STATUS OF LMFBR SAFETY TECHNOLOGY

2. Reactivity Monitoring in an LMFBR at Shutdown

March 1981

Issued by
Risley Nuclear Power Development Establishment
United Kingdom Atomic Energy Authority

on behalf of the OECD Nuclear Energy Agency

COMMITTEE ON THE SAFETY OF NUCLEAR INSTALLATIONS
OECD NUCLEAR ENERGY AGENCY
38, boulevard Suchet, 75016 Paris, France
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Nuclear Safety Division
OECD Nuclear Energy Agency
38 boulevard Suchet
F-75016 Paris
France
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Background to CSNI activities

The Organisation for Economic Co-operation and Development (OECD) was set up under a Convention signed in Paris on 14 December 1960, which provides that the OECD shall promote policies designed:

- to achieve the highest sustainable economic growth and employment and a rising standard of living in Member countries, while maintaining financial stability, and thus to contribute to the development of the world economy;

- to contribute to sound economic expansion in Member as well as non-Member countries in the process of economic development;

- to contribute to the expansion of world trade on a multilateral, non-discriminatory basis in accordance with international obligations.

The Members of OECD are Australia, Austria, Belgium, Canada, Denmark, Finland, France, the Federal Republic of Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States.

The OECD Nuclear Energy Agency (NEA) was established on 20 April 1972, replacing OECD's European Nuclear Energy Agency (ENEA) on the adhesion of Japan as a full Member.

NEA now groups all the European Member countries of OECD and Australia, Canada, Japan and the United States. The Commission of the European Communities takes part in the work of the Agency.

The objectives of NEA remain substantially those of ENEA, namely the orderly development of the uses of nuclear energy for peaceful purposes. This is achieved by:

- assessing the future role of nuclear energy as a contributor to economic progress, and encouraging co-operation between governments towards its optimum development;

- encouraging harmonisation of governments' regulatory policies and practices in the nuclear field, with particular reference to health and safety, radioactive waste management and nuclear third party liability and insurance;

- forecasts of uranium resources, production and demand;

- operation of common services and encouragement of co-operation in the field of nuclear energy information;

- sponsorship of research and development undertakings jointly organised and operated by OECD countries.

In these tasks, NEA works in close collaboration with the International Atomic Energy Agency, with which it has concluded a Co-operation Agreement, as well as with other international organisations in the nuclear field.

The Committee of the Safety of Nuclear Installations (CSNI) is an international committee made up of scientists and engineers who have responsibilities for nuclear safety research and the nuclear licensing process. The Committee
was set up in 1973 to develop and co-ordinate the Nuclear Energy Agency’s work in nuclear safety matters, replacing the former Committee on Reactor Safety Technology (CREST) with its more limited scope.

The Committee’s purpose is to organise international co-operation in nuclear safety. This is done essentially by:

(i) exchanging information about progress in safety research and regulatory matters in the different countries, and maintaining banks of specific data; these arrangements are of immediate benefit to the countries concerned;

(ii) setting up working groups or task forces and arranging specialist meetings, in order to implement co-operation on specific subjects, and establishing international projects; the output of the study groups and meetings goes to enrich the data base available to national licensing authorities and to the scientific community at large. If it reveals substantial gaps in knowledge or differences between national practices, the Committee may recommend that unified approach be adopted to the problems involved. The aim here is to minimise differences and to achieve an international consensus wherever possible.

The technical areas at present covered by these activities are as follows:

Particular aspects of safety research relative to water reactors, fast reactors and high-temperature gas-cooled reactors;

Probabilistic assessment and reliability analysis, especially with regard to rare events;

Siting research as concerns protection against external impacts;

Fuel cycle safety research;

The safety of nuclear ships;

Various safety aspects of steel components in nuclear installations;

Licensing of nuclear installations and a number of specific exchanges of information.

The Committee has set up a Sub-Committee on Licensing which examines a variety of nuclear regulatory problems, provides a forum for the free discussion of licensing questions and reviews the regulatory impact of the conclusions reached by CSNI.
FOREWORD

In November 1978, CSNI formed a Group of Senior Experts charged with undertaking certain specific tasks to further multilateral exchanges on LMFBR safety R and D. One of these tasks was to recommend areas in which international status reports on safety technology could be prepared.

The first report in the series of status reports, dealing with fission gas release from fuel pins, was completed and printed in 1980. The present report, which is the second in the series, was suggested and offered by the UK and it was agreed by the CSNI that the UK should take the main responsibility for its preparation. Comments on the first draft which was distributed to member countries in 1979 were received from experts in Germany, Japan and the USA:

Dr Shungo Iijima
NAIG Nuclear Research Laboratory,
4-1 Uchishima-cho, Kawasaki 210, Japan

Dr G Heusener
KfK, Karlsruhe
Mr Glauner

Dr Sutterlin
GRS, Glockengasse 2, Cologne

Dr F X Gevigan
Office of Reactor Research and Technology,
DOE, Washington

Dr M E Stephens of the Nuclear Safety Division, OECD Nuclear Energy Agency, co-ordinated the preparation of the report and sought for contributions internationally. One comment which came from several contributors was that the topic was connected more with operational safety than with licensing safety, but this distinction is perhaps only tentative at a time when licensing procedures for LMFBR are still at an early stage of evolution.

Within the UK, contributions or comments were made by:

Dr A R Baker
Central Technical Services, UKAEA, Risley

Mr K W Brindley
National Nuclear Corporation, Risley

Mr R J Cox
AEE, Winfrith

Mr L Maidment
Safety and Reliability Directorate, Culcheth

Dr J E Sanders
AEE, Winfrith

Mr J M Stevenson
AEE, Winfrith

Mr E B Webster
AEE, Winfrith, formerly of DNE (who was responsible for much of the calculational support)

Mr D Crowe
Dounreay Nuclear Power Development Establishment
(who made the measurements quoted on page 14)

The paper was written and revised by:

Mr R C Wheeler
Fast Reactor Development Directorate,
UKAEA(ND), Risley, Warrington, Cheshire
EXECUTIVE SUMMARY

To avoid damage to a reactor during reloading operations it is necessary not only to minimise the chances of making a sequence of loading errors but also to ensure that serious errors are detected in adequate time for corrective action. Many methods of detection are available in principle. In this paper a proposal is made to adopt 5 methods, some well known and some under development, which together provide the very high reliability which is required, for instance by a Licensing Authority. Emphasis is placed on Modified Source Multiplication (MSM), illustrated by some results from the PFR. Although the emphasis placed on the different methods varies from country to country, there is wide agreement on the conclusions reached which are that very high reliability can be achieved by adopting the right combination of methods, by correct decisions at the design stage and by reliable data handling during refuelling operations.
1. INTRODUCTION

This review is about the requirements for monitoring reactivity at shutdown, including the avoidance of damaging reactivity additions during reloading operations and the routine operational applications of reactivity measurements. It is suggested that the requirements can be met using ordinary nucleonic instrumentation, but that the only way to obtain the very high reliabilities which are needed is by using several levels of accident protection: no single simple trip system appears to be adequate. In order to illustrate the arguments, data will be presented for a large LMFBR of output around 1200 MWe, typical of several such systems being designed or built in the world, except for one example which is based on data from the PFR. But obviously while the data illustrate some of the principles quantitatively there are important variations between systems which require individual assessment. This review aims to lead towards an international consensus on some of the principles and methods (and combination of methods) which are likely to be chosen in future, detailed design and safety assessments.

2. THE PROBLEM AND THE SOLUTION

The proposed aims of a reactivity monitoring system are as follows:

i. The chance of damaging the reactor or fuel by the addition of too much reactivity following a sequence of incorrect loading operations should be made acceptably and negligibly small.

ii. During reloading operations the monitoring of reactivity should be as accurate and continuous as possible so that major loading errors or accumulated reactivity errors are detected as they occur. This not only contributes to achieving aim (i) above but also tends to minimize the economic penalties associated with shutdown times. Reactivity errors which are detected only after the critical balance point has been measured at the end of a reload usually lead to a need for further fuel moves and to wasted operational time while shutdown.

In the past, the emphasis with some systems has been placed relatively more on (i) than on (ii). For instance in the Clinch River Breeder Reactor (CRBR), the single aim of the Source Range Flux Monitoring (SRFM) system was stated to be as follows:

"The SRFM system will assure that the reactor does not reach the point where the worst single refuelling error would cause the reactor to be critical except in the instance of an intentional approach to critical".

In the CRBR and elsewhere emphasis has been placed on mechanical checks to prevent the occurrence of wrong loadings or wrong core components. This report does not seek to reduce the attention paid to identifying core components correctly and handling them reliably. Rather, it is suggested that all these checks will be needed in addition to the checks associated with (i) and (ii) above.

There are many methods of measuring shutdown reactivity: Table 1 defines and summarises 16 methods and variants of methods together with some comments on their advantages and disadvantages. More detailed explanations are contained in the references at the end of this review. Broadly, the methods can be divided into 3 groups:
A. Relative methods of calibration which do not perturb steady state reactivity and do not require detectors within the core

This group includes all the variants of source multiplication techniques which normally use the installed low-power instrumentation and are hence convenient and reliable. There is wide international agreement that the modified source multiplication (MSM) is the best method that is available for continuous use in IMFBRs.

B. Noise methods which do not perturb steady state reactivity but which require detectors within the core to achieve high counting efficiencies

There are many different methods based on the measurement of power noise, of which the 3 listed in Table 1 are examples, but the need for highly efficient detectors in the core means that the methods are mainly suitable for zero or low power systems and not for power reactors where high gamma fluxes make the counting of neutron fluxes difficult or impossible near the core. Also there are mechanical difficulties associated with moving the detectors into the core at shutdown and out of it at power. The reliability of noise methods is likely to be worse than the methods under A above because of more complicated equipment, electronics and data-handling.

C. Methods which perturb reactivity and power

In this category are listed the methods based on measuring the responses to a known reactivity or source change, caused usually by the movement of a control rod or sample or other device either once (eg rod drop) or repetitively (eg using a reactor oscillator). Apart from the operational inconvenience of the methods, all of which involve mechanisms which tend to be difficult or inconvenient to operate frequently or repetitively while reloading is in progress, it is extremely unlikely that they can be made as reliable and simple as those in category A above. However, given this proviso, many of the methods could be made to work if there were a sufficient incentive. It is usually necessary to adopt one of the methods in category C as an occasional means of calibrating the methods in category A.

As mentioned above, there is wide support for the MSM or one of its variants. It can be defined by means of the expression

\[ R = -aS/C \]  \hspace{1cm} (1)

where R is the reactivity

C is the signal at a detector, eg a count-rate

S is the effective total neutron source strength

a is a parameter which is calculated or estimated for a particular core configuration.

It can be shown that

\[ a \propto W^\nu \beta \]  \hspace{1cm} (1b)
where

\[ W = \text{the detection efficiency and represents the number of reactions in the detector per fission reaction in the core} \]

\[ \gamma = \text{the average number of neutrons produced per fission} \]

\[ \beta = \text{the total effective delayed neutron fraction} \]

It can be shown that the use of the MSM can almost achieve the 2 aims (i) and (ii) above. The MSM is most likely to be used as part of a reactivity accountancy procedure which detects discrepancies (above a certain level) between expected and measured reactivity. But because of small uncertainties in the measurements of signals, handling of data and computing, the MSM needs to be supplemented by a series of further, more approximate measurements or checks of reactivity, to be described in more detail in later sections.

The methods recommended here are summarised in Table 2 in the form of 5 protection levels. Of these, only levels 1 and 2 are ordinary measurements of shutdown reactivity. Level 3 is an indicator of increasing reactivity near critical. Levels 4 and 5 are associated with positive reactivities. In terms of the methods given above, 1 and 2 are Type A; 3, 4 and 5 are Type C.

<table>
<thead>
<tr>
<th>Protection Level</th>
<th>Title</th>
<th>Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Reactivity Accountancy</td>
<td>MSM</td>
</tr>
<tr>
<td>2</td>
<td>High count-rate trip on shutdown instrumentation</td>
<td>Source Multiplication</td>
</tr>
<tr>
<td>3</td>
<td>Power lag trip</td>
<td>Shutdown Kinetics</td>
</tr>
<tr>
<td>4</td>
<td>High level trip on intermediate and high power instrumentation</td>
<td>High power indication</td>
</tr>
<tr>
<td>5</td>
<td>Period trip</td>
<td>Divergence at positive reactivities</td>
</tr>
</tbody>
</table>

3. CORE DESIGN AND REACTIVITY MONITORING

In order to ensure that the suggested series of monitoring methods is effective it is necessary to take account of many factors at the design stage. For instance, the reactivity worths of individual control rods and fuelled subassemblies, and the withdrawal rates for unloading them, must be limited. Detectors must be designed correctly to provide a continuous measurement of neutron fluxes; and also the positioning and number of detectors must be adequate to deal with spatial effects and the requirements for redundancy.

Reliable reactor physics calculational methods must be provided both for the routine estimation of neutron flux distributions and for the effects of neutron source changes in order to provide necessary data for the MSM. Reliable handling and interpretation of count rates, computing of algorithms and the automatic initiation of trip and alarm signals are also needed. Most important
of all, it is necessary to provide for reliable reactivity reductions when required using safety rods or alternative shut-down devices. While it may be argued that a sufficiently safe procedure can be designed without the provision of safety rods or other similar devices (and some present reactors do not have safety rods), it is assumed here that safety rods will be provided in future LMFBRs, activated by the trip signals associated with the protection levels listed here.

The features of core design which are important to reactivity monitoring may be described with reference to Fig 1, on which are shown some comments on the different ranges of the reactivity scale. The range of reactivity variation during a reload is not definable very exactly but typical values are as follows:

- multi-batch fuel management scheme (eg say 6 reloads before the whole of the core is refuelled) -14.5 to -10%
- few-batch fuel management scheme (eg say 2 reloads before the whole of the core is refuelled) -23 to -10%

As well as the reactivity change during a reload, the magnitude of the natural source of neutrons from Curium 242, Curium 244 and other isotopes built up in irradiated fuel, is decreased:

- multi-batch fuel management scheme, total effective source is reduced by an estimated 16%
- few-batch fuel management scheme, total effective source is reduced by an estimated 50%

If there is no large artificial source present then the changes in the natural source will have an effect on count-rates of similar size, and of opposite sign, to the effect of the reactivity change. (It is usually inconvenient to rely on very large sources because of the handling difficulties which ensue)

In terms of the equation 1 above, C may be a very insensitive measure of R since S and R may vary in a roughly similar proportion during a reload. The count-rate C may be used to determine R only if S can be determined accurately enough during a series of fuel moves.

Below the scale in Fig 1 is shown diagrammatically typical mean and maximum incremental values for the reactivities associated with the removal of alternative shutdown devices, the removal of control rods and the insertion of fuelled sub-assemblies. To compare these reactivities with a point at which damage occurs, it is first necessary to estimate the reactivity needed to over-ride temperature increases for a typical large commercial LMFR.

The temperature coefficient of reactivity from the Doppler effect is expected to be around -0.25 cent/deg C averaged over the whole range up to the level of damage. At low flow, say 10%, damage is likely to occur first when the maximum clad temperature reaches about 900°C, at 25% of full power (say 800 MW). At this power and at an inlet temperature of 300°C the mean fuel temperature is expected to be around 710°C, or 410°C above inlet temperature. Thus a conservative estimate of the reactivity insertion required to damage the core (which omits the effect of feedback from fuel expansions and various more slowly-acting mechanisms) is about 0.25 x 10^-2 x 410 ≈ 1%. Because of variations between different systems, particularly in the choice of the primary flow rate provided at shut-down, this estimate is only approximate.
Thus provided only one move at a time can take place during reloading operations, it can be seen from Fig 1 that the important ranges of reactivities at or before which there should be an automatic trip or other preventive action are as follows:

-1 $\%$ to critical

From this range the removal of an alternative shutdown device could in principle damage the core given no protective action.

From $-0.2$ $\%$ to critical

From this range the removal of a control rod, or at least a rod with high worth, could in principle damage the core given no protective action.

For LMFBR which are smaller than the commercial-sized reactors considered here, these reactivity ranges may extend further towards much more negative reactivities. This is because the worths of control rods tend to increase with smaller core sizes.

It is noted that, with present designs, individual sub-assemblies do not have sufficient reactivity worth to take the reactor from a negative or zero reactivity to a damage level, at least in one step. Hence a chief concern is with the accidental removal of alternative shutdown devices and control rods when the initial reactivity is in the ranges given above. This sometimes leads to a useful simplification since their removal does not affect the amplitude of $S$, the source term, in equation 1.

Adequate protection from damage can be obtained in principle via the measurement of count-rates in the five ways listed above and applied as follows:

**Protection Level 1**

Over the normal shutdown range up to about $-1$ $\%$, but particularly in the normal range of routine accountancy up to about $-10$ $\%$, the MSM method may be applied. If some individual loading errors are not detectable because the reactivity discrepancy is less than about 30 cent (see section 5) then there is a progressively improved chance of detecting cumulative errors above this level.

**Protection Level 2**

The count rate $C$ may be used to provide a high level trip within the approximate range $-2$ to $-1$ $\%$, as described in section 8. The value of $C$ which initiates the trip must be above the maximum value expected during the reload, but less than the value which corresponds to $-1$ $\%$, since above this value the necessary steady state connection between reactivity and count-rate may be lost (see below).

**Protection Level 3**

The response of power to reactivity may be delayed significantly in the approximate range $-1$ to $0$ $\%$ because of the effects of delayed neutrons. It is proposed here that this delay should be made an indicator of high reactivity at shutdown. (Section 9).
Protection Level 4

In the range of positive reactivity up to about 0.8 \( \% \), a number of high level power trips associated with the intermediate and high power instrumentation are normally provided, with a response which is fast enough to prevent damage. The upper limit of 0.8 \( \% \) is associated with a steady reactor period of about 1 second, below which the response times of the trips and shut-off rods may be insufficient to prevent damage. Although of a different type, high level core outlet temperature trips provide an extra degree of protection which can be considered, if desired, as part of protection level 4.

Protection Level 5

In the range of reactivity from about 0.3 to 0.8 \( \% \) reactor period trips are normally provided as part of each range of neutronic instrumentation.

None of these five levels offers complete protection against damage since all can fail in principle or be by-passed by operators. It is necessary for the operational rules to be framed so that the chance of by-passing several trips at the wrong time is very small, while allowing scope for necessary maintenance procedures. Protection level 1 requires computing support which can be sensitive to errors in input data as well as data handling faults. Protection levels 1 and 2 may require data adjustments from run to run. Protection levels 3, 4 and 5 depend on an assumed margin to damage which depends in turn on a minimum core flow of say 10% during reloading operations: losing this flow is a source of error.

Figure 2 show the sequence of events connecting the different protection levels. Probabilities have not been placed on the diagram because these depend on the design details such as the numbers of detectors, channels and on the trip logic. It can be inferred that there is considerable scope for reducing the probability of damage to the reactor to an extremely low value by correct design of the features mentioned in this section.

4. FLUX TILTS AND NUCLEONIC DETECTORS

A case can be made for installing sensitive detectors which are especially designed and positioned for monitoring count-rates at shutdown and which are retractable during power operation. In the past, low power detectors have often been used to cover both the shutdown monitoring range and low power critical operations with the result that they have sometimes lacked sensitivity for the former. If the main purpose were to monitor fuel movements at shutdown, then a good position for the detectors would be along the axis of the core cylinder either above or below the core. This is because the relative axial flux distribution is nearly constant when control rods are either fully in or fully out, as they are for fuel moves at shut-down, while detectors outside the core on the centre-plane tend to be affected by diametric (asymmetric) flux tilts as the core is reloaded. But as explained above, the chief problem is more concerned with the withdrawal of control rods than with sub-assembly movements. Detectors on the axis of the core are subject to the axial tilts produced by the withdrawal of the control rods which can be an important consideration, so they are unsuitable for monitoring control rods. The best position for detectors is thus probably the traditional one in the radial shield on the centre plane of the core, at some optimum radius determined by the gamma and neutron sensitivities. By having at least three, but if possible more, detectors distributed around a circumference of the reflector or radial shield, some useful averaging out of diametrical flux tilts can be carried out (see below).
5. PROTECTION LEVEL 1: REACTIVITY ACCOUNTANCY

By reactivity accountancy is meant the continual comparison, probably using an on-line computer, between reactivities deduced from measured count-rates and calculated reactivities corresponding to each movement of fuel during a reload. Reactor physics studies on ZEBRA at AEE Winfrith (1) and on other zero-power mock-ups have shown that the calculational methods which are available and in general use at present are adequate for accountancy based on the MSM. The changes in the signals at detectors located far from the core can be predicted for a wide range of fuel moves (including effects both of reactivity and of source changes) to within a few percent. The reactivity, or the reactivity change, can be deduced from the signals to around \( \pm 7\% \) (lsd) or better, corresponding to a reactivity error of up to \( \pm 30 \) cents per move for typical fuel moves. The diffusion theory or transport theory calculations on which the MSM is based need not extend necessarily out to the detector positions, since it has been shown that the neutron fluxes along a radius outside the core change by closely the same proportion when a fuel move is made (except possibly in sub-assemblies adjacent to the fuel being moved). With this simplification, a reduced number of meshes is required in the calculations; and it should be possible to carry out few-group calculations very economically on a large digital computer, given sufficient advance warning of impending fuel moves. Even so, it seems unlikely that a calculation of the expected changes in signal for every move will be feasible for reasons of sheer data handling limits. More probably, the reactivity after a series of moves will be checked against a more limited number of calculations in the attempt to detect any discrepancies indicative of wrong moves. Count-rates measured after every move are likely in practice to provide a useful and continuous, but more approximate, detection process only if some simple model of the changes can be used to interpolate between more exact results from diffusion theory calculations. Two such models for accountancy are described below.

Whatever simplifications or approximations are developed, the distribution of neutron source strength needs to be specified for each stage of a reload. Source strengths in each sub-assembly can be defined to adequate accuracy as simple calculated functions of initial isotopic contents, enrichment and burn-up. It may thus be convenient to store the source data in the same data bank as the predictions from routine fuel management calculations.

6. APPROXIMATE ACCOUNTANCY

Many different approaches can probably be adopted to help interpolate between the results of diffusion theory calculations. The following approximate relationship has been proposed in connection with the PFR (2) to relate a counting rate \( C \) to reactivity \( \delta \) at shutdown.

\[
C_1R_1 = C_0R_0 (1 + A (R_1 - R_0))(1 + G (R_1 - R_0))(1 + B \phi^* \Delta S)(1 + H \Delta S)
\]  

(2)

where

- \( C_0C_1 \) are the counting rates before and after a core move (fuel or control rod movement)
- \( R_0R_1 \) are the reactivities before and after the move
- \( \Delta S \) is the change in neutron source strength as a result of the move
- \( \phi^* \) is the adjoint flux at the position of the move which provides an importance weighting for the change in source \( \Delta S \).
- \( A, B \) are constants
- \( G, H \) are series of constants defined for every combination of core position and detector position, calculated for typical core arrangements.
The terms in the four brackets on the right-hand side of equation 2 each represent small departures from unity, typical of a tightly-coupled core with a limited tendency towards flux tilting, i.e. a typical LMFBR rather than a large thermal reactor.

The first term in brackets represents approximately the symmetrical change in flux distribution for a reactivity change which is distributed rather than associated with one position in the core; the second term in brackets represents the 'flux-tilt' or symmetrical effect of a reactivity change associated with a change at one core position; the third term in brackets represents the symmetrical change in flux distribution following a change in the total weighted neutron source strength; and the fourth term represents the asymmetric effect of a change in source strength at a specific core location.

The constants $A_{B_0}$ can be shown to be relatively independent of loading changes provided the main geometric boundaries of the core are maintained constant.

The separation into symmetric and asymmetric effects here is partly for convenience in defining and tabulating (eventually) the constants $A_{B_0}$ and partly to illustrate the connection between the two approximate methods proposed:

a. **Asymmetric accountancy or 'flux-tilt' method**

   This is based on interpreting the signals separately from each detector using equation 2, and is particularly subject to the errors associated with the terms in $G$ and $H$.

b. **Symmetric accountancy or multi-detector method**

   This is based on using a mean signal from several detectors distributed uniformly around the core at the same radius, so that terms in $G$ and $H$ are averaged out, at least approximately, leading to a simpler version of equation 2. The amount of pre-calculation is much reduced by the removal of the terms based on $G$ and $H$. The larger the number of distributed detectors, the more accurate this method is likely to be.

7. **ILLUSTRATION OF APPROXIMATE ACCOUNTANCY**

   The approach mentioned above must be regarded as still under development. A partial check against measurements has been carried out with data obtained during the first reload of the PFR (3,4), which provides some support for the ideas summarised here. Only two detectors were available at the time of the reload in early 1978: one at the core centre was a fission chamber in a special, temporary core access thimble; the other was a permanently installed low power fission chamber in the radial shield of the reactor. It was assumed that the reciprocal of the count-rate of the central chamber gave estimates of reactivity changes which were nearly independent of changes in flux distribution (flux tilting) during the reload, an assumption which was however supported by a series of diffusion theory calculations. Changes in the count-rates at the installed low power fission chamber were interpreted in terms of reactivity changes using equation 2. Values of the constants $A$, $B$, $G$, $H$ were calculated in advance for an idealised version of the equilibrium core, half burnt-up. Source changes were estimated using the FISPIN program. The reactivity was normalised at the start of the reload to a calculated value ($\bar{\beta}$ 26) obtained from a TIGAR diffusion theory result (Ref 3) which was part of a standard fuel management calculation. Three partial checks of the approximate asymmetric method (method 'a' of the previous section) are reproduced here:
the total worth of the first control rod to be removed from the core (a tantalum rod in position M13) can be compared as follows:

via count rate change on the central fission chamber (where the rate reduced by 19.4%) \[3.75 \pm 0.2\%\]

via count rate change on the installed low power fission chamber B (where the rate reduced by 10.2%), using equation 2. \[3.20 \pm 0.2\%\]

calculated worth from the difference between 2 k-calculations \[3.69\]

* (NB errors here include counting statistics only)

This comparison shows fair agreement despite the fact that the percentage change in count rate varied by nearly a factor of 2 between the two fission chambers.

Figure 3 shows the detailed variations in reactivity during the reload when 130 movements of control rods and sub-assemblies took place. Results from the two fission chambers, when interpreted as above, showed the same trends. This was despite the wide variations in count-rate which sometimes showed changes of opposite sign when fuel was moved near the core edge in the sector of the installed low power fission chamber. A small systematic difference between the two estimates of reactivity was observed towards the end of the reload, believed to be due to the lack of correction factors applied to the data from the central chamber (which are not exactly independent of flux tilts).

The overall reactivity change over the whole of the reload may be compared as follows:

from the central fission chamber \[+6.8\%\]

from installed low power fission chamber corrected step by step using eq 2 \[+7.9\%\]

as measured via the difference between two critical balance points on calibrated control rods before and after the reload \[+7.3\%\]

These checks together with other comparisons based purely on calculations (2) suggest that the changes in count-rates may be modelled at least approximately without recourse to diffusion theory predictions for every move.

8. PROTECTION LEVEL 2: HIGH COUNT RATE TRIP ON SHUT-DOWN INSTRUMENTATION

If the reactivities at the beginning and end of a reload are \(R_0\) and \(R_1\), and if the total effective neutron source strengths are similarly \(S_0\) and \(S_1\) then the count-rate at a detector varies during the reload from \(aS_0/R_0\) to \(aS_1/R_1\). For some typical fuel reloading plans the count rate may change approximately linearly between the initial and final values, often with the final value being also the maximum value. More generally, the maximum can occur anywhere during the reload.

Variations in detector efficiency, etc, also cause the parameter \(a\), and hence count rates, to vary. Flux tilting is one reason why the variations in count rate become
more extreme and the position of the maximum more difficult to determine. It is suggested that the mean of two opposite, or several symmetrically arranged, detectors should always be used in connection with a high count rate trip at shut-down in order to minimise flux tilting effects. Given sufficient detectors it should still be possible to have at least a 2 out of 3 trip system to allow for false signals and maintenance periods while maintaining adequate reliability.

Suppose the maximum count rate averaged over several detectors is estimated in advance of a reload to be \( C_m \). Then if it is proposed to allow, in addition, a control rod or alternative shut-down mechanism of maximum worth \( R_{\text{max}} \) to be removed at any time during the shut-down, the appropriate reactivity level \( R_t \) for a trip setting \( C_t \) is given by

\[
R_t = \frac{aS_m}{C_m} + R_{\text{max}} + R_e
\]

and

\[
C_t = C_m \left[ 1 - \frac{C_m aS_m}{R_{\text{max}} + R_e} \right]^{-1}
\]

Where \( S_m \) is the value of \( S \) at the time \( C_m \) occurs; \( R_e \) is an allowance for errors in the quantities \( C_m, S_m, R_m \) which is designed to reduce the probability of a trip signal following the correct insertion of reactivity \( R_m \) to an acceptably low value.

Putting in typical values, eg \( \frac{aS_m}{C_m} = 3.5 \% \), \( R_{\text{max}} = 2 \% \) and \( R_e = 0.3 \% \)

it is deduced that \( R_t \approx -1.2 \% \)

However it is common practice to remove absorbing rods or devices of high worth only at the start of a reload to take advantage of the lower initial reactivity. If this rule can be enforced, then the trip level can be reduced to a value which is equivalent to \( R_t = -2 \% \) or below. The removal then of a rod or device of high worth at the wrong point in a reload, or the incorrect removal of a second rod, will trip the reactor via the insertion of shut-off rods. Protection level 2 is used in this way in many reactors at present.

9. PROTECTION LEVEL 3: POWER LAG TRIP

In order to increase the reliability of detecting high reactivity in the event of malfunction of protection levels 1 and 2, and specifically in order to prevent the continuous removal of a control rod or alternative shut-down device when starting with a high initial reactivity, a so-called power lag trip is proposed. Basically it uses the well-known method of inverse kinetics (eg Ref 14), in which changes in the shutdown count-rates are interpreted in terms of reactivity by solving the prompt and delayed neutron equations. But the adaptation of the method proposed here circumvents the problem (which is usually associated with inverse kinetics programs) which is that it is necessary to specify either the initial reactivity value at the start of a transient or (what amounts to the same) detector efficiencies and initial source terms. Specification of individual delayed neutron fractions and periods, which are also required, to adequate accuracy is usually straightforward.

The proposal is to interpret the shape of the transient following the end of a ramp insertion of reactivity. Figure 4 serves to illustrate the method. A 10-second linear insertion of reactivity is assumed to occur during which the shut-down reactivity \( R_o \) is reduced by 10%, ie the reactivity is increased from \( R_o \) to 0.9 \( R_o \). This example was chosen so that the ratio of the final (steady state) power to the initial power are the same, independently of \( R_o \). However, the shape of the transient during and after the ramp depends on \( R_o \). By analysing the shape of the transient via an inverse kinetics program, \( R_o \) may be obtained.
The method is explained in terms of one delayed neutron group and few groups in Appendix 1. The approximate analysis there is used to make some detailed comments on the method and to suggest the count-rates which will be needed in order to make it sufficiently accurate for practical use.

The prompt and delayed neutron equations to be solved are:

\[ \left\{ \frac{\beta}{\ell} \right\} (1-R)n = \sum_i \lambda_i C_i + S \]  
\[ C_i = \frac{\beta_i n - \lambda_i C_i}{\ell} \]  

where \( R \) = reactivity 
\( n \) = total neutron population 
\( C_i \) = population of delayed neutron precursors, group \( i \) 
\( \ell \) = prompt neutron lifetime 
\( \beta \) = total delayed neutron fraction 
\( \beta_i \) = delayed neutron fraction, group \( i \) 
\( S \) = production rate of source neutrons 
\( \lambda_i \) = decay constant for delayed neutrons, group \( i \)

By writing \( P = \beta n/\ell S \) and \( D_i = C_i/S \) the above expressions may be simplified to:

\[ P (1-R) = 1 + \sum_i \lambda_i D_i \]  
\[ D_i = \beta_i P - \lambda_i D_i \]  

(that is, the prompt and delayed neutrons are on a scale which is inversely proportional to the constant source term \( S \)).

A proposed practical application is as follows:

i. Count rates proportional to \( P \) are fed into an on-line computer program to obtain initial values of \( D_i \) for each delayed neutron group using equation 7, before, during and after the removal of a control rod or alternative shut-down device.

ii. The control rod or ASD is removed at any required speed, constant or variable, until, say, \( \frac{3}{4} \) removed, at which point it is stopped automatically. The \( \frac{3}{4} \) part-removal is chosen on the basis that the reactivity added at this point is less than the maximum reactivity addition associated with one fuelled sub-assembly (which is loaded without a power lag trip operational).

iii. The count rates are interpreted, from the time the rod stops, using equation 7, rearranged as follows:

\[ R = 1 - \frac{1 + \sum_i \lambda_i D_i}{P} \]  

The time required for the pause is estimated to be about one minute with typical data. An automatic trip is operated if \( R \) is greater than some chosen level (say \(-1 \beta\)).
The above steps are repeated with the rod stopping also at \( \frac{3}{4} \) and fully removed for further checks of \( R \).

10. CONCLUSIONS

Damage can be prevented (to a high level of probability) following loading errors by means of a combination of methods. Of the 5 methods recommended here, 3 are currently used in LMFBRs and have consequently been mentioned rather briefly. The power log trip is not in regular use as proposed here but it can provide in principle very useful protection against the incorrect removal of a control rod (an important safety issue). The MSM is widely used but is in a different category from the other 4 methods since the way it is applied (eg as regards detectors, data handling, calculational support and interpretation) varies, and it is currently under development. This paper has highlighted approximate methods of applying the MSM, which may be used for continuous reactivity accountancy.

The probability that damage occurs following a loading error depends basically on correct design. Items of special importance are: worths and removal rates for absorbers of high worth, numbers and positions of nucleonic detectors, coolant flow rate at shut-down and on-line computing methods. It is recommended that the effect of these items on shut-down monitoring and detection should be considered at the design stage, so that an adequately high level of reliability in detecting important reloading errors can be demonstrated. The probability of a serious loading error not being detected by the various proposed levels of protection can then be estimated at the design stage for comparison with the licensing criteria.

References

3. E B WEBSTER, DNE, Private communication.
4. D CROW, DNE, Private communication.
8. J T MIHALCZO and others. Reactivity surveillance procedures experiments with the FFTR engineering mockup core. ORNL TM 4707, 1976.


## TABLE 1
Summary of methods of measuring sub-critical reactivity

<table>
<thead>
<tr>
<th>Method and brief description</th>
<th>Typical algorithm (for nomenclature see foot of table)</th>
<th>Main advantages (A) / Disadvantages (D)</th>
<th>Very approximate estimate of range of potential usefulness for power fast reactor (- $)</th>
<th>Suitable in principle for routine use?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. RELATIVE METHODS OF CALIBRATION WHICH DO NOT PERTURB STEADY STATE REACTIVITY OR COUNT-RATES AND DO NOT REQUIRE DETECTORS IN CORE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. <strong>Source multiplication</strong> depending on steady state multiplication</td>
<td>$\frac{R = -\frac{aS}{C}}{1} ; \quad \text{or} \quad \frac{R = -\frac{a_i S}{C}}{1} ; \quad \text{or} \quad \frac{R = -\frac{a_i S}{C}}{1}$</td>
<td>A. Simplicity \nD. Without separate calibration, it is a relative method only \nD. Fails to account for flux tilting and changes in counter efficiency during reloads (cf Method 2) \nD. Requires constant source $S$, or calculated corrections.</td>
<td>0 - 2</td>
<td>No</td>
</tr>
<tr>
<td>2. <strong>Modified source multiplication</strong> (Ref 5, 12)</td>
<td>$\frac{R = -\frac{a_i S}{C}}{1} ; \quad \text{or} \quad \frac{R = -\frac{a_i S}{C}}{1}$</td>
<td>A. Useful accuracy over a very wide range (Ref 9) \nA. Applicable to asymmetric cores, detectors and sources; and works with detectors in radial shield. \nD. Requires substantial calculational support.</td>
<td>0 - 50</td>
<td>Yes</td>
</tr>
<tr>
<td>3. <strong>Modified source multiplication with flux tilt approximation</strong> (Section 6a) as Method 2 above, but flux tilts are approximated as in Equation 2 of this report, involving new constants $A$, $B$ and a library of calculated $G$, $H$.</td>
<td>$\frac{R = \frac{1 - (A + G)R_D}{C}}{1} ; \quad \text{or} \quad \frac{R = \frac{1 - (A + G)R_D}{C}}{1} ; \quad \text{or} \quad \frac{R = \frac{1 - (A + G)R_D}{C}}{1}$</td>
<td>A. Same advantages as Method 2 with reduced calculational support requirements. \nD. Less accurate than Method 2 and probably applies over a reduced range. \nD. Changes in source terms $AS$ need to be calculated.</td>
<td>0 - 35</td>
<td>Yes</td>
</tr>
<tr>
<td>4. <strong>Modified source multiplication with multi-counters</strong> (Section 6b) as Method 3 above, but terms in $G$ and $H$ are averaged out by using sum of all available detectors</td>
<td>$\frac{R = (1 - AR_D) x}{1} ; \quad \text{or} \quad \frac{R = (1 - AR_D) x}{1} ; \quad \text{or} \quad \frac{R = (1 - AR_D) x}{1}$</td>
<td>A. Much simpler than Methods 2 and 3 above, with part of their advantages. \nA. Loss calculational support needed than with above methods. \nD. Requires $3-6$ distributed detectors \nD. Less accurate than Method 2 \nD. Changes in source terms $AS$ need to be calculated.</td>
<td>0 - 35</td>
<td>Yes</td>
</tr>
<tr>
<td>Method and brief description</td>
<td>Typical algorithm</td>
<td>Main advantages (A) / Disadvantages (D)</td>
<td>Very approximate estimate of range of potential usefulness for power fast reactor (- $)</td>
<td>Suitable in principle for routine use?</td>
</tr>
<tr>
<td>-----------------------------</td>
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</tr>
</tbody>
</table>
| 5. **Asymmetric source** (Ref 13) | $R = \left(\frac{C_1}{C_2}\right)^x - 1$  
($x$ is dependent on geometry) | A. Can work in principle with a single decaying source  
D. But becomes complicated for a combination of decaying and constant sources  
D. However, it fails, like Method 1, to include the flux tilting effects caused by loading changes; these are frequently more important to $C_1/C_2$ than the departure from the fundamental flux shape monitored  
D. Significant supporting calculational effort is needed. | 0 - 2 | No |
| 8. **NOISE METHODS WHICH DO NOT PERTURB STEADY STATE REACTIVITY OR COUNT-RATES BUT REQUIRE DETECTORS WITHIN THE CORE TO ACHIEVE HIGH COUNTING EFFICIENCIES** | | | | |
| 6. **Neutronic noise amplitude** (Ref 7, 10) uses $A_{xx}$ auto power spectral density (APSD) or $A_{xy}$ cross power spectral density (CPSD) to give transfer function $G(jw)$. | $E_i A \frac{1}{\beta} G(jw)$  
where $G(jw)$ is obtained from  
$A_{xx} = a + b|G(jw)|$  
or $A_{xy} = c|G(jw)|$ | A. Fairly independent of source position in core  
D. Requires in-core detector(s)  
D. Range of $R$ is limited by size of available sources and practicable count-rates  
D. $A/\beta$ may not be constant over whole range.  
D. Requires to be calibrated at critical.  
A, D as for Method 6 above, except that there is no dependence on $A/\beta$. | 0 - 7 | No |
| 7. **Polarity spectral coherence amplitude** (PSCA) (Ref 7, 9, 10) Correlates the sign and polarity coherence between two detectors. $x, y$ whose spectral densities at the limit of low frequency is given by $A_{xx}, A_{yy}, A_{xy}$ | $(1 - R) \sigma \left( E_i \left( \frac{A_{xx} A_{yy}}{A_{xy}} \right) - 1 \right)$ | | | |
| 8. **Break frequency noise analysis (BFNA)** (Ref 6, 7, 8, 11) uses APSD, CPSD or PSCA to determine break frequency $f_b$ in noise spectra. | $(1 - R) \sigma \left( \frac{A_{xx}}{A_{xy}} \right)$ | A, D as for Method 6 above, except that it is independent of $E_i$ (potentially a great advantage) and has potentially a much wider range (Ref 8). | 0 - 35 | No |
| Method and brief description | Typical algorithm | Main advantages (A) / Disadvantages (D) | Suitable in principle for routine use?
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C. METHODS WHICH PERTURB THE CORE (ALL GIVE REACTIVITY ABSOLUTELY)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 9. Rod drop, analysed by inverse kinetics (Ref 6, 7, 9, 10, 12, 14) | \[ R(t) = 1 - \frac{S + \sum \lambda_j C_j(t)}{C(t)} \] | A. Rapid and convenient experiment to complete  
A. Conveniently calibrates other methods like WEM (Methods 2, 3, 4 above) (Sect 5)  
D. Assumes (at least in present FRP format) no change in detector efficiency before and after drop, so limiting range  
D. Although reactivity is presented as continuous output (sometimes an advantage), the averaging at constant R is often subjective  
D. Requires adequate count-rates after drop. | 0 – 5 |
| 10. Rod drop, analysed by three point method (Ref 15, 16) | See Ref 15 for integral form of above.  
C(t) = \frac{E_2 S_2}{\beta (1 - R_2)} - \frac{R_2 E_2 B_1}{(1 - R_2) \beta_1 B_2} \sum_j \lambda_j e^\lambda_j t  
+ \sum_j \lambda_j e^\lambda_j t \int_0^t C(t) e^\lambda_j t \, dt \frac{1}{1 - R_2}  
(from Ref 9) (Suffix 1 = before drop) (Suffix 2 = after drop) | A. Obtains reactivity more accurately than in Method 9 above, because 'averaging' is avoided, and is thus better tailored to the requirements of rod calibrations. Otherwise as above. | 0 – 5 |
| 11. Rod drop, analysed via post-drop transient only (Ref 9) | Similar to Method 10 above, but a fit is made to the shape of the transient after the drop only. | A. By fitting the three terms separately, \( R_2 \) can be obtained without assuming values for \( E_2/E_1, B_2/B_1, S_2 \), etc.  
A. The lack of dependence of third term of BHS on efficiency \( E \) means that this route is accurate over a wider range of \( R \) than Methods 9 and 10 above. | 0 – 15 |
| 12. Source Jerk (Ref 7, 9, 17) | Measures \( R \) via the prompt jump following removal of a source  
R = \frac{C_1 - C_2}{C_0 - C_2}  
(Suffix 0 = before jerk)  
(Suffix 1 = immediately after prompt jump)  
(Suffix 2 = final equilibrium value) | A. Nearly independent of size of source  
A. Nearly independent of counting efficiency if source is central  
D. Needs large source with mechanism to move it rapidly  
D. Needs good count-rate to obtain \( C_1 \) easily after prompt jump  
D. Not so convenient as rod drop experiments  
D. Not easy to interpret at around – 5% or below because of changes from fundamental mode. | 0-1 – 5 |


<table>
<thead>
<tr>
<th>Method and brief description</th>
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<th>Main advantages (A) / Disadvantages (D)</th>
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<th>Suitable in principle for routine use?</th>
</tr>
</thead>
<tbody>
<tr>
<td>13. Pulsed neutron source (Ref 7, 10, 18) measures prompt decay of fundamental mode after a power pulse.</td>
<td>$R = 1 + \frac{A}{B} \ln \frac{C(t)}{C_0}$</td>
<td>D. Requires high count-rates and therefore a large pulsed source of perhaps $10^{11}$ n/s is needed, in the form of 10 μs pulses. D. There are environmental problems in getting the pulsed source into the core. D. Except near critical, the decay of the fundamental mode may be contaminated by harmonics. D. Corrections for variable $B/A$ are needed.</td>
<td>0 - 3</td>
<td>No</td>
</tr>
<tr>
<td>14. Modified pulsed neutron source (Ref 7, 10, 18) by integrating the counts during and after a pulse, $R$ can be obtained from the ratio between prompt ($C_p$) and delayed ($C_d$) count-rates.</td>
<td>$R = \frac{C_p}{C_d}$</td>
<td>All above apply.</td>
<td>0 - 3</td>
<td>No</td>
</tr>
<tr>
<td>15. Reactivity oscillator the response of a detector signal (Ref 17) $\delta C$ to an oscillating input of amplitude $\delta R$ depends on $R$.</td>
<td>$R = \frac{C_p}{C_d} \sum_{j=0}^{\infty} \sum_{j=0}^{\infty} \frac{\lambda_j}{\lambda_j} \delta R$ (applies in delayed neutron range of frequencies) Reduces at steady state to Method 16.</td>
<td>A. Not dependent on source strength or position A. Not dependent on detector efficiency $E$ B. Requires mechanical oscillator in core and transfer function analyser D. Needs calibrating at critical to give $\delta R$.</td>
<td>0 - 10</td>
<td>No</td>
</tr>
<tr>
<td>16. Movement of known reactivity worth the steady-state change $\delta C$ in a detector following the insertion or removal of sub-assembly, sample or rod of known worth $\delta R$ depends on $R$.</td>
<td>$R = \frac{C_p}{C_d}$</td>
<td>A. Useful in view of possible discrepancies in $B$ B. Possible fall-back calibration method D. Depends on a calculated worth $\delta R$ (or $\delta R$ is obtained via other experiment).</td>
<td>0 - 10</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Nomenclature**

- $a_i, b_i, c_i, A_i, B_i$: Constants
- $C$: Signal at detector
- $C_j$: Signal at detector from precursor group $j$ only
- $E_i$: Efficiency of detector $i$ (count-rate per 'average' core fission event)
- $R$: Reactivity in dollars
- $S$: Source strength
- $t$: Time
- $B$: Total effective delayed neutron fraction
- $\sigma_{j, \lambda j}$: Delayed neutron fractions and decay constants, group $j$
- $A$: Prompt lifetime
- $\omega$: Frequency
- $G$: defined (differently) under methods 3, 6 above
APPENDIX 1

Analysis of the power lag trip with few delayed neutron groups

The method can be explained in terms of a single delayed neutron group starting with the following 2 equations for prompt and delayed neutrons respectively (which are the 1 group versions of equations 7 and 8):

\[
P (1 - R) = 1 + \lambda D \quad (A1)
\]
\[
\dot{D} = P - \lambda D \quad (A2)
\]

where now \( \lambda \) is the lumped decay constant for the single group.

It is required to determine the initial rate of change of reciprocal power following a ramp reactivity insertion rate \( b \), starting at \( R_0 \), ie

\[
R = R_0 + bt
\]  

(A3)

Reciprocal power is the chosen quantity here since if there were no delayed neutrons, then equations A1, A2 reduce to \( P = -1/R \); and the rate of change of reciprocal power is proportional to the reactivity insertion rate as follows:

\[
d \left[ \frac{1}{P} \right] /dt = -\dot{R} = -b
\]  

(A4)

The effect of delayed neutrons is to slow the response of power \( P \) to the ramp increase in \( R \); from equation A1, \( \frac{1}{P} = \frac{1}{1 + \lambda D} \) and so:

\[
d \left( \frac{1}{P} \right) dt = - \frac{\dot{R}}{1 + \lambda D} - \frac{(1 - R) \dot{D}}{(1 + \lambda D)^2}
\]

(A5)

substituting for \( \dot{D} \) from equation A2 and eliminating \( D \) from equation A1

\[
d \left( \frac{1}{P} \right) /dt = - \frac{1}{P (1 - R)} (\dot{R} + \lambda (1/P + R))
\]

(A6)

substituting for the initial values \( P_0, R_0 \) and remembering also that \( P_0 = -1/R_0 \), this reduces to:

\[
d \left( \frac{1}{P} \right) /dt \bigg|_{t=0} = \frac{R}{1 - R_0} \frac{R_0}{1 - R_0} = \frac{b R_0}{1 - R_0}
\]

(A7)

That is, the response of power to a ramp reactivity input is slower than the same response without delayed neutrons (equation A4) by a factor \( R_0/(1 - R_0) \). So that if the initial rate of change of reciprocal power could be measured with and without the effect of delayed neutrons the difference \( X \), obtained by subtracting equation A7 and A4 is:

\[
X = \frac{b}{1 - R_0}
\]

(A8)

where \( X \) is of interest since 1/X is just the time between the start of the ramp and the time at which damage could occur in principle at around \( R = 1 \) if the ramp continued indefinitely and if no trips took place.

It is inconvenient to obtain \( X \) in practice because of the time required to complete the steady state measurements implied by equation A4. Instead, it is much more convenient to measure the power lag implied by equation A7 via the transient response following a pause in the ramp reactivity insertion. First, it is useful to note the period over which equation A7 holds approximately.
The prompt and delayed neutron equations were solved for 2 ramp rates and a number of different values of the initial reactivity \( R_0 \) to obtain the data shown in Fig 5, using equations 7 and 8 of section 9. Delayed neutron data appropriate to a core fuelled with Pu239 were used and the only approximation made was to lump delayed neutron groups 1 and 2, 3 and 4 and 5 and 6 to give a 3 delayed neutron group representation. The higher ramp rate chosen (b = 0.1 s\(^{-1}\)) is representative of the maximum reactivity insertion rate of a control rod of high worth. The lower ramp rate chosen (b = 0.05 s\(^{-1}\)) is representative of the mean insertion rate over the whole distance moved by a rod of high worth. The calculated data show the significant difference in the quantity \( d(1/P)/dt \) for the same ramp input as the initial reactivity \( R_0 \) increases towards delayed critical \( (R_0 = 0) \). They also indicate that equation (A7) is a reasonably close representation of the delaying effect of delayed neutrons over significant periods of time up to and beyond delayed criticality.

Suppose thus that equation A7 can be applied up to a time \( t = t_1 \). Then an estimate of the power \( P_1 \) at \( t_1 \) is given by integrating equation A7 to obtain:

\[
\frac{1}{P_1} = \frac{1 + bR_0 t_1}{P_0} \frac{1 - b}{1 + P_0}
\]  

(A9)

If the ramp is stopped at \( t_1 \), the power increases from \( P_1 \) to \( P_2 \) at constant reactivity \( R_2 \) where:

\[
R_2 = R_0 + b t_1 = -1/P_2
\]  

(A10)

Equations A1 and A2 may be solved to give the transient between \( P_1 \) and \( P_2 \), i.e. with one delayed neutron group only:

\[
P = P_2 + (P_1 - P_2) \exp \frac{\lambda R_2 (t - t_1)}{1 - R_2}
\]  

(A11)

If the period \((1 - R_2)/\lambda R_2\) which characterises the transient in equation A11 could be measured, then \( R_2 \) may be obtained, at least in principle.

At low reactivities, including the range in which reloading operations normally occur, the ratio \( R_2/(R_2 - 1) \) approaches unity and the exponential term in equation A11 reduces to \( \exp \lambda (t - t_1) \). This means that there is little possibility of measuring \( R_2 \) unless \( R_2 \) is small, i.e. reactivity is high, e.g. close to critical.

For times short compared with \((1 - R_2)/\lambda R_2\), equation A11 may be approximated and rewritten in terms of an instantaneous period \( T \) (which is to be measured soon after the end of a ramp):

\[
T = P_1/dP/dt = \frac{1 - R_2}{P_1} \frac{\lambda R_2}{(R_2 + 1/P_1)}
\]  

(A12)

After rearrangement this gives the one group approximation:

\[
R_2 = \frac{1 - TA/P_1}{1 + TA}
\]  

(A13)

The error in \( R_2 \) may be found approximately by noting that in most cases of interest \( P_2 \gg P_1 \) or \( |R_2| \ll 1/P_1 \), so that equation A12 may be adapted to the form:

\[
\frac{dP}{dt} \approx \frac{\lambda}{1 - R_2} \quad t > t_1
\]  

(A14)
That is, the percentage error in $\frac{dP}{dt}$ as measured is approximately the same as the percentage error in $(1 - R_2)$. It is suggested that an automatic trip could be provided to operate at, say, $R_2 = -1 \%$. If so, it is probably unnecessary that $R_2$ should be known to better than $\pm 10\%$ (1sd) near the chosen trip level, implying that $(1 - R_2)$ should be measured to $\pm 5\%$. With this accuracy in $(1 - R_2)$, the error in determining $R_2$ at other values is as given in Table A1.

Table A1: Errors ($\delta R$) in determining reactivity when the error in $(1 - R_2)$ is $\pm 5\%$

<table>
<thead>
<tr>
<th>$R$ (dollars)</th>
<th>$\delta R$ (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 2</td>
<td>$\pm 0.15$</td>
</tr>
<tr>
<td>- 1</td>
<td>$\pm 0.10$</td>
</tr>
<tr>
<td>- 0.5</td>
<td>$\pm 0.075$</td>
</tr>
<tr>
<td>- 0.1</td>
<td>$\pm 0.055$</td>
</tr>
<tr>
<td>0</td>
<td>$\pm 0.050$</td>
</tr>
</tbody>
</table>

The accuracy to which the quantity $\frac{dP}{dt}$ can be measured depends on the counting statistics. Suppose that the actual count-rate ($c$) from the available detectors is related to $P$ via a constant $f$:

$$c = fP$$  \hspace{1cm} (A15)

An idealised way of determining $\frac{dP}{dt}$ would be to measure $c$ over 2 successive periods of integration each lasting a time $\tau$. Then:

$$\frac{dP}{dt} = \frac{1}{f} \frac{dc}{dt} = \frac{1}{f} \tau \left( C'_{\tau} - C'_0 \right)$$  \hspace{1cm} (A16)

where $C'_0$ is the count rate integrated from $t = 0$ to $t = \tau$

$C'$ is the count rate integrated from $t = \tau$ to $t = 2\tau$

The standard error on the measurement of $(C'_{\tau} - C'_0)$ is $(C'_0 + C'_\tau)\frac{1}{2}$, and thus the percentage error on the measurement of $\frac{dP}{dt}$ is (using equations A14, A15 and A16):

$$\frac{100 \left( C'_{\tau} + C'_0 \right)\frac{1}{2}}{C'_{\tau} - C'_0} = \frac{100 \left[ 2P_1 + \frac{\lambda f}{1 - R_2} \right]^{\frac{1}{2}} (1 - R_2)}{\lambda f^{\frac{3}{2}} \tau^{3/2}}$$  \hspace{1cm} (A17)

This may be solved to obtain $f$ and hence the counting rates necessary to achieve any required accuracy.

Some calculations based on equation A17 have shown that quite low counting rates at the start and end of a ramp insertion of reactivity are needed to achieve the suggested $\pm 5\%$ for $\frac{dP}{dt}$. The counting rate required for $P_1$ varies from about 30 to 50 cps depending on the reactivity at the end of the ramp, where $R_2$ is in the range -1 to 0 %. This is based on an assumption that $\tau = 30$; i.e. the counts are integrated over 30 seconds. The equivalent counting rates required at the start of the ramp are (for typical ramp rates) as follows:

- $R_0 = -3 \%$, $c_0 = f P_0 = 7 - 12$ cps
- $R_0 = -2 \%$, $c_0 = f P_0 = 10 - 17$ cps
- $R_0 = -1 \%$, $c_0 = f P_0 = 20 - 35$ cps

These count rates are achievable in most LMFBRs even before the build-up of curium isotopes. The much higher counting rates which are available in a core with near-equilibrium burn-up will allow $R_2$ to be determined in principle much more accurately than as shown in the example in Table A1.
HIGH REACTIVITY DETECTION LEVELS

LEVEL 1 (up to -1 \( \% \))
Detection of loading errors by reactivity accountancy step by step with or without automatic trip.

LEVEL 2 (-1 to -2 \( \% \))
Approximate region in which a high count rate trip on S/D instrumentation operates

LEVEL 3 (-1 to 0 \( \% \))
Approximate region in which a power trip lag trip operates

LEVEL 4 (0 - 0.8 \( \% \))
In this region high level trips(s) operate

LEVEL 5 (0.3 - 0.8 \( \% \))
Low period trips(s)

Region of special interest to safety argument since from here removal of ASD or control rod could lead to damage in one step if no trip (-1 to 0 \( \% \))

Minimum reactivity (1 \( \% \)) addition starting from low power, critical leading to damage if no automatic trips operate

Reactivity Increase During Reload (2 Batch Refuelling)

Reactivity Increase During Reload (Multi-batch Refuelling)

TYPICAL WORTHS:

- Mean Subassembly: 0.4
- Maximum Subassembly: 0.7
- Mean Control Rod: 1.0
- Maximum Control Rod: 1.5
- Mean alternative shut-down device (ASD): 1.1
- Maximum ASD: 2

FIG. 1 REACTIVITY REGIMES RELATED TO SUBCRITICAL MONITORING (TYPICAL LMFBR VALUES)
FIG. 4 POWER DURING AND AFTER 10 sec. RAMP REACTIVITY INSERTION AS A FUNCTION OF INITIAL REACTIVITY \( R_0 \). THE REACTIVITY ADDED BETWEEN \( t = -10 \) & \( t = 0 \) IS IN EACH CASE \(-0.1R_0\) (1 DELAYED NEUTRON GP)
KEY:
- $R_0 =$ INITIAL REACTIVITY (DOLLARS)
- $P =$ POWER ($= \beta_n / \delta$, SEE TEXT)

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- REACTIVITY ADDED AT 0.1 $/ \text{sec}^{-1}$
- --- --- --- REACTIVITY ADDED AT 0.05 $/ \text{sec}^{-1}$

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**FIG. 5** RECIPROCAL POWER AS A FUNCTION OF REACTIVITY AND TIME FOLLOWING RAMP INSERTIONS OF REACTIVITY

**KEY:**
- $R_0 =$ INITIAL REACTIVITY (DOLLARS)
- $P =$ POWER ($= \beta_n / \delta$, SEE TEXT)

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- REACTIVITY ADDED AT 0.1 $/ \text{sec}^{-1}$
- --- --- --- REACTIVITY ADDED AT 0.05 $/ \text{sec}^{-1}$

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**FIG. 5** RECIPROCAL POWER AS A FUNCTION OF REACTIVITY AND TIME FOLLOWING RAMP INSERTIONS OF REACTIVITY

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