Calculation of Nucleon Yields in the Nuclear Reaction of Intermediate Energy Proton with Zr,Pb and U-238

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Abstract

As part of an investigation of the bench-mark problems of neutron yield and angular-dependent neutron spectra which were set up at the PSI workshop for the specialist meeting for accelerator-based transmutation, we have studied the nuclear reaction of intermediate protons with the nuclei of Zr, Pb, and U-238, calculated by the LAHET code system. The effects of the adopting several options into the code were investigated, such as the intranuclear cascade (INC) model of Bertini's and ISABEL's models, the level density formulae, and pre-equilibrium and non-pre-equilibrium models.

1. Introduction

In this paper, we summarize our findings on the nuclear reaction of intermediate protons with the nuclei of Zr, Pb, and U-238 calculated by the LAHET code system [1].

The bench-mark problems of neutron yield and angular-dependent neutron spectra were set up at the PSI workshop at the specialist meeting for accelerator-based transmutation[2]. Since we are using the LAHET code system to plan our conceptual design for an accelerator-driven transmutor and reactor, we calculated these quantities using the several options implemented in this code.

The INC calculation in the LAHET code was set up by adopting the High Energy Transport Code (HETC)[3] which supersedes the Nucleon Meson Transport Code (NMTC)[4]. Both codes were developed at Oak Ridge National Laboratory for the spallation process, and the transport of nucleons produced by the spallation reaction are followed until their energies become less than 20 MeV. The LAHET code distributed in the United States carried the option of calculating the spallation reaction with the ISABEL code [5] which was developed at Brookhaven National Laboratory.

The original spallation codes of HMTC and NMTC have no capability for calculating the high-energy fission reaction [6]. Several high-energy fission models were developed around 1980 by Rutherford Appleton Laboratory (RAL)[7], ORNL[8], BNL[9], Japan Atomic Energy Research Institute(JAERI)[10], and Dubna[11]. They are used for analyzing neutron yield experiments which were carried out in a collaboration between Chalk River Laboratory and ORNL[12] using the cosmotron at BNL, and in the Vasilikov experiment at Dubna[13], and at Los Alamos National Laboratory[14]. Two fission models are adopted in the LAHET code; one was developed at Rutherford Appleton Laboratory by Atchison[7], and the other was developed at ORNL by Alsmiller[8.2]. The former derived an empirical formula and can calculate any nuclei; the ORNL model is based on experimental data for U-238, and target nuclei are limited to those with a mass number above U-238. Thus, fission of Pb due to intermediate energy protons cannot be calculated.

The level density formula used for the evaporation process is very important, and the three options of original HETC level-density formula, Gilbert-Cameron[15.1] Cook Ignatyuk[15.2] (GCCI) formula, and Julich level density parameterization as function of mass number[16] are incorporated into the LAHET code. However, although the level density formula for the original HETC level density is in the program, when we used this option, our computer failed to calculate the reaction, so we compared only the two options of the GCCI and Julich models.

The original HETC and NMTC codes calculate the nuclear cascade of the spallation reaction as semi-classical model, and, after emitting the nucleon and pions, the nucleus is assumed to be in a state of equilibrium. The further emission of nucleons and light nuclei, such as deuterons, triton, He-3, and Alpha particles is treated as an evaporation process. To improve the emission of nucleons during the cascade downward, the pre-equilibrium model[17] with an anisotropic process[18] was installed as an option in the LAHET code.

To study these various options used the LAHET code, we calculated the yield and energy spectra of the neutrons and protons, as well as the angular dependence of the spectra. The results are discussed in the following sections.

2. Comparison between the Bertini model and ISABEL model

Figure 1.a-f shows the cross sections of neutron production for proton injection energies of 80, 800, and 1600 MeV, calculated using the pre-equilibrium model and the GCCI level-density formula.

For the U-238 target, the cross sections calculated with Bertini's model are almost same as those obtained with the ISABEL model, but for the Zr target, and the neutron spectrum calculated by the ISABEL model is significantly smaller than that calculated by the Bertini model. However, for a 1600 MeV proton injection, the reverse is the case, the neutron spectrum calculated by the ISABEL model being the larger than the one estimated by Bertini's model.

3. The effect of the level-density formula on the cross sections of nucleon production.

Figures 2.a-f show the effect of the level-density formula for the cross section of neutron production for a p-Zr collision.
Fig.2.a shows that the Julich level-density formula gives about half the value of the one calculated by the GCCI level-density formula between 1 to 20 MeV for a 80 MeV proton injection. This difference becomes smaller as the proton energy increases to 800 and 1600 MeV. For the cross sections of proton production, the effects of level-density are large compared to the neutron production over the entire range of incident proton energy.

Figures 3.a-f show the effects of level-density effects on the differential neutron cross-sections for a p(80 MeV)-Zr collision. In the backward part of the cross section above 120 degree, the Julich formula gives a higher cross section above 10 MeV than the GCCI formula, but in the forward cross section, the Julich level density gives a smaller cross-section as the total cross section integrated over all angles (figure 2).

4. Comparison between the pre-equilibrium and equilibrium models for the production of nucleons

Figure 4-1 compares the effects of the pre-equilibrium and equilibrium models on neutron production in p-Zr collisions. For a 80 MeV incident proton energy, the pre-equilibrium model gives larger cross section above 9 MeV than the equilibrium model. At less than 9 MeV, the reverse is the case. Increasing the incident proton energies to 800 MeV and 1600 MeV, then, the cross-sections of the pre- and non-pre-equilibrium models are the same at 20 Me instead of at 9 MeV for an 80 MeV incident proton; the magnitude of the difference diminishes as the incident proton energy increases. For proton production, the energy at which the pre-equilibrium and equilibrium calculations become the same is 11 MeV, compared with 9 MeV for neutron production. However, the dynamics of the proton production show the same trends as those for neutron production.

Figures 4-2 and 4-3 show the cases for p-Pb and P-U-238 collisions. The pattern is almost same as the case for Zr targets.

Figure 5-3 shows the differential cross-section of neutron production for a p(80 MeV)-U-238 collision. Using the pre-equilibrium model, neutron production in the backward direction is a few ten times larger than the equilibrium model for neutrons of above 20 MeV energy neutrons. Toward the forward direction, this large increase of neutrons calculated by the pre-equilibrium diminishes.

For a p(80 MeV)-Zr collision, a similar trend was apparent (figure 5-1).

5. Comparisons among different nuclei

Figures 6.a-f show the comparison between the cross sections of neutrons and protons production for Zr, Pb, and U-238 calculated by the Bertini-GCCI model using the pre-equilibrium model. The dependence of the cross sections on the mass of the target nuclei mass for producing high-energy neutrons by the spallation reaction is not large as the ones for low-energy neutron where the evaporation process is dominant.

Proton production by injecting a 80 MeV energy proton to the Zr nucleus shows a larger peak at 4 MeV than for Pb and U targets due to a smaller Coulomb screening. As proton energy increases, this large difference diminishes.

6. Energy Dependence

Figures 7.a-c show the cross sections of neutron production for protons of different energy injected into Zr, Pb, and U-238 targets.

7. Neutron and Proton Yields and Number of Fission Events

Table I shows the neutron and proton yields and the number of fission events calculated using the GCCI level-density formula. p-Zr collisions do not cause any fissions. The fission events for a p-Pb collision, calculated by the pre-equilibrium model, are larger than those of the non pre-equilibrium model, but for a p-U-238 collision, the same trends are obtained for low-energy proton injection, while the opposite trends are apparent for proton energies of 800 MeV and 1600 MeV.

For neutron and proton yields, calculations with the ISABEL model give larger values than the Bertini model for 80 and 800 MeV proton energy injections for p-Zr and P-U-238 collisions except for proton production in p-U-238 collisions with a 80 MeV proton injection. In the case of an 1600 MeV energy proton injection, contrary to the injection of low-energy protons, the ISABEL model gives a large value than Bertini's model.

Neutron and proton yields calculated by the pre-equilibrium model are smaller than the values for the non equilibrium model, except in the case of p-Pb and p-U-238 collision at 80 MeV proton energy. Here, the proton yields calculated by the pre-equilibrium model are smaller than those calculated by the non-preequilibrium model.

8. Acknowledgement

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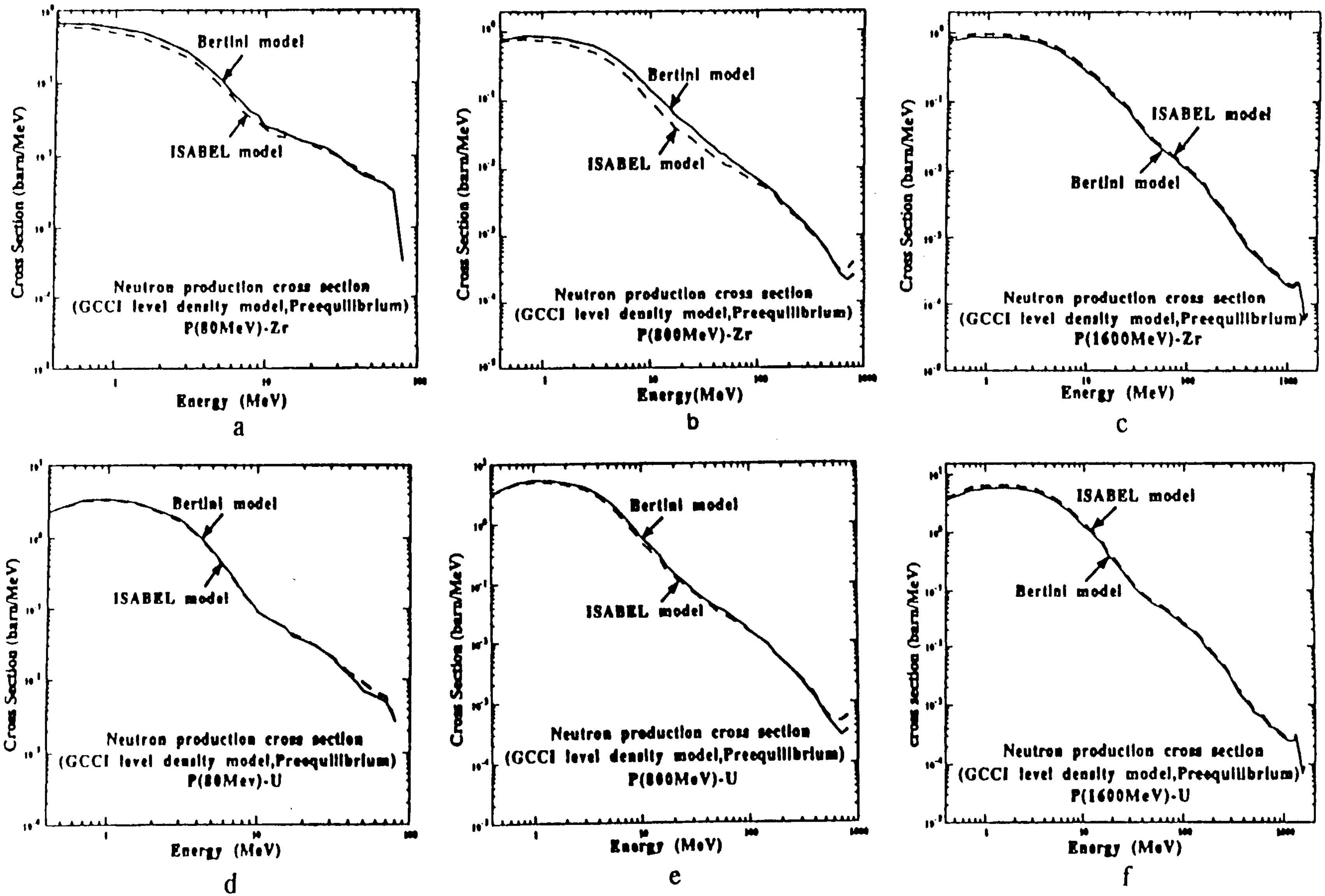
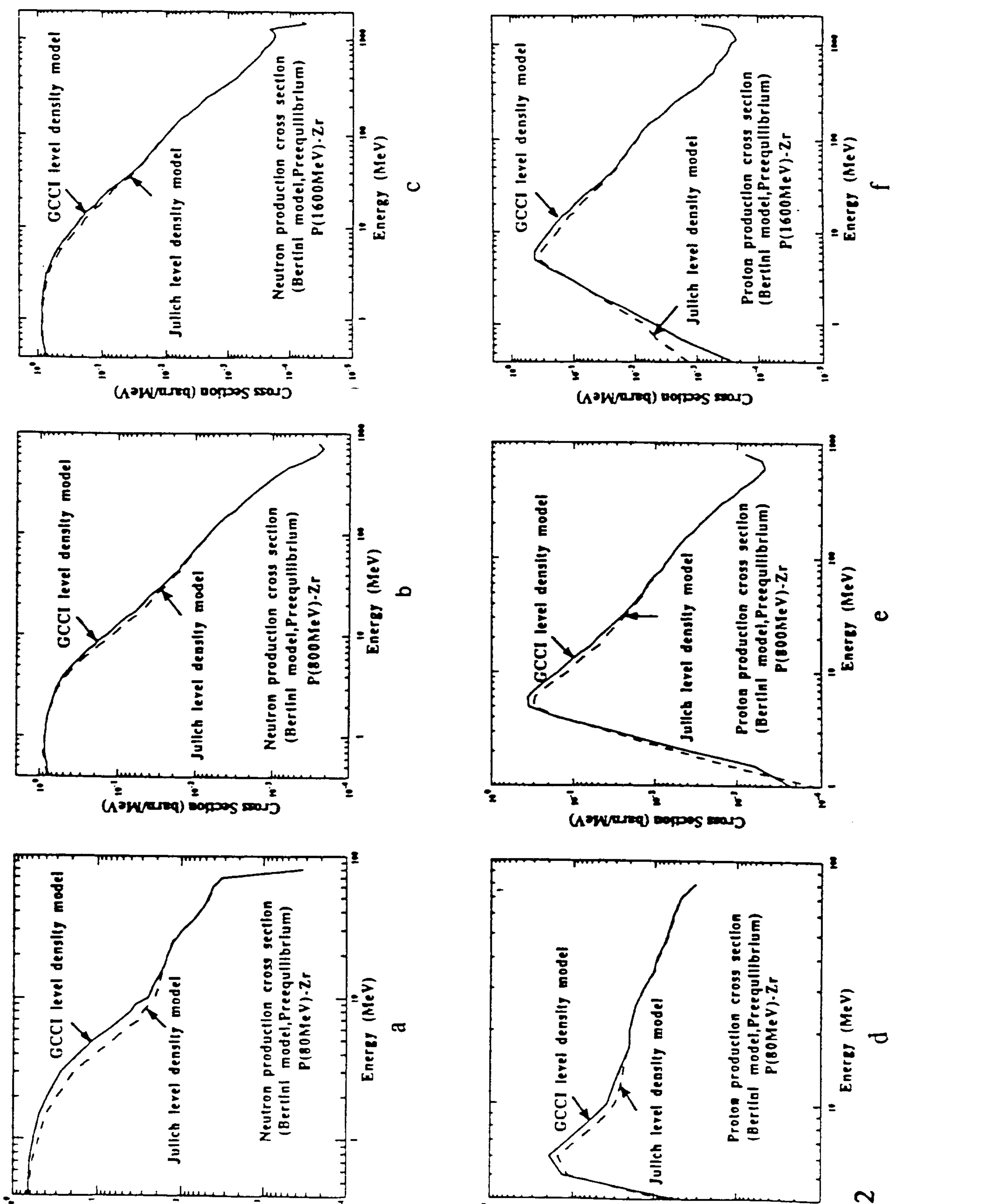


Bertini model
ISABEL model
Neutron production cross section
(GCCI level density model,Preequilibrium)
P(80MeV)-Zr
Energy (MeV)
Cross Section (barn/MeV)
a
Bertini model
ISABEL model
Neutron production cross section
(GCCI level density model,Preequilibrium)
P(800MeV)-Zr
Energy(MeV)
Cross Section (barn/MeV)
b
ISABEL model
Bertini model
Neutron production cross section
(GCCI level density model,Preequilibrium)
P(1600MeV)-Zr
Energy (MeV)
Cross Section (barn/MeV)
c
Bertini model
ISABEL model
Neutron production cross section
(GCCI level density model,Preequilibrium)
P(80Mev)-U
Energy (MeV)
Cross Section (barn/MeV)
d
Bertini model
ISABEL model
Neutron production cross section
(GCCI level density model,Preequilibrium)
P(800MeV)-U
Energy (MeV)
Cross Section (barn/MeV)
e
ISABEL model
Bertini model
Neutron production cross section
(GCCI level density model,Preequilibrium)
P(1600MeV)-U
Energy (MeV)
cross section (barn/MeV)
f

Fig. 1


GCCI level density model
Julich level density model
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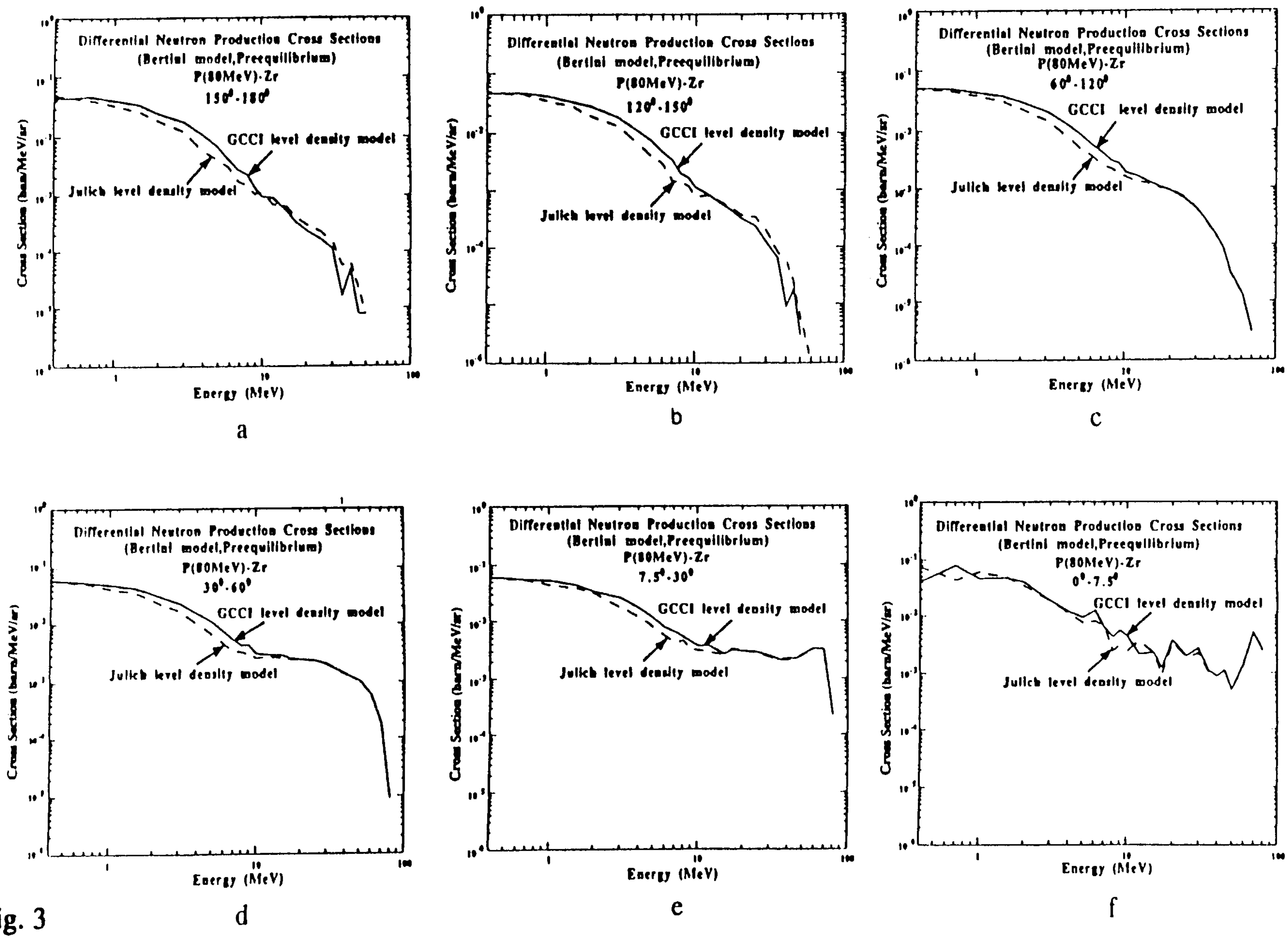
P(80MeV)-Zr
Energy (MeV)
Cross Section (barn/MeV)
a
GCCI level density model
Julich level density model
Neutron production cross section
(Bertini model,Preequilibrium)
P(800MeV)-Zr
Energy (MeV)
Cross Section (barn/MeV)
b
GCCI level density model
Julich level density model
Neutron production cross section
(Bertini model,Preequilibrium)
P(1600MeV)-Zr
Energy (MeV)
Cross Section (barn/MeV)
c
GCCI level density model
Julich level density model
Proton production cross section
(Bertini model,Preequilibrium)
P(80MeV)-Zr
Energy (MeV)
Cross Section (barn/MeV)
d
GCCI level density model
Julich level density model
Proton production cross section
(Bertini model,Preequilibrium)
P(800MeV)-Zr
Energy (MeV)
Cross Section (barn/MeV)
e
GCCI level density model
Julich level density model
Proton production cross section
(Bertini model,Preequilibrium)
P(1600MeV)-Zr
Energy (MeV)
Cross Section (barn/MeV)
f

Fig. 2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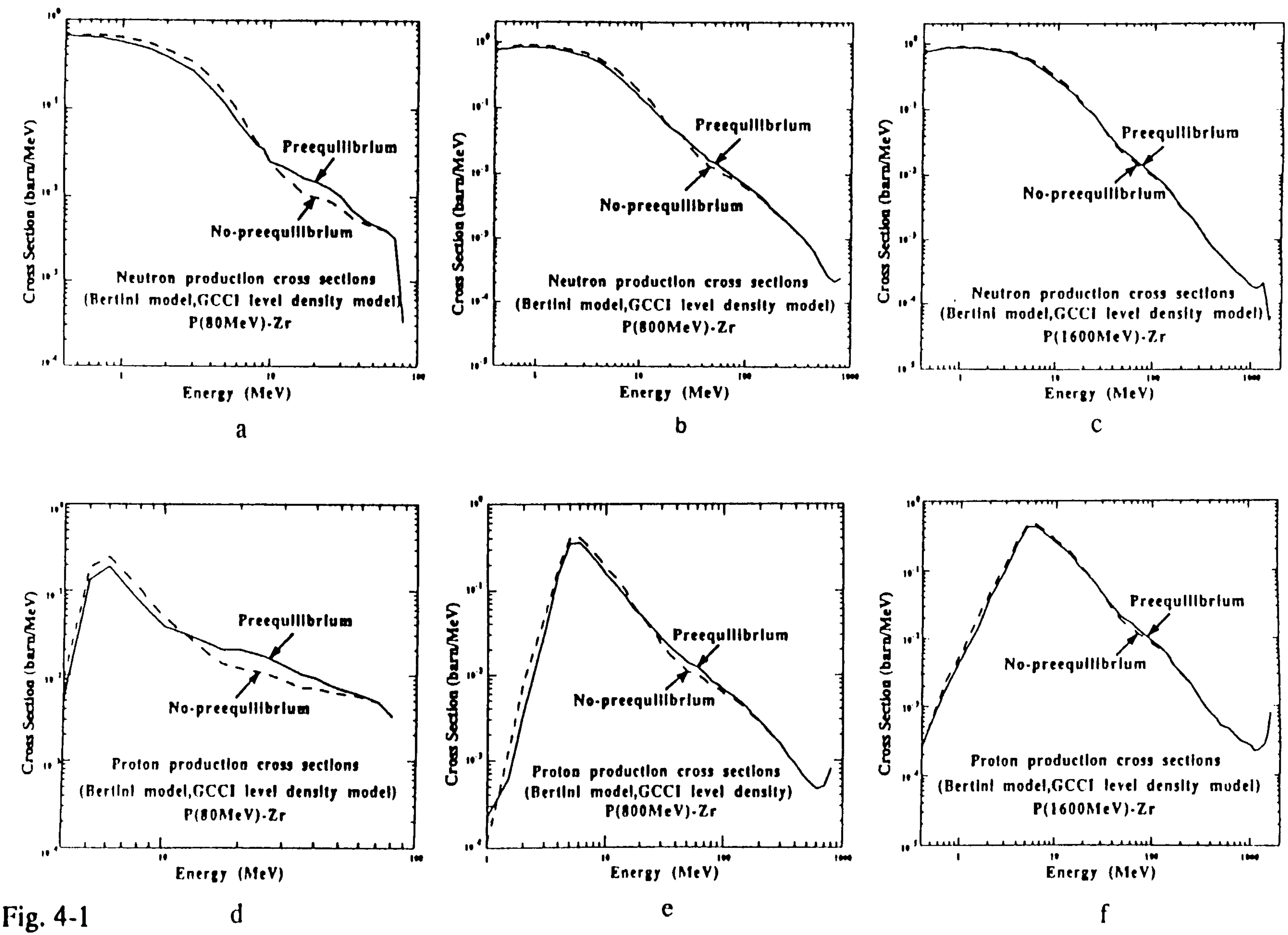
P(80MeV)-Zr
150°-180°
GCCI level density model
Julich level density model
Cross Section (barn/MeV/sr)
Energy (MeV)
a
Differential Neutron Production Cross Sections
(Bertini model,Preequilibrium)
P(80MeV)-Zr
120°-150°
GCCI level density model
Julich level density model
Cross Section (barn/MeV/sr)
Energy (MeV)
b
Differential Neutron Production Cross Sections
(Bertini model,Preequilibrium)
P(80MeV)-Zr
60°-120°
GCCI level density model
Julich level density model
Cross Section (barn/MeV/sr)
Energy (MeV)
c
Differential Neutron Production Cross Sections
(Bertini model,Preequilibrium)
P(80MeV)-Zr
30°-60°
GCCI level density model
Julich level density model
Cross Section (barn/MeV/sr)
Energy (MeV)
d
Differential Neutron Production Cross Sections
(Bertini model,Preequilibrium)
P(80MeV)-Zr
7.5°-30°
GCCI level density model
Julich level density model
Cross Section (barn/MeV/sr)
Energy (MeV)
e
Differential Neutron Production Cross Sections
(Bertini model,Preequilibrium)
P(80MeV)-Zr
0°-7.5°
GCCI level density model
Julich level density model
Cross Section (barn/MeV/sr)
Energy (MeV)
f

Fig. 3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Energy (MeV)
Preequilibrium
No-preequilibrium
Neutron production cross sections
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P(80MeV)-Zr
a
Neutron production cross sections
(Bertini model,GCCI level density model)
P(800MeV)-Zr
b
Neutron production cross sections
(Bertini model,GCCI level density model)
P(1600MeV)-Zr
c
Proton production cross sections
(Bertini model,GCCI level density model)
P(80MeV)-Zr
d
Proton production cross sections
(Bertini model,GCCI level density)
P(800MeV)-Zr
e
Proton production cross sections
(Bertini model,GCCI level density model)
P(1600MeV)-Zr
f

Fig. 4-1

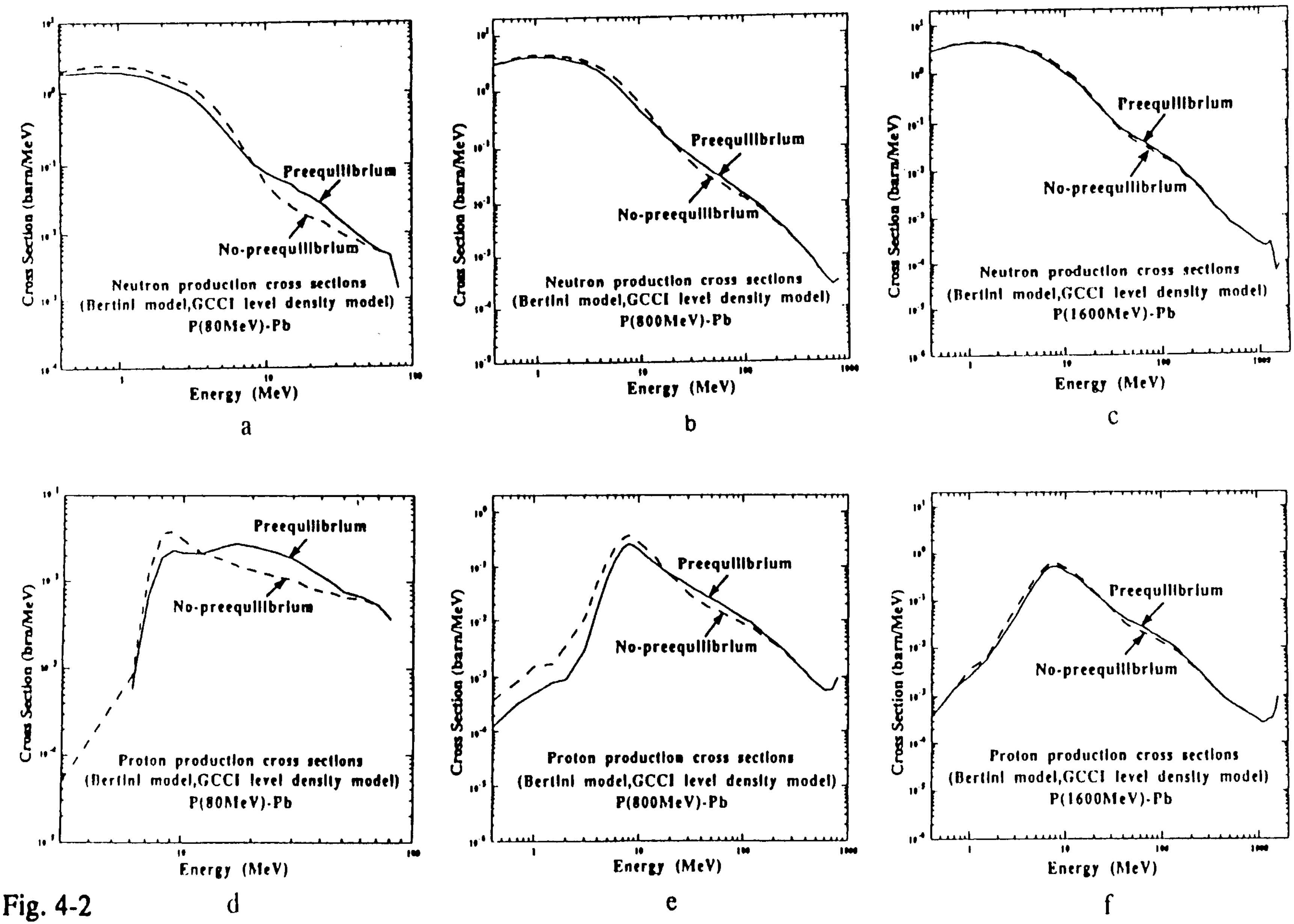
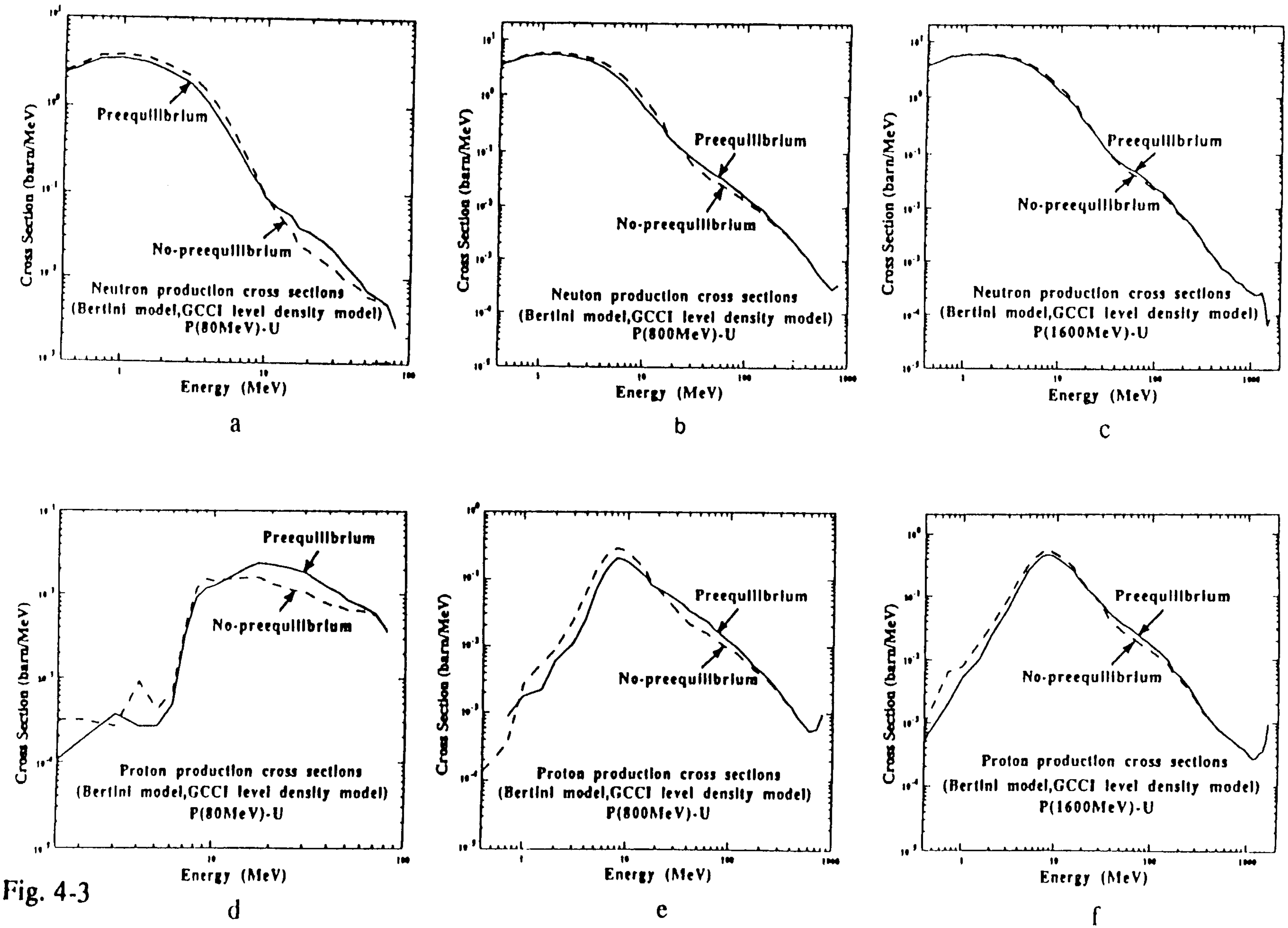

Cross Section (barn/MeV)
Energy (MeV)
Preequilibrium
No-preequilibrium
Neutron production cross sections
(Bertini model,GCCI level density model)
P(80MeV)-Pb
a
Cross Section (barn/MeV)
Energy (MeV)
Preequilibrium
No-preequilibrium
Neutron production cross sections
(Bertini model,GCCI level density model)
P(800MeV)-Pb
b
Cross Section (barn/MeV)
Energy (MeV)
Preequilibrium
No-preequilibrium
Neutron production cross sections
(Bertini model,GCCI level density model)
P(1600MeV)-Pb
c
Cross Section (brn/MeV)
Energy (MeV)
Preequilibrium
No-preequilibrium
Proton production cross sections
(Bertini model,GCCI level density model)
P(80MeV)-Pb
d
Cross Section (barn/MeV)
Energy (MeV)
Preequilibrium
No-preequilibrium
Proton production cross sections
(Bertini model,GCCI level density model)
P(800MeV)-Pb
e
Cross Section (barn/MeV)
Energy (MeV)
Preequilibrium
No-preequilibrium
Proton production cross sections
(Bertini model,GCCI level density model)
P(1600MeV)-Pb
f

Fig. 4-2


Preequilibrium
No-preequilibrium
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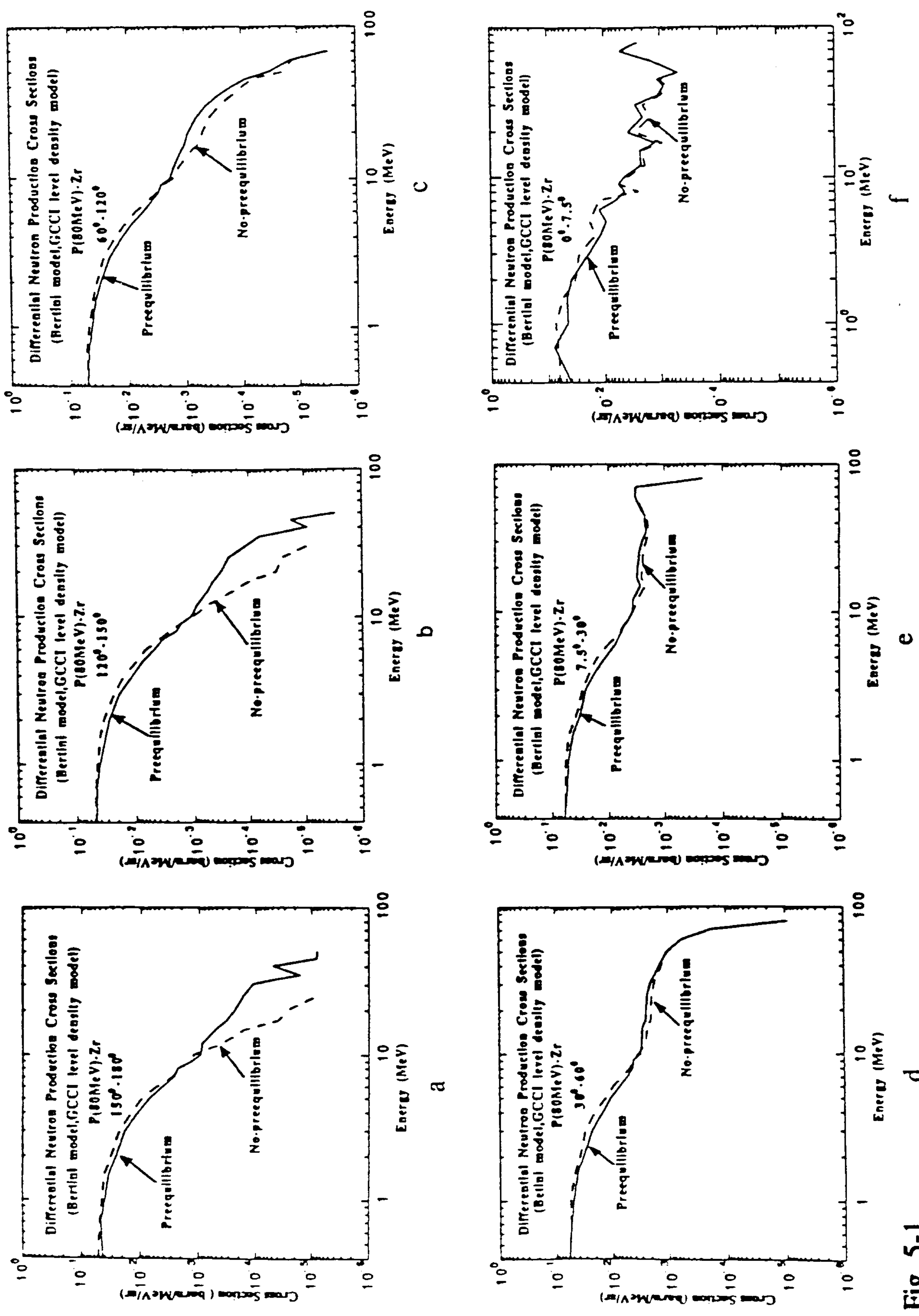
Cross Section (barn/MeV)
Energy (MeV)
a
Preequilibrium
No-preequilibrium
Neuton production cross sections
(Bertini model,GCCI level density model)
P(800MeV)-U
Cross Section (barn/MeV)
Energy (MeV)
b
Preequilibrium
No-preequilibrium
Neutron production cross sections
(Bertini model,GCCI level density model)
P(1600MeV)-U
Cross Section (barn/MeV)
Energy (MeV)
c
Preequilibrium
No-preequilibrium
Proton production cross sections
(Bertini model,GCCI level density model)
P(80MeV)-U
Cross Section (barn/MeV)
Energy (MeV)
d
Preequilibrium
No-preequilibrium
Proton production cross sections
(Bertini model,GCCI level density model)
P(800MeV)-U
Cross Section (barn/MeV)
Energy (MeV)
e
Preequilibrium
No-preequilibrium
Proton production cross sections
(Bertini model,GCCI level density model)
P(1600MeV)-U
Cross Section (barn/MeV)
Energy (MeV)
f

Fig. 4-3



Differential Neutron Production Cross Sections
(Bertini model,GCCI level density model)
P(80MeV)-Zr
150°-180°
Preequilibrium
No-preequilibrium
Cross Section (barn/MeV/sr)
Energy (MeV)
a
Differential Neutron Production Cross Sections
(Bertini model,GCCI level density model)
P(80MeV)-Zr
120°-150°
Preequilibrium
No-preequilibrium
Cross Section (barn/MeV/sr)
Energy (MeV)
b
Differential Neutron Production Cross Sections
(Bertini model,GCCI level density model)
P(80MeV)-Zr
60°-120°
Preequilibrium
No-preequilibrium
Cross Section (barn/MeV/sr)
Energy (MeV)
c
Differential Neutron Production Cross Sections
(Beiini model,GCCI level density model)
P(80MeV)-Zr
30°-60°
Preequilibrium
No-preequilibrium
Cross Section (barn/MeV/sr)
Energy (MeV)
d
Differential Neutron Production Cross Sections
(Bertini model,GCCI level density model)
P(80MeV)-Zr
7.5°-30°
Preequilibrium
No-preequilibrium
Cross Section (barn/MeV/sr)
Energy (MeV)
e
Differential Neutron Production Cross Sections
(Bertini model,GCCI level density model)
P(80MeV)-Zr
0°-7.5°
Preequilibrium
No-preequilibrium
Cross Section (barn/MeV/sr)
Energy (MeV)
f

Fig. 5-1

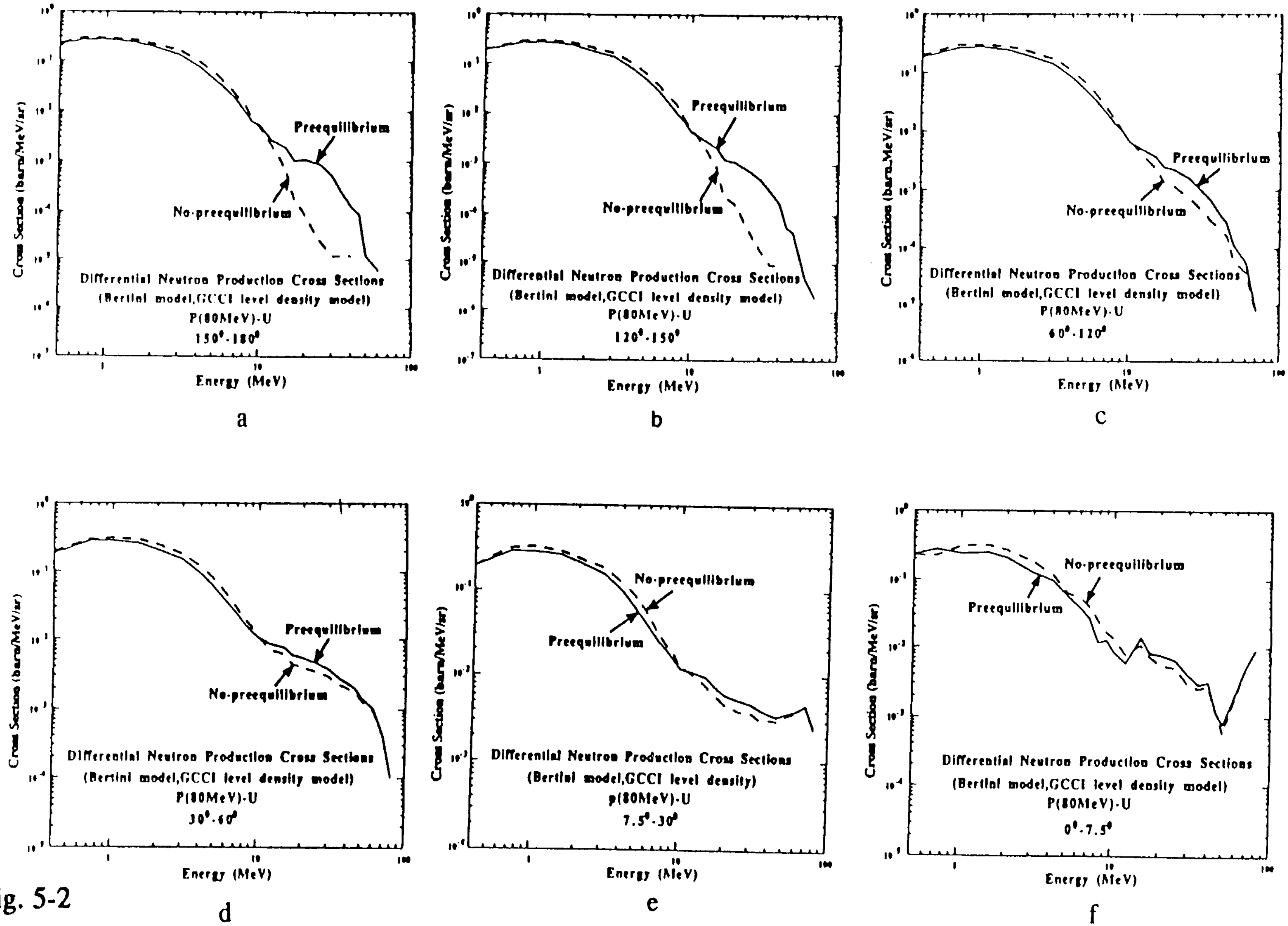
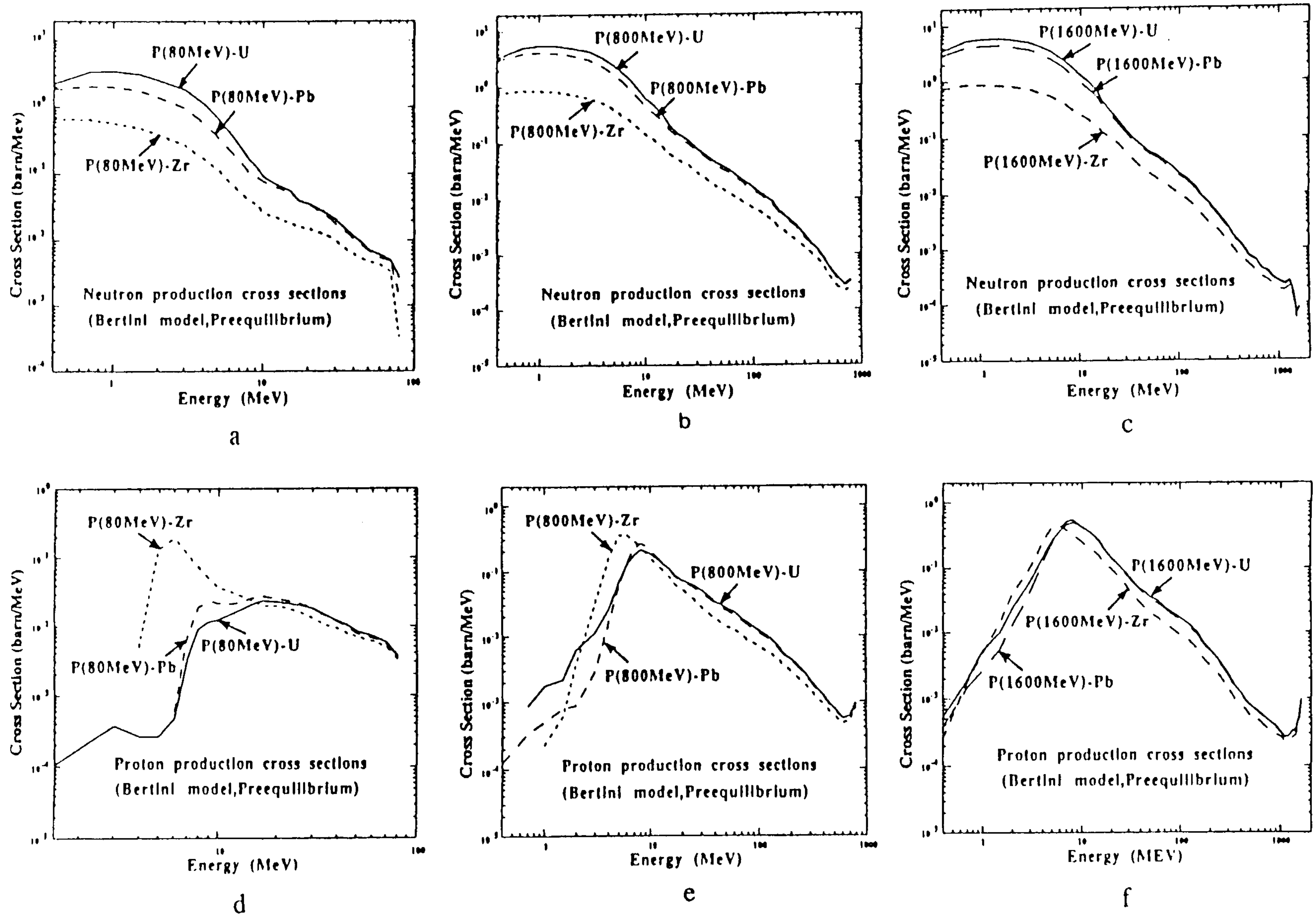

Differential Neutron Production Cross Sections
(Bertini model,GCCI level density model)
P(80MeV)-U
150°-180°
Preequilibrium
No-preequilibrium
Cross Section (barn/MeV/sr)
Energy (MeV)
a
Differential Neutron Production Cross Sections
(Bertini model,GCCI level density model)
P(80MeV)-U
120°-150°
Preequilibrium
No-preequilibrium
Cross Section (barn/MeV/sr)
Energy (MeV)
b
Differential Neutron Production Cross Sections
(Bertini model,GCCI level density model)
P(80MeV)-U
60°-120°
Preequilibrium
No-preequilibrium
Cross Section (barn/MeV/sr)
Energy (MeV)
c
Differential Neutron Production Cross Sections
(Bertini model,GCCI level density model)
P(80MeV)-U
30°-60°
Preequilibrium
No-preequilibrium
Cross Section (barn/MeV/sr)
Energy (MeV)
d
Differential Neutron Production Cross Sections
(Bertini model,GCCI level density)
p(80MeV)-U
7.5°-30°
No-preequilibrium
Preequilibrium
Cross Section (barn/MeV/sr)
Energy (MeV)
e
Differential Neutron Production Cross Sections
(Bertini model,GCCI level density model)
P(80MeV)-U
0°-7.5°
No-preequilibrium
Preequilibrium
Cross Section (barn/MeV/sr)
Energy (MeV)
f

Fig. 5-2


P(80MeV)-U
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Neutron production cross sections
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Cross Section (barn/Mev)
Energy (MeV)
a
P(800MeV)-U
P(800MeV)-Pb
P(800MeV)-Zr
Neutron production cross sections
(Bertini model,Preequilibrium)
Cross Section (barn/MeV)
Energy (MeV)
b
P(1600MeV)-U
P(1600MeV)-Pb
P(1600MeV)-Zr
Neutron production cross sections
(Bertini model,Preequilibrium)
Cross Section (barn/MeV)
Energy (MeV)
c
P(80MeV)-Zr
P(80MeV)-Pb
P(80MeV)-U
Proton production cross sections
(Bertini model,Preequilibrium)
Cross Section (barn/MeV)
Energy (MeV)
d
P(800MeV)-Zr
P(800MeV)-U
P(800MeV)-Pb
Proton production cross sections
(Bertini model,Preequilibrium)
Cross Section (barn/MeV)
Energy (MeV)
e
P(1600MeV)-U
P(1600MeV)-Zr
P(1600MeV)-Pb
Proton production cross sections
(Bertini model,Preequilibrium)
Cross Section (barn/MeV)
Energy (MEV)
f

Fig. 6

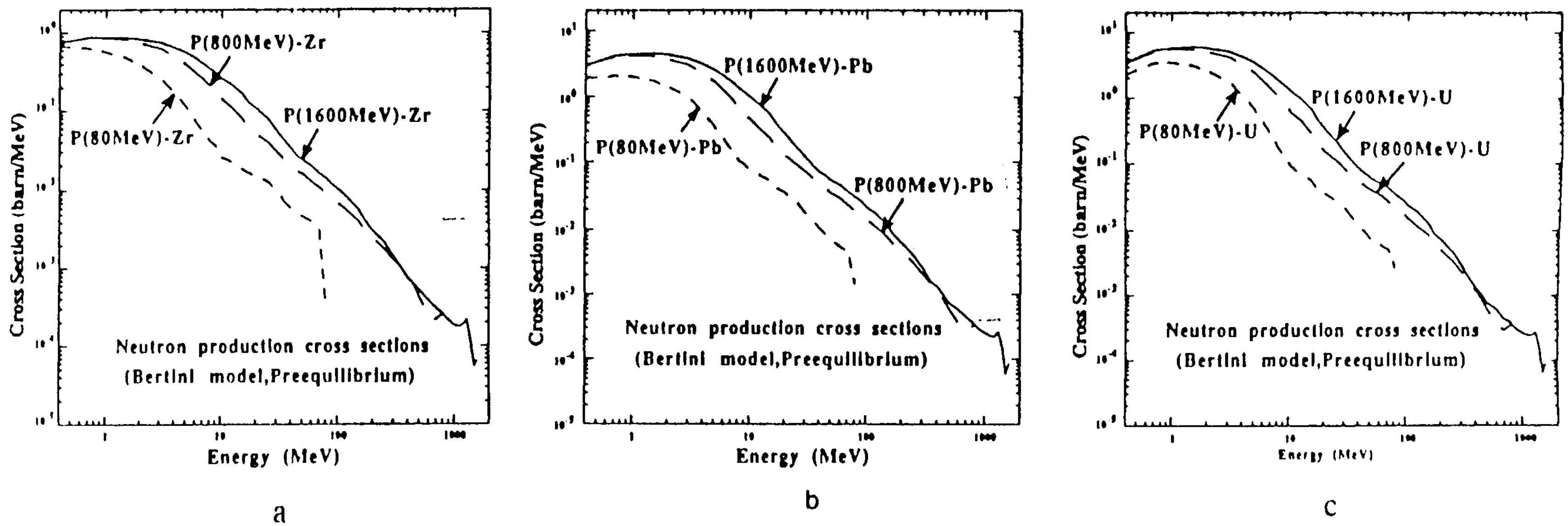
P(800MeV)-Zr
P(1600MeV)-Zr
P(80MeV)-Zr
Neutron production cross sections
(Bertini model,Preequilibrium)
Cross Section (barn/MeV)
Energy (MeV)
a
P(1600MeV)-Pb
P(80MeV)-Pb
P(800MeV)-Pb
Neutron production cross sections
(Bertini model,Preequilibrium)
Cross Section (barn/MeV)
Energy (MeV)
b
P(1600MeV)-U
P(80MeV)-U
P(800MeV)-U
Neutron production cross sections
(Bertini model,Preequilibrium)
Cross Section (barn/MeV)
c

Fig. 7

NEUTRON AND PROTON YIELDS

AND NUMBER OF FISSION EVENTS (GCCI)

			P(E)-Zr			P(E)-Pb			P(E)-U		
E (MeV)	Cascade Model	Preequilibrium	n	p	f	n	p	f	n	p	f
80	Bertini	involve	2.321	1.298	0.	7.009	0.8966	3.938×10^{-3}	11.40	0.8303	1.333
		no involve	2.537	1.369	0.	8.273	0.6935	1.506×10^{-2}	12.96	0.6069	1.413
	ISABEL	involve	2.029	1.177	0.				10.93	0.8452	1.287
		no involve	2.359	1.261	0.				13.03	0.5758	1.440
800	Bertini	involve	6.853	4.549	0.	24.12	5.233	8.669×10^{-2}	31.89	5.326	1.265
		no involve	7.333	4.777	0.	27.00	5.439	1.478×10^{-1}	35.97	5.327	1.210
	ISABEL	involve	5.415	3.631	0.				28.54	4.686	1.273
		no involve	5.549	3.670	0.				29.88	4.522	1.269
1600	Bertini	involve	9.961	6.701	0.	35.46	9.358	1.355×10^{-1}	45.80	9.657	0.9773
		no involve	10.38	6.912	0.	37.72	9.656	1.652×10^{-1}	48.89	9.879	0.9718
	ISABEL	involve	11.06	7.440	0.				51.43	10.84	1.097
		no involve	11.53	7.675	0.				54.90	11.09	1.091