REPORT OF THE SMORN IV PHYSICAL BENCHMARK

BY

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Edited by P. BERNARD (CEA - IRDI/DEDR - CADARACHE FRANCE)
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GENERAL INTRODUCTION TO THE SMORN IV - "PHYSICAL BENCHMARK"

P. BERNARD - C.E.A. IRDI/DEDR CEN/CADARACHE
(Chairman of Smorn 4 - Benchmark session)

At the last SMORN 3 meeting, in JAPAN in October 1981, the results of a benchmark exercise on noise data treatment ("Benchmark test on reactor noise analysis methods"), have been presented.

The principle of this benchmark was to send to the participating laboratories a magnetic tape (analogic or digital) containing the same signals, and to ask them to calculate, with the methods of their choice, the classical expressions used in noise data processing (PSD, CPSD, coherence functions, auto and cross correlation functions...). The results were presented in a standard format, for an easy comparison.

The magnetic tapes were transmitted by OECD/NEA and included recordings coming from:
- Signals from a simulator of BWR (artificial noise), proposed by M. MORISHIMA (JAPAN)
- Signals recorded on the BORSELE PWR, proposed by M. TURKCAN (The NETHERLANDS)
- Signals recorded on the PHENIX LMFBR, proposed by M. GOURDON (FRANCE).

23 groups of contributors (from 9 different countries) had participated in his exercise. The comparison of the results and the synthesis of the exercise had been performed by our Japanese colleagues, who accomplished an important and effective work on this benchmark compilation.

The results (1) had been very fruitful and had shown a global agreement between results of different contributors.

Due to the interest of such an exercise, it was decided to propose a new benchmark concerning physical interpretation of noise signals, called "Physical Benchmark".

The same data recordings have been used and three benchmark exercises have been defined by:
- M. MORISHIMA for artificial BWR noise
- M. TURKCAN for PWR BORSELE noise
- M. LEGUILLOU for LMFBR PHENIX noise.

The main tasks were:
- Estimation of parameters, related to physical models (time constants, gains, impulse response, resonance parameters, reactivity coefficients...)

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- Evaluation of vibrational displacement (frequencies, amplitudes, directions...)
- Identification of noise sources and of their relative contribution to the signal.

13 groups of contributors, from 7 different countries, have participated in this exercise. Comparison of results and synthesis of conclusions have been performed by the rapporteur:

- M. SUDA and M. MORISHIMA for the BWR artificial noise
- M. TURCAN for the PWR noise
- M. LE GUILLOU for the PHENIX noise

and are presented in their reports.

A special meeting took place on Monday, October 15, 1984, during the SMORN 4 meeting. Rapporteurs and contributors participated in this meeting, that included interesting and fruitful discussions. The results have been presented by the rapporteurs during the "Physical Benchmark" session, on Friday, October 19th.

Global conclusions of this physical benchmark are encouraging and satisfying, and show a global agreement of the results of the different contributions (estimation of parameters, evaluation of vibrational displacements, identification of noise sources).

Let us notice that there may exist some underlying difficulties in the treatment of very low-frequency signals, and that an effort at standardization should be beneficial to the use of auto-regressive methods.

The usefulness of a benchmark exercise is quite clear, and it can:
- Allow the contributors to compare their methods, and refine them.
- Contribute to specialized and in-depth international exchanges
- Show that application of noise techniques, performed by different laboratories, and using methods that can be different, leads to identical or similar results and interpretation.
- Hence show the maturity of these methods for reactor monitoring.
- Lead to definition of standards.

In this report are presented:
- The original definition of the benchmark exercise: tasks and related information (information concerning magnetic tapes recordings are presented in ref.)
- The reports of the rapporteurs
- The individual contributions of the participants to the benchmark.

For each of the three exercises:
- BWR artificial noise
- PWR Borselle noise
- LMFBR Phenix noise.

---

(1) SMORN 3 Benchmark test on reactor noise analysis methods
Y. SHINOHARA, J. J. HIROTA
JAERI M. 84-025 - NEACRP-L-257
BWR ARTIFICIAL NOISE
TASKS AND RELATED INFORMATION ON

THE ARTIFICIAL NOISE DATE

(BWR Artificial Noise)
Reactor Noise Analysis Physical Benchmark Test

Tasks and Related Information on the Artificial Noise Data

1. BWR dynamics included in the Artificial noise data

The BWR-noise model used for producing the Artificial noise data[1] was prepared on the basis of the linear theoretical dynamic model of a BWR[2]. Several kinds of noise sources, both white and colored, were introduced according to the following requirements:

(a) to obtain a neutron-noise spectrum similar in shape to typical results of the actually-observed spectra at frequencies below about 1 Hz[3], i.e. the so-called global component of neutron noise;
(b) to give rise to substantial coherence between neutron density and core flow, thus resulting in the introduction of an extraneous noise source bringing about fluctuations in inlet water velocity;
(c) to take into consideration also vessel-pressure dynamics particularly dominant at low frequencies of the order of 0.01 Hz.

The BWR-noise model thus obtained is, therefore, an elementary model of a whole-core dynamics relating to variations of the reactivity dependent on the neutron level itself through other state variables such as core flow and vessel pressure. Figure 1 shows the block diagram of the BWR-dynamics model, and also Table 1 gives the transfer functions actually used, together with the system variables of the BWR-dynamics model.


2. Tasks

For the Artificial noise data, you are requested to analyze dynamic characteristics of the above BWR-noise model in terms of estimation of both neutronics stability and dominant noise sources. Then the following functions and parameters should be reported:

1) Frequency response function \( H(f) \)
2) Impulse response function \( h(t) \)
3) Damping ratio \( \xi \), resonant frequency \( f_0 \) and two other parameters \( A \) and \( \tau \)
4) Cumulative power contribution ratio \( R_1(f) \)
5) Normalized covariance matrix \( \Sigma \) of residuals and variance \( \sigma^2 \) of residuals

Other findings of the analysis are to be reported as optional.
3. Definitions of the functions and parameters

1) Frequency response function $H_{13}(f)$ is a system response for neutron density to a sinusoidal excitation of unit amplitude of inlet water velocity at frequency $f$. The magnitude $|H_{13}(f)|$ and the phase $\angle H_{13}(f)$ are defined as follows:

$$|H_{13}(f)| = \sqrt{(\text{Re}[H_{13}(f)])^2 + (\text{Im}[H_{13}(f)])^2}$$

$$\angle H_{13}(f) = \tan^{-1}\left(\frac{\text{Im}[H_{13}(f)]}{\text{Re}[H_{13}(f)]}\right)$$

where $-\pi < \angle H_{13}(f) < \pi$

Note that, in the present model, there is no feedback path for inlet water velocity in terms of other system variables (see Fig. 1).

2) Impulse response function $h_{13}(t)$ is the inverse Fourier transform of the frequency response function $H_{13}(f)$, that is

$$h_{13}(t) = \int_{-\infty}^{\infty} H_{13}(f) \exp(j2\pi ft) df$$

$$H_{13}(f) = \int_{0}^{\infty} h_{13}(t) \exp(-j2\pi ft) dt$$

where $h_{13}(t) = 0$ for $t < 0$

3) Damping ratio $\zeta$, resonant frequency $f_0$, and two other parameters $\gamma$ and $\alpha$ are a set of parameters that is derived from the frequency response function $H_{13}(f)$ or the impulse response function $h_{13}(t)$ by means of a least-squares fit of the following model of a linear resonant system:

$$-\frac{d^2}{dt^2} X_1 + 2\zeta (2\pi f_0) \frac{d}{dt} X_1 + (2\pi f_0)^2 X_1 = \alpha X_3 + \frac{d}{dt} X_3$$

where $X_1$ = neutron density as an impulse response

$X_3$ = inlet water velocity as a unit-impulse input

4) Cumulative power contribution ratio $R_{1i}(f)$ is required to analyze the spectral contribution of an inherent noise source in each variable to fluctuations of neutron density, as is usual with multivariate auto-regressive analysis. For this purpose, three kinds of variables should be selected among the Artificial noise signals as follows:

Channel No. 1 : $X_1$ = neutron density

2 : $X_2$ = vessel pressure

3 : $X_3$ = inlet water velocity

Thus, the cumulative power contribution ratio $R_{1i}(f)$ (i=1 or 2) is defined as

$$R_{1i}(f) = \frac{\frac{1}{k=1} S_{1k}(f)}{\frac{3}{k=1} S_{1k}(f)}$$

where $S_{1k}(f)$ is the auto-PSD of neutron density due to the noise source of the variable $X_k$ (k=1, 2 or 3).
5) Normalized covariance matrix $\Sigma$ of residuals, which is estimated through multivariate autoregressive analysis, is required to examine the mutual independence of the noise sources inherent to the above three kinds of variables. Then the element $\Sigma_{ij}$ ($i=2$ or $3$ for $j=1$; $i=3$ for $j=2$) of the matrix $\Sigma$ corresponds to the correlation coefficient of residuals between the variables $X_i$ and $X_j$, and also $\sigma_i$ ($i=1, 2, 3$) is the variance of residuals for the variable $X_i$.

4. Format for graphical data presentation

1) The magnitude of the frequency response function should be presented on a logarithmic scale of 3 (vertical) x 3 (horizontal) decades. The scale of one decade should be equal to 5 cm. Unit should be Hz for the horizontal axis. The frequency range should be from $10^{-2}$ Hz to 10 Hz.

2) The phase of the frequency response function and the cumulative power contribution ratio should be presented on linear scale for the vertical axes, while the horizontal axes should be the same as in the case of the magnitude of the frequency response function. 10 cm. should correspond to ($-\pi$ to $+\pi$ radian) for the phase and (0 to 1) for the cumulative power contribution ratio, respectively. Note that both the results on the cumulative power contribution ratio, i.e. $R_{11}(f)$ and $R_{12}(f)$, should be presented on one graph.

3) The impulse response function should be presented on linear scale both for vertical and horizontal axes. 10 cm. should correspond to (0 to 4 sec$^{-1}$) for the vertical and 15 cm. to (0 to 3 sec) for the horizontal axis.

5. Format for numerical data presentation

1) Stability indices and related parameters

\[
\begin{align*}
\text{Damping ratio} & : \zeta = \ldots \\
\text{Resonant frequency} & : f_0 = \ldots \text{(Hz)} \\
\text{Amplitude} & : A = \ldots \text{(sec$^{-2}$)} \\
\text{Time constant} & : \tau = \ldots \text{(sec)}
\end{align*}
\]

2) Normalized covariance matrix of residuals

\[
\Sigma = \begin{pmatrix}
1.0 & * & * \\
* & 1.0 & * \\
* & * & 1.0
\end{pmatrix}
\]

3) Variance of residuals

\[
\begin{align*}
\sigma_1 &= \ldots \text{(volt$^2$)} \\
\sigma_2 &= \ldots \text{(volt$^2$)} \\
\sigma_3 &= \ldots \text{(volt$^2$)}
\end{align*}
\]
Fig. 1 Block diagram of the BWR-dynamics model used for producing the Artificial noise data.
Table 1 Transfer functions and system variables used for the BWR-dynamics model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Noise signal</th>
<th>Channel No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>neutron density</td>
<td>1</td>
</tr>
<tr>
<td>$X_2$</td>
<td>vessel pressure with additive noise</td>
<td>2</td>
</tr>
<tr>
<td>$X_3$</td>
<td>inlet water velocity</td>
<td>3</td>
</tr>
<tr>
<td>$X_4$</td>
<td>location of boiling boundary</td>
<td>4</td>
</tr>
<tr>
<td>$X_5$</td>
<td>heat flux per unit length</td>
<td>5</td>
</tr>
<tr>
<td>$X_6$</td>
<td>inlet water enthalpy</td>
<td>6</td>
</tr>
<tr>
<td>$X_7$</td>
<td>recirculation flow</td>
<td>7</td>
</tr>
<tr>
<td>$X_8$</td>
<td>void volume in core</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>noise source $f_2$</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>noise source $f_{10}$</td>
<td>10</td>
</tr>
</tbody>
</table>

The transfer functions:

<table>
<thead>
<tr>
<th>$G_{fb}(s)$</th>
<th>$k_{fb} [1 + T_{fb}s]^{-1}$</th>
<th>$G_{pl}(s)$</th>
<th>$k_{pl} [1 + T_{pl}s]^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{e1}(s)$</td>
<td>$k_{e1} [1 + T_{e1}s]^{-1}$</td>
<td>$G_{p2}(s)$</td>
<td>$k_{p2} [1 + T_{p2}s]^{-1}$</td>
</tr>
<tr>
<td>$G_{e2}(s)$</td>
<td>$k_{e2} [1 + T_{e2}s]^{-1}$</td>
<td>$G_{p3}(s)$</td>
<td>$k_{p3} [1 + T_{p3}s]^{-1}$</td>
</tr>
<tr>
<td>$G_{e3}(s)$</td>
<td>$k_{e3} [1 + T_{e3}s]^{-1}$</td>
<td>$G_{p4}(s)$</td>
<td>$k_{p4} [1 + T_{p4}s]^{-1}$</td>
</tr>
<tr>
<td>$G_{e4}(s)$</td>
<td>$k_{e4} [1 + T_{e4}s]^{-1}$</td>
<td>$G_{b1}(s)$</td>
<td>$k_{b1} [1 + T_{b1}s]^{-1}$</td>
</tr>
<tr>
<td>$G_{e5}(s)$</td>
<td>$k_{e5} [1 + T_{e5}s]^{-1}$</td>
<td>$G_{b2}(s)$</td>
<td>constant</td>
</tr>
<tr>
<td>$G_{e6}(s)$</td>
<td>$k_{e6} [1 + T_{e6}s]^{-1}$</td>
<td>$G_{b3}(s)$</td>
<td>$k_{b3} \exp(-T_{d}s)$</td>
</tr>
<tr>
<td>$G_{e7}(s)$</td>
<td>$k_{e7} [1 + T_{e7}s]^{-1}$</td>
<td>$G_{b4}(s)$</td>
<td>$k_{b4} [1 + T_{b4}s]^{-1}$</td>
</tr>
<tr>
<td>$G_{e8}(s)$</td>
<td>$k_{e8} [1 + T_{e8}s]^{-1}$</td>
<td>$G_{e9}(s)$</td>
<td>$k_{e9} G_{e2}(s) + k_{e1}s$</td>
</tr>
<tr>
<td>$G_{e10}(s)$</td>
<td>$k_{e10} [1 + T_{e10}s]^{-1}$</td>
<td>$G_{e11}(s)$</td>
<td>$k_{e11} G_{e2}(s)$</td>
</tr>
<tr>
<td>$G_{e12}(s)$</td>
<td>$k_{e12} [1 + T_{e12}s]^{-1}$</td>
<td>$G_{e13}(s)$</td>
<td>$k_{e13} G_{e2}(s)$</td>
</tr>
<tr>
<td>$G_{e14}(s)$</td>
<td>$k_{e14} [1 + T_{e14}s]^{-1}$</td>
<td>$G_{e15}(s)$</td>
<td>$k_{e15} G_{e2}(s)$</td>
</tr>
<tr>
<td>$G_{e16}(s)$</td>
<td>$k_{e16} [1 + T_{e16}s]^{-1}$</td>
<td>$G_{e17}(s)$</td>
<td>$k_{e17} G_{e2}(s)$</td>
</tr>
</tbody>
</table>
Summary of Results
of the Benchmark Test
on the Artificial BWR-like Noise

December 1984

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Summary of Results of the Benchmark Test on the Artificial BWR-like Noise

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Abstract

In conjunction with the Fourth Specialists Meeting on Reactor Noise (SMORN IV), a physical benchmark test on reactor noise analysis was made using the same data as the previous test at SMORN III. As for the artificial noise, each applicant or group of applicants was requested to estimate both neutronics stability and dominant noise sources. In this report, the contributed results are compared one another and furthermore with the calculated ones from the physical model used for the synthesis of the artificial noise. Some remarks on the deviations of the estimated results are given. The description of the physical model of BWR noise is also made in detail.

Keywords

Reactor Noise Analysis, Benchmark Test, SMORN IV, BWR Noise Model, Artificial Noise, Autoregressive Model.
1. Introduction

As is reviewed in Suda (1984), a physical model of the boiling water reactor was derived and used for the simulation of the JPDR-II (Japan Power Demonstration Reactor, Version II) noise phenomena. The model is based upon the works by Miida and Suda (1963, 1964). It was modified for a forced circulation type BWR by Matsubara, Oguma and Kitamura (1978a). Morishima and Mitsutake (1980) simulated the JPDR-II noise spectra, introducing appropriate noise sources into this model. Yamada and others set up this model on a hybrid computer and generated the analog signals which simulated the BWR noise data. The artificial BWR-like noise data, hereafter referred to as the artificial noise, thus synthesized have been used as one of the test data for the computational benchmark tests at SMORN III (Suda 1982; Shinohara and Hirota, 1984), as well as for the physical benchmark tests at SMORN IV.

In the present article, the noise model by Morishima and Mitsutake is described in more detail and then the comparison is made among the contributed results for the physical benchmark test on this type of the noise.

The first part of Chapter 2 summarizes the physical properties of the BWR noise model, the remainder of which treats the spectral representation of the model in terms of the power spectra and the cumulative power contribution ratios. In Chapter 3 the tasks for the benchmark test are described. The comparison of the contributed results is presented in Chapter 4. Some remarks are given in Chapter 5.

2. BWR Noise Model

2-1. Physical Properties
The BWR noise model is constructed on the basis of the theoretical works of Miida and Suda (1963, 1964) recently modified for a forced-circulation-type BWR by Matsumara, Oguma and Kitamura (1978a). In-vessel dynamics for neutronic and thermo-hydraulic processes is described in the frequency domain by lumped approximations. A set of linear input-output relations for several process signals is then used to express dynamic part of the present model in such ways:

(a) Neutron power density is affected by a change in reactivity only; in other words, only its global component is considered.

(b) Any changes in thermo-hydraulic process signals cause reactivity effects: that is, the reactivity-equivalent components of the signals are fully taken into account.

(c) Thus, the causal relations between and among the process signals are described in terms of their whole-core or whole-vessel components.

Three thermo-hydraulic noise sources are introduced into the BWR noise model by reference to the theoretical and experimental studies of BWR global noise presented in recent years. The following are the noise sources and their spectral properties required here:

(a) Random nature of steam void generation causing significant reactivity effects through changes in induced temperature, pressure, density and so on, and then producing a neutron noise spectral peak at about 0.4 Hz (e.g. Nomura, 1975; Fukunishi, 1977).

(b) Inlet flow fluctuation having a power spectrum with a low-pass filter characteristic (Matsumara, Oguma and Kitamura, 1978b), and also giving certain coherence between neutron density and total core flow (Seko and others, 1972; Blomberg and Akerhielm, 1975; Tsuyuki and others, 1976).

(c) Steam flow fluctuation that is assumed to be a noise signal with a
first-order low-pass filter characteristic so as to cause strong fluctuation in neutron power below about 0.1 Hz as observed in some BWRs (Tsuyuki and others, 1976; Fukunishi, 1977; Matouska, Oguma and Kitamura, 1978b).

Some of the parameters that characterize the noise sources are not determinable a priori. Then the comparison of noise spectra between experimental results and model calculation is successfully used, which is shown in Fig.1.

The BWR noise model thus determined was used to synthesize the artificial noise, though it was slightly modified for easiness of computer simulation. The block diagram of the resultant model is shown in Fig.2, in which also shown are the signals $X_i$ ($i=1$ through 8) and $f_i$ ($i=2, 10$) selected for the recording of the artificial noise. The contents of the artificial noise are presented in Table 1. The elementary transfer functions and the model parameter values used here are listed in Table 2. Note that all the transfer functions are dimensionless, because their gain constants are defined with respect to input and output signals having the same electric unit of volt.

Finally, two additional remarks should be made:

(a) In synthesizing the artificial noise, the noise inputs $C_i$ ($i=1$ through 3) were prepared as follows (Yamada and others, 1980). A signal from a noise generator was passed through a low-pass filter with a cutoff frequency $f_c$ of 43 Hz, and recorded on an analog magnetic tape. This procedure was repeated three times. Then the three statistically independent signals were stored and fed into the BWR noise model realized on a hybrid computer. Through a statistical and spectral analysis, these signals were found to show a normal distribution with sample mean of nearly zero and sample variance $\sigma^2$ of $5.3 \times 10^{-4}$ volt$^2$, and to form the power spectrum.
\[ P_E(f) = \begin{cases} \frac{7 \sigma_e^2}{16 f_c} \left( V^2 / Hz \right) & \text{for } f \leq f_c \\ \frac{7 \sigma_e^2 f^6}{16 f_c^8} \left( V^2 / Hz \right) & \text{for } f \geq f_c \end{cases} \] (1)

where \( P_E(f) \) is the two-sided spectrum, i.e. \( \int_{-\infty}^{\infty} P_E(f) df = \sigma_e^2 \).

(b) After the artificial noise having been synthesized, a newly-obtained signal by the same method as \( \epsilon_i \) was added to the vessel pressure signal. This is shown in Fig.2 by the additive noise \( \epsilon_n \), the original signal \( X_2' \) and the resultant \( X_2 \). This was done to decouple the vessel pressure signal from the others at frequencies above several Hz.

2-2. Spectral Representation

A variety of noise descriptors can be derived from the BWR noise model. In particular, this section treats the theoretical calculation of power spectra and cumulative power contribution ratios for the system of three signals, i.e. the neutron density \( X_1 \), the vessel pressure \( X_2 \) and the inlet water velocity \( X_3 \). This system is chosen according to the tasks of the artificial noise for the SMORN IV benchmark test. A set of input-output relations for the system can be obtained in the frequency domain from the block diagram of Fig.2:

\[ X_i(f) = X_i'(f) + \delta_{ij} \epsilon_n \quad \text{for } i=1, 2, 3 \] (2)

and

\[
\begin{bmatrix}
X_1'(f) \\
X_2'(f) \\
X_3'(f)
\end{bmatrix} =
\begin{bmatrix}
0 & A_{12}(f) & A_{13}(f) \\
A_{21}(f) & 0 & A_{23}(f) \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
X_1'(f) \\
X_2'(f) \\
X_3'(f)
\end{bmatrix} +
\begin{bmatrix}
\epsilon_1(f) \\
\epsilon_2(f) \\
\epsilon_3(f)
\end{bmatrix}
\]

(3)

where
Here, $\delta_{ij}$ is the Kronecker delta function, $A_{ij}(f)$ and $B_{ij}(f)$ the transfer functions expressed by using the elementary ones of Table 1, and $F_i(f)$ the driving force inherent to the signal $X_i$.

The two-sided power spectrum between $X_i$ and $X_j$ is then

$$P_{ij}(f) = \sum_{m,n=1}^{3} \left[ (E - A(f))^{-1} \right]_{mn} \left[ (E - A(f)^+)^{-1} \right]_{nj}$$

$$+ \delta_{i2} \delta_{j2} \frac{1}{4}$$

where $E$ is the unit matrix, $A(f)$ the transfer function matrix of which the $(i,j)$ element is $A_{ij}(f)$, $A(f)^+$ the transposed conjugate of $A(f)$, and $D_{mn}(f)$ the $(m,n)$ element of the driving force matrix $D(f)$ given by

$$D(f) = \begin{bmatrix}
|B_{11}(f)|^2 P_c^{(1)}(f) & B_{11}(f)B_{21}(f)P_c^{(1)}(f) & 0 \\
B_{21}(f)B_{11}(f)^*P_c^{(1)}(f) & |B_{21}(f)|^2 P_c^{(1)}(f) + |B_{22}(f)|^2 P_c^{(2)}(f) & 0 \\
0 & 0 & |B_{33}(f)|^2 P_c^{(3)}(f)
\end{bmatrix}$$

Note that all the power spectra $P_c^{(i)}(f)$ for $i = 1$ through 4 have the same shape as $P_c(f)$.

From the power spectra $P_{ij}(f)$ ($i,j = 1$ through 3), it is possible to calculate a measure of the predominance of every noise source. One method of calculation is to use the auto-power spectrum $P_{ii}(f)$ to obtain a ratio of its power contributions from the noise sources $\varepsilon_j$ ($j = 1, 2, 3$). This can be expressed as the cumulative ratio $R_{ij}^{PM}(f)$ of the physical model.
\[ R_{ij}(f) = \frac{\sum_{k=1}^{PM} S_{ik}(f)}{P_{ii}(f)} \quad \text{for } i,j = 1, 2, 3 \quad (7) \]

where

\[ P_{ii}(f) = \sum_{k=1}^{PM} S_{ik}(f) \quad (8) \]

\[ S_{i1}(f) = \sum_{m=1}^{2} \sum_{n=1}^{2} [(E-A(f))^{-1}]_{im} B_{m1}(f) B_{n1}^*(f) [(E-A(f)^+)^{-1}]_{ni} P_{c}^{(1)}(f) \quad (9) \]

\[ S_{i2}(f) = [(E-A(f))^{-1}]_{i2} |B_{22}(f)|^2 [(E-A(f)^+)^{-1}]_{i1} P_{c}^{(2)}(f) + \delta_{i2} P_{c}^{(s)}(f) \quad (10) \]

\[ S_{i3}(f) = [(E-A(f))^{-1}]_{i3} |B_{33}(f)|^2 [(E-A(f)^+)^{-1}]_{i1} P_{c}^{(3)}(f) \quad (11) \]

The expression \( R_{ij}(f) \) is obvious both mathematically and physically, but in general is not measurable experimentally. The preferable approach in this direction is to utilize the power contribution ratios from the driving forces \( F_j(f) \) \((j = 1, 2, 3)\). Especially in the case of essentially or almost independent driving forces, \( D_{mn}(f) \) being negligible for \( m \neq n \), this approach may perhaps become useful for an understanding of dominant noise sources. A similar discussion is given in multivariate autoregression (AR) analysis. The resulting cumulative ratio is then

\[ B_{ij}(f) = \frac{\sum_{k=1}^{AR} S_{ik}(f)}{\sum_{k=1}^{AR} S_{ik}^2(f)} \quad (12) \]

where

\[ S_{ik}(f) = [(E-A(f))^{-1}]_{ik} D_{kk}(f) [(E-A(f)^+)^{-1}]_{ki} + \delta_{ik} \delta_{kk} P_{c}^{(s)}(f) \quad (13) \]
3. Tasks

At the informal meeting on the SMORN III benchmark test, held in Tokyo in the evening of 27th October 1981, it was agreed to proceed to a physical benchmark test using the same data as SMORN III. It was also agreed that the physical benchmark problem for the artificial noise was to be proposed by Dr. Morishima of Kyoto University; two others for the Borssele reactor data and the Phenix reactor data by Dr. Turkcan of ECN-Petten and Dr. Gourdon of CEN-Cadarache, respectively. The tasks and related information proposed to the test were sent to NEA Data Bank in March 1982, and distributed from there to each applicant or group of applicants in October 1983, who had already sent an entry for the SMORN IV benchmark test in France in October 1984.

Before the SMORN III meeting, a large number of the analog data tapes containing the above three types of noise data were distributed to the applicants by JAERI. For the physical benchmark test, these data and also their newly-digitized versions were used. The latter data were prepared by Dr. Turkcan who digitized all the analog test data through the data collection system at ECN, and they were distributed upon request from the applicants by NEA Data Bank.

The contents of the artificial noise are presented in Table 1. The block diagram of the BWR noise model used for synthesizing the artificial noise is shown in Fig.2. These were attached to the task specification, though the three noise sources and the additive noise were not specified.

According to the task specification, the applicants were requested to estimate both neutronics stability and dominant noise sources in terms of the following functions and parameters:

1) Frequency response function $H_{13}(f)$ as a system response for the neu-
tron density $X_1$ to the inlet water velocity $X_3$.

2) Impulse response function $h_{13}(t)$ of being the inverse fourier transform of $H_{13}(f)$.

3) Damping ratio $\xi$, resonant frequency $f_0$ and two other parameters $\tau$ and $\Delta$ that are determined by fitting $H_{13}(f)$ or $h_{13}(t)$ to the model

$$\frac{d^2}{dt^2}X_1 + 2\xi(2\pi f_0) \frac{d}{dt}X_1 + (2\pi f_0)^2 X_1 = \Delta X_3 + \tau \frac{d}{dt}X_3$$

4) Cumulative power contribution ratio $R_{i1}(f)$ obtainable in a multivariate autoregressive analysis on the three signals $X_k$ ($k=1, 2, 3$) as

$$R_{i1}(f) = \frac{\sum_{k=1}^{3} S_{i1k}(f)}{\sum_{k=1}^{3} S_{i1k}(f)}$$

for $i=1$ and $2$

where $S_{i1k}(f)$ is the contribution from the residual noise of the signal $X_k$ to the auto-power spectrum of the neutron density $X_1$.

5) Variances and cross-correlation coefficients of residual noise, $\sigma_i^2$ and $\Sigma_{ij}$, to be estimated through the above autoregressive analysis.

Other findings, if any, were expected to be reported as optional.

The applicants were requested to present their results in the specified format described in the task specification, in order to facilitate detailed comparison among the results. The magnitude and phase of $H_{13}(f)$, $h_{13}(t)$ and $R_{i1}(f)$ were to be shown in a graphical form. All the parameter values were to be given in their respective physical units.

4. Comparison of Results

There were seven teams of contributors to this part of the benchmark tests, as shown in Table 3. Identification symbols, A through G, are ran-
domly assigned to these contributors in the comparison of the results.

The methods of data analysis adopted by the contributors are summarized in Table 4. A few remarks are in order.

(a) The contributor E did not report the details of the method. Moreover, his results differed from all the others drastically. Therefore they were not included in the comparison.

(b) About half of the contributors used direct spectrum estimation by FFT to obtain $H_{13}(f)$. The other half utilized autoregressive (AR) model fitting.

(c) In order to answer the tasks 4) and 5), multivariate autoregressive (MAR) analysis should be adopted. The contributor F was not prepared for the MAR analysis and hence he skipped these two tasks.

The comparison of the magnitude $|H_{13}(f)|$ and the phase, $\angle H_{13}(f)$, are shown in Figs. 3 and 4, respectively. It is observed that the results agree fairly well. The comparison of the impulse response, $h_{13}(t)$, is shown in Fig. 5. Again the results agreed fairly well.

Since this is an artificial noise, there are "true" solutions for $H_{13}(f)$ and $h_{13}(t)$, directly obtained from the model. They are shown in Figs. 6 and 7. It is noted that the results from noise analysis are quite close to the "true" solutions.

The answers to the task 3) are compared in Table 5. The estimated $\zeta$ and $f_0$, except for those by the contributor E, fall in reasonable ranges. The estimations of $A$ and $T$, however, are scattered in much wider range of values. One possible explanation to this difference is as follows. The damping ratio $\zeta$ and the natural frequency $f_0$ are estimated from $H_{13}(f)$ in the frequency range where all the results agree quite well. On the other hand in order to obtain $A$, one has to extrapolate $H_{13}(f)$ to zero fre-
frequency, or $h_{13}(t)$ to time zero, and hence the deviation in $H_{13}(f)$ or $h_{13}(t)$ may be amplified.

Comparison of the power contribution ratio is made in Fig. 8. Although the general tendencies in the results are the same, there are more deviations compared to the frequency and the impulse response functions. The "true" solution obtained from the model is shown in Fig. 9. Although they are not included in the tasks, the power contribution ratios to the vessel pressure are also shown.

Somewhat scattered results are obtained for the normalized covariances and the variances of the residuals as shown in Table 6.

5. Remarks

There is some deviation among the contributed results of the frequency response in the low frequency region, as is shown in Figs. 3 and 4. The deviations of the power contribution ratio are also pronounced in the low frequency region as in Fig. 8. Such deviations are probably due to the different frequency resolution determined by the data length, the window, the AR model order and so forth.

The frequency resolution of the AR analysis may be estimated by $\Delta f = 1/2 hM$, where $M$ is the order of the AR model and $h$ is the sampling period and thus $hM$ is the maximum lag associated with the model. The values of $\Delta f$ are summarized in Table 7, together with the Nyquist frequency, $f_N$. The $\Delta f$ and the $f_N$ define the frequency range where the particular analysis seems to be valid.

In Fig. 10, the contributions of the power contribution ratio, shown in Fig. 8, are reproduced only for the frequency range between $\Delta f$ and $f_N$. It
is noted that the results now agree fairly well. Moreover, they are close
to the "true" ones shown in Fig.9. Thus the deviation in Fig.8 seems to
be caused by extending the results beyond the limits of the validity of
the analyses.

The deviations in the frequency response will be explained in a sim-
ilar way.

There is some deviation in the phase characteristics of $H_{13}(f)$ in the
high frequency region. As is shown in Table 7 the Nyquist frequency in
some cases are 5 to 10 Hz. Hence the high frequency portion in these cases
may not be very accurate and deviate from the others.

The "true" values of the parameters were obtained by fitting the sec-
ond-order model to the $H_{13}(f)$ in Fig.6. Several trials were made with dif-
ferent frequency range of fitting and different weighting factors. A typ-
ical result is:

$$
\zeta = 0.387 \\
\omega_0 = 0.487 \\
A = 1.77 \text{s}^{-2} \\
\tau = 1.12 \text{s}
$$

and the response functions with these parameters are shown with broken
lines in Figs. 6 and 7. On comparison with the contributed results, it
is noted that the "true" damping ratio $\zeta$ is smaller than the contributed
ones, while the resonant frequency $\omega_0$ is similar. Actually the "true"
impulse response in Fig.7 is slightly less damped compared to those in
Fig.5, estimated by noise analysis. The cause of this difference is
still to be investigated.
6. Conclusion

The results of the benchmark test on the artificial BWR-like noise data are summarized. It is noted that most of the contributed results agreed fairly well with one another and also with the "true" solution by the model calculation. Obviously, it is important to pay attention to the validity of the results, limited by the frequency resolution and the Nyquist frequency.
References


Yamada, S., and others (1980). Report on the synthesis of the artificial noise using the JAERI hybrid computer, presented at the technical committee for SMORN III.
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<th>Channel No.</th>
<th>Noise signal</th>
<th>Symbol</th>
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<td>neutron density</td>
<td>$X_1$</td>
</tr>
<tr>
<td>2</td>
<td>vessel pressure with additive noise</td>
<td>$X_2$</td>
</tr>
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<td>3</td>
<td>inlet water velocity</td>
<td>$X_3$</td>
</tr>
<tr>
<td>4</td>
<td>location of boiling boundary</td>
<td>$X_4$</td>
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<td>5</td>
<td>heat flux per unit length</td>
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<td>inlet water enthalpy</td>
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<td>recirculation flow</td>
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<td>void volume in core</td>
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<td>9</td>
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<tr>
<td>10</td>
<td>noise source: steam flow fluctuation</td>
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Table 2 Elementary transfer functions of the BWR noise model

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<th>Function</th>
<th>Transfer Function</th>
<th>Constants</th>
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</thead>
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<tr>
<td>( G_r(s) )</td>
<td>( k_r1 s^{-1} + k_r2 (1+T_r2 s)^{-1} )</td>
<td>( k_r1 = 7.70E-2 \text{ s}^{-1}, k_r2 = 1.00 )</td>
</tr>
<tr>
<td>( G_f(s) )</td>
<td>( k_f1 (1+T_f1 s)^{-1} + k_f2 (1+T_f2 s)^{-1} )</td>
<td>( T_f1 = 7.76E-3 \text{ s}, k_f1 = 8.89E-1 )</td>
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<td>( G_{fb}(s) )</td>
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<td>( T_{fb} = 5.95 \text{ s}, k_{fb} = 1.01E-1 )</td>
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<tr>
<td>( G_v(s) )</td>
<td>( \text{constant } k_v )</td>
<td>( T_v = 1.07 \text{ s}, k_v = 7.70 )</td>
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<tr>
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<td>( k_{k1} = -9.66E-1 )</td>
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<tr>
<td>( G_{k2}(s) )</td>
<td>( \text{constant } k_{k2} )</td>
<td>( k_{k2} = -5.26E-1 )</td>
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<td>( G_{v1}(s) )</td>
<td>( k_{v1} (1+T_{v1} s)^{-1} )</td>
<td>( k_{v1} = 4.23, T_{v1} = T_{v4} = 1.92E-1 \text{ s} )</td>
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<td>( G_{v2}(s) )</td>
<td>( k_{v2} (1+T_{v2} s)^{-1} + k_{v'} )</td>
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<td>( G_{v3}(s) )</td>
<td>( k_{v3} (1+T_{v3} s)^{-1} )</td>
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<td>( G_{w1}(s) )</td>
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<td>( G_{b3}(s) )</td>
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<td>( \lambda_1 = 7.21E-2 \text{ s}^{-1} )</td>
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<tr>
<td>( G_{e3}(s) )</td>
<td>( k_{e3} G_{e2}(s) )</td>
<td>( \lambda_2 = 1.95E-1 \text{ s}^{-1} )</td>
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<tr>
<td>( G_{e4}(s) )</td>
<td>( k_{e4} [(s+\lambda_1)^{-1} - (s+\lambda_2)^{-1}] )</td>
<td>( k_{e4} = 8.02E-3 )</td>
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* These three functions were not used in synthesizing the artificial noise.
### Table 3  List of Contributors

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<tr>
<th>Name</th>
<th>Institution/Location</th>
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<tr>
<td>Federico, A. and R. Ragona</td>
<td>ENEA, Italy</td>
</tr>
<tr>
<td>Kanamoto, S. and S. Shimizu</td>
<td>NAIC, Japan</td>
</tr>
<tr>
<td>Kleiss, E. B.</td>
<td>IRI, The Netherland</td>
</tr>
<tr>
<td>Luo, Z. P. and S. M. Wu</td>
<td>U. of Wisconsin, USA</td>
</tr>
<tr>
<td>Morishima, N.</td>
<td>Kyoto U., Japan</td>
</tr>
<tr>
<td>v. d. Veer, J.</td>
<td>KEMA, The Netherland</td>
</tr>
<tr>
<td>Yamada, S., N. Yamamoto</td>
<td>Osaka U. (*Gifu U.), Japan</td>
</tr>
<tr>
<td>and K. Kishida*</td>
<td></td>
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<td>I.D. Symbol</td>
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AR(1,M) and MAR(1,M): I-Dimensional Autoregressive (AR) Model of Order M.

FT: Fourier Transform.

$f_s$: Sampling Rate in Hz, defined by $1/h$ where $h$ is the sampling period in sec.

$f_c$: Low-pass-filter Cut-off Frequency in Hz.

N: Sample Size (in points).
Table 5 Estimated Parameters

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<th>$A$ (sec$^{-2}$)</th>
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### Table 7: Frequency Resolution and Nyquist Frequency

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<thead>
<tr>
<th>I.D.</th>
<th>Order M</th>
<th>Sampling Period $h$ (s)</th>
<th>$\Delta f = \frac{1}{2hM}$ (Hz)</th>
<th>$f_N = \frac{1}{2h}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>0.05</td>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>80</td>
<td>1/64</td>
<td>0.4</td>
<td>32</td>
</tr>
<tr>
<td>C</td>
<td>100</td>
<td>0.1</td>
<td>0.05</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>8</td>
<td>0.1</td>
<td>0.625</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td>200</td>
<td>0.01</td>
<td>0.25</td>
<td>50</td>
</tr>
</tbody>
</table>
Fig. 1. Neutron noise spectra and ordinary coherence functions between neutron density and total core inlet flow. Model calculations are compared with the measurements in JPRD-II (Matsubara, Oguma and Kitamura, 1978b) and TEPCO-1 (Tsuyuki and others, 1976).
Fig. 2. Block diagram of the BWR noise model and the signals \( X_i \) (\( i = 1 \) through 8) and \( f_i \) (\( i = 2 \) and 10) selected for the artificial noise.
Fig. 3. The magnitude of the frequency response, $H_\text{f}_s(f)$
Fig. 4. The phase of the frequency response, $H_i(f)$
Fig. 5. The impulse response, $h_{13}(t)$
Fig. 6. Frequency response function $H_{13}(f)$ of the neutron density to the inlet water velocity, calculated from the BWR noise model.
Fig. 7. Impulse response function \( h_{13}(t) \) of the neutron density to the inlet water velocity, calculated from the BWR noise model.
Fig. 8. The cumulative power contribution ratio
Fig. 9. Cumulative power contribution ratios $R_{ij}^{AR}(f)$ to the neutron density ($i=1$: top) and the vessel pressure ($i=2$: bottom). For reference, $R_{ij}^{PM}(f)$ ($i=1,2$) are also shown.
Fig. 10. The cumulative power contribution ratio, redrawn only for the frequency region of the validity of analysis.
INDIVIDUAL CONTRIBUTIONS TO THE BWR ARTIFICIAL NOISE BENCHMARK
ARTIFICIAL NOISE BWR BENCHMARK TEST RESULTS

by Federico, Ragona ENEA, ITALY

DATA ELABORATION NOTE:

Analog processing. The signals $x_1$ (neutron density), $x_2$ (vessel pressure) and $x_3$ (inlet water velocity) were amplified by a factor of 10 when reproduced from the analog tape version of the noise data. Overall reproduction misalignment were taken into account by an A/D conversion of the whole calibration signals sequence.

The misalignments (DC offsets + gain inaccuracy) was found:

<table>
<thead>
<tr>
<th>SIGNAL</th>
<th>Offset</th>
<th>EQ. GAIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-66.7 mV</td>
<td>1.145</td>
</tr>
<tr>
<td>2</td>
<td>-86.0 mV</td>
<td>1.145</td>
</tr>
<tr>
<td>3</td>
<td>-66.0 mV</td>
<td>1.115</td>
</tr>
</tbody>
</table>

For correction of these errors a proper preprocessing of the data was done on the computer.

A/D Conversion. The sampling rate was fixed to 100 Hz for a large band analysis. After that the data were filtered with a low pass digital elliptic filter (6 poles, cutoff frequency 9.00 Hz, stop-band frequency 9.9 Hz, stop-band attenuation -40 dB, band gain 0 dB) and then undersampled with a ratio 1:5, so reaching the 20 Hz sampling rate.

Anti-aliasing analog filters (cutoff frequency 35 Hz, 120 dB/decade) were used during the A/D conversion.

Frequency response function $H_{13}$. $H_{13}(f)$ was estimated by overlapped periodograms method (also called the Welch method) with the following specifications:

- $N = 304$ periodograms
- $M = 512$ points/periodogram
- 50% overlapping
- D.C. component removed via software before FFT-transforming.

The function $H_{13}(f)$ is reported in fig. 1. Also reported (in fig. 2) is another estimate of $H_{13}$ obtained by a SISO modelling (order 10) with a least-square pro-
procedure (there is no feedback between \( x_1 \) and \( x_3 \)) on \( M = 1024 \) samples of \( x_1 \) and \( x_3 \).

**Impulse response function** \( h_{13} \). The function \( h_{13}(t) \) was obtained by Fourier-anti-transforming the \( H_{13}(f) \). A cosine tapering was performed on \( H_{13} \) in order to remove the high-frequency component. The function \( h_{13}(t) \) is reported in fig. 3.

**Dynamic parameters of the second-order equivalent model.** A least-square fitting was conducted on \( H_{13}(f) \), driven by the minimization of

\[
\sum_i \left| H_{13}(f_i) - A \frac{2\pi f_i \cdot \tau + 1}{(2\pi f_i)^2 + 2 \pi (2\pi f_i) \cdot (2\pi f_i) + (2\pi f_i)^2} \right|^2
\]

The results were as follows:

\[
\begin{align*}
\gamma^2 &= 0.5212 \\
\omega_0 &= 0.4867 \text{ (Hz)} \\
A &= 7.5404 \text{ (sec}^{-2}) \\
\tau &= 1.1982 \text{ (sec)}.
\end{align*}
\]

**Cumulative power contribution ratio.** A MAR modelling was performed on \( x_1, x_2 \), and \( x_3 \) with the method suggested by Jones R.H., "Multivariate Autoregression Estimation Using Residuals", Applied Time Series Analysis, Academic Press, 1978, with \( M = 20480 \) samples and order 10. Fig. 4 represents the estimate for \( R_{11}(f) \) and \( R_{12}(f) \) (contribution coming from \( \Sigma_{21} \) term was intentionally removed). The normalized covariance matrix of residuals resulted as follows:

\[
\Sigma = \begin{bmatrix} 1.0 & * & * \\ * & 1.0 & * \\ * & * & 1.0 \end{bmatrix}
\]

Variance of residuals:

\[
\begin{align*}
\sigma_1^2 &= 0.2212 \times 10^{-2} \text{ (volt}^2) \\
\sigma_2^2 &= 0.1316 \times 10^{-4} \text{ (volt}^2) \\
\sigma_3^2 &= 0.2516 \times 10^{-2} \text{ (volt}^2).
\end{align*}
\]

In order to reduce the off-diagonal term \( \Sigma_{21} \), an attempt was made with a MAR modelling (order 10, \( M = 20480 \) samples) with a sampling rate of 50 Hz. Results are better:
\[
\Sigma = \begin{bmatrix}
1.0 & * & * \\
-0.1541 & 1.0 & * \\
-0.0097 & 0.0350 & 1.0
\end{bmatrix}
\]

\[\varpi_x^2 = 0.0073 \times 10^{-4} \text{ (volt}^2)\]
\[\varpi_y^2 = 0.8255 \times 10^{-6} \text{ (volt}^2)\]
\[\varpi_z^2 = 0.3652 \times 10^{-4} \text{ (volt}^2)\]

but the resolution in the band 0-1 Hz is poor.
Fig. 1.a - Magnitude of the frequency response $H_{13}(f)$. 
Fig. 1.b  Phase of the frequency response $H_{13}(f)$. 
Fig. 2 - Magnitude of the frequency response $H_{13}(f)$ (via SISO modelling).
Fig. 3 - Impulse response $h_{13}(t)$.
Fig. 4 - Cumulative power contribution ratio.
SMORN-IV
(Dijon, 15 - 19, October 1984)

Physical Benchmark Test
Artificial Noise
(Data Tape: S111Q)

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Results

Task(1): Frequency Response Function $H_{1}(f)$:

The result is shown in Fig. 1 which is obtained from the 8 Hz sampling spectra of ch.1 and ch.3 given by NEA.

Task(2): Impulse Responce Function $h_{1}(t)$:

The result is shown in Fig. 2 which is obtained from $H_{1}(f)$.

Task(3): Stability Indices and Related Parameters:

\[ \xi = 0.475, \]
\[ f_s = 0.508, \]
\[ A = 2.38, \]
\[ \tau = 0.838. \]

Note: These parameters were obtained by the least squares fitting of the gain curve of $H_{1}(f)$ to the proposed 2nd order model. The gain and phase of the fitted 2nd order model were plotted by broken lines in Fig.3., and the impulse response function of this model was also drown by a broken line in Fig.4.

Task(4): Cumulative Power Contribution Ratio $R_{1}(f)$:

The result is shown in Fig. 5.

Note: The optimal order of MAR model became 3 according to the minimum AIC for the covariance sequence of 64 Hz sampling data. However, as seen in Fig. 8, the PSD predicted by this MAR model of order 3 does not fit well to the one by FFT in the lower frequency range. Hence, even though there exists no appropriate criterion to determine the model order for which the MAR model is
considered to fit well to the PSD by FFT, we adopted the MAR model of order 80 for evaluation of the cumulative power contribution ratio. For comparison, the cumulative power contribution ratio of the MAR model of order 3 and the PSD of the MAR model of order 80 are shown in Figs. 6 and 7, respectively.

Task(5): Normalized Covariance Matrix $\Sigma$ of residuals and their variance $\sigma$:

$$
\Sigma = \begin{bmatrix}
1.0 & * & * \\
-0.228 & 1.0 & * \\
-0.140 & 0.0230 & 1.0 \\
\end{bmatrix}
$$

for MAR model of order 80

with

$\sigma_1 = 0.00250$ (volt$^2$)

$\sigma_2 = 0.0000440$ (volt$^2$)

$\sigma_3 = 0.00130$ (volt$^2$).

Note: These values scarcely change for the validation of the order of the MAR model so long as the order is greater than the one determined by the AIC. Compare the values above with those for the MAR model of order 3:

$$
\Sigma = \begin{bmatrix}
1.0 & * & * \\
-0.214 & 1.0 & * \\
-0.140 & 0.0228 & 1.0 \\
\end{bmatrix}
$$

for MAR model of order 3

with

$\sigma_1 = 0.00254$ (volt$^2$)

$\sigma_2 = 0.0000463$ (volt$^2$)

$\sigma_3 = 0.00131$ (volt$^2$).
Fig. 1. Frequency Response Function $H_{13}(f)$
Fig. 2. Impulse Response Function $h_{13}(t)$
Fig. 3. Frequency Response Function of the 2nd Order Model (broken line)
Fig. 4. Impulse Response Function of the 2nd order Model (broken line)
Fig. 5. Cumulative Power Contribution Ratio of the MAR(3) Model

Fig. 6. Cumulative Power Contribution Ratio of the MAR(3) Model
Fig. 7. PSD of the neutron noise predicted by the MAR(80) model

Fig. 8. PSD of the neutron noise predicted by the MAR(3) model
Spectral Analysis of the Artificial Noise Data
for the SMORN-IV Physical Benchmark Test
on Reactor Noise Analysis

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(May 31, 1984)

1. Objective and Summary

The objective of the present report is twofold: One is to present
test results of spectral analysis of the artificial BWN-like noise data
according to the task specification, together with some description of
analyzing conditions and estimation process. The other is to study some
of the problems that arised in the SMORN-III benchmark test\(^{(1)}\) and must be
considered carefully in practical application of noise analysis methods.
The problems which will be discussed in some detail are concerned with the
following aspects:

(a) Estimation of covariance functions for the noise signals of contain-
ting very low-frequency components significantly,

(b) Quantitative comparison of different spectral estimates obtained
by two methods of noise analysis, FFT method and AR model fitting,

(c) Conjecture of a major spectral property of noise signals having
a poor signal-to-noise ratio.
The test data used hereupon are the analog ones which were sent from JAERI in March 1981. The identification number of the magnetic tape is No. 2. After the test data have been digitized by ADC, their amplitudes are calibrated using the DC signals of +0.5 and +1.0 volt.

The digitalized noise data are first analyzed statistically by means of two kinds of tests: the run test on sample mean and sample variance to examine stationarity of the data; and the Chi-square test for the degree of fittness to a normal distribution. The statistical tests on a number of samples are performed on each of six different sample sizes and also of two different sampling periods. The test results thus obtained are utilized in full for the following spectral analysis, i.e. for example, in the case of choosing both a sample size N and a sampling period h suitable for estimation of a covariance function.

Two methods of noise analysis are used: One is the AR model fitting to analyze physical aspects of the artificial noise in terms of neutronic stability and causal relation among noise signals. The other is the FFT method to obtain a variety of noise descriptors such as power spectral densities, coherence functions, transfer functions and impulse response functions. Every spectral estimate is characterized by a frequency resolution \( \Delta f \) defined as \( \Delta f = 1/2hM \) for AR model fitting of order M and \( \Delta f = (8/3)\sqrt{\pi}/hN \) for Hanning window smoothing repeated n times. Owing to this spectral characterization, the spectral estimates with the same frequency resolution are compared quantitatively with respect to their absolute values. It shows markable agreement between the estimates of the two noise analysis methods; this enables us to attach sufficient significance to the results of AR model fitting.

A major property of spectral estimates is checked carefully by a com-
parative study of those with several different frequency resolutions. It is shown that such a comparative approach, hereafter referred to as window opening/closing approach, serves as one of practical means to spectral analysis of noise signals having a poor signal-to-noise ratio and/or a slight causative relationship.

The present study of the AR spectral estimation gives the following results: (a) The Akaike's information criterion (AIC) gives an optimum or adequate value of order $M$ to an estimate in the time domain. (b) A spectral estimate in the frequency domain is characterized reasonably well by the frequency resolution, $\Delta f = 1/2hM$, that is useful in comparing quantitatively with the corresponding estimate from other noise analysis method.

2. Preparation of Digital Noise Data

The artificial noise data which are used in the present analysis are the analog ones that were sent from JAERI in March 1981. The identification number of the magnetic tape is A-2. The analog data are digitized and recorded on a digital magnetic tape. The sampling conditions are as follows.

Data recorder used for playing back: Honeywell Model 101(01)
Playback speed: $7\frac{1}{2}$ ips (1 $\frac{7}{8}$ ips in real time)
Anti-aliasing filter: an analog low-pass filter of Butterworth type having a 48 db/Oct. rolloff
: a cutoff frequency of 80 Hz (20 Hz in real time)
Noise signals digitized: neutron density in Channel 1
vessel pressure in Channel 2
inlet water velocity in Channel 3
Sampling period: 0.005 sec (0.02 sec in real time)
A/D conversion: 1211 bits for quantization
  - simultaneous sampling mode
  - stored as an integer of 2 bytes
Digitized data: total 520 blocks (104 minutes)
  - blocksize = 3600 bytes (1800 points/3 channels, 12 s)

A calibration signal is simultaneously digitized and recorded. Amplitude of the noise data is calibrated by the use of the DC signals of +0.5 and +1.0 volt. The formula used for the calibration is

\[(\text{calibrated signal}) = a \times (\text{original signal}) + b \text{ (volt)},\]

where the values of a and b are determined from the recalibration of the DC signals. Note that no corrections for a time scale are made because of no significant deviation of the estimated frequencies of the AC signals from the normal values of 10 and 100 Hz.

3. Statistical Properties of Noise data

The noise data are examined statistically by means of both the run test of sample mean and variance for a stationary property and the Chi-square test on the degree of fitness to a normal distribution\(^{(2)}\). Part of the test results is shown in Figure 1.

Two major conclusions are drawn:

(a) Vessel-pressure noise in Channel 2 is dominated by low-frequency components with a period of about 200 sec, i.e. around about 0.005 Hz. This means that, in order to estimate a covariance function for vessel pressure,
sample length of about 200 sec or more is required.

(b) The degree of fitness to a normal distribution is improved markedly with an increase in sampling period. When the normal property of a time series sample is desirable, sampling period of 0.08 sec or more is required, at least for noise signals of neutron density and inlet water velocity.

In the present AR model fitting, a sample of 204.8 sec in length and 0.1 sec in sampling period is adopted to calculate a sample covariance function. A covariance function matrix used for the AR analysis is obtained as an average of thirty sample covariance functions. Figures 2 through 7 show a set of the covariance functions, both auto- and cross-, for the three noise signals, in which $C_{ij}(\tau)$ and $\xi_{ij}(\tau)$ are the average for a covariance function and the RMS deviation from $C_{ij}(\tau)$ respectively. These functions are calculated using the formulas

\[ C_{ij}(\tau) = \frac{1}{30} \sum_{k=1}^{30} C^{(k)}_{ij}(\tau) \quad (2) \]

\[ \xi_{ij}(\tau) = \left\{ \frac{1}{30} \sum_{k=1}^{30} C^{(k)}_{ij}(\tau)^2 - C_{ij}(\tau)^2 \right\}^{0.5} \quad (3) \]

where $C^{(k)}_{ij}(\tau)$ is the covariance function of the $k$-th sample of size $N$ for two noise signals $X^{(k)}_i(n)$ and $X^{(k)}_j(n)$ in Channels $i$ and $j$, defined by

\[ C^{(k)}_{ij}(\tau=m) = \frac{1}{N} \sum_{n=1}^{N-m} X^{(k)}_i(n) \cdot X^{(k)}_j(n+m) \quad (4) \]
4. Spectral Analysis by AR model fitting

4-1. Analyzing Conditions

Two kinds of noise analysis methods are adopted here. One is the AR model fitting to analyze the noise data according to the task specification. The other is the FFT method to obtain various spectral estimates as a reference to the results of the former method. By the combined use of these two methods, the noise data are analyzed both spectrally and physically. Three kinds of noise analysis are performed in view of the following objectives:

Analysis A: to understand major spectral characteristics of the noise data by the AR model fitting; to estimate causative relations among the noise signals in Channels 1, 2 and 3 in terms of a cumulative power contribution ratio

Analysis B: to determine a transfer function and an impulse response function of neutron density to inlet water velocity

Analysis C: to determine the noise descriptors following the FFT method, such as power spectral densities, coherence functions, transfer functions and impulse response functions, in order to check the corresponding results of the AR analysis.

The analyzing conditions of these are summarized in Table 1. The frequency resolution $\Delta f$ is evaluated using the formulas

$$\Delta f = \frac{8}{3}f_T \sqrt{n} \quad \text{for Hanning window smoothing,} \quad (5)$$

$$\Delta f = \frac{1}{2} \frac{1}{n} \quad \text{for AR model fitting,} \quad (6)$$

where $f_T$: the fundamental frequency given by $1/(\text{sample length})$
n : the repetition time of Hanning spectral window \( n \geq 1 \)

h : the sampling period

M : the model order of AR analysis.

The formula (5) is obtained from the band width of Hanning window smoothing that has been repeated \( n \) times\(^{(3)} \). For reference, the derivation of Eq.(5) is presented in Appendix 1. The frequency resolution of Eq.(6) is often used in the case of ordinary spectral analysis of a covariance function having \( M \) lag points equally spaced by \( h \). Such a discrete covariance function is also utilized in the AR model fitting, though its amplitude is defined in a different way with ordinary one. Hence the present AR model fitting uses the formula (6) as a measure of frequency resolution that is determined from the model order \( M \).

4-2. Spectral Estimates

In the AR model fitting through the analyses A and B, four or five different values of order \( M \) are chosen in view of the frequency resolution. The lowest orders of 4 and 6 are the values that are determined by the Akaike's information criterion (AIC). The spectral estimates for these model orders are compared each other and checked with those of the FFT method. So that it is ascertained that, in spectral comparison of the same frequency resolution, the results of the analysis B are in good agreement with those of the analysis C. The same is said of the results of the analysis C in a relatively high frequency range, that is, above a frequency equal to approximately the frequency resolution. As a typical example of the spectral comparison, the auto PSDs of neutron density by the analyses A through C are shown in Figs. 8 through 10 respectively, in which, for example,
5. Test Results

5-1 Neutronics Stability Estimation

According to the task specification, neutronics stability is estimated in terms of a response function of neutron density to inlet water velocity. The AR analysis B gives the following results:

1) Frequency response function $H_{13}(f)$, both its magnitude and phase,
is shown in Fig. 11.

2) Impulse response function $h_{13}(t)$ is shown in Fig. 12.

3) Damping ratio $\zeta$, resonant frequency $f_0$, and two other parameters $A$ and $\tau$ are estimated as

$$\zeta = 0.51 \quad \text{(decay ratio = 0.024)}$$
$$f_0 = 0.58 \quad \text{(Hz)}$$
$$A = 4.7 \quad \text{(sec}^{-2})$$
$$\tau = 0.54 \quad \text{(sec)}$$

The estimation process and some comments are summarized:

(a) The reason for taking the largest order of 200 to estimate $H_{13}(f)$ is the best similarity to those obtained from the FFT method. Figures 13 through 16 show how $H_{13}(f)$ from the AR analysis varies in shape according to model order $M$, and how $H_{13}(f)$ from the FFT method is kept firmly in its major shape under Hanning window smoothing. A spectral comparison of the same frequency resolution leads to the conclusion that a fairly large model order should be choosen: that is, at least $M=100$ or 200 in view of $\Delta f=0.05$ or 0.025 Hz respectively. This is also supported from the coherence between neutron density and inlet water velocity, which is shown in Fig. 17. It is seen that the coherence is highly significant in a certain frequency range around about 0.3 Hz. This means that, in order to analyze such a spectral relation reasonably well, a frequency resolution much smaller than the peak width of the coherence is needed.

(b) It is noted that the estimate of $H_{13}(f)$ has much the same shape with the one of a closed-loop transfer function from inlet water velocity to neutron density. A typical example of the closed-loop transfer function is shown in Fig. 18. This suggests that there is little or no feedback path for inlet water velocity in terms of neutron density and vessel pressure.
(c) Figures 19 and 20 show a set of the estimates of \( h_{13}(t) \) from the AR model fitting and the FFT method, respectively. The function \( h_{13}(t) \) is found to have a certain property of strongly damped oscillations which is almost independent of model order as well as Hanning window smoothing. Furthermore, any significant difference of \( h_{13}(t) \) between the two methods is not observed.

(d) The values of the parameters are determined by fitting the model of a linear resonant system to the estimate of \( h_{13}(t) \). The model used here is

\[
\begin{align*}
    h_{M}(t) &= \frac{A}{\omega_0} e^{-\frac{\omega_0}{1-\zeta^2} t} \cos(\frac{\omega_0}{1-\zeta^2} t) \\
          &\quad + A(1-H) \left( \frac{1}{\omega_0} \frac{1-\zeta^2}{1-\zeta^2} \right) e^{-\frac{\omega_0}{1-\zeta^2} t} \sin(\frac{\omega_0}{1-\zeta^2} t),
\end{align*}
\]

where \( \omega_0 = 2\pi f_0 \).

From careful observation of \( h_{13}(t) \), it is found that only the first term on the right-hand side of Eq. (7) is large enough for fitting. Then the conditions of fitting are selected as

\[ h_{13}(t) = h_{M}(t) \quad \text{at} \quad t = 0.1, 0.5 \text{ and } 0.9 \text{ sec.} \]

In Appendix 3, the impulse function \( h_{M}(t) \) and its Fourier transform as a frequency response function \( H_{M}(f) \) are shown graphically.

5-2. Estimation of Dominant Noise Sources

Dominant noise sources of neutron density noise are estimated through the AR model fitting of the analysis A:

4) Cumulative power contribution ratio \( R_{1i}(f) \) for \( i=1,2 \) and 3 is shown in Fig. 21.
5) Normalized covariance matrix $\Sigma$ and variance $\sigma_i^2$ of residual noise have the following values for each of the four different model orders:

<table>
<thead>
<tr>
<th>Model order</th>
<th>6</th>
<th>20</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma(2,1)$</td>
<td>-3.79E-1</td>
<td>-3.80E-1</td>
<td>-3.74E-1</td>
<td>-3.66E-1</td>
</tr>
<tr>
<td>$\Sigma(3,1)$</td>
<td>1.20E-1</td>
<td>1.21E-1</td>
<td>1.21E-1</td>
<td>1.22E-1</td>
</tr>
<tr>
<td>$\Sigma(3,2)$</td>
<td>-1.86E-2</td>
<td>-1.86E-2</td>
<td>-1.84E-2</td>
<td>-1.81E-2</td>
</tr>
<tr>
<td>$\sigma_1^2$ (volt$^2$)</td>
<td>1.12E-2</td>
<td>1.09E-2</td>
<td>1.08E-2</td>
<td>1.08E-2</td>
</tr>
<tr>
<td>$\sigma_2^2$ (volt$^2$)</td>
<td>1.57E-4</td>
<td>1.57E-4</td>
<td>1.57E-4</td>
<td>1.58E-4</td>
</tr>
<tr>
<td>$\sigma_3^2$ (volt$^2$)</td>
<td>1.80E-2</td>
<td>1.80E-2</td>
<td>1.80E-2</td>
<td>1.79E-2</td>
</tr>
</tbody>
</table>

Note that $\Sigma(i,j)$ means the normalized cross-covariance of residual noise between the two noise signals in Channels i and j. For reference, Figure 22 shows comparatively a set of the ratios $R_{ii}(\omega)$ for the four different values of model order. It is seen that a major property of $R_{ii}(\omega)$ remains unaltered, though some fine structures appear with improving the frequency resolution.

Some comments are given:

(a) The cumulative power contribution ratios to vessel pressure and inlet water velocity are also obtained, which are shown in Figs. 23 and 24, respectively. It follows from Figs. 21, 23 and 24 that (1) the inlet water velocity noise influences the other two noise signals, but not vice versa; (2) there is a certain degree of causal relation between two noise signals in neutron density and vessel pressure. The former is consistent with the close similarity of the open- and closed-loop transfer functions from inlet water velocity to neutron density, as shown in Figs. 11 and 18. The latter
reflects in the normalized cross-covariance $\Sigma(2,1)$.

(b) By the use of the variances of both process noise and residual noise, it is possible to measure a signal-to-noise ratio

$$\frac{(S/N)_i = \frac{(V_i - \sigma_i^2)}{\sigma_i^2}}$$

where $V_i$ is the variance of process noise in Channel $i$. The AR analysis yields

$$\frac{(S/N)_1 = 3.56}{\text{where } V_1 = 5.11E-7 \text{ (volt}^2\text{)}}$$

$$\frac{(S/N)_2 = 3.58E+2}{\text{where } V_2 = 6.06E-2 \text{ (volt}^2\text{)}}$$

$$\frac{(S/N)_3 = 2.12}{\text{where } V_3 = 5.62E-2 \text{ (volt}^2\text{)}}$$

Note that the values of $V_i$ for $M=6$ are used. This S/N ratio may be useful in characterizing a departure of a reactor dynamics behavior from normal, as in the case of surveillance analysis using the ratio $\frac{\sigma_i^2}{V_i}$ on HFIR neutron noise data (4).

(c) Finally a word must be said about the auto- and cross-covariances of residual noise which exhibit a property of being almost independent of the model order $M$. Such a property is also found for estimates of the auto-regressive coefficients of the order $M$ greater than 6. These findings, together with the above-mentioned comparative study of spectral estimates, suggest that the Akaike's information criterion (AIC) gives an optimum value of model order $M$ to the estimates in the time domain rather than in the frequency domain.
References:


Appendix 1: Derivation of the frequency resolution, $\Delta f = (8/3)f_T [N]$

Expression (5) for a frequency resolution is obtained as an approximate formula to the band width of Hanning window. Consider that a Hanning window smoothing has been repeated n times ($n \geq 1$). Let $a_k^{(n)}$ (k=-n, -n+1, ..., 0, ..., n-1, n) denote a set of weighting factors operated, where the normalization and the symmetry conditions are

$$\sum_{k=-n}^{n} a_k^{(n)} = 1.0 \quad \text{and} \quad a_{-k}^{(n)} = a_k^{(n)}.$$

Note that, for $n=1$, the weighting factors are

$$a_0 = 0.50 \quad \text{and} \quad a_1 = a_{-1} = 0.25.$$

For the (n+1)-th smoothing, the weighting factors $a_k^{(n+1)}$ to be operated are obtained from $a_k^{(n)}$ as
\[ a_k^{(n+1)} = a_{-k}^{(n+1)} = 0.25a_{k-1}^{(n)} + 0.50a_k^{(n)} + 0.25a_{k+1}^{(n)} \quad \text{for } k=0,1,\ldots,n-1, \]
\[ a_n^{(n+1)} = a_{-n}^{(n+1)} = 0.25a_{n-1}^{(n)} + 0.50a_n^{(n)}, \]
\[ a_{n+1}^{(n+1)} = a_{-n-1}^{(n+1)} = 0.25a_n^{(n)}. \]

The normalization and symmetry conditions are also satisfied. Thus a set of the values of weighting factors is determined sequentially from the previous smoothing.

The band width \( b \) is generally defined as
\[ b = 1/\int_{-\infty}^{\infty} W(f)^2 df, \]
where \( W(f) \) is a spectral window function per unit frequency with the properties of
\[ \int_{-\infty}^{\infty} W(f) df = 1.0 \quad \text{and} \quad W(f) = W(-f). \]

For the Hanning window repeated \( n \) times, the band width \( b^{(n)} \) is
\[ b^{(n)} = 1.0/\left( \sum_{k=-n}^{n} \left( a_k^{(n)}/f_T \right)^2 \right)^{1/2} f_T, \]
where \( f_T \) is a fundamental frequency giving a frequency width between two adjacent weighting factors. The band width \( b^{(n)} \) is calculated numerically for repetition time \( n \) ranging from 1 up to 100, and compared graphically in Fig. A-1 with \( \Delta f = (8/3)f_T^{1/2} \). It is seen that this simple formula for \( \Delta f \) gives practically an exact value.
Appendix 2: Spectral estimates by the AR model fitting and the FFT method

Spectral estimates obtained by the AR model fitting and the FFT method are shown as follows.

Figure A-2: Auto-PSD for inlet water velocity by the AR analysis B
Figure A-3: Auto-PSD for inlet water velocity by the FFT method
Figure A-4: Magnitude of cross-PSD between neutron density and inlet water velocity by the AR analysis A
Figure A-5: Magnitude of cross-PSD between neutron density and inlet water velocity by the AR analysis B
Figure A-6: Magnitude of cross-PSD between neutron density and inlet water velocity by the FFT method (analysis C)
Figure A-7: Phase of cross-PSD between neutron density and inlet water velocity by the AR analysis B
Figure A-8: Phase of cross-PSD between neutron density and inlet water velocity by the FFT method (analysis C)

Appendix 3: Impulse and frequency response functions, $h_M(t)$ and $H_M(f)$

The impulse response function $h_M(t)$ of Eq. (7) and its Fourier transform as a frequency response function $H_M(f)$ are computed numerically for the cases:

- case A: $\varsigma = 0.61, \ f_0 = 0.58, \ A = 4.7, \ \tau = 0.54$
- case B: $\varsigma = 0.56, \ f_0 = 0.58, \ A = 4.7, \ \tau = 0.54$
- case C: $\varsigma = 0.51, \ f_0 = 0.58, \ A = 4.7, \ \tau = 0.54$
- case D: $\varsigma = 0.46, \ f_0 = 0.58, \ A = 4.7, \ \tau = 0.54$
- case E: $\varsigma = 0.41, \ f_0 = 0.58, \ A = 4.7, \ \tau = 0.54$

The results of the computation are shown in Figs. A-8 and A-9.
Table 1. The analyzing conditions of the analyses A through C

<table>
<thead>
<tr>
<th></th>
<th>Analysis A</th>
<th>Analysis B</th>
<th>Analysis C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise analysis method</td>
<td>AR</td>
<td>AR</td>
<td>FFT</td>
</tr>
<tr>
<td>Channels analyzed</td>
<td>1,2,3</td>
<td>1,3</td>
<td>1,2,3</td>
</tr>
<tr>
<td>Sampling period (s)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Sample size (points/in sec)</td>
<td>2048/204.8</td>
<td>2048/204.8</td>
<td>2048/204.8</td>
</tr>
<tr>
<td>Number of samples</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Linear trend removal</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Data window</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>AR model fitting (model order/frequency resolution in Hz)</td>
<td>6/8.33E-1</td>
<td>4/8.00E-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20/2.50E-1</td>
<td>20/2.50E-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50/1.00E-1</td>
<td>50/1.00E-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100/5.00E-2</td>
<td>100/5.00E-2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>200/2.50E-2</td>
</tr>
<tr>
<td>Hannnig window (repetition time/frequency resolution in Hz)</td>
<td></td>
<td></td>
<td>0/4.88E-3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1/1.30E-2</td>
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<tr>
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<td></td>
<td></td>
<td>4/2.60E-2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>16/5.21E-2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>64/1.04E-1</td>
</tr>
</tbody>
</table>

1) In order to create a new time series with a sampling period of 0.1 sec, the digitalized data through ADC are resampled using FFT as an anti-aliasing low-pass-filter.

2) Each of the samples is normalized to zero mean and unit variance, thus resulting in the normalized PSDs.
Fig. 1 Variances of the noise signals in Channel 1 through 3, and ratios of the degree of Chi-square fitness to normal distribution to that of 5% significance level. The variances and Chi-square values are calculated as an average of their sample estimates. The number of samples is denoted in the parentheses.
Fig. 2 Auto-covariance function for neutron density in Channel 1.

Fig. 3 Cross-covariance function between neutron density in Channel 1 and vessel pressure in Channel 2.
Fig. 4 Cross-covariance function between neutron density in Channel 1 and inlet water velocity in Channel 3.

Fig. 5 Auto-covariance function for vessel pressure in Channel 2.
Fig. 6 Cross-covariance function between vessel pressure in Channel 2 and inlet water velocity in Channel 3.

Fig. 7 Auto-covariance function for inlet water velocity in Channel 3.
Fig. 8  Auto-PSDs for neutron density in Channel 1 by the FFT method (analysis C).
Fig. 9 Auto-PSDs for neutron density in Channel 1 by the AR model fitting (analysis B).
Fig. 10 Auto-PSDs for neutron density in Channel 1 by the AR model fitting (analysis A).
Fig. 11 Frequency response function $H_{13}(f)$ of neutron density in Ch. 1 (volt) to inlet water velocity in Ch. 3 (volt) by the AR model fitting of order 200 through the analysis B.
Fig. 12 Impulse response function $h_{13}(t)$ of neutron density in Ch. 1(volt) to inlet water velocity in Ch. 3(volt) by the AR model fitting of order 200 through the analysis B.
Fig. 13 Magnitude of the frequency response function of neutron density to inlet water velocity by the AR model fitting (analysis B).
Fig. 14 Magnitude of the frequency response function of neutron density to inlet water velocity by the FFT method (analysis C).
Fig. 15 Phase of the frequency response function of neutron density to inlet water velocity by the AR model fitting (analysis B).
Fig. 16 Phase of the frequency response function of neutron density to inlet water velocity by the FFT method (analysis C).
Fig. 17 Coherence function between neutron density and inlet water velocity by the AR model fitting of order 100 through the analysis B.

Coherence = \[ \frac{|p_{13}(f)|}{\sqrt{p_{11}(f) \cdot p_{33}(f)}} \]

Fig. 18 Closed-loop transfer function of neutron density to inlet water velocity by the AR model fitting of order 200 through the analysis B.
Fig. 19 Impulse response function $h_{13}(t)$ of neutron density to inlet water velocity by the AR model fitting through the analysis B.
Fig. 20 Impulse response function $h_{13}(t)$ of neutron density to inlet water velocity by the FFT method through the analysis C.
Fig. 21  Cumulative power contribution ratio \( R_{1i}(f) \) (\( i=1,2,3 \)) of neutron density in Channel 1 by the AR model fitting of order 100 through the analysis A.
Fig. 22 Cumulative power contribution ratio $R_{1i}(f)$ (i=1,2,3) of neutron density in Channel 1 by the AR model fitting through the analysis A.
Fig. 23 Cumulative power contribution ratio $R_{2i}(f)$ ($i=1,2,3$) of vessel pressure in Channel 2 by the AR model fitting of order 100 through the analysis A.

Fig. 24 Cumulative power contribution ratio $R_{3i}(f)$ ($i=1,2,3$) of inlet water velocity in Channel 3 by the AR model fitting of order 100 through the analysis A.
Band width $\Delta f$ of Hanning window when spectral smoothing has been repeated $n$ times.

Approximate

$$\frac{\Delta f}{f_T} = \frac{8}{3} \sqrt{n}$$

Exact

Fig. A-1 Frequency resolution of Hanning window smoothing repeated $n$ times.
Fig. A-2 Auto-PSD for inlet water velocity in Channel 3 by the AR method fitting through the analysis B.
Fig. A-3 Auto-PSD for inlet water velocity in Channel 3 by the FFT method through the analysis C.
Fig. A-4 Magnitude of the cross-PSD between neutron density and inlet water velocity by the AR model fitting through the analysis A.
Fig. A-5 Magnitude of the cross-PSD between neutron density and inlet water velocity by the AR model fitting through the analysis B.
Fig. A-6 Magnitude of the cross-PSD between neutron density and inlet water velocity by the FFT method (the analysis C).
Fig. A-7 Phase of the cross-PSD between neutron density and inlet water velocity by the AR model fitting through the analysis B.
Fig. A-8 Frequency response function $H_M(f)$ for the five different cases of model parameters.
Fig. A-9 Impulse response function $h_M(t)$ for the five different cases of model parameters.
ANALYSIS RESULTS FOR REACTOR NOISE PHYSICAL BENCHMARK TEST

--- RESULTS ON THE ARTIFICIAL NOISE DATA ANALYSIS ---

May 16, 1984

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1. Analysis results for the artificial noise data

(1) Frequency response function $H_{13}(f)$
   Fig. 1

(2) Impulse response function $h_{13}(t)$
   Fig. 2

(3) Stability indices and related parameters
   Damping ratio: $\zeta = 0.546$
   Resonant frequency: $f_a = 0.481 \text{ (Hz)}$
   Amplitude: $A = 1.39 \text{ (sec}^{-1}\text{)}$
   Time constant: $\tau = 1.57 \text{ (sec)}$

(4) Cumulative power contribution ratio
   Fig. 3

(5) Normalized covariance matrix of residuals
   $$\Sigma = \begin{pmatrix}
   1.0 & * & * \\
   -0.431 & 1.0 & * \\
   0.114 & -0.032 & 1.0
   \end{pmatrix}$$

(6) Variance of residuals
   $\sigma_1 = 0.0153 \text{ (volt}^2\text{)}$
   $\sigma_2 = 0.000126 \text{ (volt}^2\text{)}$
   $\sigma_3 = 0.0222 \text{ (volt}^2\text{)}$

As for the problems (4) ~ (6), we adopted the multivariate AR model for neutron flux, dome pressure and core inlet flow. The analyzed data points were 10240 points with 0.1 sec sampling interval. The AR model order was chosen to be $M = 8$. On the other hand, we used the following 2ch ARX model to evaluate problems (1) ~ (3):

$$x_1(t) = \sum_{m=1}^{M} a_{11}(m) x_1(t-m) + \sum_{m=0}^{M} a_{13}(m) x_3(t-m)$$
As for the definition of the frequency response function (FRF) $H_{13}(f)$, we think that there are two different definitions which are shown below:

\[ (a) \quad H_{13} = G_1 \]

\[ (b) \quad H_{13} = \frac{G_1}{1 - G_2 G_3} \]

In the 3ch AR modeling, the identified open-loop FRF can be assumed to correspond to (a), since $G_1 \circ G_3$ are separately identified. On the other hand, it would be also possible to identify the closed-loop FRF (b). We think that natural definition for $H_{13}$ would be the closed-loop FRF (b). In order to evaluate the FRF (b), it is not always necessary to use dome pressure signal. Hence, we adopted the 2ch ARX model in the above analysis.
Fig. 1 Frequency Response Function by 2ch AR Model with Zero Regression Term

- --- OBSERVED T.F
- --- FITTED T.F
- X FITTING REGION

\[ \zeta = 0.546 \]
\[ f_0 = 0.481 \]
\[ A = 1.39 \]
\[ \tau = 1.57 \]
Fig. 3 Noise Power Contribution Ratio by 3ch AR Model
Fig. 2 Impulse Response Function by 2ch AR Model with Zero Regression Term
Reactor Noise Analysis Physical Benchmark Test

on the Artificial Noise Data

by

Z. P. Luo * and S. M. Wu **

1. Stability indices and related parameters (for $H_3(t)$)

Damping ratio

\[ \zeta = 0.152 \pm 0.004 \]

Resonant frequency (Hz)

\[ f_0 = 0.430 \pm 0.006 \]

Amplitude (sec$^{-2}$)

\[ A = 0.594 \pm 0.057 \]

Time constant (sec)

\[ \tau = 1.277 \pm 0.062 \]

2. Normalized covariance matrix of residuals

\[
\Sigma = \begin{bmatrix}
1.0 & -0.32 & -0.62 \\
-0.32 & 1.0 & -0.13 \\
-0.62 & -0.13 & 1.0
\end{bmatrix}
\]

* Z. P. Luo is a visiting scholar at the Department of Mechanical Engineering, University of Wisconsin, Madison, U.S.A. He comes from the Engineering-Physics Department, Tsinghua University, Beijing, China.

** S. M. Wu is with the Department of Mechanical Engineering, University of Wisconsin, Madison, WI 53706, U.S.A.
3. Variance of residuals

For \( f < 0.25 \text{ Hz} \)

\[ \sigma_1 = 5.95 \pm 0.63 \quad (10^5 \text{ volt}^2) \]
\[ \sigma_2 = 1.49 \pm 0.03 \quad (10^5 \text{ volt}^2) \]
\[ \sigma_3 = 6.05 \pm 0.11 \quad (10^5 \text{ volt}^2) \]

For \( f < 2 \text{ Hz} \)

\[ \sigma_1 = 4.30 \pm 0.20 \quad (10^5 \text{ volt}^2) \]
\[ \sigma_2 = 0.137 \pm 0.001 \quad (10^5 \text{ volt}^2) \]
\[ \sigma_3 = 4.65 \pm 0.04 \quad (10^5 \text{ volt}^2) \]

For \( f < 32 \text{ Hz} \)

\[ \sigma_1 = 0.655 \pm 0.012 \quad (10^5 \text{ volt}^2) \]
\[ \sigma_2 = 0.012 \pm 0.001 \quad (10^5 \text{ volt}^2) \]
\[ \sigma_3 = 0.337 \pm 0.008 \quad (10^5 \text{ volt}^2) \]

The noise-source of pressure is mainly present in the frequency range below 0.25 Hz.

4. Confidence interval

All of confidence interval listed above based on the sample-size \( N=2300 \).
Magnitude of the transfer function
Dear Dr. Suda,

Enclosed you will find some results of the analysis of the BWR Noise Benchmark Test.

Figure 1 (a and b) gives the magnitude ("GAIN X1/X3") and the phase ("PHASE X1-X3") of the frequency response function $H_{13}(f)$. These results were obtained from direct analysis of the recorded signals by a hardware FFT spectrum analyser.

Figure 2 (a and b) gives the functions of figure 1 filtered by a suitable filter to improve the measuring accuracy by decrease of the frequency resolution.

Figure 3 represents the overall coherence function between X1 and X3.

Figure 4 shows the impulse response function $h_{13}(t)$ as a result of the Fourier Transformation of $H_{13}(f)$.

The frequency response function $H_{13}(f)$ given in figure 2 (a and b) was fitted to the model

$$H_{13}(s) = \frac{A(1 + zs)}{(2\pi f_0)^2 + 2\zeta(2\pi f_0)s + s^2}$$

which is the Laplace formulation of the proposed linear resonant system.

2/
The fitted parameters were found to be as follows:

- Damping ratio \( \zeta = 0.59 \)
- Resonant frequency \( f_0 = 0.46 \text{ Hz} \)
- Amplitude \( A = 1.29 \)
- Time constant \( \tau = 1.78 \text{ sec} \)

Unfortunately our measuring equipment is not able to measure simultaneously the auto- and cross correlation functions which are needed to resolve the problems stated in questions 4 and 5. Furthermore, the necessary computer programs have not been completed as yet.

I hope the stated results will be sufficient to participate in the Bench Mark Test results.

Yours sincerely,

J. v.d. Veer

enclosure
- results
Fig. 1a

Amplitude $H_{13}$
Phase $H_{13}$

Fig. 1b

KEMA-NO SMORN-3 BMT
Fig. 2a
Amplitude $H_{13}$ (filtered)
Fig. 2b
Phase $H_{13}$ (filtered)
Fig. 4
Impulse Response Function $h$
IRI-contribution to the SMORN-IV reactor noise benchmark test: analysis of artificial BWR noise.

E.B.J. Kleiss

May 28, 1984

INTERUNIVERSITAIR REACTOR INSTITUUT
Mekelweg 15
Delft
1. Introduction.
In the framework of the OECD-NEA SMORN-IV conference, to be held in
Dijon, France in October 1984, the second phase of a reactor noise benchmark test is organised (see also 1). The purpose of the benchmark is the analysis of reactor noise signals (distributed on analog magnetic tape to the participants) and the subsequent interpretation and fitting of some physical reactor parameters and other quantities.

This benchmark can be considered as a good test for data processing methods and signal analysing programs used by the different contributors. This report gives the results of the BM-simulator noise analysis by the IRI Reactor Physics group, as performed with its Autoregressive (AR) modelling program package and associated programs.

The purpose of the present phase of the benchmark was:
1. simultaneous analysis of neutron flux, reactor pressure and feedwater flow noise.
2. determination of the signal contribution ratio's to the neutron noise signal.
3. determination of some AR noise source parameters.
4. determination of the transfer function between feedwater flow and neutron flux and the associated impulse response function.

The results are given in the next sections.

2. Signal analysis conditions.

Used signals: track 1, 2 and 3 of the distributed tape (neutron flux, reactor pressure and feedwater flow).

Signal conditioning: low-pass filtering at 10.0Hz, 8th order Butterworth (in some cases filtering at 2.5Hz).
10.5-bits effective ADC accuracy

Determination of spectra and correlation functions: FFT using 4096 samples/record, 142 records, 10ms sampling interval, spectral resolution 0.0122 Hz. Hanning data windowing used for spectrum calculations, correction to flat window for correlation functions.

AR-analysis: 3-dimensional AR model, presented results with model order 200 unless noted otherwise.
3. Results of Autoregression (AR) parameters.

3.1. Noise covariance properties.

It is known that the properties of the identified noise sources (correlation coefficients, variance) depend strongly on the applied sampling rate and filter frequency \((2)\). Model order is of minor influence. Table 1 shows the results for the correlation coefficients \(R_{ij}\) of the intrinsic noise source of signal \(x_i\) and \(x_j\), as derived from the normalised noise covariance matrix for different analysing conditions:

<table>
<thead>
<tr>
<th>Anal. low-pass sampling model</th>
<th>sampling rate</th>
<th>filter frequency</th>
<th>interval sampling</th>
<th>order</th>
<th>(R_{12})</th>
<th>(R_{13})</th>
<th>(R_{23})</th>
</tr>
</thead>
<tbody>
<tr>
<td>An1</td>
<td>10.0</td>
<td>100.0</td>
<td>10.0</td>
<td>200</td>
<td>-0.035</td>
<td>0.015</td>
<td>0.004</td>
</tr>
<tr>
<td>An2</td>
<td>10.0</td>
<td>25.0</td>
<td>40.0</td>
<td>50</td>
<td>-0.340</td>
<td>0.034</td>
<td>-0.007</td>
</tr>
<tr>
<td>An3</td>
<td>2.5</td>
<td>50.0</td>
<td>20.0</td>
<td>100</td>
<td>-0.026</td>
<td>0.027</td>
<td>-0.001</td>
</tr>
<tr>
<td>An4</td>
<td>2.5</td>
<td>25.0</td>
<td>40.0</td>
<td>50</td>
<td>-0.060</td>
<td>0.027</td>
<td>-0.005</td>
</tr>
<tr>
<td>An5</td>
<td>2.5</td>
<td>12.5</td>
<td>80.0</td>
<td>25</td>
<td>-0.195</td>
<td>0.100</td>
<td>-0.017</td>
</tr>
<tr>
<td>An6</td>
<td>2.5</td>
<td>12.5</td>
<td>80.0</td>
<td>50</td>
<td>-0.193</td>
<td>0.100</td>
<td>-0.017</td>
</tr>
</tbody>
</table>

Table 1. Noise source correlation coefficients for varying analysis conditions.

The results shown in the rest of this report are based on analysis \(An1\), except for the impulse response functions in Fig.8 and Fig.9 (\(An2\) and \(An4\), resp.).

The noise source variances obtained from \(An1\) are:

\[
\sigma_{i}^{2} = 1.151 \times 10^{-7} \text{ Volt}^2
\]

\[
\sigma_{j}^{2} = 2.265 \times 10^{-8} \text{ Volt}^2
\]

\[
\sigma_{k}^{2} = 1.367 \times 10^{-7} \text{ Volt}^2
\]
3.2. Cumulative Noise Contribution Ratio's

The noise contribution ratio \( NCR_{ij} \) is the relative contribution of the intrinsic noise source of signal \( x_j \) to the total power in signal \( x_i \) as function of frequency. The NCR's were obtained from the AR-model based on analysis An1. The results are presented in figure 3 as the cumulative NCR's (CNCR), defined as

\[
CNCR_{ij} = \sum_{k=1}^{i} NCR_{jk}
\]

Note that \( CNCR_{ij} \) should equal unity, if the identification is correct. Due to the non-perfect diagonalisation of the identified noise source (see Sect.2.), small deviations up to 3% exist. Furthermore, it can be seen that for signal \( x_3 \) only \( NCR_{33} \) contributes to the signal, as could be expected from the simulator model.

3.3. Spectra

Although not required for the benchmark, also the spectra, phase and coherence functions of the three signals are given in Figs.10a-c and 11a-c.

4. Response functions.

4.1. Frequency response functions.

The request for the benchmark is the determination of the transfer function \( H_{ij} \) from signal \( x_i \) to signal \( x_j \). However, different types of transfer functions can be defined. Using a model structure as given in Fig.2, we define:

- the **direct open-loop** transfer function \( H_{ij} \) (the direct signal path of \( x_i \) to \( x_j \), no effects via \( x_k \), no feedback from \( x_i \) to \( x_3 \))

- the **complete open-loop** transfer function \( F_{ij} \) (also the signal flow via \( x_k \) is taken into account). It appears that

\[
F_{ij} = \frac{H_{ij}+H_{ik}H_{kj}}{1-H_{ij}H_{ki}}
\]

\( F_{ij} \) would be obtained if only a two-signal AR analysis of \( x_i \) and \( x_j \) was performed.

- the **closed-loop** transfer function \( G_{ij} \) (also the feedback from \( x_i \) to \( x_j \) is taken into account).

Presented are the results for \( H_{ij} \) and \( F_{ij} \), computed from the fitted AR model using An1. Results are given in Fig.3a,b and 4a,b. Note that only in the low-frequency range the results differ, probably due to the
4.2. Impulse response functions.

The impulse response functions $h_{13}$ and $f_{13}$ can be defined as the Fourier transforms of the frequency response functions $H_{13}$ and $F_{13}$, resp. Because of the small contribution of $x_3$ to $x_1$ at frequencies above 2 Hz (see NCR13), the accuracy of the estimated transfer functions is small at higher frequencies. In order to obtain nice results, only the low-frequency part of the transfer function was used for the Fourier transformation (low-pass filtering). Figures 5 and 6 give the result $h_{13}$ obtained from $H_{13}$ using low-pass filter frequencies of 5.0 and 10.0 Hz. The inaccuracy in $H_{31}$ at higher frequencies causes some 'fluctuations' in the estimate $h_{13}$ which are stronger in Fig.5 than in Fig.6.

Besides the beforementioned method, the impulse response can also be obtained from the time evaluation of signal $x_1$ when using an impulse function for $x_3$ as input to the AR series. The results for different model orders and sampling frequencies are given in Fig.7 and 8 from $A_1$ and $A_2$. Also here the inaccuracies in the high-frequency (short time step) behaviour are apparent as fairly large fluctuations. Fig.9 gives the same results from $A_4$, with different filtering and modelling conditions (see table 1.).

4.3. Numerical results

The benchmark requests the determination of some parameters when fitting a second-order model to the obtained transfer function $H_{13}$. The differential equation of the model is:

$$\frac{d}{dt} A x_1 + \frac{d}{dt} (2 \tau f_0) x_1 + (2 \tau f_0)^2 x_1 = A x_3 + A \tau \frac{d}{dt} x_3$$

The parameters were obtained by a least-squares fit of this model to the transfer function $H_{13}$ in the frequency range 0 Hz to 3.0 Hz:

$$\begin{align*}
A &= 1.007 \\
\tau &= 2.130 \text{ sec} \\
A \tau &= 2.14 \text{ sec} \\
f_0 &= 0.431 \text{ Hz} \\
\frac{\tau}{f_0} &= 0.526
\end{align*}$$

We noticed that variation of fitting conditions (starting values, convergence, frequency interval) mainly affected the parameters $A$ and $\tau$ but not the product $A \tau$ (which is given independently) and $f_0$ and $\frac{\tau}{f_0}$. Estimates for the precision of the fitted parameters were not available.
References.
Figure 1a. Cumulative noise contribution ratios for signal $x_1$.

- □ CNCR$_{11}$
- ○ CNCR$_{17}$
- △ CNCR$_{13}$
SMORN-IV NOISE BENCHMARK
IRI DELFT THE NETHERLANDS

Figure lb. Cumulative noise contribution ratios for signal $x_2$

- □ CNCR$_{21}$
- ◦ CNCR$_{22}$
- △ CNCR$_{23}$
Figure 1.c Cumulative noise contribution ratio for signal $x_3$

- CNCR$_{31}$
- CNCR$_{32}$
- CNCR$_{33}$

Cumulative NCR to signal 3 at $F = 200$, $TSA=0.01$, $NS\tilde{F}_P=1$
fig. 2.
Figure 3a. Modulus of the direct open loop transfer function $H_{13}$
Figure 3b. Phase of the direct open loop transfer function $H_{13}$. 

![Graph showing the phase of the direct open loop transfer function with frequency (Hz) on the x-axis and phase (degrees) on the y-axis.](image-url)
Figure 4a. Modulus of the complete open loop transfer function $F_{13}$.
Figure 4b. Phase of the complete open loop transfer function $F_{13}$.
Figure 5. Impulse response function $h_{13}$, obtained by FT of $H_{13}$ over $0 - 10$ Hz.
Figure 6. Impulse response function $h_{13}$, obtained by FT of $H_{13}$ over 0 – 50Hz.
Fig 7. Impulse response function $f_{13}$, obtained from AR series model order 200, time step 10ms, signals filtered at 10.0 Hz.
Fig 8. Impulse response function $f_{13}$, obtained from AR series model order 50, time step 40ms, signals filtered at 10.0 Hz.
Fig 9. Impulse response function $f_{ij3}$, obtained from AR series model order 50, time step 40ms, signals filtered at 2.5 Hz.
Figure 10a.
Spectrum of signal 1.

- FFT-based spectrum
- AR-based spectrum
Figure 10b
Spectrum of signal 2.

\( NAPSD_2 (V^2/Hz) \)

- \( \times \) AR-based spectrum
- \( \diamond \) FFT-based spectrum
Figure 10c.
Spectrum of signal 3.

\[ \text{NAPSD (} \frac{v^2}{\text{Hz}}) \]

- \( \times \) AR-based spectrum
- \( \Diamond \) FFT-based spectrum

Frequency (Hz)
Fig. 11a Phase and coherence of signals $x_1 - x_2$

- $\times$ AR based
- $\Box$ FFT based
SMORN IV NOISE BENCHMARK
IRI DELFT THE NETHERLANDS

Fig. 11b Phase and coherence of signals $x_1 - x_3$

- AR based
- FFT based

Phase (°)

Coherence

Frequency (Hz)
Fig. 11c. Phase and coherence of signals $x_2 - x_3$

- AR based
- FFT based

**Phase of $x_2 - x_3$: Model Order = 200**

**Coherence of $x_2 - x_3$: Model Order = 200**
BORSSELE (PWR) NOISE
TASKS AND RELATED INFORMATION

ON THE BORSSELE PWR NOISE DATA
PHYSICAL BENCHMARK TEST FOR SMORN -IV
BORSSELE (PWR) NOISE DATA

A. EXPERIMENT

Reactor: Pressure Water Reactor with two primary coolant loops, 450 MWe, Built by KWU.

Experiment: Date 13-3-1979, Identified as B6/2E126
Full Power Operation, Boron Concentration = 750 ppm.

Detectors: -Ion Chambers model KNU-42 (Ex-core)
-Cobalt emitter self powered neutron detectors, sensitive length 20 cm (in-core).

Borssele Reactor Noise Data:

<table>
<thead>
<tr>
<th>Channel</th>
<th>Ident</th>
<th>Position</th>
<th>H(mm)</th>
<th>Mean(Volts)*</th>
<th>Gain(ac)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>In-core (IN-12)</td>
<td>F7</td>
<td>2177</td>
<td>4.75</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>Ex-core (LOG)</td>
<td>288</td>
<td>940</td>
<td>0.87</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>In-core (IN-15)</td>
<td>F7</td>
<td>573</td>
<td>8.01</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>Ex-core (LIN)</td>
<td>108</td>
<td>940</td>
<td>0.79</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>In-core (IN-14)</td>
<td>F7</td>
<td>1049</td>
<td>5.15</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>Ex-core (D62)</td>
<td>140</td>
<td>1055</td>
<td>1.53</td>
<td>200</td>
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<tr>
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<td>In-core (IN-13)</td>
<td>F7</td>
<td>1701</td>
<td>5.25</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>Ex-core (D72)</td>
<td>230</td>
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<td>1.59</td>
<td>200</td>
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<td>9</td>
<td>In-core (IN-16)</td>
<td>F7</td>
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<td>1.59</td>
<td>200</td>
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<tr>
<td>11</td>
<td>In-core (IN-11)</td>
<td>F7</td>
<td>2477</td>
<td>3.16</td>
<td>100</td>
</tr>
<tr>
<td>12</td>
<td>Ex-core (D52)</td>
<td>320</td>
<td>1055</td>
<td>1.56</td>
<td>200</td>
</tr>
<tr>
<td>13</td>
<td>Pressure YA01.P001</td>
<td>Loop 1</td>
<td></td>
<td>0.54</td>
<td>20</td>
</tr>
<tr>
<td>14</td>
<td>Pressure YA02.P001</td>
<td>Loop 2</td>
<td></td>
<td>0.55</td>
<td>50</td>
</tr>
</tbody>
</table>

Range of pressure signals: 130-180ksf/cm², 50ksf/cm² = 1 Volt, (1 ksf/cm² = 0.098 MPa).
H = mid height of detector from bottom of core (in mm), top of core is 2550 mm, length of ion chambers is 75 cm.
*) mean of the dc-signals.
B. OBJECTIVES OF PHYSICAL TEST

1. Determination of Core Barrel Motion:

From Ex-core neutron detector signals one might calculate APSD, CPSD and Phase and Coherence relations between the signals. These relations will indicate core barrel motions, Attenuation coefficient (scale factor) may be assumed equal to 0.15 cm\(^{-1}\). Azimuthal position of the ex-core detectors are given in the table.

Request:

1. Where frequency peaks in the spectra are due to beam modes of core barrel motions?
   - Amplitude of motions;
   - Direction of motions.

2. Is there any evidence of shell mode?

2. Determination of Reactivity Effect

At 9.2 Hz a peak is present in all signals. Phase and Coherence of ex-core detectors indicate a reactivity effect highly coherent with the pressure signals.

Request:

1. Determine the r.m.s. value of the reactivity effect, assuming that the reactivity transfer function is about 1/\(\beta\) (\(\beta = 700 \text{ pcm}\)).

2. Determine the reactivity/pressure coefficient at the barore concentration of this experiment, assuming that the pressure noise produces the reactivity noise.

3. Instrument Tube Motion

- In-core neutron detectors might indicate small instrument tube motion.
  
- One might try to identify the motion.

4. Identification of the driving sources of the noise (ex-in core neutron detectors and the pressure sensors) measured and identification of their mutual interaction in the range of 1-20 Hz.
C. DATA SOURCE FOR BENCHMARK TEST

Distributed among the participants of SHORN-III.

1. FM Tape in IRIG Band; 1 inch wide; 14 channels; 3600 ft long
   Real time speed 1-7/8 ips.

If needed ECN can offer the following services:

1. Analog tape recordings
   ----------------------------------------
   Copy of a Benchmark FM Tape.
   (Estimated cost £ 500.-)

2. Digital Tape (Only Borssele data)
   ----------------------------------------
   THE, 9 tracks; 0.5 inch tape; 1600 bpi.
   A copy of a experimental data on digital tape with 64 samples/s.
   Including reading program and tape specification can be obtained
   from E.C.N Petten
   (Estimated cost £500.-).

The mathematical analysis (FFT) of the experiments in the form of
Auto and Cross Spectra and coherence and phase functions for a
selected set of combinations (all ex-core or all in-core or some
some ex-core; in-core and pressure) can also be obtained from ECN.
Results will be given on digital tape (THE 9 tracks 800 or 1600
bpi), including data reading program in FORTRAN (Estimated cost
£1000.-).

Above given data can be requested from:

Netherlands Energy Research Foundation

c/o Mr. E. Türkcan

3. Westerduinweg Petten(NH)
   Postbus 1, 1755 ZG
   The Netherlands
Fig. Position of neutron detectors in and around the core of the Borssele reactor.
REPORT ON THE BORSSCHELE (PWR)  
PHYSICAL BENCHMARK TEST

by

E. TÖRTCAN

Netherlands Energy Research Fondation, ECN  
Po Box 1, 1755 ZC Petten, The Netherlands
BORGSEL REACTOR DATA PHYSICAL BENCHMARK TEST
"Comparison of Results"

E. Türkcan

Netherlands Energy Research Foundation, ECN
P.O. Box 1, 1755 ZG Petten, The Netherlands

ABSTRACT

Reactor noise analysis of the physical benchmark test has been carried out for Borgen reactor (PWR) noise data. The test is aimed at determination of physical parameters, like reactivity, reactivity to pressure coefficient, motion of core barrel and in-core instrumented tube motions. Also the identification of noise sources is requested.

Six groups of specialists contributed to this benchmark test. Results are summarized and compared in this paper with a conclusion of the successful application of the various methods of analysis by the different contributors.

KEYWORDS

Borgen Reactor; reactor noise; core barrel motion; benchmark test.

AIM AND OBJECTIVES OF THE PHYSICAL BENCHMARK TEST

Aim of the Physical Benchmark Test

Aim is to determine the actual physical parameters of the reactor through the physical benchmark. In this way different researchers can compare their methods of analysis. In SMDN-II the test was aimed at computational rather than physical intercomparison. Comparison of the computational test results is given in [1,2]. The second step in the Benchmark is the comparison of the physical quantities, where in this report results of six participants for the PWR test will be compared.

Experiment

Reactor: Pressurized Water Reactor with two primary coolant loops, 450 MWe, built by EJNU.

Experiment: Date 13-3-1979, identified as 86/22124.

Fuel operation, boron concentration = 750 ppm.

Detectors:

- Ion chambers model ENU-42 (Ex-core).
- Cobalt emitter self-powered neutron detectors, sensitive length 20 cm (In-core).

Borgen reactor noise data:

<table>
<thead>
<tr>
<th>Channel</th>
<th>Ident</th>
<th>Position</th>
<th>H (mm)</th>
<th>Mean (Volts)</th>
<th>Gain (dc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>In-core (IN-12)</td>
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<td>2096</td>
<td>4.75</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>Ex-core (Log)</td>
<td>288</td>
<td>940</td>
<td>0.87</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>In-core (IN-13)</td>
<td>F7</td>
<td>482</td>
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<td>50</td>
</tr>
<tr>
<td>4</td>
<td>Ex-core (Lin)</td>
<td>108</td>
<td>1060</td>
<td>0.79</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>In-core (IN-14)</td>
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<td>968</td>
<td>5.15</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>Ex-core (062)</td>
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<td>1033</td>
<td>1.53</td>
<td>200</td>
</tr>
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<td>7</td>
<td>In-core (IN-13)</td>
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<td>620</td>
<td>5.25</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>Ex-core (072)</td>
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<td>200</td>
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<td>9</td>
<td>In-core (IN-16)</td>
<td>77</td>
<td>192</td>
<td>3.56</td>
<td>20</td>
</tr>
<tr>
<td>Channel</td>
<td>Iden</td>
<td>Position</td>
<td>H (mm)</td>
<td>Mean (Volts)</td>
<td>Gain (ac)</td>
</tr>
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<td>----------</td>
<td>--------</td>
<td>--------------</td>
<td>-----------</td>
</tr>
<tr>
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<td>1.59</td>
<td>200</td>
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<tr>
<td>11</td>
<td>Ex-core (98-11)</td>
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<td>2396</td>
<td>3.16</td>
<td>100</td>
</tr>
<tr>
<td>12</td>
<td>Ex-core (952)</td>
<td>320</td>
<td>1055</td>
<td>1.56</td>
<td>200</td>
</tr>
<tr>
<td>13</td>
<td>Pressure YD01.P001</td>
<td>Loop 1</td>
<td></td>
<td>0.54</td>
<td>20</td>
</tr>
<tr>
<td>14</td>
<td>Pressure YD02.P001</td>
<td>Loop 2</td>
<td></td>
<td>0.55</td>
<td>30</td>
</tr>
</tbody>
</table>

Range of pressure signals: 170-190 kgf/m², 50 kgf/m² = 1 Voul, 1 kgf/m² = 0.001 MPa. H = mid height of detector from bottom of core (in mm), cop of core is 2600 mm. Length of ion chambers is 75 cm.

** = mean of the dc-signals.

**Objectives of physical test**

**Determination of core barrel motion**

From ex-core neutron detector signals one might calculate APSD, CPSD and Phase and Coherence relations between the signals. These relations will indicate core barrel motions. Attenuation coefficient (scale factor) may be assumed equal to 0.15 cm⁻¹. Azimuthal positions of the ex-core detectors are given in the table.

**Request:** 1. Which frequency peaks in the spectra are due to beam modes of core barrel motions, determine: - amplitude of motions, - direction of motions.

2. Is there any evidence of shell mode?

**Determination of reactivity effect**

At 9.2 Hz a peak is present in all signals. Phase and coherence of ex-core detectors indicate a reactivity effect, highly coherent with the pressure signals.

**Request:** 1. Determine the r.m.s. value of the reactivity effect, assuming that the reactivity transfer function is about 1/β (β = 700 sec⁻¹).

2. Determine the reactivity/pressure coefficient at the boron concentration of this experiment, assuming that the pressure noise produces the reactivity noise.

**Instrument tube motion**

5 in-core neutron detectors might indicate small instrument tube motion. One might try to identify the motion.

**Identification of noise sources**

Identification of the driving sources of the noise (ex-in core neutron detectors and the pressure sensors) measured, and identification of their mutual interaction in the range of 1-10 Hz.

**Data source for benchmark test**

Distributed among the participants of SHORN-III:

- Fe-Tape in EBIC band, ¼ inch wide, 14 channels, 1600 ft long. Real time speed 1=7/8 ips.
- TDN, 9 tracks, 0.3 inch tape, 1600 bpips.
- A copy of an experimental data on digital tape with 64 samples/s. Tape $s/10q [1].$
- C. The mathematical analysis (FFT) of the experiments in the form of Auto and Cross Spectra and coherence and phase functions for a selected set of combinations (all ex-core or all in-core or some ex-core, in-core and pressure). Tape $s/11q [2].$
PARTICIPANTS OF THE BORSELLE PHYSICAL BENCHMARK TEST

The following six groups of contributors participated in the Borselle Physical Benchmark Test.

<table>
<thead>
<tr>
<th>Identification of the group</th>
<th>Name of the participants</th>
<th>Institute and country</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>R. Sunder and J. Wach</td>
<td>Gesellschaft für Reaktorsicherheit (GDR) ZfT (Germany)</td>
</tr>
<tr>
<td>2</td>
<td>K. Saito and H. Komoto</td>
<td>University of Tsukuba (Japan)</td>
</tr>
<tr>
<td>3</td>
<td>S. Yamada, H. Tamamoto, T. Sato, N. Sugihara and H. Kanamori and K. Shishida</td>
<td>Osaka University (Japan)</td>
</tr>
<tr>
<td>4</td>
<td>J. Costa Oliveira, V. Varella and J. Góis</td>
<td>LNEET-Instituto de Energia (Portugal)</td>
</tr>
<tr>
<td>5</td>
<td>R. Giovannini, T. Martinaelli, M. Motta and M. Marques</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>E. Türkcan</td>
<td>ECN (The Netherlands)</td>
</tr>
</tbody>
</table>

Methods used by the different groups can be summarized as follows:

**Group 1:**

- Analog Benchmark tape (BM) has been used in this analysis. Through the HAPSD functions reactivity and reactivity pressure coefficients were derived. The APSD peak heights were plotted against the circumferential detector positions, a fitted cos² curve was calculated.
- The angles of reactor pressure vessel/core barrel motions were determined. Identification of fuel assembly beam mode was obtained by coherence analysis. In-core instrumented tube motions were observed by investigating the phase behaviour between in-core neutron detectors.
- Their spectrum analysis and peak assignment is given in Fig. 1-2.

**Group 2:**

- Calculated FFT results [2] have been a base for this analysis. HAPSD functions are used in determination of reactivity and reactivity pressure coefficient. Background of each peak in the power spectra was separately calculated and subtracted. Four ex-core neutron detectors (282/282/275/275) and two pressure signals were used.
- Direction of the C3-motion was calculated using annular correction factors. The analytical model for identification of pressure and neutron noise has been given.

**Group 3:**

- Digital data tape $109Q$ file 1 has been used in the analysis. Two different spectrum decomposition methods were implemented to this analysis.
- Multi-variate autoregression modelling technique with eight variables (6 ex-core, 2 pressure) was also used for identification of noise sources. Some of the results are included.

**Group 4:**

- No information was given about the analysis technique or the data source; only the results have been summarized.

**Group 5:**

- Source of data is not given. Magnitude of the CPSD functions has been used for the determination of core barrel motions.

**Group 6:**

- Digital tape $109Q$ file 3 or the FFT results of tape $111Q$ file 3 and file 4 were used in the further analysis. Core barrel motions and the reactivity effects were determined using a spectrum decomposition method, shown in Fig. 5.

**Comparison of the Results**

**Core Barrel Motions**

Results of each group are given in the following table. Direction and the amplitudes are shown in the table in graphical form for an easy comparison.
<table>
<thead>
<tr>
<th>Nr.</th>
<th>Group</th>
<th>Frequency (Hz)</th>
<th>Amplitude (µm)</th>
<th>Direction of motion (degrees)</th>
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</thead>
<tbody>
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<td>3</td>
<td>2-5.5</td>
<td>22.0</td>
<td>92</td>
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<tr>
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<td>11.7</td>
<td>117</td>
</tr>
<tr>
<td>2</td>
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<td>3.9 (5.5)</td>
<td>5.9 (5.4)</td>
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<td>4</td>
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<td>5.5</td>
<td>135</td>
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<td>180-360</td>
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<td>15.3</td>
<td>4.7</td>
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</table>
Reactivity and reactivity/pressure coefficients at 9.1 MW

The results of the reactivity and reactivity/pressure coefficients are given in the following table.

<table>
<thead>
<tr>
<th>Group</th>
<th>Reactivity (in pcm)</th>
<th>Reactivity/pressure (pcm/bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.065</td>
<td>2.40–1.70 (2.31–1.60)</td>
</tr>
<tr>
<td>3</td>
<td>0.063</td>
<td>2.10</td>
</tr>
<tr>
<td>5</td>
<td>0.058</td>
<td>2.00</td>
</tr>
<tr>
<td>6</td>
<td>0.055</td>
<td>2.00</td>
</tr>
</tbody>
</table>

( ) After correction of scale factor for pressure signals.
- Background is not subtracted, minimum and maximum value for 6 ex-core.
- Obtained from CPRS's average value.
- Calculated from modified VIBREAL program.
- Only ex-core α-decorators XAPS's were used.
- Calculated using spectrum decomposition program SPECDEC.

Instrumented Tube Motions

Investigation of phase behaviour of in-core neutron detector signals resulted in generally agreed small instrumented tube motion. The results are summarized in the following table.

<table>
<thead>
<tr>
<th>Group</th>
<th>Frequency (Hz)</th>
<th>Mode of motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.7–14.5</td>
<td>IN13/IN16</td>
</tr>
<tr>
<td>2</td>
<td>12.6</td>
<td>&quot;</td>
</tr>
<tr>
<td>3</td>
<td>12.6</td>
<td>&quot;</td>
</tr>
<tr>
<td>4</td>
<td>12.6</td>
<td>&quot;</td>
</tr>
<tr>
<td>5</td>
<td>12.6</td>
<td>&quot;</td>
</tr>
<tr>
<td>6</td>
<td>12–14</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

Some of the groups (4 and 5) observed a second mode between IN13/IN16.

Further observations and identification of noise sources

Some of the participants investigated the possible sources of noise by identifying the characteristics of the noise in the PSD functions, and for justification PSD, phase and coherence functions were used. Participant 1 gave the most detailed information about the interpretation and also phenomena observed in other reactors are mentioned.

Participants 1 and 5 have classified the observed driving sources in a few categories:

1. Thermal effects, flow instabilities.
2. Vibration of pressure vessel and core barrel.
3. Reactivity effects.
4. Reactivity effect.
5. White noise.

For details we have to refer to the original reports of the participants. Some sets of copies will be available at the meeting.

There is one poster presentation of this conference [8] where the "Signal Transmission Path Analysis-TPA" is applied for the identification of noise sources in the same reactor.

Concluding Remarks

Generally speaking the physical benchmark test was successful. Results obtained for determination of physical parameters such as reactivity and reactivity/pressure coefficient agreed well, even for the extremely low reactivity value of 3×10⁻⁷.

Noise specialists gave a good demonstration of the core barrel motion phenomena, even very small effects (~10⁻⁵) were determined by the different groups resulted in reasonably good agreement on amplitude and direction of the core barrel motions. Methods implemented for determination of core barrel motions are:

a. core-fitting to the height of the peaks of XAPS;

b. from determined peak heights in PSD functions and using an ammular correction factor, direction of core barrel motion is determined. Amplitude is determined using the peak heights derived from XAPS functions.

c. Spectrum decomposition method:

- Method described in [5] by Studnick, program VIBREAL.
- Method described in [5,7] by ECM, program SPECDEC.
REFERENCES

Fig. 1/2: Normalized APSD functions of ex-core neutron detectors, coding of frequency peaks corresponding to fig. 5.
Fig. 5. (upper) Normalized APSD functions of six in-core neutron detectors.
Fig. 6. (lower) APSD functions of two pressure noise sensors, showing frequency peaks corresponding to Fig. 5.
<table>
<thead>
<tr>
<th>Code</th>
<th>Neutron detectors</th>
<th>Frequency (Hz)</th>
<th>KCB - Interpration</th>
<th>Reactor-CB direction of motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ex-core, in-core</td>
<td>1.135</td>
<td>transient time effect due to subcooled boiling</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>1.500</td>
<td>(pressure signals in phase)</td>
<td>N-5</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>2.250</td>
<td>FUEL ASSEMBLY vibration (fundamental mode - clamped/free)</td>
<td>N-5</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>5.750 - 6.250</td>
<td>FUEL ASSEMBLY beam mode (fundamental mode - both ends clamped)</td>
<td>N-5</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>6.500</td>
<td>Fluid resonance (A/4) in volume control system</td>
<td>X-5</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>9.250</td>
<td>Fluid resonance / control rod vibration / reactivity effect</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>11.750 - 12.425</td>
<td>CORE BARREL-beam mode Separation of different vibration-directions via different frequency-peaks</td>
<td>Indef.</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>12.625 - 13.500</td>
<td>FUEL ASSEMBLY beam mode (first harmonics - both ends clamped)</td>
<td>N-5</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>13.625</td>
<td>Standing waves (A/4) on main coolant piping (reactor outlet)</td>
<td>X-5</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>14.875 - 15.875</td>
<td>REACTOR PRESSURE VESSEL-pedicular vibration Separation of different vibration directions via different frequency-peaks</td>
<td>E-W</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>17.500</td>
<td>REACTOR PRESSURE VESSEL vertical vibration **</td>
<td>N-5</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>18.250 / 19.500</td>
<td>Resonances of Barton cells</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>24.875</td>
<td>Forced vertical vibration of RPV-CB due to NCP unbalanced forces</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>29.000 Hz</td>
<td>CORE BARREL-shell mode (ovl)</td>
<td>-</td>
</tr>
</tbody>
</table>

* Due to spurious time duration used at a reactor-detectors-coupling via the main coolant piping. ** Spurious combination of RPV-pedicular vibration in E-W and direction because of the maximum coupling moment of main coolant piping by 180°. ***) combination of primary vibrations of main coolant piping by 180°.

**Fig.5.** Interpretation map and coding of frequency peaks.

**Fig.6.** Decomposed spectra, obtained using six ex-core neutron detector signals at the same height around the vessel. The reactivity spectra (upper left) and core motion amplitude (upper right), and the direction of motion (lower figure) are shown.
INDIVIDUAL CONTRIBUTIONS TO THE BORSELE NOISE BENCHMARK
SMORN-IV
Symposium on Reactor Noise

PHYSICAL BENCHMARK TEST
BORSELE (PWR) NOISE DATA

Participants to the Benchmark test of Borssele Noise Data

1. R. Sunder and D. Wach.
   Gesellschaft für Reaktorsicherheit (GRS) mbH (Germany).

2. K. Saito and H. Konno.
   University of Tsukuba (Japan).

   Osaka University (Japan).

   LNETI - Instituto de Energía (Portugal).

5. R. Giovannini, T. Martinelli, M. Motta (ENEA) and
   M. Marsequena (Politecnico di Milano) (Italy).

6. E. Türkcan.
   Netherlands Energy Research Foundation, ECN, The Netherlands.
PHYSICAL BENCHMARK TEST
BORSSELE (PWR) NOISE DATA

to be discussed during SMORN IV Symposium
Dijon (France), 15th to 19th October, 1984

Garching, May 30th, 1984

R. Sunder et. al.
PHYSICAL BENCHMARK TEST FOR SMORN-IV
BORSSEL (PWR) NOISE DATA

A SURVEY ON SIGNAL ANALYSIS AND SIGNAL INTERPRETATIONS:

All 14 Borssele noise signals have been digitized, Fourier-transformed and correlated according to Table 2. As derived from sensor positions in the Borssele primary system (fig. 1), all relevant correlation combinations have been analyzed in some detail. The following table will give a summary of calculated, selected and documented correlation combinations:

| Correlation combinations | calculated combinations (maximum possible) | selected combinations (analysed in some detail) | documentation for test combinations number of figure-
|--------------------------|-------------------------------------------|-------------------------------------------------|--------------------------------------------------
| ex-core/ex-core neutron noise | 15 | 15 | 15 | 6,7,8,9 |
| in-core/in-core neutron noise | 15 | 10 | 4 | 14 |
| pressure/ pressure noise | 1 | 1 | 1 | 15 |
| ex-core neutron/ pressure noise | 12 | 4 | 1 | 16 |
| in-core neutron/ pressure noise | 12 | 6 | 1 | 17 |
| in-core/ex-core neutron noise | 36 | 6 | - | |
| total: | 91 | 42 | 22 | |

All spectra have been calculated in a frequency range 0-100Hz, resolution 0.125Hz. Due to low pass filtering of the original benchmark-data - the information content of the signals above 50Hz can be neglected - a 50Hz frequency-band was selected for all spectra.

The normalized APSD-functions of all 12 neutron noise signals are plotted by groups in fig. 2 to 4. Fig. 5 contains the APSD functions of both pressure noise sensors. Relevant frequency peaks in the spectra are marked by numbers, the coding of peaks is uniform for all following correlation spectra, assignment to vibration phenomena and driving forces is done by an interpretation map (table 1).

The extraction of information in ex-core neutron noise signals was done by systematic correlation analysis. Characteristic combinations of phase- and coherence functions are plotted in the same figure: 180° opposite detector positions (fig. 6), opposite detector positions (fig. 7), 90° adjacent detector positions (fig. 8) and adjacent detector positions (fig. 9). The chosen sensor combinations are shown in the inserted schemes.
For estimation of RFV- and C3-vibration directions, all phase combinations between excore neutron noise signals are plotted in a coded representation (fig. 10), some selected frequency peaks (fig. 11) were analyzed in more detail: APSD peak heights were plotted against the circumferential detector positions, a cos^2-fitted curve was calculated (fig. 12), the angles of RFV- and C3-motions could be determined (fig. 13).

Identification of fuel assembly beam modes was obtained by correlation analysis of superimposed in-core neutron noise signals (fig. 14). Fluid resonances - like standing waves - could be proved by phase- and coherence analysis of pressure signals (fig. 15), pressure/ex-core neutron noise signals (fig. 16) and pressure/in-core neutron noise signals (fig. 17).

Finally, investigations with respect to incore detector vibrations - as postulated in the task list - were performed. For this purpose, the phase behaviour between adjacent in-core neutron noise signals is a very important indication (fig. 18).

**B. SHORT DESCRIPTION OF INTERPRETATION RESULTS (TABLE 1):**

1. **1.125 Hz**  
   Transit time effect due to subcooled boiling:  
   - Sink-frequencies in coherence functions of superimposed in-core neutron detector combinations (fig. 14).

2. **1.500 Hz**  
   - No interpretation -

3. **2.250 Hz**  
   **FUEL ASSEMBLY** vibration (fund. mode - clamped/free):  
   - Deduced from 4) in combination with in-core neutron analysis (fig. 4, 14 and 18).  
   - \( f_3 : f_4 = 1 : 2.7 \) (results from other KWU-PWRs)

4. **3.750 Hz**  
   **FUEL ASSEMBLY** beam mode (fund. mode - both ends clamped):  
   - Deduced from 9) in combination with in-core neutron analysis (fig. 4, 14 and 13).  
   - \( f_4 : f_9 = 1 : 2.15 \) (results from other KWU-PWRs).

5. **6.500 Hz**  
   **Fluid resonance (\( \lambda/4 \)) in volume control system:**  
   - Deduced from other KWU-PWRs: Biblis-PWR: 6.1 Hz, Neckarwestheim-PWR: 6.2 Hz, Stade-PWR: 6.9 Hz  
   - Good coherence between pressure/pressure signals (fig. 15) and pressure/ex-core neutron noise (fig. 16).

6. **9.250 Hz**  
   **Fluid resonance/control rod vib./reactivity effect:**  
   - All neutron-noise signals show good coherence and in-phase behaviour (fig. 6, 7, 3, 9, 14),  
   - In-phase behaviour between pressure sensors (fig. 15).

7. **11.750 Hz**  
   **CORE BARREL**-beam mode:  
   - Phase behaviour of ex-core neutron noise signals:  
     - Opposite detector positions show out-of-phase behaviour (fig. 6, 7)  
     - 90° adjacent detectors show alternating in-phase and out-of-phase behaviour (fig. 8).
12.625 Hz FUEL ASSEMBLY beam mode (first harm., both ends clamped):
Phase shift of superimposed in-core neutron noise signals (fig. 14 and fig. 18):
- IN11/IN12 and IN11/IN13 in phase
- IN11/IN14 and IN11/IN15 out-of-phase

13.625 Hz Standing waves (λ/4) in main coolant pipings (outlet):
- Calculated from length of outlet-piping, velocity of sound and frequency peak in pressure signals (fig. 5).
- Good coherence between pressure/pressure signals (fig. 15) and pressure/ex-core neutron noise (fig. 16).

14.875 Hz REACTOR PRESSURE VESSEL - pendular vibration:
Phase behaviour of ex-core neutron noise signals:
- 180° opposite detector positions show out-of-phase behaviour (fig. 6)
- 90° adjacent detector positions show alternating in-phase and out-of-phase behaviour (fig. 3)
- Preferred pendular directions (fig. 13) in good agreement with minimum restoring moments of main coolant pipings.

15.000 Hz REACTOR PRESSURE VESSEL vertical vibration:
- Deduced from other KWU-PWRs: Biblis-PWR: 17.1 Hz, Neckarwestheim-PWR: 18.6 Hz
- Good coherence between pressure/in-core neutron signals (fig. 4, 5 and 17)

16.250 Hz/ 19.500 Hz Resonances of Barton cells:
- Barton cells installed in other KWU-PWRs (e.g. Obrigheim-PWR 18.2/19.1 Hz) show similar resonance peaks in APSD functions (fig. 5)

24.875 Hz Forced vertical vibration of RPV-CB due to MCP unbalanced forces:
- Deterministic mechanical excitation of RPV with reaction to pressure fluctuations (fig. 5), well known from other KWU-plants with vibration/displacement-sensor instrumentation.

29.000 Hz CORE BARREL - shell mode (n=2):
Phase behaviour of ex-core neutron noise signals:
- 180° opposite detector positions show in-phase behaviour (fig. 6)
- 90° adjacent detector positions show out-of-phase behaviour (fig. 8).
C. RESULTS

1. Determination of Core Barrel Motion:

From Ex-core neutron detector signals one might calculate APSD, CPSD and Phase and Coherence relations between the signals. These relations will indicate core barrel motions. Attenuation coefficient (scale factor) may be assumed equal to 0.15 cm⁻¹. Azimuthal position of the ex-core detectors are given in the table.

Request:

1. Where are frequency peaks in the spectra due to beam modes of core barrel motions?
   Determine: - amplitude of motions,
   - direction of motions.

2. Is there any evidence of shell mode?

1.1 One specific resonance according to the 'core barrel beam mode' could be identified in the region 11.750 to 12.625 Hz, showing a distinct vibration-direction dependency.

A second core barrel motion could be separated at 14.875 to 15.875 Hz, the so-called 'pressure vessel pendular vibration'. Both kinds of core motion are marked in figure 11. Direction and maximum amplitude of motions can be extracted from cos²-fitted curves in figure 12.

Calculation of amplitude of motion:

\[
r.m.s \text{ amplitude of motion } (f_0) = \frac{1}{0.015} \times \sqrt{\text{APSD}_{\text{max}} \cdot \Delta f(fo)} \text{ [mm]}
\]

\[
\Delta f = 1 \text{ Hz}
\]

<table>
<thead>
<tr>
<th>frequency [Hz]</th>
<th>amplitude of motion [μm]</th>
<th>direction of motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>core barrel beam mode</td>
<td>11.750</td>
<td>3.71</td>
</tr>
<tr>
<td></td>
<td>12.625</td>
<td>3.74</td>
</tr>
<tr>
<td>pressure vessel/ core barrel pendular vibration</td>
<td>14.875</td>
<td>4.62</td>
</tr>
<tr>
<td></td>
<td>15.875</td>
<td>4.35</td>
</tr>
</tbody>
</table>

1.2 A core barrel shell mode (n=2) can be identified by ex-core neutron noise analysis, assuming that
- 180° opposite detector positions show in-phase behaviour (fig.5)
- 90° adjacent detector positions show out-of-phase behaviour (fig. 8).
2. Determination of Reactivity Effect

At 9.2 Hz a peak is present in all signals. Phase and coherence of ex-core detectors indicate a reactivity effect, highly coherent with the pressure signals.

**Request:**

1. Determine the r.m.s. value of the reactivity effect, assuming that the reactivity transfer function is about $1/\beta$ ($\beta=700$ pcm).
2. Determine the reactivity/pressure coefficient at the boron concentration of this experiment, assuming that the pressure noise produces the reactivity noise.

\[
\sigma_{\text{eff}} = \frac{\sigma_n}{\bar{n}} \cdot \frac{1}{G}
\]

- $\sigma_{\text{eff}}$ = r.m.s. reactivity noise
- $\frac{\sigma_n}{\bar{n}}$ = normalized r.m.s. neutronic noise
- $G$ = reactor transfer function (in approximation $1/\beta$)

<table>
<thead>
<tr>
<th>Derived from</th>
<th>$\left(\frac{\sigma_n}{\bar{n}}\right)^2$</th>
<th>$\frac{\sigma_n}{\bar{n}}$</th>
<th>$\sigma_{\text{eff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EX LOG</td>
<td>$6.962 \times 10^{-9}$</td>
<td>$8.344 \times 10^{-5}$</td>
<td>$5.84 \times 10^{-7}$</td>
</tr>
<tr>
<td>EX LIN</td>
<td>$7.980$</td>
<td>$8.933$</td>
<td>$6.25$</td>
</tr>
<tr>
<td>EX D52</td>
<td>$6.070$</td>
<td>$7.791$</td>
<td>$5.45$</td>
</tr>
<tr>
<td>EX D62</td>
<td>$8.349$</td>
<td>$9.406$</td>
<td>$6.58$</td>
</tr>
<tr>
<td>EX D72</td>
<td>$5.817$</td>
<td>$7.627$</td>
<td>$5.33$</td>
</tr>
<tr>
<td>EX D82</td>
<td>$7.492$</td>
<td>$8.653$</td>
<td>$6.05$</td>
</tr>
</tbody>
</table>

*fig. 2 and 3, calculated values

$\sigma_{\text{eff}}$: from 0.0533 to 0.0658 [pcm]

2.2 Reactivity/pressure coefficient of experiment 6/1, boron concentration 750 ppm:

<table>
<thead>
<tr>
<th>Derived from APSPD$^a$</th>
<th>PEAK frequency [Hz]</th>
<th>$\langle \delta p \rangle^2$ [bar]</th>
<th>$-\langle \delta K_{\text{eff}} \rangle^2$ [pcm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>YA 01.P</td>
<td>9.250</td>
<td>$3.162 \times 10^{-2}$</td>
<td>from 0.0533 to 0.0658</td>
</tr>
<tr>
<td>YA 02.P</td>
<td>9.250</td>
<td>$2.529 \times 10^{-2}$</td>
<td></td>
</tr>
</tbody>
</table>

*fig. 5, calculated values

$-\frac{\delta K_{\text{eff}}}{\delta p}$: from 2.501 to 1.685 [pcm/bar]
3. **Instrument Tube Motion**

In-core neutron detectors might indicate small instrument tube motion. One might try to identify the motion.

3.1 Fig. 4 shows the normalized A2PSD functions of six in-core neutron detectors. Due to spikes, the NAPSD's of IN14 and IN15 are very noisy. For all relevant peaks (corresponding to table 1) all phase and coherence relations between adjacent incore detectors were calculated. The results are summarized in fig. 13.

In the frequency range 0-10 Hz, all in-core-signals show in-phase behaviour (Coding of peaks is 3, 4, 5, 6). Driving forces may be fuel element fundamental frequencies (Coding: 3, 4).

In the frequency range 11.7-14.5 Hz, out-of-phase behaviour between IN13/IN14 represents a node of motion (Coding of peaks is 7, 8, 9, 10). Neglecting the out-of-phase behaviour between IN15/IN16 due to the noisy IN16-signal, this behaviour coincides very well with the first harmonic beam mode vibration of the fuel assembly (Coding: 9).

In the upper frequency domain (greater 14.5 Hz) there exists an in-phase behaviour between IN11/12/13/14/15 and opposite phase behaviour of IN15. This leads to the assumption that there might be a node of vibration between IN14/IN15, but the coherence is very low!

Therefore, the driving forces of instrument tube motion are the fuel assembly beam modes in the range of

- 2.25 Hz,
- 5.75-6.25 Hz and
- 12.525-13.500 Hz

(Coding: 3, 4, 9 in table 1).
4. Identification the driving sources of the noise (ex-in core neutron detectors and the pressure sensors) measured and identification of their mutual interaction in the range of 1-20 Hz.

4.1 A interpretation-scheme for driving sources of the Borssele-noise signals is given in table 1.

Four kinds of driving sources could be separated:

- **Thermo-hydraulic effects:**
  - 1.125 Hz
  - 6.500 Hz
  - 9.250 Hz
  - 13.625 Hz

- **Vibrations of pressure vessel and core barrel:**
  - 11.750 - 12.625 Hz
  - 14.875 - 15.875 Hz
  - 17.500 Hz
  - 24.875 Hz
  - 29.000 Hz

- **Vibrations of fuel assemblies:**
  - 2.250 Hz
  - 5.750 - 6.250 Hz
  - 12.625 - 13.500 Hz

- **Reactivity effect:**
  - 9.250 Hz
<table>
<thead>
<tr>
<th>H0</th>
<th>H1</th>
<th>H2</th>
<th>H3</th>
<th>H4</th>
<th>H5</th>
<th>H6</th>
<th>H7</th>
<th>H8</th>
<th>H9</th>
<th>H10</th>
<th>H11</th>
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**IRAX92 - JET SPEED SPECTRUM ESTIMATION**

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<td>Number of scans being analyzed per subs.</td>
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<td>Initial sampling rate (scans per unit time)</td>
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<td>Last frequency point to be analyzed (Hz)</td>
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<td>Detrending of each series</td>
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<td>Prefiltering of each series</td>
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**IRAX68 - MULTIPLE TIME SERIES ANALYSIS**

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<tr>
<td>Sampling Rate</td>
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<td>Evaluated series by IRAX68</td>
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<td>Total number of frequencies</td>
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<td>Frequency resolution (Hz)</td>
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Fig. 2  Normalized APSD functions of four ex-core neutron detectors
- coding of frequency peaks corresponding to table 1
Fig. 3: Normalized APSD functions of two ex core neutron detectors
- Coding of frequency peaks corresponding to Table 1
Fig. 4 Normalized APSD functions of six in-core neutron detectors - coding of frequency peaks corresponding to Table 1.
Fig. 5  APSD Functions of two pressure noise sensors
- coding of frequency peaks corresponding to table 1
Fig. 6  Correlation analysis of ex-core neutron noise signals
- 180° opposite detector positions
Fig. 7 Correlation analysis of ex-core neutron noise signals with detector positions.
Fig. 8  Correlation analysis of ex-core neutron noise signals
- 90° adjacent detector positions
Fig. 9 Correlation analysis of ex-core neutron noise signals
- adjacent detector positions
Fig. 10  Phase behaviour between ex-core neutron noise signals
- coding of frequency peaks corresponding to table 1
Fig. 11 Variations of CB- and RPV-resonance peaks with azimuthal ex-core neutron detector position
Fig. 12 Cos²-fitted curves of four CH⁻ and RPV-resonance peaks - APSD peak heights plotted against detector positions
Fig. 13 Vibration directions of four CB- and RPV-resonance peaks — coding of frequency peaks corresponding to table 1
Fig. 15 Correlation analysis of pressure noise signals
Fig. 16 Correlation analysis between pressure / ex-core neutron noise signals
Fig. 17 Correlation analysis between pressure / in-core neutron noise signals
Fuel assembly beam modes

Fig. 18 Phase behaviour between in-core neutron noise signals
- Coding of frequency peaks corresponding to Table 1
SMORN-IV

(Dijon, 15th to 19th October 1984)

Reactor Noise Analysis Benchmark Physical Test

Data Tape S111Q (E84117)
Contributors SAITO & KONNO

University of Tsukuba
Ibaraki-ken, JAPAN

Only the results are summarized.
Detailed description of these determination procedures will be sent under separate cover.
Reading of the Digital Data Tape S111Q

We have newly coded the data tape reading program, since the original files have 2048 bytes block size which must be rearranged in 80 bytes block size and listed/graphed by our FACOM-M200 digital computer.
Task No.1 Core Barrel Motion

1. Beam Modes

<table>
<thead>
<tr>
<th>Frequency Peak (Hz)</th>
<th>R.M.S. Amplitude of Motion (μm)</th>
<th>Direction of Motion</th>
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<tbody>
<tr>
<td>11.7</td>
<td>5.4</td>
<td>156°</td>
</tr>
<tr>
<td>12.7</td>
<td>4.5</td>
<td>48°</td>
</tr>
<tr>
<td>15.1</td>
<td>4.5</td>
<td>18°</td>
</tr>
<tr>
<td>16.0</td>
<td>6.1</td>
<td>123°</td>
</tr>
<tr>
<td>17.8</td>
<td>2.8</td>
<td>50°</td>
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</tbody>
</table>

2. Shell Mode has not been identified
Task No. 2 Reactivity Effect at 9.2 Hz

1. R.M.S. of Reactivity Effect
   \[ R.M.S. \text{ of Reactivity Effect} = 0.052 \text{ pcm} \]

2. Reactivity/Pressure Coefficient
   R.M.S. of Pressure Fluctuation at 9.2 Hz becomes \( 8.5 \times 10^{-2} \) bar, and
   \[ \frac{\Delta k_{eff}}{\Delta P} = -0.61 \text{ (pcm/bar)} \]
   The present contributors think the above coefficient is unreasonable. However,

   \begin{align*}
   \text{Scale factor} & \quad \text{APSD at 9.2 Hz} \\
   \text{Channel 13} & \quad 0.2663 \times 10^{-4} \quad 0.254 \times 10^{4} \\
   \text{Channel 14} & \quad 0.1067 \times 10^{-4} \quad 1.13 \times 10^{4}
   \end{align*}

   The above are the given numerical values in the present Benchmark Data Tape SIIIQ. Furthermore,

   Range of pressure signals: \( 130 - 180 \text{ ksf/cm}^2 \)
   \[ 50 \text{ ksf/cm}^2 = 49 \text{ bar} = 49 \times 10 \text{ kg/cm}^2 = 1 \text{ volt} \]
   Then
   \[ \text{APSD} \times (\text{Scale factor})^2 \times (49 \times 10)^2 \]

   Channel 13 \( 1.80 \times 10^{-6} \text{ (volt)}^2/\text{Hz} = 4.3 \times 10^{-1} \text{ (kg/cm}^2)^2/\text{Hz} \]
   This value is larger by the factor of 2, in view of the Dr. Türkcan's paper at the SMORN-3.

Is there any misunderstanding in our unit conversion process?
Task No.3 Instrument Tube Motion

Mechanical vibrations were caused with the central frequency around 12.6 Hz under the free-free boundary (top-bottom) condition with a node around the central position i.e., between IN13 (Channel 7) and IN14 (Channel 5).
Task No. 4

1. Identification of the driving source of the pressure noise

The model for identification of the local pressure noise \( \mathcal{P} \): the linear combination of four harmonic oscillators;

\[
\mathcal{P} = \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3 + \gamma_4 x_4 \quad (\gamma_j \text{ constant}), \tag{4-1a}
\]

where \( x_j + \Gamma_j \dot{x}_j + \omega_j^2 x_j = \mathcal{F}_j(t) \). \tag{4-1b}

The resonance frequencies \( \{\omega_j\} \) and the corresponding damping constants \( \{\Gamma_j\}(j=1-4) \), which are estimated from the APSD of Channel 13, are summarized in Table 4-I.

<table>
<thead>
<tr>
<th>( j )</th>
<th>( \omega_j ) [Hz]</th>
<th>( \Gamma_j ) (= ( \omega_j \xi_j ))</th>
<th>( \xi_j ) [sec(^{-1})]</th>
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<tr>
<td>1</td>
<td>18.1</td>
<td>0.64</td>
<td>( \frac{1}{\xi_1}=0.0175 )</td>
</tr>
<tr>
<td>2</td>
<td>9.26</td>
<td>0.45</td>
<td>( \frac{1}{\xi_2}=0.024 )</td>
</tr>
<tr>
<td>3</td>
<td>6.51</td>
<td>0.49</td>
<td>( \frac{1}{\xi_3}=0.037 )</td>
</tr>
<tr>
<td>4</td>
<td>1.63</td>
<td>0.59</td>
<td>( \frac{1}{\xi_4}=0.18 )</td>
</tr>
</tbody>
</table>

Assumption:

(1) \( \langle \mathcal{F}_j(t)\mathcal{F}_j(0) \rangle = \mathcal{D}_j \delta(t) \); \( \mathcal{F}_j(t) \) (j=1-4) is Gaussian white noise.

(2) \( \langle \mathcal{F}_i(t)\mathcal{F}_j(0) \rangle = 0 \) (i\(\neq j\)); Independence of the noise sources.

The estimated strength of the noise sources \( \{\mathcal{D}_j; (j=1-4)\} \) are summarized in Table 4-II.
Table 4-II

|   | $D_j = \left(\omega_j \Gamma_j\right)^2 \overline{p(\omega_j)} \text{bar}^2/\text{sec}^3 | |
|---|---------------------------------|
| 1 | $1.4 \times 10^4$ |
| 2 | 2.9 |
| 3 | $1.0 \times 10^1$ |
| 4 | $8.1 \times 10^{-2}$ |

(* $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 1$)

If the noise sources of the 4 peaks take the same strength ($D_1 = D_2 = D_3 = D_4 = D$; the same origin) and the contributions (transfer) to the pressure are different, $\gamma_1 - \gamma_4$ are the quantities to be determined. The obtained parameters are listed in Table 4-III.

Table 4-III

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>0.0015</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
</tr>
<tr>
<td>4</td>
<td>0.0007</td>
</tr>
</tbody>
</table>

The simulated APSD and ACF with use of the above coefficients are illustrated in Figs. 1 and 2, respectively.
2. Identification of neutron-pressure interaction in the range of 1 - 20 Hz

The model for identification is the non-linear dynamical power-reactor model:

\[ \frac{d}{dt} N = \frac{1}{l} \left[ \rho_0 - \gamma_p P_g - \gamma_T T \right] N + F_N(t) \quad (4-2a) \]
\[ \frac{d}{dt} T = \frac{1}{c} \left[ qN - hT \right] + F_T(t) \quad (4-2b) \]
\[ \frac{d^2}{dt^2} P + \gamma_p \frac{d}{dt} P + \omega_p^2 P = \sigma_P(t) \quad (4-2c) \]

Since Eqs. (4-2a,b) are of the global neutron and temperature fields, the pressure (Eq. (4-2c)) is also the global mode.

The scaled equations are:

\[ \frac{d}{dt} n = a \left[ 1 - \gamma_p \frac{P_g}{P} - \Theta \right] n + f_n(t) \quad (4-3a) \]
\[ \frac{d}{dt} \Theta = b \left( n - \Theta \right) + f_\Theta(t) \quad (4-3b) \]
\[ \frac{d^2}{dt^2} P + \frac{\gamma_p}{P} \frac{d}{dt} P + \omega_p^2 P = \sigma_P(t) \quad (4-3c) \]

where \[ a = \frac{\rho_0}{l} \], \[ b = \frac{h}{c} \], \[ \gamma_p = \frac{\gamma_p}{P_{Sc}} / \rho_0 \], \[ f_n = \frac{F_N}{N_S} \], \[ f_\Theta = \frac{F_T}{T_S} \], \[ f_P = \frac{F_P}{P_{Sc}} \], \[ N_S = (h\rho_0/\gamma_T q) \], \[ T_S = (\rho_0/\gamma_T) \], \[ n = N/N_S \], \[ \Theta = T/T_S \] and \[ P_g = P_g/P_{Sc} \].

The problem is reduced to the determination of the feedback coefficient \[ \gamma_p \] in the range of 1-20 Hz; the frequency-dependence of the pressure-neutron coefficient.

Assumptions:

1) Gaussian white noise; \[ \langle f_j(t) f_j(0) \rangle = \delta(t) \] (j=n, \Theta, p).
(2) Independence of the noises; $\langle f_i(t)f_j(0) \rangle = 0$ (i$\neq j$).

(3) The broad humps at 1.6 and 6.5 Hz and the peak at 9.2 Hz in APSDs of the incore neutron noise can be regarded as the contribution of the pressure noise.

The parameters for numerical simulation:

\[ a = 0.6, \quad b = 5.0, \quad N_s = 10^8, \quad T_s = 300°C, \quad \ell = 5 \times 10^{-5} \text{ sec} \]

\[ L_n = (0.01)^2, \quad L_\theta = 0.0 \quad \text{and} \quad L_p = (0.005)^2 \]

The criterion of the estimation: the simulated intensity ratio $\frac{P(\omega_p)}{P(0.2 \text{Hz})}$ = the measured ratio (Channel 7).

The result:

<table>
<thead>
<tr>
<th>$\omega_p$ [Hz]</th>
<th>non-linear feedback model $\tilde{Y}_p$</th>
<th>linear feedback model $\tilde{Y}'_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.1</td>
<td>negligible</td>
<td>negligible</td>
</tr>
<tr>
<td>9.2</td>
<td>$9.8 \times 10^2$</td>
<td>$5.9 \times 10^2$</td>
</tr>
<tr>
<td>6.5</td>
<td>$7.6 \times 10^2$</td>
<td>$4.5 \times 10^2$</td>
</tr>
<tr>
<td>1.63</td>
<td>$9.7 \times 10^1$</td>
<td>$5.8 \times 10^1$</td>
</tr>
</tbody>
</table>

\[
\frac{d}{dt} n = a \left( 1 - \theta \right) n - \tilde{Y}_p' p + f_n(t) \quad \text{(cf. Eq. (4-3a))}
\]

Figs. 3 and 4 are the simulated APSD and ACF (for $\omega_p$ = 9.26 Hz), respectively. (cf. the measured APSF and ACF curves of the incore detector (Channel 7)).
Appendix A. Procedure to Determine the R.M.S. (G)
1. Reactivity effect at 9.2 Hz
2. Pressure noise at 9.2 Hz
3. Core barrel beam motion

Appendix B. Procedure to Determine the Direction of Core Barrel Beam Motion
1. General procedure
2. Case of B612 E126

Appendix C. Procedure to Determine Peak Frequency of Core Barrel Beam Motion

Appendix D. Procedure to Identify Instrument Tube Motion

Appendix E. R.M.S. Obtained from CPSD

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Appendix A. Procedure to Determine the R.M.S. ($G$)

1. Reactivity effect at 9.2 Hz

$$G_{(pcm)} = [\pi \cdot 4f \times \text{NAPSD (at 9.2 Hz)}]^{\frac{1}{4}} \times 700$$

where

- $4f$ = full-width at half-maximum of the peak APSD. The given data of the APSD are plotted in linear-linear scale around 9 Hz. The half-width $4f/2$ is determined from the upper frequency region of the peak.

- $\text{NAPSD} = [\text{APSD (at 9.2 Hz)} - \text{Background}] 
  \times [\text{Scale factor/Mean (volts)}]^{2}$

Background is determined from the upper frequency region of the peak.

<table>
<thead>
<tr>
<th>Channel No.</th>
<th>APSD (at 9.2 Hz)</th>
<th>Background</th>
<th>NAPSD</th>
<th>$4f$ (Hz)</th>
<th>$G$ (pcm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>$1.21 \times 10^4$</td>
<td>$0.28 \times 10^4$</td>
<td>$0.366 \times 10^{-3}$</td>
<td>0.50</td>
<td>0.0531</td>
</tr>
<tr>
<td>8</td>
<td>0.973</td>
<td>0.21</td>
<td>0.226</td>
<td>0.525</td>
<td>0.0427</td>
</tr>
<tr>
<td>10</td>
<td>1.28</td>
<td>0.31</td>
<td>0.325</td>
<td>0.60</td>
<td>0.0545</td>
</tr>
<tr>
<td>12</td>
<td>1.35</td>
<td>0.58</td>
<td>0.283</td>
<td>0.80</td>
<td>0.0590</td>
</tr>
</tbody>
</table>

Mean = 0.052
2. Pressure Noise at 9.2Hz

\[ \sigma_p (\text{bar}) = [ \pi \Delta f \times \{ \text{APSD (at 9.2Hz)} - \text{Background}\} ]^{\frac{1}{2}} \times \text{Scale factor} \times 49 \]

<table>
<thead>
<tr>
<th>Channel No.</th>
<th>APSD (at 9.2Hz)</th>
<th>Background</th>
<th>( \Delta f )</th>
<th>( \sigma_p ) (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>( 0.254 \times 10^4 )</td>
<td>( 0.032 \times 10^4 )</td>
<td>0.85</td>
<td>( 10.0 \times 10^{-2} )</td>
</tr>
<tr>
<td>14</td>
<td>1.13</td>
<td>0.10</td>
<td>0.55</td>
<td>7.0</td>
</tr>
</tbody>
</table>

Mean = \( 8.5 \times 10^{-2} \)

Corrections

scale factor for ch13:
\[
\frac{5}{49} \times \frac{14}{50} = 7.477 \times 10^{-4}
\]

scale ch 14: \( = 2.991 \times 10^{-4} \)

After corrections:

<table>
<thead>
<tr>
<th>Channel No.</th>
<th>APSD (at 9.2Hz)</th>
<th>Background</th>
<th>( \Delta f )</th>
<th>( \sigma_p ) (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>0.0324 bar 1.60 pcm/bar</td>
</tr>
<tr>
<td>14</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>0.0225 bar 2.31 pcm/bar</td>
</tr>
</tbody>
</table>
3. Core barrel beam motion

\[ \sigma(\mu\text{m}) = \frac{1}{\mu} \sqrt{\pi \delta f x \text{NAPSD}(f_0, \theta_D - \theta_v)} \times 10^4 \]

where

\[ \text{NAPSD}(f_0, \theta_D - \theta_v) = [\text{APSD}(f_0) - \text{Background}] \times \frac{[\text{Scale factor} / \text{Mean}]^2}{\cos^2(\theta_D - \theta_v)} \]

\( \theta_D \) : angular position of the detector, the APSD of which is used for calculation

\( \theta_v \) : direction of motion

<table>
<thead>
<tr>
<th>( f_0 ) (Hz)</th>
<th>Ch. No</th>
<th>APSD of (f_0)</th>
<th>Background</th>
<th>( \theta_D - \theta_v )</th>
<th>NAPSD (f_0, \theta_D - \theta_v)</th>
<th>( \delta f ) (Hz)</th>
<th>( \sigma ) (( \mu\text{m} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.326 \times 10^4</td>
<td>0.15 \times 10^4</td>
<td>-16°</td>
<td>0.0767 \times 10^{-8}</td>
<td>1.7</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>11.7</td>
<td>0.766</td>
<td>0.38</td>
<td>-16°</td>
<td>0.157</td>
<td>1.9</td>
<td>6.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean = 5.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.384</td>
<td>0.08</td>
<td>2°</td>
<td>0.090</td>
<td>1.125</td>
<td>3.76</td>
<td></td>
</tr>
<tr>
<td>12.7</td>
<td>0.535</td>
<td>0.11</td>
<td>2°</td>
<td>0.142</td>
<td>1.34</td>
<td>5.15</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean = 4.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.425</td>
<td>0.138</td>
<td>32°</td>
<td>0.118</td>
<td>1.0</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>15.1</td>
<td>0.733</td>
<td>0.209</td>
<td>32°</td>
<td>0.244</td>
<td>0.73</td>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean = 4.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.730</td>
<td>0.010</td>
<td>17°</td>
<td>0.314</td>
<td>0.70</td>
<td>5.4</td>
<td></td>
</tr>
<tr>
<td>16.0</td>
<td>0.936</td>
<td>0.018</td>
<td>17°</td>
<td>0.373</td>
<td>0.85</td>
<td>6.65</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean = 6.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.165</td>
<td>0.047</td>
<td>0°</td>
<td>0.0349</td>
<td>1.25</td>
<td>2.47</td>
<td></td>
</tr>
<tr>
<td>17.8</td>
<td>0.246</td>
<td>0.061</td>
<td>0°</td>
<td>0.0620</td>
<td>1.15</td>
<td>3.15</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean = 2.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix 3. Procedure to Determine the Direction of Core Barrel Beam Motion

1. General procedure

1) Rough estimation by observing the peaks in the graphs of the CPSD's

2) Determination of angular correction factors for the pairs of detectors. For example, when roughly estimated direction of the beam motion has the angle $\theta_v$ with respect to the Channel 10 detector, the correction factors becomes as in the Table.

3) Calculation of the normalized peak value of the CPSD with the use of a pair of scale factors and a pair of the mean dc-values.

4) Determination of the direction of motion, $\theta_v$ by using the fact that if the NCPSD's are further corrected by the corresponding angular factors, the results turn out to be the same value. For example,
NCPSD \( (f_0) \) of Ch.10 and Ch.6
\[ \frac{\alpha \beta}{\alpha} \]

NCPSD \( (f_0) \) of Ch.10 and Ch.8
\[ = \frac{\alpha^2}{\alpha} \]

Then,
\[ \frac{\beta}{\alpha} = \tan \theta = \frac{\text{NCPSD of 10 & 6}}{\text{NCPSD of 10 & 8}} \]

5) The above is the general procedure of ours, and a more simplified way can be used for any particular case of the CPSD patterns (cf. Appendix B2).
2. Case of B6/2 B126

2. Procedure to determine the direction

17.8 Hz

directed along the line of Ch. 10 and Ch. 8, i.e., 50°, since the peak is most clearly observed in the CPSD of the above pair. Peaks are also observable when either Ch. 10 or Ch. 8 is paired with others.

15.1 Hz

perpendicularly to the line of Ch. 4 and Ch. 2, i.e., 18°, since the CPSD of the pair, and the other CPSD's paired by either Ch. 4 or Ch. 2 have no peak.

16.0 Hz

NCPSD is obtained for all the four adjacent pairs of 90 degree spaced ex-core detectors, and their averaged value is calculated. Its ratio to the CPSD of Ch. 12 and Ch. 6 is 3.3, which is equal to tan θν, where θν is the angle of the motion with respect to the line of Ch. 10 and the core center (cf. General Procedure). Then, \( \theta = \tan^{-1} 3.3 + 50^\circ = 123^\circ \). Peak at this frequency is well separated from the peak at 15.1 Hz for every adjacent pair. The peak in the CPSD of Ch. 12 and Ch. 6 is overlapping with that of 15.1 Hz, but the latter effects little to the former (16.0 Hz) peak, the tail of which gives, however, rise to a considerable effect to the peak at 15.1 Hz. This conclusion comes from the fact that the motion of 15.1 Hz is at 18° and its angular factor with respect to Ch. 12 and Ch. 6 becomes \( \cos 58^\circ = 0.28 \), thus the true peak at 15.1 Hz in the CPSD of Ch. 12 and Ch. 6 is about 1/3 of the virtually observed and its tail has little contribution to the peak at 16 Hz.

Direction may be between Ch. 10 and Ch. 4.

The ratio of the NCPSD of Ch. 4 and Ch. 2 to the average NCPSD of all the adjacent pairs in the right figure is 0.51.

Then, from the angular factor consideration,

\[
\cos (58^\circ - \theta_\nu) = \gamma
\]

\[
\cos \theta_\nu
\]

\[
\begin{array}{ccc}
\gamma & \theta_\nu \\
0.50 & 0^\circ \\
0.52 & -1^\circ \\
0.50 & -2^\circ \Theta \\
0.48 & -3^\circ \\
\end{array}
\]

\[\theta = 50^\circ + \theta_\nu = 48^\circ\]
Direction may be between Ch. 12 and Ch. 10, not perpendicular to the line of Ch. 4 and Ch. 2

\[
\frac{1}{2} \text{ NCPSD of Ch. 12 & Ch. 6} + \text{ NCPSD of Ch. 12 & Ch. 10 }
\]

11.7 Hz

\[
= 0.28
\]

\[
= \tan \theta_{\nu} , \quad \theta_{\nu} = 16^\circ
\]

Then, \( \theta = 320^\circ + \theta_{\nu} \)

\[
= 336^\circ \text{ or } 156^\circ
\]

(320°)
Appendix C. Procedure to Determine Peak Frequency of Core Barrel Beam Motion

<table>
<thead>
<tr>
<th>CPSD pair</th>
<th>Peak Frequency in CPSD (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 x 6</td>
<td>12.6, 15.1, 16.0, 17.8</td>
</tr>
<tr>
<td>10 x 6</td>
<td>11.7, 12.6, 15.1, 16.0, 17.8</td>
</tr>
<tr>
<td>10 x 8</td>
<td>11.7, 12.6, 15.1, 16.0, 17.8</td>
</tr>
<tr>
<td>12 x 8</td>
<td>12.6, 15.1, 16.0, 17.9</td>
</tr>
<tr>
<td>12 x 10</td>
<td>11.7, 12.7, 15.1, 16.0, 17.8</td>
</tr>
</tbody>
</table>

Mean 11.7, 12.7, 15.1, 16.0, 17.8

The above mean values are well coincided with the peak frequencies in the APSDs of ex-core detectors.
Appendix D. Procedure to Identify Instrument Tube Motion

A broad hump is observed around 13 Hz in the APSD's of Channels 1, 3 and 11, while the channels 5 and 7 have a rather flat APSD.

<table>
<thead>
<tr>
<th>CPSD pair</th>
<th>Phase around 13 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 x 1</td>
<td>-π</td>
</tr>
<tr>
<td>5 x 1</td>
<td>-π</td>
</tr>
<tr>
<td>5 x 3</td>
<td>0</td>
</tr>
<tr>
<td>7 x 1</td>
<td>0</td>
</tr>
<tr>
<td>7 x 3</td>
<td>-π</td>
</tr>
<tr>
<td>7 x 5</td>
<td>-π</td>
</tr>
<tr>
<td>11 x 1</td>
<td>0</td>
</tr>
<tr>
<td>11 x 3</td>
<td>-π</td>
</tr>
<tr>
<td>11 x 5</td>
<td>-π</td>
</tr>
<tr>
<td>11 x 7</td>
<td>0</td>
</tr>
</tbody>
</table>
Appendix E. R.M.S. Obtained from CPSD

### Core Barrel Beam Motion

<table>
<thead>
<tr>
<th>Frequency Peak (Hz)</th>
<th>R.M.S. (μm)</th>
<th>CPSD pair used for calculation</th>
<th>Δf (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.7</td>
<td>5.9</td>
<td>12 x 6</td>
<td>1.8</td>
</tr>
<tr>
<td>12.7</td>
<td>3.9</td>
<td>10 x 8</td>
<td>0.93</td>
</tr>
<tr>
<td>15.1</td>
<td>4.5</td>
<td>10 x 8</td>
<td>0.66</td>
</tr>
<tr>
<td>16.0</td>
<td>4.7</td>
<td>12 x 6</td>
<td>0.55</td>
</tr>
<tr>
<td>17.8</td>
<td>2.2</td>
<td>10 x 8</td>
<td>0.88</td>
</tr>
</tbody>
</table>

### Reactivity Effect at 9.2Hz

<table>
<thead>
<tr>
<th>R.M.S. (pcm)</th>
<th>CPSD pair used for calculation</th>
<th>Δf (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0488</td>
<td>12 x 10</td>
<td>0.52</td>
</tr>
<tr>
<td>0.0438</td>
<td>12 x 8</td>
<td>0.48</td>
</tr>
<tr>
<td>0.0406</td>
<td>12 x 6</td>
<td>0.40</td>
</tr>
<tr>
<td>0.0399</td>
<td>10 x 8</td>
<td>0.42</td>
</tr>
<tr>
<td>0.0523</td>
<td>10 x 6</td>
<td>0.50</td>
</tr>
<tr>
<td>0.0469</td>
<td>8 x 6</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Mean = 0.045
SMORN-IV
(Dijion, 15 - 19, October 1984)

Physical Benchmark Test
Borssele Noise Data Analysis
(Data Tape: M.T. S109 F1)

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Task 1: Core barrel motion

(1) Analyzing method

Core barrel motion analysis was made by a modified VIBREAL model\(^{(1)}\), which utilizes 5 ex-core neutron detectors. This model can determine 5 modes considered by Åkerhielm (reactivity mode E, two components of unidirectional beam mode A and B, multidirectional beam mode C, and shell mode D) without the assumption employed by Åkerhielm due to a new relationship among the NCPSD's \(\phi_{15}\) or \(\phi_{16}\) for \(i = 1, 2, 3,\) and \(4\)

\[
\sum_{i=1}^{4} \phi_{15} = \sum_{i=1}^{4} \phi_{16} = 4E^2
\]

where subscripts 1, 2,...,5 denote the ex-core neutron detectors LOG, LIN, D62, D72, D82, and D52, respectively. Then the r.m.s. value of displacement in each mode was determined by

\[
\sigma_j = \frac{1}{\mu} \left( \pi \cdot \Delta f \cdot P^2(f_0)/2 \right)^{1/2}, \quad j = A \text{ or } B
\]

where \(\Delta f\) is the full-width at half-maximum of the peak PSD, \(P(f_0)\), of \(A^2\) or \(B^2\) at resonance frequency \(f_0\). Then the r.m.s. value \(\sigma_j\) of displacement in unidirectional beam mode vibration and its direction \(\theta\) were determined by

\[
\sigma = (\sigma_A^2 + \sigma_B^2)^{1/2} \quad \text{and} \quad \theta = \tan^{-1}(\sigma_A/\sigma_B),
\]

respectively.

(2) Result

(2-1) Unidirectional beam mode:

PSD's of \(A^2, B^2, C^2, D^2, E^2,\) and \(A^2+B^2\) are shown in Figs. 1-6, respectively. Frequencies, directions, r.m.s. values of displacement of unidirectional beam mode vibrations are listed in Table 1. In addition to those unidirectional core barrel motions, the phases of NCPSD's between
ex-core detectors shown in Table 2 indicate that the core barrel is vibrating in the direction of LOG and LIN with fairly large r.m.s. value of displacement in the frequency range of 1-8 Hz.

(2-2) Shell mode:

Fig. 4 indicates some possibility of shell mode core barrel vibration, however, it can not be identified from the phase of NCPSD's because the PSD of the shell mode (Fig. 4) is much smaller than those of the beam modes (Figs. 1, 2, and 3).

(2-3) Note:

To check the result obtained by this method, SPEC-4R model by Dragt (2) was also applied for the same data analyzed here, and almost the same results were obtained. Fig. 7 is the PSD of $\phi_{yy}$ obtained by SPEC-4R which is equivalent to $A^2 + B^2$ in Fig. 6, and Fig. 8 is the PSD of $\phi_{rr}$ obtained by SPEC-4R which is equivalent to $E^2$ in Fig. 5.

References


Task 2: Reactivity effect

(1) Data Processing

APSD's and CPSD's of ex-core neutron detectors and pressure sensors have been calculated by FFT from the time series data given in File No.3 of the M.T. Labeled No.5109 F1. These PSD's were compared with those given in the M.T.. Some differences were observed due to background subtraction and the methods used for spectrum estimation (Blackman-Tukey method with Fourier Transform vs. FFT method).

(2) Observations

In the frequency range from 7.5Hz to 12Hz, least square fitting has been applied for APSD's, which were provided by NEA Data Bank, of pressure and neutron in the form of

$$ f_{ij}(f) = \frac{3}{n-1} \frac{\sum_{n=1}^{N} (\Delta f_n)^2 \tau_n^2}{(\tau_n^2 - \tau^2)^2 - (\Delta f_n)^2 \tau^2} + k_4 $$

where $K_n$ is the peak PSD at the resonance frequency $f_n$ with the full-width at half-maximum of the peak PSD. The results showed unreasonable wide spread of $f$ from 0.39Hz for LIN to 0.80Hz for DS2, and direct reading of the graphs of APSD also showed the same tendency. Hence, the least square fitting method is considered to be inappropriate with other reasons, such as too much parameters to be fitted, dependency on initial conditions for least square fitting and fitting frequency range, and so on. Then, CPSD's between pressure and neutron were examined and analyzed for this problem.

For notational simplicity, subscripts 1, ..., 6 will be used for ex-core neutron detectors (LDG, LIN, D52, D72, D82, D52), and 7 and 8 for pressure sensors (YAO1 and YAO2), respectively. The peak CPSD at the
resonance freq. of 9.2Hz will be denoted by $\phi_{ij}(f_0)$, and $\Delta \phi_{ij}$ is the full-width at half-maximum of the peak CPSD $\phi_{ij}(f_0)$, $i=1,...,6$ and $j=7$ or 8. Denoting fluctuations in neutron detector current, reactivity and pressure by $\Delta I$, $\Delta \rho$, and $\Delta p$, respectively, we are assuming that

$$\frac{\Delta I(s)}{I_0} = G_P(s) \delta \rho(s)$$

$$\delta \rho(s) = (\delta \rho/\delta p) \Delta p(s)$$

where $G_P(s)$ and $\delta \rho/\delta p$ are the reactivity transfer function and the reactivity /pressure coefficient, respectively and $I_0$ is the average current of neutron detector. In addition to this, we will assume that pressure is measured by two pressure sensors of different gain $H_1$ and $H_2$ at around the frequency of 9.2 Hz for YAO1 and YAO2, respectively,

$$\Delta p_1(s) = H_1 \Delta p(s) \quad \text{for YAO1}$$

$$\Delta p_2(s) = H_2 \Delta p(s) \quad \text{for YAO2}$$

Under these conditions, (1) all CPSD's among signatures of pressure sensors and neutron detectors should exhibit the same $\Delta \phi_{ij}$, and (2) for every ex-core neutron detectors, the ratio of the Peak CPSD's at 9.2 Hz with respect to YAO1 and YAO2, $\phi_{i8}(f_0)/\phi_{i7}(f_0)$, $i=1,...,6$, should be equal to $H_2/H_1$. All CPSD's between pressure(YAO1 and YAO2) and ex-core neutron detectors (LOG, LIN, D62, D72, D62 and D52) were calculated by FFT from the time series data and $\phi_{i7}(f_0)$ and $\phi_{i8}(f_0)$ are listed in Table 3, which indicates that differences in all $\Delta \phi_{ij}$'s are almost in the limit of statistical error even for CPSD's associated with D62 and D52, and $\phi_{i8}(f_0)/\phi_{i7}(f_0)$ are almost same as expected. However, $\phi_{i7}(f_0)$ and $\phi_{i8}(f_0)$ for $i=5$ and 6 are still fairly larger than those for $i=1,...,4$ due to some back ground and they were omitted for evaluations of r.m.s. values of reactivity and pressure.

From the last column of Table 3, $H_2/H_1$ is found to be 0.849. Using
this value with \( \phi_7(f_0) \), \( \phi_7(f_0) \) and \( \phi_8(f_0) \) are estimated as

\[
\begin{align*}
\phi_7(f_0) &= 2.520 \times 10^{-7} \text{ (v²/Hz)} \\
\phi_8(f_0) &= 1.816 \times 10^{-7} \text{ (v²/Hz)}
\end{align*}
\]

These values are almost equal to those found from the APSD given by NEA in the M.T.. The true peak PSD at 9.2Hz for pressure, \( \phi_{pp}('f_0') \), is evaluated by

\[
\phi_{pp}(f_0) = \left( \phi_7(f_0) + \phi_8(f_0) + 2\phi_7(f_0) \right) / 4
\]

and it was found to be \( 2.15 \times 10^{-7} \text{ v²/Hz} \). Then the r.m.s. value of pressure resonance at 9.2Hz became 0.0187 bar when evaluated by the formula

\[
\text{r.m.s. value of pressure resonance at 9.2 Hz} = \left( \times \psi_7(f_0) \times \phi_{pp}(f_0) / 2 \right)^{1/2} \times 49 \text{ (bar)}.
\]

Assuming complete coherency between pressure and neutron at around 9.2Hz, the peak NAPSD's, \( \psi_7(f_0) \), of ex-core neutron detectors are calculated and listed in Table 4. Then r.m.s. value of reactivity was evaluated by

\[
\text{r.m.s. value of reactivity} = \left( \times \psi_{ii}'(f_0) \times \phi_{ii}(f_0) / 2 \right)^{1/2} \times 700 \text{ (pcm)}
\]

and listed in Table 5.

(3) Results

- r.m.s. value of reactivity = 0.0403 \text{ (pcm)}
- r.m.s. value of pressure resonance at 9.2 Hz = 0.0187 \text{ (bar)}
- reactivity/pressure coefficient = 2.16 \text{ (pcm/bar)}.
Task 3 Instrument tube motion

(1) Data processing

APSD's and CPSD's of in-core neutron detectors have been obtained by FFT of time series data given by NEA in the M.T. No. S109 F-1, File No. 3

(2) Observations

CPSD's associated with IN-16 exhibited completely different pattern from those of others, and omitted from instrument tube motion analysis. All CPSD's, except for those associated with IN-16, exhibited broad hump in the frequency range of 11-16 Hz with phase difference of nearly 0 or π rad. as shown in Table 6. This implies that the instrument tube is vibrating with the frequency around 12.6Hz and the node between IN-13 and IN-14. Comparing peak values of CPSD in the broad hump given in Table 6 under the assumption of the same flux gradient at each in-core detector's position, the r.m.s. value of displacement of detector is greatest for IN-11 and smallest for IN-13 among the detectors IN-11, 12, and 13.

(3) Conclusion

Instrument tube vibration is observed in the frequency range of 11-16 Hz with a node between IN-13 and IN-14 and antinodes at IN-11 and IN-16. The resonance frequency is about 12.6Hz.
Task 4: Identification of driving noise sources

(1) Analyzing method

The signal transmission path analysis proposed by R. Oguma was applied for identification of driving noise sources. Eight dimensional MAR model of order 30 has been fitted for the 64 Hz sampling time series data of 6 ex- and in-core neutron detectors with two pressure sensors.

(2) Results

Following conclusion has been obtained for each peak observed in PSD's and simple coherence functions.

(2-1) around 9.2 Hz
This peak is obviously caused by a noise source in pressure, and there exists a feedback loop between YAO1 and YAO2.

(2-2) around 12.5 Hz

IN-core detectors:
Around this frequency no influence from pressure to in-core neutron detectors has been observed, but there exists strong interrelationship among in-core detectors themselves, especially among the group of IN-11, 12, and 13, and the group of IN-15 and 16. The signal is propagating from IN-16 to IN-15 and 14, and from IN-15 to IN-13 and 12. On the other hand, no signal path from IN-14 to IN-13 has been found. Furthermore, the peak around 12.5 Hz of the noise power contribution function of IN-14 is almost due to IN-14 itself. These facts support that this peak is caused by an instrument tube motion with a node at around IN-14.

Ex-core detectors:
Signal pathes among ex-core detectors were observed, especially two pairs
LOG - LIN and 072 - 082 have a signal path with large gain. However, no influence from pressure to ex-core detectors was found. Hence, the driving source of the peak is considered to be core barrel motion in the direction approximately perpendicular to the line of 052 - 062. This is consistent with the core barrel motion of angle 81 deg.

(2-3) around 13.5 Hz
Signal path is found from pressure YAO1 to ex-core neutron detectors and YAO2. Hence some noise source exists in pressure YAO1 at around 13.5 Hz.

(2-4) around 15 Hz
Large coherence has been found between two pairs of ex-core neutron detectors 052 and 062, and 082 and 072, but no remarkable coherence has been found between LOG and LIN. Furthermore, no notable signal path from pressure to neutron has been found. Hence this peak is caused by a core barrel motion in the direction approximately perpendicular to the line of LOG - LIN. This corresponds to the core barrel motion of angle 1 deg.

(2-5) around 16 Hz
Around this frequency, similar situation to the 15Hz peak was observed, and it can be concluded that this peak is caused by a core barrel motion in the direction of 052 - 062 which corresponds to the vibration angle 117 deg.

(2-6) around 17.5 Hz
Pressure YAO2 is obviously the driving source of the peak at around 17.5 Hz of ex-core neutron detectors.

(2-7) 18 - .19 Hz
A signal path from pressure to neutron has been observed. The 18 Hz peak is due to some noise source in Loop 1 (YAO1), and the 19 Hz peak is caused by a noise source in Loop 2 (YAO2).
FIG. 1  PSD of $A^2$

FIG. 2  PSD of $B^2$

FIG. 3  PSD of $C^2$

FIG. 4  PSD of $D^2$
FIG. 5  PSD of $E^2$

FIG. 6  PSD of $A^2 + B^2$ calculated from $A^2$ and $B^2$

FIG. 7  PSD of $\phi_{yy}$ obtained by SPEC-4R

FIG. 8  PSD of $\phi_{zz}$ obtained by SPEC-4R
Table 1. Unidirectional beam mode vibrations.

θ₀: direction of vibration is the angle measured from the fiducial line of direction defined in the data sheet of Borssele Noise.

Data in ( ) are results obtained by SPEC-4R.

<table>
<thead>
<tr>
<th>f₀ (Hz)</th>
<th>a₀ (μm) (D52 - D62)</th>
<th>a₀ (μm) (D72 - D82)</th>
<th>σ (μm)</th>
<th>θ₀ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.7</td>
<td>0.93</td>
<td>1.51</td>
<td>1.77</td>
<td>172</td>
</tr>
<tr>
<td>12.7</td>
<td>2.88</td>
<td>1.71</td>
<td>3.35</td>
<td>117</td>
</tr>
<tr>
<td>15.2</td>
<td>2.69</td>
<td>3.05</td>
<td>4.08</td>
<td>1</td>
</tr>
<tr>
<td>16.1</td>
<td>1.24</td>
<td>2.90</td>
<td>3.15</td>
<td>117</td>
</tr>
<tr>
<td>17.9</td>
<td>1.36</td>
<td>0.58</td>
<td>1.45</td>
<td>27</td>
</tr>
</tbody>
</table>

Table 2. The phase of NCPSD for each pair of ex-core neutron detectors in the frequency range of 1-2 Hz.
### Table 3. Peak CPSD's at 9.2 Hz resonances

<table>
<thead>
<tr>
<th>i</th>
<th>$\phi_{i7}(f_o)$ $\text{v/Hz}$</th>
<th>$\Delta f_{i7}$ $\text{Hz}$</th>
<th>$\phi_{i8}(f_o)$ $\text{v/Hz}$</th>
<th>$\Delta f_{i8}$ $\text{Hz}$</th>
<th>$\frac{\phi_{i8}(f_o)}{\phi_{i7}(f_o)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3.494 \times 10^{-8}$</td>
<td>0.458</td>
<td>$3.034 \times 10^{-8}$</td>
<td>0.470</td>
<td>0.868</td>
</tr>
<tr>
<td>2</td>
<td>$3.474$</td>
<td>0.439</td>
<td>$2.881$</td>
<td>0.445</td>
<td>0.829</td>
</tr>
<tr>
<td>3</td>
<td>$3.707$</td>
<td>0.438</td>
<td>$3.150$</td>
<td>0.435</td>
<td>0.852</td>
</tr>
<tr>
<td>4</td>
<td>$3.309$</td>
<td>0.457</td>
<td>$2.634$</td>
<td>0.462</td>
<td>0.870</td>
</tr>
<tr>
<td>5</td>
<td>$4.533$</td>
<td>0.454</td>
<td>$3.762$</td>
<td>0.473</td>
<td>0.830</td>
</tr>
<tr>
<td>6</td>
<td>$5.644$</td>
<td>0.457</td>
<td>$4.972$</td>
<td>0.430</td>
<td>0.881</td>
</tr>
</tbody>
</table>

$\phi_{78}(f_o) = 2.140 \times 10^{-7} \text{v}^2/\text{Hz}$, $\Delta f_{78} = 0.430 \text{ Hz}$  

### Table 4. Estimated peak NAPSD's of ex-core neutron detectors

<table>
<thead>
<tr>
<th>i</th>
<th>$\phi_{i7}(f_o)$ estimated from $\phi_{i7}(f_o)$ and $\phi_{77}(f_o)$</th>
<th>$\phi_{i8}(f_o)$ estimated from $\phi_{i8}(f_o)$ and $\phi_{88}(f_o)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$4.844 \times 10^{-9}$ $\text{l/Hz}$</td>
<td>$5.056 \times 10^{-9}$ $\text{l/Hz}$</td>
</tr>
<tr>
<td>2</td>
<td>$4.790$</td>
<td>$4.560$</td>
</tr>
<tr>
<td>3</td>
<td>$5.545$</td>
<td>$5.486$</td>
</tr>
<tr>
<td>4</td>
<td>$3.664$</td>
<td>$3.813$</td>
</tr>
<tr>
<td>5</td>
<td>$8.155$</td>
<td>$7.776$</td>
</tr>
<tr>
<td>6</td>
<td>$12.643$</td>
<td>$13.583$</td>
</tr>
</tbody>
</table>

$\phi_{77}(f_o) = 2.520 \times 10^{-7} \text{v}^2/\text{Hz}$ $\phi_{88}(f_o) = 1.817 \times 10^{-7} \text{v}^2/\text{Hz}$
Table 5. Estimated r.m.s. value of reactivity effect by pressure resonance at 9.2 Hz.

<table>
<thead>
<tr>
<th>Pair of In-core Detectors</th>
<th>Peak NCPSD in the Freq. Range of 11-15 Hz</th>
<th>Phase Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>IN-11 &amp; IN-12</td>
<td>0.614x10^{-9} at 12.6Hz</td>
<td>- 0</td>
</tr>
<tr>
<td>IN-11 &amp; IN-13</td>
<td>0.530 at 12.5</td>
<td>- 0</td>
</tr>
<tr>
<td>IN-11 &amp; IN-14</td>
<td>0.288 at 13.3</td>
<td>- π</td>
</tr>
<tr>
<td>IN-11 &amp; IN-15</td>
<td>0.415 at 12.7</td>
<td>- π</td>
</tr>
<tr>
<td>IN-12 &amp; IN-13</td>
<td>0.501 at 12.5</td>
<td>- 0</td>
</tr>
<tr>
<td>IN-12 &amp; IN-14</td>
<td>0.268 at 13.8</td>
<td>- π</td>
</tr>
<tr>
<td>IN-12 &amp; IN-15</td>
<td>0.410 at 12.6</td>
<td>- π</td>
</tr>
<tr>
<td>IN-13 &amp; IN-14</td>
<td>0.200 at 12.5</td>
<td>- π</td>
</tr>
<tr>
<td>IN-13 &amp; IN-15</td>
<td>0.362 at 12.6</td>
<td>- π</td>
</tr>
<tr>
<td>IN-14 &amp; IN-15</td>
<td>0.246 at 13.1</td>
<td>- 0</td>
</tr>
</tbody>
</table>

Table 6. Peak value and phase of NCPSD at resonance frequency.
1. Core Barrel Motion
   a) Beam modes

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Amplitude</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.6 Hz</td>
<td>3.5 µm</td>
<td>≈ 80°</td>
</tr>
<tr>
<td>15.2</td>
<td>5.5</td>
<td>≈ 185</td>
</tr>
<tr>
<td>15.8</td>
<td>4.3</td>
<td>≈ 125</td>
</tr>
</tbody>
</table>

   Multidirectional oscillations of the barrel manifest at 11.8 Hz and 17.6 Hz.

   b) Shell modes

   Shell mode vibrations of the core barrel's wall were not detected.

2. Reactivity effect at 9.2 Hz
   a) r.m.s. value of the reactivity effect = 0.050 pcm
   b) Reactivity/pressure coefficient = 2.23 pcm. bar⁻¹

3. Instrument tube motion

   The APSD's of the in-core neutron detectors show a strong and broad hump, from 10 Hz to 20 Hz, related to a small instrument tube motion under the influence of turbulent forces.

   This motion is such that two nodes (between IN 13 and IN 14 and between IN 15 and IN 16) are observed at 12.6 Hz and one node (between IN 15 and IN 16) is observed at 17.6 Hz.
4. Driving sources of the noise

= 1.5 Hz highly damped oscillation of the core ("seen" by ex-core neutron detectors).

3.3 weak pressure fluctuation ("seen" by pressure sensors)

6.5 strong pressure fluctuation induces a steady wave in the primary coolant system ("seen" by ex-core neutron detectors and pressure sensors)

9.2 a common cause produces both a reactivity and a pressure oscillation in the primary system; the pressure noise by itself produces a reactivity noise, via the reactivity/pressure coefficient ("seen" by all detectors)

11.8 coolant flow impact on the core barrel ("seen" by ex-core neutron detectors)

12.6 coolant flow impact on the core barrel (1) ("seen" by ex-core neutron detectors and by one pressure sensor)

13.6 pressure fluctuation (2) ("seen" by pressure sensors)

15.2 coolant flow impact on the core barrel (1) ("seen" by ex-core neutron detectors and pressure sensors)

15.8 coolant flow impact on the core barrel (1) ("seen" by ex-core neutron detectors and pressure sensor YA01)

17.6 coolant flow impact on the core barrel (1) ("seen" by ex-core neutron detectors and pressure sensor YA01)

18.2 very local effects related to mechanical properties of the sensors or their connecting wires ("seen" by one pressure sensor, 18.2 Hz for sensor YA01 and 19.6 Hz for sensor YA02).

(1) The core barrel motions induced by the coolant flow impact seem to generate pressure waves.

(2) No core motion has been identified from the neutronic noise.
- Oliveira, J.C.; Veiga, C.; Trigueiros, D.; Duarte, J.P.;
  "Data acquisition and processing system for reactor noise analysis"
  Ann. of Nucl. En., 9, 149-159 (1975)

- Thie, J.A.
  "Power Reactor Noise"

- Turkcan, E.
  "Review of Borssele PWR noise experiments, analysis and instrumentation"
  Progr. in Nuclear En. 9, 437-452 (1982)

- Oliveira, J.C.; Varela, V.; Galan, J.
  "Borssele PWR noise analysis"
  LNE/DEEN-B n° 67 (1983)
1. DETERMINATION OF CORE BARREL MOTION

REQUEST 1. Beam mode.

The beam mode was determined in the low frequency range by the Robinson et al. method [1]. In the higher frequency range by Dragt and Turkcan's method [2] (figs. 10-12).

Results are the following:

<table>
<thead>
<tr>
<th>FREQUENCY (Hz)</th>
<th>AMPLITUDE (cm)</th>
<th>DIRECTION (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 - 5.5</td>
<td>2.20E-3</td>
<td>93</td>
</tr>
<tr>
<td>12.6</td>
<td>2.91E-4</td>
<td>95</td>
</tr>
<tr>
<td>15.0</td>
<td>2.89E-4</td>
<td>180</td>
</tr>
<tr>
<td>16.0</td>
<td>1.97E-4</td>
<td>135</td>
</tr>
</tbody>
</table>

In the low frequency range a possible overestimation of amplitude could be expected, since the whole magnitude of the cross spectrum was attributed to the barrel motion.

REQUEST 2. Shell mode.

No evidence of shell mode up to 20 Hz was observed, by using the Akerhielm et al. method of investigation [3].

The figures 1-9 show the diagrams of magnitude, coherence and phase of the cross spectra for the different pairs of ex-core detectors.
2. DETERMINATION OF REACTIVITY EFFECT

REQUEST 1. r.m.s. of reactivity effect.

By the method of Turkcan in ref. [4] the value averaged over 4 ex-core neutron detectors, is:

\[ r.m.s. = \left[ \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\delta n_i}{n_i} \frac{1}{G} \right)^2 \right]^{1/2} = 45.3 \text{ pcm} \]

where \( \delta n_i/n_i \) were evaluated from the 9.2 Hz peak areas of NAPS's. The above value well agrees with \((49.3 \pm 2.5) \cdot 10^{-3} \text{ pcm}\), as calculated in ref. [41], p.441 for Boron concentration \(c_B = 750 \text{ ppm}\).

REQUEST 2. Reactivity/pressure coefficient.

The results are given in the following table:

<table>
<thead>
<tr>
<th>DETECTOR</th>
<th>( \delta \text{keff \ (pcm)} )</th>
<th>( \delta \text{p \ (kg/cm}^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex-052</td>
<td>4.50 E-2</td>
<td>-</td>
</tr>
<tr>
<td>Ex-062</td>
<td>5.11 E-2</td>
<td>-</td>
</tr>
<tr>
<td>Ex-072</td>
<td>3.90 E-2</td>
<td>-</td>
</tr>
<tr>
<td>Ex-082</td>
<td>4.34 E-2</td>
<td>-</td>
</tr>
<tr>
<td>Press-YA01.P001</td>
<td>2.241 E-2</td>
<td></td>
</tr>
<tr>
<td>Press-YA02.P001</td>
<td>1.777 E-2</td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{\langle \delta \text{keff}^2 \rangle^{1/2}}{\langle \delta \text{p}^2 \rangle^{1/2}} = 2.245 \text{ pcm/(kg/cm}^2\)

The computed value well compares with \(2.186 \pm 0.055\) as calculated in ref. [41], p.442, for Boron concentration \(c_B = 750 \text{ ppm}\).
3. INSTRUMENT TUBE MOTION

REQUEST 1. Small instrument tube motion.

Phase diagrams between IN11-IN12, IN12-IN13, IN13-IN14, IN14-IN15, IN15/IN16, given in figs. 13-17. Looking at the pairs in phase and in antiphase, it seems that two nodes are present.
One node at about 1000mm from core bottom, i.e. underneath detector IN-14.
One more node at about 400mm from core bottom, i.e. between detectors IN-15/IN-16.

Notes.
It is surprising to observe:
(a) the absence of the usual reactivity peak at the 9.2 Hz in the IN-16 detector (fig. 23).
(b) a drift toward the low frequencies of the high coherence region, while going from upper detector IN-11 to lower IN-15.
Coherence pattern IN-15/IN-16 quite different from the other pairs (figs. 18-22).

REQUEST 2. Driving sources of the noise.

Frequency around 9.2 Hz:
- YAO1/YAO2 in phase with a high coherence (no pressure nodes in the core at this frequency) (figs. 24-25).
- IN-14/EX-082 with phase displacement of 55 degrees (due to the additional filter in-in-core detectors) and high coherence (figs. 26-27).
- YAO1/IN-14 counter-phase with high coherence (taking into account the 55 degrees phase-shift due to additional Kron-Hite analog filter), consistent with negative pressure reactivity coefficient (figs. 28-29).
- YAO1/EX-082, same as YAO1/IN-14 (figs. 30-31).
The pressure variation represents a reactivity driving source.

Frequency range 12-16 Hz.

In this frequency range the behaviour of coherence and phase between YAO1/YAO2 and YAO1/EX-082 seem to indicate pressure waves generated by core barrel motion.
Pressure standing wave, with node in the core, is indicated by the high coherence and opposite phase between the sensors YAO1 and YAO2 at frequency 6.5 Hz.
The peaks within 18-24 Hz in the spectra of pressure sensors, probably are generated by mechanical vibration. There is no coherence between the signals in that frequency range (figs. 32-33).
FIGURE CAPTIONS

Fig. 1. Magnitude of NCPSD Ex.D52/Ex.D62.
Fig. 2. Coherence of NCPSD Ex.D52/Ex.D62. Phase given in turns so that 1/2 turn = 180 degrees.
Fig. 3. Phase between Ex.D52/Ex.D62.
Fig. 4. Magnitude of NCPSD Ex.D72/Ex.D82.
Fig. 5. Coherence of NCPSD Ex.D72/Ex.D82.
Fig. 6. Phase of NCPSD Ex.D72/Ex.D82.
Fig. 7. Magnitude of NCPSD Ex.Lin/Ex.Log.
Fig. 8. Coherence of NCPSD Ex.Lin/Ex.Log.
Fig. 9. Phase of NCPSD Ex.Lin/Ex.Log.
Fig. 10. $\cos^2(\phi - \phi_0)$ fit of the NAPSDs of the peak heights at 16 Hz.
Fig. 11. $\cos^2(\phi - \phi_0)$ fit of the NAPSDs of the peak heights at 12.6 Hz.
Fig. 12. $\cos^2(\phi - \phi_0)$ fit of the NAPSDs of the peak heights at 15 Hz.
Fig. 13. In-core neutron detectors 11-12 phase.
Fig. 14. In-core neutron detectors 12-13 phase.
Fig. 15. In-core neutron detectors 13-14 phase.
Fig. 16. In-core neutron detectors 14-15 phase.
Fig. 17. In-core neutron detectors 15-16 phase.
Fig. 18. In-core neutron detectors 11-12 coherence.
Fig. 19. In-core neutron detectors 12-13 coherence.
Fig. 20. In-core neutron detectors 13-14 coherence.
Fig. 21. In-core neutron detectors 14-15 coherence.
Fig. 22. In-core neutron detectors 15-16 coherence.
Fig. 23. NAPSD for In-core IN-16.
Fig. 24. Coherence between pressure YA01/YA02.
Fig. 25. Phase between pressure YA01/YA02.
Fig. 26. Coherence between IN-14/Ex.D82.
Fig. 27. Phase between IN-14/Ex.D82.
Fig. 28. Coherence between IN-14/YA01.
Fig. 29. Phase between IN-14/YA01.
Fig. 30. Coherence between YA01/Ex.D82.
Fig. 31. Phase between YA01/Ex.D82.
Fig. 32. APSD of pressure signal YA02 (physical units).
Fig. 33. APSD of pressure signal YA01 (physical units).
REFERENCES

[1] ROBINSON J.C., SHAHROKH F. and KRYTER R.C.
Quantification of core barrel motion using an analytically derived scale factor and statistical reactor noise descriptors.

Boroselle PWR noise: measurements, analysis and interpretation.

Surveillance of vibrations in PWR.

[4] TURKCAN E.
Review of Boroselle PWR noise experiments, analysis and instrumentation.
Prog. in Nucl. Energy, vol.9, p.437-452.
Fig. 9 FREQUENCY (Hz)

CORE BARREL MOTION 16 Hz

Fig. 10
Fig. 13 FREQUENCY (Hz)

Fig. 14 FREQUENCY (Hz)

Fig. 15 FREQUENCY (Hz)

Fig. 16 FREQUENCY (Hz)
PHYSICAL BENCHMARK TEST BORSSELE (PWR)
NOISE DATA

E. Türkcan

SMORN.IV, Dijon, 15-19 October 1984.
1. SIGNAL ANALYSIS AND INTERPRETATION

For the data analysis of the Barsele reactor, the digital data tape $S1094$ File 3 [1,3] has been used in present analysis.

Experimental data is analysed using mini-computer PDP11/24-array processor (AP-120B) combination of ECN. Three multichannel FFT calculations are performed:

a. All cross-combinations of ex-core neutron detectors, (S109F3.3F1).

b. All cross-combination of in-core neutron detectors, (S109F3.3F2).

c. Four ex-core neutron detectors, two-pressure and one in-core neutron detector (S109C7.3F1).

In all these calculations spectra were calculated upto 32 Hz, with frequency resolution of $df = 0.125$ Hz (256 freq. points), samples (394240 samples/channel) of data are used in the FFT and IFFT analysis; and average APSD, ACF, CCF, CPSD, PHASE and COHERENCE functions were determined.

Fig. 1, 2, 3 gives respectively APSD functions of six ex-core neutron, six in-core neutron and two pressure signals.

Calculation (a) has been used to decompose the measured spectrum to core barrel motion and reactivity [3,4,5] spectra using computer program SPECDEC (spectrum decomposition). Spectrum decomposition programs have been used for 4 ex-core detectors and for 6 ex-core detectors.

Decomposed spectra are given in fig. 4.

a. Determination of core barrel motions

Spectrum decompositions for 4-6 detectors gave very redundant results, fig. 4 indicates rms motion spectra (read upper-right scale) and the direction of the beam mode. Table 1 and the figure below are summarizing the findings.
Table 1. Summary of beam mode motions (SPECDEC)

<table>
<thead>
<tr>
<th>Beam mode frequency (Hz)</th>
<th>4 detectors</th>
<th>6 detectors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D82/D62/D72/D52</td>
<td>all ex-core neutron detectors</td>
</tr>
<tr>
<td></td>
<td>r.m.s. (in μm)</td>
<td>direction of motion degrees</td>
</tr>
<tr>
<td>11.75</td>
<td>2.1</td>
<td>167-347</td>
</tr>
<tr>
<td>12.75</td>
<td>4.1</td>
<td>81-261</td>
</tr>
<tr>
<td>15.125</td>
<td>5.7</td>
<td>180-360</td>
</tr>
<tr>
<td>16.0</td>
<td>5.1</td>
<td>118-298</td>
</tr>
<tr>
<td>17.625</td>
<td>1.84</td>
<td>37-217</td>
</tr>
</tbody>
</table>

b. Determination of the reactivity effect

Phase behaviour at 9.25 Hz for neutron detector indicates the reactivity effect (zero phase, high coherence). The r.m.s value of the reactivity can be derived from the APSD, such as calculating the peak area under the 9.2 Hz. We have calculated the reactivity spectra using program SPECDEC; (see fig. 4):

\[ \langle \delta k_{eff}^3 \rangle^{1/3} = \left( \int \rho(f).df \right)^{1/3} / 9 \text{ Hz} \]

\[ = 0.050 \text{ pcm.} \]

Pressure signals are highly coherent at 9.2 Hz. The r.m.s value of the pressure fluctuations:
\[ \langle \delta p^2 \rangle^{1/2} = (\int APSD(f) \cdot df)^{1/2} = \begin{cases} 0.026 \text{ bar (CH13)} \\ 0.022 \text{ bar (CH14)} \end{cases} \]

\( 9 \text{ Hz} \)

The derived reactivity/pressure coefficient is

\[-(\delta k_{\text{eff}}/\delta p) = 2.08 \text{ pcm/bar.} \]

Comparison of these results with the results given in [5] for 750 ppm boron concentration indicates very good agreement, table 2 summarize this results.

Table 2.

<table>
<thead>
<tr>
<th>9.2 Hz</th>
<th>results of [5] for 750 ppm</th>
<th>benchmark test</th>
</tr>
</thead>
<tbody>
<tr>
<td>reactivity in pcm</td>
<td>0.0495</td>
<td>0.050</td>
</tr>
<tr>
<td>reactivity/press.coef.</td>
<td>2.19</td>
<td>2.08</td>
</tr>
</tbody>
</table>

c. Instrument tube motion

Phase behaviour between the in-core neutron detectors were indicated small instrument tube motion. Fig. 2 shows normalized APSD functions of six in-core neutron detectors. The phase relation between all signals are indicating in-phase behaviour until 10 Hz. In the frequency range 11.5-14.0 Hz the phase behaviour between the detectors are given in the following table.
Table 3. 12-14 Hz phase behaviour between the in-core neutron detector signals.

<table>
<thead>
<tr>
<th>CH pairs</th>
<th>detector in-core</th>
<th>phase</th>
<th>in-core CH's det's</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/1</td>
<td>15/12</td>
<td>π</td>
<td>top core 2650 mm</td>
</tr>
<tr>
<td>5/1</td>
<td>14/12</td>
<td>π</td>
<td>11 . 11 -- 2396 mm</td>
</tr>
<tr>
<td>5/3</td>
<td>14/15</td>
<td>0</td>
<td>12 . 7 1 -- 2096 mm</td>
</tr>
<tr>
<td>7/1</td>
<td>13/12</td>
<td>0</td>
<td>13 . 7 1 -- 1620 mm</td>
</tr>
<tr>
<td>7/3</td>
<td>13/15</td>
<td>π</td>
<td>14 . 5 1 -- 968 mm</td>
</tr>
<tr>
<td>7/5</td>
<td>13/14</td>
<td>0</td>
<td>15 . 3 1 -- 492 mm</td>
</tr>
<tr>
<td>11/1</td>
<td>11/12</td>
<td>π</td>
<td>16 . 9 1 -- 192 mm</td>
</tr>
<tr>
<td>11/3</td>
<td>11/15</td>
<td>0</td>
<td>bottom of core</td>
</tr>
<tr>
<td>11/5</td>
<td>11/14</td>
<td>π</td>
<td>Node is inbetween detectors IN13 and IN14.</td>
</tr>
<tr>
<td>11/7</td>
<td>11/13</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

2. IDENTIFICATION OF NOISE SOURCES

Observed frequency peaks are indicated on the figs. 1, 2, 3.
Table 4 summarizes observed peaks and the interpretation. Short description of the peaks is also given in this table.

The results can be summarized as follows:

i. The reactivity effect at 9.25 Hz; pressure noise by itself produces a reactivity noise via the pressure reactivity coefficient.

ii. Vibration of core barrel and pressure vessel can be given in the following frequencies:
   11.75, 12.75, 15.1, 16.0 and 17.625 Hz.

iii. Instrument tube motion and fuel assembly beam modes:
   12.75 - 13.50 and 5.875 - 6.25 Hz.

iv. Thermohydraulic effects:
   1.5 Hz (coolant transport), 6.50, 9.25, 13.50 Hz (pressure oscillation - standing waves).

v. Pump rotation 24.85 Hz.

vi. White noise contribution of ex-core neutron detectors above 25 Hz gives information about the quality of the neutron detectors.
vii. The coherence function measured between IN-11 and other in-core detectors are shown in fig. 5. Clear coherence dips are seen at a frequency 1.50 Hz and its harmonics of 3.0 Hz. From the coherence dip, a transient time for the coolant to travel the whole core height is estimated as 0.67 s, resulting in a coolant velocity of (2.65 m / 0.67 s =) 3.98 m/s.

viii. Signal transmission path analysis will give more information about the noise sources [6].

Table 4. Observation of several frequency peaks

<table>
<thead>
<tr>
<th>code</th>
<th>ex-core</th>
<th>in-core</th>
<th>pres.</th>
<th>freq. (in Hz)</th>
<th>observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>1.125</td>
<td>Pressure oscillations</td>
</tr>
<tr>
<td>2</td>
<td>x</td>
<td>x</td>
<td>-</td>
<td>1.50</td>
<td>Ex-core (?), In-core (flow), Pressure signal in phase</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>x</td>
<td>x</td>
<td>1.875</td>
<td>Pressure oscillations</td>
</tr>
<tr>
<td>4</td>
<td>x</td>
<td>x</td>
<td>-</td>
<td>2.25</td>
<td>Fuel vibrations</td>
</tr>
<tr>
<td>5</td>
<td>x</td>
<td>-</td>
<td>x</td>
<td>3.25</td>
<td>Pressure fluctuations</td>
</tr>
<tr>
<td>6</td>
<td>x</td>
<td>x</td>
<td>-</td>
<td>3.50</td>
<td>(?)</td>
</tr>
<tr>
<td>7</td>
<td>x</td>
<td>x</td>
<td>-</td>
<td>4.75-5.75</td>
<td>Fuel vibrations</td>
</tr>
<tr>
<td>8</td>
<td>x</td>
<td>x</td>
<td>-</td>
<td>6.5</td>
<td>Standing wave, pressure effect (temperature dependent)</td>
</tr>
<tr>
<td>9</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>9.25</td>
<td>Pressure effect reactivity (standing wave)</td>
</tr>
<tr>
<td>10</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>10.7</td>
<td>Pressure oscillations</td>
</tr>
<tr>
<td>11</td>
<td>x</td>
<td>-</td>
<td>-</td>
<td>11.75</td>
<td>Core barrel</td>
</tr>
<tr>
<td>12</td>
<td>x</td>
<td>-</td>
<td>-</td>
<td>12.5-13.0</td>
<td>Fuel vibrations</td>
</tr>
<tr>
<td>13</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>12.7</td>
<td>Pressure (?2)</td>
</tr>
<tr>
<td>14</td>
<td>x</td>
<td>-</td>
<td>-</td>
<td>12.75</td>
<td>Core barrel</td>
</tr>
<tr>
<td>15</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>13.375</td>
<td>(?)</td>
</tr>
<tr>
<td>16</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>13.5</td>
<td>Standing wave</td>
</tr>
<tr>
<td>17</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>14.75</td>
<td>(?) only in-core</td>
</tr>
<tr>
<td>18</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>15.0</td>
<td>(?) in-core</td>
</tr>
<tr>
<td>19</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>15.125</td>
<td>Core barrel</td>
</tr>
<tr>
<td>20</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>15.75</td>
<td>(?)</td>
</tr>
<tr>
<td>21</td>
<td>x</td>
<td>-</td>
<td>-</td>
<td>16.0</td>
<td>Pressure vessel (core barrel)</td>
</tr>
<tr>
<td>22</td>
<td>x</td>
<td>-</td>
<td>-</td>
<td>16.75</td>
<td>Pressure (?2)</td>
</tr>
<tr>
<td>23</td>
<td>-</td>
<td>x</td>
<td>-</td>
<td>17.5</td>
<td>D82</td>
</tr>
<tr>
<td>24</td>
<td>-</td>
<td>x</td>
<td>x</td>
<td>17.75</td>
<td>Pressure vessel'</td>
</tr>
<tr>
<td>25</td>
<td>x</td>
<td>-</td>
<td>-</td>
<td>18.25</td>
<td>Resonance of press. det. (Barton cell) (?1)</td>
</tr>
<tr>
<td>26</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>19.525</td>
<td>Resonance of press. det. (Barton cell) (?2)</td>
</tr>
<tr>
<td>code</td>
<td>ex-core</td>
<td>in-core</td>
<td>pres.</td>
<td>freq. (in Hz)</td>
<td>observations</td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td>---------</td>
<td>-------</td>
<td>---------------</td>
<td>----------------------------</td>
</tr>
<tr>
<td>29</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>21.375</td>
<td>Pressure (P2)</td>
</tr>
<tr>
<td>30</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>24.875</td>
<td>Main coolant pump vibrations</td>
</tr>
<tr>
<td>31</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>25.75</td>
<td>Pressure (P2)</td>
</tr>
<tr>
<td>32</td>
<td>-</td>
<td>x</td>
<td>x</td>
<td>27.375</td>
<td>Pump vibrations (?) (P2)</td>
</tr>
<tr>
<td>33</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>29.375</td>
<td>Pump vibrations (P2)?</td>
</tr>
<tr>
<td>34</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>31.375</td>
<td>Pressure P1/P2</td>
</tr>
</tbody>
</table>

REFERENCES

Fig. 1. Normalized APSD functions of six ex-core neutron detector signals.
Fig. 2. Normalized APSD functions of six in-core neutron detector signals.
Fig. 3. APSD functions of two pressure detectors.
Fig. 4. Decomposed spectra.

Reactivity (upper figure left scale) and core motion spectra (right scale) together with the direction of motion (lower figures).
Fig. 5. Coherence functions of the in-core neutron detector signals.
PHENIX (LMFBR) NOISE
TASKS AND RELATED INFORMATION

ON THE PHENIX LMFBR NOISE DATA
PROPOSALS FOR A "PHYSICAL" BENCHMARK ON THE NOISE DATA OF THE
PHENIX REACTOR.

FIRST PROPOSAL: identification of time constants.

1°) time constant of the heat transfer in the subassembly
(fuel-cladding-sodium).

2°) time constant of the thermocouples measuring the tempe-
    rature at the outlet of the subassemblies.

In order to avoid misunderstanding, a physical presentation
of the problem seems to be useful.

Let \( \delta P(t) \) be the neutron noise and

\[ \delta T_{si}(t) \]

be the "true" thermal noise at the outlet of the
i\(^{th}\) subassembly and let us suppose that flow noise and inlet tem-
perature noise are negligible (very low frequency).

One can write the relation:

\[ \delta T_{si}(f) = F(f) \frac{\delta P(f)}{\bar{P}} \delta T_{si} + b_s(f) \]

where

- \( F(f) \): transfer function of the subassembly
- \( \bar{\delta T}_{si} \): temperature rise in the i\(^{th}\) subassembly (\( \sim 200^\circ C \))
- \( b_s(f) \): weakly correlated noise.

The transfer function of the subassembly is related to the
cut-off frequency through the relation:

\[ F(f) \approx \frac{1}{1 + i \frac{f}{f_c}} \]

The measured temperature noise is related to the "true" ther-
mal noise by the relation:
\[
\left[\delta T_{s1}(f)\right]_{\text{meas.}} = \delta T_{s1}(f) \frac{1}{1 + i \frac{f}{f_T}}
\]

where \( f_T \) is the cut-off frequency of the thermocouple.

From the analysis of neutron and temperature signals it is possible to identify the time constants of the subassembly

\[\tau_c = \frac{1}{2\pi f_c}\]

and of the thermocouple \( \tau_T \approx \frac{1}{2\pi f_T} \)

The presumed values of these time constants are

\[
\tau_c : 1 \text{ to } 5 \text{ sec.} \\
\tau_T : 3 \text{ to } 8 \text{ sec.}
\]

From analysis of the Phenix noise data, it would be possible to determine these time constants. Channels to be used for such an analysis are:

- No. 1 subassembly outlet temperature (TATA 2018)
- No. 2 " " " (TATA 2024)
- No. 4 " " " (TATA 2119)
- No. 3 ex-core ion chamber (Z1MR41)
- No. 5 " " " (Z1MR51)

SECOND PROPOSAL: determination of the components of the neutron noise.

Assuming that the neutron noise is the summation of three components:

1°) thermal noise, which is a reactivity fluctuation or a neutron flux fluctuation related to thermal fluctuations in the core (thermal noise is similar to \(1/f^\alpha\) with \(\alpha \approx 1\) and \(f\) = frequency).
2°) mechanical noise, which is due to reactivity fluctuations related to vibrations in control rods or motion of neutron detectors relatively to the core or...

3°) background noise, which is a white noise in relation with reaction rate in the neutron detector.

We propose to identify these components and to evaluate the characteristics of the mechanical resonance peaks (natural frequency and damping):

The channels to be used are:

No.3 ex-core ion chamber (Z1MR41)
No.5  "    "    " (Z1MR51)

Correlation between the two channels would be useful for a better identification.

THIRD PROPOSAL: Relation between signals related to inlet temperature and flow and signals related to neutron power and temperature at the outlet of subassemblies.

From the analysis of channels:

No.6 pump inlet temperature (PJMT25)
No.7 primary pump flowrate (P1MQ02)
No.1 subassembly outlet temperature (TATA 2018)
No.2  "    "    " (TATA 2024)
No.4  "    "    " (TATA 2119)
No.3 ex-core ion-chamber (Z1MR41
No.5  "    "    " (Z1MR51)

is it possible to determine if there any evidence of relations between inlet temperature or flow and outlet temperature or neutron power.
REPORT ON THE PHENIX (LMFBR)
REACTOR NOISE BENCHMARK

by

G. LE GUILLOU

C.E.A. IRDI/DEDR - CEN CADARACHE
Test de performance (Benchmark)

Compte-rendu des analyses réalisées sur les bruits du réacteur PHENIX

G. LE GUILLOU
CEA/CEN Cadarache

1. DONNEES

a) Les propositions

Trois propositions furent soumises aux contributeurs.

Première proposition: identification des deux constantes de temps "$\tau_{ci}$" et "$\tau_{Ti}$" qui caractérisent la dynamique du bruit de température mesuré en sortie d'un assemblage "i".

$\tau_{ci}$ = constante de temps de l'échange thermique entre le combustible et le réfrigérant.

$\tau_{Ti}$ = constante de temps du thermocouple.

L'ordre de grandeur de chacune de ces constantes de temps, connu à priori, était donné afin de permettre aux contributeurs de faire le choix convenable de la bande de fréquence à analyser:

$1 < \tau_{ci} < 5 \text{ sec}$

$3 < \tau_{Ti} < 8 \text{ sec}$

On indiquait la structure du modèle générateur du bruit de température ainsi que celle des deux fonctions de transfert concernées:

\[
\text{bruit de mesure (non corrélé)} \rightarrow \Delta T_{Si}
\]

\[
F(f) = \frac{\Delta T_{Si}}{1+i f/f_c}
\]

$\Delta T_{Si}$ = élévation de température dans l'assemblage "i" (environ 200°C)

$f_c = \frac{1}{2\pi \tau_c}$ et $f_{Ti} = \frac{1}{2\pi \tau_{Ti}}$
Cette proposition impliquait trois signaux de température (canaux 1, 2 et 4) et des bruits neutroniques (canaux 3 et/ou 5).

2ème proposition: détermination des composantes du bruit neutronique, c'est-à-dire:

- la composante de bruit thermique dont la densité spectrale est de la forme \[ \frac{C}{f^2} \]
- la, ou les, composantes dues aux vibrations mécaniques des structures (coeur, barres de commandes, détecteurs, ...)
- le bruit de détection qui est directement relié aux caractéristiques du détecteur (flux et sensibilité).

Deux canaux étaient concernés par cette proposition:
- canal No 3 chambre sous cuve ZIMR41
- canal No 5 " " ZIMR51

3ème proposition: estimer, qualitativement, la contribution des fluctuations du débit primaire et de la température d'entrée respectivement aux bruits de puissance neutronique et aux bruits de température de sortie des assemblages.

L'esprit de cette dernière proposition était de qualifier les interactions concernées en terme "d'interaction faible" ou "d'interaction nulle". Les canaux 1 à 7 étaient concernés par cette proposition.

b) La description de l'enregistrement

Les trois propositions impliquaient sept signaux (voir le Tableau I).

La durée de l'enregistrement disponible était de 1h 3/4 (dont 1,5 heure était réellement utilisable).

2. RESULTATS

a) Liste des contributions

Cinq textes de contributions nous ont été soumis. Les auteurs de ces contributions sont:

- M. Edelmann (Kernforschungsanlage Karlsruhe; R.F.A.)
- J.E. Hoogenboom (IRI Delft, Pays-Bas)
- Z.P. Luo et S.M. Wu (University of Wisconsin, États-Unis)
- N. Morishima (Kyoto University, Japon)
- T. Tamaoki et Y. Sonoda (Kawasaki, Japon)

Remarques:

- un label arbitraire a été attribué aux contributeurs dans les tableaux qui vont suivre.
- une des contributions, d'une nature trop qualitative, n'a pas pu être prise en compte dans les tableaux.

b) Résultats comparatifs

1ère proposition:
Phenix benchmark test data.

Figure A-1
<table>
<thead>
<tr>
<th>Z1MeS1 (MW)</th>
<th>P3MT25 (°C)</th>
<th>P1MQ02 (Volt)</th>
<th>S1MQ01 (Volt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>25.0</td>
<td>110×10⁻³</td>
<td>10×10⁻³</td>
</tr>
<tr>
<td>8.0</td>
<td>25.0</td>
<td>110×10⁻³</td>
<td>10×10⁻³</td>
</tr>
<tr>
<td>12.52</td>
<td>8.0</td>
<td>110×10⁻³</td>
<td>10×10⁻³</td>
</tr>
<tr>
<td>25.04</td>
<td>12.52</td>
<td>110×10⁻³</td>
<td>10×10⁻³</td>
</tr>
<tr>
<td>37.56</td>
<td>25.04</td>
<td>110×10⁻³</td>
<td>10×10⁻³</td>
</tr>
<tr>
<td>50.08</td>
<td>37.56</td>
<td>110×10⁻³</td>
<td>10×10⁻³</td>
</tr>
<tr>
<td>62.60</td>
<td>50.08</td>
<td>110×10⁻³</td>
<td>10×10⁻³</td>
</tr>
</tbody>
</table>

Phenix benchmark test data.

Figure A-2
Pour cette première proposition nous disposons d'une référence (issue des analyses d'échelons de réactivité effectuées régulièrement sur PHENIX):

\[ 1.4 < \bar{\tau}_{\text{Ci}} < 2.2 \text{ sec} \]
\[ 6 < \bar{\tau}_{\text{Ti}} < 12 \text{ sec} \] (la valeur la plus probable étant 8 sec)

Conclusion en ce qui concerne \( \bar{\tau}_{\text{Ci}} \):

L'estimation moyenne donne:

\[ \bar{\tau}_{\text{Ci}} = 2.45 \pm 0.2 \text{ sec} \]

(donc une valeur sur-évaluée par rapport à la fourchette de référence)

Conclusion en ce qui concerne \( \bar{\tau}_{\text{Ti}} \):

Si on enlève les deux valeurs extrêmes (15 sec et 16.2 sec) du canal 1 on obtient des moyennes qui se situent très valablement par rapport à la référence (les valeurs semblent sous-évaluées par rapport à cette référence):
canal 1: $\tau_T = 7.7 \pm 0.6 \text{ sec}$
canal 2: $\tau_T = 6.5 \pm 0.4 \text{ sec}$
canal 4: $\tau_T = 6.4 \pm 0.5 \text{ sec}$

Deuxième proposition: la diversité des modèles de représentation des composantes, d'une part, et les problèmes de mise à l'échelle des signaux neutroniques d'autre part, rendent difficile la comparaison des résultats.

Finalement trois contributions ont seulement pu être prises en compte (labels A, B, C). La contribution D, bâtie sur une analyse en modèle ARMA, propose une décomposition en 12 résonateurs: en l'absence de commentaires ou d'interprétations de ces résultats il nous a été impossible de les classifier.

Deux modèles physiques d'identification sont proposés:

**Modèle 1:** (contributeur B)

$$S_{NN}(f) = B + \frac{C}{f_0^\alpha} + \sum_{i=1}^{N} \frac{D_i}{f_0^\alpha (4\pi^2)^2} \cdot M(f)$$

**Modèle 2:** (contributeurs A et C)

$$S_{NN}(f) = B + \frac{1}{f_0^\alpha} \sum_{i=1}^{N} \frac{D_i}{f_0^\alpha (4\pi^2)^2} \cdot M(f)$$

avec

$$M(f) = \frac{1}{\left( f_0^\alpha f_0^2 + 2 \epsilon_i^* f_0 f_0^\alpha \right)^2}$$

- B bruit de détections
- $C_i$ paramètre d'amplitude
- $N$ nombre de résonateurs mécaniques
- $f_{0i}$ fréquence de résonance du mode "i"
- $\epsilon_i$ amortissement du mode "i"
- $S_{NN}(f)$ APSD du bruit neutronique

Pour harmoniser les deux modèles on peut remarquer que pour $f \ll f_{0i}$ on a:

$$C = \sum_{i} \frac{D_i}{f_0^\alpha (4\pi^2)^2} \cdot \frac{1}{f_{0i}^4}$$

Le tableau suivant a été obtenu en corrigeant les paramètres pour les ramener à ceux du "modèle 1".
On peut remarquer que les paramètres "C" et "α", caractéristiques des basses fréquences, sont très dispersés: ce fait doit être imputé à la trop courte durée de l'enregistrement.

La dispersion du paramètre "B" est plus surprenante: on peut penser que cela est dû à des problèmes de mise à l'échelle des signaux:

Finalement les seuls paramètres aisément comparables sont les fréquences de résonnance et les amortissements: si l'accord se fait sur la résonnance entre 5,5 Hz et 5,9 Hz les estimations de l'amortissement sont très dispersées.

On peut donc conclure que cette seconde proposition débouche sur des résultats peu satisfaisants (la contribution de label B étant toutefois très proche de notre référence). A cela trois raisons principales:

- l'enregistrement est trop court
- la complexité de la proposition (jusqu'à 9 paramètres à identifier)
- la diversité des modèles de représentation.

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- la diversité des modèles de représentation.

<table>
<thead>
<tr>
<th>Label Contributeur</th>
<th>B (hz⁻¹)</th>
<th>C</th>
<th>α</th>
<th>D₁</th>
<th>ε₁</th>
<th>D₂</th>
<th>ε₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4,110⁻⁵</td>
<td>3,7210⁻⁴</td>
<td>2</td>
<td>17,6</td>
<td>5,5</td>
<td>0,075</td>
<td>sans correction de gain après correction de &quot;Gain&quot; (correctif=45)</td>
</tr>
<tr>
<td></td>
<td>2,0510⁻⁸</td>
<td>1,8610⁻⁷</td>
<td>2</td>
<td>8,810⁻³</td>
<td>5,5</td>
<td>0,075</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1,910⁻⁸</td>
<td>3,10⁻⁹</td>
<td>1,45</td>
<td>4,10⁻³</td>
<td>5,67</td>
<td>0,13</td>
<td>channel 5 (ZIMR41)</td>
</tr>
<tr>
<td></td>
<td>1,2410⁻⁸</td>
<td>4,0310⁻⁸</td>
<td>1,08</td>
<td>id.</td>
<td>id.</td>
<td>210⁻³</td>
<td>channel 5 (ZIMR51)</td>
</tr>
<tr>
<td>C</td>
<td>non déterminé</td>
<td>2,10⁻⁶</td>
<td>0,866</td>
<td>1,2</td>
<td>5,97</td>
<td>0,245</td>
<td></td>
</tr>
</tbody>
</table>
3ème proposition:

Tous les contributeurs concluent à la faiblesse de l'interaction entre les fluctuations de puissance et de température de sortie des assemblages, d'une part, et les fluctuations de débit primaire et de température d'entrée; d'autre part. (Ce qui est conforme à toutes les analyses effectuées sur PHENIX).

3. CONCLUSIONS

Trois propositions ont été soumises et nous avons obtenu des réponses concordantes et très satisfaisantes pour l'une d'entre elle; il s'agit de l'évaluation des constantes de temps des températures de sortie du cœur. C'est un résultat intéressant qui ouvre des perspectives en ce qui concerne la surveillance en exploitation des thermocouples. Ce bon résultat doit toutefois être confirmé étant donné la brièveté de l'enregistrement utilisé.

Les deux autres propositions; par leur complexité; on prêtaient finalement peu à l'exercice "benchmark".
INDIVIDUAL CONTRIBUTIONS TO THE PHENIX NOISE BENCHMARK
For shortage of time seasons only the first problem proposed for Phenix data analysis is dealt with APSD'S, CPSD'S, coherences and transfer functions were measured. The results are shown in Figs. 2 to 8. Due to the short record length of the signals their stochastic signatures are obtained with large uncertainties only. Interpretation of the results is rather speculative, therefore. Further more, the neutron/power noise of Phenix at higher frequencies as well as the low coherence between neutron and outlet temperature noise in the relevant frequency range \( f > 0.1 \text{ Hz} \) are very unfavorable or even insufficient measure time constants in the order of 1s. Into addition, there is little chance under these circumstances to determine two time constants which are very different in magnitude. Only a tough estimate of the larger one of the two time constants can be expected from available data. If two time constants are pitted and one of them is much smaller than the other one the smaller time constant will not have any physical significance.

From Fig. 2 it is seen that auto power spectral densities of power and outlet temperature noise are quite similar except for the temperature signal TA2119 which is supposed to be strongly contaminated with extraneous noise. This is confirmed by the cross power spectral densities shown in Fig. 3 from which it is seen that the correlated noise in the three signals is the same. Fig. 4 shows that correlation of outlet temperature noise with inlet temperature is not negligible against correlation with reactor power. Obviously, both inlet temperature and power noise are driving forces for outlet temperature noise. Their individual contributions cannot be separated without more detailed knowledge of the reactor and additional measurements.

Although the low power noise level in the relevant frequency range is not favouring the measurement of power-to-outlet temperature transfer functions it was possible to obtain the rough estimates shown in Fig. 6 which allowed to approximately determine sub-assembly and thermocouple time constants of 0.5 s and 3 s, respectively. From Fig. 6 it is also seen that no more than about 60% of power noise causes outlet temperature noise. This strange result can not be explained without further detailed analysis requiring more information on the reactor, too.

The coherence functions presented in Fig. 7 directly show the low coherence between neutron and outlet temperature noise. It is suspected that the complex situation is a combined effect of high inlet temperature noise and temperature coefficient of Phenix. In Fig. 8 auto power spectral densities of inlet and outlet temperature noise are plotted in absolute temperature units. This reveals that inlet temperature fluctuations are of about the same size or even larger than outlet temperature fluctuations.
Fig. 2 Normalized auto power spectral densities of power and outlet temperature noise
Fig. 3 Normalized cross power spectral densities of neutron and outlet temperature noise
Fig. 4 Normalized cross power spectral densities of inlet temperature, power and outlet temperature noise.
Fig. 5. Phase angle of cross power spectral densities
Fig. 6 Transfer functions between neutron and outlet temperature noise

Fig. 7 Coherences of neutron noise and outlet temperature noise
Fig. 8. Absolute auto power spectral densities of inlet and outlet temperature noise.
Abstract:

According to the task specification on Phenix noise, the digital data from NEA Data Bank are analyzed both statistically and spectrally. Two methods of noise analysis are used: the FFT method to obtain noise descriptors such as power spectral densities, coherence functions and transfer functions, and the AR model fitting to estimate causal relations among several noise signals in terms of a cumulative power contribution ratio. The test results, together with some description of sampling conditions and estimation processes, are presented. It is shown that the spectral analysis is performed successfully by the combined use of the two methods and also by the window opening/closing approach that leads to comparative study of a set of spectral estimates with several different frequency resolutions.
1. Analysis Method and Estimation Procedure

The noise data of Phenix reactor analyzed here is the digital version of test data which was obtained from NEA Data Bank in March 1984. The identification number of the magnetic tape is E84135 labeled as S110.F1.FBR. Before the noise data are analyzed spectrally, all the noise data are calibrated by use of the DC signals of 0 and 1 volt. The formula used for the calibration is as follows:

\[(\text{Calibrated signal}) = a \times (\text{Original signal}) + b \text{ (volt)} \quad (1)\]

where the following values of \(a\) and \(b\) are determined from the recalibration of the DC signals,

- Channel 1 : \(a = 3.126E-4, \; b = -2.544\)
- 2 : \(a = 3.133E-4, \; b = -2.592\)
- 3 : \(a = 2.986E-4, \; b = -2.489\)
- 4 : \(a = 3.095E-4, \; b = -2.569\)
- 5 : \(a = 3.041E-4, \; b = -2.508\)
- 6 : \(a = 3.256E-4, \; b = -2.777\)
- 7 : \(a = 3.119E-4, \; b = -2.537\)
- 8 : \(a = 3.162E-4, \; b = -2.584\)
- 9 : \(a = 3.586E-4, \; b = -2.945\)
- 10 : \(a = 3.088E-4, \; b = -2.535\)

The noise data are examined statistically in terms of both the degree of stationariness of sample mean and sample variance, and the degree of Chi-square fitness to normal distribution \((1)\). It is assured that all the noise data can be used for noise analysis; it does
not contain any non-stationariness due to the control rod motion.

Part of the results of the statistical test is shown in Figs. 1 and 2.

Two important conclusions are drawn:

(a) Some of the noise signals are dominated by low-frequency components around about 0.005 Hz. This means that the sample length of about 200 sec or more is required to calculate a covariance function.

(b) The degree of Chi-square fitness to normal distribution is improved significantly with an increase in the sampling period h. The use of sampling period of 0.25 sec or more is desirable, when the normal property of a time series is required.

In the present AR model fitting, a time series of 400 sec in length and 0.5 sec in sampling period is adopted to obtain a sample covariance function, and then the arithmetic average of eleven sample covariance functions is made to obtain a covariance function estimate. Figures 3 through 5 show only three examples of the covariance functions thus obtained, in which the functions \( C_{ij}(\tau) \) and \( \triangle C_{ij}(\tau) \) are calculated using the formulas

\[
C_{ij}(\tau) = \frac{1}{11} \sum_{k=1}^{11} C_{ij}^{(k)}(\tau),
\]

\[
\triangle C_{ij}(\tau) = \sqrt{\frac{1}{11} \sum_{k=1}^{11} C_{ij}^{(k)}(\tau)^2 - C_{ij}(\tau)^2},
\]

where \( C_{ij}^{(k)}(\tau) \) is the covariance function of the k-th sample for Channels i and j, defined by

\[
C_{ij}^{(k)}(\tau=nh) = \frac{1}{N} \sum_{n=1}^{N-m} x_i^{(k)}(n+m)x_j^{(k)}(n).
\]
These covariance functions are those that caused the largest difference in the previous benchmark test(2).

Two kinds of noise analysis method are adopted here. One is the FFT method to obtain the noise descriptors such as power spectral densities, coherence functions and transfer functions. The other is the AR model fitting to estimate mutual causative relations among several noise signals in terms of a cumulative power contribution ratio. By the combined use of these two methods, the noise data are analyzed spectrally and physically, and four kinds of spectral analysis are performed according to the following objectives:

Analysis A: Noise descriptor estimation over a wide frequency range to understand major spectral characteristics of the noise data

; Determination of the value of the time constant $T$

; Estimation of the transfer functions of outlet temperature and neutron power to inlet temperature and primary-pump flowrate

Analysis B: Determination of the value of the time constant $C$

Analysis C: Evaluation of the spectral characteristics of the mechanical resonance peak in the PSD of neutron power

Analysis D: Estimation of a cumulative power contribution ratio and auto-PSD of noise source

The sampling and spectral conditions of the analysis A through D are listed in Table 1.

In the AR model fitting, four values of the model order $M$ are
chosen in view of the frequency resolution $1/2\,\text{Hz}$, that is, $M=3$, 10, 30 and 100. The order of 3 is the value that is determined by use of the AIC criterion. The spectral estimates for these four orders are compared each other and checked with those computed by the FFT method so that it is assured that there is a satisfactory agreement in both shape and magnitude between the results of the two different methods with the same frequency resolution. A typical example of the spectral comparison is shown in Figs. 6 through 9 in which, for example, A-H4 means the results of the analysis A smoothed with Hanning window 4 times, and D-M30 indicates the results of the analysis D for the model order of 30. Note that such notations are used throughout the graphical representation of the present report.

The reason for excluding Channel 3 from the AR model fitting is the almost complete coherence between the two signals of neutron noise in Channel 3 and 5. The coherence function and its phase spectrum are shown in Figs. 10 and 11, which indicate that these two noise signals fluctuate coherently in the frequency region below about 0.1 Hz. As a result of this signal selection, the AR model fitting is performed successfully. This is ascertained from the following normalized matrices of the residuals for four values of model order up to 100:

$$\Sigma = \begin{pmatrix}
1.000E+00 & -1.947E-02 & 1.828E-02 & 6.401E-02 & 6.506E-02 & -7.777E-03 \\
1.000E+00 & -7.932E-04 & 1.034E-02 & -1.997E-02 & -1.011E-02 \\
1.000E+00 & -3.608E-02 & -2.507E-02 & 5.769E-03 \\
1.000E+00 & 2.974E-02 & 3.421E-03 \\
1.000E+00 & -9.104E-03 \\
1.000E+00 & -9.104E-03 & 3.421E-03 \\
\end{pmatrix}$$

for $M = 3$, 

$$\Sigma = \begin{pmatrix}
1.000E+00 & -1.978E-02 & 1.742E-02 & 6.276E-02 & 6.511E-02 & -7.851E-03 \\
1.000E+00 & 1.000E+00 & 1.000E+00 & 1.000E+00 & 1.000E+00 & 1.000E+00 \\
1.000E+00 & -3.686E-02 & -2.449E-02 & 5.769E-03 \\
1.000E+00 & 3.057E-02 & 2.401E-03 \\
1.000E+00 & -9.038E-03 \\
1.000E+00 & -9.038E-03 & 3.421E-03 \\
\end{pmatrix}$$
2. Determination of the values of the time constants $\tau_T$ and $\tau_C$.

The time constant $\tau_T$ of the thermocouple measuring temperature noise at the outlet of the subassemblies TATA2018, TATA2024 and TATA2119 is estimated as follows. According to the power contribution ratios of the analysis D for outlet temperature in Channel 1, 2 and 4 which are shown in Figs. 12 through 14 respectively, it is understood that there is no significant contribution to the signal in Channel 4 from the other, and that there is some considerable amount of contribution to the signals in Channels 1 and 2, especially from those in Channels 5 and 6. Hence these results suggest that the values of $\tau_T$ for Channel 1 and 2 should be determined by use of auto-PSDs of the noise source from the AR model fitting. Figures 13 through 17 show the auto-PSDs for Channels 1, 2 and 4, together with the fitted curve of the form

$$(\text{constant}) \times (f^2 + f_T^2)^{-1} \quad \text{and} \quad \tau_T = 1/2\pi f_T \quad . \quad (5)$$
Note that the present fitting is made by eyeguide.

The values of $\tau_T$ and $f_T$ are determined as follows.

Channel 1 : $f_T^{(AR)} = 0.014$ Hz, $\tau_T^{(AR)} = 11.$ sec
2 : $f_T^{(AR)} = 0.022$ Hz, $\tau_T^{(AR)} = 7.2$ sec
4 : $f_T^{(AR)} = 0.025$ Hz, $\tau_T^{(AR)} = 6.4$ sec

For reference, the same fitting is applied to the auto-PSDs of all the outlet temperature noises computed by FFT method, which are shown in Figs. 6, 18 and 19. The values of $f_T$ and $\tau_T$ thus determined are as follows.

Channel 1 : $f_T^{(FFT)} = 0.011$ Hz, $\tau_T^{(FFT)} = 15.$ sec
2 : $f_T^{(FFT)} = 0.021$ Hz, $\tau_T^{(FFT)} = 7.6$ sec
4 : $f_T^{(FFT)} = 0.024$ Hz, $\tau_T^{(FFT)} = 6.6$ sec

Both of the above two kinds of the results suggest the following possibilities: (a) The temperature noise in Channel 1 at low frequencies of the order of 0.01 Hz is largely affected by the effect not described with the noise signals in Channels 2 through 7. And/or, (b) the response property of the thermocouple of Channel 1 is not the same as those in Channels 2 and 4.

The another time constant $\tau_C$ is estimated by use of the transfer functions of all the outlet temperature to neutron power in Channel 5, which are shown in Figs. 20 through 22. In order to assure that the transfer functions result from significant correlation between the
above two input and output signals, the corresponding coherence functions are obtained, which are shown in Figs. 23 through 25. The transfer function model used for fitting is of the form

\[(\text{constant}) \times (\varepsilon^2 + \varepsilon^2_C)^{-1/2} \quad \text{and} \quad \tau_C = 1/2\pi f_C. \quad (6)\]

The fitting is made by eyeguide, but keeping in mind on such frequency region that the coherence function takes relatively high values.

The values of $\tau_C$ thus determined are as follows.

Channel 1: $f_{W}^{(\text{FFT})} = 0.055$ Hz, $\tau_{C}^{(\text{FFT})} = 2.9$ sec

2: $f_{C}^{(\text{FFT})} = 0.050$ Hz, $\tau_{C}^{(\text{FFT})} = 3.2$ sec

4: $f_{B}^{(\text{FFT})} = 0.61$ Hz, $\tau_{B}^{(\text{FFT})} = .26$ sec

Note that the transfer function of outlet temperature in Channel 4 could not be fitted to the model (6). This is due to no significant correlation between outlet temperature in Channel 4 and neutron power in Channel 5, which is seen from Fig. 25 by comparison with those in Figs. 23 and 24. However it is found to be fitted to the form

\[(\text{constant}) \times (\varepsilon^2 + \varepsilon^2_B)^{-1} \quad \text{and} \quad \tau_B = 1/2\pi f_B. \quad (7)\]

The result of this fitting is presented above and also shown in Fig. 22 in dashed curve.

Two comments on the determination of the values of $\tau_C$ are given:

(a) The power contribution ratio to neutron power in Channel 5 which is shown in Fig. 26 indicates no significant contribution from
the other noise signals above about 0.02 Hz. This assures that the transfer functions by FFT method can be used as open-loop ones in order to estimate $\tau_c$.

(b) The transfer functions by the AR model fitting are obtained and checked carefully by comparison with those by the FFT method. It is exhibited that the AR transfer functions depend for their spectral shape largely upon the model order $M$, because of, probably, the low signal(below about 0.02 Hz)-to-noise(the other higher component) ratio. So the present analysis of $\tau_c$ utilizes only the results by the FFT method.

3. Determination of the mechanical characteristics of neutron noise

The auto- and cross-PSDs of neutron noise in Channels 3 and 5 are obtained through the analysis C and shown in Figs. 27 through 29. It can be seen that the auto-PSD for Channel 3 differs remarkably in shape whether or not a spectral window is applied. This means that the raw auto-PSDs for Channel 3, and probably its statistical property, vary largely from sample to sample. The neutron noise in Channel 3, therefore, is inadequate for use in the present analysis.

The normalized auto-PSD for Channel 5 is adopted to estimate the five parameters $P_0$, $\omega$, $c/m$, $f_0$ and $\varepsilon$ by the use of the formula

$$P(\omega) = P_0(\text{background noise}) + \frac{c/m}{4\pi^2} \frac{1}{\sqrt{(f_0^2 - \omega^2)^2 + (2\pi f_0)^2}} \frac{1}{\omega^5} \quad (1/\text{Hz}) \quad (8)$$
The fitting conditions and the results are as follows.

(1) $\alpha = 1.0$ by eyeguide,

(2) $P_0 = P(f=30 \, \text{Hz}) = 6.4 \times 10^{-3} \, (/\text{Hz})$,

(3) $f_0 = 5.5 \, \text{Hz}$ from the peak of auto-PSD,

(4) $c/m = 23.$ by the use of the approximation

$$P(f) = P_0 + \frac{c/m}{4\pi^2} \frac{1}{f_0^2} \frac{1}{f} \quad \text{for} \quad f \ll f_0,$$

and the condition $P(0.1) = 0.2 \, (/\text{Hz})$,

(5) $\xi = 7.5 \times 10^{-2}$ from the expression

$$P(f_0) = P_0 + \frac{c/m}{4\pi^2} \frac{1}{2\xi f_0^3} \quad \text{at} \quad f = f_0,$$

and the condition $P(5.5) = 3.0 \times 10^{-2} \, (/\text{Hz})$.

The formula (8) for $P(f)$ is computed numerically using the above values of the parameters and shown in Fig.23 in dashed curves. It is observed that there is a good agreement between the formula (8) and all the auto-PSDs of Channel 5 with and without spectral window operation. An aspect worth pointing out is to mention the possibility of some other vibration modes with resonance frequency around about 3 Hz.

3. Causative and spectral relations of outlet temperature and neutron power to inlet temperature and primary-pump flowrate

The causative relation of outlet temperature and neutron power to inlet temperature is understood by the use of a cumulative power contribution ratio. Note that, in the present chapter, only the signal in Channel 5, not both of Channels 3 and 5, is used as neutron noise.
on account of the almost complete coherence in the frequency region of interest.

The power contribution ratio to inlet temperature in Channel 6 is shown in Fig. 30 which indicates that the inlet temperature fluctuates nearly independently of the other signals, but slightly along with the subassembly outlet temperature in Channel 1. On the other hand, as shown in Figs. 12, 13 and 26, the inlet temperature noise brings about significantly the outlet temperature noises of Channels 1 and 2, and the neutron noise.

These causative relations can be understood spectrally in terms of the transfer functions of outlet temperature and neutron power to inlet temperature, together with the corresponding coherence functions. All of these estimates are shown in Figs. 31 through 42. An attention should be given to the frequency range where there is relatively high coherence, i.e. at least where significant correlation is present.

The causative relation to primary-pump flowrate in Channel 7 is shown in Fig. 43. It is clear that the noise signals in Channels 1 through 6 have no significant effect on the flowrate noise, and vice versa, as shown in Figs. 12, 13, 14, 26 and 30. Furthermore, the flowrate noise signal is characteristic of the normal distribution property and the almost white noise spectrum below about 0.5 Hz. The former has been seen from Fig. 2 and the latter is shown in Fig. 44.

Owing to the spectral property just described and, probably, slight correlation between the flowrate and the other signals, a set of the transfer functions of outlet temperature and neutron power to the flowrate are obtained somehow. These functions, in both magnitude
and phase, and the corresponding coherence functions are shown in Figs. 45 through 56.

In order to express the magnitude of the transfer functions in terms of physical units, the normalized variables $X_i (i=1 \text{ through } 7)$ which are used here are related to those in physical units as follows.

For the temperature noise signals in Channels 1, 2, 4 and 6, the measured quantity $\tilde{\Theta}_i (\text{volt})$ is expressed as

$$\tilde{\Theta}_i \propto \frac{G_i}{\sigma_i} = \tilde{X}_i \quad \text{for } i = 1, 2, 4 \text{ and } 6,$$  \hspace{1cm} (9)

where

- thermocouple sensitivity, $\alpha = 42. \times 10^{-6} \text{ volt/°C}$,
- total electronic gain, $G_1 = G_2 = G_4 = 5000, G_6 = 10000$,
- standard deviation, $\sigma_1 = 4.52 \times 10^{-2} \text{ volt}$,
- $\sigma_2 = 5.62 \times 10^{-2} \text{ volt}$,
- $\sigma_4 = 1.73 \times 10^{-1} \text{ volt}$,
- $\sigma_6 = 1.36 \times 10^{-1} \text{ volt}$.

For the neutron noise signal in Channel 5, only the mean value of the measured one, $\tilde{V}_5 (\text{volt})$, is given (3). And so, in the present report, the measured noise signal $\tilde{V}_5 (\text{volt})$ is expressed in terms of a unit of % power and related to $X_5$ as

$$\bar{P}_5 \frac{\tilde{V}_5}{100 G_5 / \sigma_5} = \tilde{X}_5,$$  \hspace{1cm} (10)

where

- power in %, $\bar{P}_5 = (\tilde{V}_5 / \tilde{V}_5) \times 100 \% \text{ power}$,
- mean power, $\tilde{V}_5 = 9.10 \text{ volt}$,
- total electronic gain, $G_5 = 5$,
- standard deviation, $\sigma_5 = 1.85 \times 10^{-1} \text{ volt}$.
For the primary-pump flowrate noise signal in Channel 7, the signal $S_7$ in the digital data from NEA Data Bank is used as it is:

$$\frac{S_7}{\sigma_7} = x_7,$$

where

standard deviation, $\sigma_7 = 1.81 \times 10^{-1}$ volt.

Then, the magnitude of the transfer functions for the normalized variables $X_1$ is expressed in terms of the corresponding physical units as

$$\Theta_1/\Theta_6 = 0.664 \times (X_1/X_6) \ (^\circ C/^\circ C),$$
$$\Theta_2/\Theta_6 = 0.826 \times (X_2/X_6) \ (^\circ C/^\circ C),$$
$$\Theta_4/\Theta_6 = 2.54 \times (X_4/X_6) \ (^\circ C/^\circ C),$$
$$P_5/\Theta_6 = 1.26 \times (X_5/X_6) \ (% \ \text{power}/^\circ C),$$
$$\Theta_1/S_7 = 1.19 \times (X_1/X_7) \ (^\circ C/V),$$
$$\Theta_2/S_7 = 1.48 \times (X_2/X_7) \ (^\circ C/V),$$
$$\Theta_4/S_7 = 4.55 \times (X_4/X_7) \ (^\circ C/V),$$
$$P_5/S_7 = 2.25 \times (X_5/X_7) \ (% \ \text{power}/V).$$

4. Some Remarks

The Phenix noise signals in Channels 1 through 6 contain low frequency components of the order of 0.01 Hz as mutually correlated noise signal. This requires long sample length of several hundred seconds to obtain various spectral estimates of sufficient significance. However, some of the signals, namely in Channels 1 through 5, contain marked changes in the amplitude of noise signal which arise from the
control rod motion. And the available data length is limited. Then, careful spectral analysis is required. The present work, therefore, uses the window opening/closing procedure, which is expected to serve as one of practical means to usual noise analysis. It is interesting and desirable to compare the present results with the design parameters and characteristics, and also with those obtained from noise data much longer than the present one.

References:

Table 1. The sampling conditions and the procedures of the spectral analysis A through D.

<table>
<thead>
<tr>
<th></th>
<th>Analysis A</th>
<th>Analysis B</th>
<th>Analysis C</th>
<th>Analysis D</th>
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<tr>
<td>Noise analysis method</td>
<td>FFT</td>
<td>FFT</td>
<td>FFT</td>
<td>AR</td>
</tr>
<tr>
<td>Channels analyzed</td>
<td>1 to 7</td>
<td>1, 2, 4, 5</td>
<td>3, 5</td>
<td>1, 2, 4, 5, 6, 7</td>
</tr>
<tr>
<td>Sampling period (s) 1)</td>
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<td>0.25</td>
<td>1/64</td>
<td>0.5</td>
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<td>Sample size (points/in sec)</td>
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<td>400/100</td>
<td>4096/64</td>
<td>800/400</td>
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<tr>
<td>Number of samples 2)</td>
<td>11</td>
<td>44</td>
<td>68</td>
<td>11</td>
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<td>59-4458</td>
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<td>59-4458</td>
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<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Data window</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Hanning window 4)</td>
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<td>0/1.00E-2</td>
<td>0/1.56E-2</td>
<td>3/3.33E-1</td>
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<td>(repetition time/frequency resolution in Hz)</td>
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<td>1/2.67E-2</td>
<td>1/4.17E-2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9/2.00E-2</td>
<td>9/8.00E-2</td>
<td>9/1.25E-1</td>
<td></td>
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<tr>
<td></td>
<td>16/2.67E-2</td>
<td>16/1.07E-1</td>
<td></td>
<td>100/1.00E-2</td>
</tr>
</tbody>
</table>

1) In order to create a new time series with a sampling period longer than 1/125 sec, the digital data from NEA Data Bank are resampled using an anti-aliasing recursive filter of Butterworth type.

2) Each of the samples is normalized to zero mean and unit variance, thus resulting in the normalized PSDs.

3) The block number shown above corresponds to that of the ECN data.

4) The frequency resolution is estimated from the formula

\[(8/3)\Delta f/n\] for Hanning window smoothing

\[1/2hM\] for AR model fitting

where \(\Delta f\) is the fundamental frequency, \(n\) the repetition time, \(h\) the sampling period, and \(M\) the model order.
Fig. 1 Variances of the noise signals in Channel 1 through 7 for sample lengths ranging from 20 to 400 s. The variances are obtained from arithmetic average of sample variances. The number of averaged samples is denoted in the parentheses.
Fig. 2 Ratio of the degree of chi-square fitness to normal distribution to that of 5% significance level for the five different sample length. The chi-square value $X^2$ is the arithmetic average of that of a sample. The number of averaged samples is the same as in Fig. 1.
Fig. 3 Auto-covariance function for outlet temperature noise in Channel 1.

Fig. 4 Auto-covariance function for neutron noise in Channel 5.
Fig. 5 Cross-covariance function between outlet temperature noise in Channel 1 and neutron noise in Channel 5.
Fig. 6 Auto-PSD for outlet temperature noise in Channel 1 by FFT method.
Fig. 7 Auto-PSD for outlet temperature in Channel 1 by AR model fitting.
Fig. 8 Coherence function between outlet temperature in Channel 1 and neutron power in Channel 5 by FFT method.
Fig. 9 Coherence function between outlet temperature in Channel 1 and neutron power in Channel 5 by AR model fitting.
Fig. 10 Coherence function of neutron noise between Channel 3 and 5 by FFT method.
Fig. 11 Phase of cross-PSD of neutron noise between Channel 3 and 5.
Fig. 12 Cumulative power contribution ratio to outlet temperature noise in Channel 1.
Fig. 13 Cumulative power contribution ratio to outlet temperature noise in Channel 2.
Fig. 14 Cumulative power contribution ratio to outlet temperature in Channel 4.
Fig. 15 Auto-PSD of noise source of outlet temperature noise in Channel 1 by AR model fitting.
Fig. 16 Auto-PSD of noise source of outlet temperature noise in Channel 2 by AR model fitting.
Fig. 17 Auto-PSD of noise source of outlet temperature noise in Channel 4 by AR model fitting.
Fig. 18 Auto-PSD for outlet temperature noise in Channel 2 by FFT method.
Fig. 19 Auto-PSD for outlet temperature noise in Channel 4 by FFT method.
Fig. 20 Magnitude of transfer function of outlet temperature in Channel 1 to neutron power in Channel 5 by FFT method.
Fig. 21 Magnitude of transfer function of outlet temperature in Channel 2 to neutron power in Channel 5 by FFT method.
Fig. 22 Magnitude of transfer function of outlet temperature in Channel 4 to neutron power in Channel 5 by FFT method.
Fig. 23 Coherence function between outlet temperature in Channel 1 and neutron power in Channel 5 by FFT method (Analysis B).
Fig. 24 Coherence function between outlet temperature in Channel 2 and neutron power in Channel 5 by FFT method (Analysis B).
Fig. 25 Coherence function between outlet temperature in Channel 4 and neutron power in Channel 5 by FFT method (Analysis 3).
Fig. 26 Cumulative power contribution ratio to neutron power in Channel 5.
Fig. 27 Auto-PSD for neutron noise in Channel 3.
Fig. 28: Auto-PSD for neutron noise in Channel 5.
Fig. 30 Cumulative power contribution ratio to inlet temperature noise in Channel 6.
Fig. 31 Magnitude of transfer function of outlet temperature in Channel 1 to inlet temperature in Channel 6.
Fig. 32 Phase of transfer function of outlet temperature in Channel 1 to inlet temperature in Channel 6.
Fig. 33 Coherence function between outlet temperature in Channel 1 and inlet temperature in Channel 6.
Fig. 34 Magnitude of the transfer function of outlet temperature in Channel 2 to inlet temperature in Channel 6.
Fig. 35 Phase of the transfer function of outlet temperature in Channel 2 to inlet temperature in Channel 6.
Fig. 36 Coherence function between outlet temperature in Channel 2 and inlet temperature in Channel 6.
Fig. 37 Magnitude of the transfer function of outlet temperature in Channel 4 to inlet temperature in Channel 6.
Fig. 38 Phase of the transfer function of outlet temperature in Channel 4 to inlet temperature in Channel 6.
In channel 4 and inter temperature in channel 6.

Figure 39: Coherence function between output temperature

Frequency (Hz)

Coherence between 4 and 6

393
Fig. 40 Magnitude of the transfer function of neutron power in Channel 5 to inlet temperature in Channel 6.
Fig. 41 Phase of the transfer function of neutron power in Channel 5 to inlet temperature in Channel 6.
Fig. 42 Coherence function between neutron power in Channel 5 and inlet temperature in Channel 6.
Fig. 43 Cumulative power contribution ratio to primary pump flowrate in Channel 7.
Fig. 44 Auto-PSD for primary-pump flowrate noise in Channel 7.
Fig. 45 Magnitude of the transfer function of outlet temperature in Channel 1 to primary-pump flowrate in Channel 7.
Fig. 46 Phase of the transfer function of outlet temperature in Channel 1 to primary-pump flowrate in Channel 7.
Fig. 47 Coherence function between outlet temperature in Channel 1 and primary-pump flowrate in Channel 7.
Fig. 48 Magnitude of the transfer function of outlet temperature in Channel 2 to primary-pump flowrate in Channel 7.
Fig. 49 Phase of the transfer function of outlet temperature in Channel 2 to primary-pump flowrate in Channel 7.
Fig. 50 Coherence function between outlet temperature in Channel 2 and primary-pump flowrate in Channel 7.
Fig. 51 Magnitude of the transfer function of outlet temperature in Channel 4 to primary-pump flowrate in Channel 7.
Fig. 52 Phase of the transfer function of outlet temperature in Channel 4 to primary-pump flowrate in Channel 7.
Fig. 53 Coherence function between outlet temperature in Channel 4 and primary-pump flowrate in Channel 7.
Fig. 54 Magnitude of the transfer function of neutron power in Channel 5 to primary-pump flowrate in Channel 7.
Fig. 55 Phase of the transfer function of neutron power in Channel 5 to primary-pump flowrate in Channel 7.
Fig. 56 Coherence function between neutron power in Channel 5 and primary-pump flowrate in Channel 7.
To: Dr. M. Leguillou
From: T. Tamaoki and Y. Sonoda

We enclose the answers of the SMORN-IV physical benchmark test and are sorry for being so late.

We wish for success of SMORN-IV.
ANALYSIS RESULTS FOR REACTOR NOISE
PHYSICAL BENCHMARK TEST

RESULTS ON THE PHENIX NOISE DATA ANALYSIS

JUNE 1984

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Yukio SONODA

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Analysis results for the Phenix noise data

1st Proposal

1) Time constant of the heat transfer in the assembly: $\tau_C$

$\tau_C = 2.30$ (sec)

2) Time constants of the thermocouples measuring the temperature at the outlet of the assemblies: $\tau_T$

TATA2018: $\tau_T = 6.29$ (sec)
TATA2024: $\tau_T = 5.57$ (sec)
TATA2119: $\tau_T = 6.81$ (sec)

2nd Proposal

Fig. 1

$c/m^2 = 4.90$
$f_0 = 5.87$
$\varepsilon = 0.245$
$\gamma = 0.866$

3rd Proposal

Fig. 2 - Fig. 9
Fig. 1 Normalized auto power spectral density function of Z1MR41 by FFT method.
Fig. 2 Open loop transfer function of Z1MR41/P3MT25 by 6-channel AR model (AR model order = 3).
Fig. 3 Open loop transfer function of TATA2018/P3MT25 by 6-channel AR model (AR model order = 3).
Fig. 4 Open loop transfer function of TATA2024/P3MT25 by 6-channel AR model (AR model order = 3).
Fig. 5 Open loop transfer function of TATA2119/P3MT25 by 6-channel AR model (AR model order = 3).
Fig. 6 Open loop transfer function of ZLMR41/PLMQ02
by 6-channel AR model (AR model order = 3).
Fig. 7 Open loop transfer function of TATA2018/P1MQO2 by 6-channel AR model (AR model order = 3).
Fig. 8 Open loop transfer function of TATA2024/PlMQ02 by 6-channel AR model (AR model order = 3).
Fig. 9 Open loop transfer function of TATA2119/PLMQ02 by 6-channel AR model (AR model order = 3).
Appendix

A-1 The Sign of Signal

It is sometimes necessary in the FBR operation to compensate reactor power, which decrease as the progress of burn-up, by drawing out control rods. This control rod adjustment makes reactor power increase stepwise. It is seen from Fig. A-1 that outlet temperature of assemblies (TATA2018, TATA2024 and TATA2119) behave as mentioned above, but neutron power and IHX primary inlet temperature (Z1MR41, Z1MR51 and P1MT01) behave inversely. Fig. A-2 also shows that Z1MR41 and TATA2024 are out of phase in very low frequency region. It is nearly impossible to explain physically the cause of this phenomenon. If this is the fact, the characteristics of transfer functions of outlet temperature/neutron power cannot be described as in the first proposal. Therefore, we concluded that the signs of following three signals were inverted when they were recorded; Z1MR41, Z1MR51 and P1MT01. In the following analysis, we inverted the signs of those three signals.

A-2 The First Proposal

It is seen from the results of auto-regressive (AR) analysis (Fig. A-3, Fig. A-5), the transfer path from neutron power $P(t)$ to outlet temperature $T_0(t)$ can be described as Fig. A-6. As for TATA2018 and TATA2024, the transfer functions by FFT method are unreliable because there is a feed-back effect from $T_0(t)$ to $P(t)$ and we cannot decide time constants by this method. On the other hand, there is no coherency between TATA2119 and Z1MR41 and so transfer function between the two is meaningless.

Under these considerations, we decided time constants of heat transfer and thermocouple by the following method which had two steps.
Step 1. We decided time constants of thermocouples ($\tau_T$) by fitting a following expression $\hat{P}$ to APSD functions of assembly outlet temperatures by FFT method in the frequency regions where effects from neutron power were small. (Fig. A-7 ~ Fig. A-9)

$$\hat{P} = \left( \frac{K_T}{1 + \tau_T \cdot S} \right)^2$$

Thus, we obtained the following values.

- TATA2018: $\tau_T = 6.29$ (sec)
- TATA2024: $\tau_T = 5.57$ (sec)
- TATA2119: $\tau_T = 6.81$ (sec)

Step 2. We obtained open loop transfer functions of outlet temperature/neutron power about TATA2018 and TATA2024 by AR method and decided time constant $\tau_C$ of heat transfer in the assembly by fitting a following expression $\hat{T}_f$ to those transfer functions. (Fig. A-10 ~ Fig. A-13)

The frequency regions for fitting were selected below 0.1Hz because the coherency between neutron power and outlet temperature is very small above 0.1Hz.

$$\hat{T}_f = \frac{K}{(1 + \tau_T \cdot S)(1 + \tau_C \cdot S)}$$

where value of $\tau_T$ is equal to that obtained in Step 1.

We obtained the following four values.

- TATA2018/Z1MR41: $\tau_C = 2.58$ (sec)
- TATA2018/Z1MR51: $\tau_C = 2.30$ (sec)
- TATA2024/Z1MR41: $\tau_C = 2.21$ (sec)
- TATA2024/Z1MR51: $\tau_C = 2.12$ (sec)
We regarded mean value of the four as the time constant of heat transfer in the assembly.

\[ \tau_c = 2.30 \text{ (sec)} \]

A-3 The Second Proposal

APSD function \( P_s(f) \) of noise source and transfer function \( H(f) \) of neutron power/noise source were given, so we fitted a following expression to a normalized APSD function of neutron power Z1MR41.

\[
\hat{P} = P_s(f) \{H(f)\}^2
\]

\[
= \frac{C}{f^Y} \left| \frac{1/(\pi^2m)}{-f^2 + 2jcf^2 + f_0^2} \right|^2
\]

The results are shown in Fig. 1. Though it is proposed to determine \( c/m \) as gain parameters of APSD, we regarded the subject as \( c/m^2 \) from the above equation.

A-4 The Third Proposal

We adopted open loop transfer functions by AR method as answers for the third proposal because there were feedback effects in the present system. However, it is seen from Fig. A-14 (a) \( \sim \) (d) that the effect of inlet temperature P3MT25 on TATA2119 and the effects of flow rate P1MQ02 on all other signals are almost nothing. Therefore, obtained transfer functions TATA2119/P3MT25 signals/P1MQ02 are thought to have no meanings.
Fig. A-1(a) Phenix benchmark test data.
Fig. A-1(b) Phenix benchmark test data.

<table>
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<tr>
<th>ZIMR51 (Mw)</th>
<th>P3MT25 (°C)</th>
<th>P1MQ02 (Volt)</th>
<th>SIMQ01 (Volt)</th>
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<tr>
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<td>-50.0</td>
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<td>6260.0</td>
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Fig. A-1(c) Phenix benchmark test data.
Fig. A-2 Open loop transfer function of TATA2024/Z1MR41
by 6-channel AR model (AR model order = 3).
Fig. A-3 Decomposed APSDs by 2-channel AR model (AR model order = 8).
Fig. A-4 Decomposed APSDs by 2-channel AR model
(AR model order = 6).

(a) ZLMR41

(b) TATA2024
Fig. A-5  Decomposed APSDs by 2-channel AR model  
(AR model order = 1).
Fig. A-6 Transfer path from neutron power to assembly outlet temperature.
Fig. A-7  Power spectral density of TATA2018
by FFT method.
Fig. A-8 Power spectral density of TATA2024
by FFT method.
Fig. A-9  Power spectral density of TATA2119
by FFT method.
Fig. A-10 Open loop transfer function of TATA2018/Z1MR41 by 2-channel AR model (AR model order = 8).
Fig. A-11 Open loop transfer function of TATA2018/Z1MR51 by 2-channel AR model (AR model order = 8).
Fig. A-12 Open loop transfer function of TATA2024/ZLMR41 by 2-channel AR model (AR model order = 6).
Fig. A-13 Open loop transfer function of TATA2024/21MR51 by 2-channel AR model (AR model order = 6).
Fig. A-14 Decomposed APSDs by 6-channel AR model (AR model order = 3).
Reactor Noise Analysis Benchmark Test (Physical) on the PHENIX Reactor

by

Z. P. Luo * and S. M. Wu **

I. Identification of time constants
1. Time constant of heat transfer in the subassembly, $\tau_c$

$$\tau_c = 1.969 \pm 0.518 \quad (\text{sec})$$

2. Time constant of the thermocouples, $\tau_t$

$$\tau_t = 5.783 \pm 0.533 \quad (\text{sec})$$

II. The components of the neutron noise

1. The mechanical resonance peaks

Note that all confidence interval listed above based on the sample size N=2400.

* Z. P. Luo is a visiting scholar at the Department of Mechanical Engineering, University of Wisconsin, Madison, U.S.A. He comes from the Engineering-Physics Department, Tsinghua University, Beijing, China.

** S. M. Wu is with the Department of Mechanical Engineering, University of Wisconsin, Madison, WI 53706, U.S.A.
2. Components

For channel No. 3, the ARMA(10,9) model is:

<table>
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<th>Natural frequency (Hz)</th>
<th>Damping ratio</th>
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<td>0.228 ± 0.003</td>
<td>0.116 ± 0.030</td>
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<td>2.40 ± 0.13</td>
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<tr>
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<tr>
<td>(4.02 ± 0.04</td>
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<td>- 0.01</td>
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<td>4.45 ± 0.02</td>
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</tbody>
</table>
\[ X_t + 0.2832X_{t-1} - 0.5692X_{t-2} - 0.4087X_{t-3} - 0.2902X_{t-4} + 0.1685X_{t-5} + 0.06707X_{t-6} + 0.1401X_{t-7} + 0.01925X_{t-8} - 0.3327X_{t-9} - 0.05105X_{t-10} = a_t + 0.3957a_{t-1} - 0.3895a_{t-2} - 0.2953a_{t-3} + 0.3412a_{t-4} - 0.251a_{t-5} + 0.1326a_{t-6} - 0.213a_{t-7} + 0.1405a_{t-8} - 0.2607a_{t-9} \]

where \( X_t \) is the measured value of neutron at time \( t \).

\( a_t \) is the residual at time \( t \).

(1) Background noise

86.87% of the neutron variance belongs to the background noise. It follows the Normal distribution and is not only related to the reaction rate of the neutron detector but something else.

\[ \tau = 4.56 \times 10^{-3} \text{ sec} \]

where \( \tau \) corresponds to the time-constant of the detector Z1MR12.

(2) Thermal noise

4.7% of the neutron variance belongs to the thermal noise. In the model, it corresponds to the time-constant

\[ \tau = 0.081 \text{ sec} \]

(3) Alternative electricity interference

50 Hz

(4) Mechanical noise

(a) Vibration from the primary pumps

13.66 Hz, 27.3 Hz, 41 Hz, and 54.66 Hz

(b) Transmission from the primary flowrate and inlet temperature

4.45 Hz and 6.03 Hz

(c) Motion of the neutron detectors

2.40 Hz for the detector No. 3

4.03 Hz for the detector No. 5
(d) Effects included by the primary flow-rate and inlet temperature

0.228 Hz, 0.412 Hz and 10.02 Hz.

III. Relations

1. Between neutron signals and flowrate

For time-step = 1/128 sec

\[ Y_t = 0.3393Y_{t-1} + 0.1107Y_{t-2} + 0.1363Y_{t-3} - 0.03903Y_{t-4} - 0.0317Y_{t-5} - 0.3011Y_{t-6} \]
\[ - 0.2578Y_{t-7} + 0.2308Y_{t-8} + 0.4085Y_{t-9} + 0.1198Y_{t-10} = X_t + 0.7835X_{t-1} + 0.3031X_{t-2} \]
\[ + 0.09547X_{t-3} + 0.1356X_{t-4} - 0.03708X_{t-5} + 0.0615X_{t-6} - 0.09543X_{t-7} - 0.1246X_{t-8} \]
\[ + 0.1874X_{t-9} \]

where output \( Y_t \) ----- neutron at time \( t \)

input \( X_t \) ----- flowrate at time \( t \)

For time-step = 0.0625 sec.

\[ Y_t = -0.2344Y_{t-1} - 0.3824Y_{t-2} - 0.3437Y_{t-3} = X_t + 0.0008995X_{t-1} - 0.06013X_{t-2} \]

For time-step = 1 sec

\[ Y_t = 0.7433Y_{t-1} + 0.152Y_{t-2} + 0.2148Y_{t-3} - 0.2663Y_{t-4} \]
\[ = X_t - 0.05448X_{t-1} + 0.07059X_{t-2} - 0.03722X_{t-3} \]

2. Between neutron signals and inlet temperature

For time-step = 1/128 sec. 25.7\% of fluctuation
\[ Y_t = 0.1056Y_{t-1} - 0.177Y_{t-2} + 0.01965Y_{t-3} + 0.2213Y_{t-4} - 0.2543Y_{t-5} + 0.256Y_{t-6} \]
\[ + 0.33Y_{t-7} - 0.345Y_{t-8} - 0.1684Y_{t-9} + 0.09234Y_{t-10} \]
\[ = X_t + 0.6324X_{t-1} + 0.0715X_{t-2} - 0.1168X_{t-3} + 0.06688X_{t-4} + 0.1302X_{t-5} \]
\[ - 0.05015X_{t-6} + 0.04927X_{t-7} + 0.1603X_{t-8} - 0.1457X_{t-9} \]

Where output \( Y_t \) ------- neutron at \( t \)
input \( X_t \) ------- inlet temperature at \( t \)

For time-step = 0.0625 sec
55.65\% of fluctuation

\[ Y_t = 0.1158Y_{t-1} + 0.4392Y_{t-2} - 0.522Y_{t-3} - 0.4174Y_{t-4} - 0.07696Y_{t-5} \]
\[ = X_t + 0.9189X_{t-1} - 0.1693X_{t-2} + 0.4002X_{t-3} - 0.4976X_{t-4} \]

For time-step = 1 sec.
12.9\% of fluctuation

\[ Y_t + 0.4845Y_{t-1} - 0.279Y_{t-2} = X_t + 0.054X_{t-1} \]

3. Between outlet temperature of subassemblies and inlet temperature

For time-step = 1/128 sec.
39.8\% of fluctuation

\[ Y_t + 1.422Y_{t-1} + 0.6714Y_{t-2} - 0.154Y_{t-3} - 0.8685Y_{t-4} - 1.393Y_{t-5} - 0.6945Y_{t-6} \]
\[ + 0.06864Y_{t-7} + 0.01868Y_{t-8} - 0.06145Y_{t-9} + 0.1902Y_{t-10} \]
\[ = X_t + 0.5316X_{t-1} + 0.4715X_{t-2} - 0.1434X_{t-3} - 0.417X_{t-4} - 0.1962X_{t-5} \]
\[ - 0.0001066X_{t-6} - 0.1661X_{t-7} - 0.0272X_{t-8} + 0.151X_{t-9} \]

Where output \( Y_t \) ------- temperature of subassemblies at \( t \)
input \( X_t \) ------- inlet temperature at \( t \)
For time-step = 0.0625 sec. 81.45% of fluctuation

\[ Y_t - 0.7377Y_{t-1} + 0.05731Y_{t-2} = X_t - 0.71X_{t-1} \]

For time-step = 1 sec 32.52% of fluctuation

\[ Y_t - 0.6218X_{t-1} - 0.1025Y_{t-2} + 0.2068Y_{t-3} - 0.8097Y_{t-4} + 0.3079X_{t-5} + 0.2262Y_{t-6} = X_t + 0.1509X_{t-1} - 0.1015X_{t-2} + 0.02187X_{t-3} + 0.008992X_{t-4} - 0.1242X_{t-5} \]

4. Between output temperature of subassemblies and flowrate

For time-step = 1/128 sec. 43.7% of fluctuation

\[ Y_t - 0.3437Y_{t-1} + 0.01872Y_{t-2} + 0.1537Y_{t-3} - 0.4106Y_{t-4} + 0.2656Y_{t-5} + 0.1555Y_{t-6} - 0.4096Y_{t-7} + 0.08082Y_{t-8} = X_t + 0.7129X_{t-1} - 0.377X_{t-2} + 0.1181X_{t-3} + 0.07263X_{t-4} - 0.2001X_{t-5} + 0.2325X_{t-6} + 0.0179X_{t-7} \]

Where output \( Y_t \) ------ temperature of subassemblies at \( t \)

input \( X_t \) ------ flowrate at \( t \)

For time-step = 0.0625 sec. 29.02% of fluctuation

\[ Y_t - 0.05808Y_{t-1} - 0.4517Y_{t-2} - 0.3636Y_{t-3} - 0.1199Y_{t-4} + 0.02618Y_{t-5} - 0.0004199Y_{t-6} = X_t + 0.05306X_{t-1} - 0.05732X_{t-2} - 0.02452X_{t-3} - 0.005065X_{t-4} + 0.007517X_{t-5} \]

For time-step = 1 sec. 4.2% of fluctuation

\[ Y_t - 0.315Y_{t-1} - 0.5342Y_{t-2} - 0.07588Y_{t-3} + 0.2123Y_{t-4} - 0.4198Y_{t-5} - 0.1643Y_{t-6} + 0.3826Y_{t-7} = X_t - 0.008017X_{t-1} - 0.006096X_{t-2} - 0.001447X_{t-3} - 0.003703X_{t-4} - 0.004744X_{t-5} + 0.001609X_{t-6} \]
IRI-contribution to the SMORN-IV reactor noise benchmark test: analysis of Phenix noise

J.E. Hoogenboom
Dear dr. Le Guillou,

Hereby I send you a report with the results of the IRI noise group for the SMORN-IV benchmark test on the Phenix noise signals.

I must apologize for my late response, but I hope you can include it in the evaluation.

Sincerely yours,

J.E. Hoogenboom.

---

<table>
<thead>
<tr>
<th>Uw kenmerk</th>
<th>Uw brief d.d.</th>
<th>Ons kenmerk</th>
<th>Delft, June 5th, 1984</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORF 80-84</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Doorkiesnummer 015 - 78 6962

encl.

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Telefoon 0 15 - 78 91 11
Telex: 38151 bhnl
Telegramadres: I.R.I. Delft

ARRIVEE
Dear dr. Le Guillou,

A short time ago I sent you a report with my results of the SMORN-IV benchmark test on the Phenix noise signals. Unfortunately, there have been an interchange in Table IV of the report. I hereby include the corrected table.

Sincerely yours,

J.E. Hoogenboom.

encl.-
IRI-contribution to the SMORN-IV reactor noise benchmark test:
analysis of Phenix noise

J.E. Hoogenboom

1984

Reactor Physics Group
Interuniversity Reactor Institute
and
Delft University of Technology
1. Introduction

At the occasion of the 3rd Specialists' Meeting on Reactor Noise SMORN-III, 1981, a computational benchmark test on noise analysis was set up. This benchmark was restricted to a comparison of the calculation of frequency spectra and correlation functions for a number of signals recorded on an analog magnetic tape originating from a simulated BWR (artificial noise, JAERI, Japan), from a PWR (Borssele, The Netherlands) and from a LMFBR (Phenix, France).

At SMORN-III it was already decided to proceed with a 'physical' benchmark test and to present the results at SMORN-IV to be held in October 1984. For this benchmark the determination of physical parameters was requested using the same signals as provided for the first benchmark. This report contains the results of the IRI Reactor Physics Group for the analysis of the Phenix noise.

2. Signal conditioning and preliminary analyses

A copy of the original benchmark tape was obtained from ECN, Petten, The Netherlands. The recording of the signals on tape has been described in the SMORN-III benchmark report (Ref. 1). From the Phenix reactor 7 noise signals were recorded with identification shown in Table I.

Table I. Phenix signals recorded

<table>
<thead>
<tr>
<th>Track</th>
<th>signal</th>
<th>identification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>outlet temperature</td>
<td>TATA 2018</td>
</tr>
<tr>
<td>2</td>
<td>outlet temperature</td>
<td>TATA 2024</td>
</tr>
<tr>
<td>3</td>
<td>neutron detector</td>
<td>Z1MR41</td>
</tr>
<tr>
<td>4</td>
<td>outlet temperature</td>
<td>TATA 2119</td>
</tr>
<tr>
<td>5</td>
<td>neutron detector</td>
<td>Z1MR51</td>
</tr>
<tr>
<td>6</td>
<td>pump inlet temperature</td>
<td>P3MT25</td>
</tr>
<tr>
<td>7</td>
<td>primary pump flowrate</td>
<td>P1MQ02</td>
</tr>
</tbody>
</table>

The Phenix noise signals were recorded on tape from footage count 2233 to 3225. However, from 3110 ft onwards a serious non-stationarity appears, as already pointed out at SMORN-III. Therefore only 850 ft are usable for analyses. With a real time tape speed of $17/8"/s$ about 1½ hour of signal is available. As
several phenomena to be analysed occur at very low frequencies, the signal duration is rather limited.

For all signals of interest auto and cross frequency spectra were determined using the FFT method. Therefore the appropriate signals were sampled with sampling time $\Delta t$ depending on the object in view. When using $N$ samples per time record the frequency resolution $\Delta f$ is given by

$$\Delta f = \frac{1}{2N\Delta t}$$

Before sampling the signals are normally low-pass filtered with 8-th order Butterworth filters with filter frequency $f_f$ given by

$$f_f = 0.7 \frac{N\Delta f}{\Delta t} = 0.35 \frac{N}{\Delta t}$$

As most signals on tape still contained a DC-component, a DC off-set was used before further amplification.

It turned out that the coherence between the outlet temperature signal TATA2119 from track 4 and the neutron detector signals, inlet temperature signal and pump flowrate signal was much smaller than for both other outlet temperature signals. Therefore the signal from track 4 has not been used in the analyses.

It also turned out that the normalised spectra for both neutron detectors do not coincide at low frequencies when the DC-values and pre-recording gains as given in Ref.1 were used, although their coherence is very close to unity. It was therefore assumed that the values for the pre-recording electronic gain had been interchanged, although there remains some discrepancy. In our analysis the following normalisation values have been used.

<table>
<thead>
<tr>
<th>track</th>
<th>$G$</th>
<th>$T(\degree C)$</th>
<th>$\bar{V}$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5000</td>
<td>571</td>
<td>24.0 mV</td>
<td>119.91</td>
</tr>
<tr>
<td>2</td>
<td>5000</td>
<td>592</td>
<td>24.9 mV</td>
<td>124.32</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td></td>
<td>8.96 V</td>
<td>44.8</td>
</tr>
<tr>
<td>4</td>
<td>5000</td>
<td>614</td>
<td>25.8 mV</td>
<td>128.94</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td></td>
<td>9.10 V</td>
<td>182.0</td>
</tr>
<tr>
<td>6</td>
<td>10,000</td>
<td>397</td>
<td>16.7 mV</td>
<td>166.74</td>
</tr>
<tr>
<td>7</td>
<td>1000</td>
<td></td>
<td>-9.0 mV</td>
<td>-9.0</td>
</tr>
</tbody>
</table>
3. Identification of time constants

The first proposal for the physical benchmark test on noise data from the Phenix reactor was to determine the time constant of the heat transfer in a subassembly and the time constants of the thermocouples measuring the subassembly outlet temperature. \( \delta P(f) \) is the Fourier transformed signal of a neutron detector and \( \delta T_{si}(f) \) the 'true' thermal noise at the outlet of the \( i \)-th subassembly we have

\[
\delta T_{si}(f) = F(f) \frac{\delta P(f)}{P} \Delta T_{si} + b_s(f)
\]

with \( F(f) \) a first-order transfer function of the subassembly with time constant \( \tau_c \), \( \Delta T_{si} \) the temperature rise in the \( i \)-th subassembly and \( b_s(f) \) weakly correlated noise. The measured temperature noise is given by

\[
[C_i^2 \delta T_{si}(f)]_{\text{meas}} = G_i(f) \delta T_{si}(f)
\]

with \( G_i(f) \) a first-order transfer function of the thermocouple

\[
G_i(f) = \frac{1}{1 + jf/f_i}
\]

The cut-off frequency \( f_i \) is related to the time constant \( \tau_i \) by

\[
\tau_i = \frac{1}{2\pi f_i}
\]

As the presumed values of the time constant \( \tau_c \) and \( \tau_i \) are in the range 1 - 5 s and 3 - 8 s, respectively, the cut-off frequencies are in the range 0.03 - 0.16 Hz and 0.02 - 0.05 Hz respectively.

If we call \( \phi_{ii} \) the auto spectrum of the measured temperature noise at the outlet of the \( i \)-th subassembly, \( \phi_{ni} \) the cross spectrum with a neutron detector, \( \gamma^2_{ni} \) its coherence and \( \phi_{bb} \) the spectrum of the weakly correlated noise, we have

\[
\phi_{ii}(1-\gamma^2_{ni}) = |G_i(f)|^2 \phi_{bb} \tag{1}
\]

\[
\frac{\phi_{ni}}{\phi_{nn}} = \frac{\Delta T_{si}}{\delta T_{si}} G_i(f) F(f) \tag{2}
\]

If it is assumed that the weakly correlated noise \( b_s(f) \) is white, the time
constant \( \tau_i \) of the \( i \)-th thermocouple can be determined by fitting the left hand side of Eq. (1) to the function

\[
\frac{A}{1 + \frac{|f|}{f_1}^2} = \frac{A^*}{1 + (2\pi f T)^2}
\]

This has been done from temperature and neutron detector spectra obtained with a sampling time of \( \Delta t = 960 \) ms and 512 samples per record. Then only 10 time records are available, which yield large statistics. The numerical results are given in table III for the thermocouple time constants of TATA 2018 and TATA 2024 (tracks 1 and 2) using the neutron detector signals from tracks 3 and 5. An example of a fit is given in Fig.1.

The functions fit rather well. However, the fit for the function obtained with the subassembly outlet temperature TATA 2024 from track 2 is very sensitive to the lower frequency bound. When weighting inversely proportional to the square of the function value is applied (assumed standard deviation proportional to the function value) this effect is absent. For the fit with outlet temperature from track 1 unit weights were used which gives a more accurate fit. The results are almost independent of the neutron detector signal used for the cross power spectrum.

### Table III. Thermocouple constants

<table>
<thead>
<tr>
<th>thermocouple</th>
<th>track</th>
<th>time constant ( \tau ) (s)</th>
<th>standard deviation(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TATA 2018</td>
<td>1</td>
<td>16.2</td>
<td>0.6</td>
</tr>
<tr>
<td>TATA 2024</td>
<td>2</td>
<td>6.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The function \( \Phi_{\text{nn}} / \Phi_{\text{n1}} \) is shown in Fig.2 with the subassembly outlet temperature taken from track 1 and the neutron detector signal from track 5. In the assumed model this function must be proportional to the product of two first-order filters in accordance with Eq.(2), so its phase should decrease from 0 to \(-180^\circ\) and its modulus should decrease as \( f^{-2} \) for frequencies larger than the highest cut-off frequency. However, the phase difference at very low frequencies is \( 180^\circ \) and the modulus is more or less constant. So, the transfer function \( F(f) \) of the subassembly cannot be described by a simple first-order low-pass filter.
Fig. 1. Fit of first-order filter to the function $\Phi_{ll}(1 - \gamma_n^2)$.
Fig. 2. Plot of the function $\phi_{n1}/\phi_{nn}$
As the phase in Fig. 2 is equal to that of the cross spectrum $\phi_{ni}$, the DC-response of the neutron signal is opposite to that of the outlet temperature, which is not expected physically. There might have been a polarity change in one of these signals.

4. Determination of the components of the neutron noise

The neutron noise is assumed to be composed of three components

1) thermal noise proportional to $1/f^3$,
2) mechanical noise related to vibrations with transfer function

$$H(f) = \frac{1}{4\pi^2 f_r^2} \frac{1}{f^2 + 2j\epsilon f_r f}$$

with $f_r$ the resonance frequency and $\epsilon$ the damping ratio,
3) white background noise.

From a preliminary analysis up to 50 Hz a coherence of almost unity between the two neutron detector signals is present, slowly decreasing above 0.1 Hz and almost vanishing above 10 Hz. The spectra decrease at low frequencies somewhat stronger than $f^{-1}$. From 10 to 35 Hz a constant background is present. At about 5.5 Hz a strongly asymmetric peak is present.

When a non-linear least squares fit is done with one vibration peak according to Eq. (3) the resonance frequency moves to lower frequencies than the top of the peak due to the asymmetry. Therefore a second vibration peak must be introduced at about 4 Hz. To be able to fit the lower and higher frequency range simultaneously the spectra of the neutron detector signals were determined for the frequency range up to 1 Hz with sampling time $\Delta t = 352$ ms (58 records) and for the frequency range up to 50 Hz with sampling time $\Delta t = 7$ ms (2916 records). The joint spectra were fitted to the function

$$S(f) = B + \frac{C}{f^{1/2}} + \sum_{i=1}^{2} \frac{D_i}{(4\pi^2)^2} \frac{1}{|f_i^2 - f^2 + 2j\epsilon f_i f|^2}$$

with the parameters $B, C, \alpha, \beta, f_1, f_2, \epsilon_1, \epsilon_2$ and $\epsilon_2$ to be determined. The parameter $D$ is equal to $1/m^2$ in the notation of Eq. (3). The authors of the benchmark test requested the parameter $C/m$ for a vibration peak, which would be equal to $C/D$. If the intensity of a vibration peak relative to the thermal noise component has to be determined the quantity $C/m^2$ (equal to $C/D$) should
be used because \( m \) appears in the denominator of the transfer function and quadratically in Eq.(4).

The fit was done for the frequency range 0.02 - 35 Hz with the spectrum at lower frequencies used up to 0.8 Hz. A weight was used proportional to the number of records and inversely proportional to the square of the spectrum value (inverse variance). The results are given in Table IV for both neutron detector signals using normalized spectra. The fit for the spectrum of the signal from track 5 is shown in Fig. 3.

Table IV. Spectrum parameters according to Eq.(4)

<table>
<thead>
<tr>
<th>parameter</th>
<th>signal from track 3</th>
<th>standard deviation (%)</th>
<th>signal from track 5</th>
<th>standard deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>( 1.89 \times 10^{-8} )</td>
<td>0.5</td>
<td>( 1.24 \times 10^{-8} )</td>
<td>1.1</td>
</tr>
<tr>
<td>C</td>
<td>( 3.0 \times 10^{-8} )</td>
<td>6.6</td>
<td>( 4.03 \times 10^{-8} )</td>
<td>2.8</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>1.45</td>
<td>2.7</td>
<td>1.08</td>
<td>2.0</td>
</tr>
<tr>
<td>( D_1 )</td>
<td>( 4.2 \times 10^{-3} )</td>
<td>15</td>
<td>( 3.7 \times 10^{-3} )</td>
<td>4.1</td>
</tr>
<tr>
<td>( f_1 ) (Hz)</td>
<td>5.69</td>
<td>0.7</td>
<td>5.65</td>
<td>0.7</td>
</tr>
<tr>
<td>( \varepsilon_1 )</td>
<td>0.13</td>
<td>16</td>
<td>0.13</td>
<td>7.4</td>
</tr>
<tr>
<td>( D_2 )</td>
<td>( 3.4 \times 10^{-3} )</td>
<td>46</td>
<td>( 1.92 \times 10^{-3} )</td>
<td>32</td>
</tr>
<tr>
<td>( f_2 ) (Hz)</td>
<td>4.66</td>
<td>6.7</td>
<td>4.32</td>
<td>3.0</td>
</tr>
<tr>
<td>( \varepsilon_2 )</td>
<td>0.35</td>
<td>6.5</td>
<td>0.22</td>
<td>9.6</td>
</tr>
</tbody>
</table>

Because the \( \chi^2 \)-values for the fits are too large for the number of degrees of freedom the given standard deviations cannot be considered as very accurate. Furthermore, it would have been better to use an intermediate frequency range in the analysis up to a few Herz for a better determination of the spectrum around 1 Hz.

5. Relation between inlet temperature or flow and outlet temperature or neutron power

From a preliminary analysis it became clear that there exists a (small) coherence between pump inlet temperature (track 6) and subassembly outlet temperature (tracks 1 and 2) as well as neutron power (tracks 3 and 5) and also between pump flow rate (track 7) and the output signals in the frequency range up to about 10 mHz.
Fig. 3. Fit of the spectrum of neutron detector signal from track 5
To determine the transfer functions between these quantities an AR-model analysis was attempted with correlation functions derived from frequency spectra determined from 28 time records with sampling time $\Delta t = 352$ ms and 512 samples per record. However, it was not possible to obtain acceptable results with the AR-analysis.

Reference
