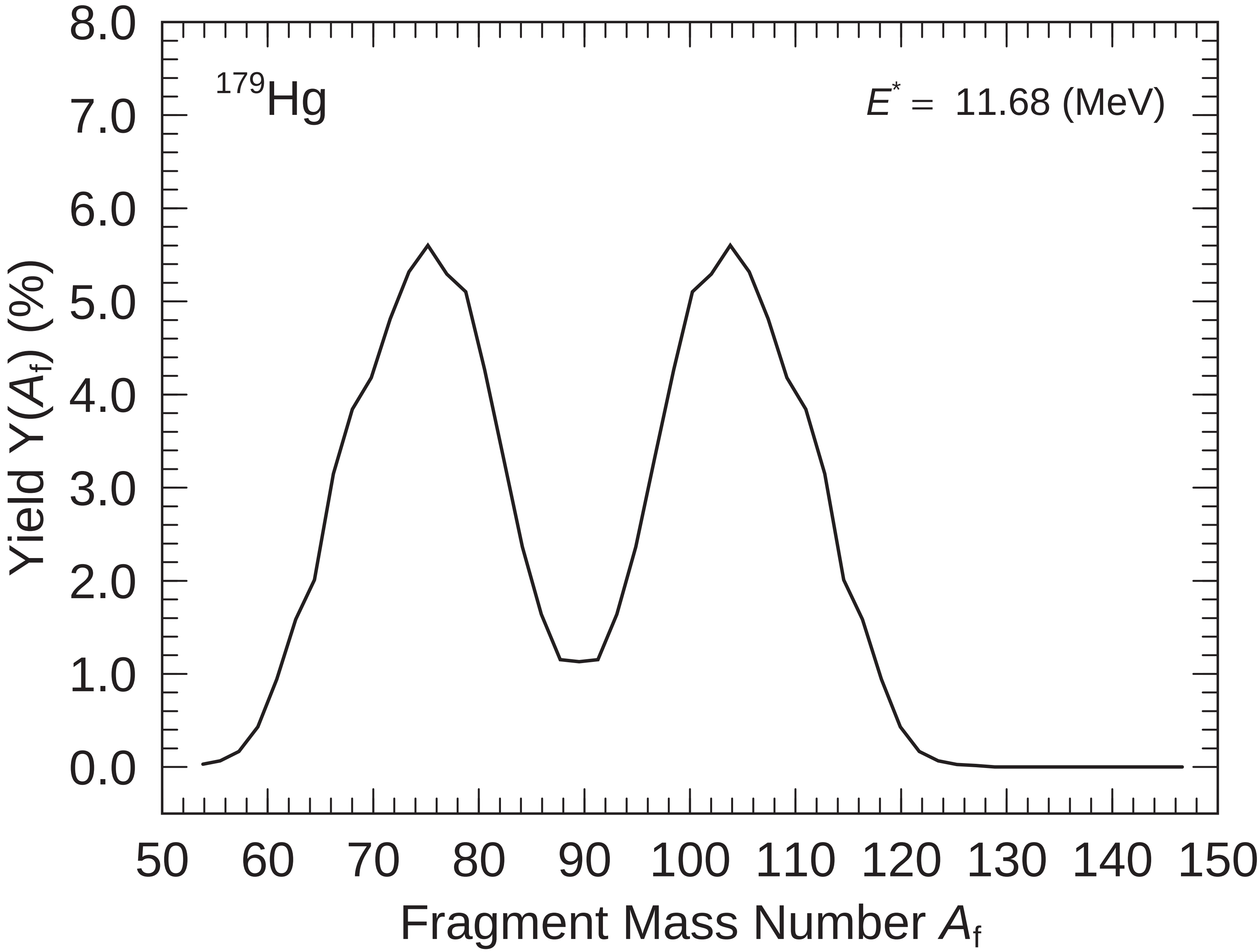
^{176}Hg
$E^* =$ 11.62 (MeV)
Yield Y(A_f) (%)
Fragment Mass Number A_f
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150



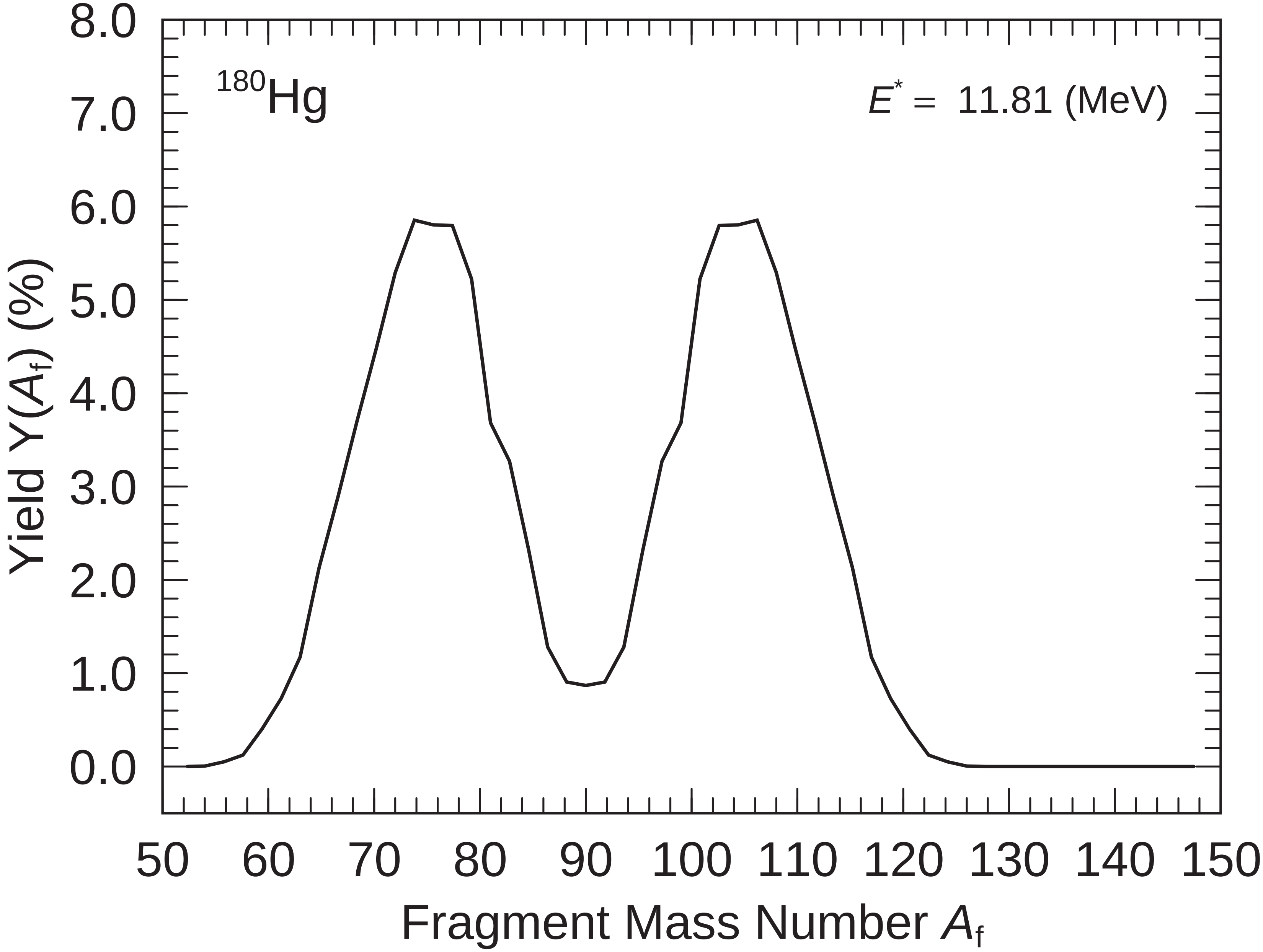
177Hg
E* = 11.41 (MeV)
Yield Y(Af) (%)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number Af



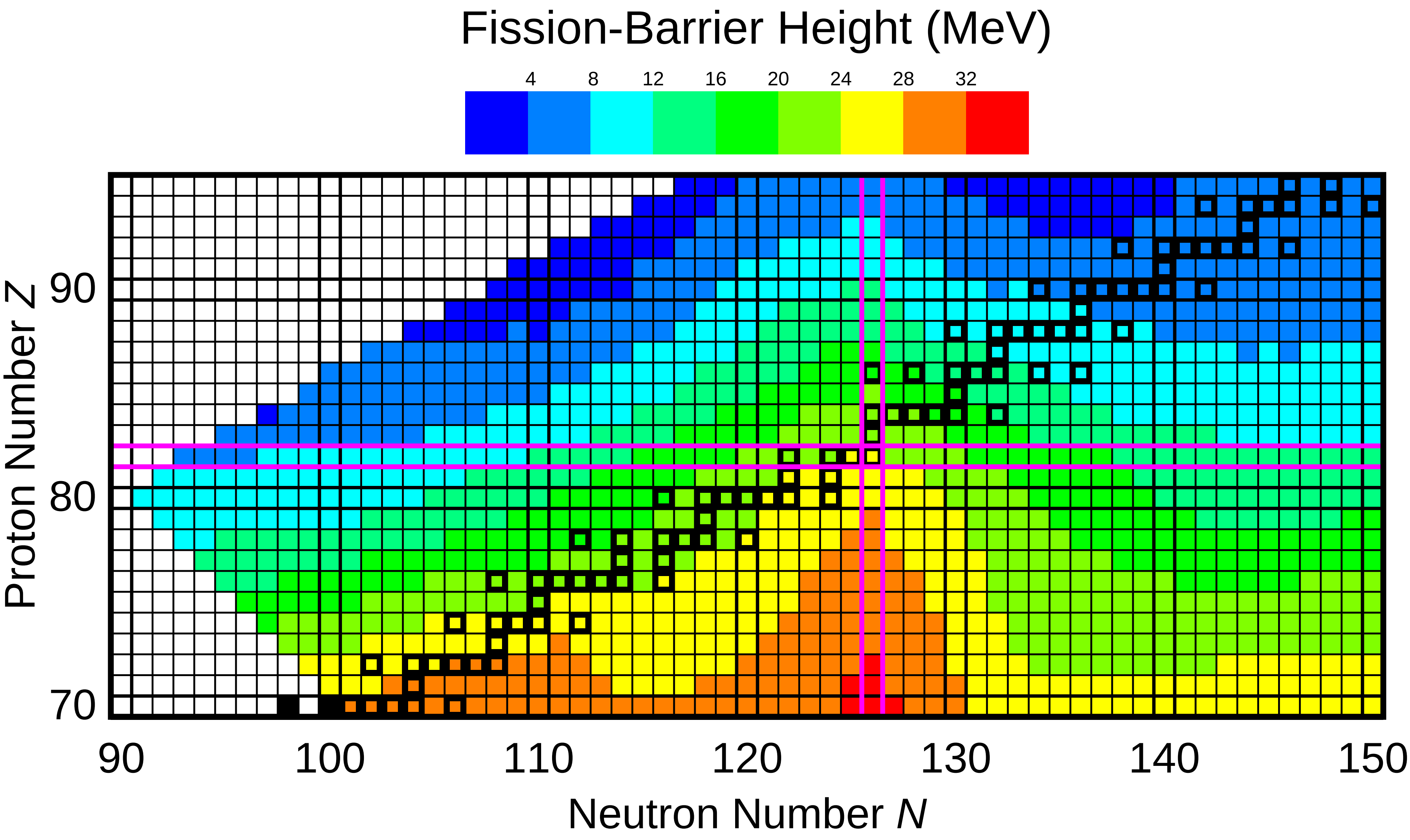
^{178}Hg
E^{*} = 11.32 (MeV)
Yield Y(A_f) (%)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number A_f



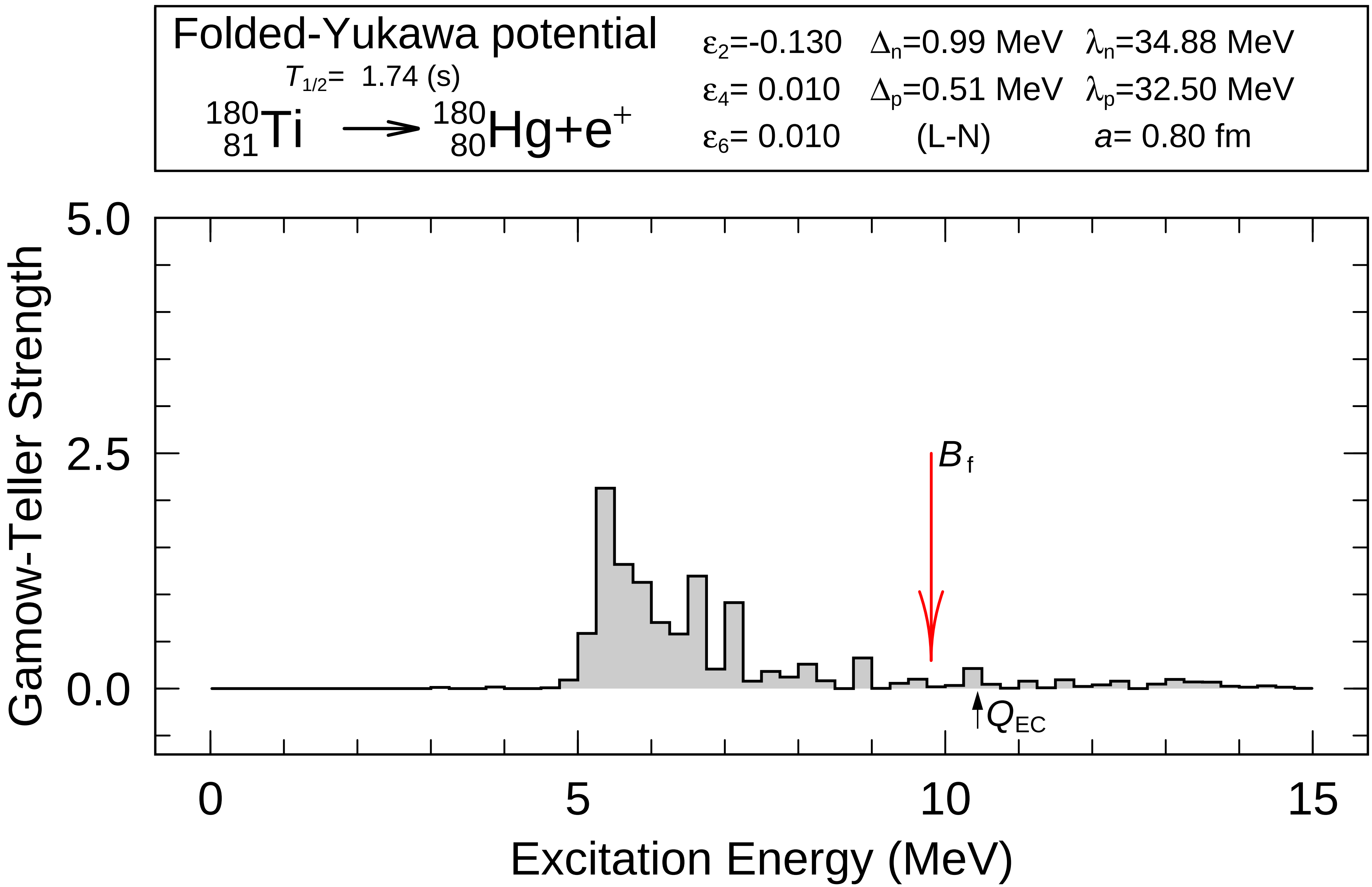
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Yield Y(Af) (%)
179Hg
E* = 11.68 (MeV)
50
60
70
80
90
100
110
120
130
140
150
Fragment Mass Number Af



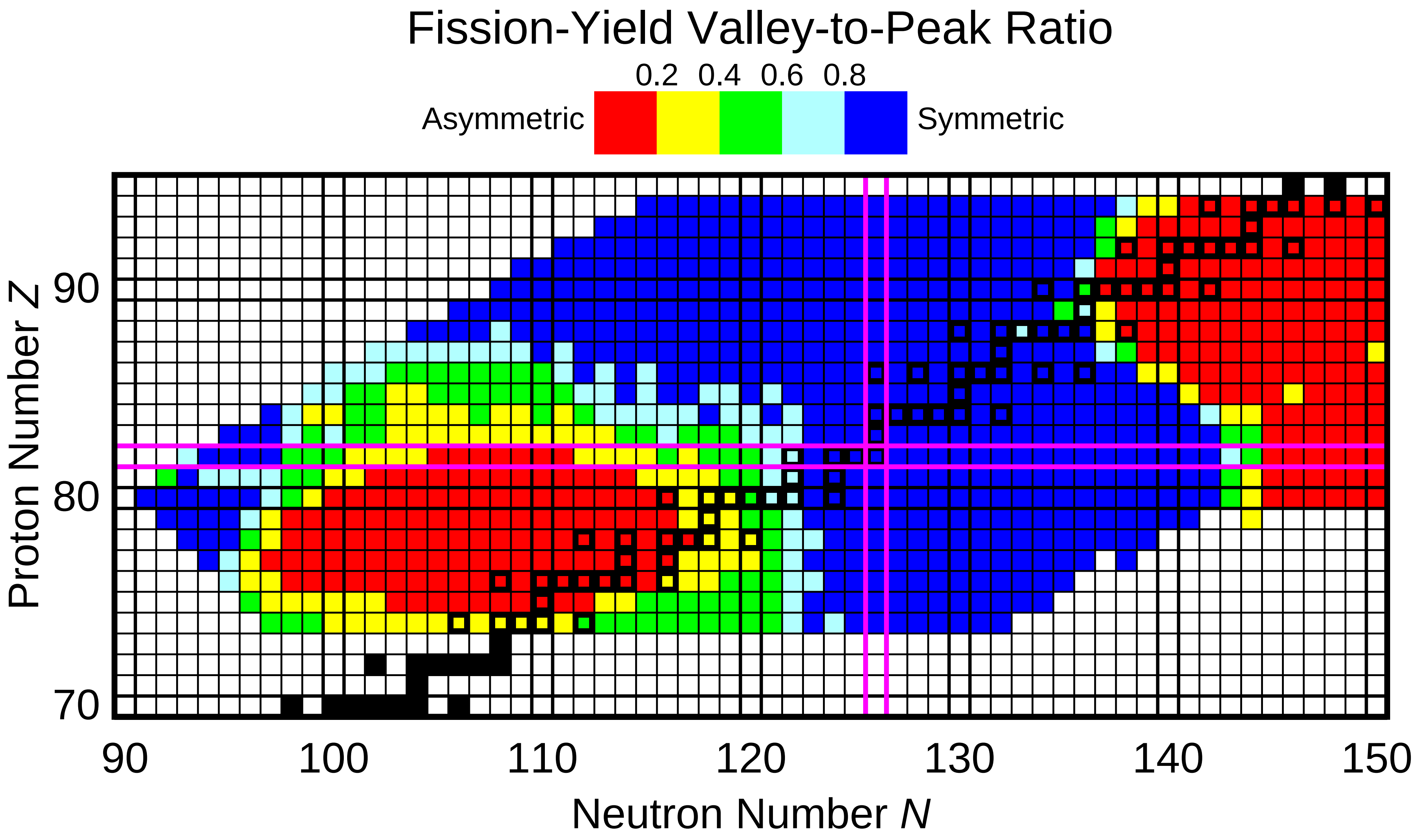
180Hg
E* = 11.81 (MeV)
Yield Y(Af) (%)
Fragment Mass Number Af
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
50
60
70
80
90
100
110
120
130
140
150



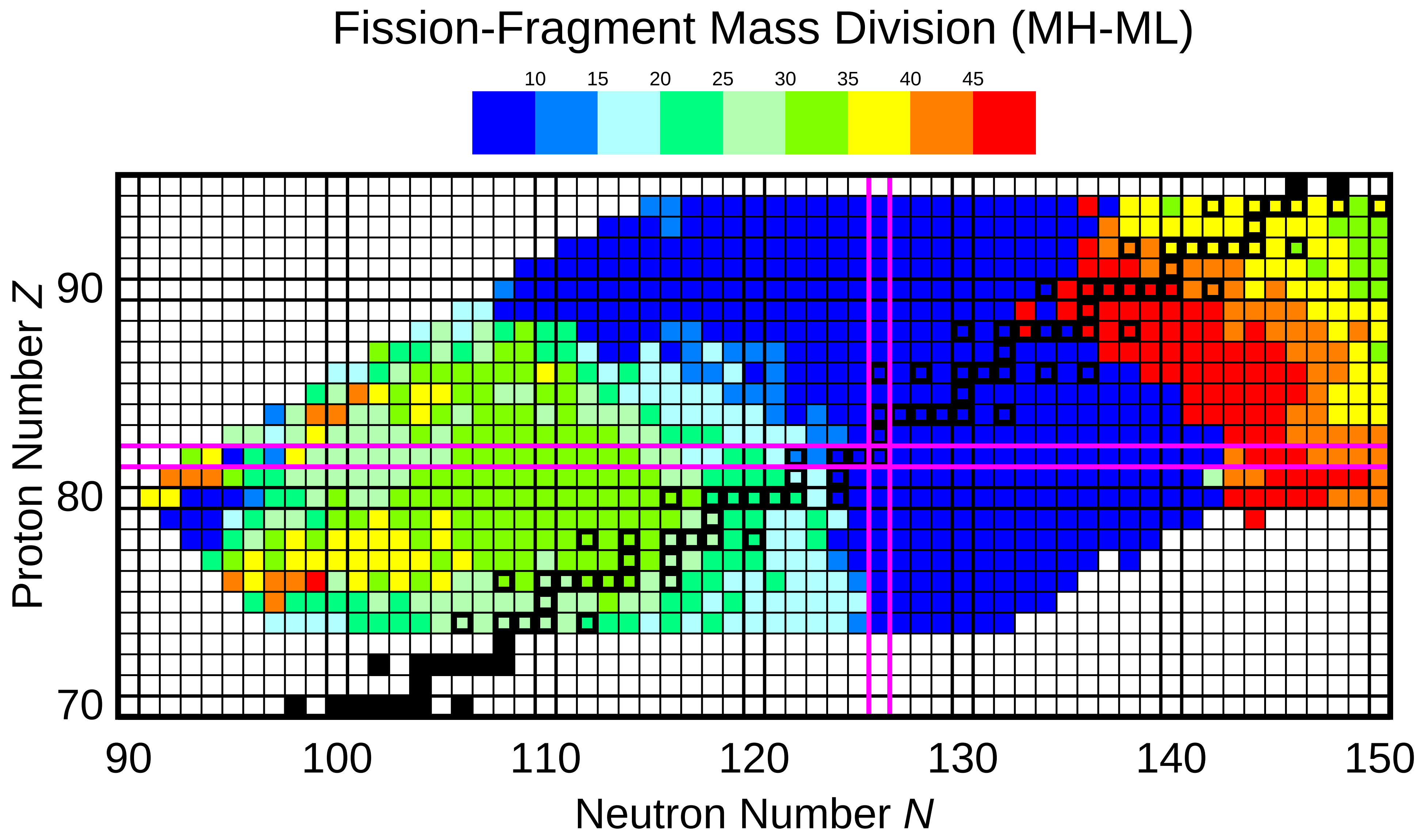
Fission-Barrier Height (MeV)
4
8
12
16
20
24
28
32
Proton Number Z
90
80
70
90
100
110
120
130
140
150
Neutron Number N



Folded-Yukawa potential
T1/2= 1.74 (s)
180 81 Ti → 180 80 Hg+e+
ε2=-0.130 Δn=0.99 MeV λn=34.88 MeV
ε4= 0.010 Δp=0.51 MeV λp=32.50 MeV
ε6= 0.010 (L-N) a= 0.80 fm
Gamow-Teller Strength
5.0
2.5
0.0
Bf
QEC
0
5
10
15
Excitation Energy (MeV)

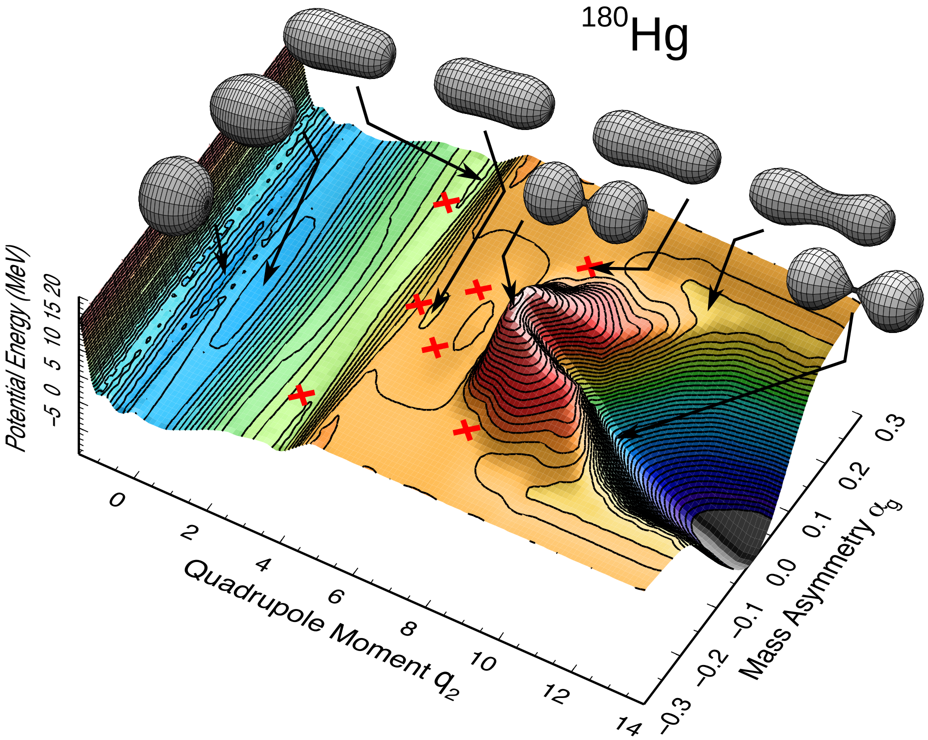


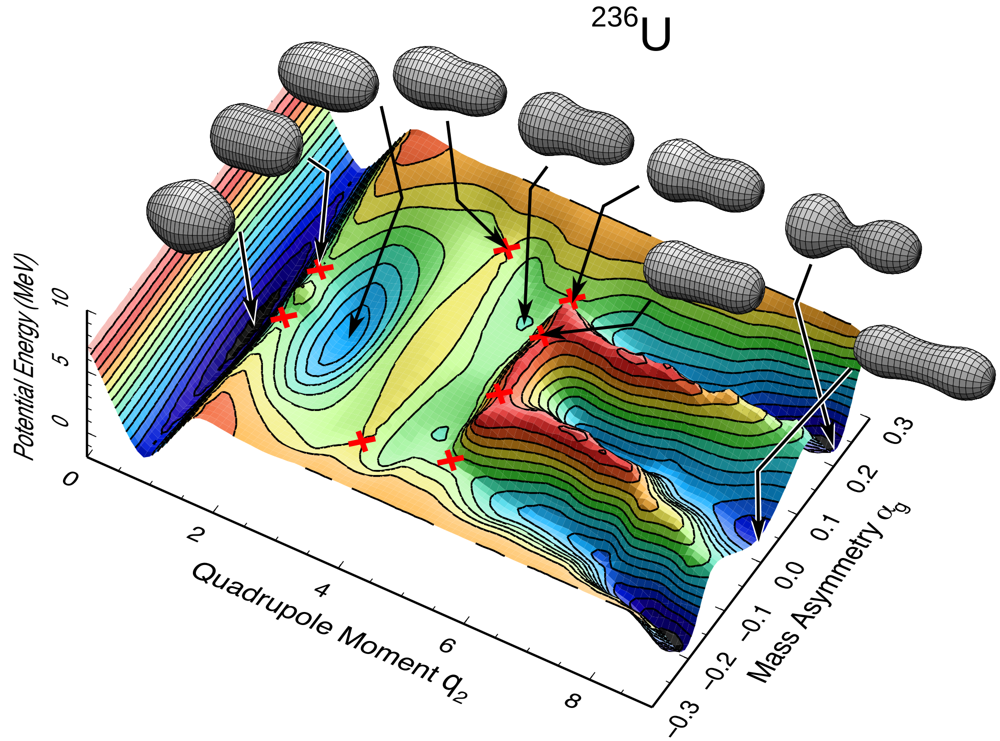
Fission-Yield Valley-to-Peak Ratio
0.2
0.4
0.6
0.8
Asymmetric
Symmetric
Proton Number Z
90
80
70
90
100
110
120
130
140
150
Neutron Number N



Fission-Fragment Mass Division (MH-ML)
10
15
20
25
30
35
40
45
Proton Number Z
90
80
70
90
100
110
120
130
140
150
Neutron Number N

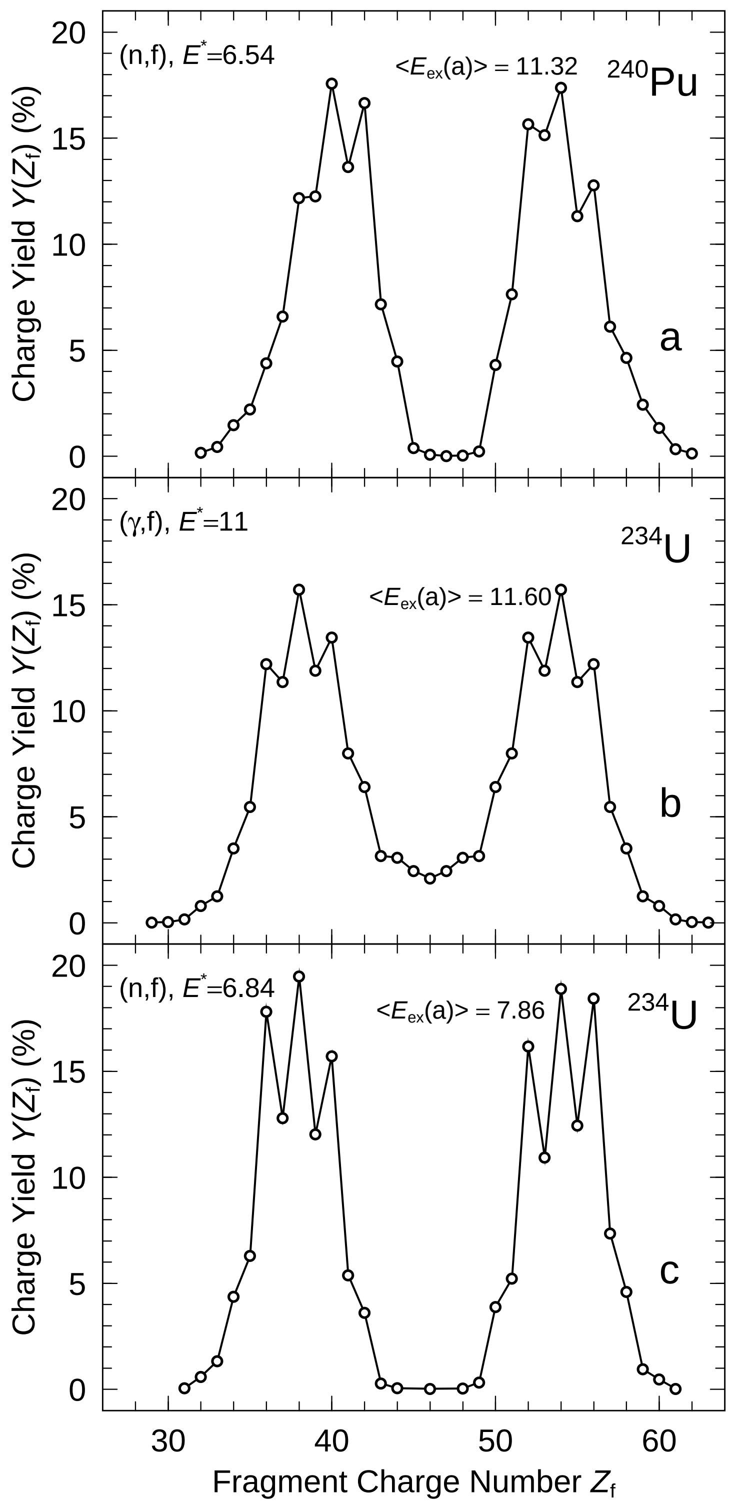
Contrasting Fission Potential-Energy Surfaces Hg↔U



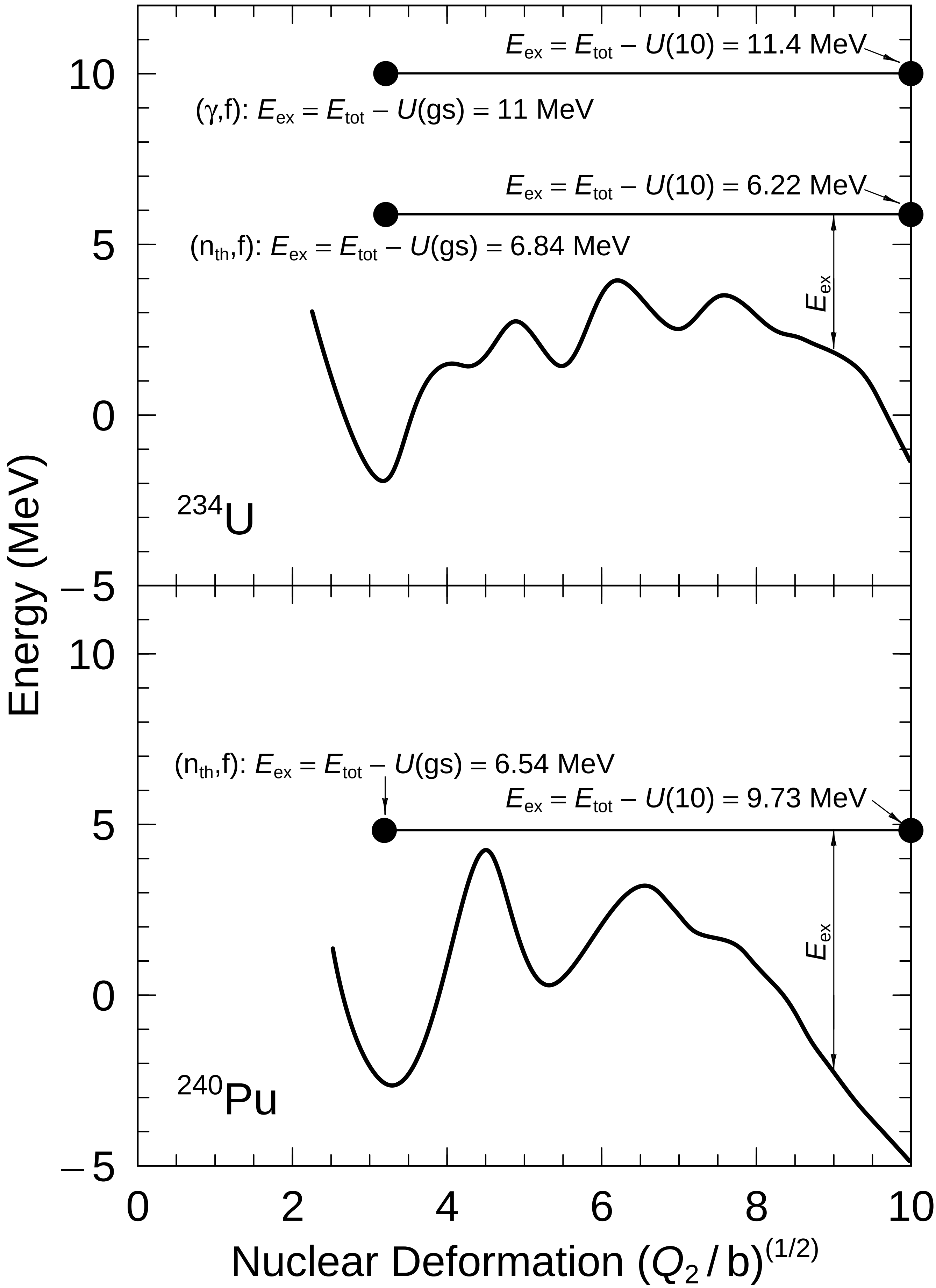


Ichikawa et al., PRC 86 024610 (2012)

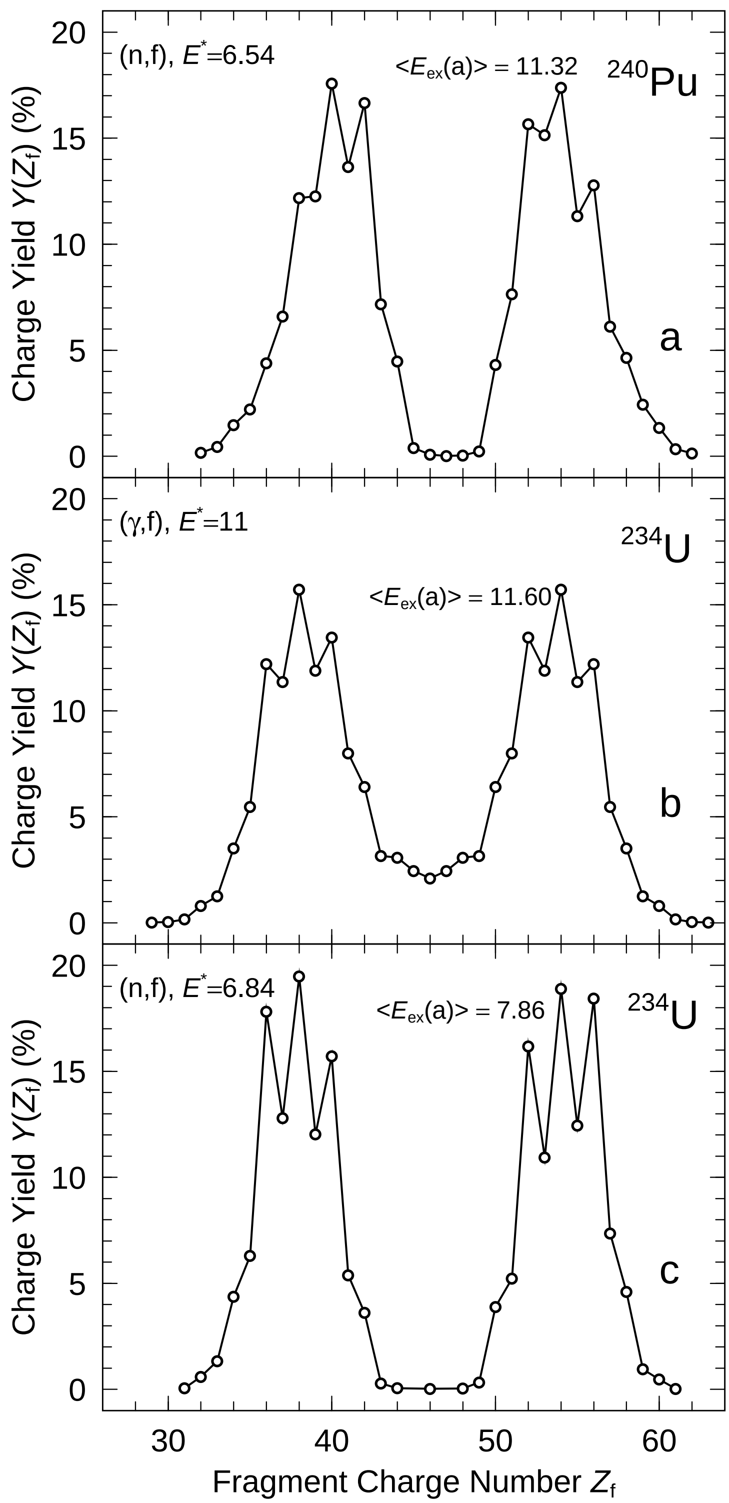
Compound		Calc.	Exp.	Calc	Exp.	Calc.
Z	A	$A_{\mathrm{L}}/A_{\mathrm{H}}$	$A_{\mathrm{L}}/A_{\mathrm{H}}$	$Z_{\mathrm{L}}/Z_{\mathrm{H}}$	$Z_{\mathrm{L}}/Z_{\mathrm{H}}$	$\epsilon_{\mathrm{L}}/\epsilon_{\mathrm{H}}$
80	180	80/100	80/100	36/44		0.60/0.50
90	222	108/114		44/46	44/46	0.55/0.55
90	228	**112/116**	89/139	**44/46**	36/54	0.55/0.55
92	234	**116/118**	95/139	**46/46**	38/54	0.55/0.55
94	240	**110/130**	102/140	**44/50**	41/53	0.55/0.65
98	252	112/140	108/144	44/54		0.55/0.75
100	252	110/142	110/142	44/56		0.55/0.75
100	258	**118/140**	129/129	**46/54**		0.55/0.75
100	260	**118/142**	130/130	**46/54**		0.55/0.65
100	264	132/132	132/132	50/50	50/50	0.65/0.65



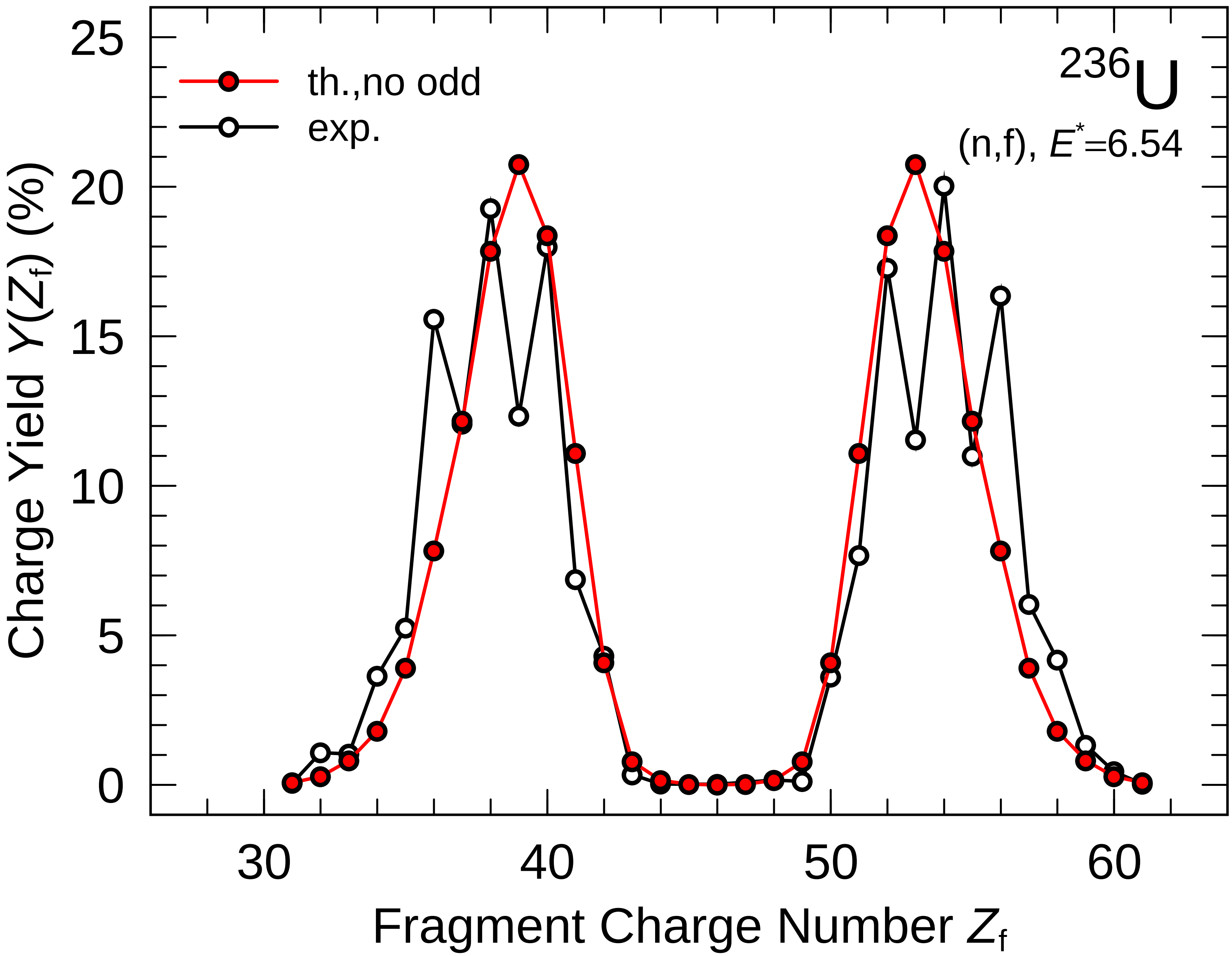
(n,f), E*=6.54
<Eex(a)> = 11.32
240Pu
a
(γ,f), E*=11
234U
<Eex(a)> = 11.60
b
(n,f), E*=6.84
<Eex(a)> = 7.86
234U
c
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
30
40
50
60



$E_{ex} = E_{tot} - U(10) = 11.4$ MeV
(γ,f): $E_{ex} = E_{tot} - U(gs) = 11$ MeV
$E_{ex} = E_{tot} - U(10) = 6.22$ MeV
(n_{th},f): $E_{ex} = E_{tot} - U(gs) = 6.84$ MeV
E_{ex}
^{234}U
(n_{th},f): $E_{ex} = E_{tot} - U(gs) = 6.54$ MeV
$E_{ex} = E_{tot} - U(10) = 9.73$ MeV
E_{ex}
^{240}Pu
Energy (MeV)
10
5
0
−5
10
5
0
−5
0
2
4
6
8
10
Nuclear Deformation $(Q_2 / b)^{(1/2)}$



(n,f), E*=6.54
<Eex(a)> = 11.32
240Pu
a
(γ,f), E*=11
234U
<Eex(a)> = 11.60
b
(n,f), E*=6.84
<Eex(a)> = 7.86
234U
c
Charge Yield Y(Zf) (%)
0
5
10
15
20
30
40
50
60
Fragment Charge Number Zf



th.,no odd
exp.
236U
(n,f), E*=6.54
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
30
40
50
60

Modeling in BSM of $Y(Z_1, N_1, Z - Z_1, N - N_1)$

We need total potential energy versus fragment proton and neutron numbers. Shell correction is actually straightforward.

Total shell correction:

$$ESH_{Z+N}(Z_1, N_1, Z - Z_1, N - N_1)$$

"Field" asymmetry $\alpha_g \rightarrow$ Asymmetry in N and Z! This means 2 asymmetry coordinates rather than a single "field" asymmetry. How?

Calculate neutron shell correction for grid of α_g corresponding to integer N values. Save the *neutron* shell corrections $ESH_N(N_1, N - N_1)$.

Calculate proton shell correction for grid of α_g corresponding to integer Z values. Save the *proton* shell corrections $ESH_Z(Z_1, Z - Z_1)$.

$$ESH_{Z+N}(Z_1, N_1, Z - Z_1, N - N_1) =$$
$$ESH_Z(Z_1, Z - Z_1) + ESH_N(N_1, N - N_1)$$

Modeling in BSM of $Y(Z_1, N_1, Z-Z_1, N-N_1)$

We need total potential energy versus fragment proton and neutron numbers. Now what about the

Macroscopic energy:

$EMAC_{Z+N}(Z_1, N_1, Z-Z_1, N-N_1)$

Start by calculating $EMAC_{\rm comp}$ for the compound nucleus for a grid in $\alpha_{\rm g}$ corresponding to integer Z_1 and $Z - Z_1$.

The neutron numbers in the fragments corresponding to the $\alpha_{\rm g}$ yielding these integer Z_1 are not integers.

Now fix Z_1 and calculate the macroscopic energy for this (fixed) Z_1 but for different integer N_ν as the sum

$EMAC_{Z+N}(Z_1, N_\nu, Z-Z_1, N-N_\nu) =$
$EMAC_{\rm comp} + \Delta EMAC$

To obtain the second term calculate the sum of the macroscopic energies for the separated fragments:

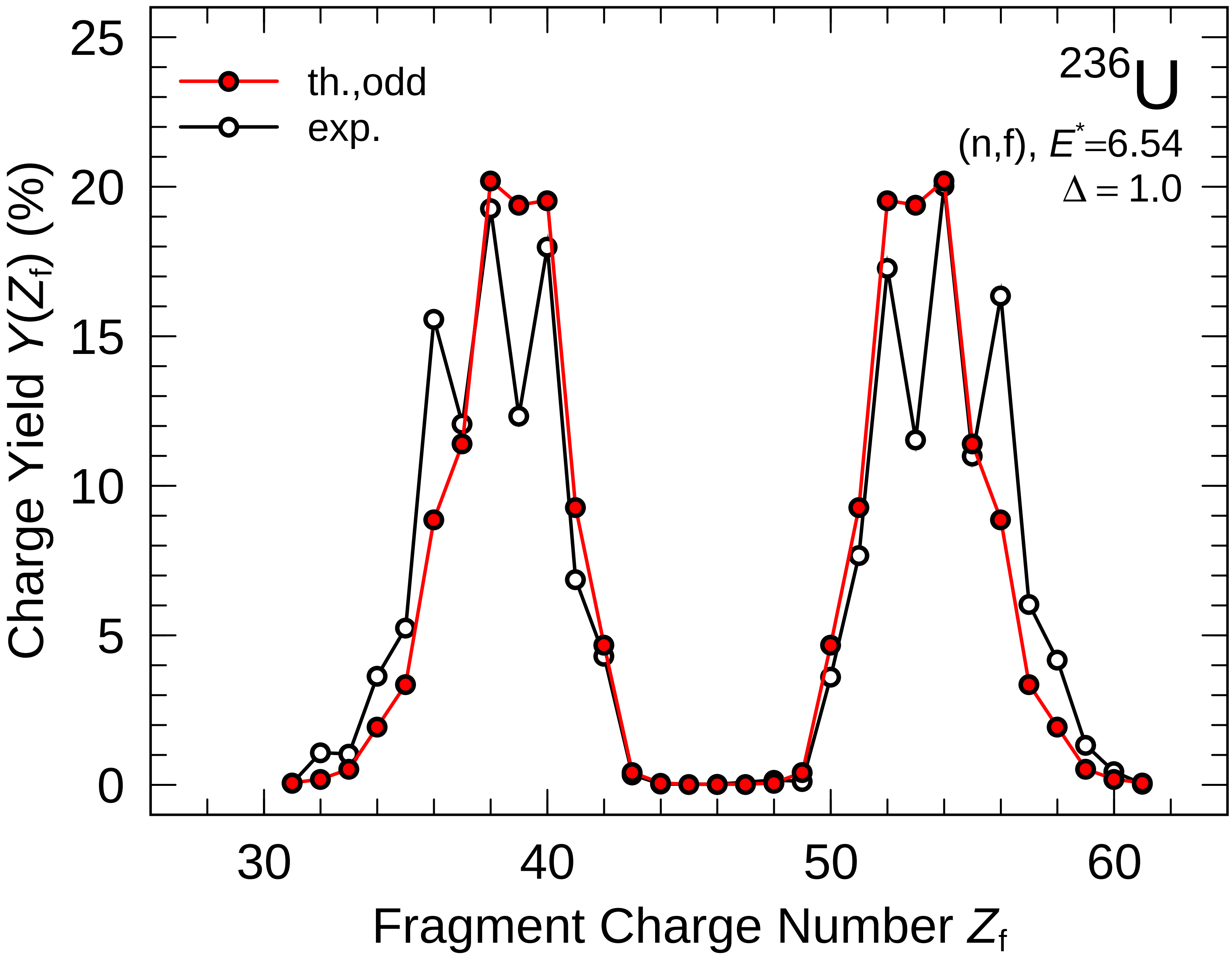
$EMAC(Z_1, N_\nu) + EMAC(Z-Z_1, N-N_\nu)$

where Z_1 is fixed, N_ν varies. $\Delta EMAC$ for various N_ν (Z_1 is still fixed) is the difference between this function at N_ν and at the noninteger N corresponding to the chosen Z_1.

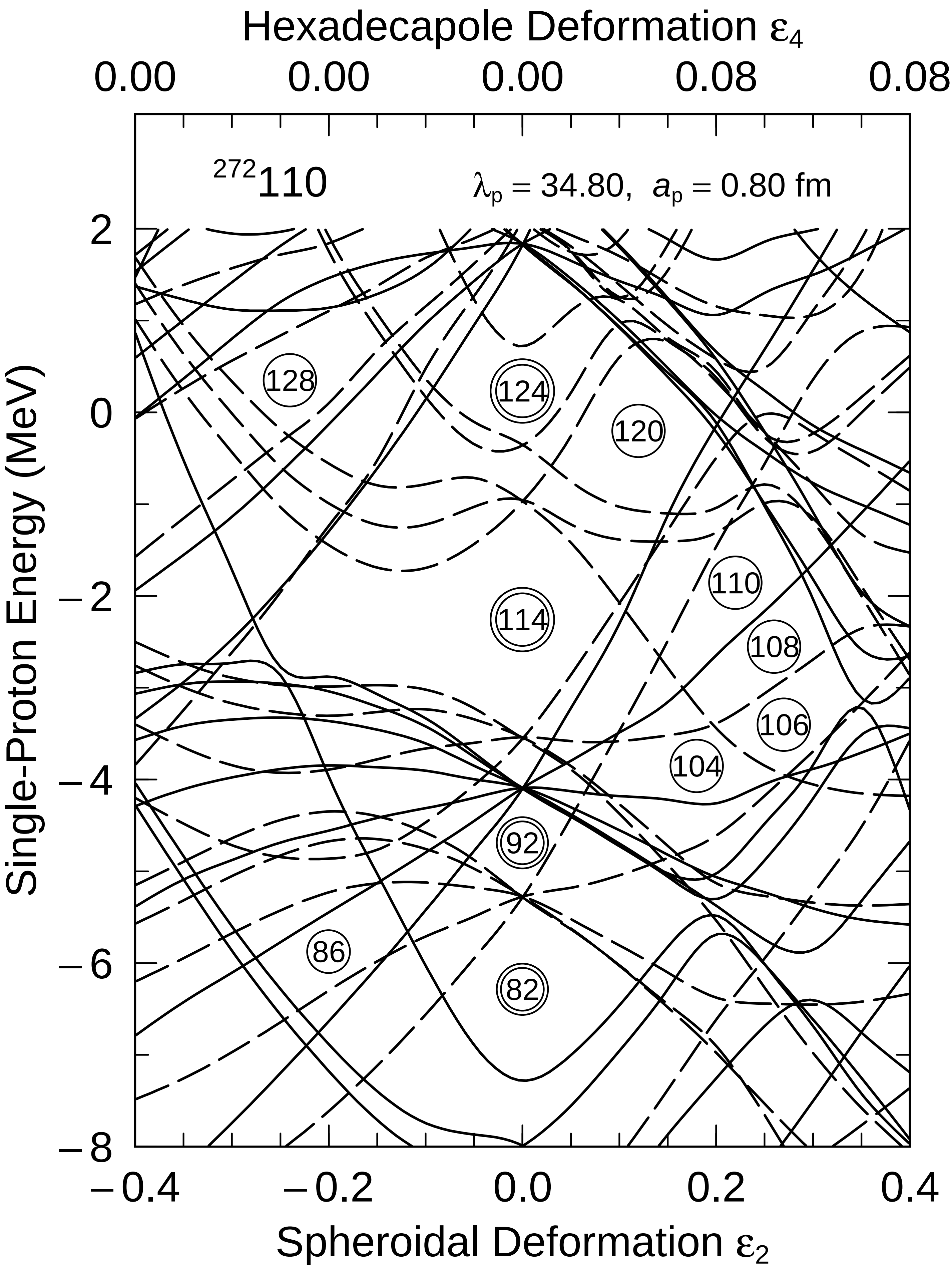
FRAGMENT MACROSCOPIC ENERGIES AND THE SUMS

Z1	N1	Ef1	Z2	N2	Ef1	Ef1+Ef2
52	96	-15.90	40	48	-84.863	-100.759
52	94	-26.21	40	50	-87.564	-113.770
52	92	-35.87	40	52	-88.817	-124.691
52	90	-44.88	40	54	-88.701	-133.576
52	88	-53.18	40	56	-87.292	-140.473
52	86	-60.77	40	58	-84.659	-145.427
52	84	-67.60	40	60	-80.865	-148.469
52	82	-73.66	40	62	-75.973	-149.635C
52	80	-78.91	40	64	-70.036	-148.945
52	78	-83.31	40	66	-63.108	-146.419
52	76	-86.83	40	68	-55.237	-142.071
52	74	-89.44	40	70	-46.470	-135.910
52	72	-91.09	40	72	-36.849	-127.938
52	70	-91.74	40	74	-26.413	-118.154
52	68	-91.35	40	76	-15.202	-106.552
52	66	-89.87	40	78	-3.250	-93.122

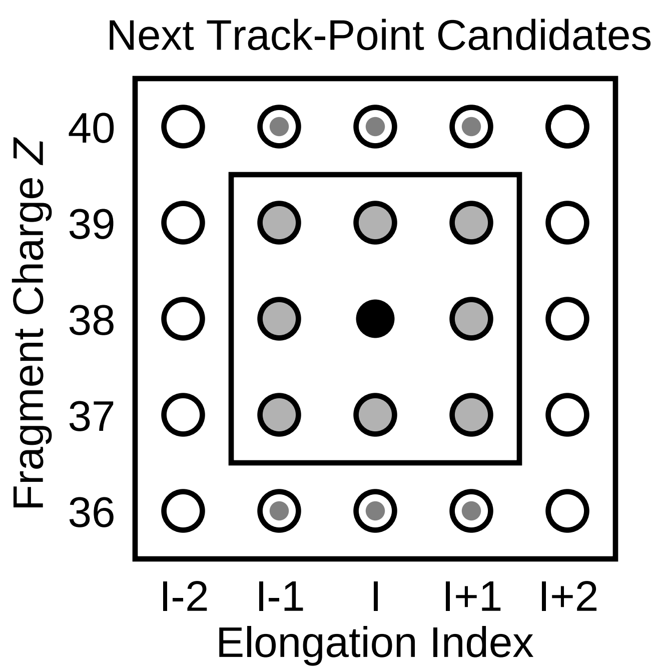
```
READ(LU,'(2f10.3)') r,rw

idiv =(N+1)/2
if(N+1 .eq. 2*idiv) r = r + (rw-1 +0.01)*2*1.0

E(I,J,K,L,N) = r
```

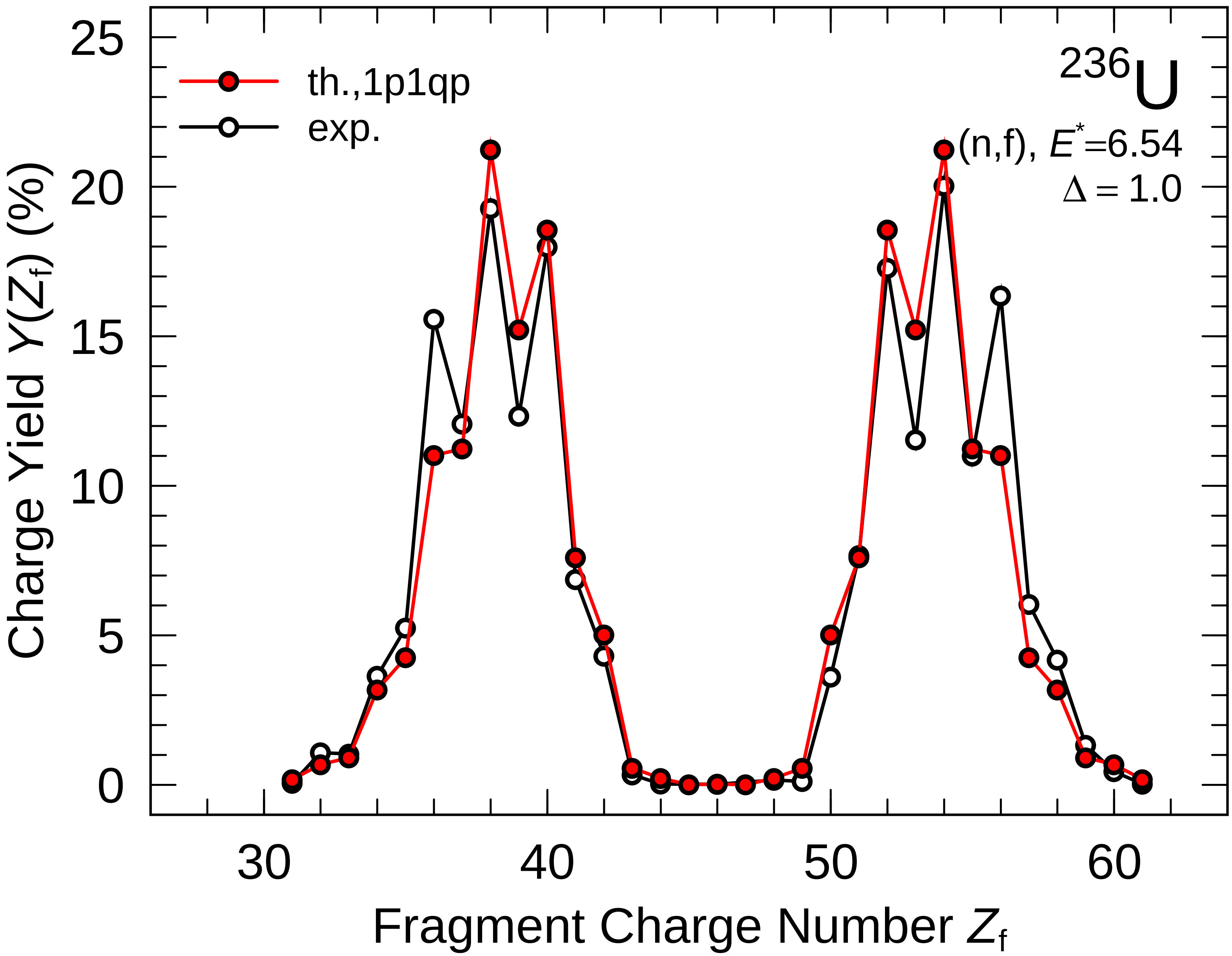


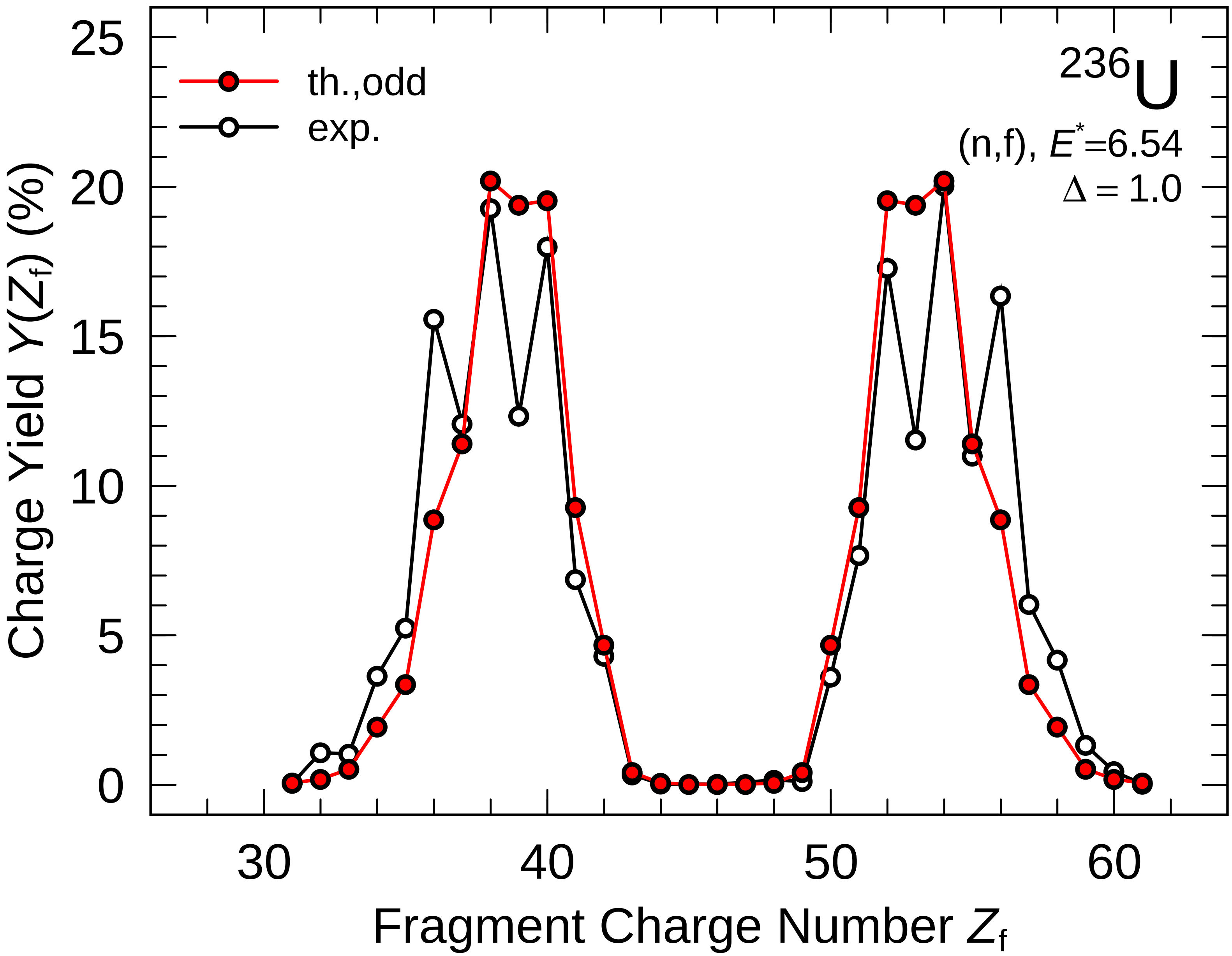
th.,odd
exp.
236U
(n,f), E*=6.54
Δ = 1.0
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
30
40
50
60



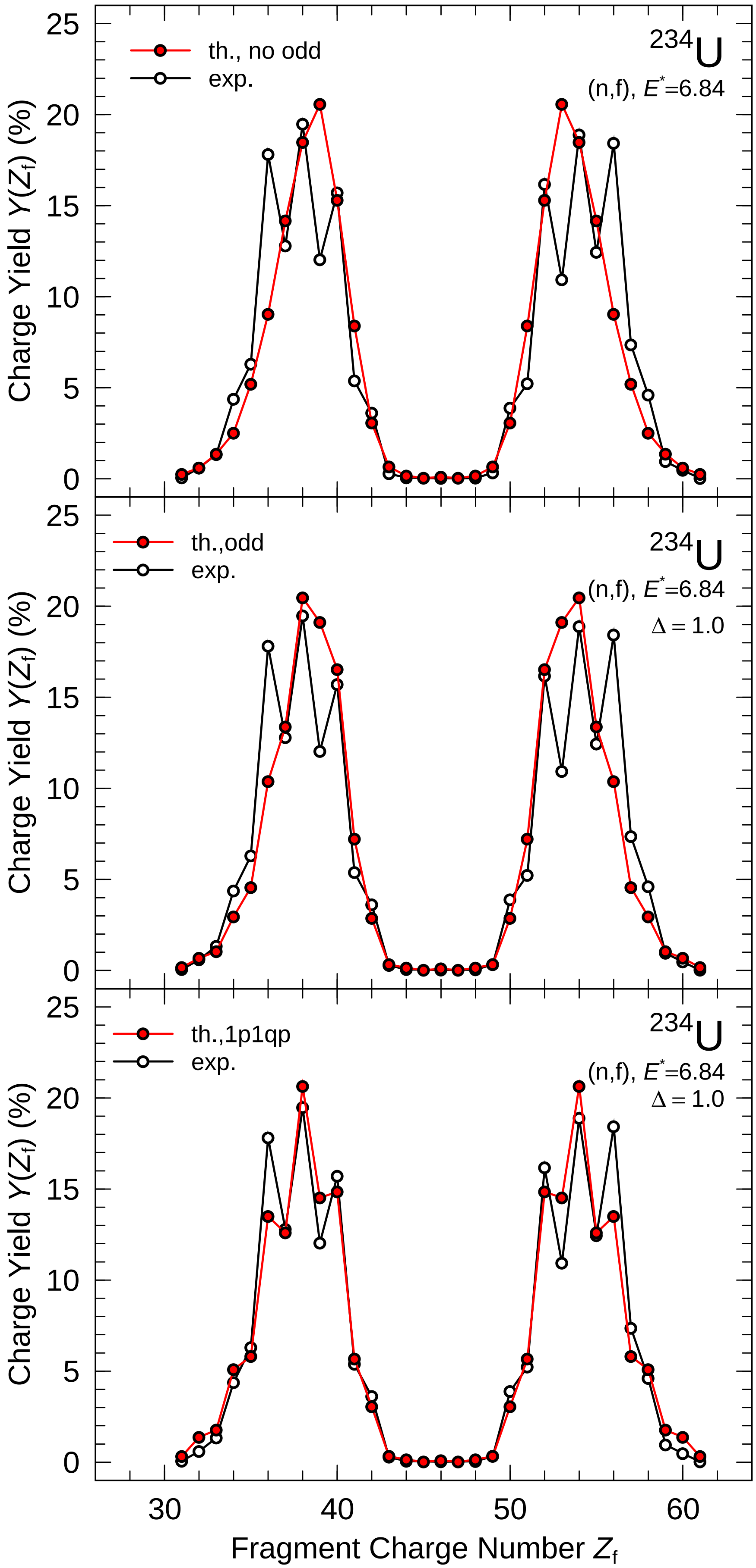
Hexadecapole Deformation ε4
0.00
0.00
0.00
0.08
0.08
272110
λp = 34.80, ap = 0.80 fm
128
124
120
114
110
108
106
104
92
86
82
2
0
−2
−4
−6
−8
Single-Proton Energy (MeV)
−0.4
−0.2
0.0
0.2
0.4
Spheroidal Deformation ε2


Next Track-Point Candidates
Fragment Charge Z
40
39
38
37
36
I-2
I-1
I
I+1
I+2
Elongation Index

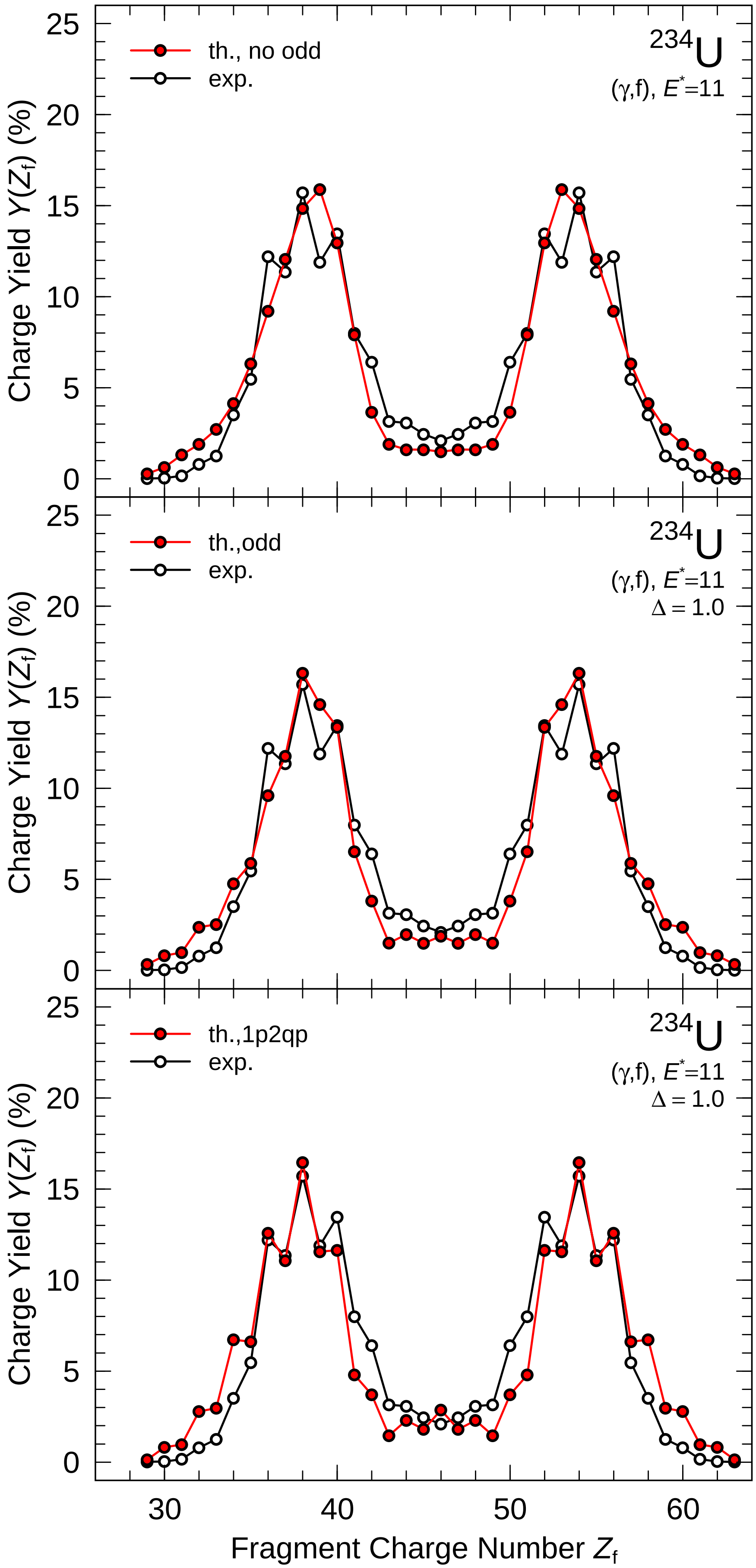

th.,1p1qp
exp.
236U
(n,f), E*=6.54
Δ = 1.0
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
30
40
50
60



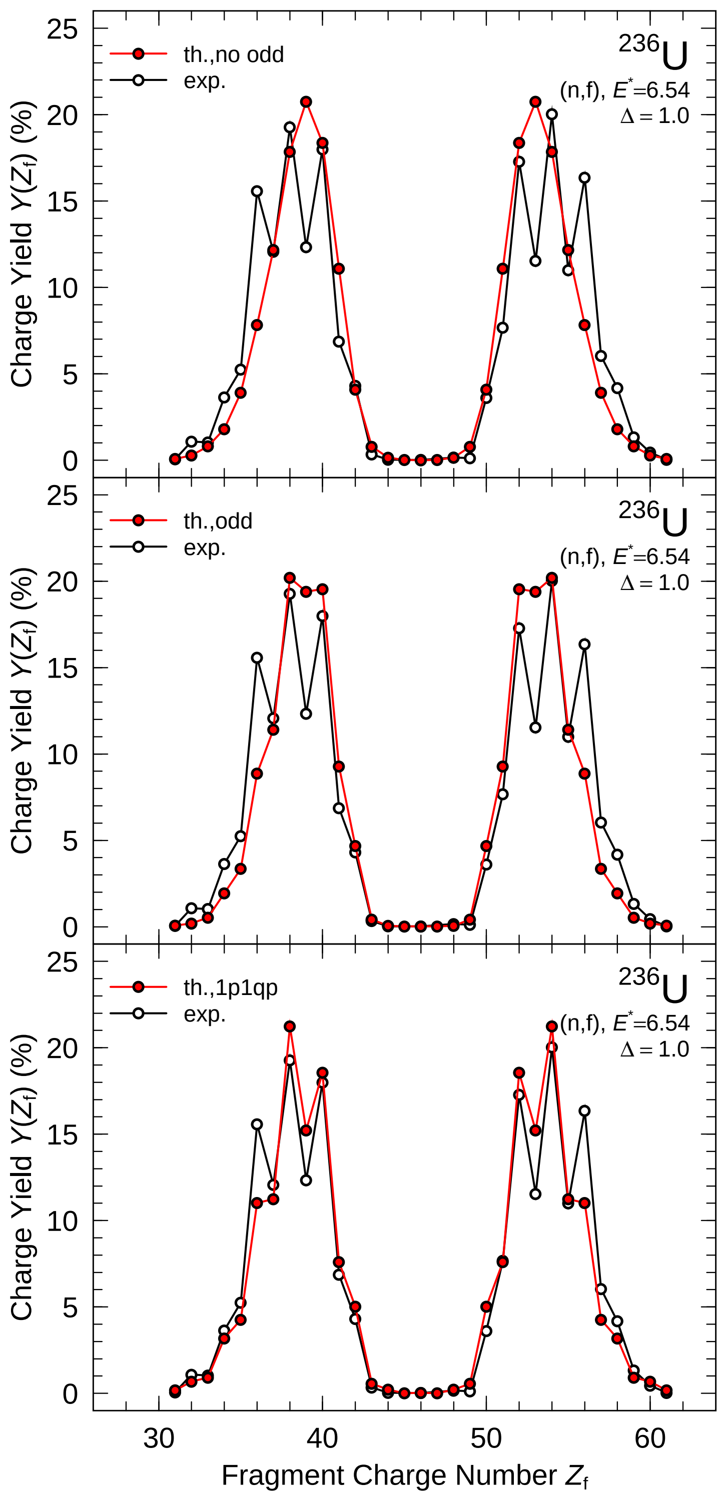
th.,odd
exp.
236U
(n,f), E*=6.54
Δ = 1.0
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
30
40
50
60



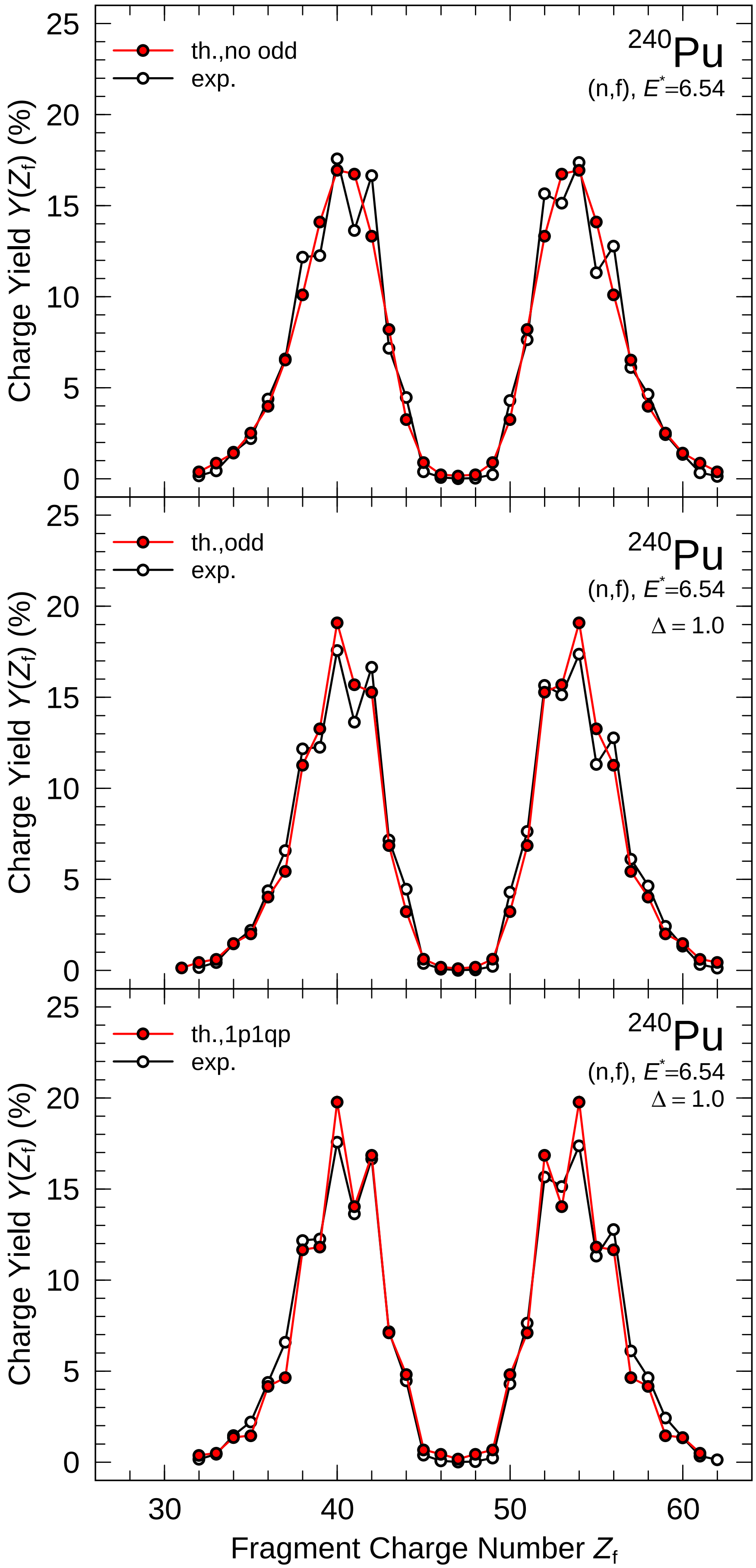
th., no odd
exp.
234U
(n,f), E*=6.84
th.,odd
exp.
234U
(n,f), E*=6.84
Δ = 1.0
th.,1p1qp
exp.
234U
(n,f), E*=6.84
Δ = 1.0
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf

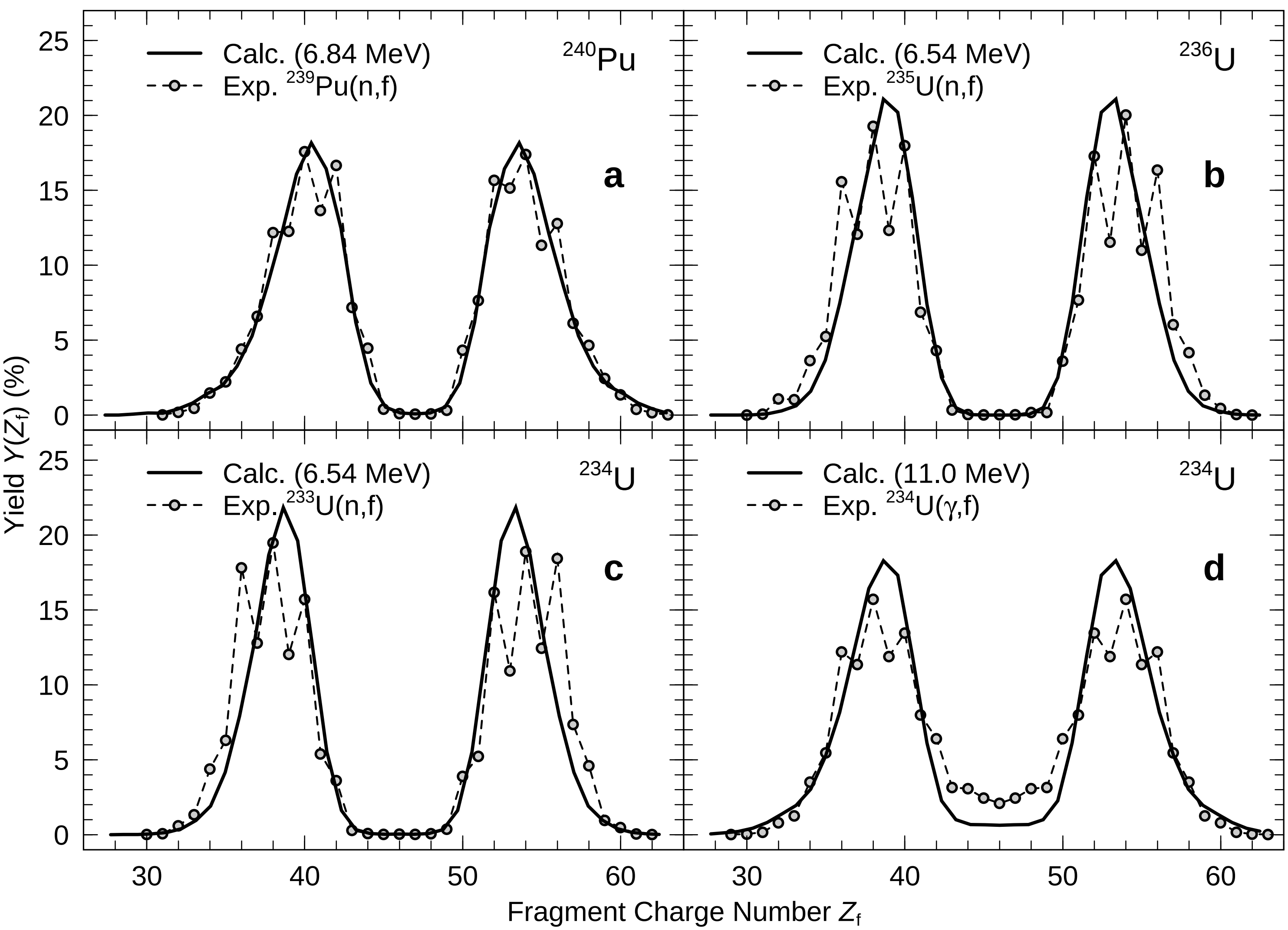


th., no odd
exp.
234U
(γ,f), E*=11
th.,odd
exp.
234U
(γ,f), E*=11
Δ = 1.0
th.,1p2qp
exp.
234U
(γ,f), E*=11
Δ = 1.0
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf

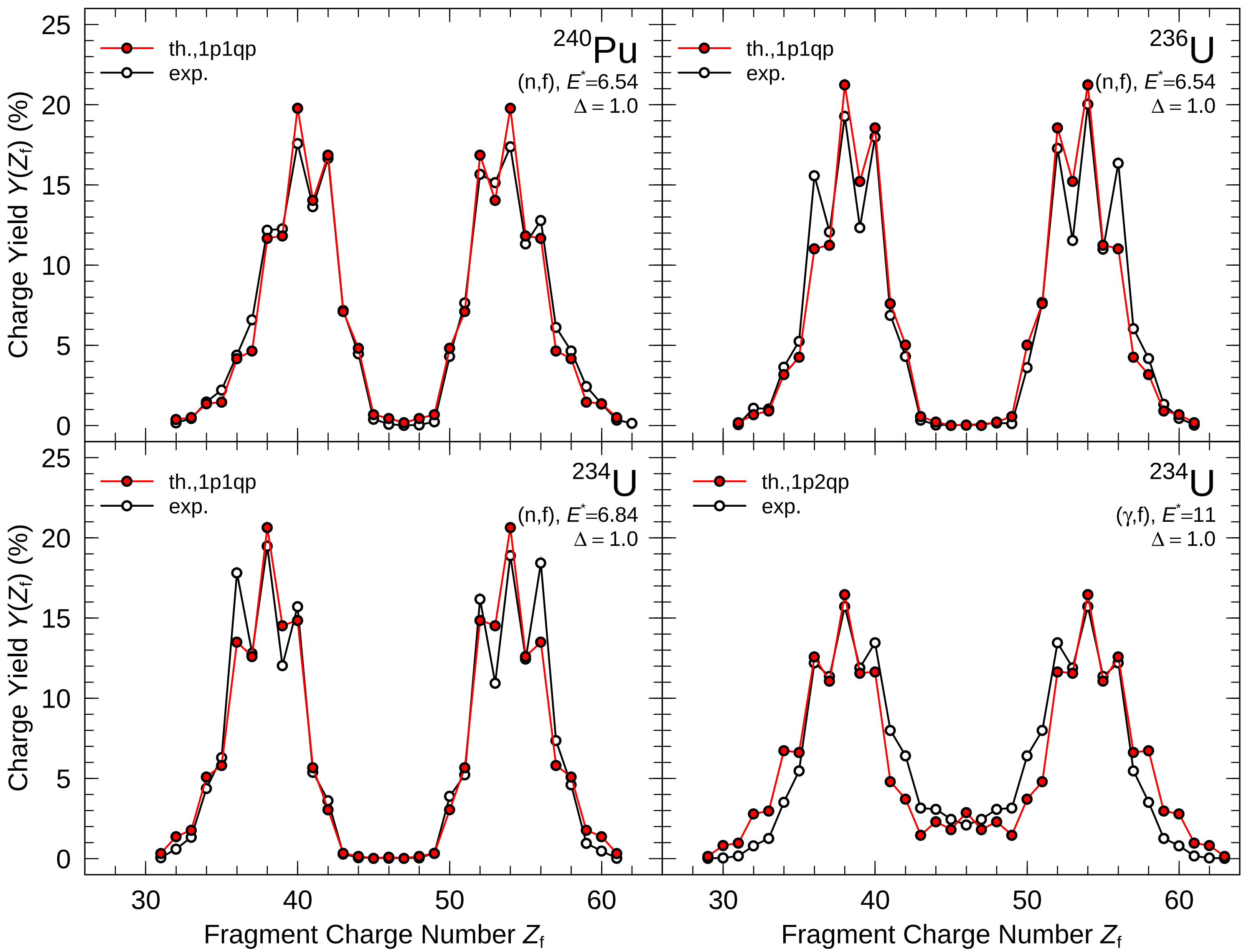


th.,no odd
exp.
236U
(n,f), E*=6.54
Δ = 1.0
th.,odd
exp.
236U
(n,f), E*=6.54
Δ = 1.0
th.,1p1qp
exp.
236U
(n,f), E*=6.54
Δ = 1.0
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf


th.,no odd
exp.
240Pu
(n,f), E*=6.54
th.,odd
exp.
240Pu
(n,f), E*=6.54
Δ = 1.0
th.,1p1qp
exp.
240Pu
(n,f), E*=6.54
Δ = 1.0
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf



Calc. (6.84 MeV)
Exp. ^{239}Pu(n,f)
^{240}Pu
a
Calc. (6.54 MeV)
Exp. ^{235}U(n,f)
^{236}U
b
Calc. (6.54 MeV)
Exp. ^{233}U(n,f)
^{234}U
c
Calc. (11.0 MeV)
Exp. ^{234}U(γ,f)
^{234}U
d
Yield $Y(Z_f)$ (%)
Fragment Charge Number Z_f



th.,1p1qp
exp.
^{240}Pu
(n,f), E^*=6.54
Δ = 1.0
th.,1p1qp
exp.
^{236}U
(n,f), E^*=6.54
Δ = 1.0
th.,1p1qp
exp.
^{234}U
(n,f), E^*=6.84
Δ = 1.0
th.,1p2qp
exp.
^{234}U
(γ,f), E^*=11
Δ = 1.0
Charge Yield $Y(Z_f)$ (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
30
40
50
60

M E T R O P O L I S

The simplicity of
the algorithm nobly
stands aside the complexity
of the problems
it successfully treats.

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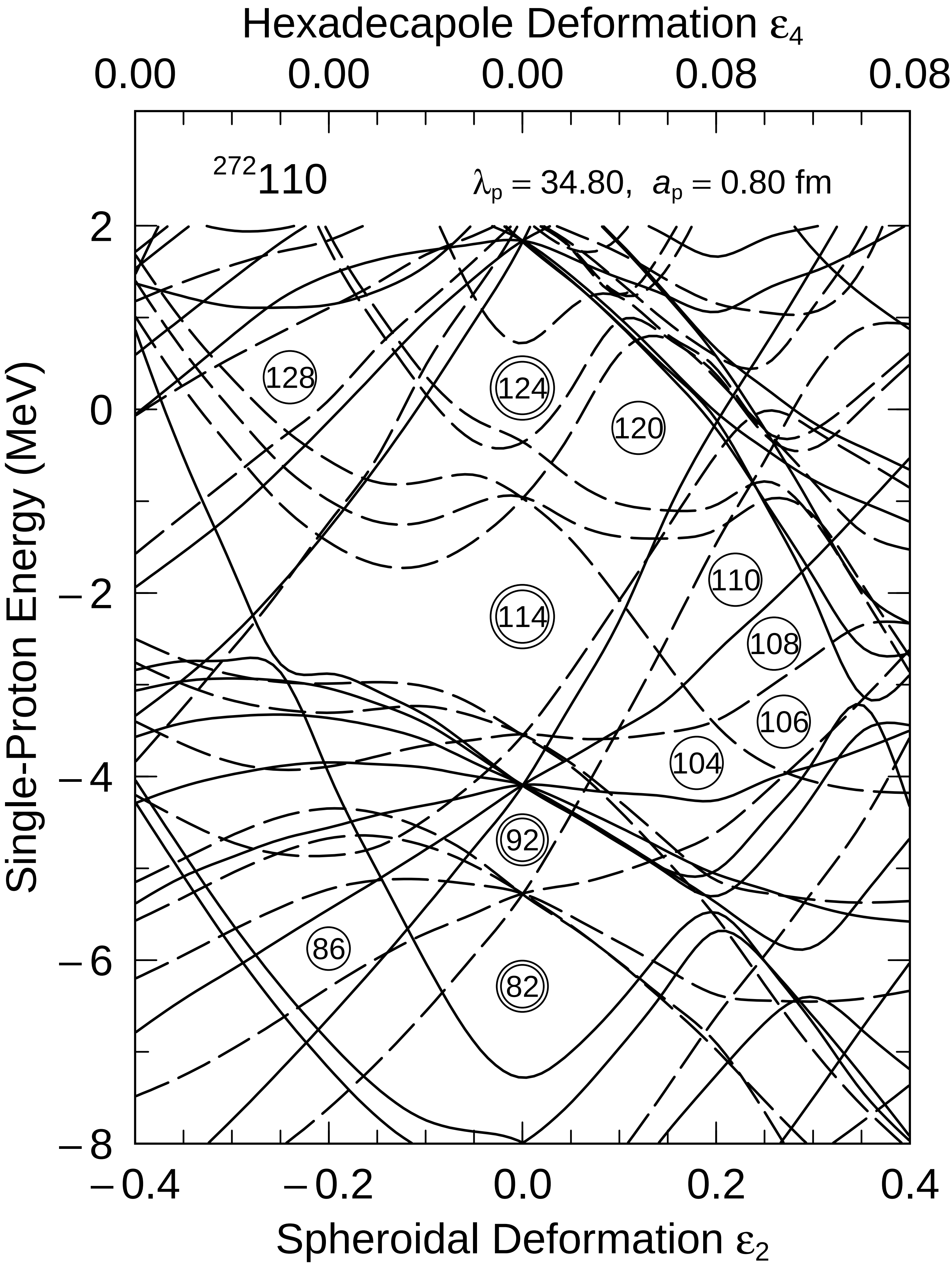
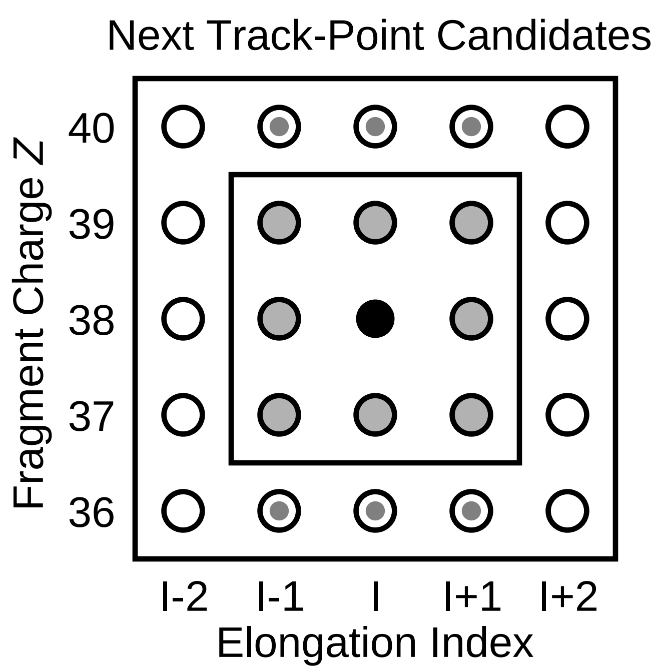
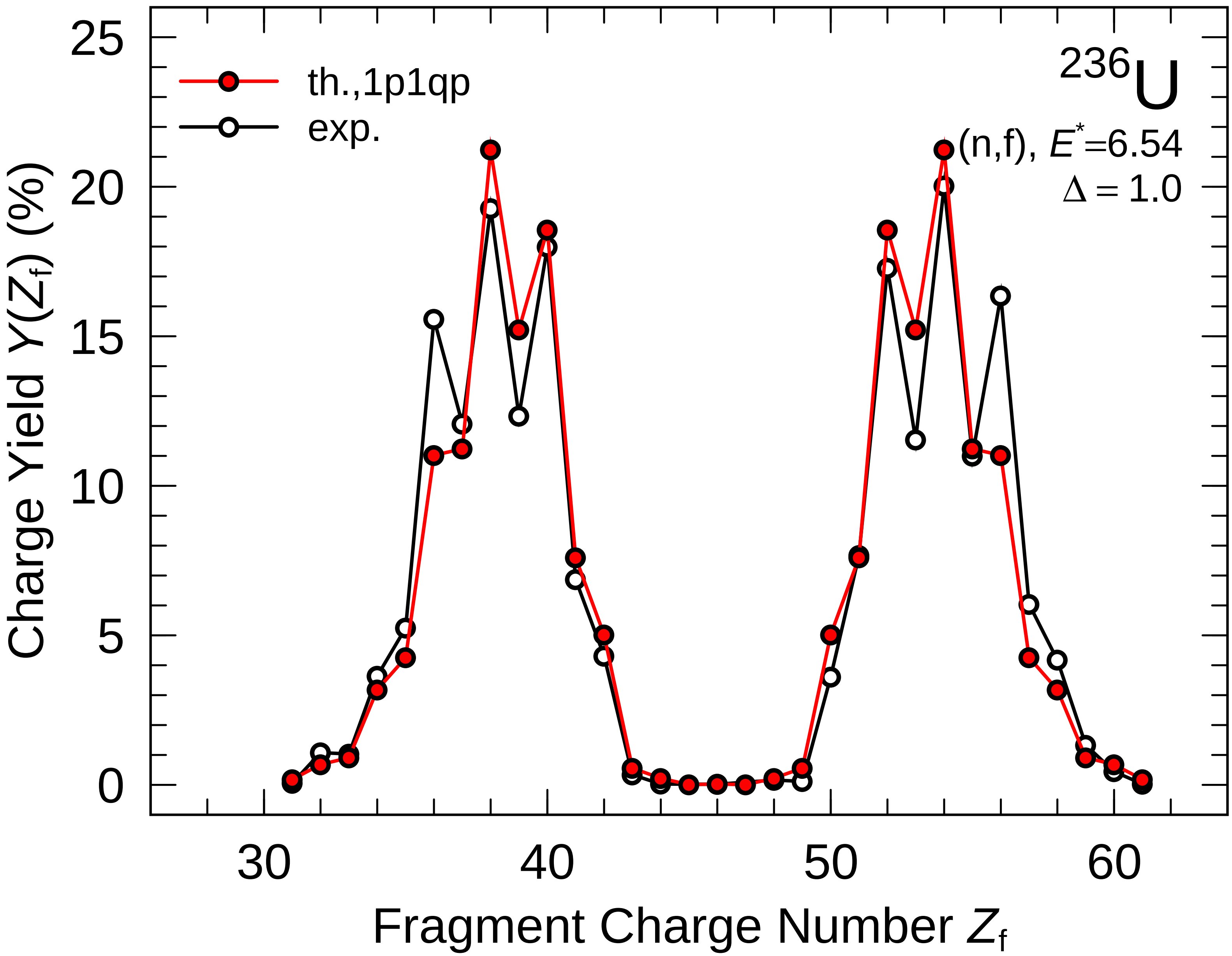
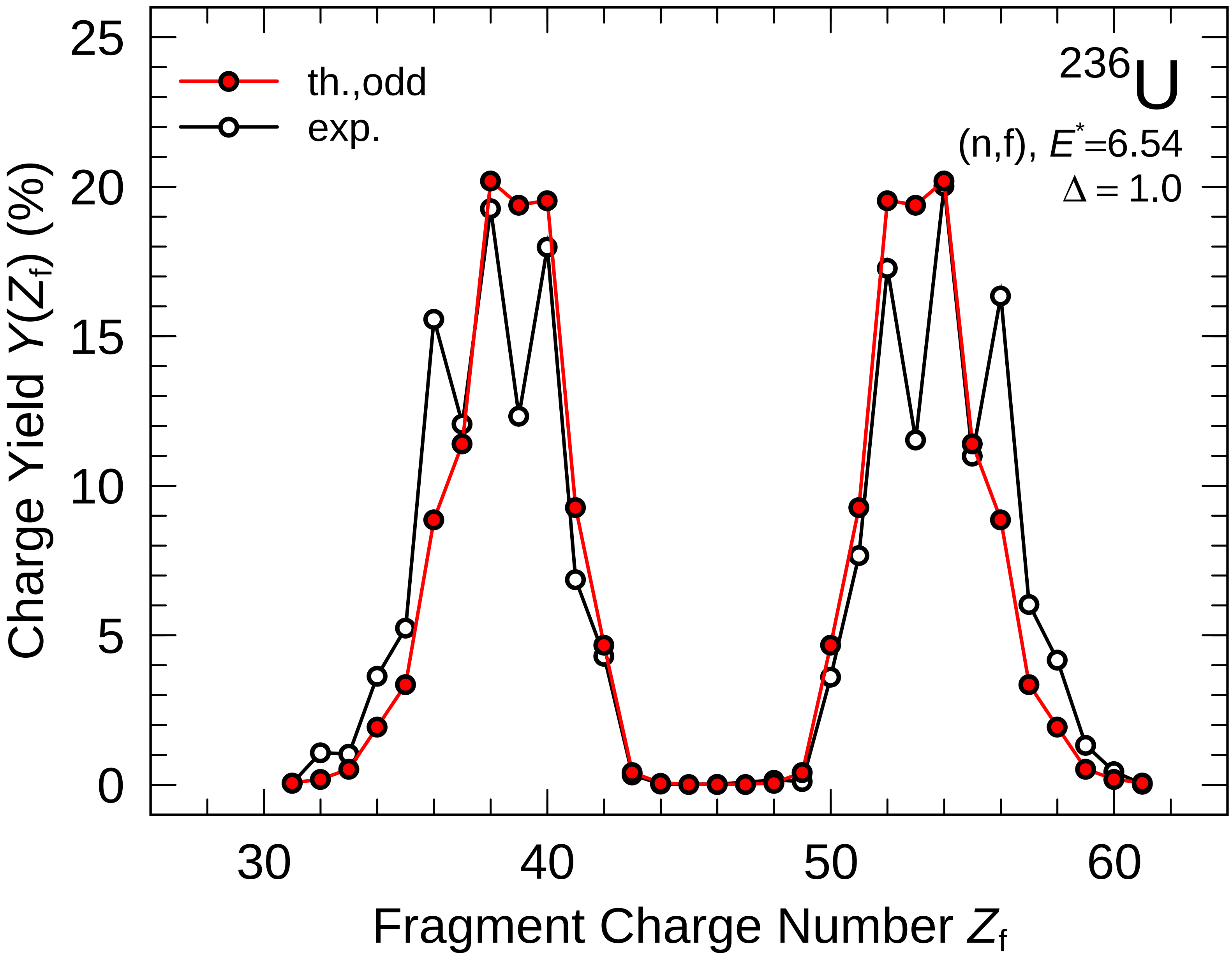

Hexadecapole Deformation ε_4
0.00
0.00
0.00
0.08
0.08
^{272}110
$\lambda_p = 34.80$, $a_p = 0.80$ fm
Single-Proton Energy (MeV)
2
0
−2
−4
−6
−8
128
124
120
114
110
108
106
104
92
86
82
−0.4
−0.2
0.0
0.2
0.4
Spheroidal Deformation ε_2

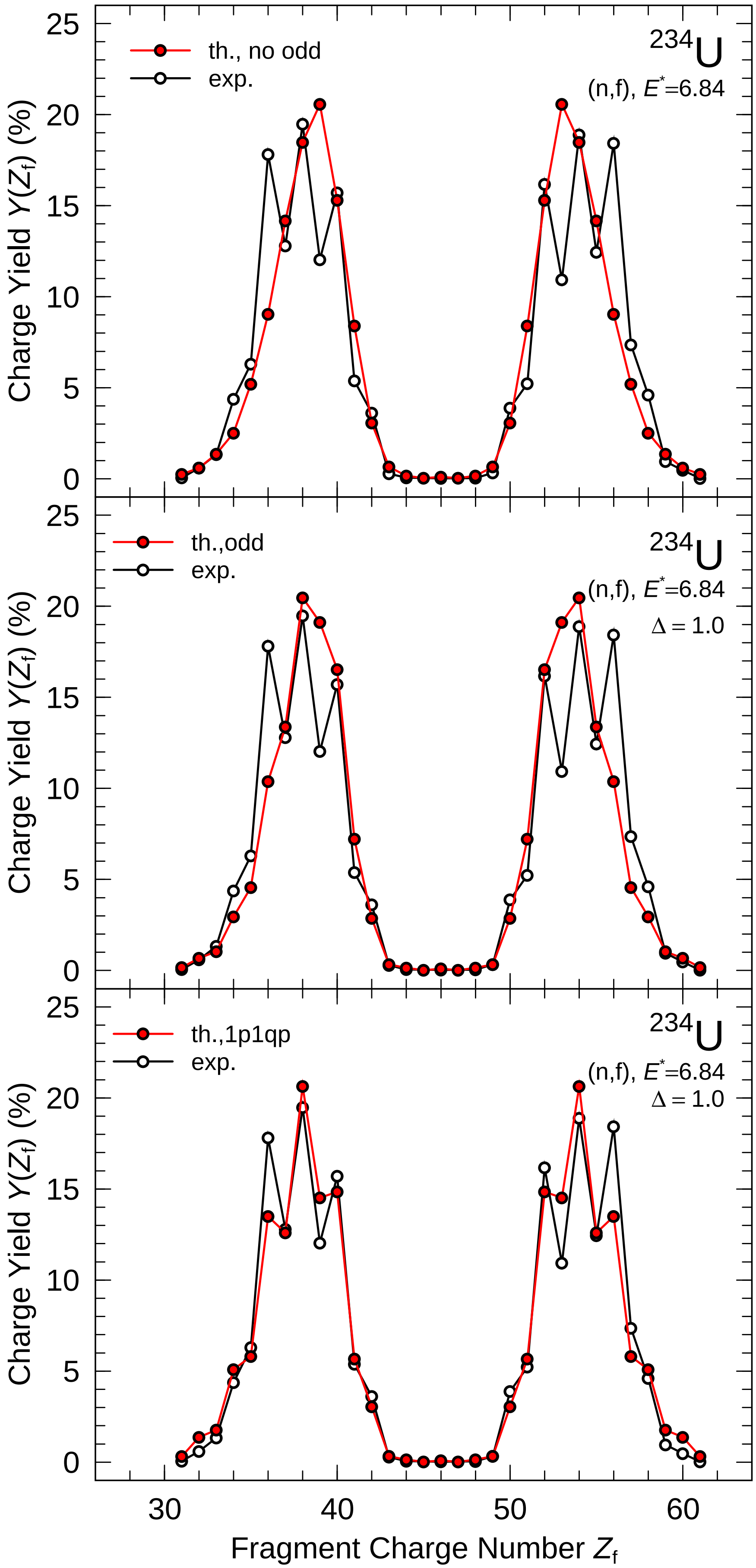

Next Track-Point Candidates
Fragment Charge Z
40
39
38
37
36
I-2
I-1
I
I+1
I+2
Elongation Index



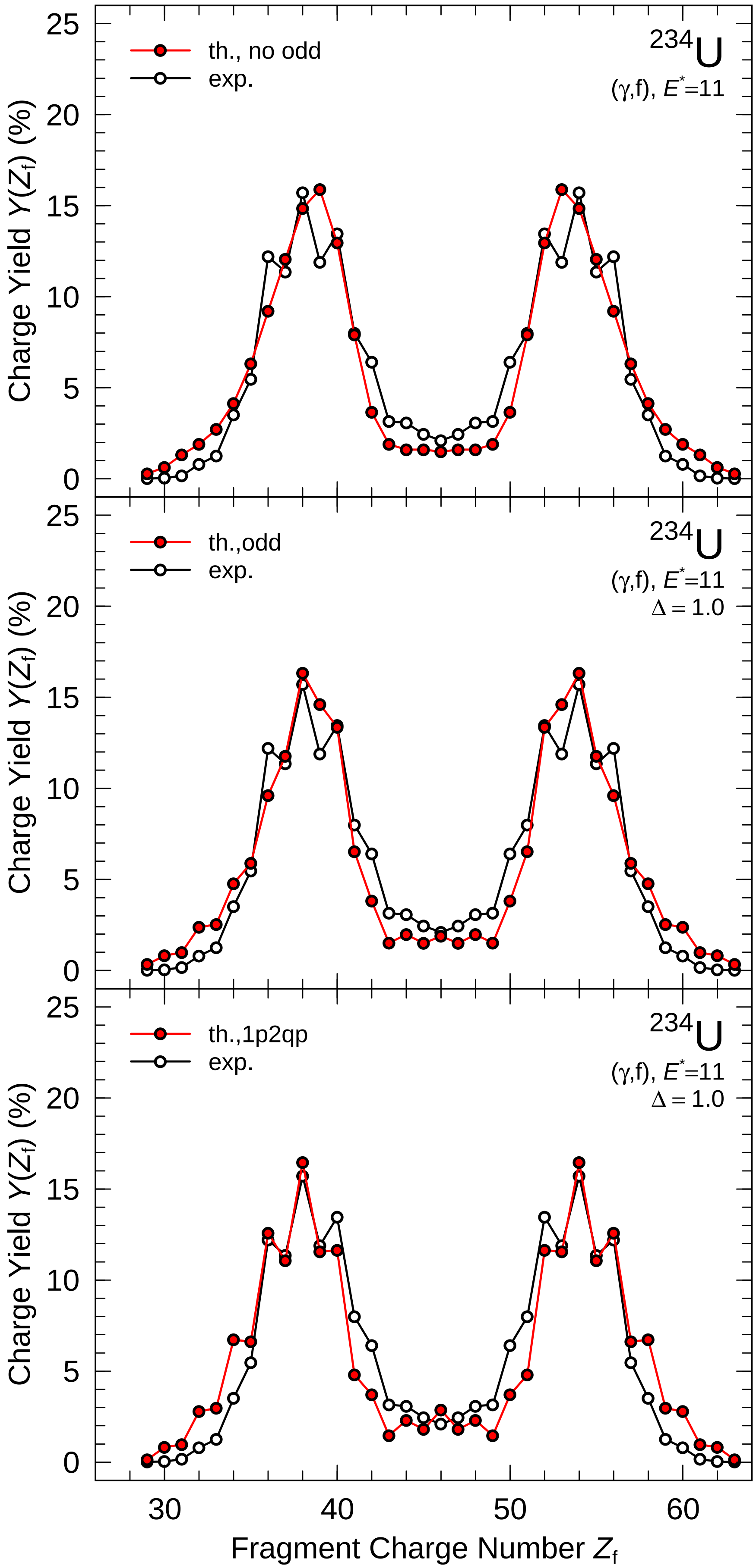
th.,1p1qp
exp.
236U
(n,f), E*=6.54
Δ = 1.0
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
30
40
50
60



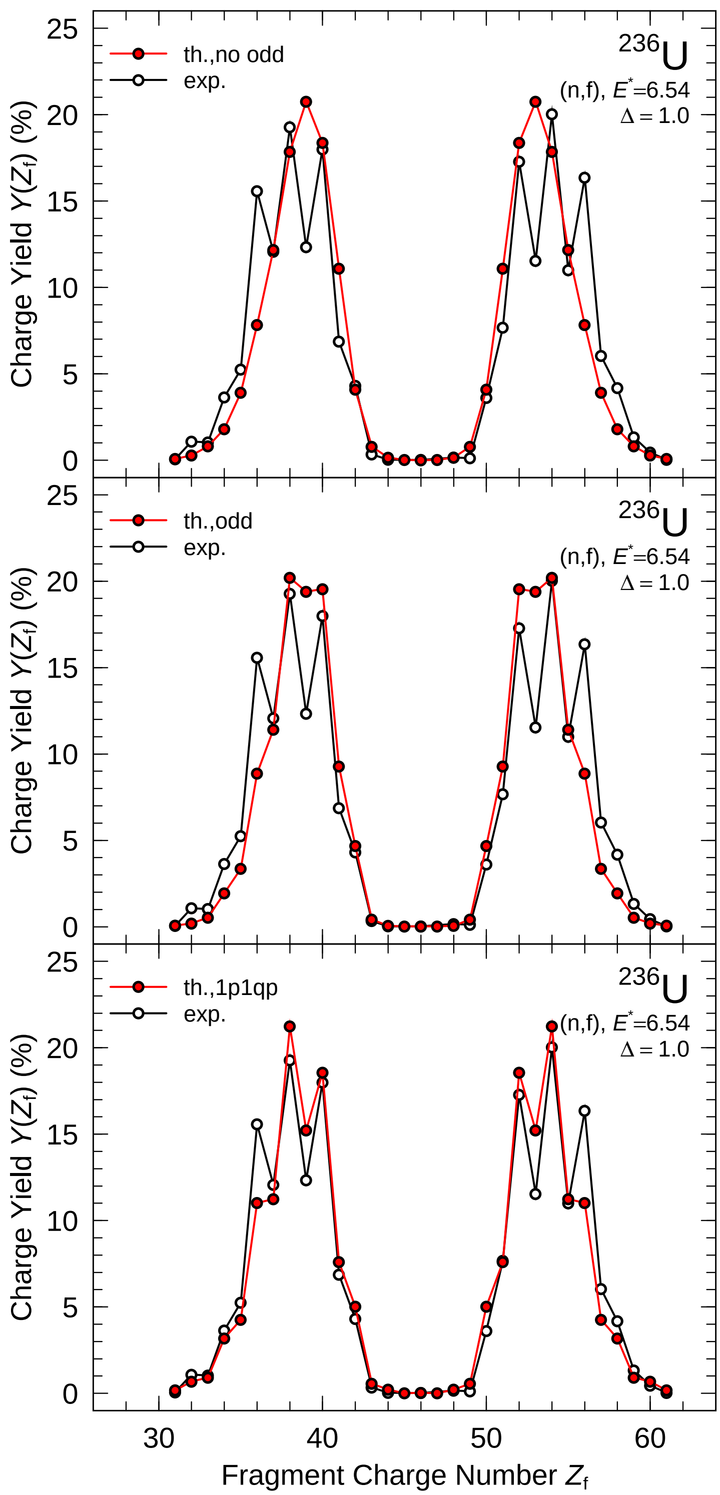
th.,odd
exp.
^{236}U
(n,f), E^*=6.54
Δ = 1.0
Charge Yield $Y(Z_f)$ (%)
Fragment Charge Number Z_f
0
5
10
15
20
25
30
40
50
60



th., no odd
exp.
234U
(n,f), E*=6.84
th.,odd
exp.
234U
(n,f), E*=6.84
Δ = 1.0
th.,1p1qp
exp.
234U
(n,f), E*=6.84
Δ = 1.0
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
15
20
25
30
40
50
60



th., no odd
exp.
234U
(γ,f), E*=11
th.,odd
exp.
234U
(γ,f), E*=11
Δ = 1.0
th.,1p2qp
exp.
234U
(γ,f), E*=11
Δ = 1.0
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf


th.,no odd
exp.
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(n,f), E*=6.54
Δ = 1.0
th.,odd
exp.
236U
(n,f), E*=6.54
Δ = 1.0
th.,1p1qp
exp.
236U
(n,f), E*=6.54
Δ = 1.0
Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
0
5
10
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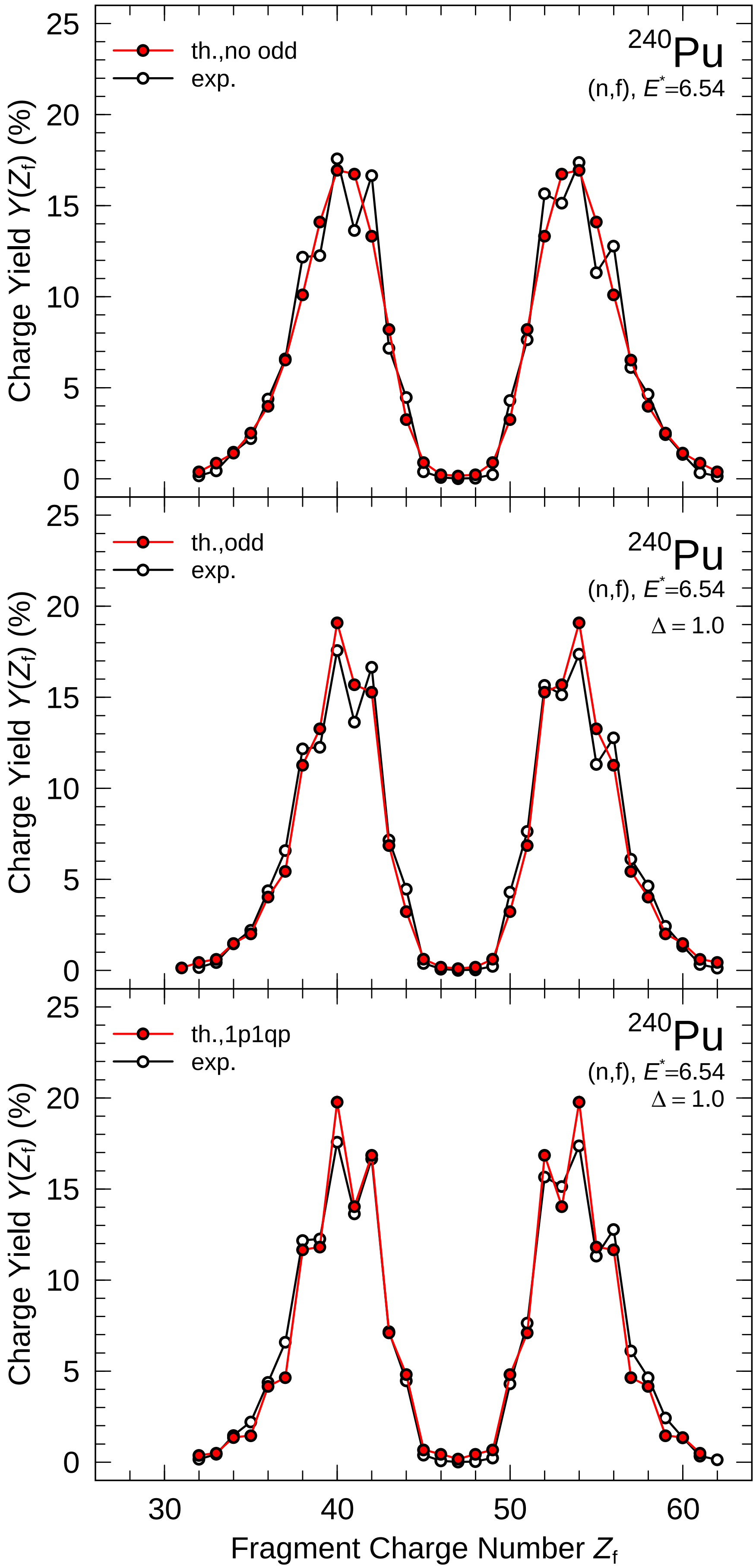
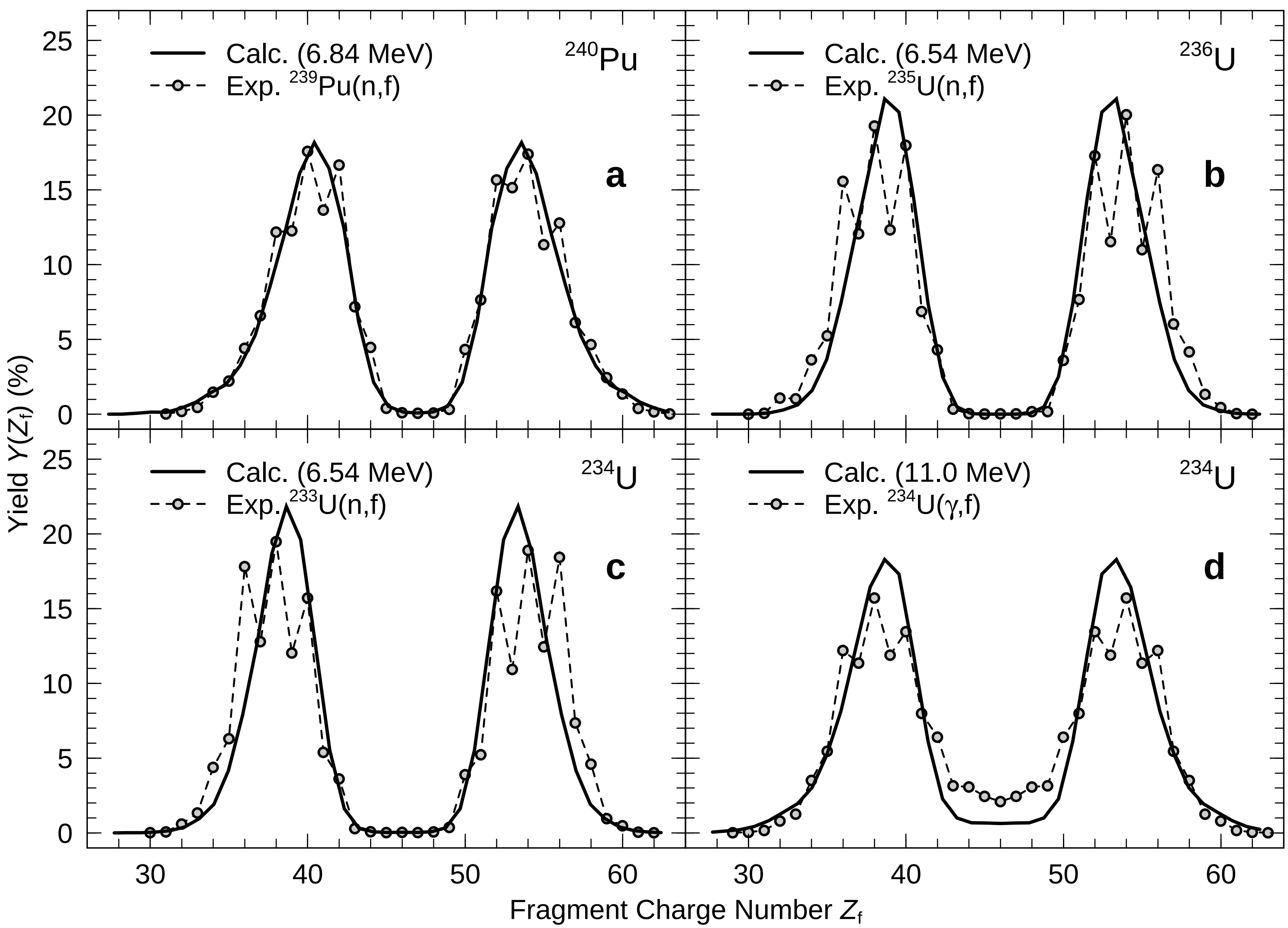
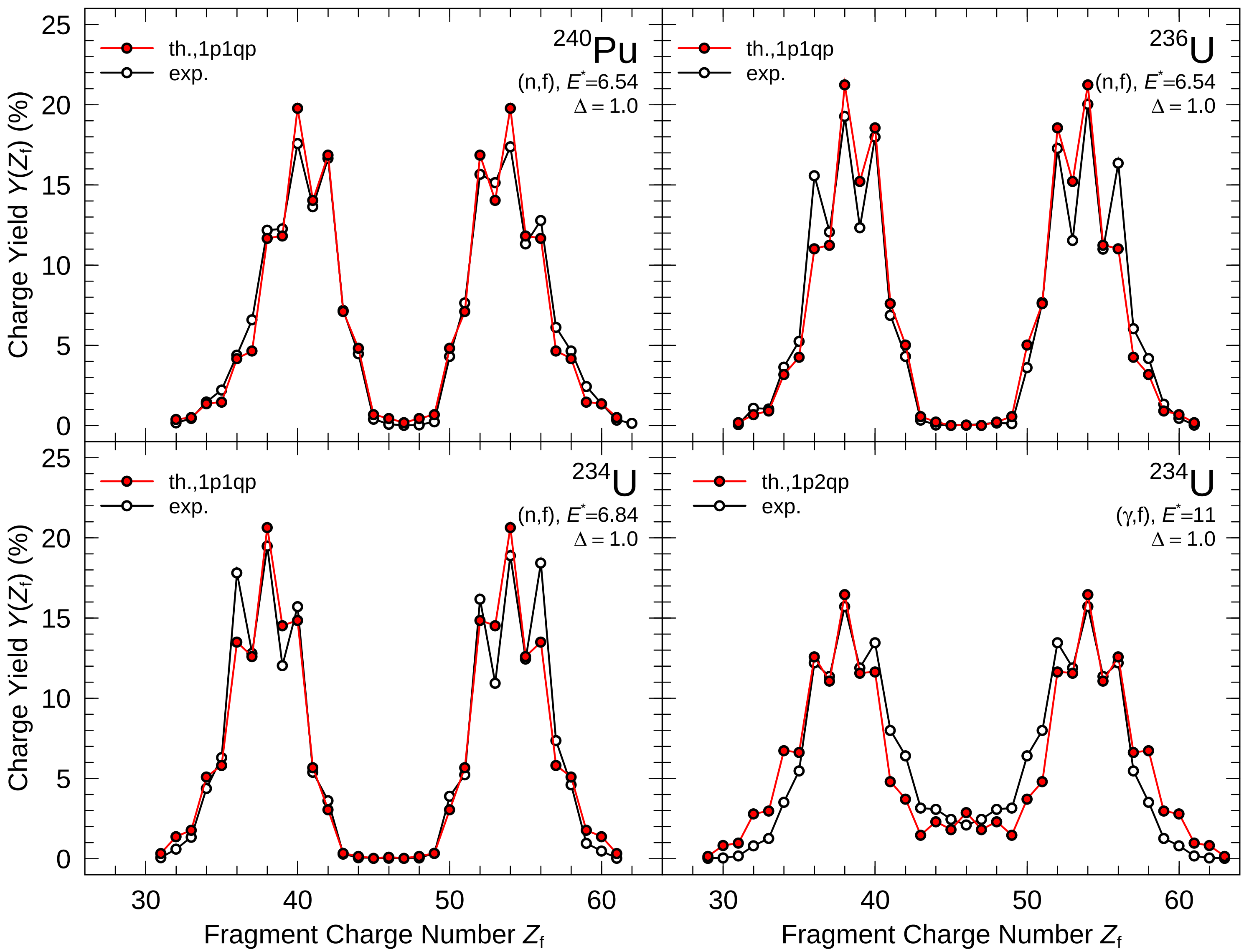
30
40
50
60



th.,no odd
exp.
240Pu
(n,f), E*=6.54
th.,odd
exp.
240Pu
(n,f), E*=6.54
Δ = 1.0
th.,1p1qp
exp.
240Pu
(n,f), E*=6.54
Δ = 1.0
Charge Yield Y(Zf) (%)
0
5
10
15
20
25
30
40
50
60
Fragment Charge Number Zf



Calc. (6.84 MeV)
Exp. ^{239}Pu(n,f)
^{240}Pu
a
Calc. (6.54 MeV)
Exp. ^{235}U(n,f)
^{236}U
b
Calc. (6.54 MeV)
Exp. ^{233}U(n,f)
^{234}U
c
Calc. (11.0 MeV)
Exp. ^{234}U(γ,f)
^{234}U
d
Yield $Y(Z_f)$ (%)
Fragment Charge Number Z_f



Charge Yield Y(Zf) (%)
Fragment Charge Number Zf
th.,1p1qp
exp.
240Pu
(n,f), E*=6.54
Δ = 1.0
236U
(n,f), E*=6.54
Δ = 1.0
234U
(n,f), E*=6.84
Δ = 1.0
th.,1p2qp
234U
(γ,f), E*=11
Δ = 1.0

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