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Paper 1.25:

Analysis of the Relationship between k-effective and Fraction Critical

Derek Putley and Stewart Hay
Serco Energy

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Presented by
Derek Putley, Serco Energy.

Analysis of k_{eff} and fraction critical

- This paper investigates the relationship between k_{eff} and the fraction critical size of fissile bodies.
- Previous recent work in this area has tended to use only numerical methods. From the UK, some examples are:
  - Venner, Haley and Bowden, (ICNC 2003)
  - Prescott and Walker, (WPC/P168, 1990)
- This paper uses both analytic and numerical methods
- Analytic methods help to explain the underlying physics.

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Definitions – k_{eff} , f_x and f_m

- k_{eff} (or k-effective) is the effective neutron multiplication factor given by

$$k_{eff} = \frac{\text{neutron production rate}}{\text{neutron loss rate}}$$

- Fraction critical is the ratio of a safe value to the corresponding critical value

$$f_{xs} = \frac{\text{limiting safe size}}{\text{critical size}}$$

$$f_{ms} = \frac{\text{limiting safe mass}}{\text{critical mass}}$$

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Safe values are used to set safety limits

- Some UK prescriptions for safety limits are:
  - k_{eff} + uncertainties < 0.95 (e.g. for Pu and HEU)
    - f_{xs} = 0.9 (safe dimension limit)
    - f_{ms} = 0.75 (safe mass limit)
    - k_{eff} + uncertainties < 0.98 (e.g. for LEU)
- We already know that there is no simple exact equivalence between k_{eff} and fraction critical.
- On most plant, operators can control mass and size (etc.) but have no direct control over k_{eff}.

Buckling gives the variation of k_{eff} with size

Buckling is an approximate (but useful) solution to the Transport Equations.

For instance, for a sphere: $$k_{eff} = \frac{k_\infty}{1 + \frac{\pi^2 M^2}{(x+\lambda)^2}}$$

or $$x + \lambda = \frac{\pi M}{\sqrt{\frac{k_\infty}{k_{eff}} - 1}}$$

so if $$z = \frac{1}{\sqrt{\frac{k_\infty - k_{eff}}{k_{eff}}}}$$ then $$x = \pi M z - \lambda$$

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Buckling in z-notation

The relationship between size and reactivity is given by:

$$x = \pi M z - \lambda$$

where $z = \dfrac{1}{\sqrt{\dfrac{k_\infty - k_{eff}}{k_{eff}}}}$

So z is the reciprocal of the square root of the leakage.

A critical size is only possible if $k_\infty > 1$

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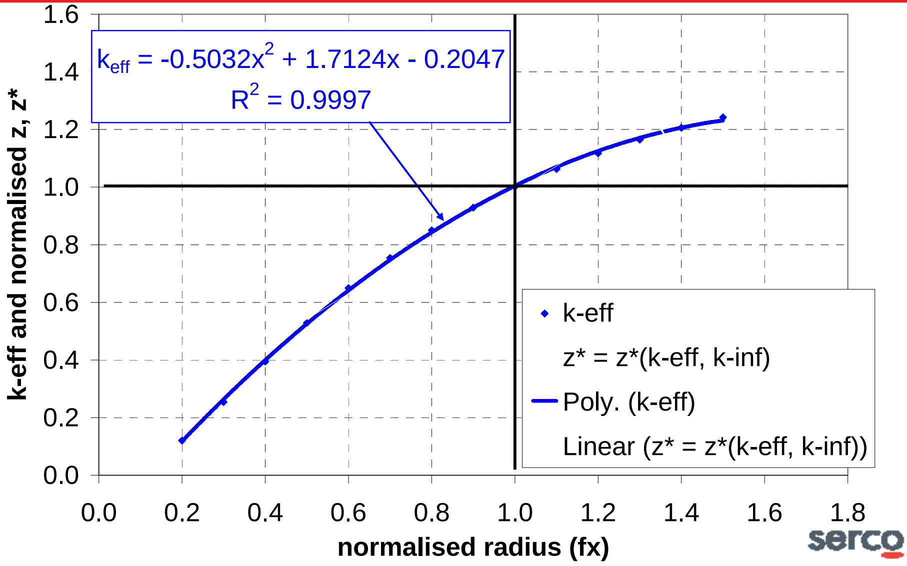
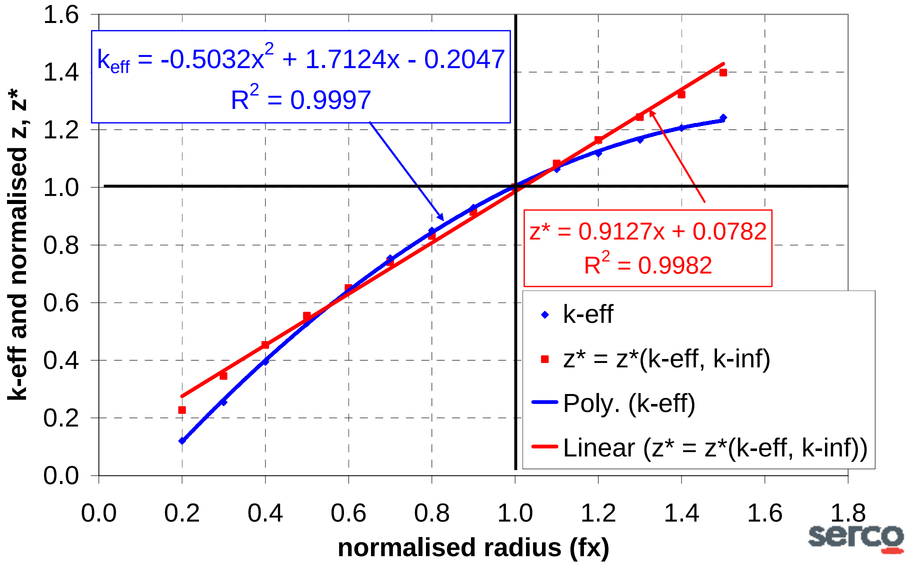
MONK results for reactivity versus size



MONK results for reactivity versus size



Safe fraction critical size, f_{xs}, in z-notation

$$x_{\text{safe}} = \pi M z_{\text{safe}} - \lambda$$

$$x_{\text{critical}} = \pi M z_{\text{critical}} - \lambda$$

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eliminating πM gives:

$$f_{\text{x, safe}} = \frac{x_{\text{safe}}}{x_{\text{critical}}} = \left(\frac{z_{\text{safe}}}{z_{\text{critical}}}\right) - \left(\frac{\lambda}{x_{\text{critical}}}\right)\left(1 - \frac{z_{\text{safe}}}{z_{\text{critical}}}\right)$$

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or, normalised relative to the critical values:

$$f_{\text{x, safe}} = x^*_{\text{safe}} = z^*_{\text{safe}} - \lambda^*\left(1 - z^*_{\text{safe}}\right)$$

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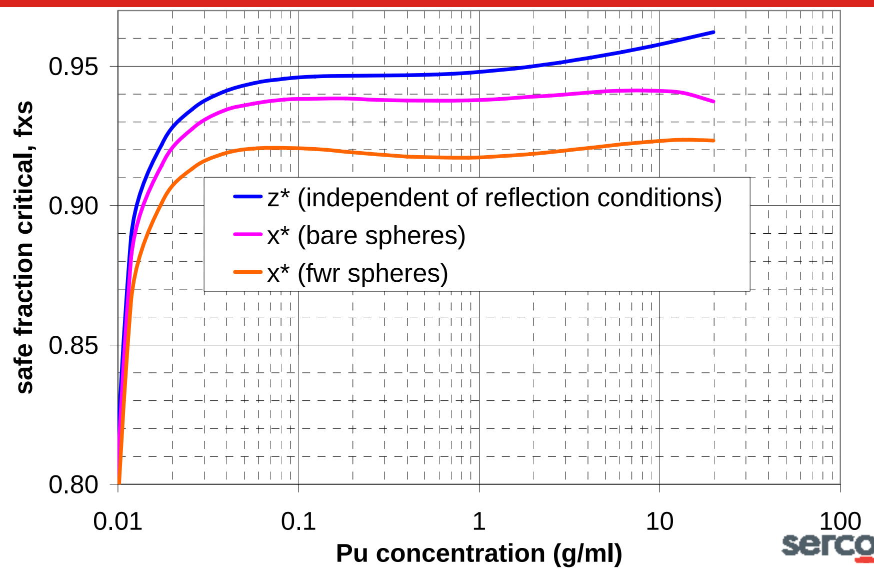
Fraction critical size for spherical Pu/water systems



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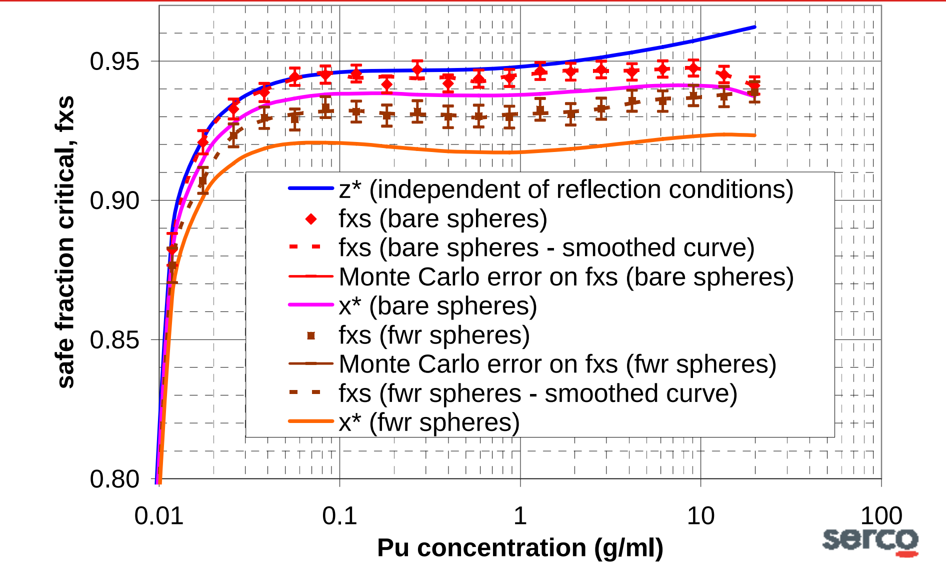
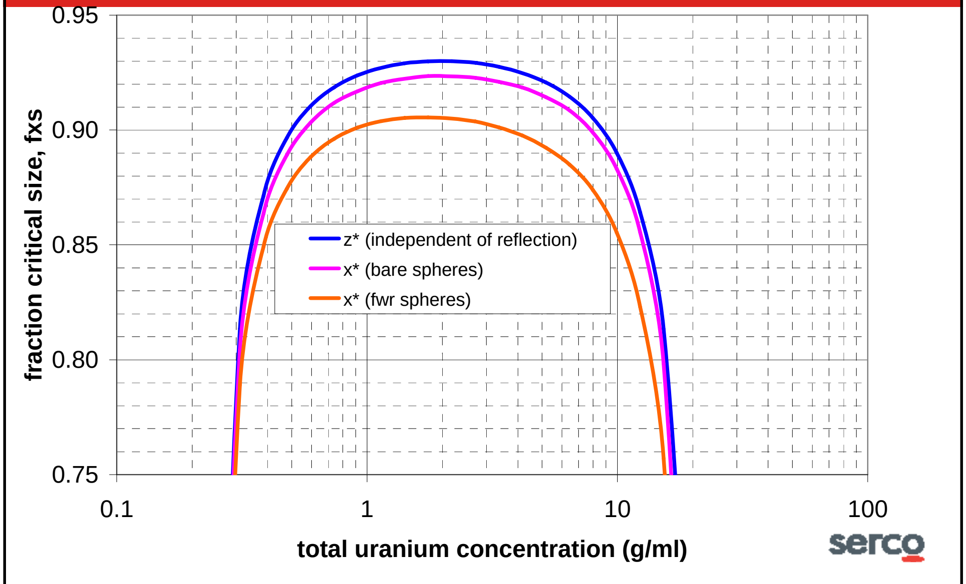


Fraction critical size for spherical U(6%)/water systems



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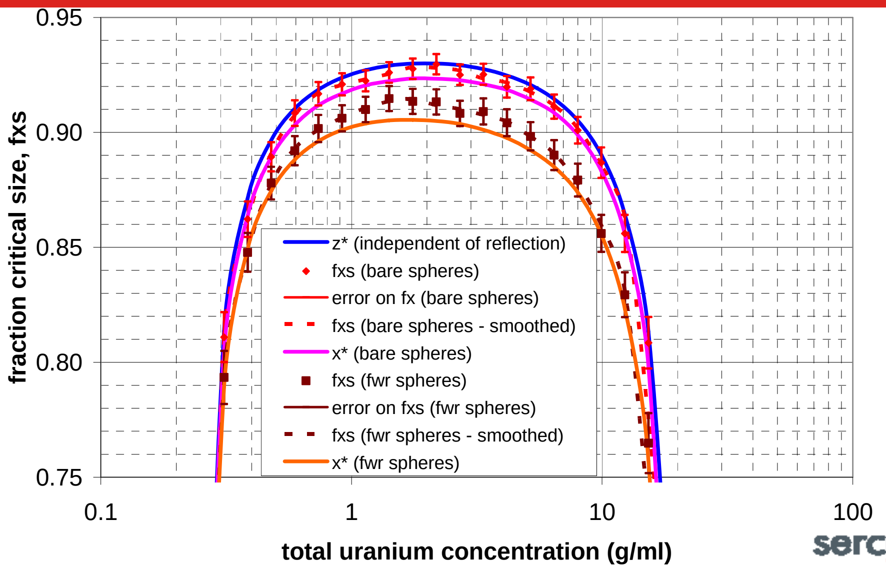

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Fraction critical in terms of Δk_{eff}

$$f_{\text{x, safe}} = x^*_{\text{safe}} = z^*_{\text{safe}} - \lambda^* \left(1 - z^*_{\text{safe}}\right)$$

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The variation of fraction critical with Δk_{eff}

$$f_{x,\,safe} = x^*_{safe} = z^*_{safe} - \lambda^*\left(1 - z^*_{safe}\right)$$

may be expanded as

$$f_{x,\,safe} = x^*_{safe} = \sqrt{\frac{k_{safe}(k_\infty - 1)}{k_\infty - k_{safe}}} - \frac{\lambda}{x_{critical}}\left(1 - \sqrt{\frac{k_{safe}(k_\infty - 1)}{k_\infty - k_{safe}}}\right)$$

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or in terms of Δk_{safe} = 1 - k_{safe}

$$f_{x,\,safe} = x^*_{safe} = \left(1 + \frac{\lambda}{x_{critical}}\right)\sqrt{\left(1 - \Delta k_{safe}\right)\left(1 - \frac{\Delta k_{safe}}{k_\infty - k_{safe}}\right)} - \frac{\lambda}{x_{critical}}$$

The variation of fraction critical with Δk_{safe}

$$f_{\text{x, safe}} = x^*_{\text{safe}} = z^*_{\text{safe}} - \lambda^*\left(1 - z^*_{\text{safe}}\right)$$

may be expanded as

$$f_{\text{x, safe}} = x^*_{\text{safe}} = \sqrt{\frac{k_{\text{safe}}(k_\infty - 1)}{k_\infty - k_{\text{safe}}}} - \frac{\lambda}{x_{\text{critical}}}\left(1 - \sqrt{\frac{k_{\text{safe}}(k_\infty - 1)}{k_\infty - k_{\text{safe}}}}\right)$$

or in terms of $\Delta k_{\text{safe}} = 1 - k_{\text{safe}}$

$$f_{\text{x, safe}} = x^*_{\text{safe}} = \left(1 + \frac{\lambda}{x_{\text{critical}}}\right)\sqrt{\left(1 - \Delta k_{\text{safe}}\right)\left(1 - \frac{\Delta k_{\text{safe}}}{k_\infty - k_{\text{safe}}}\right)} - \frac{\lambda}{x_{\text{critical}}}$$

This form can also be used to evaluate the effects of the the Monte Carlo uncertainty on k_{eff}

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Relative errors in x as a function of δk_{eff}

$$\varepsilon_R = \frac{\delta x}{x} = \frac{x - x'}{x} = 1 - \frac{x'}{x}$$

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Relative errors in x as a function of δk_{eff}

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may be expanded as

$$\varepsilon_R = \left(1 + \frac{\lambda}{x}\right)\left(1 - \sqrt{\frac{1}{1 + \delta k / k_{eff}}}\sqrt{1 - \frac{\delta k}{k_\infty - k_{eff}}}\right)$$

Relative errors in x as a function of δk_{eff}

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may be expanded as

$$\varepsilon_R = \left(1 + \frac{\lambda}{x}\right)\left(1 - \sqrt{\frac{1}{1 + \delta k / k_{eff}}}\sqrt{1 - \frac{\delta k}{k_\infty - k_{eff}}}\right)$$

and, in most cases, this can be approximated to

$$\varepsilon_R = \frac{\delta x}{x} = \left(\frac{\delta k}{k_{eff}}\right)\left(1 + \frac{\lambda}{x}\right)\left(\frac{k_\infty}{2}\right)\left(\frac{1}{k_\infty - k_{eff}}\right)$$

Summary - fraction critical size ($f_{xs} = x_{safe}/x_{critical}$)

Empirical	$f_{xs} = k_{safe}$
Simplified buckling	$f_{xs} = z^* = \sqrt{\frac{k_{safe}(k_\infty - 1)}{k_\infty - k_{safe}}}$
Buckling	$f_{xs} = x^* = z^* - \frac{\lambda}{x_{critical}}(1 - z^*)$
Direct calculation (e.g. ICASPA + MONK9A)	$f_{xs} = \frac{x_{safe}}{x_{critical}}$

k-effective and fraction critical as criticality indices

- A sub-critical k-effective is an absolute measure of reactivity but does not reveal if (or how) fault progression to criticality can occur.
- For instance, a system with k = 0.95 might progress to critical by, for instance, changing size, concentration, or shape.
- If so, a very different fraction critical value could be calculated for every possible fault progression to critical.
- Fraction critical values cannot be calculated unless criticality is possible.
- Fraction critical might not be useful in all cases – e.g. a flask filled with spent fuel.

Questions ?

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