PROCEEDINGS OF A SPECIALISTS' MEETING ON
PREEQUILIBRIUM NUCLEAR REACTIONS

Semmering, Austria, 10th - 12th February 1988

COMPTE RENDU D'UNE REUNION DE SPECIALISTES SUR
LES REACTIONS NUCLEAIRES DE PREEQUILIBRE

Semmering, Autriche, 10 - 12 Fevrier 1988

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INTRODUCTORY REMARKS

A. B. SMITH, Argonne National Laboratory, Chairman of the Nuclear Data Committee

This meeting is in cooperation with the OECD-NEA and its nuclear data committee, the NEANDC. For more than forty years, that Committee, in various reincarnations, has been an effective vehicle for stimulating the provision of nuclear data for energy development. Specialists' meetings have been an essential mechanism toward this end, and so it is with this meeting.

The NEA (and IAEA) is dedicated to safe and economic development of peaceful nuclear energy: primarily fission- and fusion energy systems. Thus the primary nuclear data concerns are at energies below \( \simeq 16 \text{ MeV} \), including neutronics, energy deposition and radiation-damage effects associated with fuels, structural components, multipliers and shielding materials. Effectively addressing these applied concerns frequently requires a broader scope of endeavor including attention to fundamental physical understanding, while at the same time encompassing a diversity of processes including both neutral and charged particle emission. A unified approach is very much in order.

Basic and applied physics are observational sciences. However, in the areas of interest here, observations have been fragmentary, difficult or even impossible. Thus the provision of data in the domain of this meeting must rely heavily upon calculational extrapolation from fragmentary experimental results. It is fitting and proper that the program deals extensively with calculational matters. To this point, calculations have led to widely discrepant results, particularly in the absence of observations. In my modest experience, the calculational problems are most acute in the region of primary NEA interest (i.e., \( \leq 16 \text{ MeV} \), and near reaction thresholds). One can hope that this meeting will resolve the more gross of these discrepancies, and offer some guidance to those who must use calculational tools to meet nuclear data needs and yet are now faced with a dicotomy of widely different calculational results.

I am not a theorist, but it is clear, even to me, that there are new and challenging questions. For example, the old "global" optical model, underlying most of the calculations, is clearly grossly deficient in the region of primary interest. There are sound theoretical reasons for this, and they are now supported by detailed experimental evidence. I do not know what the impact of these new optical model concepts on the calculation of preequilibrium emission will be. Statistical level properties have long been a major concern, and recent measurements suggest that currently accepted concepts are not particularly valid to energies where preequilibrium emission can be a consideration.

Experimentalists are not without sin. Observations, relevant to this meeting, are among the most difficult. You could well reflect on the enormous effort that has been expended in attempting to measure the "clean" fission neutron spectrum with accuracy over several intensity decades. Those observations are simple compared to the measurements associated with statistical and preequilibrium processes. It is not surprising that experimental results can vary by an order of magnitude in important regions, and large improvements will not soon come.
Our "customer", the applied user, is little concerned with our measured or calculated data. He relies on evaluated files expressed in rigid formats. Unfortunately, there is considerable uncertainty associated with these evaluated structures as they apply to the preequilibrium processes. A consequence, for example, is that major applied institutions simply have no capability to utilize the angle-energy differential preequilibrium processes that will be so much discussed here. Worse, on occasion when the information has been incorporated in the integral assessments, the results have been discouraging. The complexity, vagueness and unfortunate applied results have led to some degree of user frustration. These problems must be successfully addressed or much of the proceedings of this meeting simply will not be used. The essential linkage with the "customer" must be improved.

Finally, I am fortunate to be an elderly student these few days. I look forward to your tutorial efforts, and hope I have the ability to absorb some of your wisdom.
RECOLLECTIONS ON THE DEVELOPMENT OF
PREEQUILIBRIUM DECAY MODELS

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We are gathered in Semmering for a conference which is mainly devoted
to precompound or preequilibrium (PE) decay models. This field had its
birth 22 years ago and many of those involved in its development are pres-
ent; it is a pleasure at this time to reflect on the history of this field.

There were several early works which recognized those aspects of
nuclear reactions which one categorizes as PE decay. But I believe our his-
tory began around 1965 when H. H. Barschall presented J. J. Griffin with
some (p,n) spectra measured at the University of Wisconsin Van de Graaff
laboratory. These spectra were not amenable to interpretation by conven-
tional statistical decay models. Griffin made a breakthrough in his classic
1966 article1 by interpreting these spectra as intermediate resonances in
the continuum characterized by few quasi-particle (or "exciton") degrees of
freedom, most importantly treated by use of a few quasi-particle partial
state density expression to characterize the continuous energy distribution
of the excitons.

Aside from papers by Griffin in 1966, 19672,3 using this idea to
check the slopes of high energy components of spectra, few works appeared
testing Griffin's hypothesis until 1968, when Blann's group in Rochester
became interested in this field. They had been measuring many p, 3He and
α induced excitation functions at energies up to around 100 MeV, observing
high energy tails beyond the evaporation peaks. Blann made modifications to
Griffin's model in 1968, so that charged particle reactions could also be
treated in the exciton framework.4

This approach was used with good success in interpreting, for the first
time, the entire compound plus PE excitation functions. The dependence of
initial exciton number on projectile mass was also demonstrated. This work
interested Seeliger's group in Dresden, who began an extensive program of
d²α/dEα measurements with 14 MeV neutrons, interpreting these data
as due to PE emission.1,4 The Dresden group did much to verify and pub-
lize this new reaction model; their contributions are numerous and con-
tinue to the present time. Around 1970, the Livermore pulsed sphere program
of Wong et al./ Howerton confirmed that neutron induced reactions had a hard
component beyond the evaporation peak, and the nuclear data evaluation com-
community further became aware of and keenly interested in this problem.
Experimental data were important to the model development to follow.
Additional experimental work by Grimes et al. at Livermore and Bertrand and
Peelle at ORNL provided an important additional data base for these models,
extending the Dresden data to higher energies.
Blann's group explored and developed several aspects of PE models in this period. In 1971, Blann formulated the "hybrid model" as the first "exciton" model to permit predictive calculation of $\sigma/\omega$. This was based on using Pauli corrected nucleon-nucleon scattering cross sections for calculating the spreading rate (in 1973, use of the imaginary optical potential was given as an alternative) and detailed balance for the emission rate. Blann's formulation was strongly influenced by papers on the Boltzmann Master Equation as a nuclear reaction model. The hybrid model was used the same year (1971) by Fu at ORNL for nuclear data evaluation, which must be some record for basic research being used for applied research. In 1972, Blann published the "geometry dependent hybrid model" to evaluate the importance of limited well depth and of the nuclear surface in PE decay processes at higher (than 14 MeV) energies.

Around 1970, C. Kalbach, a graduate student in Blann's group, undertook the exploration of a master equation approach to the exciton model. It was quickly recognized that the transition rate expressions implied in the 1966 Griffin paper were incorrect; what was needed were not the total final state densities but those accessible on average in a single two-body transition. It was also recognized along the way that the Ericson expression (which had been replaced for the simpler form given by Griffin) suffered from failure to obey the Pauli principle, thereby not giving the proper equilibrium limits. These problems were quickly solved by a postdoctoral member of the group, F. C. Williams, Jr. His results are, with minor improvements, still being used in master equation approaches.

Williams also gave the first formulation of the angular momentum dependence of the partial state densities, and in 1973 demonstrated the importance of using shell model single particle levels to generate few quasi-particle densities. Large deviations from Fermi-gas models were predicted for near closed shell nuclei and in a related formulation by Kalbach; these were later confirmed by experimental spectra measured at Livermore.

Kalbach and Blann published the results of the master equation in 1971, and it is fair to say that they had a very substantial impact on the PE "movement." This work showed the relationship between PE and equilibrium decay as a time dependent phenomenon, and showed that a single (unified) model really included both processes. It was also shown that the never come back (closed form) treatment gave essentially the same result as the more tedious master equation. The unified spin dependent exciton-Hauser Feshbach codes later developed by Fu and others were a natural followup to this work.

Another major effort with great impact began around 1971 in Milano. The Milano group showed (1972) that the $1/2$ spreading rate matrix of Griffin's formula was indeed reasonably constant at fixed incident energy. A classic paper was the 1973 article of Gadioli et al. This work provided the formulation of the exciton model (using total state lifetimes, rather than single particle lifetimes, as in the hybrid model) for closed form predictive use. It included evaluation of nucleon collision times based on nucleon-nucleon scattering results, and also calculation of the hole transition rates. The Milano group and their collaborators used this formulation in successfully interpreting a broad rate of excitation functions over a wide range of incident projectile energies, verifying the predictive power of this approach. Their many contributions are too numerous to summarize in this brief historical review.

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In 1973, Kalbach published her first evaluations of $\langle M^2 \rangle$, permitting absolute predictions of spectra calculated in the exciton model master equation approach. As is well known, she continued to update and improve these empirically determined values to include the dependence on exciton number implied by Gadioli's calculation.

Here I should mention that the group in Bratislava (Obložinský, Běták, Ribanský) and Seeliger in Dresden (with some work done in collaboration with Toneev in Dubna) began explorations of their own exciton master equation approaches, initially to calculate relative spectral shapes, later calculating absolute spectra. These groups each made many other contributions to improved understanding of the exciton model, which again are too numerous to summarize adequately in this short note.

In 1975, Mantzouranis et al. presented a simplified nucleon-nucleon scattering approach for calculating PE angular distributions in the exciton model. This was followed by work of the Beijing group, in which Sun et al. used a more rigorous Fermi-gas/nucleon scattering kernel for this purpose. Additional improvements were made in Petten by considering an angle-energy correlated scattering kernel and refractive corrections. Meanwhile, Kalbach and Mann discovered simple systematics in the data from a wide variety of reaction types. A parameterization based on these systematics has been widely used.

Another important area of PE decay is in the prediction of $\gamma$-rays; the pioneering work of Plyuiko and Prokopets (1978) should be mentioned in this context, as well as the later work of Běták and Dobeš. An important area for the future will be the inclusion of partial state densities using shell model levels in our codes. Nothing much was done on this topic from the time of the early papers until Reffo took up the problem around 1980. Now from the work at ENEA, we are gaining an understanding of the angular momentum dependence of the exciton level density based on shell model levels, which was missing in the earlier work. Much work has also been done on cluster emission, but this still remains an open problem.

PE models have contributed a great deal to the important area of nuclear data evaluation, and all who have contributed to modeling effort should take justifiable pride in their contributions. The impact of this field has been quite broad, providing an interpretative basis for reactions to quite high energies, heavy ion induced reactions, photonuclear and pion induced reactions, to list a few. In short, this field has very much changed the manner in which the entire nuclear physics community views nuclear reactions. This conference shows that PE modeling continues to be dynamic and evolutionary, and obviously will continue as an active area of nuclear physics research for the next decade.

Finally, I note that the field has grown too fast for me to follow completely. For this reason, and this reason only, there are doubtless many significant works I should have referred to but have not. I give my apologies to those authors so slighted.

These are some of the main contributions to the development of PE nucleon emission models as I remember them, restricted in discussion to the earliest "introductory" works. This is not meant to overlook the many important contributions made by other groups which are numerous indeed, and for this reason hard to include in their entirety.
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PRE-EQUILIBRIUM REACTIONS INDUCED BY NEUTRONS UP TO 40 MEV
(EXPERIMENTAL DATA)

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1. Introduction

First of all, some remarks concerning the terminology used in the present paper should be explained. Frequently the term 'pre-compound' is used for nuclear reactions which take place before the compound nucleus is reached (sometimes from this reactions additionally the collective direct reactions are excluded!). Here instead of 'pre-compound' the term 'pre-equilibrium' is used, which is more general because it bases on a well defined feature of each statistical system rather than on a specific nuclear reaction model.

In our understanding the term 'pre-equilibrium reactions' designates all those nuclear reactions in which particles occur in the exit channel before the statistical equilibrium of the composite system is reached. This definition is model-independent and, of course, it includes also direct one-step and two-step processes.

Pre-equilibrium and equilibrium reactions practically never occur in a pure manner, but both of them can be more or less pronounced, pending on the type of incident particle and energy, reaction channel, the properties of target nucleus a.s.o.

The model-independent experimental evidence of pre-equilibrium processes is well-known:

i) Evidence of hard components in the spectra of emitted particles, which cannot be expected at a given excitation from fully equilibrated statistical systems;

ii) correlations between the linear momenta of incoming and outgoing particles resulting in a forward peaking of angular distributions;

iii) strong enhancement of the excitation of definite (collective) states in the final nucleus.

Pre-equilibrium reactions meant here are not connected with a definite nuclear reaction model (like the exciton model), they cover a broad range of several decades at the time scale of nuclear processes and can be caused by quite different types of physical interactions (such as single-particle, collective
vibrational or collective rotational a.s.o.).

Furthermore, it should be stressed, that the energy range between zero and 40 MeV incidence energy is a very inhomogeneous one concerning the experimental situation and results as well as the theories used for the interpretation of the data. It could be subdivided further into the following three energy intervals (of course these intervals depend also on the mass number):

i) up to 5 - 6 MeV:
The final nuclei are mainly in isolated states; pre-equilibrium reactions occur through the direct collective excitation of these states, mainly; HF- and CC- (or DWBA-) theories provide a satisfactory base for theoretical predictions, if potential and structural parameters are known;

ii) from 5 - 6 up to 15 - 20 MeV:
The final nuclei occur in isolated states (up to the binding energy) as well as in the continua; different types of reaction mechanisms contribute remarkably to the pre-equilibrium emission and the influence of nuclear structure on the reaction is still very important. Therefore, theoretical analyses in many cases are difficult and give erroneous or misleading results (at the other hand, for fusion reactor applications, this is the most important energy range).

iii) above 20 MeV:
The reactions mainly go to final states in the continua, therefore, the application of statistical pre-equilibrium and equilibrium models is well justified, making the theoretical analyses again easier; while collective interactions are almost energy independent, the strength of single-particle collision rates increases with the incident energy resulting in a dominating contribution of the later type of interaction at higher energies.

A brief review is presented here concerning neutron induced particle spectra for incidence energies above 5 MeV.
Secondary neutron emission spectra at 14 MeV incident energy since many years are used as a valuable source of experimental information on pre-equilibrium processes in neutron induced reactions /1/. A status review has been presented elsewhere /2/. Recently Pavlik and Vonach carried out an evaluation of experimental data on neutron emission cross sections of 14 MeV neutrons for medium and heavy nuclei taking into consideration the full data base from 28 experiments carried out between 1957 and 1986 /3/. In this evaluation all available data sets were critically reviewed, erroneous data were disregarded and others were reprocessed in a definite way to get evaluated data for secondary neutron energy groups having width from 0.25 MeV up to 1 MeV and being renormalized to 14 MeV incidence energy. As an example on figs. 1 and 2 both the existing data base for niobium and the evaluation of /3/ in comparison with other evaluations is shown. This new set of evaluation carried out in a unique and definite way provides a good data base for further comparisons with pre-equilibrium model calculations. Even the correlation matrix for the evaluated neutron emission cross section is given. Evaluations are carried out for Cr, Fe, Ni, Cu, Nb, Mo, Ta, W, Pb and Bi.

However, one has to be careful when these evaluations are compared with theoretical calculations at the upper end of the spectra: The evaluations are given up to 12 MeV only and due to angular integration and averaging procedures physical structures which might be reproduced by direct theories or SMD-calculations are flattened in the evaluated spectra. Examples for the strong influence of direct collective excitations on the shape of integrated neutron emission spectra are shown by SMD-calculations including collective excitations in a contributed paper to this meeting /4/.

New measurements have been reported during the last year by the groups in the Osaka University, Tohoku University, TU
Fig. 1 Compilation of all existing experimental data $\sigma_{nm}(E)$ for $^{93}$Nb renormalized to 14.1 MeV incidence energy /3/

Dresden and PEI Obninsk, which are not included so far in the evaluation.

At the Tohoku University measurements at 14.1 MeV are reported for Al, Cr, Fe and Ni with sufficient time resolution /5/.

At the TU Dresden recently new measurements on Tantalum have been finished /6/.

Although, a clear progress was achieved in this area during the last years, there still remains the situation stated in /2/ that for a quite large number of elements there are no data available and also there are almost no data measured for separated isotopes, though, they could be very helpful for the development of pre-equilibrium models.

High precision nuclear data needs for fusion transport
Fig. 2 Comparison between evaluation of experimental data by Pavlik and Vonach /3/ (step function) with ENDF/B-V and ENDL-84 for $^{93}$Nb

calculations stimulated very complex investigations of the secondary neutron spectra from natural lead /7/. These investigations included measurements of microscopic differential neutron emission cross sections using TOF-spectroscopy, integral measurements of neutron escape spectra from a lead sphere using TOF and proton recoil spectroscopy but also the theoretical analyses of microscopic neutron spectra using pre-equilibrium and equilibrium theories as well as transport calculations with different transport codes and evaluated data libraries. Results are shown on figs. 3-5. The experiments /1,7,8,10/ now allow the conclusion, that differential neutron emission is about 10% higher than pre-
Fig. 3 Compilation of recent $G_{nm}(E)$ experiments for Pb /7/ leads to an average experimental spectrum (solid line) which is higher than ENDF/B-IV prediction (dashed line).

dicted in ENDF/B-IV. The measurements at Osaka University with high resolution show clearly the strong excitation of low-lying collective states (fig. 3), whereas measurements with lower resolution give the averaged spectra. The neutron emission from lead can be well described theoretically, if single-particle statistical pre-equilibrium plus equilibrium emission is calculated with the time-integrated master equation (code AMAPRE) and additionally the direct collective
excitation of low-lying states is taken into account (fig. 4). The neutron leakage spectra from the lead sphere in comparison with transport calculations (fig. 5) again leads to the conclusion, that the neutron emission in this multiplier material is somewhat higher than predicted by ENDF/B-IV. After this complex procedure now the neutron emission spectra for lead is known.

![Graph](image)

**Fig. 4** Theoretical interpretation of the $G_{nm}(E)$ spectrum for Pb at 14 MeV incidence energy /7/; the full curve contains contributions of first neutrons as calculated by time-dependent master equation using the code AMAPRE (dotted line), second neutrons from $(n,2n)$ reaction and direct collective excitations of vibrational phonons calculated in the frame of DWBA theory.
Fig. 5 Integral neutron leakage spectrum from a lead sphere determined experimentally (solid line) is about 10% higher than predicted by transport calculations using the code MORSE and ENDF/B-IV library (dashed line) /7/ with an uncertainty less than 5%. Future substantial improvements of neutron emission spectra could follow the same 'closed-loop' line of investigations, including differential and integral measurements as well as microscopic and macroscopic calculations.

3. Double-Differential Neutron Emission Cross Sections $\sigma_{nM}(E,\Omega)$ at 14 MeV

In most of the experiments the DDNECS $\sigma_{nM}(E,\Omega)$ but not the $\sigma_{nM}(E)$ are measured directly. Therefore, the main body of experimental informations analyzed in /3/ are basically DDNECS's. At the other side, for comparisons with pre-equilibrium models the use of DDNECS has to be preferred because the physical in-
formation about the reaction mechanism in principle is inherent both to the spectral shape and the angular distribution of emitted neutrons. There are reaction models like SMD and SMC which are strongly different concerning the predicted angular distributions in the same range of energy of emitted particles. Unfortunately, the experimental situation concerning angular distributions at present is much less sufficient than it was stated for the angular integrated spectra. This is due to the following reasons: TOF scattering experiments are very time-consuming. Therefore, in many experiments only one or a few angular points have been measured. Due to the experimental arrangements, the incidence energy sometimes is angular-dependent, resulting in a complicated angular-energy-correlation of the data-points determined (in most cases this correlation is neglected!). At low scattering angles ($\vartheta < 30^\circ$) it is very difficult to distinguish between elastically scattered neutrons and the inelastic neutrons resulting in an overestimation of the forward peaking. In many cases the angular distributions look like the lead data shown on fig. 6 for incident energies around 14 MeV and emission energy ($5.5 \pm 0.1$) MeV. In these cases a broad variety of theoretical angular distributions "fit" the data and definite conclusions concerning the preference for one of the models is practically impossible. The use of a simple parametrization of the angular distributions like the Kalbach-Mann parametrization seems justified in these cases. To exclude the uncertainty of angular-energy correlations in the physical data analyses, it is recommended to compare model calculations with DDNECS at a definite angle and incidence energy rather than with angular distributions for 'energy bins', as it is shown for the OKTAVIAN data and SMC-SMD-calculations for niobium at fig. 7/8,11/.

An evaluation of the DDNECS data base in a similar way as it was done for integral data/3/ probably could improve the unsatisfactory situation mentioned for angular distributions in the continuous part of the spectra. Using DDNECS for comparisons with pre-equilibrium model predictions one has carefully
Fig. 6  Angular distribution of secondary neutrons in the energy range $5.5 \pm 0.1$ MeV after bombarding lead with 14 MeV neutrons. The data points are shown together with the weighted average (solid line) and the ENDF/B-IV evaluation /7/ (dashed line).

to check the energy resolution of experiments especially in the emission energy range 10 - 14 MeV. Otherwise erroneous conclusions might be drawn from the comparison between experiment and theory.
Fig. 7 DDNECS for $^{93}$Nb + n at 37° and 143°/8/ compared with a SMD/SMC-calculation including collective phonon excitations /4,11/
4. Secondary Neutron Spectra $\sigma_{nM}(E)$ and $\sigma_{nM}(E,\Omega)$ below 14 MeV and above 15 MeV

For medium and heavy nuclei continuous emission spectra occur above 5 ... 6 MeV incident energy. Below this energy level there was a lot of experimental information obtained about the excitation of isolated levels by inelastic scattering of neutrons (and protons). However, between 7 and 14 MeV there is still a gap of experimental information on DDNECS. The only exception is the nucleus $^{93}$Nb (and to less extent $^{56}$Fe) for which experimental spectra are available at several incident energies between 5 MeV and 12 MeV /12,13/. In the contributed paper /4/ to this meeting the experimental neutron spectra for $^{93}$Nb are shown together with SMD/SMC-calculations. These spectra especially are very suitable for testing of pre-equilibrium models. Due to the weak energy dependence of collective interactions below 14 MeV the main contribution to pre-equilibrium processes is caused by the direct vibrational excitation of low-lying states.

For testing statistical single-particle theories, like the exciton model, DDNECS at higher incident energies are preferable. Those data are very scarce, near 26 MeV incident energy measurements have been carried out at the Ohio University /14/. Again in /4/ this spectra are shown for $^{93}$Nb and $^{56}$Fe together with SMD/SMC-calculations.

In fig. 8 differential data for $^{184}$W at 26 MeV and 11.5 MeV incident energy are shown together with SMD calculations /15/.

In fig. 9 the Ohio data for $^{56}$Fe + n /14/ are compared with GNASH calculations by Arthur et al. /16/.

Recently new measurements with good resolution at 18 MeV for Fe and Ni have been reported at the Tohoku University /17/ (fig. 10). Altogether, the neutron induced neutron emission data base for testing of pre-equilibrium models at incident energies out of the interval 14 - 15 MeV remains very scarce. Above 26 MeV no experiments are available at all. For testing
the energy dependence of different reaction mechanisms at present the most suitable candidates are $^{93}$Nb + n and $^{56}$Fe + n.

![Graph](image)

**Fig. 8** DDNECS for $^{184}$W + n at 26 MeV and 11.5 MeV incident energy and different emission energy intervals compared with SMD-calculations /13,15/
Fig. 9 $\sigma_{\text{nH}(E)}$ for $^{56}\text{Fe} + n$; experimental data /14/ are compared with GNASH calculations /16/
Fig. 10 DDNECS at 18 MeV for Fe + n /17/ compared with ENDF/B-IV and JENDL-3 evaluations

5. Secondary Charged Particle Spectra from (n,x) Reactions

Secondary charged particle spectra in addition to the neutron emission spectra are very suitable for studying the pre-equilibrium reactions: on the one side equilibrium emission of charged particles is strongly reduced by the Coulomb barrier and on the other hand also the collective excitation of low-lying states is very weak in reactions of the type (n,p). Therefore, proton spectra from (n,p) reactions on heavy nuclei are especially suitable for testing of statistical single-particle pre-equilibrium reactions, which are described (f.i.) by the exciton model.

Charged particle spectra were measured successfully by means of the magnetic quadrupole spectrometer at LLL /18/ and a special 27-telescope at IRK. As an example at fig. 11 proton spectra for A ≊ 60 nuclei measured at LLL by Grimes et al. /18/ are presented together with an HF- and Hybrid-Model analyses.
Fig. 11 Neutron induced proton spectra for nuclei near mass number $A = 60$ /18/ in comparison with HF- and HM-calculations.

For molybdenum isotopes and other nuclei with $A \sim 90$ the description of experimental spectra remains problematic, as shown on fig. 12 /19/. A recent example of combined alpha and proton spectra analyses /20/ for the LLL copper data /18/ is shown on fig. 13. This picture demonstrates that different reaction channels have to be taken into account for a proper description of the charged particle spectra. An example for the consistent description of proton and neutron DDCS for $^{93}$Nb+n in the frame of the SMD/SMC-model is also given in the contributed paper /4/ to this meeting.

We can summarize that neutron induced charged particle DDCS at present are available mainly at 14 MeV incidence energy. There are a few data sets available at incidence energies much higher than 40 MeV /21,22/. For further development of
Fig. 12  Same as fig. 11 but for nuclei near $A = 90$ /18/

pre-equilibrium models it would be highly desirable to carry out new measurements at a few incidence energies between 9 MeV and 40 MeV at least for $^{93}\text{Nb} + n$ and $^{56}\text{Fe} + n$. The upper end of $\alpha$-particle spectra should be measured with high precision and resolution due to the presence of strong direct excitations of low-lying states in the residual nuclei /23/.
Fig. 13a
Fig. 13b Proton (a) and alpha particle (b) spectra for Cu + n are compared with recent calculations taking into account different reaction channels /20/
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Precompound Reactions Induced by Nucleons and Light Ions with Energies above 40 MeV

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ABSTRACT

The continuous part in the energy spectra of secondary emitted particles is discussed. Besides the gross properties, nuclear structure and charge dependent effects are considered. It is shown that a quasi-free nucleon-nucleon scattering is the first and dominating step of the reaction. A moving source parameterization of the spectra for bombarding energies up to several hundreds of GeV is discussed.
I. Introduction

If a projectile is scattered with moderate energies, its energy spectrum consists of two parts: a smooth structureless region at low energies and sharp structures at the high energy end. The first part can be attributed to emission from a compound nucleus, whereas the sharp structure belongs to more simple modes of motion excited via direct reaction processes. Both types of reaction can be described theoretically. If the bombarding energy is increased considerably, a continuous region shows up. In Fig. 1 proton spectra from the Cu(α,p) reaction at E_α = 54.8 MeV are shown\(^1\). Obviously, the yields at forward angles are larger than at backward angles. Intuitively, the spectra seem to consist of two parts: at low energies a steep decrease with increasing ejectile energy together with a high yield and at higher energies a flatter decrease with smaller cross section are observed. One attributes the low energy part to an evaporation process. Looking at the angular distribution of a bin of this region, say 8 MeV, shown in the right part of Fig. 1, we see that the distribution is symmetric to 90°. This is a clear indication for the

Figure 1:
Left side: proton spectra from 54.8 MeV α-particle bombardment of Cu from Ref.1. Measurements were carried out at the indicated angles.
Right side: Angular distributions for the three indicated proton energies.
emission from a compound nucleus. The bin at 22 MeV shows a forward peaked component together with the evaporative part at backward angles, while at 38 MeV the angular distribution is almost exponentially decreasing. It is this second region we are interested in. We will study this continuous part in the spectra by varying the experimental conditions, like projectiles, ejectiles and bombarding energy.

In this contribution we will restrict ourselves to only light ion induced reactions, mostly to nucleon induced reactions. Some data will be compared to exciton model calculations. The exciton model will not be described. This can be found in a recent review\(^2\). Furthermore we will not consider reactions with polarized projectiles. For that purpose we refer to Refs. 3-5. Little work is done to identify the residual nuclei via the \((\text{particle}, x\text{pyn})\) reactions, where the final nucleus is identified through \(\gamma\)-rays between low lying states\(^6,7\). Such data are challenging nuclear models. The third type of experiments not being considered in the present work are particle-particle correlation measurements\(^8-11\). However, there is hope that such semi-inclusive data can help in the future to seriously test models. In other words: the present work deals only with inclusive data and their properties.

II. Properties of Data in the 40 MeV to 1 GeV Range

II.1. Gross Properties

It is well known that light isotopes are weakly bound particles except, perhaps, the \(\alpha\)-particle. If we look at proton spectra from 90 MeV \(^3\text{He}\) bombardment of \(^{197}\text{Au}\) (Fig. 2) we have the same behaviour of the cross sections at backward angles as those shown in Fig. 1. However, at forward angles a bump is visible around 30 MeV. This corresponds to the energy per nucleon in the projectile and hence to the velocity of the beam. Such a bump is expected to be produced from projectile fragmentation. The spectra shown in Fig. 1 do not exhibit such a structure indicating the stability of the \(\alpha\)-particle. Stability, however, is relative as has been shown in Ref. 12. Even at 100 MeV bombarding energy the \(\alpha\)-particle dissolves into its constituents as can be seen from Fig. 3. Because these are angle integrated data\(^13\), the bump around 50 MeV in
Figure 2: Proton spectra from 90 MeV $^3$He bombardment of $^{197}$Au for the indicated angles.

Figure 3: Angle integrated cross sections for charged particles being emitted from the indicated reaction$^{[3]}$.

Figure 4: Comparisons of the laboratory proton spectra resulting from 62 MeV protons on $^{54}$Fe, 90 and 198 MeV protons on $^{58}$Ni at the indicated angles$^{[10,14,15]}$. 

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the deuteron spectrum is not as pronounced as it is in the forward angle spectra. Furthermore, the break-up into A=3 particles is more frequent and bumps are visible in the triton- and ³He spectra. From this figure even more features of the secondary particle energy spectra emerge. For energies above $E_a/2$ the inelastic scattering is by far the strongest channel although the projectile is composite. Deuterons are emitted more frequently than tritons and ³He. This is true only if the break-up bump of the two A=3 cases is neglected. The proton spectrum is the softest spectrum. Looking at the yields it is the weakest channel at high energies and the strongest at low energies. There seems to be a tendency that the spectra become softer the more nucleons are transferred in the reaction.

We will now look at the energy dependence of the continuous part of the spectra. In Fig. 4, spectra from (p,p')-reactions on $^{54}$Fe or $^{58}$Ni are shown for bombarding energies of 62 MeV, 90 MeV and 200 MeV 10,14), respectively. Again as in Fig. 1, cross sections for energies smaller than 10 MeV are dominated by an evaporative part. This cannot be seen in the data taken at 200 MeV, because of finite $\Delta E$-detector thickness. This is also true for the two previous figures. The cross sections for secondary protons with energies above 10 MeV are continuous up to the bombarding energy. They are strongly forward peaked. If we look at an exit channel energy $E_p = E_p/2$, the yields $R = \sigma(\approx 20^\circ) : \sigma(\approx 135^\circ)$ behave as 46, 66 and 220 for the bombarding energies 62 MeV, 90 MeV and 200 MeV, respectively. We can, however, also look for the same ratio at an excitation energy of, say, 40 MeV. The ratios are then 14, 178 and 4000, if we extrapolate the backward angle spectrum to the highest energy. From these comparisons we find that, if we look at the same fixed excitations in the target nucleus, the angular distribution becomes steeper with increasing bombarding energy. If we look at energy loss of the projectile we find $R/E_p = \text{const}$. This relation holds also for other cases in this energy range which we have inspected. The data taken at the lowest bombarding energy show some structure at forward angles due to the excitations of low lying states. This seems not to be the case for the two higher energies. However, the absence of structure has its origin in the poor energy resolution in the experiments. In a high resolution experiment ($\Delta E/E \sim 10^{-3}$) and employing an active aperture to discriminate slit scattering 16) we measured structures at forward angles. In Fig. 5 forward angle spectra from 200 MeV proton scattering on $^{27}$Al are shown. On top of the continuous region a huge bump shows up.
The high energy part of the bump corresponds to the excitation of giant resonances. The low energy part, however, varies as $\alpha \cos^2 \theta$. This is just the signature of the free nucleon-nucleon scattering kinematics. We can therefore attribute this bump to the quasi-free scattering mechanism.

The nucleon-nucleon cross section at this bombarding energy is at a minimum and hence the nucleon mean free path at a maximum. If the mean free path is in the order of nuclear dimensions, there is a certain probability that after one quasi-free nucleon-nucleon interaction one nucleon is outside the nuclear volume.

**Figure 5:**
Proton spectra taken at forward angles. The data are averaged over 1 MeV bins.

This nucleon then contributes to the yield contained in the bump. We now increase the size of the target nucleus considerably, i.e. a factor of two in the radius by going from $^{27}$Al to $^{197}$Au, the probability for being outside the nuclear volume must decrease strongly. This, indeed, was found experimentally and is shown in Fig. 6.

**Figure 6:**
Comparison between two proton spectra from 200 MeV protons on $^{27}$Al and $^{197}$Au.

Data, taken at the much higher energy of 800 MeV, show bumps, again with the $\cos^2 \theta$ behaviour. Also if a smooth background is assumed, the quasi-free cross section decreases with increasing mass with respect to the background. The target mass dependent mean free paths in Ref. 17 may be due to the neglect
of the background originating from multistep processes.

II.2. Structure effects

In the studies of giant resonances excited in inelastic $\alpha$-scattering we found a larger inelastic cross section for $^{24}$Mg than for $^{27}$Al. This can be understood, in terms of the underlying structure because the Mg-nucleus can be thought to be built up of six $\alpha$-particles while $^{27}$Al has not this simple structure. However, when we studied continuous spectra from 100 MeV $\alpha$-bombardment on nuclei in this mass range we found that \( \sigma(\alpha,c)/\sigma(\alpha,p) = \text{const.} \) for $c = \text{any charged particle and for fixed energies}^{16}$. The angle integrated cross sections for proton emission are shown in Fig. 7. We may try to explain the cross section in terms of DWBA:

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha K_p}{K_\alpha} |V_{fi}|^2 \rho_f(U)dU
\]

with $k$ denoting the momenta, $V$ the transition matrix element and $\rho_f(U)$ the density of final states at an excitation energy $U$. The matrix element $V$ contains the overlap integral between the initial wave function $\psi_i = \psi_a \chi_i$ and the final state wave function $\psi_f = \psi_p \chi_{res}$. In the case of Al, obviously, the overlap is much larger than for the cases with only paired protons.

However, the cross section depends also on $\rho$. For a nucleus with an unpaired proton, 3 protons and 2 neutrons have to be excited with respect to the core. For paired protons a pair has to be broken up leading to 3 protons, 2 neutrons and 1 proton hole with respect to the core. In this case one more exciton is excited and therefore the level density is considerably larger than in the previous case. The fact that the ratios for complex particle emission to Figure 7:

Angle integrated proton spectra from 100 MeV $\alpha$-particles on the indicated target nuclei$^{13}$.}
proton emission are independent of the structure indicates that emission of preformed clusters can be excluded as main source for the origin of complex particle emission.

The proton spectra have been analyzed in terms of the exciton model\textsuperscript{2,13}. A single particle state density

\[ g = \frac{A}{8\text{ MeV}} \cdot \frac{6}{\pi^2} \cdot f \]  \hspace{1cm} (2)

has been employed with \( f \) being a free parameter. The results obtained by fitting the data are given in Table I.

Table I: The obtained deviations \( f \) from the average single particle state density

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* The number of protons (neutrons) is indicated as to be \text{even} or \text{odd}.

Looking only at even-even nuclei, we find a smooth increase with increasing mass. There is a slight increase if we look at the even-odd nuclei, where first the proton number is indicated. The odd-even case which may be most sensitive for the (\( \alpha \),p) reaction gives the largest single particle state density.
In Fig. 8 this case is shown together with two exciton model calculations. One is for the average value \( f = 1.8 \) and the other for \( f = 3.2 \). Obviously, not only the absolute values of the data are reproduced by \( f = 3.2 \) but also the shape. It should be mentioned that this reproduction is on a linear scale.

Figure 8:
The \( ^{27}\text{Al}(\alpha,p)X \) data from Fig. 7 (dots) are shown together with exciton model calculations. The single particle state multiplier \( f \) (Eq. 2) was varied in the calculations to be 1.8 (dashed curve) and 3.2 (solid curve).

A similar analysis can be made for the \( \text{Ni}(n,p)X \) cross sections at \( E_n = 60 \text{ MeV} \) measured for the even isotopes\(^{18}\)). The agreement between calculations and data is less satisfactory than it is for the s-d-shell nuclei.

The obtained \( f \) values are also given in Table I. There is a smooth increase in the \( f \)-values except for the heaviest nucleus. In Fig. 9 this dependence on the target mass number is shown. Also shown are the total cross section as obtained from the experiment and the calculation. Both show the same mass number dependence which is decreasing with increasing mass number.

This result is somewhat surprising because in the \((p,p')\) reaction at the same bombarding energy an \( A^{1/3} \) dependence was found\(^{14}\) similar to that at the higher bombarding energy of 90 MeV\(^{15}\)). Also shown in Fig. 9 is the mean value of the experimental cross section and the one obtained from exciton model calculations. It is indicated that this quantity follows nearly an \( A^{-8} \) dependence. For these observations, so far, we do not have an explanation at hand.
Figure 9:
Upper part: the target mass number dependence of the mean cross section for (n,p) reaction on the even Nickel isotopes. Shown are the experimental results (full dots) and the calculations (open circles). The curves are only shown to guide the eyes. Also shown is an A^{-8} dependence (stars).
Lower part: as in the upper part the figure but for the total cross section. Also shown is the mass number dependence obtained from the single particle state density multiplier f.

II.3. Charge Exchange Effects

In chapter II.1, we have discussed the quasi-free nature of the first step in the interaction. A quasi-free nucleon-nucleon interaction may manifest itself also in cross section differences in charge exchange reactions. For 90 MeV proton induced reactions a ratio for the (p,p') and (p,n) reaction was measured\(^{19}\), which can be interpreted as a predominantly first step nucleon-nucleon interaction:

\[
\frac{\sigma(p,p')}{\sigma(p,n)} = \frac{Z\sigma_{pp} + N\sigma_{pn}}{N\sigma_{pn}}
\]  

(3)
with $q_{ij}$ the elementary nucleon-nucleon cross section and $Z$ and $N$ denoting the number of protons or neutrons in the target nucleus, respectively. We can also compare protons and neutrons in the entrance channel:

$$\frac{\sigma(n,p)}{\sigma(p,p')} = \frac{Z_\sigma_{pn}}{Z_\sigma_{pp} + N_\sigma_{pn}}.$$  \hspace{1cm} (4)

For $A \approx 58$ we get a value 0.6, while a comparison of the experimental spectra yields 0.45. Furthermore, there is striking agreement in the spectral shapes as can be seen in Fig. 10.

![Figure 10: Angle integrated proton spectra from $^{54}$Fe + protons\(^{14}\) (dots) and $^{58}$Ni + neutrons\(^{18}\) (solid curve). The former data are multiplied by 0.45.](image)

**III. Bombarding Energies Above 1 GeV**

Due to the limited space of this contribution we can not discuss here all aspects of the data. Instead we will now focus on one more point concerning higher energy data. The data are scarce and are measured mostly over only a small part of the total available momentum. Most of the data consist of spectra over an energy range of only a few hundreds of MeV. Since the spectra are smooth and structureless with nearly exponential drop-off it has become very popular to fit Maxwell-Boltzmann distributions to them. If such a
dependence is assumed in the rest frame, i.e.

\[ \frac{d^2 \sigma}{d \epsilon' d \Omega'} = \gamma \epsilon' e^{-\epsilon'/E_0} \quad (5) \]

the invariant cross section at \( \theta_{\text{LAB}} = 90^\circ \) is given as

\[ \frac{1}{p} \frac{d^2 \sigma}{d \epsilon d \Omega} = e^{-\epsilon/E_0} \quad (6) \]

with \( E_0 \) denoting the energy distribution. An example of such a procedure is
given in Fig. 11. It may be seen that Eq. 6 is not the best functional de-
pendence to describe the data. A similar analysis is made in Fig. 12 for proton
spectra measured at \( 90^\circ \) for bombarding energies ranging from 60 MeV up to 400
GeV \( 13,14,22-22 \).

![Graph of 27Al(p,p')X E_p=200 MeV](image)

**Figure 11:**

Invariant cross sections for the indicated reaction at \( \theta_{\text{LAB}} = 90^\circ \).
Data\(^{13} \) are shown as crosses, the fit Eq. (6) as solid curve.

![Graph of E0 vs E for A(p,p')X θ=90°](image)

**Figure 12:**

The dispersion \( E_0 \) (Eq. 6) as function of the proton bombarding energy. The
data which are fitted by Eq. 6 are from Refs. 13,14,20-22. The fitted values
are shown by the indicated symbols, the predictions by Eq. (7) as solid
The resulting $E_0$-values show a saturation-like behaviour for energies above 1.5 GeV. The data can be accounted for by the simple formula

$$E_0(E) = 62 \text{ MeV}(1 - e^{-0.003E}),$$

(7)

which is also shown in the figure. This might be an indication for a limiting temperature of 62 MeV in nuclei. However, recent measurements of proton induced reactions at 4 GeV/c which were able to measure up to 1.2 GeV show a second, high energy component with $E_0 = 116$ MeV (Ref. 23). It is, therefore, at the moment unclear whether a limiting temperature exists for nuclei, and whether this temperature has been observed so far or not.

IV. Conclusion

We have presented here different aspects of inclusive spectra of particles emerging from nuclear reactions with incident nucleons and light ions having energies above 45 MeV. In this energy range the smooth structureless part of the spectra is dominating. Therefore, it is of great importance to study the reaction mechanisms of this part. We have shown that the first step of the projectile-target interaction can be understood as a quasi-free nucleon-nucleon scattering. Indications are the qfs-bumps at higher energies and the proton to neutron dependences observed either in the entrance or the exit channel.

Open problems are the mass number dependences measured in different experiments. Because of the lack of space we have omitted a more detailed discussion of the data at bombarding energies around 600 MeV. Also different and contradictory models applied to these data could not be discussed. The same is true for the emission of complex particles. These problems may be attacked together with others somewhere else.
References

NEW DEVELOPMENTS IN COMPOUND NUCLEUS THEORY

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ABSTRACT

The concept of random Hamiltonians is discussed in connection with the theory of nuclear spectra and nuclear reactions. Experimental evidence is presented that suggests the Gaussian orthogonal ensemble as the basis of the theory. Some theoretical results (average cross sections, the observable in detailed balance experiments) are reported and discussed.
1. Average and fluctuation properties of nuclei

Nuclei have many aspects that can be treated with the concepts of statistical mechanics. The special interest of nuclei lies in the fact that their number of degrees of freedom is small enough so that their fluctuation properties are observable and cannot be neglected as compared to their average properties. The distinction between average and fluctuation properties separates quantities that can be interpreted in terms of nuclear models from those that seem to be universal and free from any information [1]. Let me clarify this distinction by way of an example.

Consider the spectrum on fig. 1a. The levels tend to lie ever more densely as the excitation energy increases. One can define a locally averaged level density \( \rho(E) \) growing monotonically. This is visualized best if one considers the cumulated sum \( N(E) \) of the levels on fig. 1b and approximates this irregular staircase by a smooth function \( \bar{N}(E) \). This function yields \( \rho(E) \) by differentiation. The level density \( \rho \) is an example of an average quantity. It can be interpreted in terms of the Fermi gas or more sophisticated models involving independent particle motion in a mean field. From this we distinguish the fluctuation properties of the spectrum which can be isolated by transforming [2] the variable \( E \) to the variable

\[
x = \bar{N}(E). \tag{1.1}
\]

As a function of \( x \), the level density is constant and unity, see fig. 1c. The statistical properties of this "unfolded" spectrum are the same as those of the Gaussian orthogonal ensemble of random matrices [3, 4] (GOE).

In the same spirit, we shall distinguish the average scattering matrix \( \bar{S} \) from its fluctuating part

\[
S^f = S - \bar{S}. \tag{1.2}
\]

The first one is related [5] to the fast (direct) reactions, the latter one to reactions that proceed via a long lived intermediate system (compound nucleus). Direct reactions yield spectroscopic information that can be used to construct nuclear models, compound nuclear reactions have universal properties and do not yield any information beyond the average scattering matrix. The fluctuating part of the \( \bar{S} \)-matrix can again be described by the GOE: One simulates the compound nuclear states by a random Hamiltonian. By this idea, the statistical theory of spectra and the statistical theory of nuclear reactions have been put on a common basis. The statistical theory of spectra was worked out already two decades ago [3, 4]. The statistical theory of nuclear reactions has led to an exact, tractable and globally valid analytical treatment only in recent years [6]. So this field of statistical theory of quantum systems — although based on N. Bohr's concept [7] of the compound nucleus in 1936 — is only now being formulated in terms of a unique, well-defined and tractable mathematical model.

In section 2, latest experimental evidence will be given in favour of the hypothesis that nuclear spectra and nuclear resonance amplitudes are correctly modeled by the GOE. A comment on the connection to classical chaotic motion is made in section 3.
The random matrix theory of nuclear reactions is commented on in section 4. A few of its results are discussed in sections 5 and 6.

2. Spectra and resonance decay amplitudes are modeled by the GOE

Haq, Pandey and Bohigas [8] have collected 30 sequences of resonance level energies (each sequence containing only levels with fixed \( J^\pi \)) of altogether \( \approx 1700 \) nuclear states. The information comes from slow neutron and from proton resonance scattering as well as from \((n,\gamma)\) reactions. They call this the "nuclear data ensemble" (NDE). Each sequence has been transformed in the way outlined in the introduction in order to normalize the mean level distance \( \bar{d} \) to unity. The frequency distribution of the nearest neighbour spacings \( d \) given on fig. 2 was subsequently generated [9] from the whole ensemble. The probability to find degenerate levels is compatible with zero: The levels repel each other. The curve labelled GOE is essentially the so-called Wigner-distribution [10]

\[
p(d) = \frac{\pi}{2} d \exp(-\pi d^2/4).
\]

One obtains this if one assumes that the elements \( H_{\mu\nu} \) of the real and symmetric Hamiltonian with dimension \( \Lambda \) are random variables that follow a Gaussian probability distribution with the second moments

\[
\overline{H_{\mu\nu} H_{\mu'\nu'}} = \frac{\lambda^2}{\Lambda} (\delta_{\mu\mu'} \delta_{\nu\nu'} + \delta_{\mu\nu'} \delta_{\nu\mu'}).
\]

Here, \( \lambda \) is a parameter with the dimension of an energy. It determines the mean level distance

\[
d = \frac{\pi \lambda}{\Lambda}.
\]

The Gaussian distribution of the elements \( H_{\mu\nu} \) with the second moments proportional to the Kronecker-Symbols in eq. (2.3) is a consequence [11,12] of the requirement that the joint probability distribution of the \( H_{\mu\nu} \) be invariant under orthogonal transformations of \( H \). The ensemble of random matrices \( H \) having this probability distribution is therefore called the Gaussian orthogonal ensemble (GOE).

The distribution (2.1) beautifully reproduces the experimental nearest neighbour spacings of the NDE, see fig. 2. This figure also illustrates that the comparison becomes really convincing if one takes the information from many nuclei instead of one only.

The Poisson distribution \( \exp(-d) \) which is also given on fig. 2, would result if the positions of the levels were independent of each other - as is the case for a time sequence of radioactive decay events. Hence, the levels of a GOE spectrum are not independent at all, but strongly correlated. There is not only repulsion between a given level and its immediate neighbour, there are also correlations between a given level and its \( k \)-th neighbour - in such a way that the spectrum tends to resemble an equidistant lattice. A GOE-spectrum can be distinguished from a Poisson-spectrum by inspection, see fig. 3.
The long range correlations of spectra (their "stiffness") is usually tested by a quantity called $\Delta_3$ and defined as the mean square deviation of the staircase function $N(x)$ (see sect. 1) from the best fitting straight line:

$$\Delta_3(L) = \frac{1}{L} \text{Min}_{(A,B)} \int_0^a dx \left[ N(x) - Ax - B \right]^2. \quad (2.4)$$

In fig. 4, $\Delta_3(L)$ from the NDE is compared [8] to the GOE-prediction. The dashed lines indicate a band of errors. The agreement between NDE and theory is excellent. The prediction for a Poisson spectrum (increasing linearly) is also given. The slow (logarithmic) increase is characteristic for stiff (GOE) spectra.

It seems that the fluctuation properties of complex atomic spectra are the same as those of nuclear ones. I show, on fig. 5, the nearest neighbour spacing distribution taken from [13] Nb, Sm, and Tb spectra. The data and the statistical tests are as yet by far scarcer than those from nuclear physics.

Note, that we are comparing experiments to a theory that does not contain any free parameter!

The projection $\langle \lambda | n \rangle$ of an eigenvector $| \lambda \rangle$ of a GOE matrix onto an arbitrary but fixed vector $| n \rangle$ is a random number with Gaussian distribution. This has the consequence that the decay amplitudes $\gamma_{c\lambda}$ of resonances $\lambda$ into a given channel $c$ have the same property.

Usually, not the amplitudes but rather the widths

$$\Gamma_{c\lambda} = 2\pi \gamma_{c\lambda}^2$$

are measured. Indeed a large amount of resonance decay widths has been collected by studying $(n,n)$, $(n,\gamma)$, $(p,p)$ and $(p, p')$ - resonance reactions [14-17]. Suppose that the widths $\Gamma_{c\lambda}$ of a set of resonances $\{\lambda = 1, 2, ...\}$ in a given compound nucleus - all with the same $J^\pi$ - have been measured. One removes long term variations of the penetrability [18, 19] $P_c(E)$ and normalizes the widths to unit average by going to the variable

$$x_{\lambda} = \frac{\Gamma_{c\lambda}}{P_c(E_{\lambda})} \left( \frac{\Gamma_{c\lambda}}{P_c(E_{\lambda})} \right)^{-1} \quad (2.6)$$

Here, the angular brackets denote the average with respect to $\lambda$. The variable $x = \sqrt{x}$ should then have a normal distribution

$$p_0(x) = \left( \frac{2}{\pi} \right)^{1/2} \exp(-x^2/2). \quad (2.7)$$

Fig. 6 shows the distribution of the 109 widths of one of the longest existing sequences of resonances as well as the width distribution obtained [9] from the NDE. The agreement with eq. (2.7) is good, but one sees how greatly the precision is improved if a large number of nuclear species is considered instead of one only. This means that a convincing realization of the GOE is not so much given by any single nucleus but rather by the whole of them.
Eq. (2.7) implies that the variable $z$ follows the Porter-Thomas distribution [20]

$$p_1(z) = (2\pi z)^{-1/2} e^{-z/2}.$$  (2.8)

On fig. 7, $p_1$ is compared [21] to the distribution of $\approx 1100$ proton resonance decay widths from ref. [17] which were not included in fig. 6. Again the agreement is impressive. There are even more subtle investigations – and confirmations – of the Gaussian distribution of the $\gamma$’s pertaining to their phases [22, 23].

Why does the GOE – as it seems universally – describe spectral fluctuation properties in different physical contexts? There is as yet no definite answer to this question, but there is numerical evidence for the surmise that the GOE should have a universal significance. We deal with this in the next section.

3. Chaotic motion and the GOE

Chaotic motion in classical systems (governed by Hamilton’s equations) can occur when these systems are not integrable. For the present purpose, we can characterize a completely chaotic system as follows: almost all trajectories originating in neighbouring points of the phase space separate exponentially with time. Systems with this property tend to fill the available phase space uniformly and irreversibly. They essentially obey Boltzmann’s hypothesis of molecular chaos (for details see refs. [2, 24, 25]). However, such systems need not have many degrees of freedom. Consider the following standard example of classical chaotic motion, Sinai’s billiard: A point mass moves in a plane and bounces off reflecting disks forming a periodic lattice as is shown on fig. 8. By symmetry arguments one can reduce the study of this system to the study of the motion of a particle in a square with a circular obstacle in the centre. Intuition suggests (correctly, see ref. [26]) that this system is chaotic. Indeed – as illustrated on fig. 8 – a bundle of close trajectories is drastically defocused when reflected on convex obstacles.

Now Bohigas, Giannoni and Schmit [27] have studied the quantum system corresponding to Sinai’s billiard. This amounts to solving the eigenvalue problem given by

$$(\Delta + k^2) \psi = 0$$  (3.1)

with the appropriate Dirichlet boundary conditions. From a set of 740 consecutive eigenvalues, the nearest neighbour distribution $p(d)$ and the function $\Delta_3(L)$ were generated. They are given on fig. 9. This system has a GOE spectrum.

The quantum analogs of several other classically chaotic systems have been studied, see ref. [2]. They all have GOE spectra.

This work and several similar numerical studies suggest the following surmise: The stochastic properties of quantum analogs of classical chaotic systems are completely described by Gaussian random Hamiltonians. The stochastic properties of nuclei are due to the fact that the nucleus is the quantum analog of a classical chaotic system [28].
4. Theory of Statistical Nuclear Reactions

A reaction going from channel $a$ to channel $b$ is described by the element $S_{ab}$ of the unitary and symmetric scattering matrix $S$. Let us write it in terms of the real and symmetric $K$-matrix [18]

$$S = (1 - iK)(1 + iK)^{-1}, \quad (4.1)$$

where $K$ is defined as

$$K = \gamma(E - H^B)^{-1}\gamma^T. \quad (4.2)$$

Here, $\gamma$ is a rectangular matrix with the elements $\gamma_{c\mu}$, coupling the bound state configurations $|\mu\rangle$ to the channel wave functions $|c\rangle$; the matrix $H^B$ is the Hamiltonian projected onto the space of bound (compound nuclear) states; finally $E$ is the energy of the system multiplied by the $\Lambda$-dimensional unit matrix.

If one represents the matrix $H^B$ in its eigenbasis $|\lambda\rangle$ with eigenvalues $E_{\lambda}$, the element $K_{ab}$ of eq. (4.2) reads

$$K_{ab} = \sum_{\lambda} \frac{\gamma_{a\lambda}\gamma_{b\lambda}}{E - E_{\lambda}} \quad (4.3)$$

with $\gamma_{c\lambda}$ qualitatively explained in sect. 2. For details see ref. [18].

From the results discussed so far, we infer that the compound nucleus should be described by considering $H^B$ as a random matrix drawn from the GOE. This implies – as was mentioned already – that the $\gamma_{c\lambda}$ have a joint Gaussian probability distribution.

The task of the statistical theory is now to calculate $\bar{S}$ (called the one-point function), average compound nucleus cross sections

$$\sigma_{ab}^{cn} = |S^{11}|^2 \quad (4.4)$$

(called a two-point function) and higher moments of the scattering matrix as e.g. the variance of the cross section

$$C = \bar{\sigma}_{ab}^2 - \bar{\sigma}_{ab}^{-2} = \frac{1}{|S_{ab}|^2} - \frac{1}{|S_{ab}|^4}, \quad (4.5)$$

a four-point function.

The one-point function is easy to obtain from the analytic properties of the S-matrix [29]. The two-point function and higher moments have required considerable work.

One way to obtain $\sigma^{cn}$ starts from the expansion of the factor $(1 + iK)^{-1}$ of eq. (4.1) into a geometric series. This leads to a multiple sum in eq. (4.4) which is averaged term by term and subsequently resummed. One thus obtains the answer in the two limiting cases of isolated and of strongly overlapping resonances (regime of Ericson fluctuations [30]). Extending the work of Agassi et al. [31], the above expansion has been carried
out in ref. [32] for a statistics of the $\gamma$'s that is not restricted to the Gaussian case. In the regime of overlapping resonances, one finds the Hauser-Feshbach-Formula [33] modified by an enhancement of compound elastic scattering

$$\sigma_{ab}^{cn} = \frac{r_a r_b}{\sum r_c} (1 + \delta_{ab} (W - 1)). \quad (4.6)$$

Here, $\bar{S}$ is assumed diagonal and $r_c$ is the usual transmission coefficient defined as the unitarity deficit of $\bar{S}_{cc}$, i.e.

$$r_c = 1 - |\bar{S}_{cc}|^2. \quad (4.7)$$

The enhancement factor $W$ for compound elastic scattering depends on the probability distribution of $\gamma$'s. In the limit of a very large number $N$ of open channels, one obtains

$$W = \frac{1}{2} \left( \frac{\bar{\gamma}_c^2}{\bar{\gamma}_c^2} + 1 \right). \quad (4.8)$$

In this equation, we assume the ratio of statistical moments to be independent of the channel $c$. For Gaussian statistics, one has

$$\frac{\bar{\gamma}_c^2}{\bar{\gamma}_c^2} = 3 \quad (4.9)$$

and therefore $W = 2$. These results allow for testing the statistics of the resonance decay amplitudes even in the regime of overlapping resonances.

On fig. 10, $W$ is given as a function of the number $N$ of open channels (for details see ref. [34]) for three different probability distributions of the $\gamma$'s: the rectangular, the Gaussian and the $K_0$-distribution. The last one is

$$p(\gamma_c) = \frac{1}{\pi} \gamma_c^{-1/2} K_0 \left( |\gamma_c| \gamma_c^{-1/2} \right), \quad (4.10)$$

where $K_0$ is a modified Bessel function (see sect. 9.6 of ref. [35]); the moments of this distribution are given by formula 11.4.22 of ref. [35]. The statistics of eq. (4.10) which gives an especially high probability to small values of $\gamma$, has been suggested by shell model calculations [36]. The experimental result by Kretschmer and Wangler [37] – also given on fig. 10 – agrees with Gaussian statistics.

In order to obtain $\sigma_{ab}^{cn}$ exactly and for all physical situations (for weakly overlapping resonances, too) and in order to calculate higher moments of $S$, one has to use mathematical methods other than the above mentioned expansion technique. Using methods familiar from field theory and statistical physics of disordered solid state systems, and in particular super-integration (i.e. integration over both commuting and anti-commuting...
variables), Verbaarschot, Weidenmüller and Zirnbauer [6] have obtained the two-point function. I quote from the results:

$$\sigma_{ab}^{\text{eff}} = (1 + \delta_{ab}) \tau_a \tau_b F_1 + \delta_{ab} | \bar{\sigma}_{aa} |^2 \tau_a^2 F_2. \quad (4.11a)$$

Here, $F_n(n = 1, 2)$ is the integral

$$F_n = \frac{1}{8} \int_0^\infty d\lambda_1 \int_0^\infty d\lambda_2 \int_0^1 d\lambda \mu(\lambda_1, \lambda_2, \lambda) \prod_{c} \frac{1 - \tau_c \lambda}{[(1 + \tau_c \lambda_1)(1 + \tau_c \lambda_2)]^{1/2} f_n} \quad (4.11b)$$

with

$$\mu(\lambda_1, \lambda_2, \lambda) = \frac{(1 - \lambda) \lambda | \lambda_1 - \lambda_2 |}{[(1 + \lambda_1) \lambda_1 (1 + \lambda_2) \lambda_2]^{1/2} (\lambda + \lambda_1)^2 (\lambda + \lambda_2)^2}. \quad (4.11c)$$

The quantities $f_n$ are

$$f_1 = \frac{\lambda_1 (1 + \lambda_1)}{(1 + \tau_a \lambda_1)(1 + \tau_b \lambda_2)} + \frac{\lambda_2 (1 + \lambda_2)}{(1 + \tau_a \lambda_2)(1 + \tau_b \lambda_1)} + \frac{2\lambda (1 - \lambda)}{(1 - \tau_a \lambda)(1 - \tau_b \lambda)} \quad (4.11d)$$

and

$$f_2 = \left[ \frac{\lambda_1}{1 + \tau_a \lambda_1} + \frac{\lambda_2}{1 + \tau_a \lambda_2} + \frac{2\lambda}{1 - \tau_a \lambda} \right]^2. \quad (4.11e)$$

This result involves no more than a three-dimensional integral over ordinary commuting variables independent of the number of open channels $N$. Computer programs for its evaluation are available [38].

Equations (4.11) show that $\sigma_{ab}^{\text{eff}}$ is fully determined once the set of transmission coefficients, i.e. the average S-matrix, see eq. (4.7), is known. In this sense compound nuclear reactions have universal properties as was announced in sect. 1.

Equations (4.11) give the compound nuclear cross sections for any combination of the values of the transmission coefficients $\tau_c$, especially for any value of the “effective number of open channels”

$$N_{\text{eff}} = \sum_c \tau_c \quad (4.12)$$

which measures the amount of overlap between the resonances.

By studying eqs. (4.11) in the limiting cases of isolated resonances [39] ($N_{\text{eff}} \ll 1$) and strongly overlapping resonances [40,41] ($N_{\text{eff}} \gg 1$), one retrieves the results that are known from the expansion technique and from numerical work [42]. They can be roughly summarized by eq. (4.6) with

$$W = 3 \quad \text{if} \quad N_{\text{eff}} \ll 1 \quad (4.13a)$$

and

$$W = 2 \quad \text{if} \quad N_{\text{eff}} \gg 1. \quad (4.13b)$$

For details, especially in the regime of isolated resonances, see refs. [32, 34].
There are interesting observables which cannot be expressed in terms of two-point functions, but require a variety of n-point functions with n > 2. They arise e.g. in discussing the possibility to observe the violation of fundamental symmetries (like time-reversal invariance [43, 44]) and in the discussion of a fundamental and widely used assumption of Ericson theory - the supposed Gaussian distribution of S-matrix elements [45]. Higher n-point functions that go beyond the work of ref. [6] have therefore been [43, 44] and are still being studied [46]. I turn to some results in the following two sections.

5. Inadequacy of Ericson theory

A widely used assumption out of Ericson’s original theory of fluctuating cross sections says that – in the regime of overlapping resonances – \( S_{ab}^{fl} \) has a bivariate Gaussian distribution [45]. If this is true, then odd moments of \( S_{ab}^{fl} \) can always be expressed in terms of two-point functions. This means e.g. that the four point function can be written as

\[
S_4 = S_{a_1 b_1 a_2 b_2}^{fl} S_{c_1 d_1 c_2 d_2}^{fl} = S_{a_1 b_1 a_2 b_2}^{fl} S_{c_1 d_1 c_2 d_2}^{fl} + S_{a_1 b_1 c_1 d_1 c_2 d_2}^{fl} S_{a_2 b_2 a_1 b_1}^{fl} \quad (5.1)
\]

From this follows the basic statement of Ericson theory on the variance of a fluctuating cross section:

\[
| S_{ab}^{fl} |^4 - | S_{ab}^{fl} |^2 = | S_{ab}^{fl} |^2 . \quad (5.2)
\]

An analytical expression for the four-point function has been found by Davis and Boosé [46]. Expanding it in terms of the inverse of \( N_{eff} \) one finds the behaviour in the regime of overlapping resonances, i.e. \( N_{eff} \gg 1 \). The above statements turn out to be true within the leading order \( N_{eff}^{-2} \) of \( S_4 \). The correction is \( O(N_{eff}^{0}) \). However, it is still not true that up to and including the order \( N_{eff}^{-2} \) the fluctuating part of the S-matrix has a Gaussian distribution: The three-point function

\[
S_3 = S_{a_1 b_1 c_1 d_1}^{fl} S_{a_2 b_2}^{fl} \quad (5.3)
\]

is in general not zero, it is also of the order of \( N_{eff}^{-2} \). This means that the skewness

\[
a = \frac{| S_{ab}^{fl} |^2 S_{ab}^{fl} }{ | S_{ab}^{fl} |^2 } / | S_{ab}^{fl} |^2 \quad (5.3)
\]

behaves like \( N_{eff}^{-1/2} \), going to zero quite slowly. Terms of third order in \( S_{ab}^{fl} \) appear – besides fourth order terms – in expressions for the variance of a cross section, hence typically in the procedures designed to separate direct reactions from compound nucleus reactions [47, 48]. The three-point functions are neglected. This is not justified in general.
The result of the present section can be understood – without actually calculating the three-point function – as a consequence of unitarity. It requires the identity

$$\sum \xi \frac{\delta_{\alpha \xi} \delta_{\beta \xi}}{\xi} \frac{\delta_{\alpha \xi} \delta_{\beta \xi}}{\xi} = -\overline{\delta_{\alpha \xi}} \frac{\delta_{\alpha \xi}}{\xi}. \quad (5.4)$$

Since the right hand side of this equation is of the order of $N_{\text{eff}}^{-1}$, the three-point functions on the left hand side cannot all be $O(N_{\text{eff}}^{-3})$.

6. Time reversal symmetry breaking

Blanke et al. [49] have performed the detailed balance experiment

$$^{24}Mg + \alpha \Rightarrow ^{28}Si^* \Rightarrow ^{27}Al + p \quad (6.1)$$

in the domain of overlapping resonances in order to test time reversal symmetry breaking (TRSB). The form of the measured pieces of the excitation function is – within experimental errors – the same in the forward and the backward reaction. See fig. 11. This experiment amounts to a measurement of the correlation coefficient

$$R^2 = \overline{\sigma_{ab} \sigma_{ab}} - \overline{\sigma_{ab}} \overline{\sigma_{ab}} \left[ \left( \frac{\sigma_{ab}^2}{\sigma_{ab}^2} - \overline{\sigma_{ab}^2} \right) \left( \frac{\sigma_{ba}^2}{\sigma_{ba}^2} - \overline{\sigma_{ba}^2} \right) \right]^{-1/2}. \quad (6.2)$$

If $\sigma_{ab} = \sigma_{ba}$ – which is the case if time reversal symmetry holds and, hence, the scattering matrix $S$ is symmetric – then $R^2 = 1$. Otherwise

$$R^2 = 1 - x \quad (6.3)$$

will be smaller than unity. The experiment yields

$$x \leq 10^{-6} \text{ with } 80\% \text{ confidence } \quad (6.4)$$

We want to relate this result to the TRSB matrix elements. Formally, TRSB is introduced by allowing $H^B$ – see eq. (4.2) – to be hermitean instead of requiring it to be real and symmetric. Hence, one makes the ansatz

$$H^B = H^{(0)} + H^{(w)}, \quad (6.5)$$

where $H^{(0)}$ is taken from the GOE (normalized as in eq. (2.2)) and $H^{(w)}$ from the GUE having complex Gaussian elements [3] normalized according to

$$\overline{H^{(w)}_{\mu \nu} \overline{H^{(w)}_{\mu' \nu'}}} = \frac{2\alpha_0^2}{A^2} \delta_{\mu \mu'} \delta_{\nu \nu'}. \quad (6.6)$$

The parameter $\alpha_0$ controls the relative strength of the TRSB part in the Hamiltonian. Equations (6.5) and (6.6) define the ingredients of the statistical model and one has
to calculate the four-point functions entering into eq. (6.2). The result is to leading (second) order in $H^{(u)}$

$$R^2 = 1 - 4\alpha_0^2 N_{\text{eff}}^{-1}. \quad (6.7)$$

One should express this in terms of the mean compound nucleus decay width

$$\Gamma = \frac{d}{2\pi} N_{\text{eff}} \quad (6.8)$$

and a spreading width

$$\Gamma^{(u)} = \frac{2\pi}{d} \frac{2\alpha_0^2 \lambda^2}{\Lambda^2}. \quad (6.9)$$

The last quantity is defined in analogy with the spreading width $\Gamma^1$ that appears in other symmetry breaking problems that have been treated statistically. For instance is isospin breaking described by the spreading width

$$\Gamma^1 = \frac{2\pi}{d} \overline{H_C^2}. \quad (6.10)$$

Here, $\overline{H_C^2}$ is the average square isospin breaking Coulomb matrix element. With the definitions (6.8) and (6.9), $R^2$ reads

$$R^2 = 1 - \frac{1}{2} \frac{\Gamma^{(u)}}{\Gamma}. \quad (6.11)$$

In this form, the TRSB effect is seen to be governed by the ratio of two times: the life time of the compound nucleus versus the time it takes to rearrange the levels after the symmetry breaking interaction is turned on (proportional to the inverse spreading width, cf. ref. [50]). This shows why compound nucleus reactions are especially sensitive to small symmetry breaking effects: The longer the life-time of the intermediate system, the better the symmetry breaking effect is borne out. This statement is of course true only if the spreading width is the "natural" measure of the symmetry breaking effect independent of the life-time, level density and other parameters of the compound nucleus. There are good reasons to believe that this is so. The strongest reason is the collection of isospin breaking spreading widths [51-53] on fig. 12: They vary astonishingly little all over the nuclear chart.

The detailed balance experiment of ref. [49] yields

$$\Gamma^{(u)} \lesssim 9 \times 10^{-2} \text{ eV}. \quad (6.12)$$

The enhancement of symmetry breaking effects through the life-time of the intermediate system has also been noted in theoretical treatments of parity violating reactions. It has been termed "resonance enhancement" by Bunakov and Gudkov [54].

It should be mentioned that the factor of $\frac{1}{2}$ in eq. (6.11) (and the corresponding factor in eq. (6.7)) is somewhat uncertain because the hypothesis of Ericson theory discussed in sect. 5 was used in refs. [43, 44]. The correct treatment of eq. (6.2) is currently being carried out and preliminary results indicate [55] that the quantity $\pi$ of eq. (6.3) changes by a factor of order unity.
7. Concluding remarks

The random matrix model of statistical nuclear physics has found more applications and extensions than described here. A theory of multistep compound [56] and of multistep direct [57] reactions has been formulated and partial level densities have been redefined and evaluated [58, 59]. The concept of random matrices and the mathematical techniques now relate statistical nuclear theory to other fields of physics, where disordered systems are considered [60-62].
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Fig. 1 Separation of average and fluctuation properties of a spectrum: a) The spectrum, b) the cumulated spectrum $N(E)$ with the local average $\bar{N}(E)$, c) the "unfolded" spectrum obtained by transforming to the variable $x$, see text.

Fig. 2 Nearest neighbour spacing distribution $p(d)$ given by the NDE (b) and by a single level sequence out of the NDE (a). From ref. [9].

Fig. 3 A nuclear (presumably GOE-) spectrum compared to a Poisson spectrum. The arrowheads indicate the occurrence of pairs of levels with spacings smaller than one quarter of the average. Their frequency as well as the occurrence of large gaps clearly distinguish a Poisson-spectrum from a GOE-spectrum. Taken from ref. [4].
Fig. 4 The $\Delta_3$-function given by the NDE and compared to the GOE. From ref. [8].

Fig. 5 Nearest neighbour spacing distribution from atomic spectra. See ref. [13].

Fig. 6 Distribution of resonance widths given by (b) the NDE and (a) by one sequence out of it. The experimental histograms are compared to the normal distribution $p_0$. From ref. [9].

Fig. 7 Distribution of resonance widths [21] given by the data of ref. [17]. The experimental histogram is compared to the Porter-Thomas distribution $p_1$. 

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Fig. 8 Sinai's billiard. From ref. [2].

Fig. 9 Nearest neighbour spacing distribution and $\Delta_3$-function given by the quantum analog of Sinai's billiard. From ref. [2].

Fig. 10 The enhancement factor $W$ for compound elastic scattering as a function of the number $N$ of open channels calculated with the theory of ref. [32] for three different distributions $p(\gamma)$. See text. The experimental result is from ref. [37].
Fig. 11 Excitation functions testing detailed balance from ref. [49], see text.

Fig. 12 Isospin breaking spreading widths from ref. [53], see also ref. [51].
PARTIAL LEVEL DENSITIES FOR STATES WITH SPECIFIED PARTICLE AND HOLE NUMBERS, SPIN DISTRIBUTION FOR THESE STATES

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ABSTRACT

The method for combinatorial calculation of state and level densities with fixed exciton numbers in the space of the shell-model orbitals is presented. The pairing interaction is taken into account by applying the BCS theory to each configuration. The role of the pairing interaction is discussed. The spin and parity distributions are obtained and analyzed. Effects of the restriction limiting configurations to only bound orbitals are also considered.
INTRODUCTION

The main deficiency of the formulae commonly used to estimate exciton level densities comes from the assumption of equidistant spacings for the single particle levels (s.p.l.) which is supplemented with the statistical approximations. Because only few-exciton configurations contribute significantly to preequilibrium emission and the population of low energy configurations is relevant as well, statistical approaches do not seem to be adequate for preequilibrium calculations. This has motivated calculations which provide the possibility of direct counting of the levels with a fixed number of excitons, so that at least some of the usual assumptions (equidistant single particle levels, saddle point approximation, no residual interactions) can be avoided. Combinatorial calculations seem to be well suited to this end. In the following, we outline the theoretical model and summarize most important results. We refer to Refs.1, 2, and 3 for more extensive discussion.

THE MODEL

We assume the finite set of basis vectors (single particle states) derived from the shell-model with appropriately deformed nuclear potential. The set consists of the time conjugated pairs of basis vectors. In each pair, both vectors have the same quantum numbers but differ in a sign of a spin projection. We further assume, that each nuclear state of a noncollective nature can be described by a simple sum of $N$ and $Z$ basis vectors, where $N$ and $Z$ stands for the number of neutrons or protons respectively. This assumption is equivalent to neglecting all residual interactions between the nucleons. The only exception is made for the pairing interaction which is included in terms of the BCS model with blocking according to Wahlborn /4/. This means that for each state the set of two coupled BCS equations (Eqs.(2) and (3) of Ref.1) is solved in the subspace of the basis vectors. This subspace is formed after rejection of all those pairs of the time conjugated basis vectors for which one of the vectors, and only one, is involved in the formation of a nuclear state. In the following, we refer to these rejected pairs as configurations, which can be represented by the vectors with the components enumerating removed pairs of the basis vectors. Each configuration corresponds, therefore, to a different subspace of the basis vectors leading formally to the different solutions of the BCS equations. The chemical potential and the correlation function have to be found for each configuration (but not for each nuclear state) and are used later to determine the total configuration energy (see Eq.(5) of Ref.1). The ground state of a neutron (or proton) gas is defined as a combination of $N$ (or $Z$) basis vectors which gives the lowest possible energy. The excited configurations are classified according to the number of the excitons equal to the dimension of the configuration vector. In order to account for all possible states, we also allow for the configurations which correspond to the excitation of both basis vectors in a time conjugated pair (see Ref.1). The excitons are split into 'holes' and 'particles' depending on the condition if the particular basis vector pair was, or was not involved in the formation of the ground state.
All configurations with a specified number of particles and holes are generated within the assumed set of the basis vectors. For each of them a proper coupling of the spin projections is performed to obtain the nuclear states. The parity of each state is determined as a product of the parities of the configuration vector components. The state density \( \omega(E,M,\pi) \) is found by counting states with the angular momentum projection \( M \) and parity \( \pi \) falling in an interval centered at the excitation energy \( E \). The spin cut-off parameter is derived from its definition.

The calculations were carried out in the basis of single particle orbitals derived from the harmonic oscillator well defined by parameters due to Seeger and Howard /5/. For each nucleus considered, a value of the pairing strength parameter \( G \) was determined from the mass differences.

For the purpose of identification the calculations are denoted by four integer numbers which correspond to the number of neutron particles, neutron holes, proton particles and proton holes respectively.

**PAIRING INTERACTION EFFECTS**

**Even systems**

Let us start with the simplest configuration consisting of one particle and one hole. In this case two orbitals nearest to the Fermi energy are blocked and made unavailable for pair scattering. The correlation function \( \Delta \) is decreased, and if not enough orbitals around the Fermi energy are left unblocked, superconductivity may disappear completely (\( \Delta = 0 \)). The condensation energy of the excited state \( E^c \) decreases and the total energy of the configuration approaches the free gas value. The excitation energy calculated with respect to the BCS ground state is therefore increased. When the excitons are moved away from the Fermi energy, the superconductivity reappears, and the shift in the excitation energy tends to vanish.

In Fig.1 the spectra of energy shifts caused by the pairing interaction are shown. An energy shift is defined as the difference between the excitation energies of a configuration calculated with and without the pairing interaction taken into account. In the case of the \( (1100) \) configurations in \( 58 \text{Ni} \) a peak around 1.9 MeV is observed, which corresponds to superconductivity breakdown due to blocking. The energy shifts between 0.1 MeV and 0.6 MeV correspond to configurations with less blocking. In the case of the same \( (1100) \) configurations in \( 116 \text{Sn} \), a more complicated spectrum of shifts is obtained. This is due to the higher density of orbitals around the Fermi energy which allows for solutions of the BCS equations intermediate between the \( \Delta = 0 \) and \( \Delta = \Delta_c \) limits. In the case of four exciton configurations the spectrum of energy shifts becomes even more complicated, which is easy to understand since with four excitons many more blocking possibilities exist. At this point we would like to stress that the destruction of pairing correlation always results in the energy shift.
equal to the condensation energy of the ground state and that this value is an absolute limit of the pairing effect.

In spite of the complicated nature of the pairing effect we have found that the 4,6 and 8 exciton state and level densities calculated in the frame of the free gas model can be brought into perfect agreement with the results obtained including the BCS if a shift of \( E^e - E^f \) is applied (Fig.2). This indicates that the pairing interaction affects mainly the low energy states, while at higher energies the number of states which are shifted up and out from a bin is relatively small and is to some extent compensated by the lower bin states which are shifted up and replace those which were lost.

**Odd systems**

Considerations for the odd systems run along the same lines as for the even ones but the starting point is just the opposite. A single nucleon at Fermi energy blocks an orbital important for the pairing correlation, so that \( \Delta \) and \( E^f \) decrease or disappear. In the excited states of 1 exciton structure, this nucleon is moved out from the Fermi energy. The orbitals close to the Fermi level are made available for pair scattering and superconductivity is recovered. Accordingly, the condensation energy for the excited state is higher than that for the ground state and the excited state appears at a lower excitation energy compared to the free gas model prediction. Thus, for 1 exciton configurations in odd systems, a negative energy shift is expected. These negative energy shifts were actually obtained in our calculations and are displayed on Fig.3.

The situation becomes more complicated when several exciton configurations are considered. If these excitons are placed far from the Fermi energy, their contribution to the blocking is negligible and negative energy shifts are expected as in the one exciton case. Additional excitons can have however a different effect if the ground state of the nucleus has a nonvanishing condensation energy. In this case, the blocking of orbitals close to the Fermi energy leads to a decrease of the pairing correlation and shifts of positive value appear. Examples of this type are shown in Fig.3.

**STATE AND LEVEL DENSITIES**

Shell effects and orbital degeneracy make the spectrum of the s.p.1. in spherical nuclei highly nonuniform and this feature is reflected in the state and level densities calculated. In Fig.4 we show state densities calculated for several exciton numbers for the even-even nucleus \(^{56}\)Fe. The well known feature of a rapid increase of the state density with increasing exciton number is clearly seen. Deviations from the equidistant model manifest themselves in strong fluctuations of the state densities and in the increasing threshold energy for the excitation of configurations with in-
creasing exciton numbers. Fluctuations are pronounced mostly for low exciton numbers and tend to be smoothed out when more degrees of freedom participate in the excitation. In general, the fluctuations are smeared out as the density increases with excitation energy and mass number.

The nuclear deformation is expected to smooth the fluctuations making approaches based on the statistical assumptions (e.g. formula of Williams) more realistic. Therefore, we investigated deformation effect performing calculations for $^{27}$Al, $^{100}$Mo, and $^{170}$Er in the space of the s.p.s. by Seeger and Howard /5/ obtained for different values of the deformation parameter $\alpha_2$. The calculated state densities, summed over $M$, are shown in Fig.5 for the 1-particle 1-hole configurations of neutrons in three nuclei under consideration. The predictions of the Williams' formula /6/ are given for each case to provide the reference between the results for different deformations and to compare this simple description with the microscopic calculations. For this purpose the s.p.s. density $g$ was set to the standard value $A/26$ (note that we deal with the neutron gas only).

Using the sets of basis vectors derived from the spherical potential, very strong fluctuations in the state densities are observed for all nuclei.

Introduction of the small deformation to the nuclear potential ($\alpha_2=0.05$) leads to the splitting of the spin multiplets of the basis vectors which results in a significant smoothing of the state densities. In a nucleus as light as $^{27}$Al it is however not enough to bring the calculated state densities into the form which could be reproduced by any closed form expression. This conclusion remains valid also for much higher deformations $\alpha_2=0.1$ and $\alpha_2=0.2$. In particular, the comparison of the microscopic results with the predictions of the Williams' formula shows an evident nonadequacy of the latter in the low energy region (below 10 MeV) where the preequilibrium emission usually dominates. For the heavier nuclei even the small deformation $\alpha_2=0.05$ results in the qualitative changes in the calculated results. The gaps are partially filled and a remarkable structure of roughly 5 MeV width appears in the spectra. Increasing the deformation one observes that the valleys in the spectra are gradually filled on the expense of the bumps leading to the step-like curves for $\alpha_2=0.1$ and the relatively smooth ones for $\alpha_2=0.2$.

It is surprising, how well the Williams' formula describes the general trend of the state densities for two heavier nuclei, in spite of the very low exciton number and of the fact that no attempt has been undertaken to adjust the s.p.s. density $g$. For the deformations as high as $\alpha_2=0.2$ the Williams' formula may be considered exact, while for the less deformed nuclei the shell structure is expected to show up.

**SPIN DISTRIBUTION**

The validity of the statistical law describing the spin distribution of nuclear levels must be reviewed when applied to levels with fixed exciton numbers. The formula which reads
\[ R(J) = \frac{(2J+1)}{2(2\pi)^{1/2} \sigma^3} \exp\left(-(J+1/2)^2/2 \sigma^2\right) \] (1)

is derived under the assumption of a Gaussian distribution of spin projections M. We have analysed more than 1000 spin distributions for different configurations, energies and nuclei. We have found out that Eq.1 does very well for configurations containing at least 4 excitons (Fig.6). When there are only few levels in the energy interval the agreement is random as would be expected (Fig.7).

Let us focus our discussion on the spin cut-off parameter \( \sigma \) which contains all information concerning the spin distribution. Like the level and state densities, the spin cut-off parameter reveals strong fluctuations with excitation energy, which are caused by the nonuniform distribution of the s.p.l. spin projections.

Two types of energy dependence of the spin cut-off parameter are observed. The first has a 'logarithm like' shape, while \( \sigma \) is essentially constant or slightly linearly increasing with energy in the second (see Fig.8). These features are connected with the spin structure of the s.p.l. in the vicinity of the Fermi energy.

It must be stressed that the energy variations of the spin cut-off parameter for levels with fixed exciton numbers are solely due to the sequence of shell model levels. These variations cannot be traced back to the energy dependence of the spin cut-off factor of total level density (as used in the compound model) because the latter is caused mainly by the increase in the number of excitons with increasing excitation energy.

Due to the rather weak energy dependence of the spin cut-off parameter, we can disregard it for the time being to investigate the influence of the exciton and mass numbers. For this purpose, an energy average of the spin cut-off factors was performed for each configuration type in all nuclei considered. Averaged spin cut-off factor can be approximated by

\[ \frac{2}{3} \quad \frac{2}{3} \]

\[ \sigma(n) = cnA + 0.1A + 4 \quad ; c = 0.22 \] (2)

When the intercepts in exciton and mass number dependences are disregarded, formula 2 reduces to

\[ \frac{2}{3} \]

\[ \sigma(n) = cnA \] (3)

To account for the global energy trend, a certain energy dependence can be associated with the factor \( c \). In view of the relatively large spread of the points it seems justified to assume linear dependence given by the form

\[ c = 0.24 + 0.0038E \] (4)

There is a slight odd-even effect, which we have disregarded up to now. We have observed that \( \langle \sigma^2 \rangle \) for configurations of the type (1010) are about 5 units higher than \( \langle \sigma^2 \rangle \) for the (1100) and (0011) configurations,
in spite of the equal number of excitons in each of them. This difference reduces to 2 units if 4 exciton configurations are considered and essentially disappears for 6 exciton cases. This odd-even effect is due to the difference in the average spin of the s.p.l. below and above the Fermi level. Since higher spin s.p.l. are found at higher energies, particle type excitons carry in general more angular momentum than holes.

For the sake of completeness we have to devote some attention to the yrast lines for few quasiparticle configurations. This problem has been addressed already in Ref.7, where we have pointed out that one has to deal with a separate yrast line for each exciton number. In Fig.9, the results of the present calculations for some configurations types in 58Ni are shown. One observes that the restriction imposed by yrast lines are most pronounced for low exciton numbers. Pauli principle causes yrast lines for mixed configurations to be different from those containing only a single type of nucleon. The latter appear to be shifted to lower spins by one or two units.

PARITY DISTRIBUTION

Parity distributions were investigated in 56Fe and 136Ba. A few typical results of these calculations are presented in Figs.10 and 11. As expected, very strong fluctuations are observed in the case of 56Fe. For the two-exciton configurations oscillations between nearly only positive and nearly only negative parity levels are observed throughout the whole energy range, also on the average, the amplitude of the fluctuations decreases with increasing energy. For 8 exciton levels, the fluctuations are still very striking, at least up to 30 MeV. In the case of 136Ba, the higher density of s.p.l. with a more uniform parity distribution leads to a nearly constant and equal parity distribution for the levels containing 4 or more excitons. For lower exciton numbers, the fluctuations are still significant though much weaker than for 56Fe. In general, the results have oscillating character around the equal probability value.

BOUND STATE DENSITIES

In this section we deal with the configurations which are subject to the restriction that none of the particle type excitons is allowed to occupy any orbital lying above the nucleon binding energy. The particular interest devoted to those states is connected with the multistep compound mechanism as introduced by Feshbach, Kerman and Koonin./8/.

Two examples of the combinatorial calculations for the densities of bound states are shown in Fig.12. One observes that at a certain energy, the bound state densities begin to deviate from the unconditional exciton state densities and eventually decrease with increasing energy. This effect is more pronounced for low exciton numbers.
In Fig.13, we compare the spin cut-off factors for bound states with those obtained if all configurations are taken into account. Similar to the state densities the spin cut-off factors also fall below the values calculated when no restriction had been made. To explain this result, we have to recall that the increase of the spin cut-off factor for levels with a fixed exciton number is related to the high spin orbitals which become available when the excitation energy increases. This is however not the case if only bound states are considered. The boundary condition excludes configurations with particles promoted to orbitals above the binding energy and therefore highly excited states can be formed only by deep hole excitations, that carry low spin. Neglecting the energy dependence spin cut-off parameter for bound states may be parametrized by means of Eq.3 with $c$ taking a value around 0.26.

CONCLUSIONS

We have investigated the densities of few quasiparticle states in the frame of the shell-model with the pairing interaction accounted for in terms of the BCS theory. In the method applied, the pairing is explicitly related to the exciton configuration. The spin, parity and shell effects are also accounted for directly instead of being smeared out by a statistical treatment.

We have found that, in even systems, the pairing shifts the excited states to higher energy and that the ground state condensation energy plays a key role by determining the limit for this shift. For odd systems, we have found that also negative energy shifts are possible.

Our results show that the closed formulae for state densities, as derived from the equidistant model, fail to reproduce the state densities for spherical nuclei, especially at low energies and exciton numbers, e.g. in the region where preequilibrium emission dominates. This failure is due to the shell structure in the s.p.1. which may lead to large gaps and fluctuations in the spectra of states with a fixed number of excitons. Nuclear deformation is found to remove these discrepancies only partially. In general, closed formulae may be considered accurate enough only for strongly deformed medium-heavy and heavy nuclei.

As far as the spin distribution of few exciton levels is concerned, we have found that the assumption of a Gaussian distribution of spin projections holds for exciton numbers exceeding 3. An analysis of the spin cut-off factor and of the parity reveals strong fluctuations, that are reduced with the increase of the deformation, exciton number and mass of the nucleus. No dominance of either parity over a large energy range was observed in contradiction with previous results /9,10/.

The condition of having all excitons bound shows up in the state density and spin cut-off parameter energy dependence causing them to fall below the results obtained when the condition is not imposed. Configurations with four and more excitons are less sensitive to the 'binding condition'
which, in these cases, turns out to be important only at high excitation energies.

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LIST OF REFERENCES

Fig. 1 The spectrum of energy shifts induced by the pairing interaction for several even configurations.

Fig. 2 A comparison of the state densities calculated including the pairing interaction (solid lines) with the results of the noninteracting gas model (dashed lines) shifted to make the thresholds coincide.

Fig. 3 The spectrum of energy shifts induced by the pairing interaction for two odd configurations.
Fig. 4 The state densities for some specified configurations in 56Fe calculated with the pairing interaction.

Fig. 5 The densities of 1-particle 1-hole neutron states in 27Al, 100Mo, and 170Er calculated for different values of the deformation parameter $\alpha_2$ (histograms). Solid line represents predictions of the formula by Williams with $g=A/26$. 
Fig. 6 The calculated level spin distributions (histograms) compared with the predictions of Eq. 1 when $\mathcal{G}$ is taken as a free parameter (lines). Levels contained in the 1 MeV interval centered at the depicted energy are considered.

Fig. 7 The same as Fig. 6.

Fig. 8 The energy dependence of the calculated spin cut-off factor.

Fig. 9 The yrast lines for several configuration types in $^{58}$Ni.
Fig. 10 The fraction of positive parity states as a function of the excitation energy.

Fig. 11 The same as Fig. 10.

Fig. 12 Comparison of the bound (dashed lines) and unconditional (solid lines) state densities as a function of energy.

Fig. 13 The same as Fig. 12 but for the spin cut-off factor.
MICROSCOPIC OPTICAL MODEL CALCULATIONS
IN THE ENERGY RANGE 10-100 MeV

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ABSTRACT

Together with a short introduction into the basic concepts of various theoretical formulations of the optical model, microscopic calculations on nucleon-nucleus and light-ion nucleus optical potentials are reviewed. Although numerically tough, the nucleon-nucleus optical potential is quite well understood. The main assumptions and the limits of the nuclear matter approach and the nuclear structure approach are discussed. Particular emphasis is given to recent developments in the nuclear matter approach, as e.g. the improvement of the local density approximation or the temperature dependence. The microscopic treatments of light-ion nucleus optical potentials are presented in the second part. Considering the example of α-nucleus scattering the problems, successes and failures of such calculations are presented.
1. **General formulations**

The scattering of a nucleon or a light-ion on a nucleus is a rather difficult many-body problem which can hardly be described in its full microscopic detail. The dominant process is the elastic scattering of the two colliding particles. Limiting to the elastic channel the situation is very similar to the scattering of light on an opaque sphere. In particular one observes reflection, transmission and absorption phenomena like in optics. In the fifties this simple physical picture has led to the introduction of the so-called optical model [1] which describes the elastic scattering in detail whereas all other reactions are taken into account globally. In the last decades the optical model has become a very important tool in nuclear physics, especially for the evaluation of nuclear reactions.

Due to its importance the optical model has been the subject of an immense number of investigations. In principle one has to differentiate between theoretical considerations and works on phenomenological optical potentials. The latter group is very important from the practical point of view, especially for the calculation of nuclear reactions. But the weak justification of the phenomenological ansatz renders it difficult to draw reliable physical conclusions. In this contribution only microscopic calculations of optical potentials are reviewed emphasizing recent developments. With respect to applications in preequilibrium models of nuclear reactions the survey is limited to nucleon-nucleus and light-ion - nucleus optical potentials.

There exist several theoretical formulations of the optical model [1-5] with substantial differences in the intrinsic attributes stipulated for the optical potential. The central point of nearly all treatments of the optical model is the attempt to separate the dynamics of the relative motion from the internal dynamics of projectile and target states. Thus one ends up in all formulations with an optical potential which is an effective two-body operator. In general the optical potentials contain non-Hermitian parts which can be ascribed to absorption processes. Only for energies below the first inelastic threshold the optical potential should be a Hermitian operator. In the following some definitions of the optical potentials are briefly sketched.

A very elegant operator formulation of the optical potential has been given by Feshbach [1]. This formulation is called formalism independent because the optical potential operator in the elastic channel $\beta$, $V_{opt}(\beta,z)$, is given in terms of the corresponding two-cluster Hamiltonian $H^\beta$ and the interaction $V^\beta (H=H^\beta+V^\beta)$ only,

$$V_{opt}(\beta,z) = V^\beta + V^\beta Q^\beta \left[ z - Q^\beta H Q^\beta \right]^{-1} Q^\beta V^\beta , \quad (1)$$

Here, $P^\beta$ and $Q^\beta = 1 - P^\beta$ are projection operators, where $P^\beta$ projects onto the elastic channel $\beta$. In the scattering limit $z$ corresponds to the energy. The operator $V_{opt}$ is uniquely defined and can be
calculated in different ways. The elastic transition operator $T^β(β)$ is given by a Lippmann-Schwinger type equation,

$$T^β(β) = V_{\text{opt}}(β, z) + V_{\text{opt}}(β, z) [z - H^β]^{-1} P^β T^β(β),$$

(2)

which corresponds to a specific choice of the off-shell structure of $T^β$ thus satisfying Hermitian analyticity of the optical potential [6]. From the formal expression (1) it is evident that the optical potential of the operator formalism is nonlocal in configuration space and energy dependent due to channel coupling. Furthermore it becomes complex above the first threshold. The generalisation of (1) taking into account the full antisymmetrisation among all nucleons of the colliding nuclei is not trivial [6,7].

Other commonly employed formulations are the Fock-space treatments of the optical potential. Starting from the one-particle propagator in the presence of nuclear forces Bell and Squires [2] have shown that the nucleon self-energy $M$ (or mass operator) can be identified with the optical potential. Again Hermitian analyticity is satisfied and the resulting optical potential is complex, energy dependent and nonlocal in configuration space. Although the physical content is the same as in the operator method the Fock-space formulation of the optical potential represents probably another off-shell extension. However, the associated specific off-shell structure has never been investigated. The Fock-space definition is best suited for nucleon-nucleus optical potentials where the scattering problem can be considered as an extension of the bound state problem. In these cases the mass operator can be calculated by means of the well developed techniques of the many-body field theory taking into account the full antisymmetrisation. However, composite projectiles as well as center of mass effects present serious problems in the Fock-space formulation.

Finally, the more recent formulation of an optical potential by Kuo, Osterfeld and Lee [3] should be mentioned which is based on the equation of motion approach [8]. In contrast to the previous formulations they end up with an energy independent, complex and nonlocal optical potential thus clearly demonstrating the nonuniqueness of the optical potential. In principle, one can consider this optical potential as a generalisation of the energy independent effective interaction theory for bound-state problems into the scattering regime. A numerical hint on the existence of such energy independent optical potentials has been given by Perey and Buck [9] many years ago. They found phenomenologically a nonlocal complex but energy independent optical potential which was able to reproduce the neutron elastic scattering data in the energy range from 0.4-24 MeV.

Since the actual status of microscopic calculations of nucleon-nucleus optical potentials is satisfactory compared to the light-ion - nucleus case it is preferable to treat the two subjects separately. Hence, chapter 2 is devoted to nucleon-nucleus systems, whereas optical potentials for composite projectiles are reviewed
in chapter 3. Finally a remark on the connection between phenomenological optical potentials and their microscopic counterpart is given.

![Diagram showing approximations to the nucleon-nucleus optical potential](image)

Fig.1 Approximations to the nucleon-nucleus optical potential

2. **Nucleon-nucleus optical potentials**

2.1 Introduction

The microscopic calculation of the nucleon-nucleus optical potential belongs to the most basic and most challenging problems in nuclear physics. Usually the Fock-space formulation [2] is applied since the nucleon-nucleus optical model can be considered as an extension of the nuclear shell model to scattering states. In this formulation the optical potential is identified with the nucleon self-energy which can be evaluated using the well developed techniques of many body field theory as e.g. many-body Green’s functions equations or perturbative expansions in terms of the nucleon-nucleon interaction. However, one always has to rely on
certain approximations for the self-energy because an exact solution would mean solving the many-body problem itself. Basically we can distinguish between two types of approximations, the nuclear matter and the nuclear structure approach. Fig. 1 gives a crude overview of the approximations used.

2.2 The nuclear matter approach

In the nuclear matter approach the scattering of a nucleon on an infinitely extended nuclear medium is considered. This situation corresponds to a large target limit (A → ∞) and simplifies the underlying equations essentially. Since the nucleon-nucleon force is not weak one cannot calculate the self-energy by perturbation theory directly. The generalisation of the Bethe-Brueckner theory to the scattering problem is the adequate tool to attack the problem. In the Bethe-Brueckner theory [10] one sums up all graphs of the perturbation expansion which have the same number of hole lines. This so-called hole-line expansion should be appropriate for low densities [11] as they are found in the interior of nuclei. Most of the calculations are limited to the Brueckner-Hartree-Fock approximation which takes into account one hole-line graphs (Fig. 2a) only. Higher order terms with more than one hole-line are neglected (Fig. 2b).

\[ U(k_F, k, E) = \sum_{|p| < k_F} \langle \vec{p} \mid t(\omega) \mid \vec{k}, \vec{p} \rangle \mathcal{A} = V(p, E) \]  \hspace{1cm} (3)

where \( |q\rangle \) (\( q = k, p \)) denotes a normalised plane wave of momentum \( \vec{q} \) and the index \( \mathcal{A} \) refers to antisymmetrisation of the two-nucleon wave function. From (3) it is evident that the averaged potential \( U \) depends on the Fermi momentum \( k_F \) and thus on the nuclear density \( p \) assuming nuclear matter to be an isotropic, symmetric and
noninteracting Fermi gas. The central point of the nuclear matter approach is the determination of the reaction matrix \( t(\omega) \) which is the solution of the Bethe-Goldstone equation,

\[
t(\omega) = v + v \sum_{q_1, q_2} \frac{\langle q_1, q_2 \mid q_1, q_2 | q_1, q_2 \rangle}{\omega - \epsilon(q_1) - \epsilon(q_2) + i\delta} t(\omega) \tag{4}
\]

Here, \( v \) is a realistic free nucleon-nucleon interaction, \( \epsilon(q) \) is the single particle energy of a state of momentum \( q \) and \( \omega \) is the propagation energy which should be taken selfconsistently [11]

\[
\omega = E + \epsilon(p) \quad \text{with} \quad \epsilon(p) = \frac{p^2}{2m} + \text{Re}[U(k_F, p, \epsilon)] \tag{5}
\]

The Pauli principle is taken into account by the restricted summation which eliminates the propagation in occupied states. Standard techniques are available to solve (4) numerically [12,13].

In order to construct the optical potential for finite nuclei from the nuclear matter results one has to apply some type of local density approximations. There are mainly two methods in use. Mahaux and coworkers [14] start with the simple local density approximation (LDA) which assumes that the optical potential at a given location \( \vec{r} \) in the nucleus corresponds to the nuclear matter result for the nuclear density \( \rho \) at the point \( \vec{r} \),

\[
V_{\text{opt}}^{\text{(LDA)}} (\vec{r}, E) = V(\rho(\vec{r}), E) \tag{6}
\]

This simple approximation reproduces the phenomenologically determined volume integrals per nucleon quite well [14]. However, it fails in the calculation of the root mean square radii which are always too small as compared with the experimental data. The main reason for this deficiency lies in the fact that the local density approximation is valid only for zero-range effective interactions. Therefore Jeukenne et al. [15] improved the local density approximation (ILDA) in a phenomenological way by folding with a Gaussian function

\[
V_{\text{opt}}^{\text{(ILDA)}} (\vec{r}, E) = (t \sqrt{\pi})^{-3} \int d^3 r' V_{\text{opt}}^{\text{(LDA)}} (r', E) \exp\left[-\frac{|\vec{r} - \vec{r}'|^2}{2t^2}\right] \tag{7}
\]

The new parameters \( t \) (the real and imaginary part are folded separately with different parameters \( t \)) are determined by fitting the scattering data. Usually they are of the order \( t=1.2 \text{ fm} \) and correspond to the range of the effective interaction. Although the root mean square radii can be calculated fairly well [14] by this so-called improved local density approximation the potential is probably not reliable in the tail of the nuclear surface.
Indications of the failure are wrong dependences on mass number and energies of Woods-Saxon parameters as compared with phenomenological values. The Woods-Saxon approximation recently introduced by Jaminon et al. [16] can repair this discrepancy.

Another method for the construction of the optical potential for finite nuclei from the nuclear matter results is the folding model approach [17-19]. Instead of evaluating the self-energy directly, the procedure starts with the generation of a complex effective nucleus-nucleon interaction in nuclear matter from the reaction matrix $t$ following a suggestion of Siemens [20]. In principle the reaction matrix can be divided into a direct term $t^d(|\vec{r}_1-\vec{r}_2|,\rho;E)$ and an exchange interaction $t^{ex}(|\vec{r}_1-\vec{r}_2|,\rho;E)$ and depends on the relative distance $|\vec{r}_1-\vec{r}_2|$ of the two nucleons, the nuclear density $\rho$ and the incident energy $E$ of the nucleon. Adopting the idea of the local density approximation to the folding procedure the optical potential is given by

$$V^{\text{(fold)}}_{\text{opt}}(\vec{r},E) = \int d^3s \rho(s) t^d(|\vec{r}-\vec{s}|,\rho(\frac{\vec{r}+\vec{s}}{2};E)$$

$$+ \int d^3s \rho(\vec{r},\vec{s}) t^{ex}(|\vec{r}-\vec{s}|,\rho(\frac{\vec{r}+\vec{s}}{2};E) j_0(k|\vec{r}-\vec{s}|), (8)$$

$j_0$ being the spherical Bessel function. In principle the folding potential is nonlocal due to the exchange term. For simplicity equation (8) gives the equivalent local potential which is obtained in the local momentum approximation,

$$k(r) = \sqrt{\frac{2m}{\hbar^2} (E-U(r))} . \quad (9)$$

The nuclear density $\rho(\vec{r})$ as well as the single-particle mixed densities $\rho(\vec{r},\vec{r}')$ can be easily calculated with single-particle wave functions of a phenomenological shell model or Hartree-Fock calculations. The use of experimental densities is also possible. In this case the mixed density $\rho(\vec{r},\vec{r}')$ must be determined in certain approximations [19].

The nuclear matter approach has the advantage that it is based on a realistic nucleon-nucleon interaction for a nearly parameter-free calculation of the optical potential. Usually, only the effective range parameters in the improved local density approximation or the strength parameters of the folded potential must be varied slightly in order to reproduce the experimental data fairly well. A very recent analysis of neutron-elastic scattering data [21] demonstrates the quality which can be achieved. Although it is often argued that the local density approximation is only justified at high energies, reasonable potentials are also obtained for 20 MeV nucleon-nucleus scattering. Nevertheless, one must be aware that the nuclear matter approach does not take into account
any specific effects which are due to the finiteness of nuclei, as e.g. collective shape oscillations, center of mass corrections etc.

2.3 The nuclear structure approach

The nuclear structure approach of Vinh Mau and Bouyssy [22] and Van Glai [23] is in some sense complementary to the nuclear matter calculations and corresponds to another summation of the diagrammatic expansion of the nucleon self-energy. In particular the summation is performed up to second order in the projectile-target-nucleon interaction. The particle-hole interactions are treated in all orders. This leads in contrast to the nuclear matter approach, to an optical potential which takes explicitly into account intermediate channels corresponding to inelastic excitations of the finite nucleus. Since only graphs up to second order in the particle-particle interaction are included (Fig.2a) the calculations must be performed with weak effective nucleon-nucleon interactions. The resulting potential consists of two terms,

\[ V_{\text{opt}}(\mathbf{r}, \mathbf{r}'; E) = V^{\text{HF}}(\mathbf{r}, \mathbf{r}') + V^{(2)}(\mathbf{r}, \mathbf{r}'; E) \]  \hspace{1cm} (10)

The first term \( V^{\text{HF}} \), the Hartree-Fock potential, dominates the real part of the optical potential. It is a real but energy independent term. The second order contributions are given by \( V^{(2)} \) which is generally a complex interaction and represents the leading term of the imaginary part of the optical potential. In practice only the imaginary part is calculated,

\[ \text{Im} \, V^{(2)}(\mathbf{r}, \mathbf{r}'; E) = \text{Im} \sum_{N \neq 0} <0|v|N>_{\tau} \mathcal{G}^{N}(\mathbf{r}, \mathbf{r}') <N|v|0>_{\tau'} \]  \hspace{1cm} (11)

The sum is taken over all open inelastic channels which are described by RPA wave functions \( |N> \). The matrix elements of the nucleon-nucleon interaction \( v \) between \( |N> \) and the target ground state \( |0> \) are over the internal coordinates of the target only. Furthermore, \( \mathcal{G}^{N} \) is the projectile propagator in the intermediate states at an energy \( E-E_{N} \), \( E_{N} \) being the excitation energy of the target nucleus in the state \( |N> \). For clearness, the exchange terms have been neglected in (11). Including particle identity [22,23] leads to more complicated expressions but does not change the structure essentially. The evaluation of the real part of \( V^{(2)} \) is more complicated since it does not reduce to a simple sum thus causing numerical convergence problems. However, the contribution is expected to be relatively small and is neglected.

The choice of the effective nucleon-nucleon interaction is somewhat ambiguous. From the microscopic point of view the same interaction \( v \) should be used for the Hartree-Fock problem, the determination of the RPA wave functions and the evaluation of the coupling term \( V^{(2)} \). However, such a consistency is rather hard to achieve. Furthermore, there arises also the problem of double counting due to the use of effective forces. Several very
sophisticated calculations have been performed on the n—40Ca, p—40Ca and the n—208Pb optical potentials [22-27]. The corresponding elastic cross sections resulting from these potentials show the gross structure of the experimental data but the reproduction is far from being satisfactory. All investigations indicate that the inclusion of only inelastic channels as doorways is not sufficient to obtain the right amount of absorption.

2.4 Recent developments

Since 1985 no calculation of the nucleon-nucleus optical potential in the framework of the nuclear structure approach has been reported to my knowledge. However, there have been some remarkable developments related with the nuclear matter approach recently. The investigations are focussed on the improvement of the local density approximation.

A very important work has been performed by Brugger et al. [28,29]. Based on the ladder approximation they have formulated a semiclassical expansion of the microscopic optical potential which has as its lowest order term the local density approximation of the nuclear matter approach. The theoretically derived second order terms exhibit the dependence on curvature effects and represent corrections beyond the local density approximation. A numerical calculation in a schematic model with 208 nucleons (it is not 208Pb since isospin effects have been ignored) indicates clearly that the second order corrections are restricted to the surface region. They flatten the total potential at the surface shifting the rms radii to larger values as obtained from phenomenological analyses. The global properties, such as e.g. the volume integral of the potential remain unchanged. In agreement with the physical expectation the corrections are vanishing with increasing energy. However, between 20 and 100 MeV they are still of considerable size and should be taken into account. Furthermore it has been shown that the phenomenological folding in the improved local density approximation can simulate the correction terms. Up to now the evaluation of higher order terms is not possible and therefore the question whether the expansion converges is still open.

Recently a density matrix expansion for the microscopic optical potential has been formulated [30]. The method is related to the expansion of Negele and Vautherin [31] and avoids the doubtful localization procedure for the effective nucleon-nucleon force in the folding potential.

With respect to the description of preequilibrium processes a recent investigation of the temperature dependence of the optical potential should be mentioned [32]. In this study the nucleon self-energy in nuclear matter is calculated using almost all kinds of phenomenological Skyrme-interactions which can reproduce the nuclear ground state and the optical potential at zero temperature simultaneously. Since a phenomenological effective interaction is used we must only include the lowest order contributions of the perturbation expansion in the determination of the self-energy. In
this approximation the optical potential is the sum of the Hartree-Fock potential \( V_{HF} \) and a second order term \( W^{(2)} \) which represents the lowest order contribution to the imaginary part. Both terms depend on the occupation number \( n_\alpha \) of the single particle state \( |\alpha\rangle \) with energy \( \varepsilon_\alpha \). Usually the expectation numbers \( n_\alpha \) at zero temperature are taken

\[
    n_\alpha = \begin{cases} 
        1 & \varepsilon_\alpha < \varepsilon_F \\
        0 & \varepsilon_\alpha > \varepsilon_F 
    \end{cases},
\]

(12)

\( \varepsilon_F \) being the Fermi energy. From statistical mechanics the distribution \( n_\alpha \) at finite temperature \( T \) is given by

\[
    n_\alpha = \left[ 1 + \exp \left( \frac{\varepsilon_\alpha - \mu}{T} \right) \right]^{-1},
\]

(13)

where \( \mu \) is the chemical potential which is equal to \( \varepsilon_F \) for \( T=0 \). At normal nuclear matter densities \( \rho \approx 0.17 \text{ fm}^{-3} \) the chemical potential is decreasing with increasing temperature. In order to construct the optical potential for finite nuclei a simple local density approximation is applied. Evaluations of the \( n-\text{Ca} \) and \( n-\text{Pb} \) optical potentials in the energy range from 10 to 50 MeV show an increasing absorption with increasing temperature (\( T=0-6 \) MeV). This behaviour can easily be understood by the temperature dependence of the occupation numbers. The real part becomes shallower with increasing temperature but the dependence is rather weak. These features should be taken into account when the scattering of a nucleon on an excited target is considered.

3. Optical potentials for composite projectiles

3.1 Survey and problems

Microscopic calculations of the light-ion nucleus optical potentials are not as advanced as it is the case in the nucleon-nucleus system. The main reason is the fact that in composite particle systems the center of mass distance between the colliding nuclei is a collective coordinate. Therefore many-body techniques are not applicable directly. Moreover, due to particle identity there exist partly Pauli-forbidden states in composite particle systems. As a consequence the relative motion wave function is not normalised to unity and thus arises the problem of the interpretation, especially in the overlap region. Investigations of three-cluster systems [33, 34] suggest the use of an off-shell transformation of the relative motion wave function in order to restore its interpretation as a probability amplitude. These difficulties are responsible that only crude approximations to the microscopic optical potential for composite projectiles have been realised to date. Fig. 3 gives a synopsis of the approximations used for the calculation of the light-ion nucleus optical potential.
Fig. 3 Approximations for the optical potential for nucleus-nucleus scattering. Microscopic entries are underlined.
The folding model which has often been applied in analyses of elastic scattering data [35,36] is rather phenomenological. Especially the effective nucleon-nucleon interaction is taken to be of simple form. Apart from the folding model the different approaches are discussed in more detail in the following subsections. I report only on optical potential calculations in \(\alpha\)-particle nucleus systems, but also \(d, t\) and \(^3\)He scattering have been treated microscopically. Recently, calculations in the nuclear matter approach for \(t\) [37,38] and \(^3\)He [39] have been published.

3.2 The RGM optical potential

The single channel resonating-group method (RGM) [40] is an appropriate tool to study the real part of the optical potential. The RGM is based on a weak, phenomenological effective nucleon-nucleon interaction \(v\) and takes full account of the antisymmetrisation between projectile and target nucleons. The RGM potential is given by

\[
\langle \tau | T^R + V^{RGM} - E' | \tau ' \rangle = \langle \psi_A \psi_B | \sum_{iA} \sum_{jB} \sum_{i,j} \mathcal{A} | \psi_A \psi_B \rangle,
\]

where \(T^R\) is the kinetic energy of the relative motion, \(E'\) is the scattering energy, \(\mathcal{A}\) is the antisymmetrisation operator and \(\psi_A, \psi_B\) denote the ground state wave functions of target A and projectile B, respectively. The projectile and target ground states are usually described by the harmonic oscillator shell model. In order to construct an optical potential a phenomenological imaginary term \(W\) is added.

\[
V_{\text{opt}} = V^{\text{RGM}} + \sqrt{N} \ i \ W \ \sqrt{N}
\]

where \(N\) is the norm integral kernel.

Recently, Wada and Horiuchi [41] have performed such a calculation in the \(\alpha-^{16}\)O system. They used a squared Woods-Saxon form

\[
W(r) = - W_0 / \left\{ 1 + \exp \left[ (r-R/r)/a_r \right] \right\}^2
\]

with fixed values \(W_0 = 25\) MeV and \(a_r = 0.65\) fm for the imaginary part. Adjusting only the radius parameter \(R\) they succeeded in reproducing the scattering cross sections up to 70 MeV surprisingly well. The radius parameter is slightly increasing with increasing energy thus simulating the stronger absorption. To my knowledge this is the first time that an RGM calculation gives good agreement in such an extended energy interval.

3.3 The nuclear structure approach

It is a great challenge to calculate also the imaginary part of the optical potential for composite projectiles. For this
purpose it is necessary to start from the operator formulation (1) of Feshbach [1]. The second term of (1) takes into account the coupling to other channels and is responsible for the imaginary part. There have been several attempts to calculate this term for $\alpha$-$^{40}$Ca scattering [42-44] treating the $\alpha$-particle as an elementary particle. Neglecting antisymmetrisation and considering second order contributions to the optical potential with respect to the effective projectile target-nucleon interaction only, the imaginary part $W^{(2)}$ is given by

$$W^{(2)}(\vec{r}, \vec{r}'; E) = \text{Im} \sum_{N=0}^{\infty} \langle \vec{r} \Psi_N | V_{NB} | \vec{r} \Psi_N \rangle \ g^N(\vec{r}, \vec{r}'; E) \ < \vec{r} \Psi_N | V_{NB} | \vec{r} \Psi_N \rangle$$

(17)

where $|\Psi_N\rangle$ denotes the intermediate target excited states of energy $E_N$. $V_{NB}$ is the projectile target-nucleon interaction, and $g^N$ is the intermediate projectile Green's function calculated at the energy $(E-E_N)$. These calculations take explicitly into account all energetically open inelastic channels and are therefore known as the nuclear structure approach in analogy to the nucleon-nucleus system.

The first order potential which dominates the real part corresponds to the RGM potential. For simplicity, however, first calculations [42,43] use a phenomenological potential focussing their interest completely on the imaginary part. Only the most recent calculation determines the first order potential by the so-called fish-bone model [45] which contains the essential features of the RGM potential. The complex structure of the optical potential requires a complete nonlocal calculation of the elastic differential cross section [46]. In the energy range from 26-36 MeV the calculated differential elastic cross sections show the gross structure of the experimental data. But one cannot speak of a reproduction of the data because the inelastic channel accounts only for about 50% of the reaction cross section. It is outlined that the correct treatment of the Pauli principle in the intermediate channels as well as the inclusion of transfer channels will be essential.

3.4 The nuclear matter approach

The nuclear matter approach to the nucleon-nucleus optical potential can be easily extended to nucleus-nucleus systems by performing a double folding procedure [47] using similar effective nucleon-nucleon forces,

$$V_{\text{opt}}(\vec{r}) = \lambda \int d^3 r_A \rho_A(\vec{r}_A) \int d^3 r_B \rho_B(\vec{r}_B) \ t(\vec{r}+\vec{r}_B-\vec{r}_A, \rho_A, \rho_B, E)$$

(18)

Here, $\lambda$ is a strength parameter which is adjusted to the experimental data, and $\rho_A$ and $\rho_B$ are the projectile and target ground state densities, respectively. Furthermore some phenomenological corrections are often made in order to account for antisymmetrisation effects [47].
Recently, Kobos et al. [48] have proposed a folding model for the real part based on a density dependent modification of the M3Y force [49]. It is not a complete nuclear matter approach because the imaginary part is added phenomenologically. An application to $\alpha$-nucleus scattering [50] gives not only overall good fits but shows a nearly constant value of $\lambda=1.30$. Furthermore it has been shown that the application of such potentials in three-nucleon transfer reactions improves the normalisation considerably [51].

4. **Summary**

I have tried to give a comprehensive but nevertheless concise review on microscopic calculations of optical potentials. The different possible formulations show the complexity and the ambiguous nature of the problem.

Summarising it is fair to say that the nucleon-nucleus optical potential is conceptually quite well understood and due to the well developed techniques of many-body field theory numerical calculations are feasible. The nuclear matter approach allows even quantitative calculations of the elastic and inelastic scattering observables. Furthermore global features of empirical optical potentials can be explained.

The situation is worse for light-ion nucleus optical potentials. Serious conceptual problems arise from the compositeness of the projectile. Here, microscopic calculations are still at the beginning and crude approximations are required in order to retain numerical calculations feasible. The nuclear structure approach can reproduce the gross structure of the elastic differential cross sections but suffers from a lack of absorption. Promising results can be obtained in the nuclear matter approach adjusting few parameters phenomenologically.

It is certainly a great challenge to calculate the optical potential starting from first principles. But one should always bear in mind that the optical potential is an effective operator when one tries to determine the "physical" potential. In particular this is valid in the comparison of microscopic and phenomenological optical potentials. The only reliable point of connection is and remains the elastic S-matrix.

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RECENT RESULTS IN THE DEVELOPMENT OF A GLOBAL MEDIUM-ENERGY NUCLEON-NUCLEUS OPTICAL-MODEL POTENTIAL

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ABSTRACT

Initial results are presented for the determination of a global medium-energy nucleon-nucleus phenomenological optical-model potential using a relativistic Schrödinger representation. The starting point for this work is the global phenomenological optical-model potential of Schwandt et al., which is based on measured elastic scattering cross sections and analyzing powers for polarized protons ranging from 80 to 180 MeV. This potential is optimally modified to reproduce experimental proton reaction cross sections as a function of energy, while allowing only minimal deterioration in the fits to the elastic cross sections and analyzing powers. Further modifications in the absorptive potential were found necessary to extrapolate the modified potential to higher energies. The final potential is converted to a neutron-nucleus potential by use of standard Lane model assumptions and by accounting approximately for the Coulomb correction. Comparisons of measured and calculated proton reaction and neutron total cross sections are presented for $^{27}$Al, $^{56}$Fe, and $^{208}$Pb. Medium-energy optical-model potentials for complex projectiles are briefly discussed in an appendix.
INTRODUCTION

Realistic calculations of specific medium-energy proton- or neutron-induced reactions that take into account distortion effects and total flux conservation require an optical-model potential that satisfactorily reproduces the elastic scattering and integrated scattering observables at these energies. In particular, the observed elastic differential cross section $\sigma(\theta)$ and analyzing power $A_y(\theta)$ should be well reproduced in order that distortion effects are correctly described, and the observed proton total reaction cross section $\sigma_R$ or neutron total cross section $\sigma_T$ should be well reproduced in order that the calculated specific reactions all sum to the correct physical value. In this work the status of a global phenomenological nucleon-nucleus optical-model potential for medium-energy scattering that is being developed$^{1,2}$ is presented. This potential primarily reproduces the integrated scattering observables and secondarily, the elastic scattering observables. Described here is how the potential is obtained and how its predictions compare with the measured observables.

METHOD

The starting point of this work is the global phenomenological proton optical-model potential of Schwandt $et$ $al.$$^3$ This potential is based upon experimental elastic scattering and analyzing power angular distributions for target masses $A$ and incident proton energies $E_p$ in the ranges

$$24 \leq A \leq 208, \text{ and }$$

$$80 \text{ MeV} \leq E_p \leq 180 \text{ MeV},$$

respectively. The data were analyzed in the framework of a relativistic Schrödinger-type wave equation$^4$ generated by appropriate reduction of the Dirac equation for a massive, energetic fermion moving in a localized, central potential $V(r)$. If the potential $V(r)$ is chosen to be the fourth (time-like) component of Lorentz vector potential, the reduced two-body problem with relativistic projectile mass and non-relativistic target mass then leads to a relativistic radial wave equation for the $L$th partial wave that is of the same form as for the conventional non-relativistic Schrödinger equation, namely,

$$\left\{ \frac{d^2}{d\rho^2} + \left[ \frac{1}{T_c} - \frac{\gamma V(\rho)}{\rho^2} - \frac{\kappa L(\ell+1)}{\rho^2} \right] \right\} F_L(\rho) = 0 .$$

In this equation, $\rho = kr$ where $k$ is the relativistic wave number, $T_c$ is the total center-of-mass kinetic energy, and $\gamma$ is a factor by which the potential is renormalized in the relativistic calculation,

$$\gamma = 1 + T_c/(T_c + 2m) .$$

Using this formalism, an energy-dependent complex potential of Woods-Saxon form was assumed and best-fit parameters were obtained for each individual experimental data set by performing least-squares adjustments. The resulting sets of parameters were then examined for simple dependencies with respect to the incident proton energy, the target mass number, and the target asymmetry parameter $(N-Z)/A$. In this way, Schwandt $et$ $al.$$^3$ obtained the following global phenomenological proton potential:

$$V_R = 105.5(1 - 0.1625 \ln E_p) + 16.5 \frac{(N-Z)}{A} ,$$

- 104 -
\[ r_R = 1.125 + \frac{E_p}{10^3}, \quad E_p \leq 130 \text{ MeV} \]
\[ = 1.255, \quad E_p > 130 \text{ MeV} \]
\[ a_R = 0.675 + 3.1 \frac{E_p}{10^4}, \] (6)
\[ W_V = 6.6 + 2.73 (E_p - 80)/10^2 \]
\[ + 3.87 (E_p - 80)^3/10^6, \] (7)
\[ r_I = 1.65 - 2.4 \frac{E_p}{10^3}, \] (8)
\[ a_I = 0.32 + 2.5 \frac{E_p}{10^3}, \] (9)
\[ V_{SO} = 19.0(1 - 0.166 \ln E_p) - 3.75 (N-Z)/A, \] (10)
\[ W_{SO} = 7.5(1 - 0.248 \ln E_p), \] (11)
\[ r_{VSO} = 0.920 + 0.0305 A^{1/3}, \] (12)
\[ a_{VSO} = 0.768 - 0.0012 E_p, \quad E_p \leq 140 \text{ MeV} \]
\[ = 0.60, \quad E_p > 140 \text{ MeV} \] (13)
\[ r_{WSO} = 0.877 + 0.0360 A^{1/3}, \text{ and} \]
\[ a_{WSO} = 0.62, \] (15)

where the units are MeV and fermis. The only deviations found in Eqs. (4)-(15) were that (a) \( r_{VSO} = 0.98 \) and \( r_{WSO} = 0.96 \), for \(^{24}\text{Mg}\) and \(^{28}\text{Si}\) at 135 MeV, and (b) the transition point in \( r_R \), Eq. (5), for Ca occurs at 180 MeV instead of 130 MeV.

Using this potential as a starting point, the goal is defined as a global phenomenological nucleon-nucleus optical-model potential valid for target masses and incident nucleon energies in the ranges

\[ 24 \leq A \leq 208, \text{ and} \]
\[ 50 \text{ MeV} \leq E_p, E_n \leq 400 \text{ MeV}, \] (16)

respectively. Comparing Eqs. (1) and (16), one sees that there are two tasks. The first is to extend the energy range of the proton potential downwards to 50 MeV and upwards to 400 MeV. The second is to attempt to transform the extended proton potential to a neutron potential valid for the same energy range.

The method used consists of the following: Only the parameters of the proton central absorptive potential \((W_V, r_I, a_I)\) are adjusted to optimally reproduce the experimental proton total reaction cross section \( \sigma_R \) in the extended energy range. All other parameters remain at their original values. In particular, the spin-dependent absorptive potential \((W_{SO}, r_{WSO}, a_{WSO})\) remains
unchanged due to the fact that, at this stage, experimental spin-dependent observables have not yet been included from the extended energy range in the analysis. The adjustments to the parameters of the absorptive potential are performed allowing only small changes in the calculated elastic scattering observables. Assuming a satisfactory proton potential is obtained with this method, the transformation to the corresponding neutron potential is made by using a simple Lane model, namely,

\[
\frac{N-Z}{A} \rightarrow -\frac{N-Z}{A} ,
\]

wherever this factor appears [in \( V_R \), Eq. (4), and in \( V_{SO} \), Eq. (10)], and by using a simple Coulomb correction in the real central potential, namely,

\[
V_{CORR} = 0.4Z/A^{1/3} .
\]  

(18)

Note that no Coulomb correction explicitly appears in the real central part of the original potential, Eq. (4). Thus, to obtain the neutron potential, one assumes that this correction is present implicitly in the real central part of the proton potential so that subtraction of Eq. (18) completes the transformation.

All of the calculations reported in this work have been performed using the optical-model code SNOOPY8 with relativistic wave equation (and relativistic kinematics) given by Eq. (2). A measure of the influence of relativistic effects is given in Figs. 1 and 2 where the calculated proton total reaction cross section and neutron total cross section are shown, respectively, as a function of projectile energy for three relativity options, using fixed parameter sets for protons and for neutrons. Clearly, for energies above 50 MeV, relativistic effects cannot be ignored where high accuracy is required. The magnitudes of the differences between the fully relativistic Schrödinger and nonrelativistic Schrödinger representations are 2.2% at 50 MeV, 4.6% at 100 MeV, and 10.5% at 400 MeV, for the proton reaction cross section, and are 1.3% at 50 MeV, 7.0% at 100 MeV, and 14.7% at 400 MeV, for the neutron total cross section.

Preliminary results have been obtained by considering the target nuclei \(^{27}\text{Al} \), \(^{56}\text{Fe} \), and \(^{208}\text{Pb} \), together with corresponding experimental proton total reaction cross sections and neutron total cross sections for the energy range given by Eq. (16). It was observed that the original proton potential of Schwandt et al. predicts a total reaction cross section \( \sigma_R \) that increases strongly with increasing incident energy as one approaches the upper end of the range of validity for the model, 180 MeV, and in fact, diverges as the energy is increased further (beyond the range of the potential). The source of this divergence is the strong energy dependence chosen for the strength \( W_V \) of the central absorptive potential, namely, a third-order polynomial in the incident energy, as given in Eq. (7).

This problem is addressed by dividing the modified potential into two regions: a lower energy region in which no divergence occurs in \( \sigma_R \) and an upper energy region where the divergence in \( \sigma_R \) begins and grows with increasing energy. The form of \( W_V \) is then changed in the upper region so as to remove the divergence. An energy grid of 31 points was chosen to span the range given by Eq. (16), and a number of survey calculations were performed to determine the dividing point between the two regions. In this way, the value 140 MeV was determined. Thus, the two regions of the modified potential are defined as follows:

Region I: 50 MeV \( \leq E_p, E_n \leq 140 \) MeV, and

Region II: 140 MeV \( < E_p, E_n \leq 400 \) MeV.

(19)
A study was then performed using the same energy grid, in which various forms of the absorptive potential were tested and optimized with respect to reproduction of the experimental proton total reaction cross section, while minimizing the deviations from the elastic differential cross sections and analyzing powers calculated with the original potential (the approximation has been made here that these are identical to the experimental values). This study, for protons incident on $^{27}$Al, $^{56}$Fe, and $^{208}$Pb, lead to the following adjustments in the central absorptive part of the original potential of Schwandt et al.$^3$:

**Region I:**

$W_V$ is unchanged.

$r_I$ is unchanged, and

$$a_I = 0.27 + 2.5 \ E/10^3.$$  \hfill (20)

**Region II:**

$$W_V = 7.314 + 0.0462E,$$

$r_I = 1.17$, and

$$a_I = 0.59 \ for \ ^{27}$Al \hfill (21)

$$= 0.66 \ for \ ^{56}$Fe \hfill (22)

$$= 0.79 \ for \ ^{208}$Pb . \hfill (23)

In Eqs. (20) and (21), E is either the incident proton energy $E_p$ or the incident neutron energy $E_n$. The original proton potential of Schwandt et al.$^3$, Eqs. (4)-(15), together with the adjustments given by Eqs. (20)-(23), define the modified proton potential for the energy range given by Eq. (16). Similarly, the modified neutron potential for the same energy range is given by the identical equations together with the transformation given by Eqs. (17) and (18). It is noted here that neutron scattering observables were *not* used to determine the adjustments, Eqs. (20)-(23). Thus, a good test of the assumed transformation to neutron scattering, given by Eqs. (17) and (18), will be obtained in subsequent comparisons to the neutron measurements (next section). It is also noted here that the modified potential is *discontinuous* across the 140-MeV boundary, for both proton and neutron scattering. This is a serious drawback of the modified potential and it will have to be corrected in the final version. Nevertheless, even in its present form, the discontinuous potential does *not* lead to large corresponding discontinuities in the calculated observables.

**RESULTS**

The results obtained using the modified potential for proton and neutron scattering by $^{27}$Al, $^{56}$Fe, and $^{208}$Pb are now discussed. First, the results for $^{27}$Al are shown in Figs. 3-8. In Fig. 3, experimental proton total reaction cross sections for $^{27}$Al are compared with values calculated from the original potential of Schwandt et al.$^3$, for its energy range given by Eq. (1), and with values calculated from the modified potential, for its energy range given by Eq. (16). Although the experimental data are sparse, they clearly constrain the central absorptive potential of the modified potential to an energy dependence that is well approximated by a linear assumption above 140 MeV. Overall, the agreement with experiment is good. On the other hand, the third-order assumption of the original absorptive potential clearly leads to unphysical values of $\sigma_R$ in the upper half of its energy range.

In Fig. 4, experimental neutron total cross sections for $^{27}$Al are compared with values calculated from the original potential of Schwandt *et al.*, transformed to neutron scattering via Eqs. (17) and (18), and with values calculated from the modified potential, also transformed to neutron scattering via Eqs. (17) and (18). The energy ranges for the two potentials are, again,
given by Eqs. (1) and (16), respectively. The experimental neutron total cross sections constrain the central absorptive potential of the modified potential to an energy dependence that is, again, well approximated by a linear assumption above 140 MeV. Overall, the agreement with experiment is good, except in the region between approximately 100 and 200 MeV where the calculated values are 6-7% high -- possibly indicating deficiencies in the transformation given by Eqs. (17) and (18). Similar to the proton results, the third-order assumption of the original absorptive potential tends to produce unphysical values of \( \delta_T \) in the upper third of its energy range.

In Figs. 5-8 the changes in the elastic scattering and analyzing power angular distributions that are caused by the modifications to the original potential are examined. For comparison purposes, it is assumed that the original potential predicts these experimental observables very well and therefore that a comparison of the predictions of the modified potential to those of the original potential is equivalent to comparisons with experiment. Energies that are 10 MeV below and above the boundary point of 140 MeV have been chosen to compare the elastic scattering in Figs. 5 and 6, and to compare the analyzing powers in Figs. 7 and 8. The elastic scattering hardly changes at all in Region I, for the entire angular range, and changes significantly in Region II only at angles above about 75° where it is approximately six orders of magnitude down from the forward scattering. Thus, the changes in the elastic scattering due to the modifications of the potential are small. The same conclusion is reached for the analyzing power, which hardly changes at all in Region I, and changes significantly in Region II only at angles above about 95°.

Results entirely similar to those just described for \(^{27}\text{Al}\) are obtained for \(^{56}\text{Fe}\), shown in Figs. 9-14, and for \(^{208}\text{Pb}\), shown in Figs. 15-20. Note, however, that the experimental situation for the proton total reaction cross sections on \(^{208}\text{Pb}\), Fig. 15, is somewhat ambiguous and that the tendency toward diverging \( \sigma_R \) values calculated from the original potential is just beginning, near 180 MeV. Also note that in the case of the neutron total cross sections, the modified potential yields predictions that are within approximately 5% for \(^{56}\text{Fe}\), Fig. 10, and also approximately within 5% for \(^{208}\text{Pb}\), Fig. 16, except near 50 MeV where they are systematically low by as much as 10%. Finally, note that no angular dependent neutron data have been included in the analyses due to the sparsity of such data.

The conclusions of this section are that (1) the linear absorption of the modified potential provides for accurate calculation of the proton total reaction cross section for the extended energy range, (2) the transformation of the modified proton potential to the modified neutron potential via Eqs. (17) and (18) works reasonably well, 5-10%, over the same extended energy range, and (3) the modified potential does not overly deteriorate the predictive ability for elastic scattering and analyzing powers.

CONCLUSIONS

The overall conclusions from this preliminary work on obtaining a global medium-energy nucleon-nucleus phenomenological optical-model potential are that the present potential is of sufficient accuracy that specific neutron- and proton-induced reactions can be calculated with expectations that they sum to the correct physical values, and that simultaneously the angular-dependent proton elastic scattering observables are reasonably well reproduced, while the angular-dependent neutron elastic scattering observables may be reasonably well reproduced. Further work on this potential seems justified in light of the results obtained to date.

APPENDIX: COMPLEX PROJECTILES

Approximate medium-energy optical-model potentials for deuterons, \(^3\text{He}\), tritons, and alpha particles (d, h, t, α) can be obtained using a simplified Watanabe model. Although no claims of high accuracy can be made with this approach, it has been found, nevertheless, to work
surprisingly well in producing starting parameter sets for optical-model searches on complex projectile scattering data. It has also been used with reasonable success to obtain complex projectile potentials where no data exists at all.

In simple form, one assumes that a proton-nucleus potential \( V_p(E_p) \) and a neutron-nucleus potential \( V_n(E_n) \) are given, where the geometries of the potentials are constant and the real and imaginary strengths are energy dependent, except for the spin-orbit strength, which is assumed to have the same constant value \( V_{SO} \) for both protons and neutrons. Then the simple version of the model states that the deuteron-nucleus potential is equal to the sum of the nucleon-nucleus potentials at half the energy, namely,

\[
E_p = E_d/2, \quad E_n = E_d/2
\]

\[ V^d_d(E_d) = V_p(E_p) + V_n(E_n), \text{ and} \]

\[ V^d_{SO} = V_{SO}. \]

Tensor polarization has been ignored in this approximation. Similarly, the helion-nucleus potential is equal to the sum of the nucleon-nucleus potentials at one-third the energy, namely,

\[
E_p = E_h/3, \quad E_n = E_h/3
\]

\[ V^h_h(E_h) = V_p(E_p) + V_p(E_p) + V_n(E_n), \text{ and} \]

\[ V^h_{SO} = V_{SO}/3. \]

By the same logic, the triton-nucleus potential is given by

\[
E_p = E_t/3, \quad E_n = E_t/3, \]

\[ V^t_t(E_t) = V_p(E_p) + V_n(E_n) + V_n(E_n), \text{ and} \]

\[ V^t_{SO} = V_{SO}/3. \]

Finally, the alpha-nucleus potential is equal to the sum of the nucleon-nucleus potentials at one-fourth the energy, namely,

\[
E_p = E_\alpha/4, \quad E_n = E_\alpha/4
\]

\[ V^\alpha_\alpha(E_\alpha) = 2V_p(E_p) + 2V_n(E_n). \]

Of course, in all cases the Coulomb potential is calculated appropriate to the charge of the complex projectile.

One would expect the model to perhaps work best for the deuteron, which is the least tightly bound of the four complex projectiles considered, and to perhaps work worst for the alpha particle, which is the most tightly bound of the four. At medium energies, however, the model performs reasonably well for alpha particle and \(^3\)He scattering in addition to deuteron scattering. Note that an even simpler model can be used in this approach by replacing the neutron-nucleus potential \( V_n(E_n) \) in the above equations with the proton-nucleus potential \( V_p(E_p) \).
REFERENCES


Fig. 1. Calculated proton total reaction cross section for three options on relativistic effects.

Fig. 2. Calculated neutron total cross section for three options on relativistic effects.
Fig. 3. Comparisons of measured and calculated proton total reaction cross sections for $^{27}$Al.

Fig. 4. Comparisons of measured and calculated neutron total cross sections for $^{27}$Al.

Fig. 5. Comparison of calculated elastic differential cross sections for the $p + ^{27}$Al reaction at 130 MeV (Region I).

Fig. 6. Identical to Fig. 5 except at 150 MeV (Region II).
Fig. 11. Comparison of calculated elastic differential cross sections for the $p + ^{56}\text{Fe}$ reaction at 130 MeV (Region I).

Fig. 12. Identical to Fig. 11 except at 150 MeV (Region II).

Fig. 13. Comparison of calculated analyzing powers for the $p + ^{56}\text{Fe}$ reaction at 130 MeV (Region I).

Fig. 14. Identical to Fig. 13 except at 150 MeV (Region II).
Fig. 15. Comparisons of measured and calculated proton total reaction cross sections for $^{208}$Pb.

Fig. 16. Comparisons of measured and calculated neutron total cross sections for $^{208}$Pb.

Fig. 17. Comparison of calculated elastic differential cross sections for the $p + ^{208}$Pb reaction at 130 MeV (Region I).

Fig. 18. Identical to Fig. 17 except at 150 MeV (Region II).
Fig. 19. Comparison of calculated analyzing powers for the $p + ^{208}\text{Pb}$ reaction at 130 MeV (Region I).

Fig. 20. Identical to Fig. 19 except at 150 MeV (Region II).
THE STANDARD HYBRID- AND EXCITON-MODEL FOR PREEQUILIBRIUM REACTIONS

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Abstract:
The Exciton- and Hybrid model of preequilibrium emission differ in their assumptions about the nature of particle hole states, in their computational concepts and in their predictions of emission cross sections. These differences are reviewed and shown to make the models unreconcilable. Consequences for the use of the models are discussed.

1. Introduction

Phenomenological models for preequilibrium reactions have been in use for decades both to predict cross sections for a number of practical purposes and to test ideas about the underlying physics. Among them are two models which have enjoyed special popularity, the Exciton model /1,2/ and the Hybrid model /3/. Both are particularly elegant formulations and manage to describe a rather complex type of reaction by remarkably simple closed-form equations. This makes them attractive and has no doubt contributed to their popularity, as they are
- easy to use and economic in computing time
- practical to be employed in routine calculations
- convenient as a tool to test ideas about the physics governing preequilibrium reactions.

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Although these models lack some of the rigorousness of more elaborate quantum-mechanical treatments /4/ that are now becoming applicable, their basic assumptions tend to be confirmed by such more sophisticated approaches, so that the usefulness of the phenomenological models remains unchallenged and that they are, perhaps, better than the term "phenomenological" would suggest. It should also be kept in mind that, while the quantum-mechanical approaches are parameter-free in principle, they do offer some latitude for adjustment to experimental data, e.g. via a suitable choice of model wave functions etc. On the other hand, their predictive power extends further, in my opinion, than that of either the Hybrid- or the Exciton model in that they can predict angular distributions: The physics that governs preequilibrium angular distributions is interference of partial waves influenced both by nucleon-nucleon and nucleon-nucleus interaction. This is treated explicitly in the quantum-mechanical approaches, but it is not part of the phenomenological models, and quasi-free nucleon-nucleon scattering dynamics, which have sometimes been used as a substitute, do not reproduce the experimental angular distribution data well.

Therefore, I will discuss the Exciton- and Hybrid-Model in their basic form, i.e. as predicting angle-integrated spectra. I will also not address such refinements as the inclusion of $\gamma$-emission, isospin-effects, geometry-dependence etc., as these topics are mostly covered in other lectures during this meeting. While basically only one formulation /3/ exists for the Hybrid model, the Exciton model comes in several varieties. One of these, Gadioli's formulation /2/, is particularly suitable for comparison as it uses the same ingredients as the Hybrid model, including quasi-free intranuclear nucleon-nucleon scattering cross sections to describe thermalizing collisions. I will therefore explicitly discuss only Gadioli's formulation of the Exciton model. Nevertheless, the conclusions remain valid also for Exciton model varieties that employ an average matrix element to calculate intranuclear transitions. They are just harder to pin down numerically as basic differences between the models can be
absorbed into or masked by the particular choice of that matrix element.

The Hybrid and the Exciton model differ from one another in several ways and there has been much discussion about which model is "correct" and if and how they may be reconciled with each other. Not long ago, these differences were reviewed in detail and discussed with respect to both their conceptual and practical importance /5/. The details need not be repeated here, but I will restate some of the findings and discuss what progress has been made in the meantime and what might, perhaps, be done in the future.

2. The basic difference: Configuration mixing

The expression used in both the Hybrid- and the Exciton model to calculate preequilibrium emission cross sections may be written as

\[
\frac{d\sigma}{d\varepsilon} = \sigma_F \sum_{i=0}^{\infty} D_i \varphi_i(E, e_p, p, h) \lambda_c(e_p) \tau \quad (1)
\]

with

- \( \sigma_F \) Entrance channel fusion cross section
- \( i \) Index denoting groups of emission chances (differently defined in the two models)
- \( D_i \) Depletion factor, taking into account emission from previously treated groups of emission chances
- \( \varphi_i \) Exciton distribution functions, i.e. probability of finding an exciton at single particle energy \( e_p \)
- \( E \) Total excitation
- \( e_p/h \) Single exciton energy for particle/hole
- \( \lambda_c \) Rate of emission into the continuum for a particle exciton with channel energy \( \varepsilon = e_p - S \), calculated from reciprocity in both models
- \( S \) Separation energy of particle to be emitted
- \( \tau \) Lifetime, differently defined in the two models
- \( p \) Number of particle excitons
- \( h \) Number of hole excitons
- \( n=p+h \) Total exciton number
The difference between the models which is by far the most important, both conceptually and practically, arises from the way the lifetime, $\tau$, is defined:

Both the Hybrid- and the Exciton model envision the composite system as equilibrating via a series of Markoff type nucleon-nucleon interactions and thereby passing through a corresponding series of increasingly complex particle hole states. These are characterized by the number of excited nucleons (particles) and vacant levels below the Fermi surface (holes) participating in the excitation. It is convenient to think of these intermediate states as of configurations, i.e. of combinations of single exciton excitations, where each exciton occupies a well defined single exciton level, so that the $n$-exciton state wave function can be written as a product of single exciton wave functions. Then, single exciton level occupation numbers are good quantum numbers, and the excitons may be treated as independent from one another. The intermediate states may, however, also be mixtures (linear combinations) of configurations. Each state may then decay through any of its component configurations, single exciton level occupation numbers are no longer good quantum numbers and excitons can no longer be considered as independent. It is far from obvious how much configuration mixing occurs in nature. If no configuration mixing is assumed to occur, excitons act independent of one another. The probability for an exciton residing at single particle energy $e_p$ to be emitted may then be expressed by a single particle branching ratio

$$\frac{\lambda_c(e_p)}{\lambda_c(e_p) + \lambda_+(e_p)}$$

where $\lambda_+(e_p)$ is the rate at which an exciton undergoes a thermalizing collision. This is the Hybrid model picture, in which the lifetime, $\tau$, in Eq. (1) is

$$\tau = \left[ \frac{\lambda_c(e_p) + \lambda_+(e_p)}{\lambda_c(e_p) + \lambda_+(e_p)} \right]^{-1}, \quad (2)$$
the single particle lifetime.

The exciton distribution function, $\phi_i(E, e_p, p, h)$, is the probability for an exciton to be excited to $e_p$ either in the formation of the composite system or in the history of equilibration. It is meant as an average over a large ensemble of individually different successions of particular particle hole configurations. The distribution functions are usually calculated in terms of Ericson type particle hole state densities. It is easy to verify that the number of particles assumed to undergo either emission or a thermalizing collision within one term in the sum of Eq. (1) is

$$\int_{0}^{E} \phi_i(E, e_p, p, h) \left[ \lambda_c(e_p) + \lambda_+(e_p) \right] \tau \, de_p = p \quad (3).$$

Similarly,

$$\int_{0}^{E} \phi_i(E, e_h, p, h) \lambda_+(e_h) \tau \, de_h = h \quad (4),$$

as

$$\lambda_c(e_h) = 0.$$

Consequently, evaluating one term in the sum of Eq. (1) for all possible energies $e_p$ in the Hybrid model covers the emission possibilities that arise from all $p$ particles until each one of them has undergone another thermalizing collision, thereby producing a new $2p1h$ subsystem. Similarly, the holes produce $1p2h$ subsystems, so that

$$p_{i+1} = 2p_i + h_i, \quad h_{i+1} = 2h_i + p_i, \quad n_{i+1} = 3n_i \quad (5).$$

Emission chances are thus seen to be grouped according to exciton generations in the Hybrid model. The group index $i$ denotes the number of thermalizing collisions which all excitons produced in the initial target projectile fusion have already undergone. Exciton numbers evolve according to Eq. (5) from generation to generation, and not, as the usual model formulation suggests, according to

$$p_{i+1} = p_i + 1, \quad h_{i+1} = h_i + 1, \quad n_{i+1} = n_i + 2 \quad (6).$$
A different model formulation results, if maximum rather than zero configuration mixing is assumed in the sense that the n-exciton states are assumed to have - on the average - equally strong components of all n-exciton configurations possible at total excitation E. Then, excitons are no longer independent, and the decay of n-exciton states must be considered rather than that of single excitons. Emission of a particle of energy $e_p$ can only occur if the n-exciton state decays through a component configuration that has a particle suitably excited. It may, however, equally well decay through any of its other component configurations (not having a particle at $e_p$). If the average statistical weight that configurations carry in the mixed n-exciton states is assumed to be given by $\phi_i(E,e_p,p,h)$, a branching ratio

$$\frac{\phi_i(E,e_p,p,h) \lambda_c(e_p) e_p}{\int_0^E \phi_i(E,e_p,p,h) \lambda_c(e_p) e_p \, de_p + \int_0^E \phi_i(E,e_h,p,h) \lambda_+(e_h) e_h \, de_h}$$

results. This is the Exciton model picture, and it conforms with Eq. (1) when the lifetime, $\tau$, is taken to be

$$\tau = \left[ \int_0^E \phi_i(E,e_p,p,h) \left[ \lambda_c(e_p) + \lambda_+(e_p) \right] e_p \, de_p + \int_0^E \phi_i(E,e_h,p,h) \lambda_+(e_h) e_h \, de_h \right]^{-1} \tag{7}$$

the average state lifetime.

It is crucial to note that in the Exciton model approach particle emission with channel energy $E - e_p - S$ competes against decay through n-exciton state component configurations that do not have a particle at excitation $e_p$. This is a consequence of configuration mixing and the most important difference - both conceptually and numerically - to the Hybrid model.

Integration analogous to Eq.'s (3) and (4) yields

$$\int_0^E \phi_i(E,e_p,p,h) \lambda_c(e_p) + \lambda_+(e_p) \tau \, de_p + \int_0^E \phi_i(E,e_h,p,h) \lambda_+(e_h) \tau \, de_h = 1 \tag{8}$$

in the Exciton model. This means that the transition between successive groups of emission chances (or terms in the sum of Eq.
(1)) is mediated by action of one and only one exciton. Emission chances are grouped according to \( n \)-exciton states, and exciton numbers evolve according to Eq. (6).

It was demonstrated in ref. /5/ that the most important difference between the Hybrid and Exciton model approaches is what the models - implicitly - assume about configuration mixing. This is because this difference affects the first and dominating term in Eq. (1), whereas other differences affect only higher order terms.

The Hybrid model assumes zero mixing. Excitons are assumed to be independent, much in the spirit of the intranuclear cascade model. Emission chances are grouped according to generations of excitons, and exciton numbers evolve according to \( n_{i+1} = 3n_i \) from generation to generation.

The Exciton model assumes maximum configuration mixing. Excitons are not independent of one another, similar in spirit to the evaporation model. Emission chances are grouped according to generations of \( n \)-exciton states, and exciton numbers evolve according to \( n_{i+1} = n_i + 2 \) from generation to generation.

This difference between the models is at a rather fundamental level and consists of two mutually exclusive assumptions. Therefore, a unification of the models is impossible, and a claim to the contrary /6/ is no longer maintained /7/.

3. Consequences and other differences

As a direct consequence of the differing assumptions about configuration mixing, the cross section predictions of the Hybrid and the Exciton model differ by a factor of two in a typical case /5/, when no mean free path multiplier is used in the Exciton model calculation. This is largely due to the strong hole interaction rates used in the Exciton model. They result in the prediction of a sizable probability for \( n \)-exciton states to decay by hole interaction and for a correspondingly diminished particle emission, whereas they make no difference in the Hybrid model.
These hole interaction rates were derived from quasi-free nucleon-nucleon scattering by Gadioli /8/ and recently reexamined in an independent calculation of a different type /9/. These recent calculations are in good agreement with the rates obtained by Gadioli and basically just confirm his result. It thus seems an unescapable fact that the differing assumptions about configuration mixing lead to adverse cross section predictions. There is, of course, a large number of tests of the predicted cross sections against experimental data. These tests are inconclusive, however, as differing refinements of the models and differing parameter choices were employed (e.g. a mean free path multiplier in the Exciton model, geometry dependence in the Hybrid model, differing single particle state densities etc.). To decide the question of configuration mixing, it is perhaps less suitable to look at agreement with measured absolute preequilibrium cross sections. It is rather quite conceivable that a better test is to see which assumption - or which model, when used in its most basic form and identical parameter choices - gives better agreement with the dependence of measured preequilibrium cross sections on the total excitation energy. At present, this remains an open question. Additional clues may come from quantum-mechanical treatments addressing the question of configuration mixing /10/.

Another consequence of the configuration mixing question is that the Exciton model produces exclusive spectra and the Hybrid approach does not. In the latter model, owing to the assumed independence of excitons, a distinction between single and multiple precompound emission is not readily made. This question is addressed in a separate lecture in this meeting /11/ and it may suffice here to state that at present there are two solutions. One is to introduce a few simple and plausible but not rigorously justifiable assumptions /12/. The other is a more rigorous treatment /13/ at the cost of losing the computational simplicity of the "traditional" Hybrid model.

Apart from the configuration mixing problem, there are other differences between the Exciton and the Hybrid model. One rests with the fact that calculating exciton distribution functions as a
simple ratio of Ericson-type n-exciton state densities is inconsistent with quasi-free nucleon-nucleon scattering for all terms of the sum in Eq. (1) other than the first. This is true for both models, and the use of state density ratios must be considered an approximation in either approach. It is approximative in a different way in each model, and particle hole numbers according to Eq. (6) rather than (5) are used in the Hybrid model. This problem is discussed in detail in refs. /5/ and /6/, and more accurate - and complicated - distribution functions are given. In most practical calculations, however, this is not important, as the approximation affects only higher order terms which give small cross section contributions. This was explicitly confirmed for Hybrid calculations /14/. For the Exciton model, the test remains to be done but is expected to show that level density ratios are a good approximation, just as in the Hybrid approach.

Another minor point is that the depletion factor used in the original Hybrid model formulation /3/ is incorrect. It should be

$$D_i = \frac{1}{\Pi} \frac{(p_k - c_{k-1})}{p_k} \quad (9)$$

instead of

$$D_i = \frac{1}{k-1} (1 - c_{k-1}) \quad (10)$$

with

$$c_{k-1} = \int_{0}^{E} \rho_{k-1}(E, e_p, p, h) \lambda_c(e_p) \tau_{d e_p} \quad (11)$$

Again, this affects only terms other than the first in the sum of Eq. (1) and makes only a small difference in practical calculations. More important, a correct depletion factor is given and used with the improvements suggested in ref. /12/.

As mentioned above, the Exciton model is similar in structure to the equilibrium evaporation theory. For this reason, it is easy to include angular momentum conservation into Exciton model calculations. It can be done in much the same way as is customary in evaporation theory, and it was shown /15/ that it is important to do so when cluster emission is considered in reactions involving relatively high angular momenta. In such reactions, cluster
emission is apparently largely governed by angular momentum conservation, and no special coalescence or preformation mechanisms have to be introduced. This straightforward (and parameter-free) approach /15/ does not work, however, in the domain of low angular momenta. A number of models has been suggested /16/ to overcome this problem, and their successes will be discussed in a separate lecture /17/. They involve assumptions about the preformation of clusters in the composite system or their coalescence, and they invariably contain an adjustable parameter, which is not readily overdetermined by the experimental data. It is a continuing challenge to formulate a simple and yet parameter-free model for preequilibrium cluster emission. Any ideas toward that goal are easily implemented into the Exciton model, as it deals with n-exciton states and the excitons contained therein coexist within the state life time. The Hybrid model, on the other hand, is structured according to generations of independent excitons, which may or may not coexist at any given time. That makes it more difficult to implement ideas about the mechanism of their coalescence into clusters. Similar difficulties arise, when angular momentum conservation is to be observed in the Hybrid model. It is conceivable, but not obvious, that the treatment of ref. /13/ offers a solution to these problems.

4. Summary and Conclusions

Table 1 is an attempt to summarize — in buzz word from — the points I have addressed, and parts of it reflect personal opinion rather than proven fact. The focus is on problems rather than merits of the models, as the merits are most certainly familiar to an audience of experts.

Both the Exciton and the Hybrid model continue to be powerful tools to predict preequilibrium emission cross sections. The critical reexamination presented here is by no means meant to call into question the validity of the model concepts or their usefulness for practical purposes. It is rather intended to help clarify some points which have been subject to much discussion and
<table>
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<td>Angular momentum conservation</td>
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<td>no</td>
<td>e) not parameter-free as yet</td>
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controversy. Both models are particularly elegant and computationally efficient formulations which have exhibited a remarkably large predictive power. They richly deserve to have survived the pioneering stage of pre-equilibrium calculations to which they have contributed so much.

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A REVIEW OF PHENOMENOLOGICAL MODELS FOR ANGULAR DISTRIBUTIONS AND THEIR COMPARISON WITH GENERALIZED PREEQUILIBRIUM MODELS INCORPORATING A FREE SCATTERING KERNEL

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ABSTRACT

A review of the new Kalbach systematics of continuum angular distributions extended to higher energy for nucleon-induced and -emitted preequilibrium reactions is discussed. The basis for the systematics is compared and contrasted with the generalized preequilibrium exciton model incorporating a free-scattering kernel. Comparisons of angular distributed spectra obtained from these two methods, both of which have been implemented in the GNASH preequilibrium, statistical nuclear-model code, with measured spectra encompassing an incident energy range from 25 to 200 MeV for several targets from $^{12}$C to $^{209}$Bi, are presented.
1. INTRODUCTION

Implementing a free nucleon-nucleon scattering kernel in the exciton pre-equilibrium model is both an aesthetic and logical extension of the master equation basis of this model.\(^1\) However, this resulting generalized exciton model underpredicts cross-section spectra in the backward hemisphere for reactions with incident energies of several tens of MeV.\(^2\)

In contrast to this approach based on a Legendre polynomial series, which at back angles can sum to small and sometimes even negative values, C. Kalbach has extended the Kalbach-Mann systematics\(^3\) to higher energies by fitting angular distributed spectral data over a wide range of incident energies and reaction types to angular-dependent hyperbolic functions and an energy-dependent slope parameter, a.\(^4\)

This paper has a two-fold purpose: first, to compare the predicted angular distributions of both the generalized exciton model and Kalbach's new systematics with data of nucleon-induced and -emitted reactions ranging in incident energies from 25 to 200 MeV for targets from \(^{12}\)C to \(^{209}\)Bi; and second, to determine the conditions under which the free-scattering kernel is a suitable approximation to the data.

Section 2 outlines the basic equations of the two angular distribution approaches. Section 3 compares their predicted spectra with the data. Section 4 is a summary of results followed by the bibliographic references in Section 5.

2. BASIC EQUATIONS

2.1 Generalized Exciton Model

Akkermans, Gruppeelaar, and Rello\(^5\) have demonstrated that incorporating free nucleon-nucleon scattering within the exciton model is equivalent to augmenting the functional dependence of the system mean time, \(\tau\), from \(\tau(n)\) to \(\tau(n,\Omega)\), where \(n\) is the exciton generation number and \(\Omega\) the direction of particle emission. The connection between these two is:

\[
\tau(n,\Omega) = \tau(n) \cdot Q(n,\Omega)
\]

(1)

where

\[
Q(n,\Omega) = \sum_{k}^{1/2(n-n_0)} \mu_k \cdot P_k(cos\theta)
\]

(2)

with \(P_k(cos\theta)\), the Legendre polynomials of order \(k\); \(\mu_k\) the eigenvalues associated with the free-scattering kernel obtained from

\[
\mu_k = \begin{cases} 
1 & (k = 0) \\
2/3 & (k = 1) \\
0 & (k \text{ odd, } k \neq 1) \\
(-1)^{(k+2)/2} & (k = 1) \\
2^{k-1}(k-1)(k+2)[(k/2)!]^2 & (k \text{ even})
\end{cases}
\]

(3)
and the initial condition Legendre expansion coefficient being

\[ \eta_2(n_0) = \delta_{nn_0} \frac{(2k+1)}{4\pi} \mu_t \]

(4)

where \( n_0 = 3 \) is the usual initial exciton generation number.

The preequilibrium component of double-differential cross section then becomes

\[ \frac{d^2\sigma_b}{d\varepsilon d\Omega} = \alpha_{CN} \sum_n F(n)W_b(n,\varepsilon) \tau(n,\Omega), \]

(5)

where \( F(n) \) is the depletion factor, \( W_b(n,\varepsilon) \) is the emission rate probability for a type b particle with channel energy \( \varepsilon \), and \( \alpha_{CN} \) is the compound nucleus formation cross section.

In contrast to the following systematics that represent simply an angular dependent multiplicative factor to the cross section, the generalized exciton model exhibits both an \( n \) and \( \Omega \) dependence to its angular distributions, as seen in Eqs. (2) and (5).

2.2 Kalbach's Higher Energy Systematics

To help improve predictions of angular distributions at back angles and extend her previous work to higher energies, C. Kalbach has produced a new systematics of nucleon and alpha particle induced reactions at incident energies up to 600 MeV.\(^4\)

These new systematics replace the Kalbach-Mann Legendre polynomials with hyperbolic functions, as follows:

\[ \frac{d^2\sigma_b}{d\varepsilon d\Omega} = \frac{1}{4\pi} \frac{d\sigma_b}{d\varepsilon} a \left[ \cosh(\cos\theta) + f_{msd}\sinh(\cos\theta) \right] \]

(6)

Here \( \frac{d\sigma_b}{d\varepsilon} \) is the angle-integrated differential cross section for emitted particle b and \( f_{msd} \) is the fraction of cross section that is multi-step direct (msd) compared to multi-step compound. The dimensionless slope parameter, \( a \), is found from the following equation:

\[ a = 0.04X_1 + 1.8 \times 10^{-6}(X_1)^3 + 6.7 \times 10^{-7} M_a m_b (X_3)^4 \]

(7)

where \( X_1 = (E_1 \cdot e_b'/e_a) \) and \( E_1 = \text{minimum}(e_a',E_\tau), E_\tau \) being the high energy transition energy chosen as 130 MeV; likewise \( X_3 = (E_3 \cdot e_b'/e_a) \) and \( E_3 = \text{minimum}(e_a',E_\tau_3), E_\tau_3 \) being the low energy transition energy chosen as 41 MeV. For \( e_b' = e_b + S_b \) and \( e_a' = E_{\text{inc}} + S_a \), the separation energies \( S_{a,b} \) are calculated from the liquid drop mass formula of Myers and Swiatecki.\(^6\) \( E_{\text{inc}} \) is the incident energy of projectile type a. \( M_a = 1 \) for nucleons while \( m_b = 1 \) for protons, whereas \( m_b = 1/2 \) for neutrons.
3. COMPARISONS TO DATA

Both the free scattering kernel formalism and Kalbach's new systematics have been implemented in the GNASH, preequilibrium, statistical nuclear-model code.\textsuperscript{7} Legendre polynomials to order \( l = 6 \) are used in the former, and all the following examples are considered as 100\% MSD so that \( f_{\text{msd}} = 1 \) for the latter. The cases are modeled with the GNASH evaporation version using the Ignatyuk level density and multistage preequilibrium models.\textsuperscript{8} Scattering is isotropic after the first stage of preequilibrium. The nuclear surface model,\textsuperscript{9} based upon angle-integrated spectra, was bypassed because of too much hardening of the angular-distributed spectra.

3.1 Angular-Distributed Spectra at Four Angles

A first examination of Figs. 1-6 for the various targets subjected to the 40- to 200-MeV range of proton incident energies shows that both the generalized exciton model (labeled fsk in the figures) and Kalbach new systematics (C.K. in the figures) are good approximations to the data.

Taking a closer comparative look at each of the four angles, the following observations can be made:

(a) the new systematics better approximate the data at forward angles (15\(^{\circ}\));

(b) both approaches are comparable at 45\(^{\circ}\);

(c) both approaches are fairly comparable at 90\(^{\circ}\) and 120\(^{\circ}\), except for the intermediate to higher emission energies of Figs. 5 and 6, where predictions of the generalized exciton model rapidly diverge away from both the data and new systematics results.

This last observation points to a specific pathology of the generalized exciton model, viz., oscillatory behavior of the double differential cross section through positive and negative values at back angles (\( \theta \geq 105\^{\circ} \)) for intermediate to high emission energies. In these previous examples, this effect is most noticeable in the higher incident energy cases of Figs. 5 and 6. It will be discussed further in Section 3.3.

3.2 Angular Distributions at Specific Exit Energies - The Slope Parameter

The slope parameter, \( a \), can be recovered from Eq. 6 with \( f_{\text{msd}} = 1 \) and then serve as a useful diagnostic tool for understanding the shapes of the double differential cross sections for both the generalized exciton and new systematics approaches.

Writing

\[
R(\theta) \equiv 4\pi - \frac{d^2\sigma}{ded\Omega} + \frac{d\sigma}{de} = \frac{a}{\sinh(a)} \ e^{a\cos\theta},
\]

differentiate \( R(\theta) \) with respect to \( \cos\theta \), and solve for \( a \) to obtain:

\[
a = \frac{1}{R(\theta) \ d\cos\theta} = \frac{1}{d^2\sigma \ \frac{d\cos\theta}{ded\Omega}} \cdot \frac{d(\frac{d^2\sigma}{ded\Omega})}{d\cos\theta}.
\]  

(8)
Since the present GNASH cases have been executed in angular increments of 15°, Eq. (8) is best understood as a difference equation,

\[
a = \frac{1}{\Delta \left( \frac{d^2\sigma}{d\theta d\Omega} \right)} \cdot \Delta \left( \frac{d^2\sigma}{d\theta d\Omega} \right) \Delta \cos\theta \quad (9)
\]

where \( \Delta \left( \frac{d^2\sigma}{d\theta d\Omega} \right) \) is the difference between two successive \( \frac{d^2\sigma}{d\theta d\Omega} \), starting from \( \theta = 0° \) and increasing by 15° increments to 180°; \( \frac{d^2\sigma}{d\theta d\Omega} \) is the average between the same two successive \( \frac{d^2\sigma}{d\theta d\Omega} \) and \( \Delta \cos\theta \) is the difference between the two respective values of \( \cos\theta \).

Although the 15° angular increment is coarse, it is expected that calculating the slope parameter from Eq. (9) for the total angular distribution using Kalbach's new systematics will be close to the constant value of the slope parameter calculated from Eq. (7) for a specific exit energy. However, the slope parameter obtained from Eq. (9) for the same case of total angular distribution using the generalized exciton model will, in general, not be a constant but rather a complicated function of \( \theta \). Nevertheless, the slope parameter in this latter case can be used to understand the shape of the generalized exciton's angular distribution compared with that of the new systematics.

3.3 Angular Distribution at Specific Exit Energies - Comparisons with Data

Figs. 7 through 14 present comparisons of the two calculational approaches to data for four cases ranging from incident energies of 26 MeV to 165 MeV. Each of the cases is represented by two figures: the first showing the double differential cross section at a specific particle emission (exit) energy over the entire angular range; the second comparing the slope parameter obtained from Eq. (7) (labeled "Formula" in the figures) to those obtained from the difference equation, Eq. (9), for the Kalbach new systematics and generalized exciton approaches, respectively.

In Figs. 7, the fsk curve rapidly diverges away from the data for \( \theta \geq 90° \), and as mentioned at the end of Section 3.1, the double differential cross section would oscillate through positive and negative values if continued from 150° through to 180°. This same general behavior is also true for Figs. 9 and 13. In all three of these cases, \( \epsilon > \frac{E_{\text{inc}}}{2} \).

In Figs. 8 and 10, and to a lesser extent in Figs. 12 and 14, the use of the difference equation, Eq. (9), for the new systematics approach is seen to closely approximate the straight line formula result of Eq.(7).

The use of the fsk slope parameter in Fig. 8 to diagnose the shape of the fsk double differential cross section in Fig. 7 is as follows: the larger value of the fsk slope parameter between 0° and 100° in Fig. 8 corresponds to the fsk double differential cross section having more concavity between these points than the new systematics's analogous curve; the near identity of the slope parameters close to 110° corresponds to the two curve shapes being the same in Fig. 7; finally, the rapidly increasing fsk slope parameter past 110° corresponds to the far greater concavity of the fsk curve than the C.K. curve in Fig. 7. Similar arguments can be made in the
remaining cases with smaller values of fsk slope parameter corresponding to flatter, or less concave, double differential cross-section shapes than those for C.K.

Finally, Figs. 11 and 12 show that for the condition, $\epsilon < E_{\text{inc}} / 2$, using the free scattering kernel provides a reasonable approximation to the data. This condition also holds, as a rough rule of thumb, for the cases depicted in Figs. 1 through 6. the free scattering differential cross section does not become negative. The similar shapes of the curves in Fig. 11 are consistent with the approximately similar performance of the slope parameters in Fig. 12. Because the C.K. curve in Fig. 11 increases after $\sim 120^\circ$, use of the difference equation, Eq. (9), which would yield negative slope parameters, is discontinued before $120^\circ$.

4. SUMMARY

The new Kalbach systematics have been tested for nucleon-induced and -emitted reactions up to an incident energy of 200 MeV. They provide a very good approximation to angular distributed data when incorporated within the GNASH preequilibrium, statistical nucleon-model code.

Use of the free scattering kernel within the generalized exciton model must be qualified: for emission energies, $\epsilon > E_{\text{inc}} / 2$, the cross section may go negative for back angles $> 105^\circ$; however, roughly for $\epsilon < E_{\text{inc}} / 2$, the generalized exciton model provides a good approximation to the data; finally, for any emission energies, the generalized exciton model is a reasonable predictor of angular distributed data when $\theta < 90^\circ$.

5. ACKNOWLEDGMENTS

I would like to thank Dr. Edward D. Arthur for implementing the new Kalbach Systematics in the ANGDIS2 code, helping me with the plot routines, and providing helpful suggestions during the development of this paper.

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Fig. 1. 38.7 MeV $^{54}$Fe($p,p'$); 15, 45, 90, 120 degrees. Data from Bertrand and Peelle.\textsuperscript{10}

Fig. 2. 62 MeV $^{12}$C($p,p'$); 15, 45, 90, 120 degrees. Data from Bertrand and Peelle.\textsuperscript{10}
Fig. 3. 90 MeV $^{90}\text{Zr}(p,p')$; 15, 45, 90, 120 degrees. Data from Wu, Chang, and Holmgren.\textsuperscript{11}

Fig. 4. 90 MeV $^{90}\text{Zr}(p,n)$; 15, 45, 90, 120 degrees. Data from Kalend et al.\textsuperscript{12}
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Fig. 9. 62 MeV $^{209}$Bi(p, p'); 50-MeV exit. Data taken from Kalbach.\textsuperscript{4}

Fig. 10. $^{209}$Bi(p, p'); 50-MeV exit slope parameter.
Fig. 11. 90 MeV $^{27}$Al(p,n); 20-MeV exit. Data taken from Kalbach.\textsuperscript{4}

Fig. 12. $^{27}$Al(p,n); 20-MeV exit slope parameter.
Fig. 13. 165 MeV $^{208}$Pb$p,p'$; 145-MeV exit. Data taken from Kalbach.4

Fig. 14. $^{208}$Pb$(p,p')$; 145-MeV exit slope parameter.
EMISSION OF COMPLEX PARTICLES IN PRECOMPOUND REACTIONS

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ABSTRACT

Various theoretical models that have been developed to describe emission of complex particles in nucleon induced reactions are discussed and their predictions compared with the experimental data.
Production of complex particles \((d,t,^3He,\alpha)\) represents a sizeable contribution to the total reaction cross section of light projectiles.

As an example of the features that are characteristic of the spectra of all these different ejectiles, the angle integrated energy distribution of \(\alpha\)-particles emitted following the interaction of a medium energy proton with \(^{118}\text{Sn}\) is shown in Fig. 1 [1]. In this spectrum, the contributions of different mechanisms, that, for the purpose of the present discussion, I will call one-step, multi-step and evaporative components are apparent.

One-step processes feed low energy states of the residual nucleus that, partly, are resolved. They are dominated by structure effects and the angular distributions for transitions to discrete final states greatly depend on the transferred angular and total momenta \(L\) and \(J\). These properties make these processes an invaluable tool for nuclear spectroscopy studies.

An example of the influence of the shell structure of the target nucleus on the energy distribution of emitted particles is offered by the comparison of the \(\alpha\)-particle spectra from \((p,\alpha)\) reactions on adjacent nuclei, one magic, with a magic neutron and/or proton shell, and one near magic with one further nucleon outside the magic shell. In Fig. 2 [2] the angle integrated spectra of \(\alpha\)-particles from \(^{208}\text{Pb}(p,\alpha)^{205}\text{Tl}\) and \(^{209}\text{Bi}(p,\alpha)^{206}\text{Pb}\) reactions, at 24 MeV incident proton energy, are compared as a function of the ejectile energy. This comparison shows that, while transitions to the lowest energy levels of \(^{205}\text{Tl}\) dominate the spectrum of \(\alpha\)-particles from \(^{208}\text{Pb}\), levels of \(^{206}\text{Pb}\) below \(\approx 3.25\) MeV are populated very weakly in the \(^{209}\text{Bi}(p,\alpha)^{206}\text{Pb}\) reaction. Further, transitions to the levels of \(^{208}\text{Pb}\) with energy between \(\approx 3.25\) and 7 MeV have an intensity similar to that of the transitions to the low lying levels of \(^{205}\text{Tl}\) \((0 \leq E \leq 4\) MeV), and a striking correspondence exists between the structures appearing in the two spectra, in the two different excitation energy intervals.

This excitation of homologous states in the two residual nuclei (in \(^{205}\text{Tl}\), proton hole and two neutron-one proton hole states; in \(^{206}\text{Pb}\), the hole states of the \(^{205}\text{Tl}\) core weakly coupled to the \(1h_\frac{1}{2}\) proton outside the magic shell) is indicative of the spectator role of the 83th proton of \(^{209}\text{Bi}\).

This effect, occurring also at the closure of other shells, is valuable for investigating the structure of states at an excitation energy of some MeV in near-magic nuclei, and has a notable influence also on the continuous part of the spectra.

This is demonstrated by the comparison (see Fig. 3) of the \(\alpha\) spectra in the \(^{91,90}\text{Zr}(p,\alpha)^{88,87}\text{Y}\) reactions at about 25 MeV [2]. When, as in the previous case, the two spectra are superimposed as a function of the \(\alpha\)-particle energy one finds that, in spite of the widely different \(Q\) values, the spectrum of \(\alpha\)-particles from \(^{91}\text{Zr}\) looks like an energy average of the more structured spectrum from \(^{90}\text{Zr}\), showing that the total strength for transitions to states in a given energy interval is nearly the same when the excitation energy of the odd-odd \(^{87}\text{Y}\) nucleus exceeds by about 2.16 MeV that of the odd-even \(^{87}\text{Y}\), the opposite of a pairing energy effect.

As a consequence of these structure effects, an accurate reproduction of the measured spectra in the region corresponding to the excitation of low energy levels of the residual nucleus requires the use of microscopic or semi-microscopic models. As an example of such a calculation, in Fig. 4 are compared the experimental and the calculated angle integrated spectra of \(\alpha\)-particles from the \(^{91,90}\text{Zr}(n,\alpha)^{88,87}\text{Sr}\) reactions at 18.15 MeV incident neutron energy [3]. The \(\alpha\)-particle spectrum, in the case of \(^{90}\text{Zr}\), displays a sharp maximum around one MeV of excitation energy; it has been reproduced, in the framework of the the semi-microscopic pick-up model [4], taking into account all known low energy levels of \(^{87}\text{Sr}\) that may be populated in the \((p,\alpha)\) process. In case of \(^{91}\text{Zr}\), only the contribution of the transition to the to g.s. of \(^{88}\text{Sr}\) and to the excited states between \(\approx 4\) and 6.5 MeV of excitation energy that are homologous of the states of \(^{87}\text{Sr}\),
Fig. 1 - Angle integrated spectrum of α particles from the $^{118}$Sn(p,α)$^{115}$In reaction at $E_p = 44.3$ MeV, as a function of the α-particle channel energy [1].

Fig. 2 - Angle integrated α-particle spectra from the reactions $^{208}$Pb(p,α)$^{205}$Tl, histogram, and $^{209}$Bi(p,α)$^{206}$Pb, full line, at $E_p \approx 24$ MeV [2]. The bars indicate the excited levels of $^{208}$Pb below $\approx 3.2$ MeV that are very weakly populated.

Fig. 3 - Angle integrated α-particle spectra from the reactions $^{90}$Zr(p,α)$^{87}$Y, histogram, and $^{91}$Zr(p,α)$^{88}$Y, full line, at $E_p \approx 24$ MeV [2].
in the energy interval 0 - 2.5 MeV, has been calculated. In these calculations there is only one free parameter, normalizing the calculated to the experimental spectra, that is the same for both nuclei and different incident energies. All other input parameters are optical model parameters obtained by best fit analyses of elastic scattering data and spectroscopic factors that are calculated as suggested by Smits and Siemssen [4] using for the neutron spectroscopic amplitudes the experimental values obtained in the analysis of neutron pick-up reactions [3].

Results similar to those just discussed are obtained in the analysis of the α-particle spectra in the (n,α) reactions induced on $^{142,143,144}$Nd by 12-20 MeV neutrons [5].

In the multi-step part of the spectra, structure effects are washed out to a large extent.

Tamura, Udagawa and Lenske [6] have developed a multistep-direct-reaction (MSDR) theory to describe these processes. Without entering into mathematical details, the main assumptions that have been introduced by these authors are the following:

(i) these processes feed states of the residual nucleus that are a complicated superposition of pure shell model states, so in principle one should expect, in measured cross sections, interference between amplitudes for transitions to different shell model states. However, at high excitations, the residual nucleus states are highly overlapped and interference terms cancel to a large extent. Then, to reproduce the data, one needs only to evaluate the cross sections for transitions to pure shell model states and sum them incoherently. Assuming that a given state can only be excited in a particular process, also interference between one-step and multi-step amplitudes need not to be considered and the contributions of the various steps to the total cross sections are added incoherently;

(ii) the cross section for the transitions to the levels within a given excitation energy interval is proportional to a spectroscopic density that is evaluated for any given process in the hypothesis of a dominant reaction mechanism. For instance, in the case of one-step (p,α) reactions, one considers the pick-up of a triton. In this case, of the many possible three nucleon-hole final states which might be excited only those corresponding to three particle wavefunctions which have a non vanishing overlap with a triton wavefunction of the correct spin-isospin nature and zero relative internal momentum must be considered. This overlap integral $G([\tilde{n}_i])$ is a function of the quantum numbers $\tilde{n}_i = 2n_i + l_i$ of the transferred nucleons and may be evaluated following a procedure suggested by Ichimura et al. [7] in the framework of the harmonic oscillator shell model. By summing $G^2([\tilde{n}_i])$ over the possible triads of nucleons $[\tilde{n}_i]$ that may be excited in the unit excitation energy interval one obtains the desired spectroscopic density $\rho^{(3h)}_l(U)$, where $l$ and $U$ are the transferred orbital angular momentum and the excitation energy of the residual nucleus. Then, the cross section is simply given by

$$\sigma_{R}^{(p,\alpha)}(E_{\alpha}, \theta) = \sum_{l} \rho^{(3h)}_l(E_p - E_{\alpha} + Q_{S,T}) \sigma_{R,l}^{(B\Lambda)}(E_{\alpha}, \theta),$$

where $\sigma_{R,l}^{(B\Lambda)}(E_{\alpha}, \theta)$ is the first order DWBA cross section;

(iii) when one considers two step processes also the spectroscopic density appropriate for the other step of the reaction has to be considered. So, in the case of a (p,α')($\alpha',\alpha$) process the total cross section becomes

$$\sigma_{R}^{(p,\alpha')}(E_{\alpha}, \theta) = \sum_{l_1,l_2} \int_{l_3} \rho^{(ph)}_{l_3}(E_{\alpha'} - E_{\alpha}) \rho^{(3h)}_l(E_p - E_{\alpha'} + Q_{S,T}) \sigma_{R,l_1,l_2}^{(B\Lambda)}(E_{\alpha}, E_{\alpha'}, \theta) dE_{\alpha'},$$

where $\rho^{(ph)}_{l_3}(E_{\alpha'} - E_{\alpha})$ is the spectroscopic density for creating a ph pair appropriate for inelastic scattering, and $\sigma_{R,l_1,l_2}^{(B\Lambda)}(E_{\alpha}, E_{\alpha'}, \theta)$ is the second order DWBA cross section.
Fig. 4 - Comparison of the experimental angle integrated \( \alpha \)-particle spectra from the reactions \(^{90}\text{Zr}(n,\alpha)^{87}\text{Sr}\) and \(^{91}\text{Zr}(n,\alpha)^{88}\text{Sr}\), black points with error bars, at \( E_n \approx 18 \text{ MeV} \), with those calculated by pick-up theory [3].

Fig. 5 - Comparison of \((p,\alpha)\) cross sections and analyzing powers calculated with MSDR theory [6] with the experimental data of Ref. [8]. Solid lines include one step and two-step contributions \([\alpha,\alpha'](\alpha',\alpha)\) and \((p,p')(p',\alpha)]\).

Fig. 6 - Triton formation factor \( F_{t,m} \) as a function of triton energy \( \epsilon_t \) (the triton energy \( E_t \) in the Fermi gas is also shown) [11].
In spite of all these simplifying assumptions, at the increase of the number of steps, the calculation becomes very time consuming so, in practice, only one and two-step terms have been considered, in a few instances, and also in these cases only a few of the possible reaction paths have been taken into account [6].

A systematic study of the possibilities offered by this approach is highly advisable; in fact, though the various authors who employed this theory to reproduce double differential spectra and analyzing powers all claim to have obtained a very satisfactory reproduction of the data (a typical result is shown in Fig. 5), the many different approximations and the different choices of basic parameters that have been made do not allow one to conclude that one can make reliable predictions of measured quantities. Also the relative importance of one- and multistep contributions cannot be convincingly established (see Ref. [9] for a detailed discussion of this point).

Other, more phenomenological approaches, assume that particle emission occurs, before evaporation, at various stages of an equilibration cascade of nucleon-nucleon interactions that distributes the initial excitation energy among an ever increasing number of degrees of freedom. The basic assumption is made of a statistical competition between the various decay modes of the composite (projectile plus target) nucleus during this equilibration cascade.

Here I will discuss three different theoretical models of complex particle emission based on this general description of the multistep mechanism.

(a) In a statistical decay, it is possible that \( p_x \) excited particles condense into a cluster \( x \) which is emitted with energy \( \epsilon_x \). The probability of coalescence of \( p_x \) particles into the cluster \( x \) is given by the square modulus \( f(p_x, \epsilon_x) \) of the overlap between the wavefunction of the cluster (factorised into the product of a center of mass motion wavefunction \( \chi(\epsilon_x, \mathbf{R}) \) times an internal motion wavefunction \( \psi_x \)) and the \( p_x \)-nucleon wavefunction [10]

\[
f(p_x, \epsilon_x) = |\langle \chi(\epsilon_x, \mathbf{R}) \psi_x | \psi_1 \psi_2 \cdots \psi_{p_x} \rangle|^2.
\]  

The sum of \( f(p_x, \epsilon_x) \) over all possible combinations \( 1, 2, \ldots, p_x \) is equal to unity since the \( p_x \) nucleon wavefunctions form a complete orthogonal set.

The cluster \( x \) may be made of \( l \) particles above the Fermi level and \( m = x - l \) particles below. The probability \( f(l, m, \epsilon_x) \) of forming this cluster is

\[
f(l, m, \epsilon_x) = \sum_{1, \ldots, l \leq c_x^t, l+1, \ldots, p_x > c_x^t} |\langle \chi(\epsilon_x, \mathbf{R}) \psi_x | \psi_1 \psi_2 \cdots \psi_{p_x} \rangle|^2,
\]  

where \( \sum_{l, m} f(l, m, \epsilon_x) = 1 \).

Indeed, since the cluster \( x \) should be formed at the nuclear surface, one has to impose further restrictions when evaluating the overlap integrals in (3) and the corresponding probability of forming the cluster \( x \) will be \( F(l, m, \epsilon_x) \leq f(l, m, \epsilon_x) \).

The calculation of \( F(l, m, \epsilon_x) \) may be made in the framework of Fermi gas model, assuming that the internal wavefunction of cluster \( x \) is the ground state of an harmonic oscillator intrinsic Hamiltonian whose constant \( \nu = \omega/\hbar \) is that corresponding to the measured rms radius of \( x \). When one evaluates the phase space volume that divided by \( h^3(x-1) \) gives the number of states contributing to \( F(l, m, \epsilon_x) \), one requires that the centre of mass of the \( x \) particles be at the surface of the residual nucleus \( (|\mathbf{R}| = R_{res}) \) and that, for each particle, \(|r_i| < R_{res} + \Delta R\), where \( \Delta R \) is a free parameter.
The decay rate for continuum emission of the cluster $x$ made by $l$ particles above and
$m$ particles below the Fermi level, when the composite nucleus is in an $n$ exciton
configuration, is now given by

$$W_{e}^{n,x}(l,m)(E, \epsilon_{x})d\epsilon_{x} = \frac{2s_{x} + 1}{\pi^{2}}\mu_{x}^{2}\sigma(\epsilon_{x})\epsilon_{x}F(l, m, \epsilon_{x})\frac{\omega_{n(l,m)}(U)}{\omega_{p,h}(E)}d\epsilon_{x}$$  \hspace{1cm} (5)

where $s_{x}$, $\mu_{x}$, $\sigma(\epsilon_{x})$ are, respectively, the spin, the reduced mass and the inverse cross
section of $x$ and $\omega_{n(l,m)}(U)$ and $\omega_{p,h}(E)$ the residual and composite nucleus state
densities. The total decay rate for emission of cluster $x$ is

$$W_{e}^{n,x}(E, \epsilon_{x}) = \sum_{l,m}W_{e}^{n,x}(l,m)(E, \epsilon_{x}).$$  \hspace{1cm} (6)

The emission of the cluster $x$ leads from a $p,h$ configuration to a $p-1,h+m$ configuration. However
in evaluating $F(l,m,\epsilon_{x})$ one has already taken into account the state density of the newly created
$m$ holes and Iwamoto and Harada [10] postulate that the residual nucleus state density be
$\omega_{n(l,m)}(U) \approx \omega_{p-1,h}(U)$.

The calculation of $F(l,m,\epsilon_{x})$ shows that for ejectile energies smaller than $\approx$50 MeV the
probability that a cluster be made by coalescence of particles all above the Fermi level is very
small. In addition the residual nucleus state density $\omega_{p-1,h}(U)$ further enhances the probability
of emission of clusters with $l$ as small as possible.

Because the dominant contribution to the complex particle yield, at low energies, corresponds
to $l=1$, this approach may be considered a statistical description of multi-step pick-up.

Fig. 6 shows the calculated formation factors $F(l,m,\epsilon_{x})$ for tritons as a function of the triton
energy both outside and inside the nucleus [11]. A comparison between the experimental data
of Bertrand and Peelle [12] and the calculated angle-integrated spectra of complex particles
from 62 MeV proton bombardment of $^{197}$Au shows a large discrepancy at the highest energies,
especially evident for $d$ and $^{3}$He (Fig. 7) [11]. This is attributed to the presence of one-step
pick-up processes (the incident nucleons picks up directly the requisite number of nucleons
without creating an intermediate composite system) that are not calculated by the model. It is
interesting to note that, according to these calculations, this contribution does not seem to
contribute noticeably to the $\alpha$ spectrum.

One remarkable aspect of these calculations is the fact that they reproduce quite naturally
the ratio of triton to $^{3}$He yield showing that in this model the reduced helion emission is due
to the $^{3}$He being a little more loosely bound than the triton, so that the phase space volume
accessible to the condensing particles is substantially reduced;

(b) A second approach is based on the hypothesis (found to be valid for explaining the
production of complex particles in relativistic light and heavy ion reactions [13,14] and in
relatively low energy heavy ion reactions [15]) that excited nucleons have an high probability of
coalescing into a cluster if their relative momentum is low, i.e. if they share the same volume
of momentum space.

The probability $P$ of finding a nucleon within the coalescence volume centered around a
momentum $p$ is given by [15]

$$P = \frac{4\pi}{3}p^{3} \int_{0}^{1} \frac{dN(p)}{dp}$$  \hspace{1cm} (7)

where $\bar{m}$ is the average nucleon multiplicity and $N(p) \int_{0}^{1} \frac{dN(p)}{dp}$ represents the differential nucleon
multiplicity normalised to unity. $P_{o}$ is the coalescence radius usually treated as a free parameter.
Fig. 7 - Comparison between experimental, bar graph [12], and calculated, full line [11], angle integrated energy spectra for charged particles emitted in reactions induced by 62 MeV protons on $^{197}$Au.

Fig. 8 - Comparison of experimental differential cross sections of $^3$He from $(p, ^3\text{He}X)$ reactions at $E_p=72$ MeV with those predicted by relation (12) using proton differential cross sections from Ref. [8] (from Ref. [16]).
If the nucleons that are excited are \( m \), the probability for \( n \) of them to be within the coalescence volume is

\[
P_{m,n} = \binom{m}{n} P^n (1 - P)^{m-n}.
\]  

(8)

If \( f(m) \) is the probability distribution of the excited nucleon multiplicity, the probability of coalescence of \( n \) nucleons, \( P_n \), is given by \( P_n = \sum_{m \geq n} f(m) P_{m,n} \). Assuming for \( f(m) \) a Poisson distribution one easily finds

\[
P_n = \binom{\bar{m} P}{n} \frac{\exp(-\bar{m} P)}{n!} \approx \frac{(\bar{m} P)^n}{n!}.
\]

(9)

since the product of the average nucleon multiplicity times \( P \) is usually small. The probability of coalescence of \( N \) neutrons and \( Z \) protons is then

\[
P_{N,Z} = \frac{(\bar{m}_N P_N)^N (\bar{m}_P P_Z)^Z}{N! Z!}
\]

(10)

where \( P_N \) and \( P_Z \) are the values of \( P \) for, respectively, the neutrons and the protons. If one assumes that the protons and the neutrons have the same momentum distribution \( \frac{dN(p)}{dp} \) except for a numerical coefficient which simply affects its absolute value that is assumed to be unity for the protons and equal to \( \frac{N_T + N_P}{Z_T + Z_P} \) for neutrons \( (T=\text{target}, P=\text{projectile}) \) one obtains

\[
\frac{dN_{N,Z}(p_A)}{d^3 p_A} \approx \frac{dN_{N,Z}(p_A)}{d^3 p_A} = \left( \frac{N_T + N_P}{Z_T + Z_P} \right)^N \frac{1}{N! Z!} \frac{4\pi}{3} \frac{p_0^3}{m_e} A^{-1} \left( \frac{dN(p)}{dp} \right)^A.
\]

(11)

\( A=N+Z \) is the cluster mass and \( \vec{p}_A = A \vec{p} \). Further refinements taking into account the Coulomb energy per unit charge \( E_c \) of the cluster created near the nuclear surface lead to the following expression for the double differential spectrum of the cluster at large distances from the nucleus, as a function of the proton distribution

\[
\frac{d^2N_{N,Z}(E_A)}{dE_A d\Omega} = \left( \frac{N_T + N_P}{Z_T + Z_P} \right)^N \frac{A^{-1}}{N! Z!} \frac{4\pi}{3} \frac{p_0^3}{m_e} (E - E_c)^\frac{3}{2} \left( \frac{d^2N(p)}{dE d\Omega} \right)^A.
\]

(12)

where \( m \) is the nucleon mass and \( E_A = AE - NE_c \).

The cluster double differential spectrum may thus be evaluated if the proton spectrum \( \frac{d^2N(p)}{dE d\Omega} \) is known.

In Fig. 8, as an example, the experimental double differential cross sections for \( (p,^3\text{He}X) \) reactions are compared with those calculated with relation (12) [16]. Results of the same quality are obtained also in the case of \( (p,\alpha) \) reactions [16].

Assuming that (12) holds also when one considers the polarization of particles after the interaction process, one easily relates the analysing power of the cluster with energy \( E_C \), emitted in a \( (p,\text{CX}) \) reaction induced by a proton beam with polarisation \( p_p^C \), to that of protons of energy \( E_p = \frac{E_A}{A_C} + \frac{N_O}{A_C} E_c \) \( (N_O \text{ and } A_C \text{ are, respectively, the number of neutrons and the mass number of the cluster } C) \), emitted in a \( (p,p'X) \) reaction induced by protons of the same energy and polarisation \( p_p^C \) [17]. For instance, in the case of a \( (\vec{p},^3\text{He}X) \) reaction the relation is

\[
A_{\vec{p}}(\vec{p},^3\text{He}X) = \frac{p_{p}^{2} A_{\vec{p}}(\vec{p},p'X)}{p_{p}^{2}^{\text{He}} 1 + 3(p_{p}^{2} A_{\vec{p}}(\vec{p},p'X))^{2}}.
\]

(13)
If the proton analysing power $A_p(p, 3\text{He}X)$ is small $A_p(p, 3\text{He}X) \approx \frac{p^2}{p_{\text{cm}}} 3A_V(p, p'X)$ [17]. To assume that relation (13) holds means to neglect the effect of the spin dependent final state interaction of the nucleons which coalesce in the $\alpha$-particle. Koziolowski et al. [16,17] assume that this final state interaction may alter only the absolute value of the measured analysing power of the cluster without modifying its angular dependence. In Fig. (9) from Ref. [16] the analysing power of $^3\text{He}$ from $(p, ^3\text{He})$ reactions measured at 72 MeV is compared to that predicted with previous approach using data on proton analysing power at 65 MeV reported by Sakai et al. [8] for the same or neighbouring nuclei.

(c) Many calculations of $\alpha$ spectra and angular distributions and of excitation functions of $(n, \alpha)$ and $(p, ^2\alpha \alpha \text{np})$ reactions have been based on the hypothesis of a knock-out mechanism [18,19]. Here, I will discuss briefly the calculations described in Ref. [19] made in the framework of the Exciton model.

It is assumed that the $\alpha$-particle is already present in the nucleus and is knocked out by the projectile or by an excited nucleon during the equilibration cascade. Thus the picture is that of the usual exciton model with the further assumption that one of the excitons may be an $\alpha$-particle. The possible configurations at each stage are divided in two groups: that which contains only nucleon-excitons and that which also contains $\alpha$-excitons. Since the number of $\alpha$-exciton states is small compared with the total number of possible excited states, the decay rates for exciton-exciton interactions, $W_{\text{eq}}^n$, are assumed to be unaffected by the presence of $\alpha$-excitons. On the other hand, the decay rates for $\alpha$-particle emission into the continuum strictly depend on the number of $\alpha$-exciton configurations and the energy distribution of excited $\alpha$-particles resulting from nucleon-$\alpha$ scattering inside the nucleus. To do the calculation one introduces the basic hypothesis that if it is not immediately emitted an excited $\alpha$-particle dissolves into its constituents, so the $\alpha$-particles excited in the $k$-th stage of the de-excitation cascade are produced in the decay of states of the $(k-1)$-th stage through the interaction of an excited nucleon with a preformed $\alpha$-cluster. The nucleon energy distribution in the $(k-1)$-th stage is assumed to be that resulting from a statistical partition of the excitation energy so it may be evaluated using state density functions $\omega_{p,h}(E)$. If $p$ and $h$ are the number of particles and holes in the $k$-th stage $(p+h=n)$, $E$ is the composite nucleus excitation energy, $\Phi_\alpha = N_\alpha/A$ the density of preformed $\alpha$-clusters in terms of nuclear density, the $\alpha$-particle energy distribution $P_n(E, \epsilon_\alpha)$, is given by

$$P_n(E, \epsilon_\alpha) = \frac{(p-1)\Phi_\alpha \lambda_{n-2}^+(E, \epsilon_\alpha)}{(p-1)\Phi_\alpha \lambda_{n-2}^+(E) + W_{\text{eq}}^{n-2}(E)}$$

(14)

where the average nucleon-$\alpha$ collision probability per unit time $\lambda_{n-2}(E, \epsilon_\alpha)$ is given by

$$\lambda_{n-2}^+(E, \epsilon_\alpha) = \frac{1}{(p-1)\omega_{p-1,h-1}(E)} \int_{\epsilon_\alpha}^E \omega_{p-2,h-1}(E - \epsilon_p) \phi(\epsilon_p) \lambda^+(\epsilon_p, \epsilon_\alpha) d\epsilon_p$$

(15)

and the average total nucleon-$\alpha$ collision probability per unit time $\lambda_{n-2}(E)$ by

$$\lambda_{n-2}^+(E) = \int_{0}^{\epsilon_{\alpha}^{\text{max}}} \lambda_{n-2}^+(E, \epsilon_\alpha) d\epsilon_\alpha,$$

(16)

where $\epsilon_{\alpha}^{\text{max}}$ is the maximum $\alpha$ energy. The nucleon and $\alpha$ energies ($\epsilon_p$ and $\epsilon_\alpha$) are those in excess of respective Fermi energies. The nucleon-$\alpha$ collision probability per unit time $\lambda^+(\epsilon_p, \epsilon_\alpha)$ is given by

$$\lambda^+(\epsilon_p, \epsilon_\alpha) = \sigma(\epsilon_p + \epsilon'_p, \epsilon_\alpha + \epsilon'_\alpha) \nu \rho$$

(17)
Fig. 9 - Comparison of ($\bar{\beta}$, $\beta^3$HeX) and ($\bar{\beta}$, p'X) analysing powers based on relation (13). $\beta^3$He data are from Ref. [16], proton data from Ref. [8] (from Ref. [16]).

Fig. 10 - Comparison of the measured angle integrated $\alpha$ spectra in (p,$\alpha X$) reactions on $^{118}$Sn (black points and triangles) with those calculated in the hypothesis of $\alpha$ knockout (histograms) [20].

Fig. 11 - Comparison of the measured excitation function for the $^{50}$Ti(p,4p5n)$^{42}$K reaction with that calculated in the hypothesis of $\alpha$ knockout (full line). The dashed line is the result of a calculation made by assuming that the $\alpha$-particles may be only emitted in the evaporation of an equilibrated compound nucleus [22].
where $\epsilon_f^N$ and $\epsilon_f^\alpha$ are the nucleon and $\alpha$ Fermi energies, $\sigma(\epsilon_f^N+\epsilon_f^\alpha, \epsilon_f^N+\epsilon_f^\alpha)$ is the angle-integrated cross section (for the free interaction of a nucleon, with energy $\epsilon_f^N$ inside the nucleus, with a preformed $\alpha$-cluster leading to an excited $\alpha$-particle with energy $\epsilon_f^\alpha$) averaged over the $\alpha$ momentum distribution, $v$ is the nucleon velocity and $\rho$ the nuclear density. The Pauli principle is approximately taken into account by requiring that the nucleon and the $\alpha$-particle after a nucleon-$\alpha$ interaction have an energy greater than, respectively, $\epsilon_f^N$ and $\epsilon_f^\alpha$. In the initial 2p-1h stage of the de-excitation cascade, the $\alpha$ energy distribution is given by

$$P_\alpha(E, \epsilon_\alpha) = \frac{\Phi_\alpha \lambda^+(E; \epsilon_\alpha)}{\varphi_\alpha \lambda^+(E) + W_{1P}(E)}$$  \hspace{1cm} (18)

where $W_{1P}(E)$ is the collision probability of the incident nucleon with energy $E$ (in excess of $\epsilon_f^N$) with a nucleon of the target nucleus.

As a function of $P_n(E, \epsilon_\alpha)$, the continuum decay rate for $\alpha$ emission is given by

$$W_{c^{0,\alpha}}(E, \epsilon_\alpha') = \frac{1}{\pi \hbar^2 \mu_\alpha} \epsilon_\alpha' \sigma(\epsilon_\alpha') P_n(E, \epsilon_\alpha) \frac{d\epsilon_\alpha'}{g_\alpha}.$$  \hspace{1cm} (19)

where $\epsilon'_\alpha = \epsilon_\alpha - B_\alpha$ is the $\alpha$-particle energy outside the nucleus, $B_\alpha$ and $g_\alpha$ are the binding energy and single state density of the $\alpha$-cluster, $\mu$ and $\sigma$ the reduced mass and the inverse cross section.

$W_{c^{0,\alpha}}$ and $W_{1P}$ are the leading terms in (14) and (18), respectively, so (19) shows that the continuum $\alpha$ decay rate, and, as a consequence, the absolute yield and the spectra of emitted $\alpha$-particles depend essentially on the ratio $R=\Phi_\alpha/g_\alpha$ (usually a free calculation parameter) and on the preformed $\alpha$ momentum distribution. This dependence has been studied in Ref. [20].

A measure of $g_\alpha$ independent of $\Phi_\alpha$ may be obtained by the analysis of the spectrum of inelastically scattered $\alpha$-particles in $\alpha$-particle induced reactions as shown in Ref. [21]. The value thus found is $g_\alpha \approx A/10.36$ MeV$^{-1}$. The values of $\Phi_\alpha$ that correspond to this single alpha particle state density depend, as previously discussed, on the preformed $\alpha$-particle momentum distribution, but in all cases are unexpectedly high (to give a quantitative information, values smaller than $\approx 0.06$ have never been reported). So, even if the previous theory provides a reasonable reproduction of a large body of data using a unique set of parameters, as shown, for instance, in Figs. 10 and 11, a well founded theoretical justification of the values found for the $\alpha$ density seems to be a necessary requisite for assuming that the mechanism described above is the dominant physical process contributing to the hard component of the continuous $\alpha$-particle spectra.

These phenomenological models are only a few of those that have been proposed to describe multi-step processes. Though, with an appropriate parametrization, they give a reasonable description of a large set of data, each of them may at most, reflect one aspect of the real process occurring. They remain only a first order approximation of more fundamental microscopic theories whose application in the multi-step region presents formidable computational difficulties.

The MSDR approach is an attempt to apply semi-microscopic theories to transitions to highly excited states. The many approximations and the lack of a comprehensive analysis of the existing data do not allow one to judge its ability to provide accurate quantitative predictions.

Further progress in this field might be also obtained through exclusive experiments that allow a more accurate test of theoretical predictions.
Previous arguments are discussed in greater detail in a review article on Nucleon-Alpha Reactions in Nuclei submitted to Rep. Prog. Phys. by Peter E. Hodgson and the author. I wish to thank Dr. Hodgson for the extensive quotations from this review article.

[1] Ferrero A. M., Iori I., Molho N. and Zetta L. : "(p,α) Reactions on $^{93}$Nb, $^{107}$Ag, $^{118}$Sn, $^{165}$Ho, $^{169}$Tm from $\approx$20 to $\approx$44 MeV Incident Proton Energy", Report INFN/BE-75/6


PREEQUILIBRIUM GAMMA EMISSION

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ABSTRACT

Review of preequilibrium $\gamma$ ray emission is given. Thanks to considerable recent progress the preequilibrium $\gamma$ emission emerges as a viable concept in surprisingly broad spectral energy range and variety of reactions. Two mechanisms of $\gamma$ emission during preequilibrium cascade can be distinguished. Single-particle radiative transitions dominate the spectral energy range of giant resonances. This is discussed in terms of the exciton as well as hybrid model. Emission of $\gamma$ rays in excess of 30 MeV becomes dominated by two-particle radiative processes. This is shown via neutron-proton bremsstrahlung radiation as well as using inverse quasi-deuteron photoabsorption. The above points are illustrated on reactions induced with 14 MeV neutrons, 27 MeV $\alpha$ particles and 140 MeV protons.
1. Introduction

It is recognized that the statistical model underestimates hard parts of $\gamma$ ray spectra ($\geq 10$ MeV) observed in a number of nuclear reactions often by several orders of magnitude. This resembles the earlier situation with high energy parts of particle emission spectra that led to development of several models for preequilibrium decay [1]. In spite of that, $\gamma$ emission has been neglected as a domain for applying the ideas of preequilibrium decay for years. Thanks to exciting recent progress, however, the preequilibrium $\gamma$ emission is emerging as a viable concept in surprisingly broad spectral energy range and variety of reactions.

Until very recently, the preequilibrium $\gamma$ emission was studied only in the frame of the exciton model via single-particle radiative transitions. Two earlier formulations from 1978 and 1979 [2,3] were followed in 1985 by a paper of Akkermans and Grupelaar [4] giving, for the first time, a plausible solution consistent with the equilibrium statistical limit. This was subsequently extended by introducing angular momentum coupling [5]. In meantime, several analyses of hard parts of $\gamma$ ray spectra using these ideas were attempted in 14 MeV neutron induced reactions [6-9] and some analyses were concerned with n-gamma coincident data [10-11]. Single-particle radiative transitions were introduced into the hybrid model by Reffo et al. in 1987 [12] followed by another suggestion [13] consistent with the exciton model result [4].

The above mechanism should be limited to $\gamma$ ray energies in the region of giant resonances, where $\gamma$ ray emission as well as absorption proceed via dipole interactions embodied in single-particle transitions. For $\gamma$ ray energies in excess of 30 MeV, a two-body mechanism becomes dominant. This understanding was triggered by recent observation of high energy $\gamma$ rays, even above 100 MeV, in heavy-ion reactions (ref. 14 and references therein). One of the interpretations of this interesting phenomenon, pursued by Remington et al. [15-17], suggests that these $\gamma$ rays come from neutron-proton bremsstrahlung radiation during the equilibration process as described by the Boltzmann master equation. This was shown to be the case also in reactions due to energetic protons [16,17]. A more recent approach uses a quasi-deuteron photoabsorption cross section to describe the inverse process of n-p collision yielding a $\gamma$ ray and employs the hybrid model to treat preequilibrium cascade [18].

The paper is organized as follows. In sect. 2 we discuss the physics of preequilibrium $\gamma$ ray emission. This is followed in sect.3 by a more detailed discussion of single-particle radiative transitions in the exciton as well as the hybrid model. In sect.4 we proceed with two-body radiative processes to explain high energy $\gamma$ rays. Conclusions are given in sect.5.
2. **Mechanisms of preequilibrium \( \gamma \) emission**

A basic element in modeling preequilibrium \( \gamma \) emission is emission rate. In preequilibrium models this rate should be evaluated from the principle of detailed balance in a way similar to that used for nucleon and complex particle emission rates. Two questions arise when applying this procedure: (i) What is the cross section for an inverse process (photoabsorption in the present case) and (ii) what is the microscopic picture of the interaction under discussion in terms of excited particles and holes. On the microscopic level, there is a direct link between the photoabsorption and photoemission, therefore, the physics of photoabsorption provides a key to understanding of preequilibrium \( \gamma \) emission.

Shown in fig. 1 is the photoabsorption cross section as a function of \( \gamma \) ray energy calculated for \( ^{53}_{\text{Cr}} \). Three \( \gamma \) ray energy ranges can be distinguished, low (\( \leq 10 \text{ MeV} \)), medium (\( \sim 10-30 \text{ MeV} \)) and high (\( \geq 30 \text{ MeV} \)). This is sufficient for the purposes of the present paper since we do not wish to discuss the processes above the pion threshold (\( \geq 150 \text{ MeV} \)).

Photoabsorption in medium and low energy ranges is described by the giant dipole resonance (Lorentzian) lineshape and the mechanism of the photoabsorption can be identified with a single-particle transition. In other words, coordinates of only one nucleon can be changed. This is because of the one-body character of the electromagnetic transition operator (see, e.g., ref. 19). Since low spectral energy range is dominated by equilibrium statistical \( \gamma \) rays emitted after multiparticle emission, preequilibrium \( \gamma \) rays of single-particle origin should be found in the giant resonance range, \( \varepsilon_\gamma \sim 10-30 \text{ MeV} \). This picture was already incorporated into the exciton model in refs. 2-5.

In the high energy range a \( \gamma \) ray is increasingly absorbed by a neutron-proton pair (a quasi-deuteron) in a nucleus. This is because of the wavelength of the incident \( \gamma \) ray is comparable to the intranucleonic distance in the nucleus. The quasi-deuteron mechanism proposed by Levinger [20] is now thought to be a dominant process for the absorption of high energy \( \gamma \) rays [21]. The incident \( \gamma \) ray interacts with a n-p pair rather than n-n or p-p because they have no dipole moment. The cross section is proportional to the number of n-p pairs per unit volume, \( N/Z/A \), and to the photoabsorption of a free deuteron, \( \propto \exp(-30/\varepsilon_\gamma) \), reduced by a damping factor \( \exp(-30/\varepsilon_\gamma) \). In the high energy range, preequilibrium \( \gamma \) ray emission should thus have a two-body character and can be described as the inverse quasi-deuteron photoabsorption [18] or via incoherent n-p bremsstrahlung [15].

3. **Single-particle radiative transitions**

3.1. **The exciton model**

The \( \gamma \) emission rate as proposed by Akkermans and Gruppenlaar [4] is most easily derived from the principle of detailed
balance. Two assumptions are made. Firstly, the Brink–Axel hypothesis is invoked implying that the photoabsorption cross section for an n-exciton state can be identified with that for the ground state. Secondly, the photoabsorption microscopically means either creation of a particle–hole pair, $\Delta n=+2$, or transition of one excited particle, $\Delta n=0$. A branching ratio for these possibilities is assumed to be given by the available phase space

$$b_{n^+2}^{n-2} = \frac{q^2 E_b}{g_{n^+2}^2 E_b}, \quad b_n^n = \frac{g_n}{g_{n+2}^2 E_b},$$

(1)

where $E_b$ is the X-ray energy and $g$ is the single-particle state density. Therefore, the absorption cross section is split into two parts and the detailed balance applied individually to $n=n-2$ and $n=n+2$ gives

$$\omega_n (E_b) = \frac{\epsilon_0^2 \sigma (E_b)}{\sqrt{h^3/2}} \frac{\omega_{n-2}(E-E_b) b_{n-2}^{n-2} + \omega_{n}(E-E_b) b_n^n}{\omega_n(E)},$$

(2)

where $\omega_n(E)$ is the Ericsson state density and

$$\sigma (E_b) = \frac{\epsilon_0^2 \sigma (E_b)}{\sqrt{h^3/2}} \frac{E_b^2 \Gamma_R^2}{(E_b^2 - E_R^2)^2 + E_R^2 \Gamma_R^2},$$

(3)

with $\epsilon_R$, $E_R$, and $\Gamma_R$ being the giant resonance cross section, energy and width, respectively.

An important consequence of eqs. (1,2) is that in the equilibrium limit they lead to the $\gamma$ emission rate of the statistical model

$$\omega_{eq}(E_b) = \sum_n \frac{\omega_n(E)}{\omega(E)} \omega_n (E_b) = \frac{\epsilon_0^2 \sigma (E_b)}{\sqrt{h^3/2}} \frac{\omega (E-E_b)}{\omega(E)},$$

(4)

where $\omega(E)$ is the total state density. This proves consistency between the pre-equilibrium and equilibrium results.

A comparison of experimental and calculated spectra is shown in fig. 2 for $^{93}$Nb($n,\gamma$) at 14.1 MeV. The calculated primary spectrum includes pre-equilibrium plus equilibrium components. The accord with the hard part of the data is encouraging. Also shown is the result following earlier suggestion of Betáč and Dobeš [3] that does not fulfill the consistency test of eq.(4), however. It should be pointed out that the dominant contribution to the pre-equilibrium part around the giant resonance energy, $E_R = 16.5$ MeV, is due to the direct term $n=1$.

A more systematic study of ($n,\gamma$) spectra at 14 MeV was conducted [6]. It was found that the calculated ($n,\gamma$) cross sections as integrated over bound final states give about 500–700 $\mu$b for nuclei $A=45–209$. This is less by a factor of $\sim 1.5–2$ com-
pared to the experimental data. The spreading width $\Gamma_t \sim 1.9$ MeV was used for the most important direct term in these calculations.

3.2. Angular momentum coupling

Angular momentum coupling in the pre-equilibrium $\gamma$ emission was studied by Obložinský [5]. It was shown that no simple solution, like adding the angular momentum part of level densities into $\omega_m(E)$ terms of eq. (2), is possible. Rather, the angular momentum coupling terms are complicated expressions, where one has to take into account the angular momentum structure of the electric dipole matrix elements, perform summation over available single-particle states and averaging over initial states. The structure of eqs. (1,2) is still preserved, but in the branching ratios three coupling terms appear. For example, instead of $b^{n\gamma}$ one has

$$b^{n\gamma} = \frac{q^{n\gamma} x^{n\gamma} \gamma_{mS}}{q^{n\gamma} x^{n\gamma} + q^{E_\gamma} x^{n\gamma+2J}}$$

(5)

for the emission $J \rightarrow S$ (or absorption $S \rightarrow J$), $J$ and $S$ being spins. Here,

$$x^{n\gamma}_{mS} = \frac{3(2J+1)}{R_m(S)} \sum_{j_1 j_2} (2j_{1\gamma}+1) R_{j_1 j_2} R_{j_1 j_2} R_{j_2 j_2} (\begin{array}{ccc} j_2 & j_1 & 1 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{array}) (j_1 j_2 S) \right)^2$$

(6)

where the notation is explained in fig. 3a and $R_m(S)$ stands for the spin part of the level density. Importantly, consistency is again achieved with the equilibrium statistical result.

Without going into further details we show in fig. 4 that the effect of the angular momentum coupling on the $^{56}$Fe($n$,\gamma) spectrum at 14.6 MeV is rather small. We note, however, that the matrix element $|M_{n\gamma}|^2$ appearing in the spin dependent exciton model was, after averaging over $J$, adjusted to the non-spin matrix element $|M_{n\gamma}|^2$ so that

$$\langle |M_{n\gamma}|^2 \rangle = |M_{n\gamma}|^2 \langle x_{n\gamma} \rangle = |M_{n\gamma}|^2$$

(7)

where $\langle x_{n\gamma} \rangle$ is the coupling term for intranuclear transitions evaluated as 0.0208 in the reaction studied.

3.3. The hybrid model

In the hybrid model one needs the $\gamma$ emission rate for a particle with the excitation energy $E$. The rate can be obtained by the procedure similar to that outlined in sect. 3.1 and explained in detail in ref. 13. Considering an $n$-exciton state the absorption of a $\gamma$ ray with the energy $E_\gamma$ can proceed via $(gn+g-E_\gamma)/g$ different single-particle excitations. The branching ratio for one excitation is the inverse of this number. Using
the Brink-Axel hypothesis and the detailed balance one gets for the $\gamma$ emission rate, see also fig.3b,

$$\omega_n(E,\gamma) = \begin{cases} \frac{E_n^2 \bar{\gamma}(E_n)}{x^2 + c^2} q_n \frac{\omega_{m-2}(E-\varepsilon_n)}{\omega_{m-2}(E-\varepsilon_n)} & \text{for } \Delta n = -2, \varepsilon_n \geq \varepsilon \\ \frac{E_m^2 \bar{\gamma}(E_m)}{x^2 + c^2} q_n \frac{\omega_{m-n-1}(E-\varepsilon_n)}{\omega_{m-n-1}(E-\varepsilon_n)} & \text{for } \Delta n = 0, \varepsilon_n < \varepsilon \end{cases}$$

(8)

This rate is consistent with the exciton model result (2) because of

$$\int_0^E \omega_n(E,\gamma) \frac{\omega_{m-1}(E-\varepsilon_n)}{\omega_m(E)} dE = \omega_n(E_n).$$

(9)

Another relation for the $\gamma$ emission rate was suggested by Reffo et al. [12]. It reads

$$\omega_n(E,\gamma) = \frac{\langle q \rangle_{\gamma}^2}{2 \pi \hbar} \frac{E_n^2 \bar{\gamma}(E_n)}{x^2 + c^2} \frac{\omega_{m-2}(E-\varepsilon_n)}{\omega_m(E)}$$

(10)

where the effective charge $\langle q \rangle_{\gamma}$ is $N/A$ for proton excitons and $-Z/A$ for neutron excitons. It is clear, however, that expression (10) is not consistent with exciton model result of Akkermans and Gruppelaar [4] and it should be based on different physical assumptions. This point needs further clarification.

Calculated preequilibrium spectra are compared with the $^{93}$Nb$(n,\gamma)$ data at 14.1 MeV in fig.5. It is seen that eqs. (9) and (10) lead approximately to the same results. However, they are several times smaller than the exciton model spectrum, see fig. 2. This is because of substantial differences between the spreading width $\Gamma_{\gamma}$ in the exciton and the hybrid models. In the former one had about 1.9 MeV while in the latter the width is 4-5 times larger. We thus see that the old discrepancy between the two models [22] manifests itself in the hard parts of $(n,\gamma)$ spectra with fast neutrons. Also shown in fig. 5 is the contribution due to two-body processes, see sect. 4. It proves that indeed they can be neglected in this energy region.

Another example shown in fig.6 concerns reactions $\alpha +^{159}$Sm and He$^{4}$ + $^{146}$Sm, both at 27 MeV. The accord with the data is nice and eqs. (8) and (10) give close results in the hard parts of the spectra. The two-body mechanism is again marginal.

4. Two-particle radiative processes

Two approaches were proposed to treat the two-particle radiative processes. Remington et al. [15-17] assume that high energy $\gamma$ rays result from (incoherent) neutron-proton collisions
in the interacting nuclei via a n-p bremsstrahlung (dipole) radiation process. The bremsstrahlung cross section is taken in
the semiclassical form [15,23] 

\[
d\frac{d^2 \sigma_{\gamma \gamma}^p}{d\epsilon_\gamma d\omega_\gamma} = \frac{\alpha}{4 \pi^3} \epsilon_\gamma \left( \frac{\sqrt{\hat{\beta}_z}}{1 - (\hat{\beta}_z)^2} \right)^2 \frac{1 - (\hat{\beta}_z)^2}{1 - (\hat{\beta}_i)^2} P_{\gamma\gamma}(1+X),
\]

where \( \alpha = 1/137 \), \( \sigma_{\gamma \gamma}^p \) is the n-p elastic cross section, \( \beta_z \) and \( \beta_i \)
are the proton initial and final velocities, \( \hat{n} \) is the unit vec-
tor in the direction of propagation of the \( \gamma \) ray and the last
factors are two quantal corrections. \( P_{\gamma\gamma} \) is related to the rel-
avtivistic contraction and the meson-exchange correction \( X=1 \)
was applied in [15-17]. Time dependent intranuclear nucleon-nucleon
collision process was treated by the Boltzmann master equation
(the Harp-Miller-Berne model).

The approach of ref. 18 follows closely the spirit of the
preequilibrium models, i.e., phase space considerations, prin-
ciple of detailed balance and the treatment of preequilibrium
cascade (the hybrid model). Absorption of high energy \( \gamma \) rays by
a n-p pair (quasi-deuteron) in the nucleus means in terms of the
exciton number that \( \Delta n=\pm 4, \pm 2, 0, -2 \). Considering only the lead-
ing term, the available phase space is simply \( \omega_\gamma(\epsilon_\gamma) = q^4 \epsilon_\gamma^2/6 \)
and the branching ratio of the photoabsorption corresponding to
one excitation is \( g/\omega_\gamma(\epsilon_\gamma) \). For the inverse process, the \( \gamma \) emis-
ion, one has \( \Delta n=-4, -2, 0, +2 \), the leading term being \( \omega_{\gamma 2}(\epsilon_\gamma - \epsilon_\gamma) = q^3(\epsilon_\gamma - \epsilon_\gamma)^2/8 \). The detailed balance implies, see also fig.3c,

\[
\omega_\gamma(\epsilon_\gamma, \epsilon_\gamma') = \frac{\epsilon_\gamma^2}{\pi^2 \hbar^2 c^2} \sigma_{\gamma\gamma}(\epsilon_\gamma') \frac{\omega_\gamma(\epsilon_\gamma')}{\omega_\gamma(\epsilon_\gamma)} \frac{\omega_\gamma(\epsilon_\gamma - \epsilon_\gamma)}{\omega_\gamma(\epsilon_\gamma')},
\]

where

\[
\sigma_{\gamma\gamma}(\epsilon_\gamma') = 6.8 \frac{N Z}{A} \sigma_{\gamma\gamma}(\epsilon_\gamma') e^{-30\epsilon_\gamma'/\epsilon_\gamma'}, \quad \sigma_{\gamma\gamma}(\epsilon_\gamma') = 6.12 \frac{(\epsilon_\gamma' - 2.22)^{3/2}}{\epsilon_\gamma'}
\]

are given in mb and \( \epsilon_\gamma \) in MeV, and \( g(\epsilon) \) differs from \( g \) because of
high excitations [11]. Although eq.(12) already gives reasonable
results, it should include also other \( \Delta n \) terms as well as correc-
tions for the bottom of the potential well [18].

Comparison of experimental and calculated spectra of high
energy \( \gamma \) rays for two reactions, p+C and p+Pb with 140 MeV pro-
tons, is shown in fig.7. It is seen that the two approaches dis-
cussed above give fairly equivalent results that agree nicely
with the experimental data.

5. Conclusions

The preequilibrium \( \gamma \) ray emission seems to be presently
the most rapidly developing part of the theory of preequilibrium
decay. We have shown that two mechanisms are important. Single-
particle radiative transitions dominate the spectral energy range.
of giant resonances and two-body neutron-proton radiative processes dominate the high energy spectral range (exceeding even 100 MeV). These ideas, incorporated into standard exciton and hybrid models seem to be supported by a broader set of experimental data. While a number of questions still remains to be solved, the preequilibrium $\gamma$ emission represents a viable concept that apart of nucleon and light particle induced reactions successfully entered new domains opened up by heavy-ion reactions.

References


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Fig. 1. Photoabsorption cross section as a function of γ-ray energy calculated for $^{53}$Cr. Shown are 3 energy regions and the interactions involved.

Fig. 2. The primary γ-ray emission spectrum for the reaction $^{93}$Nb(n,γ) at 14.1 MeV. Taken from ref. 4.
Fig. 3. Diagrams of selected single-particle and two-particle transitions. (a) $\Delta n=0$ single-particle absorption of a dipole $\gamma$-ray with angular momentum coupling. (b) Single-particle transitions for a particle $\varepsilon$ in a 3p2h state. (c) Two-body radiative process for a particle $\varepsilon$ in a 3p2h state. Shown are $\Delta n=+2$ and $\Delta n=0$ transitions.

Fig. 4. Experimental and calculated $\gamma$-ray spectra for $^{56}$Fe+n at 14.6 MeV. Taken from refs. 5 and 18.

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Fig. 5. Experimental and calculated γ-ray spectra for the reaction $^{93}\text{Nb}(n,\gamma)$ at 14.1 MeV. Shown is calculated npγ (bremsstrahlung) yield, one-body preequilibrium component according to Reffo et al. [12] as well as the hybrid model result of ref. 13.

Fig. 6. Experimental and calculated γ-ray spectra for $\alpha$ and $^3\text{He}$ reactions (27 MeV lab) on Sm isotopes. Shown is compound nucleus spectrum, npγ yield, one-body spectrum by Reffo et al. [12] as well as the primary spectrum in the hybrid model. (Refs. 12, 13).
Fig. 7. Experimental and calculated high energy γ ray spectra for p+C and p+Pb at 140 MeV. Shown is the spectrum calculated via npγ (bremsstrahlung) mechanism [15] as well as using inverse quasi-deuteron photoabsorption during preequilibrium cascade [18].
MULTIPLE PREEQUILIBRIUM DECAY PROCESSES

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ABSTRACT

Several treatments of multiple preequilibrium decay are reviewed with emphasis on the exciton and hybrid models. We show the expected behavior of this decay mode as a function of incident nucleon energy. The algorithms used in the hybrid model treatment are reviewed, and comparisons are made between predictions of the hybrid model and a broad range of experimental results.

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1. Introduction

This presentation is intended first, to compare different treatments of precompound (PE) decay, secondly, to show the importance of multiple decay versus energy, and finally, to compare calculated and experimental results where multiple PE decay is an important component of the calculated yield.

In Section II, we will describe the INC approach [1-3] to the multiple PE decay problem, and then two different exciton model [4] approaches: the Exciton [5,6] and Hybrid models [7,8]. In Section III, we will compare predictions of the latter two approaches versus incident neutron energy, and we will present comparisons of hybrid model calculations with a range of experimental results. Conclusions will be presented in Section IV.

2. Models for Multiple Pre-equilibrium Decay

2.1 Intraneutron Cascade Model

Although we are primarily concerned with exciton model formulations for multiple PE decay, it is worth remembering that the first treatment was the intraneutron cascade model [1-3]. In Fig. 1, we show three cascade processes in which one and only one, two and only two, and three and only three, PE particles are emitted. By following the history of each reaction separately (rather than by an average ensemble) the distribution of PE multiplicities naturally follows. Here the treatment of multiple PE decay is very clear; the calculation, however, is tedious, so we wish to consider exciton model approaches.

2.2 The Exciton Model

The Exciton model may be expressed in the following familiar form: [5,6]

\[
\frac{d\sigma}{dc} = \sigma_R \sum_{n=n_0}^{n} \left( \frac{\chi_{\gamma} \rho_{n-1}(U)}{\rho_n(E)} \right) \cdot \left( \frac{\lambda_c(e)}{W_c(E) + W_s(E)} \right) D_n \quad \text{(Eq. 1)}
\]

where the first set of square brackets represents the number of excitons of type \( \gamma \) which could be emitted with channel energy \( e \), and the second set represents the probability that the particle will be emitted before any \( n \) exciton configuration either emits a particle or undergoes an intraneutron (two-body) interaction. Here the decay normalization is based on any action of all members of the n-exciton configuration ensemble, a two-body interaction or particle emission. One can calculate precisely the fraction of the ensemble which will emit a particle, and the fraction which will make a two-body transition, and these two fractions sum to unity for the n exciton ensemble. This mode is shown pictorially in Fig. 2. In this work we use Exciton model with a capital "E" to refer to formulations which may be used to predict absolute cross sections; we use the lower case to refer to any models using exciton densities.

Under this proper and civilized normalization, the ensemble at excitation \( E \) will either (in the never-leave-back approximation) go to an \( n+2 \) exciton configuration, or emit an exciton at energy \( e \) leaving the residual nucleus with a residual excitation and one particle exciton fewer. Multiple emission is then treated in a completely straightforward fashion by following the decay of the daughter nuclei with \( p-l \) particles and \( h \) holes, and
distribution of excitation energies $\mathcal{U}$ corresponding to the initial excitation $E$, reduced by the particle binding energy $B_p$ and distribution of channel energies $\epsilon$. This formulation of PE decay has been elegantly formulated and exploited by Gadioli and his collaborators [6,9] for single PE decay. Akkermans and Gruppelaar have explored the importance of the multiple precompound decay mode at incident energies up to 50 MeV, and their results will be presented shortly [10,11]; first, it is instructive to describe the much more convoluted approach to this problem which is taken in the hybrid model.

### 2.3 Hybrid Model

The hybrid model is a semi-classical approach analogous to the INC, where each particle interacts independently of all others. A consequence of this is a different normalization than that of the Exciton model, with:

\[
\frac{d\sigma}{d\epsilon} = \sigma_R \sum_{n=n_0}^{n} \left[ \frac{\chi \rho_{n-1}(U)}{\rho_{n}(E)} \right] \left[ \frac{\lambda_c(\epsilon)}{\lambda_c(\epsilon) + \lambda_+ (\epsilon)} \right] D_n \tag{Eq. 2}
\]

where the second set of square brackets has a denominator consisting of the two-body transition rate and continuum emission rate of the exciton under consideration, rather than integrated over all excitons as in Eq. 1. The consequence of this difference for decay normalization is great, for now, e.g., a 2p1h configuration could decay by both particles (and the hole) making a two-body transition (which would give a 3n exciton final configuration), or by one and only one exciton being emitted, or by two and only two excitons being emitted [12,13]. With two types of excitons (neutrons and protons) the number of exclusive possibilities increases. The choices for a one exciton gas are illustrated in Fig. 3.

The distribution of these many inclusive channels is not clear and exact as it is in the Exciton model formulation. Rather, the differentiation is made on statistical arguments, which is a euphemistic manner of stating that we make an educated guess, and hope that intuitive arguments will not leave us too far from the truth. We will present a discussion of the algorithms used which will be quoted almost completely from Ref. 12, without explicitly indicating quotation.

Multiple precompound decay processes must be considered at higher excitations since they are important in determining the cross section surviving to the (equilibrium) compound nucleus, and in determining yields of products which require multiple precompound emission for population, e.g., a (p, 2p) reaction on a heavy element target. There are two types of multiple precompound decay which might be considered. Type I results when a nucleus emits more than one exciton from a single exciton hierarchy (see Fig. 3). It may be seen that, e.g., in a two-particle-one-hole configuration, up to two particles could be emitted; in a three-particle-two-hole configuration up to three particles could be emitted, etc. The particle density distribution of these excitons, as given in the first set of brackets in Eq. 1, may be seen to be governed by the total composite system excitation. For illustrative purposes, we show the number of excitons expected at excitations above 8 MeV (taken as an estimate of average particle binding energy) versus composite nucleus excitation in Fig. 4. The importance of considering this "type I" multiple decay mode at excitations above 50 MeV is evident from Fig. 4.
The second type of multiple precompound decay (type II) would be described by the sequence "particle emission, one or more two body intra-nuclear transitions in daughter nucleus, particle emission." If the intervening two-body transitions are omitted from this sequence, it becomes type I multiple emission.

In the type II sequence for nucleon induced reactions, the leading term would be two-particle-two-hole. The particle density for this hierarchy for nucleons above 8 MeV is shown as a function of residual nucleus excitation energy in Fig. 4. It should be recognized that the relevant residual excitation of this population curve should be reduced by the nucleon binding energy and by the kinetic energy of the first emitted nucleon before comparing with the type I curve. Then it may be seen that at excitations below ~50 MeV for the residual nucleus following one particle emission, type II multiple precompound decay should rapidly become small compared with type I decay. We have investigated type II decay quantitatively in unpublished work. Results confirm the speculation that type I multiple precompound decay is far more important than type II for most reactions at moderate excitations. Because the first particle emission leaves a range of residual excitations and exciton numbers, a calculation of type II emission becomes more complex and time consuming than for type I emission. Nonetheless, one version of the ALICE code has been written to compute both type I and type II PE emission.

To extend Eq. 2 to higher energies and maintain its simplicity, we have made some arbitrary assumptions to estimate type I multiple particle emission branches. We define these assumptions based on simple probability arguments.

If \( P_n \) and \( P_p \) represent the total numbers of neutron and proton excitons emitted from a particular exciton number configuration, we assume that

\[
P_{np} = P_n P_p \tag{Eq. 3}
\]

is the number of either type of particle emitted in coincidence with the other from the same nucleus and exciton hierarchy. This definition covers \( P_{pn} \) since in an emission from the same exciton number there is no distinction to be made.

We assume that the number of neutrons which are emitted in coincidence with another neutron from a particular exciton number configuration is given by

\[
P_{nn} = 2^{-\frac{n}{2}} P_n P_n \tag{Eq. 4}
\]

with the fraction of the reaction cross section decaying by the emission of two coincident neutrons being \( P_{nn}/2 \). The value of \( P_{nn} \) is restricted to be \( \leq P_n P_p \). Similar expressions are used for proton-proton coincident emissions.

The number of neutrons (protons) emitted from the \( n \)-exciton configuration, which were not in coincidence with another particle, would be given by

\[
P_n \text{ (n only)} = P_n - P_{nn} - P_{np}, \tag{Eq. 5a}
\]

\[
P_p \text{ (p only)} = P_p - P_{pp} - P_{np}, \tag{Eq. 5b}
\]

and the fraction of the population \( F_n \) which had survived decay of the exciton number in question would be

\[
F_n = 1 - P_n \text{ (n only)} - P_p \text{ (p only)} - P_{pp}/2 - P_{nn}/2 - P_{np}. \tag{Eq. 6}
\]
This fraction would multiply the fractional population which had survived to the n exciton state, i.e., is the depletion factor multiplier.

The treatment of multiple emission is completed by storing spectra of excited nuclei into the appropriate daughter nucleus buffers following the emission of one neutron only, one proton only, one neutron and one proton, two neutrons only, and two protons only. The sum of these cross sections plus the cross section predicted to survive to the original parent compound state, must equal the reaction cross section. This aspect of the calculation will have very little effect on the predicted emission spectra (none on the precompound spectra) but will have major impact on the predicted excitation functions for products for which one or two neutrons or protons, or one n and one p are emitted in the precompound mode. We describe next the method used for this last step of the precompound calculation, following which the evaporation calculation is performed within the code.

Within each exciton hierarchy, we calculate the number of neutrons (protons) emitted in singles, in coincidence with protons, or in coincidence with neutrons, as the product of the nucleon numbers from Eqs. 3-5 multiplied by the surviving population cross sections and the reaction cross sections. These cross sections ([C_{\nu}(Cp), C_{np}, C_{nn}, etc.] are defined in Table I.

From the calculated total precompound neutron emission spectrum \( \sigma_n(\varepsilon)/d\varepsilon \), the cross section which could be involved in the emission of two neutrons is calculated as

\[
\sigma_{2n} = \int_{U=0}^{E-B_{2n}} \frac{d\sigma_n(\varepsilon)}{d\varepsilon} d\varepsilon \quad ,
\]

(Eq. 7)

where \( B_{2n} \) represents the sum of first and second neutron binding energies.

Similarly the neutron cross section which could be emitted in coincidence with protons is given by

\[
\sigma_{np} = \int_{U=0}^{E-B_n-B_p} \frac{d\sigma_n(\varepsilon)}{d\varepsilon} d\varepsilon \quad ,
\]

(Eq. 8)

where \( B_n \) is the first neutron out binding energy and \( B_p \) is the proton binding energy of the daughter nucleus following neutron emission. Similar integrals are made for the proton emission cross section which could consist of two coincident protons, \( \sigma_{pp} \), and of a proton in coincidence with a neutron \( \sigma_{np} \). The cross section available for the emission of a single nucleon \( \sigma_p(p) \) is, of course, the sum of all \( d\sigma(\varepsilon)/d\varepsilon \) (the integrals are replaced by sums since the code computes spectra at fixed energy intervals).

For the daughter nucleus following emission of one and only one precompound neutron, we store

\[
\sigma_{A-1,Z(U)} = \frac{d\sigma_n(\varepsilon)}{d\varepsilon} \frac{C_n}{\sigma_n} \quad ,
\]

(Eq. 9)

where \( U=E-B_n-c \); for the daughter nucleus following the coincident emission of two neutrons, we store

\[
\sigma_{A-2,Z(U)} = \frac{d\sigma_n(\varepsilon)}{d\varepsilon} \frac{C_{nn}/2}{\sigma_{nn}} \quad ,
\]

(Eq. 10)
where \( U=E-B_2 n - \varepsilon - \varepsilon_n \).

where \( \varepsilon_n \) is the average kinetic energy of the second neutron for a given energy \( \varepsilon \) of the first neutron. For the case of the daughter nucleus produced by the coincident emission of a neutron and a proton,

\[
\sigma A-2,2-1 (U) = \frac{C_n p}{2 \sigma_{np}} \left[ \frac{d\sigma_n (\varepsilon)}{d\varepsilon} \right] + \frac{C_p}{2 \sigma_{pn}} \left[ \frac{d\sigma_p (\varepsilon)}{d\varepsilon} \right],
\]

(Eq. 11)

where U = E - B_2 n - \varepsilon - \varepsilon_p(n) as previously defined, and where \( \varepsilon_p(n) \) is the average kinetic energy of the proton (neutron) emitted in coincidence with a neutron (proton) of kinetic energy \( \varepsilon \). An expression analogous to Eq. 10 is used for the case of two proton emission.

3. Comparisons of Different Approaches with Data

The algorithms presented limit multiple precompound decay to two particle emission. For nucleon induced reactions at energies below 200 MeV this does not provide a serious shortcoming. The types of algorithms employed could be extended beyond the two particle limit, if necessary, by someone with greater energy.

The calculated contributions to single vs. multiple PE decay are shown versus neutron energy for the system \( n^{12} \)H for neutrons up to 300 MeV in the hybrid model approach (in the geometry dependent form) [8] and for neutrons up to 50 MeV in the Exciton model formulation of Ref. 10 in Fig. 5. There is a quite reasonable agreement between these two approaches for the energy range of overlap. These comparisons are for a one Fermion gas, and for type I multiple PE decay in the hybrid model approach; type II decay is relatively less important. Results for a two Fermion system are summarized in Fig. 6.

Tests of the algorithms for multiple PE decay must ultimately be made by reference to experimental data. One set of comparisons is presented in Fig. 7, where we have made comparisons for the reactions \( ^{202} \)Hg(p,2p) and \( ^{202} \)Hg(p,p2n) using the new multiple emission algorithms versus the older single precompound particle emission decay code [14]. These excitation functions should provide a fairly rigorous test of the multiple decay assumptions, as proton evaporation is very highly inhibited in nuclei of high atomic numbers. The proton emission yields should therefore result primarily from the precompound process. The earlier GDM-evaporation calculation may be seen to give poor shapes for the excitation functions, and more significantly to underestimate yields of the (p,2p) and (p,2pn) products by 3 and 5 orders of magnitude, respectively. The new algorithm gives cross sections to the correct order of magnitude, and of quite satisfactory shapes over nearly the entire energy range. It should be emphasized that these cross sections are only around 0.3% of the total reaction cross section, so that the fraction of the reaction cross section calculated to populate these yields is given surprisingly well.

In Fig. 8, we present calculated (p,pxn) and (p,2pxn) yields from \( ^{62} \)Ni targets for incident proton energies of 80 to 164 MeV [15]. The relatively good agreement of the (p,p) and (p,2p) yields indicates that the algorithms used are quite successful in estimating the multiple yields. Similar comparisons are shown in Fig. 9 for the \( ^{64} \)Ni(p,xpyn) reactions [15]; again the multiple decay algorithms work quite well.

A very interesting test of multiple PE decay is the case of reactions following the capture of stopped negative pions. For these reactions, both
mass yields and nucleon spectra are available. An unusual aspect of these reactions is that the neutron spectra result primarily from the initial exciton distribution, whereas the proton spectra result primarily from decay of configurations following an intranuclear two-body transition. Then we have a good test of the contribution of higher order (than \( n_0 \)) terms to the PE decay; in terms of the hybrid model this also means a large contribution from type II PE decay.

The reason for this result is the quasi-deuteron mechanism for stopped pion capture; the pion may be captured either by a pn or by a pp pair

\[
\pi^- + p + n \to n + n \quad \pi^- + p + p \to n + p \tag{Eq. 12}
\]

The ratio of these reactions becomes a parameter to determine from the emitted nucleon spectra; results indicate that the first reaction dominates, consistent with the observation that the np interaction is stronger than the pp interaction.

In Fig. 10-12, we show calculated and experimental neutron spectra following stopped pion capture \([16,17]\), and in Figs. 13-15 \([18-21]\), we show proton spectra. In Figs. 16 and 17, we show the neutron and proton spectra from a \( ^{12}C \) target divided into type I and type II precompound decay. We see that the different slopes of the neutron and proton spectra are given quite nicely by the hybrid model, reflecting the contributions of first vs. higher order (for protons) contributions. We see also that the type II contributions are important to the total proton spectrum, and that the multiple decay \((I+II)\) contributions dominate the proton spectra. The success in reproducing these proton data so well is strong evidence in support of the algorithms adopted for treating multiple PE decay in the framework of the hybrid model. Detailed results for the division of single and multiple PE decay for nine targets between \( ^{12}C \) and \( ^{208}Pb \) (following \( \pi^- \) capture) are tabulated in Ref. 22.

In Figs. 18-20, we show calculated and experimental yields following stopped \( \pi^- \) capture by \( ^{209}Bi \), \( ^{197}Au \) and \( ^{181}Ta \) \([23,24]\). The generally good agreement in both the xn and pxn channels, once more supports the validity of the algorithms adopted, testing both type I (xn channels) and type II decay (pxn channels).

4. Conclusions

The Exciton PE model offers a precise normalization for considering multiple PE decay processes. There are, however, unanswered questions about the consistency of rates and partial state densities used in some formulations. It is important to compare results of exciton model codes with experimental results for which multiple PE decay is important. To the author's knowledge, this has not yet been done.

The hybrid model is decidedly less satisfactory for treating multiple PE decay than is possible in principle using the Exciton model normalization. Intuitive statistical arguments were made to estimate the contributions of exclusive reactions involving multiple PE decay. These algorithms have been tested against a very broad range of experimental results which are sensitive to the correctness of these algorithms, with quite satisfactory results. Similar comparisons are needed for Exciton formulations before reaching conclusions on that approach, which could offer a preferable alternative if the same predictive power were shown to be present.
References

Intranuclear Cascade

1 and only 1

2 and only 2

3 and only 3

Exciton model

multiple decay

$E$

$2p1h$

$3p2h$

$E$

$U = E - B - r$

This ensemble may decay
(multiple P.E.)

$3p2h$

Figure 2 Diagrammatic representation of the Exciton model. A change in exciton configuration occurs when any member of the hierarchy either emits a particle or makes a two body transition. If a nucleon of energy $c$ is emitted, the daughter nucleus of one fewer particle at excitation $U$ may be put into an ensemble to treat secondary P.E. decay.

Figure 1 Diagrammatic representation of the intranuclear cascade calculation. Each projectile-target interaction is individually followed so that each reaction is treated on an exclusive basis.

Hybrid Model

No emission

One and only one

Two and only two (type I multiple)

Type II multiple

Figure 3 Diagrammatic representation of type I and type II multiple precompound decay processes. This figure is supplemented by the discussion in the text.

Figure 4 Number of particle excitons at excitations greater than 8 MeV above the Fermi energy versus composite nucleus excitation and particle exciton number. The heavy solid curve is for a 2$p1h$ configuration, the dotted-dashed curve is for 3$p2h$, and the dashed curve is for 4$p3h$. The thin solid curve is for a 2$p2h$ configuration which would be relevant for type II multiple precompound decay as discussed in the text.
Figure 5  Calculated PE emission for one and only one, two and only two particles, vs. incident neutron energy for $^{127}$I. These results are for a one component Fermi gas. The upper scale gives one minus the fraction of the reaction cross section surviving to the compound nucleus. The solid line is the result of the GDH model calculation. The open circles represent one minus the sum of first plus second chance PE emission from the exciton model as reported in Ref. 10. The lower scale gives one particle emission (solid line) and two particle emission (dashed) in mb as predicted by the hybrid model.

Figure 6  As in Fig. 5, for a two component Fermi gas. The solid curve gives the reaction cross section versus incident neutron energy (abscissa). Other curves give the excitation functions for emission of one neutron only, two neutrons only, etc.

Figure 7  Calculated and experimental $^{202}$Hg(p,2p) and (p,2pn) excitation functions. The points represent experimental yields from Ref. 14. The long dashed curve is the (p,2pn) prediction of this work, and the solid line the (p,2p) result. Multiple precompound decay algorithms are used in these results. The dotted-dashed curve is the GDH result ($\times 10^3$) for (p,2p) from the precompound formulation without multiple precompound decay, and the short dashed curve is the same ($\times 10^2$) for the (p,2pn) reaction.

Figure 8  Calculated and experimental $^{62}$Ni(p,pxn) and (p,2pxn) excitation functions from 100 and 136 MeV proton bombardment of $^{44}$Ni. Experimental points are from Ref. 15. The calculated GDH model results have been connected by solid line segments. The dashed line is the result when the equilibrium level density parameter is varied from A/9 to A/12.
Figure 9  Experimental and calculated product yields from 80-164 MeV proton bombardment of $^{64}$Ni. Points and lines are as in Fig. 8.

Figure 10  Calculated and experimental ($\alpha$,xn) spectra following the capture of stopped pions on Cu, $^{27}$Al, and $^{24}$O. Data are from Ref. 16. The calculated result is for the hybrid model with 1.90 primary neutron excitons and 0.10 proton excitons. Other details of the calculation are given in the text. The arrows indicate the thermodynamic end points for the spectra.

Figure 11  Calculated and experimental ($\alpha$,xn) spectra for $^{181}$Ta and Pb targets. Data are from Ref. 16; solid curve as in Fig. 10. The dotted curve is the predicted hybrid model spectrum if a 2p2h primary excitation is assumed (with 1.75 neutron excitons); the dashed curve results if a 2p2h primary excitation is assumed.

Figure 12  Calculated and experimental $^{197}$Au($\alpha$,xn) spectra. Experimental results are from Ref. 17. The solid curve is as in Fig. 10. The dotted curve is calculated for capture in nuclear matter for which maximum energy per hole is 10 MeV, assuming 1.95 neutron and 0.05 proton excitons following $\alpha$ capture.
Figure 13 Calculated and experimental $^{12}$C$(\gamma^{-},xp)$ spectra. Experimental results are from Refs. 18-20. Results of the hybrid model calculation (with maximum hole depth of 30 MeV) due to different initial proton exciton numbers are shown. A result with maximum hole depth of 10 MeV is also shown.

Figure 14 Experimental and calculated $^{40}$Ca$(\gamma^{-},xp)$ spectra. Experimental results are from Refs. 20 and 21. Calculations are as in Fig. 10, plus a result in which the initial proton exciton number is reduced to 0.05.

Figure 15 Calculated and experimental $^{59}$Co$(\gamma^{-},xp)$ and $^{197}$Au$(\gamma^{-},xp)$ spectra. Experimental results are from Ref. 19. The solid line is the result of the calculation described in Fig. 10. Different initial proton exciton numbers are used for the dotted and dashed curves, maintaining the 30 MeV maximum hole depth. The dotted-dashed curve assumes 0.05 initial proton excitons, and a maximum hole depth of 10 MeV.

Figure 16 Contributions to the calculated $^{12}$C$(\gamma^{-},xn)$ spectrum from several components. Results are as described for Fig. 17.
Figure 17 Contributions to calculated $^{12}$C($\pi^-$,xp) spectrum due to several components. The dotted line represents the primary ($n=3$) proton spectrum, including type I multiple precompound decay. The dashed line represents contributions from all precompound decay terms ($n=3$ to $n$) including type I multiple decay. The dotted-dashed curve represents only the contribution of type II multiple precompound decay. The solid line gives the sum of type I (total) plus type II precompound decay. No equilibrium component has been added to these spectra. Calculations were performed for the parameters used in Fig. 10.

Figure 18 Calculated and experimental ($\pi^-$,xn) and ($\pi^-$,pxn) yields for stopped pions on $^{181}$Ta. Experimental results are from Ref. 23. The solid line is the intranuclear cascade result reported in Ref. 23. The dashed lines for an emission spectrum multiplied by $2.4 \times \exp \left( - (\sqrt{3} - 5.5)^2 / 16 \right)$. The dotted curve is for $\epsilon_f = 10$ MeV, $1.95n$, $0.05p$.

Figure 19 Calculated and experimental ($\pi^-$,xn) and ($\pi^-$,pxn) yields for stopped pions on $^{197}$Au. Experimental yields are from Ref. 24. Calculated results given by the dotted and dashed lines are as in Fig. 18. The solid line represents a calculation with $1.95n$ and $0.05p$ (primary) and with the maximum hole depth of 5 MeV (10 MeV maximum for the hole pair assumed in the 2p1h primary excitation).

Figure 20 Calculated and experimental ($\pi^-$,xn) and ($\pi^-$,pxn) yields for stopped pions on $^{209}$Bi. Experimental results (points with error bars) are from Ref. 24. Calculated points have been connected by line segments. The dotted line results from the parameters ($\epsilon_f = 10$, $1.95n$, $0.05p$). The dashed line results from the parameters giving the dashed line yields in Fig. 18.
THE PRE-EQUILIBRIUM MASTER EQUATION WITH ANGULAR-MOMENTUM CONSERVATION
AND ITS MICROSCOPIC ASPECTS

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ABSTRACT

We discuss some aspects of the unification of pre-equilibrium models. The incorporation of angular-momentum conservation in the exciton model is reviewed, as well as the relationship of the resulting model with the Hauser-Feshbach formula. Several possibilities for extending this model to multistep direct angular distributions are described. Finally, we consider its microscopic foundations, commenting upon various existing approaches to this problem.

1. INTRODUCTION

The creation over the years of a variety of exciton models [1] has been mainly motivated by the desire for a phenomenological description of pre-equilibrium reactions, i.e., of reactions with characteristics inbetween those of direct and of compound reactions. From the practical point of view, these efforts have been highly successful. At the same time, they have actually given rise to quite different formalisms for these three reaction types. One would intuitively expect, however, that the pre-equilibrium exciton formalism can in some way be naturally linked to compound-reaction theory (in particular the Hauser-Feshbach model) and at the same time also to direct-reaction models (at least in the continuum). The development of such a unified theory appears to be a difficult problem, which has pragmatic as well as fundamental aspects, as it touches upon the microscopic foundations of macroscopic nuclear-reaction theory. The present authors believe that a fruitful starting point for unification attempts is the master-equation approach [2], due to its omnipresence in both classical and quantal nonequilibrium statistical mechanics.

In this paper we shall discuss some aspects of the unification of pre-equilibrium models. Section 2 surveys the recent progress in the incorporation of angular-momentum conservation into the exciton model. It shows that the resulting spin-dependent master equation embodies both the Hauser-Feshbach and the exciton models as limiting cases. Section 3 discusses various possibilities for extending this approach to the description of precompound angular distributions, of interest for
making empirical contact with multistep direct models. Section 4 considers the problem of a microscopic, quantum-statistical justification of the master-equation model: without such a foundation any unified reaction theory will remain incomplete.

2. CONSERVATION OF ANGULAR MOMENTUM

A long-standing criticism regarding the exciton model is its neglect of angular-momentum information. This is undesirable for at least two reasons. Firstly, angular-momentum effects may influence the calculated pre-equilibrium excitation functions, spectra and angular distributions. Secondly, seen from a unification point of view, it renders impossible the merging of pre-equilibrium theory with Hauser-Feshbach theory, since the standard exciton model can only have as its equilibrium limit the Weisskopf-Ewing evaporation model. Recently, two papers have appeared [3,4] that solve both these problems for the angle-integrated case. These papers build upon earlier investigations by Plyuiko [5], Reffo et al. [6], Fu [7], Gruppelaar [8] and upon ideas taken from PKK theory [9].

An extensive exposition is given by Shi Xiangjun et al. [3]. They introduce a master equation that also includes the total angular momentum $J$ and the parity $\Pi$ of the composite nucleus (target plus projectile) as a characterization of the nuclear state under consideration, in addition to the excitation energy $E$ and the exciton number $n$. The resulting master equation reads:

$$\frac{dq^{J\Pi}(n,t)}{dt} = \sum_{m} \lambda^{J\Pi(m+n)} q^{J\Pi(m,t)} - \left[ w^{J\Pi}(n) + \sum_{m} \lambda^{J\Pi(n+m)} \right] q^{J\Pi}(n,t).$$

(1)

We see that the master equation decomposes into a set of uncoupled equations, one for each value of $J$ and $\Pi$. This is related to a general property that can be mathematically proved, stating that the master equation decouples in a manner as indicated above for each (macroscopically) conserved quantity. Incidentally, here we also find a physical explanation for the decoupling into various $\ell$ of the phenomenological angular-distribution models as discussed in Refs. [10-16]: in these spinless models the orbital angular momentum of the system is conserved. Consequently, if we identify $\ell$ with the orbital angular momentum, the master equation is decomposable into a set of separate equations for each value of $\ell$. Thus, we have given a natural physical interpretation to the purely mathematical proof originally given in [12].

Returning to the spin-dependent master equation, we remark that its further treatment is as usual. Upon integration over time we obtain:

$$-q^{J\Pi}(n,t=0) = \sum_{m} \lambda^{J\Pi(m+n)} \tau^{J\Pi(m)} - \left[ w^{J\Pi}(n) + \sum_{m} \lambda^{J\Pi(n+m)} \right] \tau^{J\Pi}(n),$$

(2)
with the initial condition

$$q^J(n,t=0) = \left[ \frac{\sigma^J_a}{\sigma_a} \right] \delta_{n3},$$

(3)

where $\sigma_a$ is the composite-formation cross-section. The average emission spectra and cross-sections are given by:

$$\frac{d\sigma(a,b)}{d\epsilon} = \sigma_a \sum \Sigma \frac{W^J_a (n,\epsilon)}{W^J_a (n,\epsilon)} \tau^J(n).$$

(4)

The transition and emission rates are calculated from time-dependent perturbation theory according to Fermi's golden rule, but they now include spins and parities. Here, one takes advantage from the fact that the densities of final states can be factorized into a spin part, a parity part (assumed to be equal to 0.5) and an energy-dependent part. The treatment of the angular-momentum couplings occurring in the expression for the transition rates [4] is similar to that in the FKK model [9]. The expressions for the emission rates are very similar to those used in Hauser-Feshbach theory, although the level density has become $n$-dependent.

The approach taken by Oblozinsky [4] is essentially the same, except for the fact that a never-encore-back approximation has been introduced to the above equations (1,2). He extends the model by also considering the case that the emitted particle $b$ is a photon. This is done by means of a generalization of the nonspin approach by Akkermans and Gruppelaar [17] to the case of angular-momentum conservation.

Furthermore, Shi Xiangjun et al. [3] discuss a number of methods in order to speed up the numerical model computations. Partly, these are based on the factorization properties of the level density formulae. Consequently, as demonstrated by Akkermans et al. [18], nested summations over spins can be replaced by very accurate analytical approximations. In addition, assuming a weak spin dependence of the transition and emission rates, one is able to find a very simple approximation for the spin-dependent mean lifetimes in terms of the spinless ones, as follows [3]:

$$\tau^J(n) = \frac{\sigma^J_a}{\sigma_a} \frac{W^J_t(n)}{W^J_t(n)} \tau(n),$$

(5)

This effectively reduces the set of master equations (1) (totalling a number $J_{\text{max}}$) to a single one. Also this approximation turns out to be very accurate [3]. The combined result of these computational optimizations is that the CPU time for a precosmpound/compound master-equation calculation reduces to that of a Hauser-Feshbach calculation or even less.

The observation that the transition and emission rates generally show a weak angular-momentum dependence is a basic one. This finding is reported in both Refs. [3] and [4] as well as by various authors in the FKK line of work. Physically, this weak spin dependence can be explained by noting
that even for simple exciton states a large number of angular-momentum couplings is possible. This washes out much of the selectivity in the angular-momentum structure. Accordingly, the calculated impact of spin effects on the angle-integrated emission spectra and cross-sections appears to be small. It is shown in [3] and [4] that this applies to both nucleon and γ-ray emission in nucleon-induced reactions, cf. Fig. 1. Thus, one is led to the conclusion that from a practical viewpoint the standard spinless models give quite reliable results.

Nevertheless, the master-equation model with conservation of angular momentum has definite conceptual advantages over the usual exciton model. It can be rigorously demonstrated [3] that in the equilibrium limit it goes over into the continuum Hauser-Feshbach theory. This result is obtained upon writing out Eq. (4), inserting the equilibrium conditions and employing the conservation of probability. Hence, one can genuinely speak of an integrated precompound-compound theory. Also the standard exciton model and the Weisskopf-Ewing model can be derived from it, by neglecting any spin dependences. The relationships between these models are displayed in a transparent fashion in Fig. 2. Moreover, one can show that the angular-momentum conserving master equation integrated over time (i.e., Eq. (2)) can be brought in the form of the AWM theory of Agassi et al. ([19], see especially Eqs. (5.10) and (5.23)), if in the latter the so-called "external mixing" is neglected.

Hence, the angular-momentum conserving master-equation theory seems to us of interest for a number of reasons:

a. It is sufficiently specific and simple to be amenable for practical applications (thereby showing that in many cases the results of spinless models are reliable).

b. Many useful extensions can be envisaged which are straightforward and do not affect the basic structure of the theory. For instance, it enables extensions to describe cross-sections for the excitation of discrete levels if their structure is known.

c. At the same time, it is closely related to the AWM theory ([19], see also [20]) so that a more fundamental quantum-statistical justification seems possible. If this could be supplied (for a further discussion, see Sec. 4), the exciton master equation would no longer count as purely phenomenological.

d. It unifies in a single formalism several important nuclear-reaction mechanisms and theories, notably the Hauser-Feshbach, Weisskopf-Ewing and exciton models.

e. In order to describe angular distributions in a satisfactory way the starting point should be an angular-momentum conserving model.

3. EXTENSION TO ANGULAR DISTRIBUTIONS

The statistical reaction model represented by Eqs. (1)-(4) can be extended in several directions in much the same way as has been done for the non-spin exciton model. This applies for example to multiple emission, complex-particle absorption and emission, isospin (two-component) problems, more realistic level densities, etc. A very important generalization would be the inclusion of angular distributions. For the symmetric (multistep compound) part of the angular distributions this is
quite simple: since the model can be cast in a Hauser-Feshbach-like form, the symmetric angular distributions are obtained by an immediate generalization of Hauser-Feshbach theory, like performed by Plyuiko [5]. However, the main problem resides in the modeling of the observed forward-peaked (multistep direct) angular distributions. Here, we also encounter the question of the relationship with direct-reaction models. This problem appears to be a hard one to solve.

Several different approaches presently in existence may be useful here, but they all seem to have their drawbacks, too. The most obvious procedure would be to in some way insert the "leading-particle" approach [10,11], see also [12-16], into the spin-dependent master equation, cf. Ref. [6]. However, on several points this approach has a classical flavour and its microscopic justification is not entirely satisfactory.

Another possibility is to follow the methods used in the FKK theory [9]. However, in that case a separation should be made between multistep-direct and multistep-compound reactions. Although this is an improvement over the fast-particle assumption, it leads to two quite different formalisms. This is not very attractive from a unification point of view. The same applies to the never-look-back approximation employed in the FKK theory: the link with the Hauser-Feshbach model may be intuitively clear, but it is not really embedded in the theory. In addition, the microscopic postulates are much more obscure in the FKK model [9] than in the AWM theory [19,20].

A third approach has been developed by Fu [21] who notes (following Plyuiko [5]) that in two limiting cases the phases of the states connected by the residual matrix elements are known: at $n = 1$ (1p0h) there is a complete correlation, while at $n = n_{eq}$ (equilibrium) they are entirely random. Introducing an extended Hauser-Feshbach equation and using a phenomenological $n$-dependent weighting function, these two components can be linked. The disadvantage here is that this weighting function is purely empirical, at present not being calculated on a theoretical basis. Perhaps it would be useful to relate this weighting function to the number of collisions $k$ rather than to $n$. It is noted that progress is made if one could say something about the correlation of phases just after the first collision. In fact, Plyuiko [5] has already suggested to consider only $n=n_0$ (=3), albeit that he has adopted the unrealistic assumption of fully correlated entrance and exit channels for $n=3$. At present the proposal by Fu seems to be the most practical one available in the spirit of a "unified" precompound model, albeit that there is one fit parameter. Further work on this method is underway [21].

Finally, we want to mention the work of Tamura et al. [22] on multistep direct reactions. It is more general than that of FKK and its starting point resembles that of AWM (statistical properties of the nuclear Hamiltonian). On the other hand, the connection with multistep-compound reactions has not been studied. Also, the formalism is quite complicated in the higher steps. From the present point of view, the best solution would be to merge in some way the Tamura and AWM theories in a time-dependent (master equation) fashion. It is not clear, however, how this can be achieved, the more so if one aims at a theory amenable for routine applications.
4. MICROSCOPIC ASPECTS OF PRE-EQUILIBRIUM MODELS

At first sight it may seem that this subject is somewhat esoteric. However, in applications usually combinations of various models and related parameters are employed, the consistency of which not always being evident. This is an unsatisfactory situation and it may at times give rise to unreliable results. Working towards a better integration of the respective models may be one of the remedies. In this context, study of the microscopic foundations of pre-equilibrium theory will be (and has been) profitable. Such a foundation is practically important, since it demonstrates how to reduce the treatment of the many millions of nuclear eigenstates to one with a small number of macroscopically significant variables. Precisely this feature (although with a phenomenological justification only) is a major factor in the popularity of the exciton model. Ideally, in our view, statistical continuum nuclear-reaction theory might be established as one of the branches of nonequilibrium quantum statistical mechanics.

The foundational problem can be formulated in a compact manner as follows. Our starting point is the Schrödinger equation for the composite nuclear system (target plus projectile):

\[ \text{inh} \frac{d\Psi}{dt} = (H_0 + V)\Psi. \]  

(6)

Here, $H_0$ is the shell-model Hamiltonian and $V$ represents the residual binary interaction. It is convenient to work in the evolution operator formalism:

\[ \Psi(t) = U(t,t_0)\Psi(t_0), \]  

(7a)

where

\[ U(t,t_0) = \exp \left[ -i(H_0 + V)(t - t_0)/\hbar \right]. \]  

(7b)

as follows from Eq. (6). The evolution operator can be expanded in a power series in $V$ (the Born series)

\[ U(t,t_0) = \Sigma_{s=0}^{\infty} U^{(s)}(t,t_0), \]  

(8a)
where

\[
\langle \nu | U(s)(t, t_0) | \mu \rangle = (im)^s \int_{t_0}^{t} dt_s \cdots \int_{t_{a-1}}^{t_{a_1}} dt_{a_s} \Sigma_{a_s-1} \cdots \Sigma_{a_1} \times \left[ b_\nu(t - t_s) V_{a_{s-1}} \cdots V_{a_2 a_1} b_\mu(t_2 - t_1) \right] .
\]

and

\[
b_\mu(t) = \exp \left[ -iE_\mu t / \hbar \right].
\]

Note the coupled time and energy integrals in Eq. (8b). Next, we expand the wave function in terms of the eigenstates of \( \hat{H}_o \) (which in the present case may be thought of as the individual particle-hole configurations of the target nucleus):

\[
\Psi(t) = \sum_{\mu} c_{\mu}(t) | \mu \rangle .
\]

In order to make contact with the exciton master equation, we are interested in the probability \( q(n,t) \) that at time \( t \) the system occupies exciton state \( n \), located in a small interval around energy \( E \) (for convenience we drop the \( E, J \) and \( \Pi \) labels). From the above it follows that

\[
q(n,t) \overset{\text{def}}{=} \sum_{\nu \in n(E)} |c_\nu(t)|^2 = \sum_{\nu \in n} \sum_{\rho \in n} \langle \nu | U(t, t_0) | \rho \rangle^* \langle \nu | U(t, t_0) | \mu \rangle c_\rho(t_0) c_\mu(t_0) .
\]

This equation is still exact (and thus time-reversible). It is seen that the full microscopic information is needed for the calculation of the exciton occupation probability. In order to obtain a master equation, only occupation probabilities may occur in the right-hand side of Eq. (10), instead of the coefficients \( c \). This can only be achieved by throwing away part of the microscopic information. Qualitatively speaking, this can be done by neglecting the "improbable" solutions of Eq. (10). This should result in an equation which is irreversible but nevertheless adequate because it will be representative of the majority of the solutions to Eq. (10). An essential ingredient to such an argument is that one is dealing with a large number of states. Any procedure to derive a master equation (or something similar) must be statistical in nature.
In the framework of pre-equilibrium theory various attempts in such a
direction have been made [9,19,20,23,24]. The FKK work [9] has generated
models that are used in practice, but its microscopic assumptions are made
in the course of the derivation and are therefore not always transparent.
According to McVoy and Tang [23], they are similar to but less general
than those of AWM [19,20]. By extending the stationary AWM theory to the
time-dependent domain, McVoy and Tang [23] have sought to justify the
master-equation formalism. Bunakov [24] has proposed a quite different
procedure to the same end. We will discuss these studies in more detail
below.

The AWM approach seems to us to be an elegant one, since (unlike FKK) its
statistical postulates are immediately related to the matrix elements of
the nuclear Hamiltonian. Instead of carrying out an energy average with
respect to the physical Hamiltonian, an ensemble of Hamiltonians (macro-
scopically similar to the physical one) is introduced. The statistical
assumptions are made with respect to this ensemble, whereby the calculated
physical properties of the ensemble are identified (through the principle
of "stationarity") with the physics of the real-world system under consi-
deration. This is the fundamental and most distinguishing characteristic
of the AWM approach. An additional hypothesis, of considerable technical
interest, is that the ensemble matrix elements have a Gaussian distribu-
tion. As a result, the full Born series can be summed. This yields a
probability-balance equation which is very close to the time-integrated
master equation (2). We stress that the AWM approach is not a
perturbation treatment in the residual interaction V, unlike the usual
master-equation models.

McVoy and Tang [23] have adapted the AWM theory to the time-dependent
quantum equations, cf. Eqs. (6)-(10). They employ the same two postulates
of AWM as outlined above. In addition, they introduce an approximation in
order to solve the energy-time integrals of Eq. (8b), assuming that the
energy integrals yield delta functions in time. As a consequence of this
assumption the time integrals can be easily solved. Thus, a master
equation is obtained which is the time-dependent analogon of the AWM
probability-balance equation.

Bunakov [24] has tried to justify the use of master equations for nuclear
reactions along different lines. He introduces an ensemble of composite
nuclei (target plus projectile) and points out the lack of correlations
between the particles in the incident beam. From this he concludes that
the density matrix is diagonal, whence master equations for various
nuclear-reaction processes can be derived. Here, a perturbation analysis
in leading (second) order in V is utilized. The treatment as a whole is
interesting but rather fragmentary and, in our opinion, sometimes con-
fusing (cf. below). In terms of Eq. (10) given above, one may say that
Bunakov makes his statistical assumptions with respect to the initial
coefficients c occurring in the right-hand side, whereas the hypotheses of
AWM and McVoy and Tang refer to the Hamiltonian, i.e., to the matrix
elements of the evolution operator U.
Clearly, the final results of these approaches are all quite similar and close to the master equation (1). In the Bunakov approach it would result if one introduces there the usual hierarchy of state classes of increasing complexity (i.e., the exciton number). Furthermore, one can state that the exciton master equation derives from the AWM/McVoy-Tang theory if a perturbation treatment in the residual interaction is carried out. This is evident from the calculation of the transition rates by means of Fermi’s golden rule, as well as from the neglect in Eq. (1) of external mixing.

Accordingly, here the contours are visible of a quantum-statistical, microscopic justification of the pre-equilibrium master equation with angular-momentum conservation. The exciton model on its turn derives from this as a consequence of the neglect of all spin effects, cf. Fig. 2. Nevertheless, this research programme cannot be considered as completed yet, since there are still a number of less satisfactory elements in the microscopic theories outlined above. A point in common is that they explicitly introduce some ensemble. The way this is done by Bunakov resembles that of classical mechanics. This is actually a superfluous procedure, since in quantum mechanics the notion of such an ensemble is already contained in the wave function itself. The confusing aspect mentioned above lies in the fact that the proposed energy-averaging (coarse-graining) procedure does not really need such a classical ensemble. Further, it is not true that, as is implied by Bunakov, correlations remain absent for all times if they can be shown to be so for some initial time.

AWM and McVoy and Tang construct an additional ensemble of Hamiltonians in order to make their statistical assumptions. However, the availability of many states within each cell of a coarse-grained phase space (= exciton state) should be sufficient to do so. A method directly related to the physical Hamiltonian would certainly be preferable. Also, the way the time-energy integrals of Eq. (8b) are decoupled by McVoy and Tang through an approximation by delta functions in the time is rather drastic, because in that case the associated energy integrals average over all possible values of the energy. A coarse graining in time, like the Kirkwood procedure quoted by Bunakov, is perhaps more physical here.

In all of the discussed approaches it remains to be seen whether the needed coarse graining in energy and time can be carried out in a manner compatible with the intuitive physical notions underlying the phenomenological models. This may be particularly problematic for the very first stage of the reaction process (far from statistical equilibrium or multistep direct-like), where the relevant time intervals are very short. Therefore, the investigation of pre-equilibrium angular distributions, discussed in Sec. 3, is of much interest, since these observables are very sensitive to the characteristics of the first phase of the reaction. Finally, we want to point out that the present problem is one of quantum (diffusion-like) transport. In this sense it is quite general and not very specific to nuclear physics. Hence, nonequilibrium nuclear theory may well profit from related work in a variety of other disciplines.

Summarizing, we believe to have indicated that there need not be an
insuperable gap between statistical microscopic theory and models such as the exciton master-equation model suited for routine calculations. It seems possible to eventually obtain a microscopically acceptable model with full angular-momentum conservation, that is at the same time sufficiently simple to be utilized in nuclear-data evaluations as a better alternative to the current generation of Hauser-Feshbach codes with a correction for pre-equilibrium effects.

5. REFERENCES

Fig. 1. Comparison between results of the angular-momentum conserving master equation (UM) and the standard spinless exciton model (EM) for the reaction \(^{93}\text{Nb}(n,n'x)\) at 14.6 MeV. The dashed curve corresponds to a fictitious target spin \(I=1/2\) (instead of \(I=9/2\)). This figure has been taken from Ref. [3].

[ spin effects; pre-equilibrium effects included? ]

[yes,yes] UM \rightarrow HF [yes,no]

[no,yes] EM \rightarrow WE [no,no]

Fig. 2. Schematic of the relationships between the master-equation model with angular-momentum conservation (UM), the Hauser-Feshbach model (HF), the Weisskopf-Ewing model (WE) and the usual exciton model (EM). The arrows denote a mathematical reduction by means of simplifying assumptions. The horizontal arrows represent the assumption of equilibrium (\(dq/dt = 0\)), while the vertical arrows stand for the limit of an (infinitely) large spin cut-off parameter. (Figure taken from [3])
THE TWO-COMPONENT EXCITON MODEL, ISOSPIN CONSERVATION AND OTHER SECOND ORDER EFFECTS

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ABSTRACT

Recent developments in preequilibrium reaction studies require moving from the one-component version of the exciton model to the two-component version in which proton and neutron degrees of freedom are considered separately. This model is described and is shown to be equivalent to the one-component model for simple calculations. The important question of the relative interaction rates between like and unlike nucleons is also reviewed. Finally the question of isospin conservation (which can only be treated adequately in a two-component preequilibrium model) is addressed using particle-hole state densities with good isospin quantum numbers.
1. Introduction

The one-component exciton model was introduced in the late 1960's beginning with the now classic paper by Griffin [1]. Its beauty is that it incorporates the major pieces of physics needed to describe preequilibrium reactions in a simple, physically transparent set of equations. This produces a flexible model, readily extended to include additional phenomena.

There are, however, some phenomena which cannot reasonably be included until neutron and proton degrees of freedom are explicitly distinguished from one another, as in the two-component version of the exciton model. Among the effects to be considered are shell structure, pairing effects, and the possible conservation of isospin as a quantum number.

This paper reviews work [2-4] on the two-component exciton model, including the open question of the relative matrix elements for the p-p, n-n, and p-n residual interactions. The place in the model that shell and pairing effects can be included is noted, and the modifications required for the study of isospin as a possibly conserved quantum number are described. Finally, a few preliminary results are presented.

2. The Two-Component Model

Figure 1 (taken from ref. [4]) compares the two versions of the exciton model. In the one-component model, the degrees of freedom (or excitons) are divided into particles and holes. Each class of states is labelled by its exciton number, $n$, or by the numbers of particle and hole degrees of freedom, $p$ and $h$, that it has. Particle and hole degrees of freedom are always assumed to be created and annihilated in pairs so that all of the states fall into a hierarchy, with neighboring classes differing by $\Delta n=2$ or $\Delta p=\Delta h=1$. In the two-component model, proton particle and hole degrees of freedom are distinguished from their neutron counterparts. Thus each class of configurations becomes a family of subclasses, and the description of the energy equilibration of the nucleus changes from a one dimensional problem to a two dimensional one. The different subclasses of states are labelled by the numbers, $p_n$, $h_n$, $p_\nu$, and $h_\nu$ where the subscripts $\nu$ and $\nu$ refer to protons and neutrons, respectively.

The arrows in Fig. 1 show the residual two-body interactions which take the system from one kind of state to another, thus bringing about energy equilibration in a reaction. In the one-component model only the creation and destruction of particle-hole pairs are considered. In the two-component model the creation of proton and neutron pairs are treated separately, and similarly for

![Figure 1. Schematic diagrams of proton-induced reactions in the one- and two-component exciton models. The labels are $(p,h)$ and $(p_n,h_n,p_\nu,h_\nu)$.](image-url)
pair annihilation. In addition, interactions which involve changing a neutron particle-hole pair into a proton pair or vice versa must also be considered.

In spite of these differences, the same basic information must be supplied in each version of the model. This includes the initial number of excitons, \( n_0 \), a matrix element normalization factor, and the relative probabilities for exciting proton and neutron degrees of freedom. The basic tools for the calculations are also similar: the particle-hole state densities, the rates for the residual two-body interactions, and the particle emission rates. The equilibration process is described using either a set of coupled master equations or a closed form reaction equation, and the resulting time-integrated occupation probabilities for the various classes of states are used to calculate the particle emission spectra.

2.1 State Densities

The state densities used in the two-component exciton model are based on the work of Williams [5] and have the form

\[
\omega(p_\pi, h_\pi, p_\nu, h_\nu, E) = \frac{g_\pi^n g_\nu^n [E - A(p_\pi, h_\pi, p_\nu, h_\nu)]^{n-1}}{p_\pi! h_\pi! p_\nu! h_\nu! (n-1)!} f(p, h, V) \tag{1}
\]

where \( E \) is the excitation energy of the system, \( g_\pi \) and \( g_\nu \) are the densities of single particle states for protons and neutrons, and \( f(p, h, V) \) is a correction for the finite depth, \( V \), of the nuclear potential [6]. Surface effects can be introduced in the reaction calculations by using a shallower well depth for the initial composite nucleus and residual states [7].

The quantity \( A(p_\pi, h_\pi, p_\nu, h_\nu) \) is the Pauli principle correction function which occurs in the transition rates as well. In most applications it represents only a small fraction of the total excitation energy. On the other hand, this is where the main shell [8] and pairing [9,10] corrections are included, and in those applications the Pauli correction term can become significant. The two critical factors that often go unrecognized in determining \( A(p_\pi, h_\pi, p_\nu, h_\nu) \) are the presence of passive particles or holes adjacent to the Fermi level, and the placement of the Fermi level between the last filled and first vacant single particle states in the ground state of the nucleus [4]. When these two factors are considered, the Pauli correction function is given by

\[
A(p_\pi, h_\pi, p_\nu, h_\nu) = E_{\text{Pauli}}^{(\pi)}(p_\pi, h_\pi) + E_{\text{Pauli}}^{(\nu)}(p_\nu, h_\nu) - \frac{p_\pi^2 + h_\pi^2 + n_\pi}{4g_\pi} - \frac{p_\nu^2 + h_\nu^2 + n_\nu}{4g_\nu}. \tag{2}
\]

Here the proton Pauli energy in the simple equi-spacing model is

\[
E_{\text{Pauli}}^{(\pi)}(p_\pi, h_\pi) = \frac{q_\pi^2}{g_\pi} \tag{3}
\]

with \( q_\pi = \text{maximum}(p_\pi, h_\pi) \), with similar expressions for neutrons.
2.2 Transition Rates

The transition rates considered are $\lambda_{\pi^+}$ and $\lambda_{\pi^-}$ for the creation and annihilation of proton particle-hole pairs; $\lambda_{\nu^+}$ and $\lambda_{\nu^-}$ for neutron pairs; and $\lambda_{\nu\pi}$ and $\lambda_{\nu\pi}$ for the conversion of a proton pair into a neutron pair and vice versa. The interactions are assumed to be residual so that perturbation theory applies. Thus each rate is written in terms of the average matrix elements for the appropriate interactions and the average densities of accessible final states. Allowing for the possibility that the matrix elements for p-p and n-n residual interactions will be different from those for p-n and n-p interactions, the rates are

$$\lambda_{\pi^+}(p_\pi^+, h_\pi^+, p_\nu^+, h_\nu^+, E) = \frac{2\pi}{\hbar} \left[ M_{\pi\pi}^2 \omega_{\pi^+}(p_\pi^+, h_\pi^+, p_\nu^+, h_\nu^+, E) + M_{\nu\pi}^2 \omega_{\nu^+}(p_\pi^+, h_\pi^+, p_\nu^+, h_\nu^+, E) \right]$$

(4)

$$\lambda_{\pi^-}(p_\pi^-, h_\pi^-, p_\nu^-, h_\nu^-, E) = \frac{2\pi}{\hbar} \left[ M_{\pi\pi}^2 \omega_{\pi^-}(p_\pi^-, h_\pi^-, p_\nu^-, h_\nu^-, E) + M_{\nu\pi}^2 \omega_{\nu^-}(p_\pi^-, h_\pi^-, p_\nu^-, h_\nu^-, E) \right],$$

(5)

$$\lambda_{\nu\pi}(p_{\pi^+}, h_{\pi^+}, p_{\nu^+}, h_{\nu^+}, E) = \frac{2\pi}{\hbar} M_{\pi\nu}^2 \omega_{\nu0}(p_{\pi^+}, h_{\pi^+}, p_{\nu^+}, h_{\nu^+}, E),$$

(6)

with analogous expressions for $\lambda_{\nu^+}$, $\lambda_{\nu^-}$ and $\lambda_{\nu\pi}$.

Two different methods [3,4] have been used to derive the transition state densities (or densities of accessible final states). Their results differ in form, but the major parameter dependences are basically the same and are similar to those in the one-component model.

As for the evaluation of the mean square matrix elements, it is typically assumed that $(M_{\pi\pi})^2 = (M_{\nu\nu})^2$ and $(M_{\pi\nu})^2 = (M_{\nu\pi})^2$ and that all four matrix elements will have the same dependences on the other reaction parameters (i.e. target mass, excitation energy and exciton number). Virtually all the assumptions made about these dependences in the one-component model can be carried over into the two-component model. Thus constant matrix elements [2], matrix elements derived from nuclear matter calculations [3] and semi-phenomenological matrix elements [4] have all been used.

Another important factor is the ratio $R=(M_{\pi\nu})^2/(M_{\pi\pi})^2$ which effects the relative preequilibrium yields in the inelastic and charge exchange channels in nucleon induced reactions. This question is discussed in Section 3.

2.3 Particle Emission Rates

The particle emission rates for nucleons of type $b$ and energy $\epsilon$ are fairly unambiguous and are given by the relation

$$W_b(p_{\pi^+}, h_{\pi^+}, p_{\nu^+}, h_{\nu^+}, E, \epsilon) \, d\epsilon = \frac{(2S_b + 1)}{2} \frac{\mu_b}{\hbar^3} S_b < \frac{\omega(p_{\pi^+}, h_{\pi^+}, p_{\nu^+}, h_{\nu^+}, U)}{\omega(p_{\pi^+}, h_{\pi^+}, p_{\nu^+}, h_{\nu^+}, E)} > \sigma_b(\epsilon),$$

(7)
Here $s_b$, $\mu_b$, $N_b$, and $Z_b$ are, respectively, the spin, reduced mass, neutron number and proton number of the emitted particle, and $\sigma_b(e)$ is the reaction cross section for the inverse process of particle absorption. This same expression holds for complex particle emission if all of the constituent nucleons of the particle are assumed to start out as particle degrees of freedom. In that case, Eq. (7) predicts too little complex particle emission, and the target mass dependence of the cross section is also wrong. Additional mechanisms such as nucleon transfer and cluster knockout are probably needed.

2.4 Reaction Calculations

The time-integrated occupation probabilities, $S(p_\pi, h_\pi, p_\nu, h_\nu)$, for the various subclasses of particle-hole states can be evaluated either for the preequilibrium phase alone (giving $S_{\text{pre}}$) or for the full reaction (giving $S_{\text{tot}}$). In principle either set can be obtained by solving the master equations for the equilibration process, but these equations are so numerous and complex that they have not actually been solved. Dobš and Běták [3] use an iterative approach starting from solutions to the zeroth order time-integrated master equations to estimate $S_{\text{tot}}$. Unfortunately, the model assumption that p-h is always equal to $A_A$ in the composite nucleus and $A_B-A_b$ in the residual nuclei is inappropriate in the equilibrium limit [4], making it advantageous to determine $S_{\text{pre}}$ rather than $S_{\text{tot}}$. This is most readily done using closed form reaction equations.

Closed form expressions typically assume that in the early stages of the reaction the system undergoes a series of pair creation interactions. Pair annihilation is ignored, and only competition from particle emission is considered. In the two-component model this is modified [4] to include the possibility of a single $\pi \nu$ or $\nu \pi$ exchange interaction between pair creations. Additional exchanges can easily be included but are unlikely.

Two slightly different sets of occupation probabilities are used in the closed form calculations. Simplifying the state notation from $(p_\pi, h_\pi, p_\nu, h_\nu)$ to $(p_p, p_\pi)$, the first set is denoted $P_1(p_p, p_\pi)$ and represents the strength populating the specified states by pair creation from simpler states. The total strength which passes through these configurations is greater than $P_1$, however, because of the exchange interactions. It is denoted $P_2(p_p, p_\pi)$. The quantities $P_1$ and $P_2$ are calculated using the recursion relations [4]

\begin{align}
P_1(p_p, p_\pi) &= P_2(p_{-1}, p_{-1} - 1) \Gamma_{\pi+}(p_{-1}, p_{-1} - 1) + P_2(p_{-1}, p_{-1} - 1) \Gamma_{\nu+}(p_{-1}, p_{-1}) \quad (8a) \\
P_2(p_p, p_\pi) &= P_1(p_p, p_\pi) \\
&\quad + [P_1(p_p, p_{-1}) \Gamma_{\nu+}(p_{-1}, p_{-1}) + P_1(p_p, p_{+1}) \Gamma_{\pi+}(p_{+1}, p_{+1})] L(p_p, p_\pi). \quad (8b)
\end{align}

Here the four $\Gamma$'s are branching ratios each having the form \( \Gamma_{\pi+}(p_p, p_\pi) = \lambda_{\pi+}(p_p, p_\pi) \tau(p_p, p_\pi) \), and the quantity $L(p_p, p_\pi) = \tau'(p_p, p_\pi)/\tau(p_p, p_\pi)$. There are two sets of lifetimes in these relations. The $\tau(p_p, p_\pi)$ are the partial lifetimes against pair creation and exchange interactions and against particle emission, while the $\tau'(p_p, p_\pi)$ consider only pair creation and particle emission.

The initial conditions for the recursion relations are
\[ P_1(A_a+1,Z_a) = \frac{\lambda_\nu+(A_a,Z_a)}{\lambda_\nu+(A_a,Z_a) + \lambda_\pi+(A_a,Z_a)} \]  \hspace{1cm} (9a) \\

\[ P_1(A_a+1,Z_a+1) = 1 - P_1(A_a+1,Z_a) \]  \hspace{1cm} (9b)

and correspond to an initial configuration of \((p_0,h_0)=(2,1)\) for nucleon induced reactions. For complex particle induced reactions it is probably more appropriate to start with the initial condition of \(P_1(A_a,Z_a)=1\) or \((p_0,h_0)=(A_a,0)\). The recursion process is continued until the most probable particle number at equilibrium is reached or until essentially all of the strength in the reaction has gone into primary preequilibrium emission.

The time-integrated strengths are then given by \(S_{\text{pre}}(p,p_\pi) = P_2(p,p_\pi) \tau(p,p_\pi)\) and the energy differential preequilibrium cross section for a particle of type \(b\) is

\[ \frac{d\sigma_b(\epsilon)}{d\epsilon} = \sigma_a(\epsilon_a) \sum_p \sum_{p_\pi} S_{\text{pre}}(p,p_\pi)^b(\epsilon) \]  \hspace{1cm} (10)

where \(\sigma_a(\epsilon_a)\) is the reaction cross section in the entrance channel of the reaction.

### 2.5 Results of Reaction Calculations

Figure 2 (taken from ref. [4]) compares first-particle-out preequilibrium energy spectra calculated in the one- and two-component exciton models. A value of \(R=1\) was used in the two-component model, and the distinguishability factor, \(Q_b(p)\), chosen for the one-component model emission rates likewise assumed equal matrix elements for the \(p-p\), \(n-n\) and \(p-n\) residual interactions.

The results of Fig. 2 show that when the same physical assumptions are made about the relative probabilities of exciting proton and neutron degrees of freedom, the two versions of the exciton model produce nearly identical preequilibrium energy spectra. Different values of \(R\) in the two-component model correspond to different forms for \(Q_b(p)\) in the one-component model and yield different relative yields for protons and neutrons, but the two forms of the model are essentially equivalent. The advantage of the

![Figure 2. Comparison of preequilibrium energy spectra in the one- and two-component models.](image)
two-component model comes only in the ability to correctly include things like shell structure, pairing effects and the possible conservation of isospin as a quantum number.

3. The Effective Matrix Elements for the Residual Interactions

The two suggested values of R are three [3] and one [4]. The value R=3 is derived from free nucleon-nucleon scattering cross sections and reflects the fact that the p-n cross section is about three times as big as either the p-p or n-n cross sections. On the other hand, the present matrix elements are thought to be residual in nature so that arguments based on free cross sections are not necessarily valid.

In the exciton model it is assumed that the main part of the two-body interaction between all A(A-1)/2 pairs of nucleons has gone into the potential well in which the single particle states exist, but neither the system Hamiltonian nor the parts of it which have gone into the potential well are ever defined. Thus it is impossible to choose an a priori value of R. In particular, if all of the isospin dependent part of the interaction is in the well, a value of R=1 would be quite realistic.

The practical consequences of varying R are fairly small because the pair creation rates, \( \lambda_{n^+}(p,p_n) \) and \( \lambda_{n^+}(p,p_n) \), each depend on both \((M_{\pi^+})^2\) and \((M_{\pi^0})^2\). The results in [3] indicate that changing from R=1 to R=3 causes a 20-30\% enhancement of the preequilibrium yield in the charge exchange channel of a (nucleon,nucleon) reaction relative to the inelastic channel.

At a more fundamental level, the value of R affects the validity of the equal probabilities hypothesis. The derivations of both the interaction rates and the particle emission rates in all versions of the exciton model make the implicit assumption that all the states in a given class (or subclass) are equally likely to be populated. While there is little hope of ever rigorously testing this hypothesis, work with the version of the one-component model which differentiates between closed and open configurations shows that these two subclasses for each particle number, \( p \), are populated in proportion to their state densities. Similarly, it would be reassuring if the various subclasses of states for a given \( p \) in the two-component model were

![Figure 3. Test of the equal probabilities assumption in a sample system. The R=3 results are shown only for states with p=2.](image-url)
also populated in proportion to their state densities.

Figure 3 (taken from ref. [4]) shows that when R=1 this condition is indeed fulfilled to a high degree of accuracy while for R=3 it is seriously violated for the initial \((p_0,h_0)=(2,1)\) states. (It has not been evaluated for higher configurations.) Thus if a value of R=3 is indicated by the data, the equal probabilities assumption would automatically be invalidated for the one-component model, and serious questions would be raised about its validity in the two-component case.

4. The Two-Component Model with Isospin Conservation

If a choice for R is to be based on the relative preequilibrium yields of protons and neutrons, it is vital that all other factors capable of producing an effect of similar or greater size be understood and correctly included in the model calculations. These effects are pairing, shell structure, and the possible conservation of isospin as a quantum number of the system.

The importance of using the two-component version of the exciton model to treat isospin conservation was indicated in the work of ref. [11] on particle-hole state densities with good isospin. An obvious first step in implementing isospin conservation is to consider the two extreme cases of complete mixing and complete conservation of the isospin quantum number to see the sensitivity of the calculated spectra to these assumptions. Considering isospin conservation requires doing multiple calculations, one for each value of the total isospin in the composite nucleus, and adding the results together incoherently. The calculations proceed as for the mixed case, but the transition and particle emission rates are now derived from the particle-hole state densities with good isospin.

4.1 Particle-Hole State Densities

Densities of states with good isospin are usually derived assuming that states with good isospin exist in isospin multiplets. The members of a given multiplet have all of the same quantum numbers except for the \(z\)-component of the total isospin, \(T_z=(N-Z)/2\), and thus they exist in neighboring isobaric nuclei. The same single particle states are occupied in all the members of a multiplet, but not always by the same combination of protons and neutrons.

It was expected [12] that this would also be true when the numbers of particle and hole degrees of freedom were considered as quantum numbers. Figure 4 (from ref. [11]) shows that this is not quite the case. We start with a state having

\[
T_z = T_0 - 1
\]

\[
(1,1,2) \rightarrow (2,1,2) + (2,0,2,3)
\]

Figure 4. Schematic drawing of the different components in the isobaric analog of the indicated starting configuration.
$(p_\pi, h_\pi, p_\nu, h_\nu) = (2, 1, 2, 2)$ and $T = T_0$ in a nucleus with $T_z = T_0$ and we wish to find its analog in the neighboring nucleus with $T_z = T_0 - 1$. This analog state will have different components each corresponding to the conversion of a neutron in the original configuration into a proton in the same single particle state.

There are three types of neutron states to consider. Case 2 in Fig. 4 corresponds to neutrons in the neutron excess orbitals. Isospin flip creates a proton particle and neutron hole, but they are not really degrees of freedom since in order for the isospin of the state to be conserved, they must continue to occupy corresponding single particle states. Thus, in some sense the degrees of freedom in the state are unchanged. On the other hand, in case 1 a neutron particle becomes a proton particle, while in case 3 a proton hole becomes a neutron hole. In both instances the total number of degrees of freedom is the same, but the set of "quantum numbers" $p_\pi, h_\pi, p_\nu, h_\nu$ has changed.

Approximate state densities have been derived [11] which should be fairly accurate when pairing and shell corrections are small. Dropping the $T_z$ labels (which always refer to the nucleus whose state density is being calculated), they can be written

$$\omega(p, p_\pi, E, T) = \omega(p, p_\pi, E - E(T)) - F_t(p, p_\pi, E, T) \omega(p, p_\pi, E - E(T + 1)) \quad (11)$$

where $F_t$ is a correction to the result of [12] and is given by

$$F_t(p, p_\pi, E, T) = \frac{p_\pi + h_\nu}{2T + p_\pi + h_\nu} + \frac{2T}{2T + p_\pi + h_\nu} \frac{p_\pi h_\nu}{g_\pi g_\nu [E - E(T + 1) - A(p, p_\pi)]^2} \quad (12)$$

For the states of primary interest in pre-equilibrium calculations, $F_t$ is always less than unity, and for states with 2 or 3 excitons, $F_t = 0$ is a better approximation than $F_t = 1$ (corresponding to the results of [12]).

### 4.2 Internal Transition Rates

The internal transition rates are derived ignoring the energy dependence of $F_t$ using the general method of refs. [13,4]. For pair creation, the $F_t$ value in the numerator is adjusted to conform to the energy terms in the numerator. The previously unpublished rate expressions which result are:

$$\lambda_{\pi^+}(p, p_\pi, E, T) = \lambda_{\pi^+}(p, p_\pi, E - E(T)) \frac{f_t(p + 1, p_\pi + 1, E, T)}{f_t(p, p_\pi, E, T)} \quad (13a)$$

$$\lambda_{\nu^+}(p, p_\pi, E, T) = \lambda_{\nu^+}(p, p_\pi, E - E(T)) \frac{f_t(p + 1, p_\pi, E, T)}{f_t(p, p_\pi, E, T)} \quad (13b)$$

$$\lambda_{\pi\nu}(p, p_\pi, E, T) = \lambda_{\pi\nu}(p, p_\pi, E - E(T)) \frac{f_t^{(\pi\nu)}(p, p_\pi)}{f_t(p, p_\pi, E, T)} \quad (14a)$$

$$\lambda_{\nu\pi}(p, p_\pi, E, T) = \lambda_{\nu\pi}(p, p_\pi, E - E(T)) \frac{f_t^{(\nu\pi)}(p, p_\pi)}{f_t(p, p_\pi, E, T)} \quad (14b)$$

$$\lambda_{\pi^-}(p, p_\pi, E, T) = \lambda_{\pi^-}(p, p_\pi, E) \Theta(E - A(p, p_\pi) - E(T)), \quad (15a)$$

$$\lambda_{\nu^-}(p, p_\pi, E, T) = \lambda_{\nu^-}(p, p_\pi, E) \Theta(E - A(p, p_\pi) - E(T)) \quad (15b)$$

where $\Theta$ is the Heavisher function. The $\lambda$'s on the right sides of the
equations are evaluated as in the normal two-component model [4] and the correction factors are given by the relations

\[ f_{\nu}(p, p_{\pi}, E, T) = 1 - F_{\nu}(p, p_{\pi}, E, T) \left\{ \frac{E - A(p, p_{\pi}) - E(T+1)}{E - A(p, p_{\pi}) - E(T)} \right\}^{n-1} \]  

(16a)

\[ f_{\nu}(p, p_{\pi}, E, T) = 1 - F_{\nu}(p, p_{\pi}, E, T) \left\{ \frac{E - B_{\pi\nu}(p, p_{\pi}) - E(T+1)}{E - B_{\pi\nu}(p, p_{\pi}) - E(T)} \right\}^{n-1} \]

\[ \frac{2[E - B_{\pi\nu}(p, p_{\pi}) - E(T+1)] - n[A(p, p_{\pi}) - A(p, p_{\pi} - 1)]}{2[E - B_{\pi\nu}(p, p_{\pi}) - E(T)] - n[A(p, p_{\pi}) - A(p, p_{\pi} - 1)]}. \]  

(16b)

The expression for \( f_{\nu}(p, p_{\pi}) \) is obtained from Eq. (16b) when \( A(p, p_{\pi} - 1) \) is replaced with \( A(p, p_{\pi} + 1) \) and \( B_{\pi\nu}(p, p_{\pi}) \) is replaced by \( B_{\pi\nu}(p, p_{\pi}) \). The B's are given by the relation

\[ B_{\pi\nu}(p, p_{\pi}) = B_{\nu\pi}(p, p_{\pi} - 1) = \text{maximum}(A(p, p_{\pi}), A(p, p_{\pi} - 1)). \]  

(17)

4.3 Particle Emission Rates

The particle emission rates with good isospin have the same general form as in the isospin mixed case. Thus

\[ W_{\nu}(p, p_{\pi}, E, T, \epsilon, T_{b}) = \frac{(2s_{b} + 1)}{r_{c}^{2}} \frac{\mu_{b}}{\hbar^{3}} \frac{\epsilon}{\sigma_{b}(\epsilon, T, T_{b})} \frac{\omega(p_{\pi}, h_{\pi}, p_{\nu}, N_{\nu}, h_{\nu}, U, T_{b})}{\omega(p_{\pi}, h_{\pi}, p_{\nu}, h_{\nu}, E, T)} \]  

(18)

where \( T_{b} \) is the isospin in the residual nucleus and \( \sigma_{b}(\epsilon, T, T_{b}) \) is the usual inverse reaction cross section multiplied by the exit channel isospin coupling Clebsch-Gordan coefficient, \( C_{2} \).

If the emitted particle has \( T > 0 \), however, there may be more than one allowed isospin in the residual nucleus. Figure 5 (adapted from ref. [14]) shows the states which need to be considered. For neutron induced reactions, only the lower isospin states in the composite nucleus are populated. When these emit protons only the lower isospin in the residual nucleus is allowed, and when they emit neutrons, two isospin values are allowed, but the lower one is favored both by the coupling coefficients and by the state densities. The higher isospin can be ignored. For proton induced reactions, two \( T \) values are generally populated in the composite nucleus. Decay of the lower isospin (\( T < \)) states in

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the composite nucleus follows the same considerations as for reactions with incident neutrons. For the higher isospin (T> states, neutron emission gives a unique isospin in the residual nucleus, but proton emission can yield two different T values. The ground state T is favored by the state density while the higher T is favored by the Clebsch-Gordan coefficients. Both should be considered.

4.4 Reaction Calculations and Preliminary Results

Separate calculations are performed for each allowed isospin value in the composite nucleus, and the results are added incoherently. The results for each composite nucleus T value have the form of Eq. (10) so that

\[
\left[ \frac{d\sigma_b(\epsilon,T)}{d\epsilon} \right]_{\text{pre}} = \sigma_a(\epsilon_a,T_a) \sum_p \sum_{p_n} S_{\text{pre}}(p,p_n,T) \sum_{T_b} W_b(p,p_n,\epsilon,T,T_b) \tag{19}
\]

Such calculations have been run using the two-component exciton model code PRECO-E assuming [12] that \( E(T) = 110 \text{ MeV} \, (T^2-T_2^2)/A \). Unfortunately, the code currently only allows one isospin value in each residual nucleus, so that for proton induced reactions two T> calculations were run, one for each residual isospin in the \((p,p')\) channel, and the proton spectra were added. As a result the depletion of strength is not correctly calculated. Since the resulting spectra are only being compared with the corresponding PRECO-E isospin-mixed results to see how big a difference isospin conservation could make, this error should not be serious.

The results of Figure 6 demonstrate that the maximum effect due to isospin conservation depends on both the target mass and the bombarding energy. It is also generally larger for incident protons than for incident neutrons. (No results are shown for neutrons on heavy targets because the T-conserved and T-mixed results are virtually identical.) Isospin conservation always works in the direction of enhancing the yield in the inelastic channel relative to the exchange channel when compared with the corresponding isospin mixed results. This enhancement, though small, is often comparable in size to the reductions caused by going from \( R=1 \) to \( R=3 \). Thus it is important that isospin be considered when this or similar phenomena are studied using comparisons with experimental data.

![Figure 6: Preequilibrium energy spectra calculated assuming both complete conservation and complete mixing of isospin.](image-url)
5. Summary and Conclusions

It has been demonstrated that the two-component exciton model yields the same calculated particle energy spectra as the one-component model when similar physical assumptions are made. The calculations with the two-component model are, admittedly, more complicated, but have the advantage that phenomena such as pairing, shell structure and isospin conservation can be treated in a more realistic way. All of these phenomena must be considered in a coherent way if we are ever to gain an understanding of such fundamental quantities as the relative matrix elements for the residual p-p and p-n interactions. A method for including isospin as a good quantum number in the two-component exciton model has been outlined.

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References

THE MULTI-STEP DIRECT REACTIONS

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Abstract

The current status of the multi-step direct reaction theories and their application in evaluation of cross sections of nucleon induced reactions at energies between 14 and 62 MeV are reviewed. The interplay of the one-step process with respect to the multi-step processes are discussed.

1. INTRODUCTION

The earliest important steps towards theoretical description of the forward-backward asymmetry, observed in the preequilibrium angular distributions were the formulation of the intranuclear cascade model (INC) [1,2] and the quantum-statistical master equation approach of Mantzouranis et al. [3] (MWA). Both models succeeded to provide forward peaked double differential cross sections, which we now relate to the statistical multi-step direct reactions, introduced by Feshbach et al. [4] (PKK) and meant as reaction mechanism rather than a theoretical model only.

The INC was applied to finite nuclei and has proved well in describing cross sections taken for protons with hundreds MeV incident energy. When applied in analyses at tens of MeV the model underpredicted the reaction yield at backward angles. This was connected perhaps with assuming a classical path between collisions of nucleons and with the neglect of diffraction and reflection effects.

The MWA model exploits the leading particle concept and calculates the cross sections also in a cascadelike way applied however to infinite nuclear matter. The resulting cross sections fit the angular distributions of preequilibrium nucleons particularly well at lower emission energies. At the high energy end of the emission spectrum, where the surface region of the nucleus plays a dominant role in direct processes, the MWA calculations give too little chance for particle emission [5].

The adoption of the semiclassical preequilibrium models for description of the angular distributions uses...
scattering kernels based on nucleon-nucleon scattering kinematics for an incident nucleon on a Fermi gas. These trials even when accounting for refraction and reflection effects failed in reproducing satisfactorily the back angle yields (see Blann et al.[6]) which appear to be dominated by quantum-mechanical interference. Thus one could expect that only the application of fully quantum-mechanical theories will remove the difficulties in predicting the double-differential cross sections in full extent.

The theories, which have been validated by numerous comparisons with experimental data are the DWBA based multi-step direct reaction method (MDR) developed by Tamura and his coworkers [7-9] (TU) and the statistical theory of FKK [4], which distinguishes the statistical multi-step direct reactions (SMDR) in parallel to the multi-step compound ones (SMCR).

2. THE MULTI-STEP DIRECT REACTION CONTINUUM CROSS SECTIONS

2.1. DWBA continuum cross sections

The justification for the use of the DWBA method in continuum transitions is based on statistical assumptions. In the continuum a large number of very complicated eigenstates |N⟩ are involved, even when a narrow energy interval is considered. The measured cross sections per unit energy are energy averages of the sum of cross sections for excitation of these eigenstates. To calculate all these cross sections is unpractical, however, one can replace this sum by another sum of cross sections for populating a much smaller number of quite simple model states |B⟩. For the single-step cross section one has

\[
\frac{d^2 \sigma^{(1)}(E_b)}{dE_b d\Omega_b} = \sum_N |T_N^{(i)}|^2 \delta(E_N - E_x) = \sum_N \delta(E_N - E_x) \sum_{B_i} c_B^N a_B^N t_{ba}^{B^*} t_{ba} (B)
\]

(1)

with the assumption of random phases \( \sum_N a_B^N c_B^N \delta(E_N - E_x) = \delta_{BB} c_B(E_x) \), which removes interference between transitions to different final states [9]. In eq. (1) \( E_x = (E_a - Q^B) - E_b \), \( E_a \) and \( E_b \) are the energies of excitation of the incident and exit channels, respectively. The transition matrix is \( T_N^{(i)} = \sum_B a_B^N t_{ba} \). The DWBA is just appropriate for calculation of such an average cross section. In applications the model states appear to be shell model particle-hole states or the RPA collective states.

For the two-step transitions the cross section reads

\[
\frac{d^2 \sigma^{(2)}(E_b)}{dE_b d\Omega_b} = \sum_{BC} \int_{E_b^{(Q^B - Q^C)}}^{E_b^{(Q^C)}} C_B(E_b'|) C_C(E_x) \left\{ \frac{d \sigma^{(2)}_{BC}(E_b, E_c)}{d\Omega_b} \right\} dE_c
\]

(2)
with \( E_b' = (E_b + Q_B - Q_C) - E_b \).

The elementary cross sections \( d\sigma^{(1)}_{B}/d\Omega_b \) and \( d\sigma^{(2)}_{BC}/d\Omega_b \) are the DWBA first and second order cross sections. The statistical assumptions prohibit interference between one-step and two-step amplitudes, too. The \( C_B(E_b) \) is a probability per unit energy that a state \( B \) is located at the excitation \( E_b \). The continuum cross sections expressed by eq. (1) and (2) look simple but in numerical calculations they are still too much involved especially in case of the two-step calculations.

One way to make the calculations feasible is to remove the dependence on \( B \) and \( C \) of the elementary cross sections. One can see from DWBA consideration that this dependence arises from the dependence of the form factors \( f_J^{BA}, f_J^{BC} \) on \( B \) and \( C \). In practice one uses averaged \( f_J \) over large numbers of states or replaces them drastically by \( f_j \) an average over the transferred angular momentum \( J \), too (this is the LIFF (1 indep. form factor) approximation). This reduces (1) and (2) to

\[
\frac{d^2\sigma^{(1)}(E_b, \theta_b)}{dE_b d\Omega_b} = \sum_J p_J(E_b) \frac{d\sigma^{(1)}_{B}(E_b, \theta_b)}{d\Omega_b},
\]

\[
\frac{d^2\sigma^{(2)}(E_b, \theta_b)}{dE_b d\Omega_b} = \sum_{J_1, J_2} \int dE_c p_{J_1}(E_c) p_{J_2}(E_c) \sum_J \frac{d\sigma^{(2)}_{BC}(E_b, E_c, \theta_b)}{d\Omega_b}
\]

(3)

(4)

Here the definition \( p_J(E_b) = \sum_{E_b} C_B(E_b)(f_J^{BA})^2 \) for the spectroscopic density holds. By assuming LIFF the model state dependence of eqs. (3) and (4) occurs only through the square of the spectroscopic amplitude \((f_J^{BA})^2\). All but one numerical calculations of TU have assumed LIFF and an appropriate model for \( f_J \). In eqs. (3) and (4) \( J = \{j_l j_f\} \) denotes a set of orbital, spin and total transferred angular momenta, respectively. For residual interaction of Wigner type there is no spin transfer \( s = 0 \) and \( J \) is replaced by 1. In case of inelastic scattering or charge exchange reactions the averaging over the \( \{B = \{j_l j_f\} \ell m\} \) one-particle states provides a form factor \( f_{j_l}(r) \), which is peaked at the surface of the nucleus, thus justifying introduction of a macroscopic, collective form factor (CPP)

\[
\overline{f_{j_l}(r)} = \langle f_{j_l} \rangle = \langle j_l \rangle \bar{R}_{j_l} \frac{dU}{dr}
\]

(5)

The 1-dependent deformation parameter may now be absorbed into the spectroscopic density \( \rho_{j_l} \) leaving the rest as LIFF. A choice of the spectroscopic density appropriate to the CPP (eq. (5)) means to obtain it as the distribution of \( \rho_{j_l} \) in \( E_b \). Allowing the probability distribution \( C_B(E_b) \) to take the Lorentzian form \( C_B(E_b) = \frac{1}{\sqrt{\pi}} \frac{1}{(E_b - E_B)^2 + \gamma^2} \) and remembering that \( d\sigma^{BA} \) is a purely geometrical factor \( \langle j_l l j_f | Y_{\ell m} \rangle \) one can express \( \rho_{j_l} \) explicitly in terms of the single-particle response function \( \chi_{sp} \) [10].
\[ \phi_{\ell}(E_x) = \beta_{\ell}^2 \text{Im} \chi_{\ell}^{sp}(E_x), \]

with

\[ \chi_{\ell}^{sp} = \frac{i}{\pi} \sum_B (\phi_{\ell}^{BA})^2 (E_x - E_B - i\Gamma)^{-1}, \]

or after setting \[ \phi_{\ell}^{BA} = 1, \quad \phi_{\ell}(E_x) = (\beta_{\ell}^2 \sum_B c_B(E_x), \]

which has been used by Traxler et al. [11]. When RPA collective states B are chosen the spectroscopic density to be used is expressed by the RPA response function [12].

2.2. The Statistical MDR Cross Sections

The statistical assumptions made by FKK resulted in a double differential cross section for reaction from an unbound state having a particle of momentum \( k_i \) to one of momentum \( k_f \), which is a sum of terms corresponding to single-step and multi-step transitions. The multi-step cross section reads,

\[ \left[ \frac{d^2 \sigma(k_f, k_i)}{dU d\Omega} \right]_{\text{multi}} = \sum_n \sum_{m=n-1}^{n+1} \int \frac{d\kappa_n}{(2\pi)^3} \int \frac{d\kappa_f}{(2\pi)^3} \frac{d^2 W_{n,n}(\kappa_n, \kappa_f)}{dU_1 d\Omega_1} \times \frac{d^2 W_{m,n}(\kappa_n, \kappa_m)}{dU_2 d\Omega_2} \times \frac{d^2 \rho(k_f, k_i)}{dU_3 d\Omega_3} \]

with the double differential transition probability,

\[ \frac{d^2 \rho(k_n, k_{n-1})}{dU_n d\Omega_n} = 2^{n-2} \rho(k_n) \rho_2(U) < \psi_{n,n-1}/2 \]

for a transition between the continuum states excited in the \( (n-1) \)-th and \( n \)-th stage, leading to a change in the momentum of the continuum particle from \( k_{n-1} \) to \( k_n \). The transition matrix element \( \psi_{n,n-1} \) is of DWBA type

\[ \psi_{n,n-1} = \int \chi_n^{(s)}(k_n) \chi_{n-1}^{(t)}(k_{n-1}) d\tau \]

whereas \( \rho(k) \) and \( \rho_2(U) \) in (11) are the space phase and the one ph (2 excitons, repeatedly formed at each stage of reaction) state density.

The characteristic feature of the multi-step cross section given by eq. (9) is that it is written as a convolution of single-step direct cross sections, thus making the calculations rather simple to carry out. The angular distribution is forward peaked, as is the single-step cross section although the multiple folding will broaden it. The averaging in eq. (10), over many final
states, is straightforward since we expect the interference terms between different states and different orbital angular momentum transfers to cancel out (no spin transfer is assumed for simplicity). One then obtains

\[ \langle | \nu_{n,n-1}^2 | \rangle = \sum_{\ell} (2\ell + 1) \langle | \nu_{n,n-1} \rangle^2 \rangle \tag{12} \]

where \( R_2 (l) \) is the spin distribution function of the one-ph configurations \( \sum_{\ell} (2\ell + 1) R_2 (l) = 1 \). In the same way the averaged single step source term in (10) reads

\[ \left[ \frac{d^2 \sigma (k_f, k_i)}{d\Omega d\Omega} \right]_{\text{single}} = \sum_{\ell} (2\ell + 1) \beta_2 (v) R_2 (l) \langle \frac{d\sigma}{d\Omega} \rangle^{\text{DWBA}}_l \tag{13} \]

In practical calculations one assumes that the only contribution to the spins of final states is due to the transferred orbital angular momentum. For each transferred \( l \)-value the DWBA angular distributions are calculated microscopically for given values of ejectile energies and all possible ph-pairs, created by scattering of the particle in the continuum with a bound nucleon, in accordance with energy conservation. Spherical shell model orbitals are used to define the states. An average of a reasonable number of these cross sections gives the last term in (13). For the interaction \( V(r) \) a Yukawa potential of 1.0 fm range is adopted. Its strength \( V_0 \) is a free parameter.

3. RELATIONS BETWEEN THE MULTI-STEP DIRECT REACTION THEORIES

In spite of the fact that the physical pictures behind the TU and FKK formulations of the continuum cross sections appear to be very close, only the one-step cross sections are derived in both cases in the same way. The multi-step terms differ significantly. The elementary DWBA cross sections for two-step transitions in eq. (2), as well as the higher-order DWBA cross sections contain \( S^{-1} \)-terms in the transition matrix elements (is the scattering matrix). When absorption is large \(|S|\) is small and \( S^{-1} \) is large again. This means that the cross sections given by TU may differ by large from those of FKK, expressed in terms of matrix elements free of the \( S^{-1} \) factor. This fact and the redereivation of the FKK formulas by starting with formula (2) led Udagawa et al. [13] to a claim that the extreme simplicity of the FKK cross sections for the multi-step processes has been obtained at the expense of three successive approximations, namely:
- assuming an on -energy shell approximation by neglecting the principal value integral part of the Green's function,
- neglecting the \( S^{-1} \)-term in the transition matrix elements,
- inserting a delta function \( \delta (\vec{R} - \hat{R}) \) under a double integral in the cross section expression.

The effects of these differences are individually large but have the tendency to cancel out in certain calculations [13]. However
in the light of discussion of the optical model in presence of strong absorption by Feshbach [14,15], the S\(^{-1}\) term seems to be an artifact connected to the second Born approximation. It disappears after energy averaging. The energy averaging destroys also the nonstatistical phase relations among various components in the spectral decomposition of the Green's function and facilitates the application of the random phase approximation. Bearing this in mind the FKK result is better suited for continuum and the use of DWBA matrix elements, free of the S\(^{-1}\) term in calculation of the SMDR cross sections is justified.

4. NUMERICAL CALCULATIONS AND COMPARISON WITH EXPERIMENT

4.1. The DWBA Continuum Cross Section Calculations

The DWBA MDR method has been proved by TU in a number of analyses of (p,p') and (p,α) data obtained at 62 MeV and some (n,n') and (p,n) data obtained at lower energy. The early analyses used LFF and both single-particle and RPA response functions to construct the spectroscopic density. Despite of the overall good description of the angular distributions by using only the one-step and two-step cross sections this theory appeared not free of the magnitude problem. The excellent fit to the \(^{27}\)Al(p,p') and \(^{208}\)Bi(p,p') data [16] was questioned [17] and subsequent analysis, which switched to use the RPA states instead of the shell model pb-states did not improve enough [9]. It has been concluded that the simplifying use of LFF, which in fact made the calculations commonly accessible, causes the difficulty in reproducing the absolute cross sections satisfactorily. The investigation of the use of microscopic form factors MFF together with (1) and (2) are underway [9]. The same pertains to the analysis of the \(^{208}\)Pb(p,n) reaction. The (p,α) reaction has been studied first by TU [10] and also by Dragun et al. [19] at lower proton energies 44.3 and 34.6 MeV. A good agreement with experiment has been reached considering only the one-step direct cross sections in addition to the compound nuclear ones (see fig. 1). Recently Tamura presented an analysis of some (n,n') data and the \(^{63}\)Cu (n,p) data both taken at about 26 MeV (fig. 2). The low incident energy justifies the one-step calculations only. An interesting case was the application of the MDR theory at incident energies as low as 14 MeV, for inelastic neutron scattering on targets ranging from Fe to Au [20]. It was found that the two-step contribution amounted to only 10% of the one-step cross section. The predicted shapes of angular distributions were right but the absolute cross sections too low. Also Traxler et al. [11] report on the use of the ORION-TRISTAR code in their study of the \(^{93}\)Nb (n,p) reaction data measured at 14.1 MeV. These authors treated \(\beta_2\) as an adjustable parameter kept independent on l. The same conclusions have been drawn as in previous case. The cross sections fell down with decreasing spectral energy in contradiction to experiment. Inconsistency has also been stated in parameterization for different projectile energies.

The comparisons presented in figs. 1-2 characterize the present state of art; good angular shapes and magnitude problems.
The limited experience of these analyses indicates that up to 30-40 MeV of projectile energy the one-step process describes the experiment satisfactorily, though already at 62 MeV the two-step contribution dominates in the excitation region 20-30 MeV at backward angles.

4.2. The Statistical MDR Cross Sections

The SMDR theory of FKK has been used in studying $(p,n)$ reactions at energies ranging from 25 to 45 MeV on a number of target nuclei Ca, Zr, Sn, Pb [21,22] as well as on Cu and Mo [23,24]. All these papers report a rather very good reproducibility of the experimental angular distributions and energy spectra (fig. 3). The shortcomings at high excitation energies, in fig. 4, are attributed to the omission of the SMDR contribution. The strength $V_0$ of the Yukawa interaction potential was the only free parameter. $V_0$ values extracted from comparison with experiment range from 25 to 27.5 MeV for the single-step calculations, and from 15.5 to 17 MeV for the average over unlike and like nucleon interactions in the multi-step calculations. The latter extends up to 6 steps providing at 45 MeV 53% of the total cross section for 208-Pb, 45% for 48-Ca and 90-Zr, 28% for 120-Sn at 35 MeV and 19% for 120-Sn and 65-Cu at 25 MeV.

The FKK theory has been also applied by the author in interpretation of the recently measured $(n,n')$ cross sections on 194-Ag at neutron energies 11.5 and 26 MeV [25]. In this analysis difficulties have been encountered which could then be traced to the averaging of the elementary DWBA cross sections. It has been found that the elementary cross sections, corresponding to different $np$-configurations do not necessarily display such a regular pattern like shown in ref. [22], where only particular $np$-pairs have been considered. Moreover the average appeared to fluctuate from one spectral energy bin to another. This was caused by the fact the some of the $np$-configurations dominated the average over a definite energy interval. In order to overcome this difficulty the average had to be extended over almost the whole energy range considered, ignoring energy conservation, which is equivalent to assuming a broad ($\sim 10$ MeV) uniform response function.

5. CONCLUSIONS

Both the FKK and DWBA theories evaluate the one-step direct emission in the same manner and agree in that up to 30 MeV of incident energy the higher order contributions play little role. At these low energies however consideration of the multi-step compound emission, which even at 45 MeV contributes significantly at high excitations, has to be included in the analysis of experimental data. Above 30 MeV the multi-step direct component becomes important and here the two theories differ in its evaluation. The DWBA MDR theory provides two step contributions, which add enough cross section for fitting the experimental data up to 62 MeV, whereas the FKK statistical theory indicates that up to six steps may contribute considerably. Though the overall
A good description pertaining especially to the angular distributions indicates that basically the two approaches are correct there remain magnitude problems related probably to the assumed approximations. For the purpose of nuclear data evaluation specially interesting is the description of the strong forward peaking of experimental double differential neutron cross sections at energies as low as 11-14 MeV [25-27].

REFERENCES


Figure 1. Comparison of the single-step direct plus compound nucleus cross sections with the experimental data of ref. [19] for the \((p, \alpha)\) reaction. The dashed lines in the left-hand side picture are the compound nucleus contributions. The dot-dashed lines are the single-step direct cross sections.
Figure 2. Comparison of the single-step direct cross sections with the Ohio neutron scattering data for $^{56}$Fe and $^{209}$Bi, taken at incident energy 25.7 MeV, and with the $^{65}$Cu(p,xn) cross sections measured at Hamburg. The dashed lines represent the pre-equilibrium calculations based on the hybrid model.
**Fig. 3** Experimental angular distributions (circles) of neutrons from $^{100}$Mo(p,xn) for the bins $(E_n,E_n+0.5$ MeV$)$. Calculations are first step SMDE (dash-dotted line), SMDE of the first three steps (dashed), and the sum SMDE plus SMCE (solid line).

**Fig. 4** Comparison between calculated and experimental differential cross sections for the $^{40}$Ca(p,n) reaction at several excitation energies of the residual nucleus. At the highest excitation energy ($\omega = 28$ MeV), the experimental cross section exceeds the calculated value due to the presence of multiple particle emission.

--- single-step contribution; ——— total.
MULTISTEP COMPOUND REACTIONS

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Abstract: The quantum-mechanical theory of pre-equilibrium reactions due to Feshbach, Kerman and Koonin is applied to analyse the interactions of 14 MeV neutrons and protons with $^{59}$Co, $^{89}$Y and $^{93}$Nb. The accuracy of the calculations has been improved in several respects, in particular by the use of realistic wavefunctions in the nuclear interior and by a more refined treatment of the neutron-proton cascade including the distinguishability of neutrons and protons. The results of the improved multistep compound calculations are compared with neutron and proton emission spectra.
1. Introduction

The theory of Feshbach, Kerman and Koonin (FKK) [1] has been extensively applied to calculate the pre-equilibrium contributions to a range of multistep compound [2] and multistep direct [3] reactions. On the whole the calculations are in good accord with the data, but the fits are often achieved by some parameter adjustment and there are also some approximations in the way the theory is implemented. It is likely that the effects of these approximations are absorbed by the parameter adjustment so that it becomes difficult to make a severe test of the theory or to use it to predict unknown cross-sections.

In this work we describe in Section 2 several improvements that have been made to the theory, and then test the improved theory by comparison with experimental data in Section 3. Some conclusions are presented in Section 4.

2. Improvements to the Feshbach-Kerman-Koonin theory

The essential physics of the theory is the same as for the exciton model. It is assumed that the interaction between the incident nucleon and the target nucleus takes place in a number of stages of increasing complexity. The nucleus is excited by a series of nucleon-nucleon collisions between the projectile and the target nucleons, each producing a particle-hole pair. This process continues until the energy is spread through the nucleus to produce a fully-equilibrated nucleus which then decays statistically. At each stage of the excitation there are three possibilities: excitation of an additional particle-hole pair, de-excitation of a particle-hole pair and emission into the continuum. The particles emitted into the continuum from the first few stages of the reaction together constitute the pre-equilibrium cross-section. This is calculated by evaluating the matrix elements for the transitions from one stage to the next, together with the probabilities at each stage for emission into the continuum. The detailed formalism is given by Feshbach, Kerman and Koonin and in many other papers [1–3]. Here we are concerned with the improvements to the formalism that have been made recently, and their effects on the cross-sections.

In the work of Feshbach, Kerman and Koonin the wavefunctions of bound nucleons were assumed to be constant, thus considerably simplifying the calculation. The strength $V_0$ of the effective interaction is set to a low value to compensate for the high overlap of the constant wavefunctions. Following previous work by Bonetti [2], we have removed this approximation by using for the wavefunctions of the bound nucleons the eigenfunctions of a harmonic oscillator potential and for the scattering wavefunctions those appropriate to a Saxon-Woods potential with the parameters of Becchetti and Greenlees [4].

The residual two-body interaction was taken to have the zero-range form

$$ V(r_1, r_2) = V_0 \frac{4}{3} \pi r_0^3 \delta(r_1 - r_2) $$

(2.1)

This considerably simplifies the calculations and it has been shown by Bonetti and Colombo [5] that it gives essentially the same results as the more realistic Yukawa form, providing the strength parameter $V_0$ is appropriately decreased.

The original formulation of this multistep theory does not explicitly include the distinguishability of neutrons and protons within the nucleus. At each stage in the multistep process the numbers of particles and holes are specified, but not whether the particles and holes are neutrons or protons. We now show how this distinction can be introduced, and how the corresponding correction factors to the FKK expression are calculated.
The damping and escape widths give the probabilities of the excitation of an additional particle-hole pair and of escape into the continuum. They can be written as a product of three factors

\[ \Gamma = 2\pi I^2 XY \]

(2.2)

where \( X \) is an angular momentum coupling factor, \( Y \) is a statistical factor, including density of states effects, and \( I \) is the overlap integral between initial and final wavefunctions.

When calculating the transition matrix elements, no exchange contributions are included. Even when identical particles interact, a fully antisymmetric wavefunction for these particles was not used. Nuclear structure calculations suggest that this is a reasonable approximation, especially as the interaction assumed here is a delta function.

At each stage the density of states \( \omega(p, h, E) \) is given by the equidistant single-particle model of Williams [6]

\[ \omega(p, h, E) = \frac{g^n E^{n-1}}{p!h!(n-1)!} \]

(2.3)

where \( g \) is proportional to the level density parameter, \( p \) and \( h \) are the numbers of particles and holes, \( n = p + h \) and \( E \) is the total energy including the Pauli correction factor.

The processes contributing to damping and escape are most conveniently shown diagrammatically. In Figs.2.1 and 2.2 an upward arrow represents a particle, a downward arrow a hole, and vertical lines indicate spectator nucleons. Since the two-body interaction is dominant, the damping process is represented by the diagrams shown in Fig.2.1.

Diagram (a) describes a hole interacting with a bound nucleon and exciting it to produce a particle-hole pair. In diagram (b) it is a particle above the Fermi level which excites a particle-hole pair. There are three possible types of escape modes, in which the number of excitons changes by \(-2, 0, \) or \(+2\), and these are shown in Fig.2.2.

The \( Y \) functions may be calculated with the aid of these diagrams. If the incident projectile particle is a proton, the original Feshbach formulation describes the processes as

\[ \begin{array}{c}
1p \\
\rightarrow\\
\rightarrow\\
\rightarrow\\
\rightarrow\\
\rightarrow
\end{array} \quad 1p \rightarrow 2p1h \rightarrow 3p2h \rightarrow 4p3h \rightarrow \text{Towards equilibrium} \]

(2.4)

However, treating the neutrons and protons as distinguishable particles leads to the possible reaction chains

\[ \begin{array}{c}
\text{Possible emission} \\
\rightarrow\\
\rightarrow\\
\rightarrow\\
\rightarrow\\
\rightarrow
\end{array} \quad 1p_\pi \rightarrow 1p_\nu, 1h_\pi \\
\rightarrow\\
\rightarrow\\
\rightarrow\\
\rightarrow\\
\rightarrow\quad 1p_\pi, 2p_\pi, 1h_\pi, 1h_\nu \]

(2.5)

where \( \pi \) indicates a proton and \( \nu \) a neutron.

Thus if a proton is the incident projectile the only possible \( 2p1h \) states are \( 2p_\pi 1h_\pi \) and \( 1p_\pi 1p_\nu 1h_\nu \), and states such as \( 2p_\pi 1h_\nu \) are not possible. This selective population
2.1 Diagrammatic representation of the two damping processes.

2.2 Diagrammatic representation of the four escape processes.

2.3 Diagrammatic representation of the damping processes in the two-component model. The nature of the interaction (i.e. whether it is between two neutrons, two protons, or a proton and a neutron) is indicated above the residual interaction in the above diagrams.
of exciton states leads to a modification of the relative numbers of neutrons and protons
emitted in the reaction. Inclusion of this effect is particularly important when analysing
neutron emission and proton emission data for a given incident nucleon in a reaction.

There are two factors which will affect the relative population of the allowed
exciton states:

(a) The distinguishability of nucleons enables the unlike nucleon to be more easily
excited than the like one.
(b) The interaction between unlike nucleons is different from that between like ones.
The ratio of their strengths is treated as a parameter.

Since pre-equilibrium emission from the first 2p1h stage is usually the most impor-
tant contribution to the pre-equilibrium cross-section, the necessary correction factors
are calculated only for this stage.

The simple FKK ('one-component model') expression for the first stage emission is

\[
\frac{d^2\sigma}{d\Omega de} = \pi \lambda^2 \sum_J (2J + 1) \sum_{\ell \ell'} C_{\ell \ell'} \rho_\lambda(\cos \theta) \sum_\nu \frac{< \Gamma_{t\ell}^{\ell\nu}(U) \rho_\nu(J)^* >}{< \Gamma_{\lambda J} >} \cdot \frac{2\pi < \Gamma_{t\ell}^{\ell\nu} >}{< D_{t\ell} >}
\]

(2.6)

When the neutron and proton distinguishability is included, this becomes

\[
\frac{d^2\sigma}{d\Omega de} = \pi \lambda^2 \sum_J (2J + 1) \sum_{i=0}^1 \sum_{\ell \ell'} C_{\ell \ell'} \rho_\lambda(\cos \theta) \sum_\nu \frac{< \Gamma_{t\ell}^{\ell\nu}(U) \rho_\nu(J_i)^* >}{< \Gamma_{\lambda J_i} >} \cdot \frac{2\pi < \Gamma_{t\ell}^{\ell\nu} >}{< D_{t\ell} >}
\]

(2.7)

where the sum over \(i\) represents the sum over emissions from the two possible substates.
The approach used by Dobeš and Béták [7] for the two-component exciton model can be
extended to calculate the \(Y\) functions and the corresponding escape and damping
widths for the two-component FKK theory. It was found that, to a good approximation,
the first stage two-component expression (2.7) can be reduced to a one-component
expression (2.6) with corrected reduced widths. As the \(X\) functions contain only angular
momentum coupling factors they remain unchanged in the two-component theory.

To illustrate the method of calculation, we describe in detail the calculation of the
damping widths.

The one-component FKK model

The two possible damping processes are shown in Fig.2.1.

In diagram (a) a particle above the Fermi level interacts with a bound nucleon,
exciting it to produce a particle hole pair. The bound particle itself loses energy in the
process. The \(Y_{n+21}^{n}\) function can be written as a product of the probability of finding
one specific particle of energy \((E - z)\) amongst all the possible configurations of particles
and holes at an energy \(E\), the state density of this particle, and the state density of two
particles and one hole after the interaction.

Thus for process (a)

\[
Y_{n+21}^{n} = \int_{z=0}^{E} \frac{\omega(p-1,h_1,z)}{\omega(p,h,E)} \omega(1,0,E-z) \omega(2,1,E-z) dz = \frac{pg^3 E^2}{2n(n+1)}
\]

for process (b)

\[
Y_{n+21}^{n} = \int_{z=0}^{E} \frac{\omega(p,h-1,z)}{\omega(p,h,E)} \omega(0,1,E-z) \omega(1,2,E-z) dz = \frac{hg^3 E^2}{2n(n+1)}
\]

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Adding these two gives

$$Y_{n+21}^n = \frac{g^2 E^2}{2(n+1)}$$

so

$$\Gamma_{n+21}^n = 2\pi X^4 \frac{g^2 E^2}{2(n+1)} I^2$$

(2.8)

The two-component reformulated model

Neglecting the Coulomb interaction, for nucleons in the same state the $\pi - \pi$ interaction is the same as the $\nu - \nu$ and the $\pi - \nu$ interaction. For distinguishable protons and neutrons the state density expression becomes

$$\omega (p_\pi, h_\pi, p_\nu, h_\nu, E) = \frac{(g/2)^n E^{n-1}}{p_\pi!h_\pi!p_\nu!h_\nu!(n-1)!}$$

$$n = p_\pi + h_\pi + p_\nu + h_\nu$$

(2.9)

Four diagrams contribute to the $\Delta n = +2$ processes (Fig. 2.3): Diagram 1.

$$Y_{n+21}^n = \int_0^E \frac{\omega (p_\pi - 1, h_\pi, p_\nu, h_\nu, z)}{\omega (p_\pi, h_\pi, p_\nu, h_\nu, E)} \omega(1, 0, 0, 0, E-z) \omega(2, 1, 0, 0, E-z) dz = \frac{p_\pi (g/2)^3 E^2}{2n(n+1)}$$

(2.10)

Diagram 3.

$$Y_{n+21}^n = \int_0^E \frac{\omega (p_\pi, h_\pi, p_\nu - 1, h_\nu, z)}{\omega (p_\pi, h_\pi, p_\nu, h_\nu, E)} \omega(0, 0, 1, 0, E-z) \omega(1, 1, 1, 0, E-z) dz = \frac{p_\nu (g/2)^3 E^2}{n(n+1)}$$

(2.11)

Diagram 5.

$$Y_{n+21}^n = \int_0^E \frac{\omega (p_\pi, h_\pi - 1, p_\nu, h_\nu, z)}{\omega (p_\pi, h_\pi, p_\nu, h_\nu, E)} \omega(0, 1, 0, 0, E-z) \omega(1, 2, 0, 0, E-z) dz = \frac{h_\pi (g/2)^3 E^2}{2n(n+1)}$$

(2.12)

Diagram 7.

$$Y_{n+21}^n = \int_0^E \frac{\omega (p_\pi, h_\pi, p_\nu, h_\nu - 1, z)}{\omega (p_\pi, h_\pi, p_\nu, h_\nu, E)} \omega(0, 0, 0, 1, E-z) \omega(1, 1, 0, 1, E-z) dz = \frac{h_\nu (g/2)^3 E^2}{n(n+1)}$$

(2.13)

The overlap integrals will be different for $\pi - \nu$ and $\pi - \pi$ (or $\nu - \nu$) processes. Adding the above contributions gives

$$\Gamma^1(\Delta n = 2) = 2\pi X^4 \frac{(g/2)^3 E^2}{n(n+1)} \left[ \frac{1}{2} n_\pi I^2(\pi\pi) + n_\nu I^2(\pi\nu) \right]$$

(2.14)

By symmetry

$$\Gamma^1(\Delta n = 2) = 2\pi X^4 \frac{(g/2)^3 E^2}{n(n+1)} \left[ \frac{1}{2} n_\nu I^2(\pi\pi) + n_\pi I^2(\pi\nu) \right]$$

(2.15)

The total $\Delta n = 2$ damping width is

$$\Gamma^1(\Delta n = 2) = 2\pi X^4 \frac{(g/2)^3 E^2}{n(n+1)} \left[ \frac{1}{2} I^2(\pi\pi) + I^2(\pi\nu) \right]$$

(2.16)

this may be compared with (2.8).
The modified expressions for $\Gamma_l^l(\Delta n = 0)$ and $\Gamma_l^l(\Delta n = 2)$ as well as the entrance width $\Gamma^{in}$ have been calculated in the same way. The two-component expression is equivalent to the one-component expression with corrected escape widths. The correction factors depend on the ratio of the $\pi - \nu$ to the $\pi - \pi$ interaction strengths. If this is taken as unity they are, for $(n,n)$ and $(p,p)$ processes

$$K_1^l(\Delta n = 2) = 1.27 , \quad K_1^l(\Delta n = 0) = 1.36$$

and for $(n,p)$, and $(p,n)$ processes

$$K_2^l(\Delta n = 2) = 1.09 , \quad K_2^l(\Delta n = 0) = 0.73$$

The validity of the assumption of equal $\pi - \nu$ and $\pi - \pi$ interaction strengths is hard to determine. For free nucleon-nucleon scattering the $n - p$ cross section is about three times larger than the $n - n$ cross section due to isospin effects. In this case the nucleons are bound and their distinguishability is incorporated into the density of states expression. Charge independence would suggest that the basic interaction strength ratio for the $\pi - \nu$ to $\pi - \pi$ force should be unity. It is likely that the true effective ratio would lie somewhere between one and three. In any case the correction factors are not very sensitive to this ratio, and the factors for a ratio of three are not significantly different to those given above.

These correction factors show that emission of particles of the same type as the incident particle is enhanced over the emission of the opposite type of particles.

![Diagram](image)

2.4 Comparison between the $Y$-functions calculated as described in the text with those obtained from the analytical expressions of Stankiewicz.

In the calculations we have also incorporated the restriction of excitons to bound states. The Williams expression (2.3) for the exciton state density gives the number of configurations that a system of particles and holes with a given total energy can have, but does not impose any restriction on the individual particle and hole energies. For multistep compound processes the particles above the Fermi level are restricted to energies smaller than their binding energy, and holes are restricted to energies less negative than the nuclear potential well depth.

The reactions considered in the next section are induced by 14 MeV nucleons, so
the excitation energy of the composite system is of the order of 23 MeV. The lowest energy level that a bound nucleon can be excited from is thus about 23 MeV below the Fermi level, so that the nuclear well depth restrictions are unimportant here. However the restriction to bound states must be included in the calculation of the $Y$ functions for the damping and escape widths. The numerical cut-off technique developed by Bonetti [8] has been used to determine the $Y$ functions. The $Y$ functions calculated using this method are compared with $Y$ functions obtained using the analytic expressions of Stankiewicz et al [9] and Obložinský [10] in Fig.(2.4) and this shows the consistency of these two methods.

3. Comparison with Experimental Data

Several multistep compound calculations carried out as described in the previous section were compared with the cross sections of $(p,zn)$ and $(n,zp)$ reactions, and the results are shown in Figs.3.1–3.3. The experimental cross sections are nearly symmetric

3.1 Differential cross-sections for the $^{89}$Y $(p,xn)$, $^{59}$Co $(p,xn)$ and $^{59}$Co $(n,xn)$ reactions compared with multistep compound calculations. In the case of $^{59}$Co $(n,xn)$ the difference between the data of Salnikov et al and that of Hermsdorf et al makes reliable comparison impossible. In the case of the $^{89}$Y $(p,xn)$ reaction, neutrons from the $(p,pn)$ reaction contribute to the measured cross-section at outgoing proton energies below that indicated by the arrow; these were not included in the calculation.
3.2 Differential cross-section for the $^{93}$Nb ($n, nx$) reaction compared with multistep compound calculations. The theory and experiment agree well for outgoing energies between 4 and 6 MeV but at higher energies the data are discrepant. Below 3.5 MeV the data include contributions from the $(n, 2n)$ and $(n, pn)$ reactions, which are not included in the calculation.

3.3 Angle-integrated differential cross-section for the $^{93}$Nb ($n, xp$) reaction compared with multistep compound calculations. The data have been corrected to remove the direct component.
about 90° indicating that they are predominantly due to the multistep compound process, especially in the backward direction. The level density parameters of Facchini [11] were used where available and otherwise those of Gilbert and Cameron [12]. The value of the residual interaction parameter strength, \( V_0 \), was found to be rather constant for the reactions considered, lying between 7.9 and 10.5 MeV. The parameters used are shown below in table I.

A serious problem is the discrepant nature of some of the experimental data. Thus in Figs.3.1 and 3.2 the \((n,xn)\) data of Salnikov et al [13] and of Hermsdorf et al [14] diverge markedly, making it impossible to assess the success of the theory in these cases. In other cases the comparison is satisfactory, particularly for the \(^{89}\)Y \((p,xn)\) and \(^{59}\)Co \((p,xn)\) reactions shown in Fig.3.1 [15]. The calculations also agree well with the \(^{59}\)Co \((n,xn)\) data of Hermsdorf et al. If these data are correct, it is then notable that the theory fits very well the data for both the \(^{59}\)Co \((p,xn)\) and \(^{59}\)Co \((n,xn)\) reactions.

The comparison in Fig.3.2 for the \(^{93}\)Nb \((n,nx)\) reaction is satisfactory for outgoing neutron energies from 4 to 6 MeV where the data are consistent. At lower energies there are additional neutrons from the \((n,2n)\) and \((n,pn)\) reactions and previous calculations show that the observed cross-section is in accord with calculations of two-particle emission using the Weisskopf-Ewing theory for the second stage. At high outgoing neutron energies there may be an additional contribution to the cross section due to collective inelastic processes, but it would be premature to investigate this until the data discrepancies are removed. The calculations also agree with the \(^{93}\)Nb \((n,\pi p)\) cross section shown in Fig.3.3 at outgoing proton energies above 7 MeV.

**Table I**

Parameters used in the calculations: \( V_0 \), the strength of the two body interaction (in MeV); \( a_c \), the single particle level density of the compound nucleus; \( a_{r1} \), the same as \( a_c \) but for the residual nucleus; \( a_{r2} \), the same as \( a_c \) but for the residual nucleus of the reaction in competition.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>( a_c )</th>
<th>( a_{r1} )</th>
<th>( a_{r2} )</th>
<th>( V_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{89})Y ((p,n))</td>
<td>12.0</td>
<td>10.0</td>
<td>9.0</td>
<td>7.9</td>
</tr>
<tr>
<td>(^{59})Co ((n,n))</td>
<td>7.2</td>
<td>8.0</td>
<td>7.3</td>
<td>10.0</td>
</tr>
<tr>
<td>(^{59})Co ((p,n))</td>
<td>6.5</td>
<td>6.0</td>
<td>8.0</td>
<td>9.4</td>
</tr>
<tr>
<td>(^{93})Nb ((n,p))</td>
<td>12.5</td>
<td>11.8</td>
<td>10.8</td>
<td>10.5</td>
</tr>
<tr>
<td>(^{93})Nb ((n,n))</td>
<td>12.5</td>
<td>10.8</td>
<td>11.8</td>
<td>10.5</td>
</tr>
</tbody>
</table>

4. **Conclusions**

The results presented in the previous section show that the improved FKK theory is able to give satisfactory fits to a range of experimental data. The level densities were not adjusted, and tabulated values were used in each case. The constancy of the values of \( V_0 \) in Table I show that quite good fits could be obtained with an average value of \( V_0 = 10 \) MeV. It remains necessary to test the theory over a wider range of energies, projectiles, target nuclei and emitted particles, and to evaluate its predictive power.
References


THE CONTRIBUTION OF DIRECT PROCESSES
TO INELASTIC NEUTRON SCATTERING

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ABSTRACT

The analysis was made for the results of direct process contribution to the observed spectra of inelastic neutron scattering with the initial energies 14 and 26 MeV. The uncertainties of theoretical description for integrated direct transitions cross-sections associated with the available data on spectral distribution of nuclear coherent multipole excitation intensities are discussed.

Subdividing nuclear reactions into direct and compound ones proceeding through the compound nucleus state is related to resolving open and closed configurations in the stationary wave function, which describes the system under analysis /1/. The direct processes within this approach are defined as the transitions due to relation of diverse open configurations (or channels) whereas the compound processes correspond to the transitions from the open configurations to the closed ones and vice versa. If there is a hierarchy of closed states concerning their coupling with the open channels, then long-lived states of the compound nucleus will correspond only to the late stage of evaluation of transitions between various intermediate configurations. Particle emission at an early stage of the evolution process in this case can be called a preequilibrium decay of closed configurations. The resulting spectrum of particles emitted in the reaction in the general case should consist of three components: a hard spectrum of direct transitions, Maxwell spectrum of equilibrium compound nucleus decay and a spectrum of preequilibrium compound nucleus configurations decay with intermediate characteristics /2, 3/. All the models adopted in the practical tasks of nuclear cross-sections description /4, 5/ incorporate the above three mechanisms of nuclear processes. However, essential disagreements occur in the quantitative analysis of integrated contributions of direct and preequilibrium processes. On the example of neutron spectra analysis we intend this work to deal with up-to-date theoretical evaluations of direct process contributions and conclusions on the role of preequilibrium compound processes drawn on the basis of these evaluations.
Born approximation of distorted waves (DWBA) is a traditional and commonly used method of direct reactions description /1/. For inelastic scattering the corresponding ratio can be given in the form:

$$G_{ij}^{ij}(\theta) = \sum_{j_i,j_f} \sum_{\lambda} C_{j_i,j_f}^{\lambda} (\theta) \int_0^\infty U_{j_i}^*(r) F_{\lambda}(r) U_{j_f}(r) dr \right)^2,$$

(1)

where $$C_{j_i,j_f}^{\lambda} (\theta)$$ are the kinematic factors determined by the angular moments addition, $$U_{j_i}^*(r)$$ and $$U_{j_f}(r)$$ are the radial wave functions of the optical model for neutrons in entrance i and exit f channels, $$F_{\lambda}(r)$$ is the form-factor of the excited level of nucleus-target.

At the inelastic scattering the direct mechanism excites basically the coherent collective degrees of freedom, and at the phenomenological approach based on the relationships of the integrated nuclear model the form-factor is normally written in the form:

$$F_{\lambda}(r) = \frac{\beta_{\lambda} R_o}{(2\lambda + 1)^{1/2}} \frac{dV(r)}{dr},$$

(2)

where $$V(r)$$ is the optical model potential, $$R_o = r_0^3$$ is the nucleus radius and $$\beta_{\lambda}$$ is the dynamic deformation parameter for the corresponding multipole excitations.

The first $$2^+$$ and $$3^-$$ levels are excited for even-even nuclei-targets at the inelastic direct scattering. There is a wealth of experimental data on the parameters $$\beta_2$$ and $$\beta_3$$ for these levels, which is extracted from the analysis of the reduced gamma-transitions probability, as well as differential cross-sections of charged particles and neutrons.

It appears possible to obtain the data on the spectral distribution parameters $$\beta_{\lambda}$$ for a number of nuclei in the experiments with high resolution even for higher levels. The most complete information of this kind seems to be obtained for a double-magic nucleus $$^{208}\text{Pb}$$, where the intensities of all direct transitions in the nuclear excitation energy range up to 7 MeV were analyzed in the experiments on 35 MeV proton scattering /6/.

If these data are used for the calculations of direct transition intensity in inelastic scattering of neutrons with the initial energy 14 MeV, then the description of neutron spectra shown in fig. 1 /7/ can be obtained. The hard non-statistical part of the observed spectra in fact is seen to be totally determined by the direct processes. Spectral distribution intensities $$\beta_{\lambda}$$ in a magic lead isotope can also be adopted for the calculation of the part of inelastic neutron scattering spectra on neighbouring odd nuclei. In particular these calculations give a true representation of the observed spectra of neutron scattering on $$^{209}\text{Bi}$$ nucleus /7/. The above approach naturally results in a fair descrip-
tion of high energy neutron angular distributions.

For a much lighter non-magic nucleus $^{56}$Fe a rather abundant information on the parameters $\beta_\lambda$ is also available. These experimental values $\beta_\lambda$ were used by us for direct-transition spectra calculations of inelastic scattering of 14.5 MeV neutrons. The resulting integral spectra are shown in fig.2 together with the corresponding experimental data. Ref. /9/ contains a more detailed comparison of the calculations with the double-differential spectra observed. Note that for non-magic nuclei the coupling of the lowest collective excitations $2^+$ and $3^-$ with the elastic scattering channel is not weak. That is why the calculations were held within the method of strongly-coupled channels rather than DWBA.

This approach permits the effect of collective transitions on an imaginary part of optical potential to be taken into account consistently, as well as the contributions of two-step direct transitions developed at sequential excitation of various coherent modes (phonons) to be calculated correctly. In fig.2 the theoretical calculations are averaged over the corresponding experimental resolution of the measured neutron spectra, and the resulting integral cross-section of direct transitions really describes the whole hard part of the observed spectra above the energy 9 MeV.

In spite of fairly high resolution of proton experiments /6, 8/, they are helpful for obtaining information which is, however, related only to a small region of coherent multipole nuclear excitations. An essential amount of data on isoscalar quadrupole and octupole giant resonances have been collected in the active investigations of giant resonances of the recent years /10/. The experimental information, however, is almost missing for the resonances of higher multipolarity. That is why we are forced to rely on the theoretical calculations $\beta_\lambda(U)$ results when considering the total spectral intensity of diverse multipole nuclear excitations. By now a great number of the like calculations have been performed which differ basically in the choice of effective forces applied in the microscopic calculations of coherent multipole nuclear excitations intensities /11 - 13/. Explicit or implicit fitting of the calculations to the experimental data on the lowest collective nuclear excitation intensities being employed in any of the microscopic approaches, as well as those for the known giant resonances, the disagreement of the calculation results for different groups appear not too large. Fig.3 is an illustration of the results of two calculations for the spectral intensity $\beta_\lambda(U)$ for $^{56}$Fe nucleus. Calculations /12/ adopt the agreement conditions to determine the effective forces, and the residual force constant remained unchanged in the calculations of excitation intensity with different multipolarity. Alternatively, a more differentiated determination of force constants was specified in calculations /13/, the constants were selected independently for each chosen multipolarity on the basis of the experimental information on the low-lying collective nuclear levels. Despite the diversity of approaches the main features of
\( \beta_{\lambda}^2 \) distributions do not differ greatly. The disagreements are displayed in energy bias of the most intensive excitations by 1 - 2 MeV, and these differences seem to specify a realistic error of theoretical spectral intensity calculations.

Fig. 3 represents also experimental data on the values \( \beta_{\lambda}^2 /8 \) together with the calculations. The calculations can be seen to agree with the experiment for the lowest collective \( 2^+ \) and \( 3^- \) excitations only, whereas the disagreement in the intensity of excitations \( 2^+, 3^+, 6^+ \) in the excitation energy range 3 to 7 MeV appear rather significant. There is an impression, that the direct transitions multipolarity is not always identified unambiguously in the analysis of the experimental data /8/, and a portion of excitation with multiplicities \( 2^+, 3^- \) and \( 4^+ \) should well be referred to higher multiplicities. A more accurate experiment only is able to overcome these doubts. It is essential to note, that multipolarity variations of direct transitions recorded in the experiments will not influence dramatically the results of the integral contribution of direct processes to neutron spectra (Fig.2), and consequently the general conclusions on the mechanism of this spectra portion forming.

Fig. 4 shows the calculation results for \( ^{93} \text{Nb} \) nucleus inelastic neutron scattering spectra.

Experimental information on the parameters \( \beta_{\lambda} \) for this nucleus is far more deficient than for the above nuclei, and the calculations of direct transition intensities are primarily based on the theoretical description of spectral intensity in multipole coherent excitations for the neighbouring even-even nucleus \( ^{92} \text{Zr} /12/ \). The comparison of the calculations with the observed spectra implies the same conclusions as for the previous nuclei.

Fig. 5 demonstrates the results of direct transitions spectra calculations at the inelastic 25.7 MeV neutron scattering. These calculations employ the same spectral distributions \( \beta_{\lambda}^2(U) \) as in the analysis of neutron spectra with the initial energy 14 MeV. In addition to low-lying collective excitations and pigny-resonances with excitation energy about 5 - 9 MeV, giant multipole resonances with the energies above 10 MeV start making essential contribution to the integral spectrum of direct transitions as well as superposition of giant resonances with low-lying collective excitations. The comparison of the calculations with the experimental spectra /14/ enables us to draw a conclusion on the direct processes to be absolutely responsible for inelastic neutron scattering spectra shaping in the excitation neutron energy range up to 5 MeV, and their contribution to the formation of non-statistical spectra component for the excitation energies above 5 MeV is as low as 40 - 50%.

Nowadays rather simple relations of the preequilibrium nucleon evaporation model are widely used for the analysis of a hard component of nucleon inelastic scattering spectra /4, 5/. Simplicity of the calculations certainly is an attractive property of the models, but it fails justifying the errors in the conclusions on nuclear process mechanisms. The above calculations as
well as the analysis results of 39 and 62 MeV proton inelastic scattering spectra /15, 16/ indicate the dominant contribution of the direct processes in the hard component of spectra on the one hand, and the need of taking into account the coherent collective properties of nuclei in the correct analysis of direct transitions on the other hand. Therefore, a quantitative analysis of multistep direct transitions cannot be justified without taking into account coherent phenomena. These phenomena are due to fairly strong effective interaction of nucleons, and their effect extends in fact to all nuclear properties. Without the direct processes being allocated a physically justified evaluation of multistep compound transitions cross-sections cannot be obtained. In any case the contribution of these transitions to inelastic neutron scattering spectra in Refs. /17/ is overestimated.

For the consistent description of preequilibrium compound processes cross-sections to be reached, their analysis should be made not only with regard to the integral cross-section of direct transitions obtained in this work, but with the use of sequential microscopic description of thresholds and multiquasiparticle nuclear excitation densities /18/ as well.
REFERENCES


Fig. 1. Spectrum of inelastically scattered neutrons with initial energy 14 MeV on $^{208}$Pb nucleus:

Dotted line - spectrum of direct transitions,
solid curve - sum of equilibrium and preequilibrium evaporation of neutrons,
dashed curve - resulting calculation of spectrum taking into account reactions (n,2n).
Fig. 2. The same for $^{56}$Fe nucleus. Experimental data are compilation of measurements by various authors. 
Solid line - spectrum of direct transitions, 
 dash-dotted line - spectrum of equilibrium transitions.
Fig. 3. Spectral distributions of multipole coherent excitations $\beta^L_\lambda$ intensities for $^{56}$Fe:
- $\bullet$ - experimental data /8/,
- $\cdot$ - calculations /12/,
- $\ldots$ - calculations /13/.
Fig. 4. The same as in Fig. 2 for $^{93}$Nb nucleus.
Fig. 5. Hard portion of inelastic neutron scattering spectra with the initial energy 25.7 MeV:
- experimental data /14/,
solid histogram - direct transition calculations,
dotted histogram - calculations of equilibrium and preequilibrium evaporation of neutrons.
PRECOMPOUND EMISSION OF NUCLEONS IN HEAVY-ION REACTIONS

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ABSTRACT

A summary of inclusive and exclusive data of precompound or preequilibrium nucleon emission in heavy-ion induced reactions is given. The influence of impact parameter and mass asymmetries in the entrance channel is discussed. Preequilibrium nucleon emission in heavy-ion induced reactions are primarily studied in order to obtain information on the early stages of the energy and linear-momentum dissipation in these reactions. Neutron and proton data can be used to study isospin dependencies of the nucleon-nucleon interaction inside excited nuclei. Phenomenological approaches to parametrize the preequilibrium nucleon emission are discussed and the experimental data are compared with microscopic and macroscopic reaction models.
1. Introduction

Already in the early fifties the study of neutron and proton induced reactions indicated that the deexcitation of nuclei disagree with the idea of the formation and decay of a compound nucleus. For instance Eisberg [1] noted in 1954 that in order to describe the observed proton energy spectra from inelastic proton scattering from Au at 135° a temperature of 7 MeV is required which results in the assumption that the total excitation energy would “be confined to only ~3 nucleons in Au”. This statement is an early implicit formulation of the exciton model [2,3] which was and is very successful in describing precompound light-ion and also heavy-ion induced light-particle emission as we will see in this talk and the one by Blann. About ten years later Britt and Quinton [4] observed highly energetic α-particles also in heavy-ion induced reactions. However, since the emission of complex particles is more complicated (e.g. preformation probability) than nucleon emission I will restrict myself in this talk to nucleon emission. Because light-ion induced nucleon emission has already been studied extensively I briefly state the expectations we have in studying in addition the precompound emission also in heavy-ion induced reactions:

- In order to study the thermal and statistical properties of hot nuclei it is absolutely necessary to know the thermal excitation energy left in the fragments after preequilibrium emission.

- To understand the mechanisms of preequilibrium emission and energy dissipation in the interaction of systems consisting of a large number of nucleons. There is a conceptual difference between this and light-ion induced precompound emission in the sense that for the emission of a nucleon which carries away a large fraction of the total excitation energy brought into the system by A nucleons a coherence of several degrees of freedom is necessary. These questions are related to the subthreshold pion production and high energy gamma-ray emission.

- Since the models describing precompound emission use the nucleon-nucleon cross section inside cold or hot nuclear matter as an essential quantity one might hope to extract this quantity from the study of precompound nucleon emission. By studying entrance channel mass-asymmetries the degree of heating prior to the nucleon absorption of subsequently injected nucleons can be varied.

- The isospin dependencies of the nucleon-nucleon cross section influence the neutron-to-proton ratio in preequilibrium emission. Furthermore different N/Z ratios in the projectile and target can be exploited to get information on the spatial origin of the precompound nucleons.

Thus there are many additional interesting questions which we hope to answer in studying precompound nucleon emission in heavy-ion reactions. Any theory or model is of course simplest for neutrons since the long-range Coulomb interaction can be neglected.

2. Separation between Compound and Precompound Emission

For nucleon or light-ion (n,p,d,...α) induced reactions on heavy nuclei the separation between direct and/or precompound reactions and compound nucleus evaporation is experimentally considerably simpler than in the reaction of two heavy ions. In the latter case we have in principle two excited nuclei after the reaction both of which can deexcite via light-particle emission. Since the projectile-like compound nucleus will move still at high, that is almost beam velocity to angles smaller than the grazing angle of the reaction (8° or 30° for Ho + 30 or 10 MeV/nucleon Ne, respectively) any compound nucleus evaporation from the projectile-like nucleus will result in high-energy particles due to the sum of the velocities of the source and the evaporated particles which are, however, not due to precompound emission. This process is usually called sequential emission [5,6]. One possibility to maintain the simple separation of compound and precompound emission based on the emission energy is to confine the study to fusion-like processes where the situation is very similar to the light-ion induced reactions. However, this requires exclusive experiments.
Figure 1: Double differential neutron multiplicity measured [7,8] in coincidence with evaporation residues detected at $\Theta_{ER}$; the neutron detection angles $\Theta_n$ are indicated for the different spectra which were multiplied by $10^n$ with $n=4, 3, 2, 1, \text{and } 0$, starting with the smallest neutron angle. The dashed and dotted curves are fits to the high and low energy component, respectively. The solid line is the sum of both components. At the highest bombarding energy of 600 MeV the spectra are shown in coincidence with a mean linear momentum transfer of 93%.

Yet, the majority of data of precompound nucleon emission has been collected in inclusive experiments in which case the precompound emission at forward angles smaller than about 30°, where the angular distribution of the projectile-like fragments are peaked, had to be disregarded due to the above described sequential emission. But in order to be sensitive to various models of precompound emission the nucleon energy spectra at such small angles are important. That is why I will confine myself in the following essentially to exclusive experiments but in the discussion I will also include the large body of inclusive data.

3. Phenomenology of Preequilibrium Nucleon Emission

Systematic studies of preequilibrium (PE) neutron emission in heavy-ion reactions have been performed by measuring neutrons in coincidence with evaporation residues (ER) from fusion-like reactions of light heavy-ions ($A=12-20$) on heavy targets ($A=141-175$). Sequentially emitted neutrons from projectile-like fragments are rejected by detecting the ER at small angles and requiring that their mean velocity is close to the velocity of the center-of-mass. A study of such reactions over a wide range of bombarding energies has been carried out [7,8] for the reaction $^{165}\text{Ho} + (11-30\text{MeV/nucleon})^{20}\text{Ne} \rightarrow \text{ER} + \text{n}$. The ER were detected at small angles ($5° - 16°$)
with the condition that their velocity was close to that corresponding to full linear-momentum transfer. Thus events in which projectile-like fragments sequentially emit neutrons were rejected. The results of this systematic study is shown in figure 1 where neutron energy spectra in the lab. system are shown at four bombarding energies and several neutron detection angles ($\Theta_n$). The qualitative appearance of the heavy-ion induced neutron emission is very similar to that observed by light-ion induced reactions. We observe a well separated low and high energy component. The spectra of the latter which we associate with preequilibrium emission show (i) an exponential decrease with emission energy, (ii) a strong decrease with angle (much stronger than one would expect from the transformation from the c.m. to the lab. system) and (iii) the intensity as well as the mean energy is increasing with bombarding energy (see also figures 2 and 3). It is also interesting to note that though with increasing bombarding energy an increasing amount of the initially available excitation energy is carried away by preequilibrium neutrons the equilibrated excitation energy left in the system is also increasing. The inverse exponential slopes representing the thermal temperature of the low energy component are (figure 1) increasing with bombarding energy, that is, the nuclei are getting hotter.

3.1 Moving-Source Parametrization

In order to make the phenomenological characterization of the preequilibrium emission more quantitative the spectra are parametrized by assuming isotropic emission of neutrons with a spectral shape of $M_{PE} \sqrt{E/e^{-E/T_{PE}}}$ in a rest frame moving with velocity $v_{PE}$.

The experimental data is fitted in the lab. system by treating the multiplicity $M_{PE}$, the temperature (inverse exponential slope) $T_{PE}$, and the velocity $v_{PE}$ as parameters. The result of all such fits is that the velocity $v_{PE}$ is about half the velocity of the incident projectile. This is true, however, only if the sequential emission has been excluded from the fit. In inclusive measurements the small angles are not included in the fit. A possible physical interpretation of this finding ($v_{PE} \approx 0.5v_{P}$) is that the preequilibrium nucleons are due to a nucleon-nucleon scattering process since $0.5v_{PE}$ corresponds to the nucleon-nucleon center-of-mass velocity. The fact that for asymmetric systems $v_{PE}$ is much larger than the c.m. velocity ($v_{cm} = \frac{A_p}{A_p + A_t} v_{p} \ll \frac{1}{2} v_{p}$) reflects the experimental finding of a strong forward emission of the preequilibrium neutrons which is also shown in figure 3. However, the forward peaking is not uniquely characteristic of precompound emission, since the angular distribution for smaller linear-momentum transfers (LMT=52% compared to 93% in figure 3) where sequential emission is contributing shows an even stronger forward peaking.

The multiplicity $M_{PE}$ obtained in this way is shown in figure 2a as a function of the energy per nucleon above the Coulomb barrier of the projectile. The multiplicity for different reactions scatter considerably in particular at small energies. But for one reaction (Ho+Ne, o in figure 2a) the multiplicity seems to increase linearly with bombarding energy as indicated by the dashed line. The solid line is the result of a Fermi-jet calculation [15] for this reaction which will be discussed later.

The neutron multiplicities shown in figure 2a are obtained by fitting the observed neutron energy spectra with two components: an evaporative and a preequilibrium one. Thus the PE component is extrapolated underneath the evaporative component by employing the used spectral shape $\sqrt{E/e^{-E/T_{PE}}}$ (see also figure 1). Other procedures to deduce $M_{PE}$ were discussed by Gavron et al. [10] and shown to result in smaller values. However, the above described procedure agrees essentially with most of the preequilibrium models as we will see below (similar spectral shapes). The multiplicity for the reaction $^{40}$Ca + 20MeV/nucleon$^{40}$Ar → ER + n [12] (Δ figure 2a) was obtained somewhat differently: the low energy component was fitted only and the obtained multiplicity of evaporated neutrons was subtracted from the total measured multiplicity which results in a somewhat smaller PE multiplicity. In figure 2a the filled triangle shows the proton multiplicity.
of the reaction $^{nat}$Ag + $^{40}$Ar → ER + p which was obtained by integrating the anisotropic (in the c.m.) coincident proton yield [13]. The proton multiplicity is considerably smaller than the neutron multiplicity, which is expected and predicted by many models due the Coulomb barrier for proton emission, whereas in the nearly symmetric system $^{48}$Sc + $(768\text{MeV})^{48}$Ti the PE proton multiplicity (not shown in figure 2a) is as large as the neutron multiplicity which is due to difficulties in preventing in a symmetric system contributions from sequential emission [21].

![Graph](image)

Figure 2: (a) Preequilibrium neutron multiplicity [7-14] in coincidence with ER: squares C, o Ne-induced fusion, △ Ca+Ar; ▲ proton multiplicity. Dashed line drawn through the Ho+Ne o, solid line Fermi-jet calculation for Ho+Ne [15]. (b) Preequilibrium slope parameter of exclusive neutron [7-11] (o) and inclusive proton (●) data [16-19]. The open triangles result from exclusive proton data [20,21]. The open squares are neutron data [8] for different linear momentum transfers which are identified in the figure. The solid line is the free Fermi gas expectation assuming equal number of projectile and target nucleons.

Since the energy spectra are essentially decreasing exponentially up to the maximum measured energies it is interesting to investigate how those spectra continue to higher emission energies. For $\alpha$-particles there are some indications [22] in the reaction $^{197}$Au + $(178\text{MeV})^{22}$Ne that up to the kinematical limit $\alpha$-particles were observed. Holub et al. [9] found that in the reaction $^{165}$Ho + $(300\text{MeV})^{12}$C → ER + n the probability to emit neutrons with energies larger than 100 MeV ($E_{\text{n-max}} = 268 \text{ MeV}$), or larger than twice the beam velocity, is $(3 \pm 1)10^{-3}$.

In heavy-ion induced fusion the fraction of linear-momentum transfer (LMT) to the composite system is decreasing with increasing bombarding energy due to an increase in light-particle emission prior to the attainment of a thermal equilibrium. In figure 4 the preequilibrium neutron multiplicity is shown to be essentially independent of LMT $\sim v_{ER}$ and at the highest LMT almost all the missing LMT is due to nucleon emission, indicating that for the most central collisions preequilibrium nucleon emission is the essential process whereas for less central collisions the emission of composite particles must be responsible for the missing LMT. Gamma-ray multiplicities in coincidence with evaporation residues and preequilibrium proton emission show a broad range of entrance channel partial waves leading to prompt proton emission and thus are also favoring a central jetting mechanism [23].
Figure 3: Angular distribution of neutrons with energies larger than 30 MeV in the lab. system for the reaction $^{165}$Ho + (600 MeV)$^{20}$Ne $\rightarrow$ ER + n with LMT = 52 and 93%, (see also figure 4). The long dashed, short dashed, and solid curves are the results of a moving-source fit for the preequilibrium, evaporative component, and the sum of both, respectively.

Figure 4: The parameters deduced with the hot-moving-source fit for the high energy component [8] for $^{165}$Ho + (600 MeV)$^{20}$Ne: the PE neutron multiplicity $M_{PE}$, temperature parameter $T_{PE}$, and ratio of the source velocity $v_{PE}$ to the beam velocity $v_{P}$ as function of the ER velocity $v_{ER}$. The dashed line is drawn through the points and extrapolated to zero ER velocity. The arrows indicate full LMT.

Finally the temperature parameters $T_{PE}$ obtained by the moving-source fit of the preequilibrium nucleon energy spectra are shown in figure 2b. This figure represents a compilation of neutron measurements in coincidence with ER (open circles and squares) and inclusive proton data ($\bullet$). The temperature parameter for PE protons in coincidence with heavy residues ($\triangle$ with error bar in figure 2b) from Keim et al. [21] includes also contributions from sequential emission in the almost symmetric system $^{45}$Sc + $^{48}$Ti. The inclusive proton data ($\bullet$) results systematically in lower values than the exclusive neutron data. One exclusive proton point [20] ($\triangle$ in figure 2b without error bar) is consistent, however, with the exclusive neutron data. This indicates that exclusive nucleon data show stiffer energy spectra than inclusive measurements where peripheral reactions are contributing. This is shown more explicitly in figure 4, where the deduced $T_{PE}$-parameter is plotted versus the ER-velocity $v_{ER}$ which is proportional to LMT. This shows that $T_{PE}$ is increasing with $LMT \approx v_{PE}$ which can be associated with decreasing impact parameter. The simplest explanation of this finding is probably due to the superposition of sequential emission and true preequilibrium emission at smaller $v_{ER}$ or LMT. The dashed line is an extrapolation to zero $v_{ER}$ or purely peripheral interactions, where $v_{PE} \approx v_{P}$ and $T_{PE} \approx$ equilibrium temperature of the projectile-like fragment.

3.2 Angular Momentum Effects on Preequilibrium Nucleon Emission

In heavy-ion induced fusion reactions large amounts of angular momentum can be transferred to the composite system. Thus it is interesting to investigate the dependence of preequilibrium nucleon emission upon the spin of the composite system. The spin transferred to ER is typically
smaller than about 50-60h. The detection of the direction of the ER does not define the direction of the spin of the composite system, in these reactions we only know that the spin is perpendicular to the beam direction. No anisotropy has been observed [7,13,14] in PE nucleon emission in coincidence with ER.

Figure 5: Reaction geometries for an "in-plane" (left) and “out-of-plane” (right) measurement: (F) fission plane and (n) neutron plane. Neutron energy spectra as measured in the in-plane (○) and out-of-plane (△) geometry for the reaction $^{175}\text{Lu} + (192\text{MeV})^{12}\text{C}$.

Since nuclei fission perpendicularly to the spin the detection of a fission fragment is a possibility to determine in a fusion-reaction the spin direction. Furthermore the angular momentum of the composite system is much larger than for that leading to ER. Zank et al. [14] have measured the anisotropy of preequilibrium neutron emission in coincidence with ER ($I \leq 49 \pm 6h$) and fusion fission ($49 \pm 6 \leq I \leq 62 \pm 6h$) for the reaction $^{175}\text{Lu} + (16 \text{MeV/nucleon})^{12}\text{C}$. The schematical setup is shown in figure 5. In the case where the plane defined by the fission fragments is approximately coplanar with the neutron detector plane the neutrons are always emitted perpendicularly to the spin. In the case where the fission plane is perpendicular to the neutron plane the neutrons emitted at small and large angles are also emitted perpendicularly to the spin whereas at angles around 40-70° the emission is parallel to the spin. This is clearly exhibited in figure 5 which shows that the yield of high energy neutrons at ±13° is the same for both geometries whereas at ±(40 to 70°) considerably less neutrons are emitted parallel to the spin. A more detailed analysis by fitting the spectra with the above moving-source parametrization shows that the neutrons in coincidence with ER can be described by the assumption of isotropic emission in the rest frame moving with half the beam velocity, whereas this is not possible for neutrons in coincidence with fission events. More specifically Zank et al. [14] have introduced an anisotropic neutron emission with respect to the spin given by $\exp(-a_{n}^{PE}\cos^{2}\Delta)$ in the rest frame. The quantity $\Delta$ is the angle between the spin vector of the composite system and the direction of the emitted neutron. Zank et al. [14] find a very large value for $a_{n}^{PE} = 2.2 \pm 0.6$. The curves in figure 5 were calculated with this anisotropy.

### 3.3 Ratios of Preequilibrium Neutrons and Protons

In figure 6 the total cross section for the free nucleon-nucleon scattering is shown as a function of lab. energy. This figure emphasizes the importance of the isospin dependence of $\sigma_{nn}$ or $\sigma_{pp}$
and $\sigma_{np}$. By comparing the preequilibrium neutron and proton emission one might in principle get information of the nucleon-nucleon cross section in nuclear matter or even in excited nuclear matter [14].

This information could be deduced from the ratios of the double differential multiplicities of neutrons to protons. For the above discussed reaction Ho + (600 MeV) Ne it is found [8] that it is more probable to emit neutrons than protons at nucleon velocities larger than the projectile velocity. These ratios are, however, associated with large errors due to uncertainties of the neutron detection efficiencies at high neutron energies. It is possible to determine the ratios for each nucleon-species separately for targets with different number of neutrons. Very recently Lehmann et al. [25] have measured the ratio of nucleons produced in the reaction $^{144,154}$Sm + (838 MeV)$^{32}$S $\rightarrow$ FF + n.p. Preliminary results for these ratios are shown in figure 7. We note that the probability to produce protons is about equal in both reactions having the same number of protons in the projectile as well as in the target, whereas it is more likely to emit neutrons in the reaction with the more neutron-rich target nucleus $^{154}$Sm. The effect is larger than the 12% increase in neutron number in the target (82 to 92). It is interesting to note that the neutron (proton) binding energy plus Coulomb barrier in $^{144}$Sm is about 30% larger (15 % smaller) than in $^{154}$Sm. Though these effects have not been quantitatively understood yet they provide a further observable to test preequilibrium models and may in conjunction with these models provide us with new insight into the physics of preequilibrium emission and result in information on the above mentioned nucleon-nucleon cross section inside (excited) nuclear matter. Furthermore, as we will see below, essentially all models can reasonably describe the measured preequilibrium energy-spectra and thus these additional observables will enable a more sensitive test of these models. In relativistic nuclear collisions the inclusive (no coincidence condition) neutron-to-proton ratios for 300-400 MeV/nucleon Ne induced reactions on U and Pb could be accounted for by a cascade-coalescence model for nucleon velocities which were smaller or equal to the projectile velocity [26]. In particular Stevenson [26] has shown that the coalescence of nucleons to complex particles tends to increase the neutron-to-proton ratio.

![Figure 6: Nucleon-nucleon cross section as a function of incident lab. energy [24].](image)

![Figure 7: Ratio of the double differential multiplicity of neutrons and protons for 838 MeV $^{32}$S induced fission on $^{154}$Sm and $^{144}$Sm, 9° $\leq \Theta_n \leq$ 27°, 5° $\leq \Theta_p \leq$ 32° [25]. Central collisions were selected with the folding angle method.](image)
4. Theoretical Models for Pre-equilibrium Nucleon Emission

So far I have presented only a phenomenological description of the characteristics of pre-equilibrium nucleon emission. Though the moving-source parametrization corresponds implicitly to a physical picture I have used it only to obtain quantities which can be employed for a systematic comparison of a large body of data and different reactions. In the following I will now briefly mention the various models which have been applied to describe and thus to understand the mechanisms of pre-equilibrium nucleon emission. I will discuss, however, only the Fermi-jet model in more detail whereas the other models I will only list and mention their successes and failures.

The exciton model or Boltzmann master equation [3] is able to describe the integrated energy spectra by setting the initial exciton number equal to the number of nucleons in the projectile. By exploiting a similar master equation for the angular dependence [27] it was also possible to describe angular distributions.

Model calculations with the Vlasov-Uehling-Uhlenbeck equation, a combined treatment of mean field effects and a nucleon-nucleon collision term, show similar characteristics as the experiment [28]. However, a more sensitive test would be to compare the neutron-to-proton ratios with such calculations including an exact treatment of the isospin dependencies of the nucleon-nucleon cross section [29].

Quantal phase space approach [30] is able to reproduce extremely well the double differential neutron multiplicities as a function of angle and energy for asymmetric [28] and symmetric systems [12].

![Graph showing double differential neutron multiplicity](image)

**Figure 8:** Double differential neutron multiplicity at three bombarding energies [7] from the reactions $^{165}$Ho $+^{20}$Ne $\rightarrow$ ER $+ n$ compared to an evaporation calculation (short dashed), Fermi-jet calculations with a zero (long dashed) or finite T=3 MeV (dashed dotted curve) temperature Fermi-distribution [7], and a Fermi-jet calculation [15] with a dynamical varying Fermi-distribution and taking into account nucleon rescattering (solid line).
4.1 Fermi-jet or Promptly Emitted Particles

Bondorf [31] and independently Robel and Swiatecki [32] proposed that the highly energetic particles might be due to the coupling of the relative velocity of two interacting Fermi-spheres (projectile and target) and the Fermi-motion. They coined the term, respectively, for these particles promptly emitted particles (PEP) or Fermi-jet. In this picture nucleons are being transferred from the projectile to the target, or vice versa, through a window between the two interacting heavy-ions with a velocity given by the sum of the relative velocity and the Fermi-velocity. The particles are following linear trajectories until they reach the surface of the receptor nucleus. If the energy is larger than the nuclear potential they will escape the receptor nucleus promptly, that is within times of the order of the transit time of a nucleon through the receptor \( \sim 2r_0A_T^{1/3}/(v_F + v_{rel}) \approx 1 \cdot 10^{-22}\) sec assuming 20 MeV/nucleon \(^{20}\text{Ne} + ^{165}\text{Ho}\). The radial deceleration time of the projectile of about \(10^{-22}\) sec has to be added to this time. The radial deceleration is calculated for classical trajectories.

![Graph](image)

Figure 9: Temporal evolution of the reaction \(^{20}\text{Ne} + ^{165}\text{Ho}\): \(R\) internuclear separation, \(c\) neck radius, \(c_{eff}\) neck radius including a diffuse surface, \(\tau_{Ne}, \tau_{Ho}\) temperature in projectile and target, \(\nu_{proj}\) time differential neutron multiplicity induced by transfers from the projectile.

The comparison of the exclusive neutron data for the central collisions of \(^{20}\text{Ne} + \text{Ho}\) with the results of the code of Bondorf et al. [33] is shown in figure 8 by the long dashed line. The dashed dotted line was obtained by Holub et al. [7] by a modification of this code by the ad hoc assumption that the Fermi velocity distribution is given by a finite temperature \((T=3\text{ MeV})\) Fermi-distribution. This assumption yields higher velocities and consequently also higher neutron energies in agreement with experiment. However, the angular distribution is too forward peaked, resulting in a discrepancy at 60° of 1-2 orders of magnitude. This discrepancy resulted in the conclusion [7-9] that the Fermi-jet calculations without taking into account the rescattering of transferred nucleons is in disagreement with experiment. Better treatment of the dynamics of the heavy-ion reaction
as was done by Leray et al. [34] could not improve the agreement [8]. In two recent publications [15,35] the rescattering of nucleons was explicitly taken into account plus a heating of the donator nucleus due to the induced excitation by transferred nucleons from the receptor nucleus (and vice versa) prior to the injection of a Fermi-jet nucleon. This is an explicit treatment of the ad hoc assumption by Holub et al. [7] of a finite temperature Fermi-distribution. The resulting agreement with preequilibrium neutron data is shown for Ho + (402 MeV) Ne by the solid line in figure 8. Considering the fact that no normalization factor has been applied the agreement is reasonable though at higher neutron energies the deviations between experiment and model calculations are increasing. The total multiplicity of the forward emitted preequilibrium neutrons which are due to Fermi-jet neutrons from the projectile is underestimated by a factor of two as is shown in figure 2a. However, Randrup and Vandenbosch [15] have added in a note to their paper that the inclusion of the radial motion in calculating the transmission coefficients of the escaping neutron can increase the preequilibrium neutron multiplicity by almost a factor of two.

The temporal evolution [15] of a fusion-like collision of Ho + Ne at 402 MeV and \( l=46\ h \) is shown in figure 9. The radial kinetic energy is dissipated in about \( 10^{-22} \) sec in which the neck radius \( c \) opens up and thus the transfer of nucleons and consequently also the emission of Fermi-jet neutrons sets in. The temperature \( \gamma_n \) in the projectile reaches about 5 MeV in \( 10^{-22} \) sec. In the same time the total emission probability reaches its maximum. Randrup and Vandenbosch [15] noted that the fact that considerable kinetic energy damping has occurred prior to the most probable time of preequilibrium emission might be the cause that moving-source fits result in a source velocity of only half the beam velocity.

In summary, the Fermi-jet model can reproduce the measured preequilibrium neutron spectra in central collisions except for maybe an underestimation of the total neutron multiplicity by a factor of two and some disagreement at higher energies. Better agreement than shown in figure 8 was obtained with less exclusive data [36] of \(^{238}\text{U} +^{16}\text{O} \) which was measured in coincidence with fission fragments. Since the fission barrier of \( \text{U} \) is very small U fissions sequentially also in peripheral reactions, thus in this case fission coincidences do not effectively reject sequentially emitted neutrons.

5. Conclusions

In summary we have seen that it is possible to separate experimentally genuine preequilibrium nucleon emission in central collisions if sequential emission from projectile-like or intermediate-mass fragments is excluded. This requirement can easily be fulfilled in asymmetric collisions whereas for symmetric systems [12,21] the separation is considerably more complicated [21]. The preequilibrium nucleon emission shows (i) an exponential decrease of particle yield as a function of energy and angle, (ii) the spectra can be fitted with a moving-source parametrization and the obtained parameters show a systematic behaviour with bombarding energy for many reactions, (iii) in central collisions preequilibrium nucleon emission is the dominant process compared to the emission of complex particles, (iv) preequilibrium emission is strongly anisotropic with respect to high spins (50-60\( h \)) transferred to the composite system, and (v) the emission of neutrons is larger than for protons and depends on the N/Z ratio of the target.

Theoretical models have reached a stage where all are essentially more or less successful in describing the characteristics (i) to (iii) but so far have not been compared to the characteristics (iv) and (v). In particular, I think that the neutron-to-proton ratios will be a sensitive test of theoretical
models and, maybe, will provide us with some knowledge on the nucleon-nucleon cross section or mean free path length in excited nuclear matter.

Acknowledgement

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COMPUTER CODES
FOR PRE-EQUILIBRIUM CALCULATIONS
AVAILABLE FROM THE NEA DATA BANK

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ABSTRACT

The computer codes for pre-equilibrium calculations which are available from the NEA Data Bank are listed and their differences are briefly outlined. Reference is made to international comparison exercises which try to assess the quality and performance of these and other nuclear model codes.

INTRODUCTION

The NEA Data Bank has collected over the years a set of computer codes for nuclear model calculations which cover the different interaction phenomena of light incident particles with matter. These codes are distributed to requesters after they have been verified or validated on at least one computer. At present the set consists of about fifty codes which is about four percent of the complete program library.

Most of the early codes were designed for energies below 10 MeV and for the neutron as projectile. The upper energy limit in cross-section libraries for nuclear reactor applications was chosen in fact at 10 MeV (the lethargy of a neutron is still set to zero at this energy value). Nuclear model codes that produced cross section data for reactor applications contained typically the optical and statistical model. As technology applications extended their interest to higher energies and other particles new phenomena had to be considered to be able to describe correctly the experimental cross section values.

The pre-compound or pre-equilibrium model was then introduced to fill the gap between phenomena described by the statistical model and those of the direct reaction. The first code acquired by the NEA Data Bank containing the pre-compound option in addition to the Hauser-Feshbach model was STAPRE [18] in 1976.
As new formalisms were developed (i.e. exciton or hybrid models), codes featuring these were also collected. The codes so far acquired contain only phenomenological models. Codes including more recent developments such as quantum mechanical approaches are used for fundamental research and some are of an experimental nature; they are therefore not suitable yet for inclusion in the model code library and are not meant for a wider distribution.

The NEA Data Bank relies on the generosity of code developers for maintaining an up-to-date nuclear model code collection. Many codes were obtained through the National Energy Software Center (NESC), Argonne and the Radiation Shielding Information Center (RSIC), Oak Ridge. An important set of codes was developed in the NEA Data Bank member countries and an interesting set of codes has recently been acquired from the Chinese Nuclear Data Centre.

**MODEL CODE COMPARISONS**

Nuclear model computer codes like any product of science and technology require quality assurance and maintenance. A high degree of verification, validation and quality assurance carried out on the model codes can make up at least partially for the reduced efforts available now worldwide for cross section measurement and evaluation.

Confidence in the predictive power of the nuclear model codes in cross section data regions where experiments are lacking or inexistet can be improved through international code comparison exercises. Agreement between codes developed independently by different researchers on a well selected set of problems contributes strongly to the buildup of this confidence.

The model-code comparison activity had been initiated by the Cross Section Evaluation Working Group in the U.S. and has been further internationalized at the NEA Data Bank for the lower energy range. For the intermediate energy region the NNDC at BNL conducts the intercomparison activity.

Different comparison exercises with their achievements are summarized in Ref. [1]. A first exercise, which included both the optical model and statistical model with an option for pre-equilibrium effects [2] has pointed out considerable discrepancies in the higher energy parts for reactions with emission of charged particles. It was inconclusive as to whether this was caused by discrepancies in the optical or statistical model part. Precision comparisons [3] were therefore carried out to assess the quality of the optical model coding and benchmark results were published. Two additional exercises were designed and are now near completion which address precision calculations in the different statistical model approaches. They have attracted many participants and the results will be reported when the study is completed.
In order to contribute to improvements in the quality of the data bases required for technological applications other than fission reactors (i.e. fusion blanket studies etc.) special efforts have been devoted to the comparison of pre-equilibrium models. Here in addition to computer codes also the different approaches and formalisms were intercompared. A first exercise [4] was conducted for neutrons interacting with $^{93}$Nb nuclei. Because different parameters are used in different formalisms, participants had to fit their pre-equilibrium model parameters on an experimental angle-integrated neutron emission spectrum induced by neutrons with a given incident energy. Compared were angle and energy-integrated cross sections, total neutron emission spectra at different energies and to some extent double-differential cross sections and photon production spectra.

The overall results of the calculations were quite consistent for inelastic scattering and neutron emission. Large deviations were found for charged particle emissions. The predictive power of these models were studied in a blind comparison in which the experimental results are disclosed to the participants only after they have carried out the calculations. Except for the optical model part, participants were asked to use their best apriori choice of the parameters. Details of this comparison are reported at this meeting [5].

The publications issuing from these comparisons are added as validation reports to all participating nuclear model codes that are available from the NEA Data Bank. Users can then better familiarize themselves with the codes developed by others and gain a better feeling of the confidence they can attribute to results obtained therefrom.

The dream of unifying the different formalisms to one model capable of describing in all its aspects both the pre-equilibrium and equilibrium phenomena with an overall smaller set of free parameters seem slowly to take now a real shape. It will take some time however before an universal computer code will materialize, capable of describing satisfactorily all interaction phenomena over a wide energy range.

**PRE-EQUILIBRIUM PROGRAMS**

The following Table lists all computer codes available from the NEA Data Bank which include pre-equilibrium modelling. Together with their names a program package identifier is given which can be helpful to search for more details in the Nuclear Programs Abstracts publications and to place orders. Some additional features of the codes are described; the bibliographic reference is given and to which comparison exercise they have participated.
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<td>DI</td>
<td>[15]</td>
<td>[4,5]</td>
</tr>
<tr>
<td>PREM</td>
<td>NEA 0888</td>
<td></td>
<td>[16]</td>
<td>[4]</td>
</tr>
<tr>
<td>SC2N3N</td>
<td>IAEA0917</td>
<td>SYS SM</td>
<td>[17]</td>
<td></td>
</tr>
<tr>
<td>STAPRE</td>
<td>NEA 0461</td>
<td>SM</td>
<td>[18]</td>
<td>[4,5]</td>
</tr>
<tr>
<td>TNG</td>
<td>USCD0678</td>
<td>SM</td>
<td>[19]</td>
<td>[4,5]</td>
</tr>
</tbody>
</table>

**Table. Pre-Equilibrium Model Codes**

| DI | Direct Interactions, |
| OM | Optical Model, |
| SM | Statistical Model, |
| SYS | Systematics |

**origin:**

| NEA | NEADB member countries, |
| IAEA | non-OECD countries |
| NESC, PSR, USCD | codes of US origin |

The same classification as in Ref. [4][5] is adopted here for the pre-compound formalisms.

1. **Hauser-Feshbach Codes with Pre-compound Option**
   
   These codes ensure angular-momentum conservation and allow the cross sections of reactions to discrete final states to be calculated. They are applicable over a large energy range but are expensive to run.
   
   The following codes belong to this category:
   
   **GNASH, HAUSER-5, MUP-3, STAPRE, TNG.**

2. **Exciton Model Codes (Spin-Independent)**

   These codes do not include explicit angular momentum conservation and have no large memory requirements and so are simple and fast. The Weisskopf-Ewing model is used including pre-equilibrium. They are restricted to energies above 5 MeV.
since they do not give cross sections to discrete final states. The following codes belong to this category:

AMALTHEE, GRAPE, IPEET-103, PREANG, PRECO-D2, PREM.

3. **(Geometry Dependent) Hybrid Model Codes.**

The hybrid model is based on a combination of the exciton model and the Harp-Miller-Berne model. Different treatments of the outgoing energy dependence causes differences in the shapes of the emission spectra. In geometry-dependent versions of the model a decomposition is made according to incoming angular momentum in order to account for the effects of the nuclear density distribution leading to increased higher energy particle emission.

The following code belongs to this category:

ALICE/85/300.

4. **Systematics Codes.**

They contain analytical formulae with parameters based on experimental data and derived by using a constant temperature evaporation model for statistical phenomena and the exciton model for the pre-compound part.

A set of three codes is available for quick estimations of cross sections for the following reactions:
- \((n,2n)\), \((n,3n)\) : SC2N3N [17]
- \((n,p),(n,\alpha)\) : NX-1 [13]
- \((n,d),(n,^3\text{He})\) : NX-2 [13]

Their validity range is for masses \(23 < A < 197\) and for energies up to 25 MeV, they reflect data published up to 1984, precompound effects are limited to exciton number \(n=3\).

**CONCLUSIONS**

Developers of new model codes or improvements and generalizations to existing codes are invited to continue to make these available for general distribution to one of the code centres. New codes or new versions should possibly undergo first a validation not only on the standard problems used by the developers but also on the ones designed for the international intercomparisons. Feedback on possible errors detected by other users will be reported to them. This may lead to revisions of the codes and in general to improvement of the internationally available nuclear model code library. This ensures also that the effort in cross section evaluation shared at international level is of accepted quality and improves standardization in the methodologies used.
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RESULTS OF RECENT CODE COMPARISONS

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Abstract

The results of a recent international "blind intercomparison", that is a comparison of nuclear model calculations with each other and with experimental results unknown at the time of the calculations, are presented and discussed. Participants from 7 laboratories calculated the secondary neutron energy and angular distribution from the interaction of 25.7 MeV neutrons with the nucleus $^{184}$W, before these measured double-differential neutron emission cross-sections were made available.

The comparison demonstrates that these models can reasonably describe the neutron energy spectrum except for the highest energy part, where an additional contribution from direct excitation of collective levels is definitely needed. The angular distributions can at present only be described adequately by phenomenological and semi-phenomenological methods.

In addition some questions connected with the 1984 preequilibrium code intercomparison on $^{93}$Nb are reexamined. Especially the description of the neutron emission spectra at 14 and 25 MeV and the total proton emission cross-sections at 14 MeV are discussed in comparison with new measurements. It is pointed out that the spread in the reported total proton emission cross-sections needs further investigation.
1. Blind intercomparison for $n + ^{184}_{\text{W}}$ at 25.7 MeV

1.1 Condition of the intercomparison and participants

Two years ago, the NEA reported on the results of an international nuclear model and code comparison for neutron induced reactions on $^{93}_{\text{Nb}}$ for incident neutron energies in the range between 10 and 25 MeV /1/. In this comparison the parameters needed to determine equilibrium particle emission were fixed, whereas for the pre-equilibrium part the participants were only asked to fit the experimental neutron emission spectrum at 14 MeV incident neutron energy. Thus one of the main aims of this exercise was to test the ability of the various precompound models to describe pre-equilibrium neutron emission over a rather wide range of incident neutron energies. The comparison had produced the encouraging result that most codes (using either the exciton or the hybrid model) were able to successfully predict the precompound neutron emission at 25.7 MeV after their parameters had been adjusted to fit the 14 MeV data.

In a further test of the predictive power of the existing precompound models the comparison described in this report attempts to test the accuracy attainable with such model calculations for nuclei where no data at all exist for neutron emission.

For this purpose, the participants were asked to calculate the angle-integrated secondary neutron spectrum from the interaction of 25.7 MeV neutrons with $^{184}_{\text{W}}$ and also the angular distributions for selected secondary neutron energies (13.5, 15.5, 18.5 and 21.5 MeV) using their best a priori choice of all parameters except for the optical model parameters. For these parameters it was recommended to use the deformed optical potential derived by J. P. Delaroche /2/ for the studied nucleus by a fit to the recent measurements /3/ of the sum of the elastic scattering and first (2+) and second (4+) excited level inelastic scattering cross-section. These measurements were part of an experiment performed at Ohio University by Marcincowski et al. and included neutron emission cross-sections.

Scientists from seven laboratories participated in this comparison; most of them used the global optical potentials built into their codes instead of the recommended potential of reference /2/.

The following contributions were received:

Class A, modified Hauser-Feshbach codes unified models - GNASH (LAS); P.G. Young /4/
- STAFRE (LL-2); D.G. Gardner and M.A. Gardner /5/
- TNG (ORL); C.Y. Fu /6/

Class B, Exciton-model codes (spin-independent) - PRECO-D3 (DKE); C. Kalbach /7/
- PEQCM (SLO); E. Běták and J. Dobeš /8/
- EXIM (TRM); A. Chatterjee and S.K. Gupta /9/

Class C, Hybrid and geometry dependent hybrid (GDH) codes - ALICE GDH + evaporation model (LLL-1); M. Blann /10/

Total neutron emission spectra (including neutrons from successive evaporation processes) were calculated by the codes GNASH, STAFRE, TNG, ALICE and PEQCM, whereas only first neutron emission is given by PRECO-D3 and EXIM.

Angular distributions were calculated by 5 of the codes: GNASH, TNG, PRECO-D3, EXIM and ALICE.
The classification of the codes into classes A, B, C, the assumptions and models underlying these classes of codes and the details of most of the codes used in this exercise have been discussed extensively in sections 7, 12, 13 and 16 of reference 1. There are only two points to be mentioned in addition to this discussion.

i. PRECO-D3: C. Kalbach has slightly modified her code PRECO-D2 by replacing her angular distribution systematics with an improved version, which uses exponentials instead of Legendre polynomials for the description of the angular distributions and this new version PRECO-D3 (otherwise identical to PRECO-2) was used in this intercomparison.

ii. EXIM: The only new code not described in reference 1 in this exercise is the code EXIM /9/ used by A. Chatterjee. It is based on the exciton model and uses an exact solution of the time integrated master equation in order to calculate the energy spectrum of the first emitted particle by both pre-equilibrium and equilibrium emission. It uses the Williams formula for particle-hole densities with $g = A/13$ MeV$^{-1}$ and the Kalbach prescription /11/ for internal transition rates. For the angular distributions the code applies the leading particle approach of Mantzouranis et a. with distinction of multistep in compound and multistep-direct-processes /12/. It neglects angular momentum effects and $\gamma$-emission and cannot describe multiple particle emission.

1.2 Total neutron emission cross-sections

Total neutron emission cross-sections are essentially determined by the total reaction cross-sections and the average neutron multiplicity.

Table 1 shows the neutron total and reaction cross-sections resulting from the different optical potentials used by the participants. In addition to the reaction cross-section $\sigma_r$ we also give the cross-section $\sigma'_r = \sigma_r - (\sigma_2^+ + \sigma_4^+)$, the reaction cross-section available for pre-equilibrium plus equilibrium processes. As the table shows, there may be differences of up to 25% in $\sigma'_r$ between the results of the global optical potentials like Wilmore-Hodgson and potentials derived specifically for the nucleus or mass range considered; thus the latter should be used whenever possible.

<table>
<thead>
<tr>
<th>Participant</th>
<th>$\sigma_t$ (mb)</th>
<th>$\sigma_r$ (mb)</th>
<th>$\sigma_2^+$ (mb)</th>
<th>$\sigma_4^+$ (mb)</th>
<th>$\sigma'_r$ (mb)</th>
<th>Optical Model used</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>DKE/PRECO-D3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2253</td>
<td>Chatterjee</td>
<td>[43]</td>
</tr>
<tr>
<td>LAS/GNASH</td>
<td>5145</td>
<td>2545</td>
<td>109.8</td>
<td>13.</td>
<td>2427</td>
<td>Delaroche</td>
<td>[2]</td>
</tr>
<tr>
<td>LLL1/ALICE</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2167</td>
<td>??</td>
<td>[40]</td>
</tr>
<tr>
<td>LLL2/STAPRE</td>
<td>5115</td>
<td>2522</td>
<td>99.7</td>
<td>10.8</td>
<td>2414</td>
<td>Delaroche</td>
<td>[2]</td>
</tr>
<tr>
<td>ORL/TNG</td>
<td>5364</td>
<td>2125</td>
<td>196</td>
<td>9.4</td>
<td>1920</td>
<td>Wilmore-Hodgson</td>
<td>[44]</td>
</tr>
<tr>
<td>SLO/PEQGM</td>
<td>5264</td>
<td>2430</td>
<td>-</td>
<td>-</td>
<td>2430</td>
<td>Delaroche</td>
<td>[2]</td>
</tr>
<tr>
<td>TRM/EXIM</td>
<td>-</td>
<td>2164</td>
<td>-</td>
<td>-</td>
<td>2164</td>
<td>Chatterjee</td>
<td>[43]</td>
</tr>
</tbody>
</table>

$\sigma'_r = \sigma_r - (\sigma_2^+ + \sigma_4^+)$ is the total reaction cross-section for continuum particle emission.
Table 2 shows the angle- and energy-integrated first neutron ($\sigma_{nnx}$) and total neutron ($\sigma_{nem}$) cross-sections (not including the cross sections for the first two discrete levels). The first two columns give the values reported by the authors, in columns 3 and 4 we have renormalized these data to the reaction cross-section of the Delaroche potential for those participants which used other optical potentials and finally the average neutron multiplicity ($\bar{V} = \sigma_{nem}/\sigma_{nnx}$) is also reported. As the table shows there is a considerable (~25%) spread in the reported $\sigma_{nnx}$ and $\sigma_{nem}$. This spread is mainly due to the use of different total reaction cross-sections resulting from the different optical potentials; as shown by columns 3 and 4 there is a very good agreement of the total neutron emission cross-sections between the results of all codes if the same total reaction cross-section is used in the input. Accordingly also the predictions of the average neutron multiplicity agree within 6% between all codes. There is no recognizable difference between the Hauser-Feshbach codes and the spin-independent codes indicating that the total neutron emission is not influenced much by either angular momentum effects or $\gamma$-competition.

<table>
<thead>
<tr>
<th>Participant</th>
<th>$\sigma_{nnx}(mb)$</th>
<th>$\sigma_{nem}(mb)$</th>
<th>$\sigma_{nnx}^{'}(mb)$</th>
<th>$\sigma_{nem}^{'}(mb)$</th>
<th>$\bar{V}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DKE/PRECO-D3</td>
<td>2100</td>
<td>-</td>
<td>2256</td>
<td></td>
<td>2.89</td>
</tr>
<tr>
<td>LAS/GNASH</td>
<td>2427</td>
<td>7006</td>
<td>2427</td>
<td>7006</td>
<td>2.91</td>
</tr>
<tr>
<td>LLL1/ALICE</td>
<td>2040</td>
<td>5935</td>
<td>2278</td>
<td>6623</td>
<td>2.65</td>
</tr>
<tr>
<td>LLL2/STAPRE</td>
<td>2443</td>
<td>6466</td>
<td>2443</td>
<td>6466</td>
<td>2.90</td>
</tr>
<tr>
<td>ORL/TNG</td>
<td>1920</td>
<td>5568</td>
<td>2420</td>
<td>7016</td>
<td>2.72</td>
</tr>
<tr>
<td>SLO/PEQGM</td>
<td>2401</td>
<td>6534</td>
<td>2401</td>
<td>6534</td>
<td></td>
</tr>
<tr>
<td>TRM/EXIM</td>
<td>2111</td>
<td>-</td>
<td>2420</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\sigma_{nem}^{'}$ and $\sigma_{nem}^{''}$ are the cross-sections renormalized to reaction cross-section $\sigma_{r}^{'}$ of Delaroche for those participants using different optical potentials.

1.3 Angle-integrated total neutron emission spectrum

The angle-integrated total neutron emission cross-sections are shown in Figures 1 and 2. Fig. 1 shows the $d\sigma_{nem}/dE_{n'}$ values from all calculations and the measured neutron emission cross-sections /3/ for $E_{n'} > 10$ MeV, as the cross-sections have been measured for $E_{n'} > 12$ MeV only. Fig. 2 shows the low energy range of the neutron emission spectra for all calculations including multiple particle emission.

From Figures 1 and 2 we can conclude the following:

1. All codes based on the exciton model give a reasonable prediction of the absolute neutron emission cross-section (within about 25%) in the neutron range of 12-18 MeV, but fail to describe the high energy part of the measured spectrum. It appears that an adequate description of the complete spectrum is only possible by addition of a considerable contribution from direct excitation of collective levels up to an excitation energy of about 7 MeV. This situation has also been found before
by several authors /15, 16/ in their attempts to describe neutron emission spectra at 14 MeV incident energy with the exciton model.

The results of the calculations with the geometry dependent hybrid model are in better agreement with the shape of the measured angle-integrated spectrum (which is reproduced in shape up to about 21 MeV), but also with this model there is a need for an additional direct reaction contribution at the highest neutron energies. The geometry-dependent hybrid model calculations underestimate the absolute neutron emission cross-sections by about a factor of 1.8 (see table 3). In this connection one has to keep in mind, however, that this model is strictly based on free nucleon-nucleon scattering cross-sections and - contrary to the exciton model - does not contain any free parameter like the average transition matrix element in the exciton model. A somewhat better fit to the data could be obtained if a small empirical renormalization would be used in the GDH model (by using a value larger than 1 for the so-called mean free-path multiplier), as it has been observed before that this model tends to predict somewhat too small absolute cross-sections /1, 10/, although the observed differences have been smaller in those cases.

Figure 2 shows that there are considerable discrepancies between the predictions of the different calculations for the shape of the evaporation spectrum. Part of this spread is due to the different total reaction cross-sections $\sigma_r$ used by different participants. There are, however, in addition considerable differences in the shapes of the evaporation spectra, which are probably due to small differences in the level density parameters used by the different participants and reflect the present uncertainties in these parameters. In addition we observe striking differences in the relative cross-sections calculated for the first (0 - 0.5 MeV) and second (0.5 - 1.0 MeV) energy bin between the different codes, which cannot be explained by small differences of level density parameters, but seem to indicate deficiencies of the codes, possibly connected with a too coarse bin structure. There is no recognizable difference between the Hauser-Feshbach codes (GNASH, TNQ, STAPRE) and the Weisskopf-Ewing codes (ALICE, PEQOM): the scatter within each group is about as large as the difference between the averages of the two groups.

1.4 Angular distributions

The angular distributions calculated for the energy bins 13-14, 15-16, 18-19 and 21-22 MeV are compared to the measured double-differential neutron-emission cross-sections in figures 3-6. As the experimental cross-sections do have rather large statistical errors we have used for this comparison experimental cross-sections averaged over 3 MeV bins centered around the bin used in the calculations. As the purpose of this comparison is to judge the shape of the calculated angular distributions, all calculated cross-sections were renormalized to give the same angle-integrated value as the experiment before plotting. The corresponding renormalization factors are given in table 3.
Table 3: Normalization factors used for the comparison of calculated neutron angular distributions with the experimental results of reference [3].

<table>
<thead>
<tr>
<th>Calculation</th>
<th>( E'_n ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>13.5</td>
</tr>
<tr>
<td></td>
<td>15.5</td>
</tr>
<tr>
<td></td>
<td>18.5</td>
</tr>
<tr>
<td></td>
<td>21.5</td>
</tr>
<tr>
<td>DKE/PRECO-D3</td>
<td>1.28</td>
</tr>
<tr>
<td>LAS/GNASH</td>
<td>1.19</td>
</tr>
<tr>
<td>LLLI/ALICE</td>
<td>1.85</td>
</tr>
<tr>
<td>ORL/TNG</td>
<td>1.05</td>
</tr>
<tr>
<td>TRM/EXIM</td>
<td>1.17</td>
</tr>
</tbody>
</table>

From these figures we conclude:

i. The empirical angular distribution (original Kalbach-Mann systematics in GNASH and the new Kalbach systematics in PRECO-D3) fit the data in the angular range \( 30° - 140° \). The same is true for the semi-empirical partially random phase model of Fu /6/ which uses only one adjustable parameter to describe the angular distributions. The differences between these predictions are so small that the uncertainties of the data do not allow to make any statements on the relative merits of these 3 distributions.

ii. For the most forward angles there is an indication that the measured cross-sections may become somewhat higher than the predictions of the mentioned systematics for the highest-neutron energies. It may, however, be that the experimental data at the highest-neutron energies and smallest angles have additional systematic errors due to the vicinity of the large elastic peak.

iii. For very large backward angles there is a factor of two difference between the angular distributions of Kalbach-Mann /18/ and the partially random phase model /6/. Unfortunately, there are only two data points in this angular region. These indicate a weak preference for the angular distribution assumed in ref. 6.

iv. The agreement between the shape of the measured angular distribution and the results of GNASH, PRECO-D3 and TNG is about equally good for all energy bins although for the highest energy bin \( (21-22 \text{ MeV}) \) the reaction mechanism should be different from the other bins (see discussions in section 1.3). The postulated direct collective contributions seem to have about the same angular distribution as for the precompound particles.

v. The angular distributions derived from nucleon-nucleon scattering kinematics in ALICE /10/ and EXIM /9/ are not confirmed by the data. Experimental cross-sections are seriously underestimated at large emission angles and in case of ALICE also overestimated at very small angles.

2. Special questions arising from the 1984 NEA Data Bank pre-equilibrium intercomparison on \( ^{93}\text{Nb} \)

2.1 Angle-integrated neutron emission spectra at \( E_n = 14.6 \) and 25.7 MeV

As already mentioned, in this intercomparison /1/ participants were asked to calculate a number of neutron cross-sections on \( ^{93}\text{Nb} \) for incident energies between 10 and 25.7 MeV using a common set of parameters (optical
potentials, level densities etc.) for the equilibrium decay. Concerning
pre-equilibrium emission participants were asked to use their specific
models with parameters adjusted to fit the measured neutron emission
spectra of Hermsdorf et al. for 14.6 MeV incident energy /19/ in the range
E_n' = 6–9 MeV where pre-equilibrium emission dominates.

It was one of the main results of this exercise that the neutron
emission spectra calculated with these parameters (fitted to the 14 MeV
experimental neutron emission spectrum) also gave a good description of the
measured neutron emission spectrum at E_n = 25.7 MeV except at the highest
secondary neutron energies, where direct collective excitations become
important.

This statement has to be modified to some extent. In the mean time
more measurements have been reported on the \(^{93}\)Nb neutron emission spectrum
at E_n = 14 MeV and all data have been combined in a recent evaluation /20/.
This evaluation shows that the \(d\sigma/dE_n\), values of ref. /19/ are too high by
about 25% as shown in Fig. 7. The figure also shows one of the calcula-
tions done within the intercomparison (most of the other calculations are
similar /1/ and have not been plotted for clarity). Fig. 8 shows the
spacing situation at 25.7 MeV. As obvious from the two figures there
is now some discrepancy between the two energies. The original GNASH
calculations give a good fit to 25.7 MeV data /21/ but overestimate
precompound emission at 14 MeV. If on the other hand the calculations were
fitted to the 14 MeV data, they would underestimate the 25.7 neutron
emission spectrum (s. fig. 8). It should be pointed out, however, that the
effect is just marginal and that there exists only one \(d\sigma/dE_n\) at
measurement at E_n = 25.7 MeV. Experience at E_n = 14 MeV has shown that errors larger
than estimated may occur. Thus more measurements also at 25 MeV
are needed before definite conclusions can be drawn.

2.2 Proton emission cross-sections and neutron-proton emission ratios

Within the discussed intercomparison also proton emission cross-
sections (\(\sigma_{\text{n-pem}}, \sigma_{\text{n-px}}, \sigma_{\text{n-py}}\), etc.) have been calculated. These results,
however, have not been discussed as yet /1/ apart from the statement that
there are large discrepancies between different calculations.

It is an important question, however, how well the pre-equilibrium
models can describe both neutron and proton emission, that is how
accurately the proton emission cross-section can be predicted from
calculations adjusted to fit the neutron emission spectra and vice versa.

The nucleus \(^{93}\)Nb is especially suited for the study of this question
as both the neutron emission spectrum (s. fig. 7) and the total proton
emission cross-section \(\sigma_{\text{n-pem}}\) are known quite accurately. There exist three
mutually consistent measurements /15, 22, 23/ of \(\sigma_{\text{n-pem}}\) at E_n = 14–15 MeV and
a value of \(\sigma_{\text{n-pem}}^{\text{exp}} = 47.4 \pm 2\) mb at E_n = 14.6 MeV can be derived as
weighted average from these results.

Therefore, all \(\sigma_{\text{n-pem}}\) values calculated in the \(^{93}\)Nb intercomparison
are compared in table 4. Also given in the table (col. 3) are the so-called
R-factors which describe the difference between neutron and proton emission
rates because of the predominance of neutrons in the initial 2p1h states
formed by the incident neutrons, as the use of different such R-factors may be one of the causes for different results.

Column 4 gives the calculated $\sigma_{n, pem}$ values as reported in ref. 1. As already discussed these calculations were adjusted to fit the $d\sigma/dE_n\gamma$ values of ref. 19 which since have proved to be too high by about 25%. Thus it is to be expected that these calculations will also overestimate the $\sigma_{n, pem}$ cross-sections as this reaction proceeds to about 70-80% by precompound emission (see tables 2a-2c of ref. 1). This is shown clearly in column 6 of table 4.

Therefore a rough estimate was made of the cross-sections $\sigma_{n, pem}$ that would result if the precompound matrix elements in the calculations were adjusted to fit the true $d\sigma/dE_n\gamma$ values of ref. 20. These values called $(\sigma_{n, pem})_{corr}$ are given in column 5 of table 3 and their ratios to $(\sigma_{n, pem})_{exp}$ are listed in column 7.

An examination of these ratios shows the following:

i. About half of the calculations give excellent results (R' value equal to unity within ±10%) showing that a good simultaneous description of neutron and proton emission is possible in principle. There seems to be no big difference which kind of R-factors are used in the calculations and also good agreement was obtained for one of the codes based on the geometry dependent hybrid model.

ii. The other half of the calculations produced rather poor results deviating from the experimental value by up to a factor of three. The reason for this is not obvious. Using the same parameters (s. Appendix A of ref. 1) one would expect that all calculations of the same type e.g. exciton model calculations or geometry dependent hybrid model calculations should give very similar results for $\sigma_{n, pem}$ too.

One trivial cause for these discrepancies may be the use of parameters different from the specifications of the intercomparison. One such case was detected: In the PREM calculations very crude approximations were used for the inverse cross-sections instead of calculating these values from the specified optical potentials. The calculation HAUSER V gives unreasonable results also for $\sigma_{n, 2n}$ and $\sigma_{nn, \gamma}$ and thus must be in error as a whole. For the GNASH(LAS) calculations the rather low value of $\sigma_{n, pem}$ is probably due to the use of different single particle densities $g_n$ and $g_p$ ($g_p = .9 g_n$) in the exciton model. For the remaining cases the information in ref. 1 was not sufficient to find the causes of discrepancies.

Thus it would be very desirable if the $^{93}$Nb exercise could be continued in order to clarify the discussed problems and also to address the problem of $\alpha$-emission which could not be discussed here because of the limited time. Probably an additional calculation of the proton and $\alpha$-spectra including a break-up into their equilibrium and pre-equilibrium parts would help to identify the reasons for the differences between the different calculations.
3. References

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/6/ C. Y. Fu, Approximation of precompound effects in Hauser-Feshbach codes for calculating double-differential (n,xn) cross-sections (submitted to Nucl. Science Eng., Dec. 1987)
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<table>
<thead>
<tr>
<th>CALCULATION</th>
<th>P-E Model</th>
<th>R-Factor</th>
<th>( \frac{\sigma_{\text{np}x}}{\sigma_{\text{np}}^\text{exp}} )</th>
<th>( R' = \frac{\sigma_{\text{np}x}^\text{corr}}{\sigma_{\text{np}}^\text{exp}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>STAPRE (ILR-1)</td>
<td>EM</td>
<td>/24/</td>
<td>53.5</td>
<td>.94</td>
</tr>
<tr>
<td>STAPRE (ILL-1)</td>
<td>EM</td>
<td>/25/</td>
<td>56.6</td>
<td>1.13</td>
</tr>
<tr>
<td>CRASH (LAS)</td>
<td>EM</td>
<td>?</td>
<td>47.1</td>
<td>1.19</td>
</tr>
<tr>
<td>CRASH (JAE)</td>
<td>EM</td>
<td>?</td>
<td>39.1</td>
<td>.99</td>
</tr>
<tr>
<td>CRASH (ED)</td>
<td>GDH</td>
<td>?</td>
<td>66.4</td>
<td>1.09</td>
</tr>
<tr>
<td>PERIM (ECN-1)</td>
<td>EM</td>
<td>/26/</td>
<td>56.4</td>
<td>.82</td>
</tr>
<tr>
<td>HAUSER-F functional (TPM-1)</td>
<td>EM</td>
<td>/26/</td>
<td>53.2</td>
<td>.85</td>
</tr>
<tr>
<td>TNG (ORNL)</td>
<td>EM</td>
<td>/26/</td>
<td>62.2</td>
<td>1.40</td>
</tr>
<tr>
<td>PRANG (ECN-2)</td>
<td>EM</td>
<td>/26/</td>
<td>51.9</td>
<td>1.17</td>
</tr>
<tr>
<td>PREM (TUH)</td>
<td>EM</td>
<td>/26/</td>
<td>55.4</td>
<td>.99</td>
</tr>
<tr>
<td>PREM (ECN-2)</td>
<td>EM</td>
<td>/25/</td>
<td>47.1</td>
<td>.35</td>
</tr>
<tr>
<td>PRECO 2 (TNU-1)</td>
<td>EM</td>
<td>/26/</td>
<td>16.7</td>
<td>.93</td>
</tr>
<tr>
<td>PRECO 2 (TNU-2)</td>
<td>EM</td>
<td>/27/</td>
<td>44.2</td>
<td>.87</td>
</tr>
<tr>
<td>ALICE (ILL-1)</td>
<td>EM</td>
<td>?</td>
<td>47.7</td>
<td>1.04</td>
</tr>
<tr>
<td>AY: (CNS-1)</td>
<td>EM</td>
<td>?</td>
<td>67.7</td>
<td>1.71</td>
</tr>
<tr>
<td>Hősz (CNS-1)</td>
<td>EM</td>
<td>?</td>
<td>13.1</td>
<td>.82</td>
</tr>
<tr>
<td>( (\sigma_{\text{np}x})^\text{exp} = 47.4 + 2 \text{ mb} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Figure 1: Calculated angle-integrated total neutron emission cross-sections and experimental data from reference 3, high-energy part

Figure 2: Calculated angle-integrated total neutron emission cross-sections, low-energy part
Figure 3: Comparison of calculated angular distributions for $E' = 13-14$ MeV with experimental results for 3 MeV wide energy bin centred at middle of the bin used in the calculations. All calculated double-differential cross-sections have been renormalized so as to give the same angle-integrated cross-sections as the experiment. The normalization factors are given in Table 3.

Figure 4: Comparison of calculated angular distributions for $E' = 15-16$ MeV with experimental results for 3 MeV wide energy bin centred at middle of the bin used in the calculations. All calculated double-differential cross-sections have been renormalized so as to give the same angle-integrated cross-sections as the experiment. The normalization factors are given in Table 3.
Figure 5: Comparison of calculated angular distributions for $E' = 18-19$ MeV with experimental results for 3 MeV wide energy bin centred at middle of the bin used in the calculations. All calculated double-differential cross-sections have been renormalized so as to give the same angle-integrated cross-section as the experiment; the normalization factors are given in table 3.

Figure 6: Comparison of calculated angular distributions for $E' = 21-22$ MeV with experimental results for 3 MeV wide energy bin centred at middle of the bin used in the calculations. All calculated double-differential cross-sections have been renormalized so as to give the same angle-integrated cross-section as the experiment; the normalization factors are given in table 3.
Figure 7: Comparison of the experimental values used in $^{93}$Nb evaluations with the recent evaluation of ref. /20/. Histogram: evaluation of ref. /20/.

- $\times$ Experimental values given to participants of $^{93}$Nb intercomparison
- Solid curve: GNASH calculation of $^{93}$Nb intercomparison

Figure 8: Impact of new experimental data on simultaneous fit of neutron emission spectra at 14.6 and 25.7 MeV. $^{93}$Nb intercomparison (fitted to Hermsdorf n-emission cross-sections at 14.6 MeV (s, fig. 7)).

- Solid line: GNASH calculation of $^{93}$Nb intercomparison
- Dashed line: GNASH calculation modified as to fit the evaluation of ref. /20/ at 14 MeV
- $\square$ Experimental data of ref. /24/
THERMODYNAMIC DESCRIPTION OF PREEQUILIBRIUM EMISSION OF NUCLEONS IN NUCLEUS-NUCLEUS COLLISIONS*

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ABSTRACT

Nucleon yields from central-like collisions of heavy ions at 15-30 MeV/nucleon have been compared with Boltzmann Master-Equation-type calculations. The comparison has shown that the excitation energy per initial degree of freedom, $E^*/n_0$, is essentially independent of the projectile and target masses and depends only on the per nucleon energy brought in by the projectile, $(E_{\text{inc}}-V_{CB})/A_p$. The linear relationship thus obtained between $E^*/n_0$ and $(E_{\text{inc}}-V_{CB})/A_p$ leads to introducing a quantity $T_{PF}$, "preequilibrium" temperature. It has been shown that $T_{PF}$ is related to the exciton density in the same way as the temperature for the equilibrated systems is related to the total level density.

*This paper is part of a study under the US-Yugoslavia collaboration project JFP 554 DOE/IRE.
1. Introduction

We have met here, in snowy Semmering, to study an interesting and challenging subject: the equilibration of a nuclear many-body system after the initial projectile-target collision and the accompanying preequilibrium particle emission. Much of our present knowledge about this subject comes from light-ion induced reactions where preequilibrium phenomena \(1,2\) were observed several decades ago and subsequently studied. However, because of their very nature, light-ion induced reactions are limited to asymmetric projectile-target combinations and relatively low angular momenta. The recent development of powerful heavy-ion accelerators has made available beams of heavy projectiles with energies per nucleon corresponding to light-ion energies at which preequilibrium phenomena are present. And, indeed, at these energies \(E_{\text{inc}}/A_p \sim 10-15\) MeV/nucleon, the preequilibrium emission of particles, manifesting itself by high spectral energies and anisotropic angular distributions, has also been observed in heavy-ion reactions \(3\).

In this paper we investigate the sharing of energy among various degrees of freedom in the early stages of nucleus-nucleus collisions \(4\) based on the energetic nucleon data from heavy-ion reactions. This investigation has inspired us to introduce an exciton-based "preequilibrium" temperature \(T_{\text{pp}}\) which is related to the exciton level density in the same way as the temperature is related to the total level density.

2. Equilibration of a nuclear system; initial number of degrees of freedom

The description of the evolution of a nuclear system after the projectile-target collision is performed using a Foltzmann Master Equation \(\text{FME}) \(5\) which is schematically represented by the expression

\[
\frac{dn}{dt} = \text{gain term} + \text{loss term} + \text{escape term} + \text{mixing term (fusion)}
\]  

(1)

The evolution is fully determined if one defines (i) the initial conditions of the system (e.g. the initial number of degrees of freedom) and (ii) the transition rates between its subsequent states.

In our analysis of the high-energy part of nucleon spectra from heavy-ion reactions we took the transition rates from nucleon-nucleon scattering scaled to experimental multiplicities by a factor of \(1/4 \) \(6,7\); this scaling, however, does not influence the slope of the calculated spectra. The initial number of degrees of freedom, \(n_0\), was then in turn determined from the comparison of the experimental and calculated slopes of nucleon spectra. The FME code RELAX written by M. Flann and G.D. Harp was used in actual calculations.

The values of \(n_0\) were extracted from both exclusive and inclusive spectra. For inclusive spectra, the part corresponding to central-like collisions was singled out using a refined multiple-source analysis of the data. In this analysis we used a
new ansatz \[8,9\] which explicitly takes into account the anisotropy of the emitted preequilibrium particles already in the source frame (c.m. system). The anisotropy parameter \(\Delta \theta\) depends on the energy of the emitted particles through the relation \(R \sim \Delta \theta \sim 2\pi/K\) \[10\], with \(K\) the nucleon wave number and \(R\) the radius of the compound system.

The best-fit values of \(n_0\) obtained from a systematic set of proton data from heavy-ion collisions are collected in Table I. The main feature is the dependence of \(n_0\) on the entrance channel:

<table>
<thead>
<tr>
<th>Projectile Energy (MeV)</th>
<th>(27\text{Al})</th>
<th>(46\text{Ti})</th>
<th>(60\text{Ni})</th>
<th>(120\text{Sn})</th>
<th>(124\text{Sn})</th>
<th>(197\text{Au})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(16\text{O})</td>
<td>16</td>
<td>19</td>
<td>19</td>
<td>21</td>
<td>-</td>
<td>22</td>
</tr>
<tr>
<td>(32\text{S})</td>
<td>23</td>
<td>28</td>
<td>29</td>
<td>35</td>
<td>35</td>
<td>37</td>
</tr>
<tr>
<td>(58\text{Ni})</td>
<td>23</td>
<td>28</td>
<td>29</td>
<td>35</td>
<td>35</td>
<td>37</td>
</tr>
<tr>
<td>(876.5)</td>
<td>26</td>
<td>35</td>
<td>-</td>
<td>46</td>
<td>46</td>
<td>61</td>
</tr>
</tbody>
</table>

The obtained values are grouped around the mass number of the projectile \((A_T)\) viz. around the mass number of the lighter partner. Furthermore, the values of \(n_0\) show an increase with the mass of the system (i.e. with the target mass \(A_T\) for a given projectile).

3. Sharing of energy in the initial stages of nucleus-nucleus collisions. "Preequilibrium" temperature

A nuclear many-body system in the early stages of nucleus-nucleus collisions is characterised by its excitation energy \(E^* = E_{c.m.} + E_{\text{fus.}}\) in addition to the initial number of degrees of freedom, \(n_0\). Now, a natural question arises: how is this excitation energy shared among various degrees of freedom? The answer to this question, stemming from our analysis, can be seen in Figures 1a and 1b, where the values of \(E^*/n_0\) are plotted vs. \(A_T\) and vs. the available energy, respectively. The striking feature in the figures is that this average excitation energy per initial degree of freedom, \(E^*/n_0\), depends only on the per nucleon energy of the incident projectile. It is, in fact, constant for a given projectile at a given energy and also for two different projectiles (e.g. \(32\text{S}\) and \(58\text{Ni}\)) having the same per nucleon energies.

The quantity \(E^*/\bar{n}\) (\(\bar{n}\) being the number of degrees of freedom in equilibrium) has a precise physical meaning in equilibrium thermodynamics, i.e. it is related to the temperature by

\[
E^*/\bar{n} = \frac{1}{2} kT,
\]

with \(k\) the Boltzmann constant. Based on the similarity of \(E^*/n_0\) and \(E^*/\bar{n}\) behaviour (equal sharing of energy among various degrees of freedom), one can introduce a quantity \(T_{PE}\) that is the analogue for nonequilibrated systems of what "normal" temperature is for
equilibrated systems \[4,11\]. This can be done starting from the standard definition of temperature

\[
T^{-1} = \frac{d}{dE^*} \ln \rho(E^*) ,
\]

with, however, \(\rho(E^*)\) taken as the exciton level density at the appropriate excitation energy \[12\]

\[
\rho(E^*,p,h) = \frac{q^{p+h}(E^*)^{p+h-1}}{p!h!(p+h-1)!} .
\]

It is easily shown that the quantity \(T_{PE}\) depends linearly on \(E^*\). Combining (3) and (4), one gets

\[
E^* = (p+h-1)T_{PE} = (n-1)T_{PE} ,
\]

hence

\[
E^*/n \propto T_{PE} ,
\]

which reproduces the straight line in Figure 1b.

In analogy to the equilibrium temperature \(T\), related to the
average energy per equilibrated degree of freedom, the newly introduced "pre-equilibrium" temperature, $T_{PP}$, is simply related to the average energy per initial degree of freedom. Being essentially determined by the size of the projectile and the total available energy and thus independent of the microscopic properties of the system, this quantity $T_{PP}$ can be used in the thermodynamic description of the early stages of nucleus-nucleus collisions.

References

A SIMPLIFIED UNIFIED HAUSER-FESHBACH
/PRE-EQUILIBRIUM MODEL FOR CALCULATING
DOUBLE DIFFERENTIAL CROSS SECTIONS†

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ABSTRACT

A unified Hauser-Feshbach/Pre-Equilibrium model is extended and simplified. The extension involves the addition of correlations among states of different total quantum numbers (J and J') and the introduction of consistent level density formulas for the H-F and the P-E parts of the calculation. The simplification, aimed at reducing the computational cost, is achieved mainly by keeping only the off-diagonal terms that involve strongly correlated 2p-1h states. A correlation coefficient is introduced to fit the experimental data. The model has been incorporated into the multistep H-F model code TNG. Calculated double differential \((n, xn)\) cross sections at 14 and 25.7 MeV for iron, niobium, and bismuth are in good agreement with experiments. In use at ORNL and JAERI, the TNG code in various stages of development has been applied with success to the evaluation of double differential \((n, xn)\) cross sections from 1 to 20 MeV for the dominant isotopes of chromium, manganese, iron, nickel, copper, and lead.

†Research sponsored by the Office of Basic Energy Sciences, U. S. Department of Energy under contract DE-AC05-84OR21400 with Martin Marietta Energy Systems Inc.
INTRODUCTION

Hauser-Feshbach/pre-equilibrium model codes have been widely used for calculating neutron-induced reaction cross sections from a few keV to several tens of MeV. The ability of the H-F model, based on the random phase approximation, in calculating angular distributions for the equilibrium part is well-established. Plyuiko\(^1\) has attempted to unify the two models for the purpose of calculating angular distributions for the P-E part with the same components as for the H-F part by allowing fully correlated phases in the partial waves forming states of a fixed total angular momentum \((J)\) in the first pre-equilibrium stage \((n_0)\). We have found it necessary to allow cross terms involving different \(J\)'s and to change the full correlation into partial correlation by introducing a correlation coefficient. Deviations in the spin distributions in the composite nucleus in the P-E stage from the H-F stage are accounted for and lead to enhanced peripheral emission in the pre-equilibrium stage. The model is then simplified by removing many second order terms. Good agreement of calculated double differential \((n, xn)\) cross sections with experimental data for two incident neutron energies and for both even-even and odd-A targets over a wide mass range was obtained. In the calculations for the odd-A targets, the target spin was assumed to be inactive in the P-E stage but active as usual in the H-F stage.

THE UNIFIED FORMULA

The double-differential cross section due to both the H-F and P-E models, derived by Plyuiko\(^1\) and here modified by adding a summation over \(J'\), is given by

\[
d^2\sigma(a, b; J_x)/dE_b d\Omega = k \sum_L (D_L + B_L) P_L(\cos \theta),
\]

(1)

with

\[
D_L = \sum (-1)^{j_a - j_b} Z_a Z_b T_a T_b \sum_{n, \Delta n = 2} f(n, U, J, \Pi) \rho(n - n_b, U_y, J_y, \Pi_y),
\]

(1a)

and

\[
B_L = \sum' (-1)^{j_a - j_b} Z'_a Z'_b (T_a T'_a T_b T'_b) \frac{1}{2} [f(n_0, U, J, \Pi) f(n_0, U, J', \Pi')] \frac{1}{2} \rho(n_0 - n_b, U_y, J_y, \Pi_y),
\]

(1b)

where

- \(a\) = the incident particle
- \(x\) = the target nucleus
- \(b\) = the outgoing particle
y = the residual nucleus
k = \lambda_a^2 / [4(2J_a + 1)(2J_x + 1)]
Z = Z coefficients defined in ref. 2; e.g. \( Z'_a = \epsilon_a - \epsilon'_a - L Z(\ell_a J_a \ell'_a J'_a; j_a L) \)
T = optical model transmission coefficients; e.g. \( T'_b = T(E_b, j_b, \ell'_b) \)
f = pre-equilibrium weighting function = \tau(n, U, J, \Pi)Q(n)/\[h\rho(n, U, J, \Pi)\]
\( \tau = \int_0^\infty P(n, U, J, \Pi, t)dt \) = mean lifetime of states with exciton number \( n \), excitation energy \( U \), spin \( J \), and parity \( \Pi \)
P = occupation probability of exciton states
Q = a factor defined by Kalbach\(^a\) for neutron-proton distinguishability (not used by Plyuiko but added here to enhance the applicability of the model)
\[ \sum = \text{sum over } J, \Pi, J_y, \Pi_y, j_a, j_b, \ell_a, \text{ and } \ell_b \]
\[ \sum' = \text{sum over } J, \Pi, J'_y, \Pi'_y, j_a, j_b, \ell_a, \ell'_a, \ell_b, \ell'_b \text{ with the restriction } J \neq J', \text{ and/or } \ell_a \neq \ell'_a, \text{ and/or } \ell_b \neq \ell'_b. \]
The prime is used to denote a different value of the unprimed quantity. The quantities without a subscript, such as \( n, U, J, \Pi \), are for the composite nucleus. The remaining variables are recognizable and are defined elsewhere\(^4\).

The reason for adding the sum over \( J' \) is now explained. The assumption that states with different total angular momenta in the composite nucleus are not correlated was not explained by Plyuiko. The assumption probably originated from a desire for saving computing effort. However, this assumption cannot be justified. For example, the assumption breaks down for an alpha-particle induced reaction on even-even targets. Since the incident channel spin for such a reaction is zero, none of the incoming partial waves can interfere for a fixed \( J \) to generate any odd Legendre coefficients. Consequently, all alpha-particle-induced reactions on even-even targets would have to emit particles symmetrically about 90 degrees in the center-of-mass frame, a prediction contrary to forward-peaked angular distributions commonly observed in the pre-equilibrium energy ranges. Therefore, we added the summation over \( J' \) throughout Plyuiko’s derivations. By removing the sum over \( J' \) and \( \Pi' \) in Eq. (1b) and by replacing \( J' \) and \( \Pi' \) with \( J \) and \( \Pi \) in \( f(n_0, U, J', \Pi') \), Eq. (1) becomes the original Plyuiko’s formula. Unfortunately, the newly added summation much increased the necessary computation. To compensate, other simplifications are sought, as discussed next.

Simplifying Approximations

Three simplifying approximations were made and are summarized below. The third one has to do only with an odd-A or odd-odd target.

1. The deviations of the spin and parity distributions in the pre-equilibrium weighting function \( f \) from the equilibrium distributions are
considered only for the initial exciton states \((n_0 = 3\) in the present calculations). These \(n_0\) spin and parity distributions are taken to be an average of those at \(t = 0\) and \(t = T\) (the equilibration time). The deviations of these distributions from the equilibration distributions lead to an enhancement of pre-equilibrium emission from large \(J\)-states (peripheral emission). For \(n > n_0\), the usual H-F spin and parity distributions are used, saving a lot of computations for the minor components. Advanced treatments of pairing correction\(^5\) and pairing-corrected spin distributions\(^6\) for the particle-hole level densities are adopted to ensure that the unified model does not require any more level density parameters as already required by the H-F model.

2. Only those terms in Eq. (1) that can contribute to the \(P_0\) and \(P_1\) Legendre coefficients are kept. This approximation represents a first order relaxation of the RPA approximation used for the H-F model and leads to very large savings in computation. A simple way to think of the approximation is as follows. Those terms that can give \(P_0\) are the usual H-F terms (the diagonal terms), therefore kept, while those that can give \(P_1\) (the near-diagonal terms) approximately represent the "memory" of the incident direction by the outgoing particle in a strongly localized pre-equilibrium collision. The latter is weighted by a correlation coefficient (between 0 and 1) to account for a partial dissipation of this memory.

3. For an odd-A target, the odd nucleon and its spin have little chance of participation in the first pre-equilibrium stage but will take part in the H-F stage. This consideration is accounted for by calculating the P-E and the H-F components separately, setting the true target spin to zero in the pre-equilibrium run. The TNG code was programmed to do this.

VALIDATION OF THE MODEL

Selected double differential \((n, xn)\) data at 14 and 25.7 MeV for \(^{56}\)Fe, \(^{93}\)Nb, and \(^{209}\)Bi were used to test the present model. This choice provided two incident neutron energies, a wide mass range, and both even-even and odd-A targets. Extensive numerical experiments were performed to check the above simplifying approximations. Good agreement with data was obtained with a constant correlation coefficient of 0.5 and a standard set of optical model, level density, and P-E parameters. A detailed description of this work\(^4\) is available upon request.

The same correlation coefficient and parameters as above were applied to the Blind Pre-equilibrium Benchmark\(^7\) for neutron emission from 25.7-MeV incident neutrons on \(^{184}\)W. (See also proceedings of this meeting for the many interesting contributions to this intercomparison.)

Our contribution to the \(^{93}\)Nb intercomparison\(^8\) was made with an earlier version of the code before the third approximation was implemented. The calculation was done by setting the target spin to zero for both the P-E and H-F parts. This, of course, led to less valid results for the H-F part, which,
in turn, affected the P-E part of the calculation because the two competed. However, the results remained satisfactory in comparison with the widely spread data. Our new calculation\textsuperscript{4} yielded better agreement.

**APPLICATION OF THE TNG CODE**

The model described here has been applied to the ENDF/B-VI evaluations of iron and lead. Preliminary versions of the code have been used for the chromium, manganese, nickel, and copper evaluations with considerable success. Reports, preprints, or summaries on these are available upon request. However, we encountered a difficulty in the lead evaluation which is described below.

As shown in Fig. 1, the neutron emission spectrum from 14-MeV Pb($n, xn$) reaction has a flat portion for $E_n'$ between 6 and 9 MeV. This $E_n'$ range is usually dominated by the P-E reaction which cannot yield a flat spectrum, a spectrum possible only in collective reactions. To meet our ENDF/B-VI schedule, we tentatively added a constant state density (a parameter) to the $1p-1h$ state density for the residual nucleus, thus flattening the P-E spectral shape. The result is shown in Fig. 1, together with the ENDF/B-V evaluation and the recent data of Takahashi\textsuperscript{9}. This approach is unfounded and is an area we plan to improve in the near future. Fortunately, the double differential cross-section data\textsuperscript{9–11}, shown in Fig. 2, are still well reproduced with the global correlation coefficient of 0.5.

The same difficulty encountered in the lead calculation also exists (see proceedings of this meeting) in our $^{184}$W calculation\textsuperscript{7}. Strong collective excitation (vibration for Pb and rotation for W) is expected in each case and a simple approximation for such an excitation is desirable. In the calculation shown in Fig. 1, DWBA results for many discrete levels were already included but there is still too big a gap between the collective range and the H-F/P-E range.

**REFERENCES**


Fig. 1. Neutron emission spectrum from 14.1-MeV Pb(n,xn) reaction.
Fig. 2. Double differential neutron-emission cross sections from 14.1-MeV Pb\((n, xn)\) reaction.
PROGRESS ON THE APPLICATION OF PRE-EQUILIBRIUM AND
EQUILIBRIUM STATISTICAL THEORY TO NUCLEAR DATA EVALUATION

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ABSTRACT

The double differential cross section, spectrum, angular distribution and cross section of all more important reaction channels in the energy range between 3 and 20 MeV for both structural and fissionable nuclides are described simultaneously and consistently by using the optical model, exciton model, and evaporation model. In exciton model, the importance of taking into account the influence of both the Fermi motion and the Pauli principle has been emphasized. The systematic behavior of level density at saddle point has been obtained by means of the approximation of effective single hump fission barrier.
The nuclear data of reactions induced by neutrons in the energy range of 1 keV - 20 MeV have been described by different theoretical models, namely optical model, Hauser-Feshbach statistical theory with width fluctuation correction, exciton model and evaporation model. The pre-equilibrium statistical theory has been proved to be very successful in the description of light projectiles induced reactions in the energy range of several tens of MeV. We have first derived an exact closed form solution for the time-integrated master equation of exciton model with emphasis on the importance of taking into account the influence of both the Fermi motion and the Pauli principle (1) in the energy range of several tens of MeV, which is comparable with or less than Fermi energy. In generalized master equation of exciton model the Fermi gas model has been adopted for nucleon-nucleon interaction inside the nucleus for the derivation of the kernel, thus the effects of Pauli principle and Fermi motion are properly taken into account as proposed in ref.(1). The effects of the finite nuclear size and the refraction of the incident wave at the nuclear surface underlying the exciton model are also properly taken into account by cutting 1 values and considering the refraction kernel given by E. Gadioli. (2) The calculated results (3) show that the theory can explain the presently existing experimental (4) data, especially solve the problem of double differential cross section for emitted neutrons with high energy at backward angles. The theory has been used for the calculation of MP-6 in ENDF/B-VI for both structural and fissionable nuclides and analysing new experimental data. (5-6) In this respect we have unified optical model, exciton model and evaporation model consistently to calculate FM3,4,5,6 of ENDF/B of multi-particle emission process in the energy range between 3 and 20 MeV. We assume that for the structural nuclides, charged particles, such as p,d, 3He, T, α can be emitted, but the fission channel is not open; for the fissionable nuclides, the fission channel is open instead of charged particle emission. In the code the direct reaction mechanism can be considered separately. We suppose only considering the first and second pre-equilibrium emissions in the calculations. (7)

First of all, we define the tendency to equilibrium probability from exciton number n_1 to n_2 by λ_{±2} transition of the composite system with mass number A, atomic number Z, excitation energy U for k-th emission process as the following

\[ D_k(n_1,n_2,Z,A,U,n) = \prod_{n=n_1}^{n_2} \frac{\lambda_+(Z,A,U,n)}{(\lambda_+(Z,A,U,n)+\lambda_-(Z,A,U,n)+\lambda_0(Z,A,U,n)+W_k(Z,A,U,n))} \]

(1)

here, \( \lambda_{±0} \) are the exciton transition rate for \( n = ±2,0 \) processes, which can be derived by taking into account the exact Pauli exclusion effect given by Zhang Jing Shang in ref.(8). \( W_k(Z,A,U,n) \) is the k-th total emission rate of the n-th exciton state.
Another definition is the probability of emitting particles with mass number $A_\nu$, atomic number $Z_\nu$, energy $E_\nu$ in direction $\Omega_\nu$ at the composite system state with mass number $A$, atomic number $Z$, excitation energy $E$ after from exciton number $n_1$ to $n_2$ by $\lambda_+$ transition for $k$-th emission process as below

$$P_k(z, A, U, n_1, n_2; Z_\nu, A_\nu, E_\nu, \Omega_\nu)$$

$$= t_k(z, A, U, n_1, \Omega_\nu) * W_\nu(z, A, U, n_1; Z_\nu, A_\nu, E_\nu)$$

(2)

with

$$t_k(z, A, U, n_1, \Omega_\nu) = \sum_{\Omega_\nu} P_{\Omega_\nu}(\cos \theta_k)$$

here, $t_k$ is the life time and $W_\nu$ is the total emission rate of per MeV for $\nu$ particles in which the complex particle formation mechanism can be considered as given in ref. (9). $Z_\nu$ is the exact closed form solution to the time-integrated master equation in the form of partial wave, (1) in which the effects of the finite nuclear size and the refraction of the incident wave at the nuclear surface have been included by cutting $l$ values (1) and calculating the refraction kernel as given in ref. (2, 10).

We take $(n_1, 2n_2)$ reaction as an example to give the cross section $\sigma_{n, 2n_2}(E_\nu)$ as follows

$$\sigma_{n, 2n_2}(E_\nu) = \sigma_{\nu}(E_\nu) \int \int \int \int dE_{n_1} dE_{n_2} dE_{n_3} dE_{n_4}$$

$$= \sum_{n=n_0}^{\infty} P_1(z, A+1, E_0 + 8n_1, n_0, n; 0, l, E_{n_1}, \Omega_1)$$

$$\sum_{n=n_1}^{\infty} P_2(z, A, E_0 - E_{n_1}, n-1, n', 0, l, E_{n_1}, \Omega_1)$$

$$+ D_{n_0}(E_0, 0, n_1, 2, A, E_0 - E_{n_1}) * \delta_{n_1}(z, A, E_0 - E_{n_1}, 0, l, E_{n_1})$$

$$+ D_1(n_0, n_1, z, A+1, E_0 + 8n_1) * \delta_{n_1}(z, A+1, E_0 + 8n_1, 0, l, E_{n_1})$$

$$\sum_{n=n_2}^{\infty} P_3(z, A, E_0 - E_{n_1}, 0, l, E_{n_1})$$

$$\sum_{n=n_3}^{\infty} P_4(z, A-1, E_0 - E_{n_1} - 8n_2 - E_{n_2})$$

(3)
Here, \( E_0 \) is the energy of incident neutron. \( B_{n1}, B_{n2}, E_{n1}, E_{n2} \) are the binding energy and the emission energy of the first and second emitted neutrons, respectively. \( q_1, q_2 \) are the probabilities of the evaporated first and second neutron per unit of MeV* Sr. \( \Gamma \) is the \( \gamma \)-ray emission probability after two neutrons emitted. \( \sigma_a(E_0) \) is the formation cross section of the compound system. Thus one can have the doubly-differential cross section \( \sigma_{n,2n}(E_0,E_n,\theta) \), angular distribution \( \sigma_{n,2n}(E_0,\theta) \) and normalized spectrum \( n_{n,2n}(E_0,E_n) \) from eq. (4), respectively.

For actinide nuclei bombarded by neutron with energy in the range of 3 - 20 MeV, we have used the unified evaporation model and exciton model presented by probability of finding the system in the state of exciton number \( n \) at time \( t \) with never come back assumption to calculate cross sections and spectrum of \((n,n'\gamma),(n,2n\gamma),(n,3n\gamma),(n,n'f),(n,2nf)\) reactions for U, Pu elements. In the calculations the compound and residual nucleus level densities are adopted in the form of Gilbert-Cameron (11). The saddle point state density is adopted in the same form, but the level density parameter \( \alpha_f \) is no longer a constant for a highly deformed nucleus, but according to ref. (7) given by

\[
\frac{\alpha_f}{a} = 1 + \frac{K_2}{(E-\Delta)}. \quad (4)
\]

The values of \( \beta_f(E,K_1,K_2) = K_1\beta(E,K_2) \) which we have obtained (7) is the same as the one proposed by J.E.Lynn (12) in the same energy range of \( E=3-4 \) MeV. Here \( K_1, K_2 \) are the new fitting parameters. The systematic behavior of level density on saddle point for U, Pu elements is obtained by fitting all the known experimental fission cross section based on the approximation of effective single hump fission barrier assumption shown as

\[
\eta = \begin{cases} 
1.675 & \text{for e-e nuclei} \\
4.624 & \text{for odd nuclei} 
\end{cases} \quad (5)
\]

with \( \eta = \beta_f(E,K_1,K_2)/\beta(E,K_1=1,K_2=0) \).

From the calculated results we can conclude that the model described above and the systematic behavior of level density on saddle point are applied satisfactorily. Based on the models described above and combined with Hauser-Feshbach formula with width fluctuation correction, we have made more useful codes named MUP1,2 and FUP1 for calculating files MF4-5 of ENDF/B for both structural and fissionable nuclides from 1 keV to 20 MeV. Calculations of the files MF3,4,5,6 from 1 keV to 20 MeV for both structural and fissionable nuclides have also been made successfully.
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CAPABILITIES AND PLANS FOR PREEQUILIBRIUM NUCLEAR REACTION MEASUREMENTS AT TARGET-4 AT LAMPF

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ABSTRACT

The Target-4 neutron source at the Los Alamos Meson Physics Facility (LAMPF) has recently been commissioned and used as a "white" source of fast neutrons from 1 to 500 MeV. A 90-meter flight path at a neutron production angle of 15 degrees has been characterized for neutron flux and spectral shape. The flux and shape agree well with calculated values showing that this is indeed an intense and useful white neutron source. Initial measurements of the $^{12}$C(n,p) reaction at forward angles have been made in the energy range 50 to 500 MeV. The results point the way to improve detector schemes and to a program of (n,p) cross section measurements in the energy range where preequilibrium processes dominate much of the charged-particle emission spectra.
1. Introduction

A new facility for measuring (n,p) and other (n, charged particle) reactions is being developed at the Target-4 white neutron source at LAMPF. Preequilibrium charged-particle emission is a particular focus of this project which also is concentrating on direct processes to detect Gamow-Teller strength and on compound nuclear processes for level density determinations. This report describes the features of the facility relevant to preequilibrium studies from about 10 MeV neutron energy to over 100 MeV.

Neutron-induced preequilibrium reactions complement proton-induced reactions and, from a comparison of the two, isospin effects can be isolated. Single-particle effects (1) can be different for proton and neutron-induced reactions simply because the preequilibrium systems are different. Furthermore, as the incident particle energy increases, the nucleus becomes more transparent, and the difference between proton and neutron-induced effects might change.

Only a small body of data exists on neutron-induced preequilibrium proton emission. At 14 MeV there have been many high quality measurements over the past decade, but the emission spectra are often dominated by the equilibrium component. At higher energies, studies generally have focused on direct processes to obtain spectroscopic information on low lying states or giant resonances. Spectra at back angles are usually not taken in these works although they are essential in deriving the magnitude of preequilibrium components. Thus there is a wide range of parameter space where there are essentially no data for neutron-induced preequilibrium charged particle emission. The new Target-4 facility is ideally suited to address this area.

2. The Target-4 Facility

At Target-4 (2), neutrons are produced by spallation of 800 MeV protons on a tungsten target 3 cm in diameter and 7.5 cm long. The continuous-in-energy or "white" neutron spectrum resulting from spallation extends from low energies to several hundred MeV. Flight tubes view the source from angles of 15° to 90° and detector stations are located along these flight tubes at suitable distances (see Fig. 1).

The pulse characteristics of the source are determined by the LAMPF beam and the demands of all the users for this beam. LAMPF accelerates three beams concurrently: \( H^+ \) (1 mA average), \( H^- \) (0.1 mA), and a lower intensity polarized beam, \( P^- \). Part of the \( H^- \) beam is directed to Target 4. The beam has both macropulse and micropulse structure. Macropulses are typically 700\( \mu \)s long and at present 40 macropulses per second can be diverted to Target 4. Micropulses occur with a spacing of 5 ns and it is easy to increase the separation of micropulses in units of 5 ns by selecting individual pulses at the source. In our studies so
far, micropulse spacings of 1 to 8µs are typical. Beam currents of 30-60 nA were used in 1986 and 300-600 nA last year. We expect microampere beam currents in 1988.

Two of the flight paths will be used for preequilibrium and direct reaction studies. One of these, at 15° has been completed and tested. The other, at 90°, is under construction.

3. 15-Degree Flight Path

A 90-meter flight path has been constructed at a neutron production angle of 15° left (Fig. 1). This flight path and associated detectors are designed to measure (n,p) reactions from about 30 to 500 MeV. During the LAMPF cycle that ended December 1, 1987, initial measurements were made at this facility.

The neutron flux spectrum was measured by n-p elastic scattering at a fixed angle (15°). The target was a thin plastic scintillator. Recoil protons were detected by a ΔE-E coincidence system with a plastic ΔE scintillator and a 15 cm diameter by 25-cm-long NaI(Tl) E-scintillator. Wire chambers served to define the trajectory of each proton. Although final corrections need to be made to the data, we can state that the neutron flux spectrum is close to that calculated (Fig. 2). The area of the collimated neutron beam was 10 cm x 10 cm.

Data were collected for $^{12}\text{C(n,p)}^{12}\text{B}$ at forward angles with the same detectors as for the n-p elastic scattering study. At 0° a magnet was employed to deflect the protons from the incident neutron beam direction. Proton spectra and angular distributions indicate the importance of direct reactions. Data over a wider angular range need to be taken to investigate preequilibrium particle emission.

4. 90-Degree Flight Path

A new flight path is now being constructed at 90° to the incident beam direction (Fig. 1). Here the detector station will be only 10 m from the source and will therefore have the greatest flux, if anisotropy of the source is neglected. In fact, for neutron energies below 20 MeV, more neutrons are produced at 90° than at forward angles (Fig. 2). This anisotropy is due to the production target being thinner toward 90°. For much higher neutron energies, more neutrons are produced per steradian at forward angles, however. This 90° flight path is therefore being designed for neutron energies below 50 MeV. For experiments where the high energy neutrons are a problem, e.g. as background or high singles rates, this production angle should reduce the magnitude of the problem.

Detectors for this flight path are being designed. A set of simple ΔE-E coincidence counters is one approach. To increase the counting rate, detectors of larger solid angle are being considered. Multi-wire proportional chambers will likely
be used to measure $\Delta E$ and to define the reaction angle. The $E$ detector must be thick enough to stop the maximum energy, 50-MeV, protons. Scintillators will likely be used for this.

5. Summary

Target-4 at LAMPF is a new spallation neutron source that promises to be ideal for preequilibrium ($n,p$) and other ($n$, charged particle) reaction studies in the range from 10 to over 100 MeV. Initial measurements show that the flux is equal to or greater than that calculated. Experimental stations are now being established for this measurement program.

6. References


Fig. 1. Target-4 facility showing the heavily shielded neutron production target and flight paths viewing the source. Freeequilibrium \((n, \text{ charged particle})\) reaction studies will be made at the 90 and 10 meter stations at 15° and 90° respectively.
Fig. 2. Calculated flux of neutrons from 800-MeV protons on a 3 cm diameter by 7.5 cm long tungsten target at angles of 15° and 90° (3).
SMD - Plus SMC-Calculations Below 30 MeV Neutron Incidence Energy
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1. Introduction

The statistical reaction model of the TU Dresden, firstly proposed in /1,2/, was used for calculations of neutron-induced particle emission spectra and angular distributions for different nuclides. This model was derived from first principles (Green's function formalism; Gaussian Ensembles) /3/. It bases on a clear separation between statistical multistep compound (SMC) and statistical multistep direct (SMD) processes.

The double-differential cross section has the form

\[
\frac{d^2C_{\alpha,\beta}(\varepsilon)}{d\varepsilon d\Omega'} = \frac{1}{4\pi} \left[ \frac{dC_{\alpha,\beta}^{\text{SMD}}(\varepsilon)}{d\varepsilon} \sum L (2L+1) \mathcal{Q}_L(\varepsilon') \mathcal{R}_L(\cos \theta') + \frac{dC_{\alpha,\beta}^{\text{SMC}}(\varepsilon)}{d\varepsilon'} \right]
\]  

(1)

where the incidence nucleon of type \( \alpha = \pi \) or \( \gamma \) (proton or neutron) has the kinetic energy \( \varepsilon \), and \( \beta, \varepsilon' \) refer to the outgoing particle. Here, the simple assumption is made, that the single-step and two-step contributions are of pure direct type, whereas all next following steps (beginning at exciton number \( n_0 = 5 \)) are pure SMC-processes. Further, in (1) the angular distribution of the SMC-emission is assumed to be isotropic while for the SMD-processes the empirical systematics of Kalbach and Mann /4/ is used.

2. SMD-cross section

Within the SMD-description we restrict ourselves to the excitation of non-collective particle-hole states (excitons) and the two low-lying surface vibrations (\( 2^+, 3^- \)-phonons). The differential cross section of the single-step processes can be written for the inelastic neutrons scattering as /3/
\[
\frac{dG^{(\text{ex})}(\epsilon)}{d\epsilon} = \left( \frac{m V_N}{2\pi \hbar^2} \right)^2 4\pi (2s+1) \overline{I}^2 \left( \frac{g}{2} \right)^2 (\epsilon-\epsilon') \sqrt{\frac{\epsilon'}{\epsilon}} 
\]
(2)

\[
\frac{dG^{(\text{vib})}(\epsilon)}{d\epsilon} = \left( \frac{m V_{\text{surf}}}{2\pi \hbar^2} \right)^2 \sum_{\lambda=2,3} \left( \frac{1}{\lambda} \right)^2 \beta^2 \nu V_0^2 \sqrt{\frac{\epsilon'}{\epsilon}} \delta(\epsilon-\epsilon'-\omega_{\lambda}) 
\]
(3)

Appropriate expressions are used for the direct two-step process which is a sum of (ex,ex),(ex,vib),(vib,ex) and (vib,vib)- contributions.

The non-collective particle-hole matrix element \( \overline{I}^2 \) in (2) was estimated from the OM-reaction cross section for neutrons \( G_y(\epsilon) \) which can be decomposed into the (energy-integrated) single-step and one multistep component,

\[
G_y(\epsilon) = G^{(\text{ex})}(\epsilon) + G^{(\text{vib})}(\epsilon) + G^{(\text{M})}(\epsilon) 
\]
(4)

The multistep contribution

\[
G^{(\text{M})}(\epsilon) = \frac{2\pi V_N}{\hbar \nu} \left( \frac{A+Z}{2A} \right) \overline{I}^2 \left( \frac{g}{2} \right)^3 \frac{E^2}{2} 
\]
(5)

includes all processes in which the incoming particle produces a composite system that decay by further collisions into more complex states. \( g = A/13 \) and \( E = \epsilon + B \) are the single particle state density and excitation energy. In using (4) it turns out that \( \overline{I}^2 \) depends on incidence energy and can fairly be approximated by

\[
\overline{I}^2 = \overline{I}^2(\epsilon) \approx 1800 A^{-3} \epsilon^{-1} \quad \text{for} \ A \leq 100, 
\]
(6)

and for \( \epsilon \approx 5 \text{ MeV} \). Even the parametrization (6) provides simple analytical expressions for the direct two-step processes similar
to those given in Appendix B of /1/.

Expression (3) describes the excitation of surface vibrations of multipolarity $\lambda$ at energy $\omega_\lambda$ and with deformation parameter $b_\lambda$.

In contrast to the assumption of refs. /1, 2/ where

$$\overline{I}^2_{\text{coll}} = \infty \overline{I}^2$$

and $\infty = \text{const} \gg 1$, the present model ansatz (3) bases on the particle-vibration coupling matrix element taken from Bohr and Mottelson /5/. Here, $V_0 = 52 - 0.3 \varepsilon$ is the real QM well depth of the nucleus, and the factor (1/2) is the asymptotic value of the Clebsch-Gordan coefficient. Since we deal with surface interactions the nuclear volume $V_N = \frac{4}{3} \pi R^3$ was replaced by $V_{\text{surf}} = \frac{4}{3} \pi a R^2$ in (3), where $R = 1.2 \text{fm}$ and $a = 0.65 \text{ fm}$. Contrary to (2) there is no spin factor $(2s + 1)$ in (3). It was dropped because in phonon excitations spin-flip processes are negligible /5/.

3. SMC-cross section

The SMC-emission

$$\frac{dG^{SMC}_{\lambda \beta}(\varepsilon)}{d\varepsilon'} = G^{SMC}_{\lambda}(\varepsilon) \sum_{n=5}^{n} \tau_n \left[ \Gamma^{(+)}_{n \beta}(\varepsilon') \uparrow + \Gamma^{(0)}_{n \beta}(\varepsilon') \uparrow + \Gamma^{(-)}_{n \beta}(\varepsilon') \uparrow \right] / \hbar$$

was calculated beginning at excitation number $n_0 = 5$ up to the equilibrium stage $n = (2gE)^{1/2}$. The mean life-time $\tau_n$ is taken from the time-integrated (up to $t = \infty$ ) master equation. The damping and escape widths (entering (7) and the Pauli master equation) are given both by the Golden Rule

$$\Gamma^{(\Delta n)}_{n \beta}(\varepsilon') \uparrow = 2 \pi \overline{I}^2 S^{(\Delta n)}_{n \beta}(\varepsilon') \uparrow , \quad \Delta n = -2, 2$$

$$\Gamma^{(\Delta n)}_{n \beta}(\varepsilon') \uparrow = 2 \pi (2s+1) \overline{I}^2 S^{(\Delta n)}_{n \beta}(U) P_{\beta}(\varepsilon') \uparrow , \quad \Delta n = -2, 0, 2$$

For the explicit expressions of the state densities $S^{(\Delta n)}_{n \beta}$ and $S^{(\Delta n)}_{n \beta}(U)$ we refer to /1/. $S(\varepsilon')$ is the state density of a particle in the nuclear volume $V_N$, whereas the penetration factor $P_{\beta}(\varepsilon')$ in (8b) is 1 for neutrons and $G_{\pi}(\varepsilon') / G_{\gamma}(\varepsilon')$ for protons. In
using (8b) there is no reference to the detailed balance principle or inverse cross sections in this model. Moreover, since \( \Gamma_n(\varepsilon') \uparrow \) and \( \tau_n^{-1} \) are both proportional to \( \overline{I}^2 \) all matrix elements cancel exactly within the sum of (7). The only mean square matrix element that survives is that in \( G_{\alpha}^{\text{SMC}}(\varepsilon) \). In this way the shape of the SMC-emission cross section becomes independent of \( \overline{I}^2 \).

The normalization constant \( G_{\alpha}^{\text{SMC}}(\varepsilon) \) in (7) will not be calculated from matrix elements but will be obtained in the present model from the condition of flux conservation, i.e.,

\[
G_{\alpha}^{\text{SMC}}(\varepsilon) \equiv \sum_{\beta = \pi, \nu} \int d\varepsilon' \frac{dG_{\alpha \beta}^{\text{SMC}}(\varepsilon)}{d\varepsilon'} = G_{\alpha}(\varepsilon) - \sum_{\beta = \pi, \nu} G_{\alpha \beta}^{\text{SMD}}(\varepsilon). \tag{9}
\]

It should be noticed that the escape mode \( \Gamma_{n\beta}^{(+)}(\varepsilon') \uparrow \) in (7) is energetically impossible and should be neglected in SMC-processes where all particles are in bound configurations. However, it was proved in calculations that even this mode has a small influence on the SMC-emission spectra for \( n \geq 5 \).

4. Results

Calculations of \((n,n')\) and \((n,p)\) emission spectra within the SMD/SMC-model (code EXIFON /6/) were performed for four nuclides at different incidence energies. The input data are summarized in Table 1. For the binding energies we use the linear-shell-term values in /7/. The parameters \( \omega_\lambda \), \( \beta_\lambda \) for the lowest \( 2^+ \) phonon state are taken from /8/, while \( \omega_3 \) is taken from the systematics given in /5/. The \( \beta_3 \)-parameter was approximated by \( (0.007 \omega_3)^{1/2} \). All delta functions in (3) are replaced by Gaussians of width 1 MeV for \( \varepsilon \approx 20 \) MeV, and 2 MeV for \( \varepsilon = 25.7 \) MeV. In using \( g = A/13 \) we ignore shell- and pairing-effects. For \( G_{\alpha}(\varepsilon) \) the reaction cross section of the OM (Wilmore-Hodgson for neutrons; Becchetti-Greenlees for protons) is used in a parametrized form /9/.

Results are depicted in Figs. 1 to 9. The meaning of the curves.
is the same in all figures: SMC-contributions (dotted line), SMD-contribution (dashed-dotted line), the sum of SMC and SMD (full line). Further, the individual single- and two-step contributions (broken lines) are denoted by the appropriate label. For \( \xi = 25.7 \) MeV also the Legendre-coefficients of the double-differential cross section \( (L=1,2) \)

\[
I_l(\xi,\xi') = \left( \frac{d\sigma_{SMD}(\xi)}{d\xi'} \right) \left( \frac{d\sigma_{SM}(\xi)}{d\xi'} \right) \alpha_l(\xi')
\]  

(10)

are shown. Finally, for 93-Nb also \((n,p)\) energy spectra and Legendre-coefficients are depicted for \( \xi = 14 \) MeV in Fig. 9. Experimental data are taken from \( /10 - 14/ \).

The incidence-energy dependence of SMC and SMD processes, as well as the individual SMD-contributions (in mb) are summarized for 93-Nb in Table 2. As we see, the non-collective particle-hole excitations rise with increasing energy whereas the phonon-contribution is almost incidence-energy independent. The direct three-phonon excitation which was also included in the calculations turns out to be negligible.

In summary, the predicted SMD/SMC-model is successful in reproducing experimental data. (At \( \xi = 14 \) MeV the discrepancy in the low energy part is due to the neglect of \((n,2n)\)-processes in our calculations till now.) Despite the fact that it includes besides the SMC-processes also the multistep direct excitation of non-collective and collective modes this model is extraordinarily simple.

References


Table 1 Parameters used for calculations
(explanations given in the text)

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### Table 2

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**Fig. 1.**

**Fig. 2.**
Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.
An Analysis of Some Inconsistencies in the Calculation
of Level Densities with the Fermi Gas Model

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Abstract

The Fermi gas model is widely used in calculating nuclear level densities. With
the use of modern computers, calculations can be made including both pairing and shell
effects. Despite these improvements, there are indications that the model fails to
properly predict the parity ratio even in situations where the level density itself is
correctly described. A detailed examination of the discrepancies for some specific
systems is presented.

Level densities for nuclei regarded as consisting of non-interacting nucleons are
usually made with the Fermi gas model. First introduced by Bethe, it has been
reformulated to include both shell and pairing effects. Comparisons with data and
with exact calculations provide an indication that the Fermi gas model gives a
generally reliable approximation for the total level density. A recent comparison^1 of
the Fermi gas model with exact results, however, does suggest that this model does fail
to describe the parity ratio. An earlier examination^2 of the predictions for this
parameter suggested problems, although at that time the discrepancies were attributed
to a breakdown in the non-interacting Fermion assumption rather than a failure of the Fermi gas formalism to give the correct numerical result within the assumptions of the model.

The parity ratio denotes the relative fraction of the levels at a given energy which are of positive parity. It and a related parameter, the spin cutoff parameter, are necessary in order to decompose the total level density into densities of levels with specific spins and parities. Since most applications and many model comparisons involve level densities for specific J and π values, this apparent failure is significant. At present, no information is available on whether the breakdown observed by Jacquemin and Kataria is more general or is limited to a few nuclear systems.

The present investigation focuses on two systems. The first is a system consisting of 10 Fermions distributed among ten pairs of levels with energies of 2, 4, 6, 8, . . ., 20. A calculation of the exact state density was made using the method of Ref. 1. This was compared to the results of a Fermi gas calculation as shown in Fig. 1. The direct comparison provides a test of two assumptions used in the Fermi gas calculation: the Fermi–Dirac occupation function and the saddle–point approximation. The generally good agreement suggests that both of these assumptions are valid. A comparison of the exact occupation probabilities with those of a Fermi–Dirac occupation function (Fig. 2) provides a separate check on the first assumption; the agreement is rather good.

To obtain a parity ratio for this system, it is necessary to specify the parity of each level. Take the parity of each level to be + except for the two levels at 10 MeV. This yields the interesting result that the parity ratio is only sensitive to one parameter: the occupation probability of the 10 MeV level. This is because a positive parity orbital will not, whether occupied or not, change the parity of the level, while a negative parity orbital will change the parity when filled but not when empty. As a consequence, the parity ratio becomes a function only of the occupation probability of
those orbitals with negative parity. Moreover, the orbitals for which spin projection partners \((+J_z, -J_z)\) are both occupied do not change the parity ratio, since two negative parity orbits occupied yield positive parity. Thus, the parity ratio becomes extremely sensitive to the occupation probability of those negative parity orbitals nearest the Fermi level, since these are most likely to have one but not both of the pair of states occupied. In Fig. 3 we show the parity ratio for this system calculated exactly and that determined from the Fermi gas model; the comparison shows good agreement, although the Fermi gas result tends too rapidly to .5. It also never dips below .5, although the exact result does show small oscillations.

This system, however, is not similar to that studied in Ref. 1 because the latter system showed substantial oscillations in the parity ratio with energy for the exact results, with essentially no structure in the Fermi gas result. To try to see if this behavior could be reproduced, we examined a more realistic system. This was a \(^{24}\text{Mg}\) nucleus treated in a \(d_{5/2}\), \(s_{1/2}\), \(d_{3/2}\), \(f_{7/2}\) basis. Single particle energies were \(-4.15\), \(-3.29\), \(+.93\) and \(+3.55\) MeV, respectively. This system, like the one first studied, has only one orbital of negative parity, but unlike the earlier one, has irregular spacing of the single particle energies. Also, if done in a spherical basis, it has varying degeneracies, with the \(f_{7/2}\) representing four pairs of \((+J_z, -J_z)\) orbits for both protons and neutrons. Fig. 4 shows the comparison of the exact and the Fermi gas level density for this system. Obviously, the detailed structure does not appear in the Fermi gas calculation, but the average behavior is well described. This indicates that, despite the shell structure, the Fermi gas formalism can give reasonable values for the total level density.

In Fig. 5, we show a comparison of the spin cutoff parameter as calculated exactly with that evaluated from the Fermi gas formalism. As was the case for the total level density, the agreement is quite satisfactory. Finally, a comparison between the parity ratio as calculated with the Fermi gas formalism and that calculated exactly
is presented in Fig. 6. Here, the same discrepancies are found as were seen in Ref. 1. The correct parity ratio shows oscillations about .5 which are quite substantial even 20 MeV above the lowest state, while the Fermi gas calculation simply goes smoothly to .5. Moreover, the ratio for a Fermi gas never goes below .5, so it obviously incorrectly estimates the total ratio averaged over energy.

As was discussed previously, this has to be a function only of the occupancy of the \( f_{7/2} \) state. Fig. 7 shows the comparison between the occupancies as exactly calculated and as given by the Fermi gas model. This comparison suggests that fluctuations occur which are not reproduced by the Fermi gas model, but this actually does not explain why the Fermi gas model never predicts a ratio less than .5 at any energy.

To understand this problem, consider a comparison of the parity ratio calculated as a function of the occupancy of the \( f_{7/2} \) orbit. Obviously, for even numbers of particles in the orbit, the parity is positive, while for odd numbers, the parity is negative. Thus, the actual ratio oscillates between 0 and 1 as the occupancy changes. The Fermi gas result gives a continuous function, since the occupancy is an average quantity.

The probability of a negative parity state is simply the sum over the binomial distribution for 1, 3, 5 and 7 particles for a given occupancy \( f \), while the corresponding positive parity sum is over 0, 2, 4, 6, and 8. These will be perfectly balanced if \( f = .5 \), but for all other values (including \( f \) values which correspond to 1, 3, 5, or 7 particles on the average), the ratio will be greater than .5. The skewing in parity ratio can be seen in Fig. 8a. The origin of the discrepancy lies not in small discrepancies in occupancy, but rather the averaging provided by the binomial distribution over particle number.

The same averaging occurs also for the spin cutoff parameter but appears to cause fewer problems. In Fig. 8b we show the result of a comparison of the spin cutoff parameter for various numbers of particles in the \( f_{7/2} \) shell with Fermi gas
calculations. A small but systematic discrepancy also occurs here but is probably small enough not to cause problems. Similar results occur for other degeneracies. The results for a $p_{1/2}$ orbital are shown in Fig. 9. Note that even when only three possible occupancies are included (0, 1 or 2 particles, respectively), the averaging has very significant consequences for both the parity ratio and the spin cutoff parameter.

These problems are completely independent of any difficulties with the state density itself. For the reasons already indicated, the discrepancies are significant in comparing level densities with experiment. The conclusion is that discrepancies in parity ratio predicted by the Fermi gas model are not necessarily an indication of the importance of two-body forces but rather a mathematical limitation of the formalism. Interestingly, the tendency of the Fermi gas model to average over particle number in some sense simulates the two-body interaction. Even for systems in which a two-body force is included, however, it appears that the parity ratio shows more structure than predicted by the Fermi gas model without interactions. For reliable calculations of the parity ratio for non-interacting Fermions, a method like that of Ref. 1 should be used.

REFERENCES

Fig. 1 State density of the system consisting of pairs of orbitals at 2, 4, 6, ..., 20 MeV filled with ten Fermions.
Fig. 2 Occupancies of the orbital for the system described in Fig. 1 as a function of total energy. The orbitals are labeled with the appropriate single particle energy. The +, 0 and Δ marks give the exact values at excitation energies of 2, 6 and 17 MeV, respectively, while the smooth curves give the Fermi–Dirac predictions at the appropriate temperature.
Fig. 3 Parity ratio for the system described in the caption to Fig. 1. The + marks give the Fermi gas predictions, while the smooth line represents the exact values.
Fig. 4 State density for $^{24}\text{Mg}$ as described in the text. Exact values are calculated using the method of Ref. 1, while the values marked statistical mechanical are from the Fermi gas formalism.
Fig. 5  Spin cutoff parameter for $^{24}\text{Mg}$ as described in the text. The smooth line is the prediction of the Fermi gas model, while the + marks denote exact values.

Fig. 6  Parity ratio for $^{24}\text{Mg}$ as described in the text. The smooth line is the Fermi gas prediction, while the + marks are exact values.
Fig. 7  Orbital occupancies for \(^{24}\text{Mg}\) as described in the text. The smooth lines are Fermi gas predictions; the marks are exact values.

Fig. 8  

a) Positive parity fraction as a function of occupancy of the \(f_{7/2}\) shell. Continuous curve is from the Fermi gas; \(\Delta\) marks are exact results.

b) Spin cutoff parameters as function of occupancy of the \(f_{7/2}\) shell. Continuous curves are for the Fermi gas, while the \(\Delta\) marks are exact results.
Fig. 9  

a) Same as Fig. 8a, except for the $p_{1/2}$ orbital.

b) Same as Fig. 8b, except for the $p_{1/2}$ orbital.
INELASTIC SCATTERING OF 14 MeV NEUTRONS
BY IRON NUCLEI.

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ABSTRACT

Double differential neutron emission spectra from iron have been measured by time of flight method with a time resolution about 0.45 ns/m. The direct contribution to the neutron emission spectra have been calculated using DWBA and CC theory. Contributions from statistical and direct reactions are found to reproduce well the experimental data.

A number of the works [1-2] deal with a theoretical description of inelastically-scattered neutron spectra. The availability of experimental data obtained with high time resolution [4,5] was encouraging their more detailed theoretical analysis [3]. Regardless of the structural features of the two references being in an agreement, however, they disagree by the absolute value at certain angles. That is the reason of the authors' rerunning of the earlier experiment with an aim to obtain higher resolution and more detailed analysis of the results.

The time-of-flight spectrometer used for the measurements, the features of their procedure and experimental data processing are described in Ref. [8]. A path length was 7.1 m (90°), a neutron detection threshold ~ 1 MeV, time resolution for all angles was as high as 0.45 ns. An iron sample of natural isotope composition had a hollow cylinder form with an outer diameter 45 mm, inner diameter 35 mm, height 55 mm. The neutron detection efficiency from the threshold up to 6 MeV was determined by comparing the measured spectra of Cf252 spontaneous fission with Maxwell distribution with a temperature 1.42 MeV. In addition the neutron detection efficiency in an energy range 3.5 to 12 MeV was measured by hydrogen scattering. The resulting efficiency was extrapolated up to the energy 14 MeV, where the efficiency value was obtained through the target yield measurement. A cor-
rection for attenuation and multiple scattering of neutrons in the sample was calculated by the Monte-Carlo method according to Ref. [9]. The absolute value of cross-sections was determined by normalization on scattering cross-section on hydrogen, carbon and on Al(n, \alpha) reaction cross-section.

![Graph showing double-differential inelastic scattering cross-sections](image)

**Fig.** Comparison of experimental and calculated double-differential inelastic scattering cross-sections.
- ○ - present work data,
- ● - data of Ref [5],
- - - CC + HF calculation,
- - - - DWBA + HF calculation.
The figure shows double-differential cross-sections of inelastic neutron scattering on iron nuclei as compared with the data from Ref. [5], which demonstrate a good agreement both by the absolute value and by the structure observed in the spectra. The spectra calculated by the present representation authors' averaged by the real experimental resolution are included in this figure as well. The inelastic scattering cross-sections have been represented as a sum of two independent scattering processes - the direct and equilibrium ones. The equilibrium component has been calculated in frame of the Hauser-Feshbach model (H-F) with a code SMTO-80 [10]. The contribution of direct transitions has been calculated in the first order of Born distorted waves approximation (DWBA) with a code BAP-82 [11] and coupled channels approximation (CC) with a code ECIS-79 [12]. The parameters of neutron optical potential for all theoretical models employed have been determined according to Rapeport et al. global systematic [13].

The calculations in frame of the statistical model for neutron, proton and alpha - channels have taken into account all known discrete states of residual nuclei (46 states for $^{56}$Fe, 12 for $^{56}$Mn, 14 for $^{53}$Cr). A nucleus excitation region above the discrete states with the known characteristics ($\sim 5$ MeV) have been described by the level density in the back-shifted fermi gas model with Dilg et al. systematic parameters [14].

The required information on the parameters of dynamical deformation for each direct transition of $^{56}$Fe even-even nucleus was based on the available data on inelastic scattering of protons in $^{56}$Fe [6]. For DWBA calculation (see the figure for the angle $90^\circ$) the direct transitions with an excitation of 27 vibrational single-phonon states of the target-nucleus were adopted. The CC calculation also took into account 10 two-phonon states $\{2_1^+ \otimes 2_1^+\}$, $\{3_1^- \otimes 3_1^-\}$. The imaginary part of the optical potential was chosen so as to yield the best description of experimental data on excitation of strong-collective states $2_1^+$ and $3_1^-$ for $^{56}$Fe nucleus for incident neutron energies from 11 to 26 MeV [15] and for the energy 14 MeV it was $\hbar \omega_{SF} = 6.4$ MeV.

In the figure you can see that the experimental data for all the angles are well reproduced in the calculations for energy ranges corresponding to strong collective states $2_1^+$ and $3_1^-$ with the energies 0.845 MeV and 4.51 MeV. In the excitation energy range from 1 MeV to 2 MeV the disagreement of experimen-
tal data and calculated ones can be due to the isotopic composition of the sample. The disagreement in the excitation energy range above 5 MeV may well be caused by two reasons:

a) a lack of reliable experimental data on the structure of states in the energy range,

b) an employment of global systematics of level density parameters, which can undergo significant modifications when considering specific nuclei.

Hence, the (n,n') reaction analysis demonstrates that at an energy of incident neutrons of 14 MeV on the whole we can have the experimental data to be qualitatively and quantitatively described virtually over the whole nuclear excitation energy range under investigation making use only of the direct and equilibrium mechanisms of the reaction. In this case the coupled-channels method enables the inelastic scattering process to be better described in the low-energy part of the nuclear excitation spectrum. In addition more detailed consideration should be given to the equilibrium part of the spectrum taking into account the collective enhancement of level density.

References