Physical basis for the Kalbach angular distribution systematics

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Abstract

We show how the conservation of linear momentum is fundamental to the description of angular distributions in preequilibrium nuclear reactions. By using state densities with linear momentum to describe the phase space during the preequilibrium cascade, angular distributions can be derived in a transparent way. Fermi-motion and Pauli-blocking effects are included, and correlations between the emission particle's energy and angle are obtained for all orders of scattering. Our model provides a physical basis for the widely used phenomenological systematics of Kalbach, and provides a framework for understanding the systematical properties of continuum angular distributions.

Particles ejected during the early stages of a nuclear reaction are typically of high energy and have forward-peaked angular distributions, since they are emitted prior to nuclear equilibration and partially preserve the incident projectile's direction of motion. Refs. [1-10] describe a selection of the semiclassical theories that have been presented to account for the angular distributions. Many works use angle-integrated preequilibrium emission spectra from the semiclassical exciton or hybrid models, and obtain angular distributions from the Kikuchi-Kawai (KK) [11] nucleon-nucleon scattering kernel in a Fermi-gas, convoluting this quantity for multiple-scattering processes [1-7]. The KK kernel implicitly includes momentum conservation, yet most works hypothesize a fast leading-particle that carries all the directional information during the cascade. This is in contradiction to the equiprobability assumption used in the exciton model which puts all the excited particles and holes on an equal footing, and does not follow the individual particle's motion [2]. Another widely-used approach is to apply the phenomenological systematics of Kalbach [12] to obtain continuum angular distributions. While these systematics are useful for describing and predicting differential cross sections, their basis in physics has remained obscure. The fact that observed continuum preequilibrium cross sections tend to vary smoothly with angle and energy, and lend themselves to simple parameterizations [12, 13], suggests that they should be describable using a relatively simple model of the reaction process. In this letter we show how Kalbach's parameterization of the forward-peaking can be derived theoretically using an exciton model that explicitly conserves momentum, and that does not assume a leading particle.

Our derivation relies on the use of a state densities with linear momentum, which we introduced in Refs. [9,10]. These densities describe the linear momentum structure of the phase space of excited particles and holes (excitons), and are closely related to angular-momentum-dependent state densities [10]. We use an exciton preequilibrium model, though use of the hybrid model [4] would result in the same conclusions. The rate for particle emission from a given preequilibrium stage containing p particles and h holes is obtained by applying detailed balance. By explicitly conserving linear momentum we obtain a rate for emission with energy ϵ and direction Ω given by

$$\frac{d^2\lambda_n(\epsilon,\Omega)}{d\epsilon d\Omega} = \frac{2m\epsilon\sigma_{\rm inv}R(p)}{\pi^2\hbar^3}\frac{\rho(p-1,h,E-\epsilon_\Omega,\mathbf{K}-\mathbf{k}_\Omega)}{4\pi\ \rho(p,h,E,\mathbf{K})}, \tag{1}$$

where m is the ejectile mass, and the reaction cross section for the inverse process is $\sigma_{\rm inv}$. The composite system total energy and momentum before particle emission are E and $\mathbf{K}$, respectively, and the residual nucleus energy and momentum after emission are $E-\epsilon_\Omega$ and $\mathbf{K}-\mathbf{k}_\Omega$, respectively, all these quantities being measured relative to the bottom of the nuclear well. The energy and momentum of the emitted particle relative to the bottom of the nuclear well are $\epsilon_\Omega = \epsilon + B_{\rm em} + \epsilon_F$ and $\mathbf{k}_\Omega$, where $|\mathbf{k}_\Omega| = \sqrt{(2m\epsilon_\Omega)}$, $B_{\rm em}$ being the emission particle separation energy and ϵ_F the Fermi-energy. $R(p)$ is a correction factor to account for neutron-proton distinguishability. The forward-peaked angular variation for a given emission energy follows directly from the variation of $\rho(p-1, h, E-\epsilon_\Omega, \mathbf{K}-\mathbf{k}_\Omega)$ with angle Ω in Eq. (1). This in turn follows from the inclusion of Fermi-motion and Pauli-blocking in the state-densities, and ignores deviations from center-of-mass isotropy in nucleon-nucleon scattering. During the preequilibrium cascade our model assumes that particle-hole states can be populated providing that both energy and momentum are conserved, and the "memory" of the initial projectile direction is not maintained solely by a fast leading-particle, but rather it is carried by both the excited particles and the holes.

The state density with linear momentum can be expressed [10] as the product of a state density in energy space, $\rho(p, h, E)$, and a linear momentum distribution function $M(p, h, E, \mathbf{K})$,

$$\rho(p, h, E, \mathbf{K}) = \rho(p, h, E)\ M(p, h, E, \mathbf{K}), \tag{2}$$

in analogy to the usual partitioning of the angular-momentum state density. It has units of $\mathrm{MeV}^{-1}(\mathrm{MeV/c})^{-3}$, is independent of the direction of $\mathbf{K}$, and yields energy-dependent state density when integrated over all momenta, $\int \rho(p, h, E, \mathbf{K}) 4\pi K^2 dK = \rho(p, h, E)$. The individual momenta of the particles and holes are oriented in random directions, and the state density with linear momentum counts all configurations which sum to the required total energy and total momentum. The Central Limit Theorem implies that the ensemble of the various particle and hole momenta sum to yield a distribution of total momenta which follows a Gaussian,

$$M(p, h, E, \mathbf{K}) = \frac{1}{(2\pi)^{3/2}\sigma^3} \exp(-K^2/2\sigma^2), \tag{3}$$

where we call σ the "momentum cut-off" (representing the width of the distribution). This Gaussian solution has been shown to accurately describe the momentum distribution even when the number of excitons is small [10]. The momentum cut-off can be obtained by considering the average-squared value of the exciton momentum projections on the direction of $\mathbf{K}$ in a Fermi-gas nucleus, giving

$$\sigma^2 = n\left(\frac{2m\epsilon_{\mathrm{av}}}{3}\right), \tag{4}$$

where the number of excitons is $n = p + h$, and ϵ_{av} is the average exciton energy relative to the bottom of the nuclear well. Thus, as n increases with more excited particles and holes, the width of the total momentum distribution increases. If the excitation energy is not too high and $p \approx h$, then $\epsilon_{\mathrm{av}} \approx \epsilon_F$, but in general in an equidistant single-particle model with well-depth restrictions it is given by

$$\epsilon_{\mathrm{av}} = \frac{2p(p+1)}{ng}\frac{\rho(p+1, h, \tilde{E})}{\rho(p, h, \tilde{E})} - \frac{\tilde{E}}{n} + \epsilon_F, \tag{5}$$

where we use the notation that $\tilde{E}$ denotes the excitation energy relative to the Fermi-level, $\tilde{E} = E - (p-h)\epsilon_F$, and the state densities in Eq. (5) are taken from the finite-well-depth restricted Williams formula [14].

The expressions outlined in Eqs. (2-5) can be substituted into Eq. (1) to yield the angular distributions. Before doing so, though, we make use of a result shown in Ref. [10] that when the angle-dependent emission rate in Eq. (1) is integrated over all angles, it is approximately equal to the usual exciton model emission rate that ignore linear momentum effects. Since the cross section for preequilibrium emission is proportional to the emission rate, this enables the double-differential cross section to be written as

$$\frac{d^2\sigma_n(\epsilon,\theta)}{d\epsilon d\Omega} = \frac{d\sigma_n(\epsilon)}{d\epsilon} G(n,\theta), \tag{6}$$

where $d\sigma_n(\epsilon)/d\epsilon$ is the usual angle-integrated exciton model cross section for emission leaving n excitons, and the angular distribution kernel $G(n,\theta)$ is normalized so that $\int G(n,\theta)d\Omega = 1$. The total preequilibrium emission is a sum of the above contributions for all preequilibrium stages.

When preequilibrium emission of a particle with momentum $\mathbf{k}_\Omega$ occurs, the squared absolute value of the residual nucleus momentum is $|\mathbf{K} - \mathbf{k}_\Omega|^2 = K^2 + k_\Omega^2 - 2Kk_\Omega \cos\theta$, where θ is the angle of emission in relation to the projectile direction. When this is substituted into the numerator of Eq. (1), and Eqs. (2-6) are used we obtain the angular distribution kernel,

$$G(n,\theta) = \frac{1}{4\pi}\frac{2a}{e^a - e^{-a}} \exp(a\cos\theta). \tag{7}$$

This is exactly the same expression that Kalbach used to describe the "multistep direct" preequilibrium angular distribution. The variable "a" that she parameterized by comparing with measurements (which governs the degree of forward-peaking) is predicted theoretically in our model to be stage-dependent,

$$a = \frac{3Kk_\Omega}{2nm\epsilon_{\text{av}}}, \tag{8}$$

and we reiterate that the momenta in the above equation are measured from the bottom of the nuclear well. The angular variation as the exponential of the cosine of the scattering angle results from the Gaussian accessible phase space, and the vector addition of momenta using the cosine formula. We list some features of our angular distribution Eq. (7): while the Kalbach-systematic formula is of the same functional form as our result, her expression applies to the full preequilibrium spectrum whereas ours applies to each preequilibrium stage component. But since the 1-step scattering is often dominant, one would expect Eq. (7) to be a good approximation for the full spectrum. Also, even when higher steps are significant, the summation of contributions from Eq. (6) with various values for a still yields a form which can be well approximated by a shape given by Eq. (7); the conservation of linear momentum, and hence angle-energy correlation, is maintained for all orders of scattering; the forward-peaking increases with incident and emission energy, and decrease with increasing n as the incident momentum is shared among more particles and holes.

Our model also provides a framework for beginning to understand other previously-unexplained features of the systematic behaviour of angular distributions: why the approximate independence of Kalbach's a parameter on incident energy below 130 MeV? This would arise by the approximate canceling of the incident energy dependence in our expression for a with the increasing number of preequilibrium stages (each with successively flatter angular distributions) that contribute; why the approximate independence on projectile mass for energies below 130 MeV? In our approach, a increases as the mass increase, but this is (partly, at least) compensated by the increased number of excitons n in the preequilibrium cascade for cluster-induced reactions. A complete understanding of the angular distributions of preequilibrium clusters reactions would require a detailed analysis of the mechanisms involved [15]. Recent analyses by Zhang *et al.* [16] have investigated a preequilibrium alpha emission model, paying close attention to momentum effects. Use of the present approach may yield a straightforward description of angular distributions in these reactions.

There are similarities between our model and exciton models which use the KK angular kernel. If instead of using our Gaussian (statistical) solution, the state densities with linear momentum are determined in a Fermi-gas by convoluting single-particle densities while conserving energy and momentum, the KK result follows for 1-step scattering [10]. But our result for multistep scattering differs from a convolution of KK kernels since we do not make a leading-particle assumption. We showed in Ref. [10] that the Gaussian solution approximates the exact Fermi-gas result very well even when the number of excitons is small. We are further encouraged to use the Gaussian solution since Reffo and Herman [17] found that a Gaussian angular momentum distribution described shell-model with BCS pairing calculations well, even when there are just two excitons.

We compare angular distributions predicted by our linear-momentum conserving exciton model with a sample of experimental measurements for nucleon reactions. Even though our model includes the quantum phenomena of Fermi-motion and Pauli-blocking, it does not account for other quantum effects such as refraction and diffraction from the nuclear potential, and finite-size effects. At low incident energies these have been shown to be important for obtaining sufficient backward-angle emission [2, 4, 5], and result in a flatter angular distribution. A simple applications-oriented way to account for these effects is to modify a in Eq. (7) so that it is decreased by an energy-dependent parameter ζ. Writing a in terms of channel energies we then obtain

$$a = \frac{3\sqrt{(\epsilon_{\rm in} + B_{\rm in} + \epsilon_F)(\epsilon + B_{\rm em} + \epsilon_F)}}{\zeta \; n\epsilon_{\rm av}}, \tag{9}$$

and we take the Fermi-energy as 35 MeV. By analyzing a few experimental data sets, and by making parallels with the systematics that Kalbach obtained for a, we have found that the simple parameterization $\zeta = 9.3/\sqrt{\epsilon}$ where the emission energy ϵ is in MeV, works fairly well up to 85 MeV. This factor tends to 1 for the higher emission energies where the quantum effects become small, and increases to 2 at 20 MeV. We calculate exciton model cross sections using the GNASH [18] nuclear model code and analyze three different preequilibrium reactions which span a range of energies and projectile and ejectile types: the 80-MeV induced ^{90}Zr(p,p') reaction measured by Cowley *et al.* [19]; the 45-MeV induced ^{90}Zr(p,n) reaction measured by Galonsky *et al.* [20]; and the 26-MeV induced ^{93}Nb(n,n') reaction measured by Marcinkowski *et al.* [21]. Our results are compared with these data in Figs. 1-3, and show that our model is able to account for the angular distributions well.

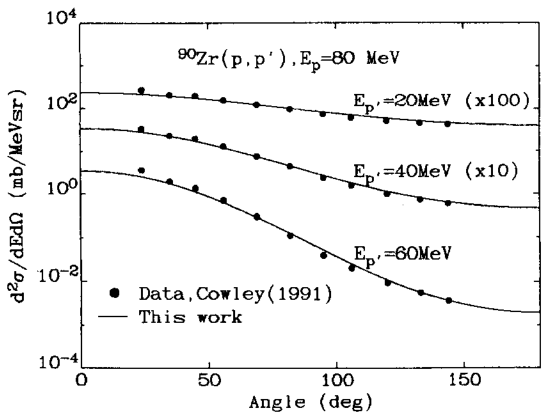

Fig. 1. Calculated angular distributions in the 80 MeV ^{90}Zr(p,p') reaction compared with experimental data [19].

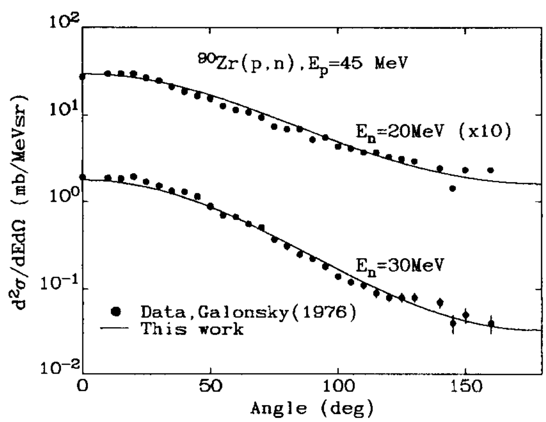

Fig. 2. Calculated angular distributions in the 45 MeV ^{90}Zr(p,n) reaction compared with experimental data [20].

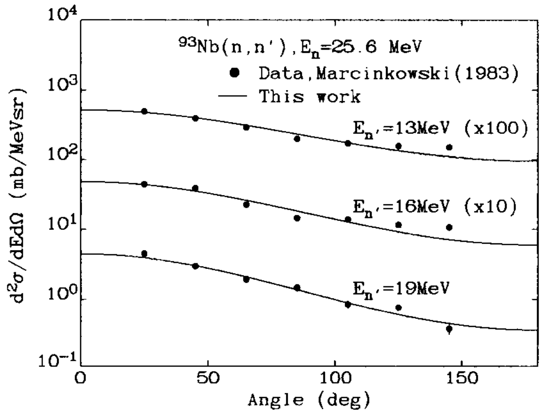

Fig. 3. Calculated angular distributions in the 26 MeV ^{93}Nb(n, n') reaction compared with experimental data [21].

In summary, our linear-momentum conserving exciton model can account for many features of preequilibrium angular distributions, as embodied in the Kalbach systematics. Use of state densities with linear momentum enables these distributions to be obtained easily, once the reaction mechanism has been established, using simple (and exact) expressions. The model is straightforward to apply computationally since the usual exciton model can be used for the angle-integrated cross section, and describes measurements well when modifications which approximate quantum effects are included.

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REFERENCES

[1] Sun Ziyang, Wang Shunuan, Zhang Jingshang, Zhou Yizhong, Z. Phys. A**305**, 61 (1982).

[2] C. Costa, H. Gruppelaar, and J.M. Akkermans, Phys. Rev. C**28**, 587 (1983).

[3] A. Iwamoto and K. Harada, Nucl. Phys. **A419**, 472 (1984).

[4] M. Blann, W. Scobel, and E. Plechaty, Phys. Rev. C **30**, 1493 (1984).

[5] K. Sato, Phys. Rev. C**32**, 647 (1985).

[6] H. Machner, Z. Phys. A**327**, 175 (1987).

[7] E. Gadioli, in *Proceedings of the International Conference on Nuclear Reaction Mechanisms*, Saha Institute of Nuclear Physics, Calcutta, 1989, edited by S. Mukherjee (World Scientific, Singapore, 1989), p.145.

[8] P. Madler and R. Reif, Nucl. Phys. **A 337**, 445 (1980).

[9] M.B. Chadwick and P. Oblozinsky, Phys. Rev. C **44**, R1740 (1991)

[10] M.B. Chadwick and P. Oblozinsky, Phys. Rev. C **46**, 2028 (1992)

[11] K. Kikuchi and M. Kawai, *Nuclear Matter and Nuclear Reactions* (North-Holland, Amsterdam, 1968), p. 44; M.L. Goldberger, Phys. Rev. **74**, 1269 (1948).

[12] C. Kalbach, Phys. Rev. C **37**, 2350 (1988).

[13] C. Kalbach and F.M. Mann, Phys. Rev. C **23**, 112 (1981).

[14] F.C. Williams, Nucl. Phys. **A166**, 231 (1971); E. Betak and J. Dobes, Z. Phys. **A 279**, 319 (1976).

[15] E. Gadioli, E. Gadioli-Erba, J.J. Hogan, and B.V. Jacak, Phys. Rev. C**29**, 76 (1984).

[16] Zhang Jingshang, Nucl. Sci. Eng. **116**, 35 (1994).

[17] G. Reffo and M. Herman, Nuovo Cimento Lett. **34**, 261 (1982).

[18] P.G. Young, E.D. Arthur, and M.B. Chadwick, Los Alamos National Laboratory Report No. LA-12343-MS, 1982.

[19] A.A. Cowley, A. van Kent, J.J. Lawrie, S.V. Fortsch, D.M. Whittal, J.V. Pilcher, F.D. Smit, W.A. Richter, R. Lindsay, I.J. van Heerden, R. Bonetti, and P.E. Hodgson, Phys. Rev. C**43**, 678 (1991).

[20] A. Galonsky, R.R. Doering, D.M. Patterson, and H.W. Bertini, Phys. Rev. C**14**, 748 (1976).

[21] A. Marcinkowski, R.W. Finlay, G. Randers-Pehrson, C.E. Brient, R. Kurup, S. Mellema, A. Meigooni, and R. Taylor, Nucl. Sci. Eng. **83**, 13 (1983).