

From low to high energy nuclear data evaluations Issues and perspectives on nuclear reaction models and covariances

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State of the art of methodologies for Cross Section evaluation in the resonance range

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General Actual Framework



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General Actual Framework





General Actual Framework

Reactor context :

- PWR, BWR ; thermal and epithermal energy range
- FR ; epithermal and fast range
- Needs of reliable Uncertainties (not too optimistic/ not too pessimistic)

Objectives/perspectives

- Proper link between Resonance range and continuum
- Take benefit of nuclear reaction models progress in high energy range
- Increase physical contents of resonance parameters
- Get rid of some "free" parameters
- Find guidelines for new evaluation
- Avoid compensations (Fresnel representation of Morillon)

NUCLEAR REACTION THEORIES R-MATRIX THE ORIGIN



General Hypothesis

- I. Non-relativistic Quantum Mechanics
- II. Only process with two product nuclei
- III. No processes of creation/destruction
- IV. Channel $c = \{J^{\pi}, \alpha_1 \alpha_2, \{q_i\}\}$
- V. For $r > a_c$ (in configuration space) : V = V(r)

Additional Considerations in RRR

- A. Compound nucleus
- B. Potential square well (Hard sphere)
- C. Level Approximations (Breit-Wigner, Reich-Moore)
- D. Fission ; Capture
- E. Averaged R-Matrix \rightarrow URR

R-Matrix

$$R_{ab} = \sum_{\lambda} \frac{\gamma_{a\lambda} \times \gamma_{b\lambda}}{E - E_{\lambda}}$$

Collision Matrix *

$$U_{ab} = e^{i(\Omega_a + \Omega_b)} (\delta_{ab} (1 - 2iP_a / L_a) + 2i\sqrt{P_a} (I - RL)_{ab}^{-1} \sqrt{P_b} / L_b) = e^{i(\Omega_a + \Omega_b)} (\delta_{ab} + i\sum_{\lambda\lambda'} \Gamma_{\lambda a}^{1/2} \Gamma_{\lambda' b}^{1/2} A_{\lambda\lambda'})$$

where
$$\Gamma_{\lambda a}^{1/2} = (2P_a)^{1/2} \gamma_{a\lambda}$$

and
$$(A_{\lambda\lambda'})^{-1} = (E_{\lambda} - E)\delta_{\lambda\lambda'} - \sum_{a}\gamma_{\lambda a}L_{a}^{0}\gamma_{\lambda a}$$

* Lane and Thomas for details : Rev. Mod. Phys. **30** (2) p.275 (1958)

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NUCLEAR REACTION THEORIES R-MATRIX THE ORIGIN AND BEYOND

 γ 's / E_{λ} are real numbers independent of E + Physical Meaning of Γ 's



ac is framing the resonance parameters ; Boundary Conditions ;

RRR/URR/Continuum ; Averaged Parameters ; Link to Optical Model

Modelling of Fission



We will present a few perspective that could be achieved in the future



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Step Forwards for Cross Section evaluation in the resonance range



What about Optical Potential in the Resonance range ?

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R-MATRIX WITH AVERAGE PHENOMENOLOGICAL POTENTIAL

Collision Matrix $U_{ab} = e^{i(\Omega_a + \Omega_b)} (\delta_{ab} + i \sum_{\lambda \lambda'} \Gamma_{\lambda a}^{1/2} \Gamma_{\lambda b}^{1/2} A_{\lambda \lambda'}) \text{ where } \Gamma_{\lambda a}^{1/2} = (2P_a)^{1/2} \gamma_{a\lambda}$

Penetrability is calculated via a potential square well for entrance channel (neutron most of the time) +Coulomb barrier for charged particles



Arbitrary choice of ac and several Boundary cond.

Effect of a **diffuse edge optical potential** ?*

$$\Gamma_{\lambda a}^{1/2} = (2P_a^{OM})^{1/2} \gamma_{a\lambda} \qquad \text{Calculate Penetrability with Optical Potential}$$

$$\Gamma_{\lambda a}^{1/2} = (2P_a^{SW})^{1/2} \gamma_{a\lambda} f \qquad \text{Correction factor keeping Square Well}^*$$

$$\Gamma_{\lambda a}^{1/2} = (2P_a^{ESW})^{1/2} \gamma_{a\lambda} \qquad \text{Equivalent Square Well} \rightarrow \text{choice of proper ac}$$

* Vogt for details : Rev. Mod. Phys. **34** (4) p.723 (1962)

R-MATRIX WITH AVERAGE PHENOMENOLOGICAL POTENTIAL

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Calculate Penetrability with Optical Potential

1st Perspective

Correction factor keeping Square Well*

Equivalent Square Well → choice of proper ac

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Is it working ? → look at Unresolved resonance range

Averaged Collision Matrix (over a limited energy domain)





No direct reaction contributions considered here (absorption)



In R-Matrix ; ac is arbitrary Choose to give ac a proper physical interpretation using models coming from high energy



Choice of channel radius a_c





²⁴¹Am+n

Forward : from OM to R-Matrix



Choose a_c using phase shift ϕ_c coming from optical model calculations

241**Am+n**

Forward : from OM to R-Matrix



Choose a_c using phase shift ϕ_c coming from optical model calculations

$$\begin{cases}
\phi_{0}(\rho) = \rho \\
\phi_{1}(\rho) = \rho - \tan^{-1}(\rho) \\
\phi_{2}(\rho) = \rho - \tan^{-1}(\frac{3\rho}{3 - \rho^{2}})
\end{cases}$$

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Forward : from OM to R-Matrix



Choose a_c using phase shift ϕ_c coming from optical model calculations



Backward \rightarrow Verify ac choice with transmission factor T_c



Choose a_c using phase shift ϕ_c coming from optical model calculations



$$(\mathbf{f} \mathbf{f} \mathbf{f}_{c} = 1 - \left| \overline{U}_{c} \right|^{2} \longrightarrow T_{c} \approx 4\pi P_{L} s_{c}$$

241**Am+**N

Backward \rightarrow Verify ac choice with transmission factor T_c



Choose a_c using phase shift ϕ_c coming from optical model calculations



$$T_{c} = 1 - \left|\overline{U}_{c}\right|^{2} \longrightarrow T_{c} \approx 4\pi P_{L}s_{c}$$

$$\begin{cases}
P_{0}(\rho) = \rho \\
P_{1}(\rho) = \frac{\rho^{3}}{1 + \rho^{2}} \\
P_{2}(\rho) = \frac{\rho^{5}}{9 + 3\rho^{2} + \rho^{4}}
\end{cases} \qquad (\rho = ka_{c})$$

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 $P \Longrightarrow P^{ESW}$

Backward \rightarrow Verify ac choice with transmission factor T_c

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Backward \rightarrow Verify ac choice with transmission factor T_c

241**Am+n**



 \Rightarrow Very good agreement between ECIS and « hard sphere » up to 200-300 keV \Rightarrow Different ac for different orbital momenta

Find Strenght functions and R^{\pi} with SPRT*



Confirmation of empirical rule (F. Frohner, O. Bouland, NSE, 2001) $S_0 \approx S_2 \approx cst$ $\overline{R}_0^{\infty} \approx 0 \Rightarrow$ Effective Radius R' equal to channel radius (Averaged R-Matrix formalism)

$$\sigma_p = \lim_{E \to 0} \sigma_{e_c}(E) = 4\pi R'^2 \qquad R' = a_0(1 - \overline{R}_0^\infty) \Longrightarrow R' \approx a_0$$

*Delaroche et Lagrange (IAEA-190, 1976) *E.Rich et al. NSE, **162** (2009) 76-86 | PAGE 11

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Apply equivalent methods to Resolved Resonance range



Fertile nuclei ok ; What about fissile nuclei ?



Step Forwards for Cross Section evaluation in the resonance range



What about Fission in the Resonance Range?

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 $V(\eta)$



In the Unresolved resonance range and Continuum Fission Barriers calculations used for Transmission coefficient $\pi T_r \cdot T_f$

 $\sigma_{n,f} = \frac{\pi}{k^2} \frac{T_n \cdot T_f}{T} W_{nf}$

+ Fission ingredients (J,K, Rotational band, class II, etc ...) Fission barriers calculation in the Resolved Resonance Range ?



1380

1400

1420

Incident neutron energy (eV)

1440

1460

1480

Simple Cramer-Nix Barrier

0.8 0.6 0.4 0.4 0.4 0.4 0.4 0.4 0.4 0.6 0.4 0.6 0.7 0.8 0.340 0.340 0.360 13400.360

R-MATRIX WAS IS MAINLY DEDICATED TO PARTICLE CHANNEL



 P_c and S_c are defined from φ_c and evaluated at $r = a_c$, statistics can be done on $E_{\lambda}, \gamma_{\lambda c}$ $S_c + iP_c = \left[\frac{r_c}{2}\frac{\partial \varphi_c}{\partial z}\right]$

$$c_{c} + iP_{c} = \left[\frac{r_{c}}{\varphi_{c}}\frac{\partial\varphi_{c}}{\partial r_{c}}\right]_{r_{c} = a_{c}}$$

Is a similar approach possible for fission?

EXAMPLE OF EXPLICIT TREATMENT OF THE FISSION BARRIER

 $V^{(\mu)}(\eta)$; η characterizes a collective fission coordinate (e.g. $Q_2, \beta_2, ...$)



Ces



Cez



Cez









Image: Second stateFission analysis in resolved resonance range
towards better evaluation of fission widths

Generic issues for defining fission channels: Reich-Moore allows a good description of fission \rightarrow For Heavy Nuclei ; 1 radiative capture channel with Hypothesis: many photons interferences cancel out

$$(A_{\lambda\lambda'})^{-1} = (E_{\lambda} - E) \delta_{\lambda\lambda'} - \sum_{a} \gamma_{\lambda a} L_{a}^{0} \gamma_{\lambda a}^{\dagger} \xrightarrow{\text{Reich-Moore}} (A_{\lambda\lambda'})^{-1} = (E_{\lambda} - E - i\Gamma_{\lambda, \gamma tot} / 2) \delta_{\lambda\lambda'} - \sum_{a \neq \gamma} \gamma_{\lambda a} L_{a}^{0} \gamma_{\lambda a}^{\dagger} \xrightarrow{\text{Reich-Moore}} (A_{\lambda\lambda'})^{-1} = (E_{\lambda} - E - i\Gamma_{\lambda, \gamma tot} / 2) \delta_{\lambda\lambda'} - \sum_{a \neq \gamma} \gamma_{\lambda a} L_{a}^{0} \gamma_{\lambda a}^{\dagger} \xrightarrow{\text{Reich-Moore}} (A_{\lambda\lambda'})^{-1} = (E_{\lambda} - E - i\Gamma_{\lambda, \gamma tot} / 2) \delta_{\lambda\lambda'} - \sum_{a \neq \gamma} \gamma_{\lambda a} L_{a}^{0} \gamma_{\lambda a}^{\dagger} \xrightarrow{\text{Reich-Moore}} (A_{\lambda\lambda'})^{-1} = (E_{\lambda} - E - i\Gamma_{\lambda, \gamma tot} / 2) \delta_{\lambda\lambda'} - \sum_{a \neq \gamma} \gamma_{\lambda a} L_{a}^{0} \gamma_{\lambda' a}^{\dagger} \xrightarrow{\text{Reich-Moore}} (A_{\lambda\lambda'})^{-1} = (E_{\lambda} - E - i\Gamma_{\lambda, \gamma tot} / 2) \delta_{\lambda\lambda'} - \sum_{a \neq \gamma} \gamma_{\lambda a} L_{a}^{0} \gamma_{\lambda' a}^{\dagger} \xrightarrow{\text{Reich-Moore}} (A_{\lambda\lambda'})^{-1} = (E_{\lambda} - E - i\Gamma_{\lambda, \gamma tot} / 2) \delta_{\lambda\lambda'} - \sum_{a \neq \gamma} \gamma_{\lambda a} L_{a}^{0} \gamma_{\lambda' a}^{\dagger} \xrightarrow{\text{Reich-Moore}} (A_{\lambda\lambda'})^{-1} = (E_{\lambda} - E - i\Gamma_{\lambda, \gamma tot} / 2) \delta_{\lambda\lambda'} - \sum_{a \neq \gamma} \gamma_{\lambda a} L_{a}^{0} \gamma_{\lambda' a}^{\dagger} \xrightarrow{\text{Reich-Moore}} (A_{\lambda\lambda'})^{-1} = (E_{\lambda} - E - i\Gamma_{\lambda, \gamma tot} / 2) \delta_{\lambda\lambda'}$$

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→Fission channels allow fission interference
 →No fundamental physical meaning
 →²³⁹Pu

 $0^+ \rightarrow 2$ Fission channels $1^+ \rightarrow 1$ Fission channel

→Statistics ? ; v_{eff} ??? ; (n, γ f) process ?

FISSION ANALYSIS IN RESOLVED RESONANCE RANGE TOWARDS BETTER EVALUATION OF FISSION WIDTHS

Use an additional quantum number* \rightarrow K (proj. of J on the fission axis) Fission channels defined by $c_{f,K} = \{J^{\pi}, fission, \{K\}\} \longrightarrow \Gamma_{cf,K}$

$$\sigma_{n,f}(E) = \sum_{J^{\pi}} \sigma_{n,f}^{J^{\pi}}(E) = \sum_{J^{\pi},K} \sigma_{n,f}^{J^{\pi},K}(E)$$

Need of new experiments:

- Polarized neutron/target
- Angular Distribution of F.F.

Development in Analysis codes:

- Polarized neutron/target
- K contributions
- Angular Distribution of fission fragments

For ²³⁵U :

some experiments

N.J. Pattenden et al.; Nucl. Phys. A **167** (1971)

G.A. Keyworth et al.; Conf. On nuclear cross section and technology,

Washington D.C., USA, NBS Special Publication 425 (1975) p.576

few evaluations

M.S. Moore, L.C. Leal, et al., Nulc. Phys. A 502 (1989)

Evaluation in Resonance Range

Disentangle fission channels Evaluation of J, K and $\Gamma_{cf,K}$

* W.I. Furman: FJ/OH Spring Session'99, Neutron data measurements & evaluation May 17 1999, Geel
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For ²³⁵U :3rd Perspective : 3.1N.J. Pattenden et al.; Nucl. Phys. A **167** (1971)G.A. Keyworth et al.;Conf. On nuclear cross section and technology,
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FISSION ANALYSIS IN RESOLVED RESONANCE RANGE TOWARDS BETTER EVALUATION OF FISSION WIDTHS

- Investigation of the two-step $(n,\gamma f)$ process*
- Still a topic of discussion
- No direct measurements of this reaction \rightarrow challenge
- WPEC/SG34 provides some recent explanations for ²³⁹Pu



Future evaluation artworks on ²³⁹Pu and others \rightarrow include explicitly the two-step (n, γ f) reaction (additional dedicated partial reaction width, $\Gamma_{\gamma f}$); usual fitted fission width becoming clear \rightarrow one-step fission component only.

* E. Lynn, Rev. Mod. Phys. **52** (1980)

FISSION ANALYSIS IN RESOLVED RESONANCE RANGE TOWARDS BETTER EVALUATION OF FISSION WIDTHS

3rd Perspective: 3.2

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Step Forwards for Cross Section evaluation in the resonance range



What about Uncertainties from the Resonance Range to the Continuum?

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CROSS SECTIONS "KNOWLEDGE" EVALUATION IN THE RESONANCE RANGE AND HIGHER



Issues :

- Systematic experimental uncertainties
- Phenomenological Nuclear reaction model theories + Parameters
- Model defects (Syst. Uncertainties)
- Integral experiment assimilation
- Common Physics from RRR to Continuum (previous slides)

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C23

Bayes' theory



> Description of \vec{x}







Bayes' theory



> Description of \vec{y}

- Microscopic experiments (TOF)
 - Transmission,
 - GELINA, nTOF, DANCE, ...
- Integral experiments
 - ICBEP
 - PROFIL, PROFIL-2, PROFIL-R et PROFIL-M
 - Spectral indices MASURCA





$$\boldsymbol{\sigma}(E_n, \vec{x}_{RRR}, \vec{x}_{URR}, \vec{x}_{OM}, \vec{x}_{fission}, \dots)$$

- How to deal with it ?
 - Several teams with internal constraints
 - Several analysis methodologies
- Solutions :
 - Share Physics (see previous slides)
 - → coupling between RRR/URR/Continuum
 - External constraints (Experiments ; Mathematics)
 - Extensive use of Monte-Carlo / look at pdf's



4th Perspective

 $\sigma(E_n, \vec{x}_{RRR}, \vec{x}_{URR}, \vec{x}_{OM}, \vec{x}_{fission}, ...)$

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4th Perspective

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4th Perspective

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Unified model on an energy domain

 $[\mathsf{E}_{\mathsf{L}}, \mathsf{E}_{\mathsf{R}}] + \text{Boundary at } \mathsf{E}_{\mathsf{C}}^{:}$ $\vec{t} = \vec{t}_{L}(x_{\mu}) \text{ if } \mathsf{E}_{L} \ge E \ge E_{c}$ $\vec{t} = \vec{t}_{R}(x_{\mu}) \text{ if } \mathsf{E}_{R} \le E \le E_{c}$ @ Conrad

Two models



- Unified model on an energy domain
- $[\mathsf{E}_{\mathsf{L}}, \mathsf{E}_{\mathsf{R}}] + \text{Boundary at } \mathsf{E}_{\mathsf{C}^{:}}$ $\vec{t} = \vec{t}_{L}(x_{\mu}) \text{ if } \mathsf{E}_{L} \ge E \ge E_{c}$ $\vec{t} = \vec{t}_{R}(x_{\mu}) \text{ if } \mathsf{E}_{R} \le E \le E_{c}$

© Conrad

Two models

- Given a microscopic experiment with statistical and systematic uncertainties on $[E_L, E_R]$
 - See effect of systematic uncertainties on nuclear model parameters
 - See effect of systematic uncertainties on cross section

- Unified model on an energy domain
- $\begin{bmatrix} \mathsf{E}_{\mathsf{L}}, \, \mathsf{E}_{\mathsf{R}} \end{bmatrix} + \text{Boundary at } \mathsf{E}_{\mathsf{C}^{:}} \\ \vec{t} = \vec{t}_{L}(x_{\mu}) \text{ if } \mathsf{E}_{L} \ge E \ge E_{c} \\ \vec{t} = \vec{t}_{R}(x_{\mu}) \text{ if } \mathsf{E}_{R} \le E \le E_{c} \end{bmatrix}$

@ Conrad

Two models

- Given a microscopic experiment with statistical and systematic uncertainties on $[E_1, E_R]$
 - See effect of systematic uncertainties on nuclear model parameters
 - See effect of systematic uncertainties on cross section
- Imposing constraints on boundary E_{c} :
 - Mathematical framework
 - Cross sections continuity
 - See effect of constraint on cross section



- Unified model on an energy domain
- $\begin{bmatrix} \mathsf{E}_{\mathsf{L}}, \, \mathsf{E}_{\mathsf{R}} \end{bmatrix} + \text{Boundary at } \mathsf{E}_{\mathsf{C}^{:}}$ $\vec{t} = \vec{t}_{L}(x_{\mu}) \text{ if } \mathsf{E}_{L} \ge E \ge E_{c}$ $\vec{t} = \vec{t}_{R}(x_{\mu}) \text{ if } \mathsf{E}_{R} \le E \le E_{c}$

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Two models

- Given a microscopic experiment with statistical and systematic uncertainties on $[E_L, E_R]$
 - See effect of systematic uncertainties on nuclear model parameters
 - See effect of systematic uncertainties on cross section
- Imposing constraints on boundary E_C:
 - Mathematical framework
 - Cross sections continuity
 - See effect of constraint on cross section
- Use of Integral experiments impacting several energy domains:
 - See effect of Integral Data Assimilation on cross sections/models





- × Didactic example : Sodium inelastic cross sections
 - \times Energy Range studied [1.9 2.1 MeV]; Boundary at 2 MeV.
 - \times Below 2 MeV : Resolved resonance range (Jeff3.2)
 - × Above 2 MeV : Jeff3.2 (Optical Potential + Partial models)

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"Simulated" experimental Data :

- Based on theoretical points (red)
- 3% statistical uncertainties
- No/0.5/1/3% systematic uncertainties

4.1th Perspective Analysis of wide range experiments

- Didactic example : Sodium inelastic cross sections
 - Energy Range studied [1.9 2.1 MeV]; Boundary at 2 MeV.
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Syst. Uncertainty
 Tends to ensure cross section continuity
 1st attempt with normalization
 → Generalize to other experimental
 (background, resolution parameters., isotopic concentration)



*Rouki et al., NIM in Physics Research Section A, **672** (2012)

x 10⁶

2.15

2.1

(e </ 2.05 Euergy

1.95

1.9

1.9

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0.6 0.4 0.2 -0.2 -0.4 -0.6 -0.8 1.95 2.1 2.15 2.05 Energy (eV) x 10⁶ 4.1th Perspective Analysis of wide range experiments PAGE 31

Statistical Uncertainty 3%

Normalization Uncertainty 0.5%

*Rouki et al., NIM in Physics Research Section A, **672** (2012)

0.8

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Normalization Uncertainty 1% x 10⁶ 0.8 2.15 0.6 2.1 0.4 0.2 Energy (eV) -0.2 -0.4 -0.6 1.95 -0.8 1.95 2.1 2.15 1.9 2 2.05 Energy (eV) x 10⁶ 4.1th Perspective Analysis of wide range experiments

Statistical Uncertainty 3%

*Rouki et al., NIM in Physics Research Section A, 672 (2012)

PAGE 31

x 10⁶

2.15

2.1

Energy (eV)

1.95

1.9

1.9

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0.8

Statistical Uncertainty 3%

Normalization Uncertainty 3%

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"Real" Evaluation done for time being JEFF3.2 ²³Na Based on new (n,n') measurements* (Syst. Unc. 2.6%)



— 0,00. 011001.0110,

Tends to ensure cross section continuity

- 1st attempt with normalization
- \rightarrow Generalize to other experimental

(background, resolution parameters., isotopic concentration)



*Rouki et al., NIM in Physics Research Section A, **672** (2012)





$$\chi_{GSL}^{2} = \left(\vec{x} - \vec{x}_{m}\right)^{T} M_{x}^{-1} \left(\vec{x} - \vec{x}_{m}\right) + \left(\vec{y} - \vec{t} \left(\vec{x}\right)\right)^{T} M_{y}^{-1} \left(\vec{y} - \vec{t} \left(\vec{x}\right)\right)$$



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$$+ (\vec{y} - \vec{t})^{T} M_{y}^{-1} (\vec{y} - \vec{t})$$

$$+ 2|C^{T} (\vec{x})| \cdot \lambda$$



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- × Difficult Mathematical resolution
 - × Based on Uzawa algorithm
 - \times Slow convergence
 - \times Constraints calculations \rightarrow time consuming



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4.2th Perspective Impose constraints

IMPOSING CONSTRAINTS ON SEVERAL MODELS LAGRANGE MULTIPLIERS ; ²³⁸U EXAMPLE

- Didactic example : Uranium Total cross section
 - Energy Range studied [25 750 keV] ; Boundary E_C at 150 keV.
 - Below 150 keV : Average R matrix
 - Above 150 keV : Average R matrix or Optical Potential

Considered parameters : <u>Unresolved Resonance Range</u> : Optical Model :

Effective Radius (**R**'), Strength ($S_{l=0,1}$), Distant level ($R^{\infty}_{l=0,1}$) Reduced Scattering Radius (Γ_0) and Diffusiveness (a_0)

- Considered Constraint on Cross sections at $E_c : C(x) = \langle \sigma_t^R \rangle_{E_c} \langle \sigma_t^L \rangle_{E_c} = 0$
- "Real" experimental Data :
 - Based on C.A.Uttley et al., 1966
 - 1% statistical uncertainties
 - No systematic uncertainties
- Difficulty arises if :
 - Parameters are not well chosen
 - Boundary is not well chosen : too high or too low making one model outside its scope
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UNCERTAINTIES : EVALUATION

$$\boldsymbol{\sigma}(E_n, \vec{x}_{RRR}, \vec{x}_{URR}, \vec{x}_{OM}, \vec{x}_{fission}, \dots)$$

What ever is the methodology σ and x are considered as random variables (pdf)

Monte-Carlo Sampling is a natural ingredient

Estimation of Uncertainties with Monte-Carlo during the evaluation process *:

$$p(\vec{x} \mid \vec{y}, U) = \frac{p(\vec{x}, U) \cdot p(\vec{y} \mid \vec{x}, U)}{\int p(\vec{x}, U) \cdot p(\vec{y} \mid \vec{x}, U) d\vec{x}}$$

Sample of $p(\vec{x} | M, U) \rightarrow \vec{x}_k$

For each \vec{x}_k calculation of Likelihood $\ell_k[p(\vec{y}|M, \vec{x}_k, U)]$



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*R. Capote and D. Smith, Nucl. Data Sheets **109**, 2768 (**2008**) * and **C. De Saint Jean et al., Nuc. Sci. Eng., **161**, 363 (**2009**). ** P. Schilleebeck et al., Nucl. Data Sheets (to be published) Sample of $p(\vec{x} | M, U) \rightarrow \vec{x}_k$

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UMC for the whole energy range with integrated analysis tools covering [0eV ; 200MeV] With shared Physics (parameters) With constraints With Experiments → Treatment of Experimental parameters and marginalisation** ("get rid of them properly") FULL BAYESIAN**




$$\sigma(E_n, \vec{x}_{RRR}, \vec{x}_{URR}, \vec{x}_{OM}, \vec{x}_{fission}, ...) + \left\{ \delta \left\langle \sigma_{i} \sigma_{j} \right\rangle \right\}$$

What about propagation of uncertainties and/or integral data assimilations?



Need of a new generation of simulation codes :

- Need of Integrated tools : Nuclear reaction codes ; data treatment codes ; transport codes (Analog Monte-Carlo simulation,...)
- Need of "parallelized" codes → HPC horizon (for example Conrad is multithreaded)
- Use Pdf of parameters ; Sampling →bunch of random numbers (1000 is a magic number ?) ; Statistical methods ; correlations (Cholesky) ;

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Reactors





INTEGRAL DATA ASSIMILATION

- Evaluation libraries are judged to their ability to reproduce public benchmark (ICSBEP,IRPHE,...)
- Some of these benchmarks are even used as judged of a single evaluation (JEZEBEL)
- Consistent Nuclear Data Evaluation ; Integral Data Assimilation *



- Generalization ; extensive use of Monte-Carlo neutron transport code (MCNP,Tripoli4,...)
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6th Perspective 6.1 IDA of Int. Experiments 6.2 HPC for Reactors !

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Cea

²³⁹PU COVARIANCE MATRICES

Prior Correlation on Fission





Multigroup cross section Data Assimilation Nuclear model parameters Data Assimilation

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Post Correlation on Fission with Feedback on Parameters



"Public" Integral Experiments



Multigroup cross section Data Assimilation Nuclear model parameters Data Assimilation



²³⁹PU COVARIANCE MATRICES



 σ_g^r $\vec{x} = \{OMP, Fission, ...\}$

Multigroup cross section Data Assimilation Nuclear model parameters Data Assimilation



²³⁹PU COVARIANCE MATRICES





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²³⁹PU COVARIANCE MATRICES



 σ_{g}^{r} and $\chi_{g}, \upsilon \dots$ + TRENDS

Multigroup cross section Data Assimilation

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+ TRENDS

Multigroup cross section Data Assimilation

Cea conclusions

- Several kind of Nuclear Data
- Several kind of Nuclear Reaction Models
- □ Several kind of Experiments
- Several kind of Covariance Matrices
- Progress on Methodologies needed:
 - Data assimilation techniques
 - Adding physical constraints (On several models)
- □ Progress on Experiments needed:
 - Reduction of systematic uncertainties for microscopic measurements
 - Integral experiments to target limited energy domain / reactions / isotopes
- Progress on Nuclear models needed:
 - Share Common physic features
 - Microscopic models
 - Avoid compensations
- □ Needs to define Covariance estimation benchmarks:
 - Fixed experiments
 - Fixed a priori (on parameters and/or cross section & uncertainties)
 - Incremental complexity
 - Compare covariance evaluation methodologies



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8th





Cea OTHER PERSPECTIVES

- 1. Other theories (S,K,...?)
- 2. Reich-Moore alternatives / Progress for Fission
- 3. Resonance shape analysis with double differential data (sodium/Fe/U,Pu,etc...)
- 4. Direct reaction treatment even in RRR
- 5. Microscopic Measurement :
 - a) Systematic uncertainties
 - b) Long range experiments (from RRR to Continuum)
 - c) Cold/hot experiments (few K to 1000 K)
 - d) Surrogate
 - e) Multi-Observables
 - I. fission+Capture, Fission xs / Fission yields ;
 - II. Spectra ...
- 6. Microscopic theories

7.