CSNI Specialists Meeting on Plastic Tearing Instability

Held at the
Center for Fracture Mechanics
Washington University
St. Louis, Missouri, U.S.A.
September 25-27, 1979

Prepared by P.C. Paris

Office of
Nuclear Regulatory Research

U.S. Nuclear Regulatory Commission
NOTICE

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, or any of their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for any third party's use, or the results of such use, of any information, apparatus product or process disclosed in this report, or represents that its use by such third party would not infringe privately owned rights.

Available from

GPO Sales Program
Division of Technical Information and Document Control
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555

and

National Technical Information Service
Springfield, Virginia 22161
CSNI Specialists Meeting on Plastic Tearing Instability

Held at the
Center for Fracture Mechanics
Washington University
St. Louis, Missouri, U.S.A.
September 25-27, 1970

Prepared by P.C. Paris

Division of Reactor Safety Research
Office of Nuclear Regulatory Research
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555
The CSNI Specialists Meeting on Plastic Tearing Instability

PREFACE

The Specialist Meeting on Plastic Tearing Instability was held at the Center for Fracture Mechanics, Washington University, St. Louis, USA, and was sponsored by the Committee on the Safety of Nuclear Installations (CSNI) of the OECD Nuclear Energy Agency. The meeting was hosted by the Center for Fracture Mechanics and by the United States Nuclear Regulatory Commission.

The Committee on the Safety of Nuclear Installations maintains a Working Group to advise on specific safety aspects of steel components; the Group's activities fall within the three principal areas of fracture mechanics, welding and heat treatment, and non-destructive testing. In this context three Specialist Meetings have already been held on the general topic of elasto-plastic fracture mechanics: in Brussels (1974); San Francisco (1976) and Daresbury, United Kingdom (1978). The present meeting arose from a proposal made by the Working Group in 1978 to the effect that little or no international work had been done specifically with regard to tearing instability. Its purpose is to review recent developments and to provide CSNI, in these proceedings, with a status report and specific recommendations as to areas where further work appears desirable.

The Meeting constituted a forum for the 45 participants to make presentations and to thoroughly discuss details of the material presented. Copies of the material presented appear in this report. In addition, each Session Chairman has provided a Summary in this report of his session, including significant points made in discussion. Finally, at the end of the meeting a set of Consensus Points were drafted, discussed, and modified and as modified found agreeable to all participants, as presented herein.

It is felt that the Consensus Points speak for themselves in indicating a considerable international agreement on the "State of the Art" of how to address the main areas of Elastic-Plastic Fracture Mechanics Analysis. Therefore, it is concluded that this meeting and report demonstrate the considerable progress in developing Elastic-Plastic Fracture Mechanics which has occurred in the past few years.

Paul C. Paris
St. Louis, Missouri
1 October, 1979
# TABLE OF CONTENTS

Preface ................................................................. ii

Consensus Points for the Meeting ................................. 1

Session Chairman's Summaries

I  Summary Points by A. Pellissier-Tanon  
   Session Chairman of "Theory and General  
   Considerations, I"  ........................................ 3

II  Summary Points by J. Carlsson  
    Session Chairman of "Theory and General  
    Considerations, II"  .................................... 5

III Summary Points by C. E. Turner  
     Session Chairman of "Experimental Results" ---- 7

IV  Summary Points by E. T. Wessel  
    Session Chairman of "Experiments and  
    Applications"  .......................................... 10

V  Summary Points by G. R. Irwin  
    Session Chairman of "Applications"  .............. 13

Presentations* at the Meeting:

I (1) "Foundations of Tearing Instability Theory"  
     Invited Lecture by J. W. Hutchinson  
     (Based on manuscript entitled "Recent  
     Developments in Non-Linear Fracture  
     Mechanics") ............................................ 16

I (2) "Plastic Tearing Instability - What Is It?"  
     Paper by W. H. Irvine and A. Quirk .......... 29

I (3) "Study of Instability of Growing Cracks  
     Using Damage Function: Application to Warm  
     Prestress Effect"  
     Paper by F. M. Beremin; Presented by A.  
     Pineau ..................................................... 39

II (1) "Remarks on Unstable Ductile Crack Growth"  
      Invited Lecture by C. E. Turner ............ 74

II (2) "A Technique for Analyzing Fracture Toughness  
      Test Data During Slow Growth"  
      Paper by I. Milne and G. G. Chell (see also  
      Papers submitted in discussions) .......... 100

*Note - Underlined Author presented paper
II (3) "Stable Crack Growth Estimates Based on
Effective Crack Length and Crack Opening
Displacement"
Paper by J. G. Merkle and C. E. Hudson ------ 115

II (4) "An Engineering Approach for Examining
Growth and Stability in Flawed Structures"
Paper by C. F. Shih -------------------------- 144

III (1) "Size and Geometry Effects on Elastic
Fracture Characterization"
Invited Lecture by J. D. Landes ---------- 194

III (2) "Crack Growth Resistance - Geometry Effects
and Structural Predictions in A533B Class I
Steel"
Paper by S. J. Garwood ------------------ 226

III (3) "Issues in Developing a Plane Strain J-R
Curve Test Procedure"
Paper by J. T. Gudas, J. A. Joyce and P.
Albrecht ------------------------------- 270

III (4) "J-R Curve Characterization of Irradiated
Nuclear Pressure Vessel Steels"
Paper by E. Loss, B. H. Menke, R. A. Gray,
Jr., and J. R. Hawthorne ---------------- 292

IV (1) "Correlation Between General Yielding and
Tearing Initiation in C. T. Specimens"
Invited Lecture by A. Pellissier-Tanon
Coauthored by B. Marandes and J. C.
Devaux ------------------------------- 316

IV (2) "The Measurement of Ductile Crack
Initiation"
Paper by T. Ingham and E. Moreland ------- 330

IV (3) "Formulas Giving the J-R Curve from Results
from One Experimental Test"
Paper by R. L. Roche --------------------- 358

IV (4) "A Simple Method for Determining the
Ductile Instability of Cracked Structures"
Paper by G. G. Chell and I. Milne ------- 376

V (1) "Fracture Proof Design"
Lecture by P. C. Paris and G. Zahalak
(Based on manuscript entitled "Progress in
Elastic-Plastic Fracture Mechanics and its
Applications") ----------------------- 401
V (2) "Condition of Stability of an Axial Through Crack Appearing on PWR Secondary Piping in Reduced Scale (1/10)"
Paper by J. Cheissoux (See also papers submitted in discussions) 429

V (3) "An Analysis of Ductile Crack Extension in BWR Feedwater Nozzels"
Paper by B. Szabo, G. G. Musicco and M. P. Rossow 440

V (4) "A Preliminary Fracture Analysis on the Integrity of H.S.S.T. Intermediate Test Vessels"
Paper by A. Zahoor and P. C. Paris 477

Post-Meeting Presentations of Recent Work at Washington University:

(1) "Some Considerations in the Application of J-Integral Analysis to Structures Subjected to Multiple Loads"
By G. Zahalak and P. C. Paris 519

(2) "Fracture Characterization of A-36 Steel and Structural Applications"
By S. Baldini and P. C. Paris 544

(3) "A Stability Analysis of Circumferential Cracks for Reactor Piping Systems"
By H. Tada, P. C. Paris and R. Gamble (See NUREG CR-0838) 598

(4) "Techniques of Analysis of Load-Displacement Records by J-Integral Methods"
By H. Ernst and P. C. Paris (Separate NUREG to be released in the future) 599

Papers Submitted in Discussions of Other Presented Papers:

(1) "A Plastic Zone Instability Phenomena Leading to Crack Propagation"
By Jose A. Vazquez and Paul C. Paris (as discussion of Paper V (2) by Cheissoux) 601

(2) "The Application of the Plastic Zone Instability Criterion to Pressure Vessel Failure"
By Jose A. Vazquez and Paul C. Paris (as discussion of Paper V (2) by Cheissoux) 632

(3) "Prediction of Ductile Tearing of Compact Specimens using the R-6 Failure Assessment Diagram"
By J. M. Bloom (as discussion of Paper II (2) by Milne and Chell) 655

List of Participants at the Meeting 686
Program Followed at the Meeting 688
CONSENSUS POINTS FOR THE MEETING
(All Participants)

Subsequent to the presentations by authors and the open discussions that followed, the Chairman of each session met together with Professor Paris and Mr. Oliver. Together they drafted the statements below that both outlined the scope of the meeting and described the features of current work on which there was general agreement. No attempt was made to discuss topics not addressed during the meeting or to assess any balance of opinion on matters that were still subject to different interpretations. The draft statement was then put before the whole meeting and amended, where necessary to reach agreement by all as follows:

Introduction
The following statements address the subject of Tearing Instability and related analysis methods applied in general to nuclear problems:
1. A single parameter approach to crack stability (such as the J approach) can adequately characterize conditions associated with crack instability (and crack initiation and crack growth) within certain limitations.
2. For this purpose J is the most general (current) parameter and approach.
3. It is recognized that J is convertible to or from other parameters, for example COD.

Beginning of Crack Extension
1. It is agreed that it is appropriate to attempt to define a measurement-point pertaining to the beginning of crack extension (such as $J_{IC}$).
2. The approaches to determining this onset of extension value (such as $J_{IC}$) are essentially the same.
3. The details and some limitations of measuring a practical initiation point remain to be resolved.
4. The practical engineering initiation value (such as $J_{IC}$) is a measure of the extent of loading or deformation (such as measured by J) below which tearing instability is not expected.

Growth
1. Stable Crack Growth can be characterized in terms of a single parameter (such as J and the J-R Curve) within certain limitations.
2. The limitations on these characterizations for plane strain are associated with similarity of field conditions and avoiding substantial amounts of unloading (such as Hutchinson's conditions for J-controlled growth).

3. These characterizations apply uniformly from small-scale yielding through fully plastic conditions within the limitations.

4. For structural steels, cleavage may intervene on the condition for stable crack growth, but is currently expected only in or near the transition temperature range.

Instability of Tearing
1. The occurrence of tearing instability of cracks can be characterized through use of the same parameters as those for describing growth (such as \( J_{\text{appl}} \) and the J-R curve).

2. The crack instability problem can be characterized as the occurrence of a tangency between the driving parameter and the material's resisting values of that parameter (such as \( J_{\text{appl}} \) giving tangency to the J-R curve).

3. The several approaches to instability (such as energy-balance, J-tangency, etc.) are basically the same except for details or extrapolation beyond the recognized limitations.

Micromechanisms
It was agreed that the study of micromechanisms (for example the damage function and other approaches) can enhance our understanding of fracture processes and the limitations of current analysis.

--- End of Consensus Agreement ---

Acknowledgements
The Session Chairmen wish to express their thanks to CSNI as organizing authority, the Center for Fracture Mechanics, Washington University as hosts, and to authors and participants for their technical contributions. An informal consensus clearly existed that such small working party type meetings provided an excellent forum for discussion. Real advances in the understanding of different viewpoints has emerged at this meeting and a basis has been provided for the (never ending) advancement towards further understanding of the nature of fracture, tearing and unstable behavior.
I. SUMMARY POINTS BY A. PELLISSIER-TANON, Session Chairman for
"THEORY AND GENERAL CONSIDERATIONS - I"

I. J-Integral related crack growth and instability criteria (from
Dr. J.W. Hutchinson presentation)

The theoretical conditions under which the J-integral can be used as a unique
field parameter to characterize crack initiation and growth have been
established.

Various analytical and numerical calculations show that the extent at which
these conditions can be met varies strongly with the geometry of the specimen
and improves markedly with increasing strain hardening. These conditions are
best satisfied in small scale yielding and with bend type specimens in fully
plastic situations, but require very large sizes to be met for the centered
cracked panel.

The influence of the specimen geometry and of the material properties on the
features of the J-Δa resistance curve have been investigated by numerical
simulation of crack growth in small scale yielding and fully yield bend.
The initial slopes as well as the steady states values differ from small
scale yielding to fully plastic, where it depends on the specimen geometry.
The tear resistance increases markedly with strain hardening, and the dependence
on the specimen geometry decreases for high tear resistance materials.

The tearing modulus instability criterion has been validated by experimental
programs showing well the interrelation between the stability of a structure
and its elastic compliance. Although its validity is restricted strictly to
conditions where the J-Δa resistance curve can be strictly a material
characteristic, that is to the conditions under which the J integral remains the
dominant field parameter, wider areas of practical engineering applications are
investigated.
2. Numerical simulation or crack growth and instability using elastoplastic crack tip separation damage functions (from Dr. A. PINEAU presentation)

This approach is looked at as the ultimate mean to take into account all the complexities arising from complex geometries and loadings (such as thermal stresses) in a stability analysis, and to check the degree of precision of simplified models and criteria.

As the possibility of success of this approach is thought to depend on the quality of the matching of the microscopic damage examinations and of the local stress and strain history computations, a two-dimensional approach using notched and cracked axisymmetric tensile specimens is being pursued, as it combines ease of computation with the avoidance of spurious three-dimensional effects.

3. The understanding of the patterns of the instability of an actual structure under contained yielding conditions (from Mr. W.H. IRVINE presentation)

A diagram picturing the variation of stored energies or compliance with crack size and loading, such as the Fracture LOCUS is an appropriate means for revealing the mechanisms explaining actual failure in service of or checking the possible failure conditions of a designed structure.
II. SUMMARY POINTS BY JANNE CARLSSON, Session Chairman for "THEORY AND GENERAL CONSIDERATIONS - II"

Development of crack growth and instability criteria for cracked structures of ductile materials is following several lines:

1. One line of development is a logical continuation of LEFM. It is theoretically well founded and based on the J-Integral and solutions by Rice, Hutchinson and co-workers of elastic-plastic work problems for stationary and quasi statically growing cracks. Both the J-Integral criterion for onset of stable crack growth and the Paris tearing modulus criterion for stable growth and instability are based on the characterizing property of J for the crack tip field. The limitations of the validity of these criteria can be well specified precisely because of their solid theoretical foundation. It is interesting to note the comment by Hutchinson, referring to theoretical work of Rice and Shih that there exists at least approximately a geometry independent material characterizing tearing modulus which governs stable crack growth.

The work of Shih reported here shows promise that the T-concept can be made accessible to engineers as a design tool analogous to the linear elastic stress intensity factor. The discussion has shown that the limitations on the validity of the tearing modulus may not be too serious with regard to applications. The T-approach is valid for materials with high tearing resistance and that is where one would need the concept. For materials with low tearing resistance an onset of crack growth would be limiting in applications.

Both the J-Integral and the tearing modulus have the advantage that they are easily incorporated in a normal structural analysis.

The Turner approach to stable crack growth seems essentially equivalent to the T-modulus approach, although it is based on a generalized energy release rate consideration. There are different ways of defining J when it comes to formulas for J-calculations. These are in some cases more or less approximate, and on this level there seems to be some disagreement about the equivalence of the two approaches reviewed.
2. Other lines of development of criteria are tied to the direct needs in structural design of tools for estimation of permissible defects and safety margins. The approach presented by Milne and Chell is such an engineering method. From the well known cases corresponding to linear elastic fracture mechanics and the limit load analysis they interpolate a design curve using a Dugdale-type modelling of the plastic effects. Theories of this type are attractive since they are made so as to be easy to apply. However, one may have doubts whether the design curve takes proper account of all parameters entering problems of this type, as e.g. geometry of the crack and structure. Engineering or empirical theories need extensive experimental verification to determine validity and limitations. This verification has to involve not only specimen testing but also full scale testing of structures.

Having said this, one must point out that engineering approaches to these problems are necessarily awaiting the development of more well founded theories. Many engineering approaches imply shortcuts in a development of a theory. There are indications that the empirical results presented by Merkle are of this nature, and that they contain some deeper truth. Anyhow, these results seem very useful.
III. SUMMARY POINTS BY C.E. TURNER, Session Chairman of
"EXPERIMENTAL RESULTS"

It is not my intention to review this morning's papers, since they have
just been presented and discussed in some detail. I shall use my position as
Chairman to select a few of the topics mentioned today, and I choose first
some that bear directly on matters discussed yesterday, namely the various
models of tearing stability.

In introducing the four stages in his study of cracking (i.e. the severity
of the crack tip stress field, blunting, initiation of growth and instability)
John Landes said he preferred a characterising rather than energetic view-
point. I believe this is widely accepted and would emphasize that it is only
in connection with instability that I advocate the energetic interpretation
outlined yesterday. For non-linear elastic (nle) material there is, of course,
no controversy since $J$ is both a characterising parameter and a measure of
energy absorption or release rate. It is only in the interpretation of $J$ for
"real" elastic plastic loading-elastic unloading (epe) material that the problem
arises on whether to accept the use of $J$ in a characterising and dissipative
role, or reject $J$ for use of a term that is more relevant for energy release
rate in epe material, namely the value $I$, where $G \leq I \leq J$. John Gudas has
just shown that by modifying the compliance of the testing system, as indeed
introduced in tests some time ago by Paul Paris and his colleagues, the tearing
instability behavior of a test piece is greatly affected. The energy release
rate of the system is highly dependent on compliance whilst the stress field
intensity is not, thus predisposing me to take the energetic view of instability.
Of course, the rate of change of $J$ is altered, moving from near $\partial J/\partial a|_q$ for a
stiff machine to $\partial J/\partial a|_0$ when the machine is modified by the leafsprings. How-
ever, if those using $\partial J/\partial a$ as a characterising parameter also allow $J$ to have
an energetic meaning, then it is clear that it serves as an upper bound since
$G \leq I \leq J$, and thus gives a conservative estimate of instability, as discussed
by Fong Shih yesterday. With that interpretation there is no difference in the
characterising and energetic viewpoints, and for small amounts of growth
typified by $\omega \gg 1$, or as I argued yesterday by $\omega \gg f_1(\eta)$ which has a value such as
1 or 2 for deep notch test pieces, the results are identical to within the
approximations of making best estimates of the various terms in the theory.
It is only when straying beyond this common foundation point that the various
viewpoints differ on what is the most appropriate model to adopt.
In describing his philosophy for obtaining J-R curves relevant to the loading conditions of a structure, Steve Garwood threw in as an aside remark that certain results are very similar when examined by the extension to the Two Criteria method described yesterday by Ian Milne. This prompted a hasty coffee time discussion, from which, if I correctly understood him, I now appreciate that Ian Milne sees the boundary of the extended Two Criteria diagram as a driving force curve for constant load with the constructed curve that is used to give the tangency point for maximum load servings as a resistance curve. Naturally, because of the simplicity of the one (log sec) curve used in the model, it is not as detailed as J theory in its application to various cases, but that is consistent with its intention of being a quick assessment method. I had not hitherto perceived the "applied" and "resistance" interpretation of the Two Criteria diagram when used to predict maximum load and suggest that the log sec curve that forms the diagram (and which strictly relates to COD or J via the Dugdale model) serves as a measure of δJ/δa applied because (in the n notation) δJ/δa = (J/b)(f_n(η+ n)), i.e., δJ/δa is just proportional to J, so that for any given problem the log sec curve accepted for J or δ can be scaled to serve for δJ/δa or δd /δa. There is thus a very much closer measure of agreement in fundamental concepts, between the J characterising, the energetic and the extended Two Criteria interpretation of instability than appeared to be the case before this meeting started.

Frank Loss inserted a timely reminder that in some of his irradiated steels ductile tearing turned to cleavage failure. That aspect of the problem is still rather beyond the scope of the methods just described, at their present stage of development. Loss, and indeed all of today's authors, also mentioned the use of side grooved pieces, which with suitably chosen groove depths can give a lower bound on the R-curve. Steve Garwood took this idea further to suggest the rationalization of earlier proposals that δ_max or J_max could be used rather than δ or J values. If this side grooved data is itself obtained on bend pieces of high contrast and perhaps with due respect to anisotropy, he found it was a conservative estimate of the lowest part-through thickness R curve data obtained on the same steel. He therefore argued that without using a full R curve analysis, the maximum load δ or J value, taken from side grooved (lower bound) data obtained from tests conducted under something approaching load controlled conditions, is a safe value to use in the COD design curve type of analysis of a structure that is itself near displacement controlled, and for the particular steels in question, this gave a four-fold or greater advantage over the use of conventional initiation
data. In short, this procedure relaxes the severity of measurement of toughness sufficiently, for some steels, to avoid the argument of instability via R curves once the general argument of lower bound material data and over estimate of structure driving force has been demonstrated.

Clearly, the last few years have seen a great development in the concepts available for discussing stable crack growth, and these have now progressed sufficiently for the similarities in their arguments to be appreciated. Thus, despite their rather different starting points, and at first sight diverse presentations, they are now tending to be complementary, each reinforcing by alternative arguments the general trends and conclusions of the other.
IV. SUMMARY POINTS BY EDWARD WESSEL, Session Chairman for "EXPERIMENTS AND APPLICATIONS"

The applications aspect of the information presented in this session focused on two areas.

1. The use of elastic-plastic fracture mechanics to assess margins of safety in nuclear components, and
2. The use of small, single specimens to procure the pertinent material parameters ($J_{IC}$ and $J_R$ curves) of irradiated materials for use in safety margin analyses.

The general philosophy being pursued in item (1) above is that nuclear structures are always designed to insure the maximum applied $K$ or $J$ will never exceed the critical material parameters of $K_{IC}$ or $J_{IC}$ under normal or accident conditions. In the sub-transition and transition temperature ranges, the margin of safety can be readily established using state-of-the-art linear-elastic fracture mechanics technology by considerations of the relative values of maximum applied $K$ levels to the critical values ($K_{IC}$).

However, in the upper-shelf or ductile behavior temperature regime, the elastic-plastic fracture mechanics technology is not yet sufficiently developed to permit quantitative evaluations of the margin of safety beyond the $J_{IC}$ level. It is recognized that for most cases, structures can tolerate applied $J$ levels that are substantially in excess of $J_{IC}$. This is associated with the stable crack growth behavior which for nearly all cases requires an increase in applied $J$ to cause further crack advancement. Further, the maximum extent of crack extension and the associated maximum tolerable level of applied $J$ is governed by a critical tearing instability condition which is dependent upon both the materials resistance to ductile crack growth and the specific structural loading conditions that exist. Hence, the
current limitations in assessing the margin of safety (beyond $J_{IC}$) for the upper shelf temperature regime are associated with uncertainties in the ability to predict stable crack growth and instability behaviors. Therefore, the focus of current research in elastic-plastic fracture mechanics is on analysis models, experiments and material-parameter test-techniques for characterizing stable crack growth and instability conditions. The progress in this area during the past one to two years has been dramatic; and despite some of the current limitations and uncertainties the present technology does facilitate at least some qualitative assessment of margin of safety. Continued research will undoubtedly provide increased confidence and increased quantitative prediction capability.

Some of the current limitations or areas of uncertainty involve the following considerations:

1. the applicability of $J$ as a single crack tip characterizing parameter for large amounts of stable crack extension;
2. The experimental capability to measure a fundamental $J$ resistance curve that can be considered a basic material property for relatively large amounts of crack extension;
3. The models for predicting instability conditions in structures;
4. The possible interruption of the stable crack growth by a change in fracture from ductile tearing to brittle (unstable) cleavage behavior; and
5. The development of standard test methods for obtaining the basic material parameters with a single, relatively simple, small specimen, especially for the irradiated condition.

The papers presented in this session dealt primarily with Items 2 and 5 above. Relative to experimental methods for determining $J_{IC}$ and early stages of stable crack growth (Item 2), there is general agreement in concept and approach. Some differences relative to the experimental details and the precise choice of the $J_{IC}$ point from experimental results is evident from the papers, and further refinement is necessary. However,
these are differences in detail and do not distract from the concept or general usefulness of the $J_{IC}$ approach to defining the initial growth of a stable crack (from an engineering viewpoint).

Relative to Item 5, it is evident that there is a strong interest and associated high activity level on developing analysis and experimental methods that will facilitate measurement of the pertinent material parameters (especially for irradiated material, using a single, small simple specimen. While several approaches are being investigated, there is a general similarity among them. They all attempt to use a simple load-deflection record from a test specimen to obtain a reasonable estimate of the $J_{IC}$ point and the $J$ resistance curve. Both state-of-the-art specimen types as well as older types of specimens (such as those in reactor surveillance capsules) are being studied. Results to date are encouraging in that there appears to be reasonable agreement between the results obtained by these less sophisticated methods and the more rigorous methods that have general acceptance by those working in the field. In view of the critical need for extracting all possible significant information (today's technology) from the limited and expensive irradiated test specimens from surveillance programs, continuing research in this area is encouraged.

In conclusion, this session dealt primarily with details associated with current approaches or concepts, rather than introducing new or conflicting viewpoints. Therefore, there were no conflicts with the overall or general conclusion established for the conference.
V. SUMMARY POINTS BY G. R. IRWIN, Session Chairman for "APPLICATIONS"

A selected group of T(applied) estimations was presented by P.C.Paris. These served to demonstrate advantages of the non-dimensional form of dJ/da termed T. Comparisons of T(applied) showed similarities with regard to dependency upon length ratios in various classes of crack problems. It was suggested that enough T(applied) estimates are becoming available so that a collection of these in handbook form would be feasible and helpful. The illustrations presented were over-simplified through considerable use of proportionality between J and 6 and by estimates of 6 from plastic slip fields. Presumably the suggested handbook would contain more exact results where available.

B. Szabo discussed a finite element calculation for a nozzle corner crack. He concluded that such cracks would be stable relative to tearing at a size equal to 95 percent of the section thickness. Previous studies by Paris and Tada showed that tearing instability is not expected after initial extension of such a crack through the section thickness. Presumably such cracks should be detectable by inspection or leakage prior to rapid fracturing. These conclusions assume a high enough temperature so that cleavage fracturing does not occur prior to tearing instability.

One-tenth scale model studies of a pressurized cylinder with an axial crack were discussed by J.L. Cheissoux. The results corresponded to semi-emperical equations which accounted for pressure, out-of-plane bending, and crack size. The equations were similar to those used previously by R.Eiber and co-workers at Battelle Columbus Laboratories. Such results are expected to predict full-scale behavior if no substantial change of toughness occurs with increase of wall thickness to full scale.

A tearing instability analysis was presented by A.Zahoor, specialized to the axial part-through cracks in two of the ORNL-HSST tests, using cylindrical steel vessels with 6-inch wall thickness. In one case, due to the large crack depth, leakage without rapid crack extension occurred at a moderate pressure. In the other test, the crack depth was much smaller,
In describing his philosophy for obtaining J-R curves relevant to the leading conditions of a structure, Steve Garwood threw in as an aside remark that certain results are very similar when examined by the extension to the Two Criteria method described yesterday by Ian Milne. This prompted a hasty coffee time discussion, from which, if I correctly understood him, I now appreciate that Ian Milne sees the boundary of the extended Two Criteria diagram as a driving force curve for constant load with the constructed curve that is used to give the tangency point for maximum load servings as a resistance curve. Naturally, because of the simplicity of the one (log sec) curve used in the model, it is not as detailed as J theory in its application to various cases, but that is consistent with its intention of being a quick assessment method. I had not hitherto perceived the "applied" and "resistance" interpretation of the Two Criteria diagram when used to predict maximum load and suggest that the log sec curve that forms the diagram (and which strictly relates to COD or J via the Dugdale model) serves as a measure of $\frac{\partial J}{\partial a}$ because (in the $n$ notation) $\frac{\partial J}{\partial a} = (b/n) J (f_1(n))$ i.e. $\frac{\partial J}{\partial a}$ is just proportional to $J$, so that for any given problem the log sec curve accepted for $J$ or $\delta$ can be scaled to serve for $\frac{\partial J}{\partial a}$ or $\frac{\partial \delta}{\partial a}$. There is thus a very much closer measure of agreement in fundamental concepts, between the J-characterising, the energetic and the extended Two Criteria interpretation of instability than appeared to be the case before this meeting started.

Frank Loss inserted a timely reminder that in some of his irradiated steels ductile tearing turned to cleavage failure, That aspect of the problem is still rather beyond the scope of the methods just described, at their present stage of development. Loss, and indeed all of today's authors, also mentioned the use of side grooved pieces, which with suitably chosen groove depths can give a lower bound on the R-curve. Steve Garwood took this idea further to suggest the rationalization of earlier proposals that $\delta_{max}$ or $J_{max}$ could be used rather than $\delta_i$ or $J_i$ values. If this side grooved data is itself obtained on bend pieces of high constraint and perhaps with due respect to anisotropy, he found it was a conservative estimate of the lowest part-through thickness R curve data obtained on the same steel. He therefore argued that without using a full R curve analysis the maximum load $\delta$ or $J$ value, taken from side grooved (lower bound) data obtained from tests conducted under something approaching load controlled conditions, is a safe value to use in the COD design curve type of analysis of a structure that is itself near displacement controlled, and for the particular steels in question, this gave a four-fold or greater advantage over the use of conventional initiation
the failure pressure was much higher, and rapid crack propagation occurred. The test results corresponded well with results computed using J and dJ/da measurements and analysis. It was of interest to note that, in the high pressure test, cleavage fracturing started at about the expected end point of axial slow-stable tearing extension of the crack.
RECENT DEVELOPMENTS IN NONLINEAR FRACTURE MECHANICS

J. W. Hutchinson
Professor of Applied Mechanics
Division of Applied Sciences
Harvard University
Cambridge, Massachusetts 02138
USA

INTRODUCTION

Within the past few years research into the nonlinear mechanics of fracture has started to have a practical payoff. I would like to use the opportunity of this CANCAM lecture to describe some of these recent developments. I will start by reviewing some of the fundamentals of nonlinear crack problems. Then the initiation of crack growth will be discussed, followed by a discussion of a new approach to the growth and stability analysis of small amounts of crack advance in the presence of large scale plastic yielding. The last part of the lecture deals with the limited success which has been achieved to date in employing a single basic near-tip fracture criterion in the analysis of both initiation and growth under general conditions of yielding. Other recent survey articles which cover some of the ground reviewed here have been given by Carlson [1], Paris [2] and Rice [3]. My coverage will emphasize the theoretical side of the subject, but I will try to bring out the vital interaction between theory and experiment which has been so characteristic of much of the development of nonlinear fracture mechanics.

THE J-INTEGRAL AND CRACK-TIP FIELDS

The unifying theoretical idea behind the extension of linear elastic fracture mechanics into the range of large scale plastic yielding is the J-integral introduced for crack problems by Rice [4] in 1968 and, independently, by Cherepanov [5] in Russia. A small strain, nonlinear elastic (deformation theory of plasticity) material is assumed with strain energy density \( W(\epsilon) \) such that the stress is

\[
\sigma_{ij} = 2W'\epsilon_{ij}
\]

The proto-type body shown in Fig. 1 is assumed to be in conditions of either plane strain or plane stress. The material is taken to be homogeneous and isotropic. Let \( P \) denote the generalized force per unit thickness acting on the body and let \( \Delta \) be the generalized displacement quantity through which \( P \) works. For reasons which will be clear later, a linear spring with compliance (per unit thickness) \( C_M \) is placed in series with the cracked body such that the total generalized displacement of the system is

\[
\Delta_T = \Delta + \frac{C_M}{E} P
\]

With \( PE \) defined as the potential energy of the system per unit thickness, \( J \) is defined as the energy release-rate per unit advance of the crack in its plane (per unit thickness) with \( \Delta_T \) held fixed, i.e.
\[ J = \frac{3PE}{3a} \sqrt{\frac{\Delta}{a}} \]  

(3)

Fig. 1 Cracked body in series with a linear spring

The energy release rate defined above is easily shown to be independent of the compliance of the spring \( C_M \). Since \( C_M \rightarrow \infty \) corresponds to dead load with \( P \) prescribed and \( C_M = 0 \) corresponds to prescribed \( \Delta \), \( J \) is the same for these limiting cases, as well as all those in between. With \( P \) regarded as a function of \( \Delta \) and \( a \), (3) reduces to

\[ J = \frac{\Delta}{3P} \int_0^\Delta \frac{d^2 \Delta}{a} (\Delta, a) d\Delta \]  

(4)

Or with \( \Delta \) as a function of \( P \) and \( a \), (3) becomes

\[ J = \frac{P}{3a} \int_0^P \frac{d^2 P}{a} (P, a) dP \]  

(5)

These latter expressions are given by Rice [6]. His path-independent line integral expression for \( J \) is

\[ J = \int (W n_1 - \sigma_{ij} n_1 u_{ij}) ds \]  

(6)

where \( \Gamma \) is any contour encircling the tip of the crack in a counter-clockwise direction, \( u_i \) is the displacement vector, \( n_1 \) is the outward unit normal to \( \Gamma \) and \( ds \) is the length of the line element.

While \( J \) is the energy release-rate for the cracked deformation theory body, it has another role which is more pertinent to non-linear fracture mechanics. It can be regarded as the amplitude of the singularity fields at the tip of the crack. As an example, assume that the uniaxial stress-strain curve is represented by

\[ \frac{c}{\epsilon_o} \sim a (\sigma/\sigma_o)^n \]  

(7)

for \( \epsilon >> \epsilon_o \), where \( \sigma_o \) is the yield stress and \( \epsilon_o = \sigma_o/E_o \) the yield strain. Furthermore, assume the \( J_2 \) deformation theory generalization of (7) to multi-axial states, i.e.

\[ \frac{\epsilon_{ij}}{\epsilon_o} \sim \frac{3}{2} (\sigma/\sigma_o)^{n-1} s_{ij}/\sigma_o \]  

(8)

\[ \epsilon_{ij} = \frac{\sigma_{ij}^2}{2 \epsilon_o} \]  

where \( s_{ij} \) is the deviator stress. Then the asymptotic crack-tip fields are [7, 8]

\[ \sigma_{ij} \sim \sigma_o \left[ \frac{J}{\epsilon_o n} \right]^{1/(n+1)} \epsilon_{ij}(\theta, n) \]  

(9)

\[ \epsilon_{ij} \sim \frac{1}{\epsilon_o} \left[ \frac{J}{\epsilon_o n} \right]^{n/(n+1)} \epsilon_{ij}(\theta, n) \]  

(10)

where \( r \) and \( \theta \) are polar coordinates centered at the tip. The dimensionless \( \theta \) variations, \( \sigma_{ij} \) and \( \epsilon_{ij} \), depend on the symmetry of the fields with respect to the crack and on whether plane strain or plane stress prevails, as does the normalizing constant \( J_n \).

The separation of the two crack faces varies like \( r^{1/(n+1)} \) as \( r \to 0 \). Defining an effective crack-tip opening displacement \( \delta_t \) as the separation where the 45° lines intercept the crack faces, as in Fig. 2, gives

\[ \delta_t = d(\epsilon_o, n) \frac{J}{\sigma_o} \]  

(11)
Values of $d$ have been given by Shih [9] for plane strain and plane stress. In plane strain $d$ ranges from about .8 for $n = \infty$ to .3 for $n = 3$ with a relatively weak dependence on $\varepsilon_0$; in plane stress the same variation is from 1.0 to about .4. In the range of low strain hardening $d$ is a fairly strong function of $n$. Reported results [9, 10] for $d$ for plane strain obtained from finite element calculations for low and zero strain-hardening materials range from .8 to about .5. Although the connection between $\delta_L$ and $J$ is not as well established as it should be, the implication of (11) is that $\delta_L$ also measures the intensity of the crack-tip fields.

Fig. 2 Crack-tip opening displacement

ZONE OF DOMINANCE OF CRACK-TIP FIELDS AND LIMITATIONS OF SINGLE PARAMETER CRACK-TIP CHARACTERIZATIONS

For a stationary crack subject to a monotonically increased single loading variable, it is expected that plastic loading will not depart radically from proportionality. Thus, the deformation theory solution should be a good approximation to the corresponding solution based on incremental plasticity theory. A number of numerical studies have shown this to be the case. In particular, the line integral representation of $J$ (6) is found to be essentially independent of the path in the plastic zone when calculated using the standard incremental theories. The argument for using a critical value of $J$ or of $\delta_L$ to identify the onset of crack propagation, independent of other geometric and loading parameters, assumes that the crack-tip fields (9) and (10) dominate (i.e., are a good approximation to) the behavior over a zone at the tip which surrounds the region of finite strains and fracture processes where (9) and (10) break down. Since the region of finite strains (and also usually the fracture process zone) is on the order of $\delta_L$, the zone of dominance of (9) and (10) must therefore be sufficiently large compared to $\delta_L$.

McMeeking [10] employed a finite element method, based on a finite strain version of $J_2$ flow theory of plasticity, to study the near-tip behavior in small scale yielding under mode I plane strain conditions. (Small scale yielding is the asymptotic situation where the plastic zone is small compared to the crack length and other relevant in-plane length quantities. In mode I the fields are symmetric with respect to the line of the crack.) McMeeking found that finite strain effects are important over distances of about 2 or 3 times $\delta_L$ for values of the initial yield strain less than .01. For distances from the tip greater than $3\delta_L$ the small strain theory predictions were accurate and the J-integral was essentially independent of the path. We will use $R$ to characterize the size (radius) of the zone of dominance of the crack-tip fields (9) and (10) in the small
strain problem. From McMeeking's work one concludes that a necessary condition for using $J$ or $\delta_c$ as a single, configuration-independent parameter to characterize the near-tip behavior in plane strain is approximately

$$R > 3\delta_c$$  \hfill (12)

Very recently there have been several efforts [11, 12] to ascertain the size of the zone of dominance $R$ under large scale yielding conditions where the cracked body has become fully yielded. As background to these studies we recall that when $J$ was first discussed a possible intensity measure for fracture analysis under large scale yielding conditions, McClintock [13] pointed out the following limitation on $J$ (or on any other single parameter such as $\delta_c$). He noted that neither the stress nor the strain fields near the tip of a crack can be configuration-independent in elastic-perfectly plastic bodies under fully yielding conditions. Examples of two plane strain slip line fields with fundamentally different near-tip stress and strain fields are sketched in Fig. 3. The edge-cracked strip in bending develops a high triaxial and normal stress ahead of the crack, similar to that associated with the well-known Prandtl slip-line field. The stress ahead of the crack in the center-cracked tension strip is the plane strain tensile yield stress which is significantly below that attained in the other case. The strain fields are different, as well, with strain concentrating on planes emanating from the tips at 45° to the crack in the center-cracked strip.

Fig. 3 Fully yielded edge-cracked strip in bending and center-cracked strip in tension

These observations would appear to be at odds with the assertion that the stress and strain fields, (9) and (10), uniquely determine asymptotic conditions at the tip once $J$ is given. That assertion relies on the existence of some strain hardening (i.e., finite $n$). In the limit of elastic-perfectly plastic behavior ($n=\infty$), singular terms not considered become potentially as important as (9) and (10). Put differently, the $\theta$-variations $\delta_{ij}$ and $\tilde{\gamma}_{ij}$ are only unique for finite $n$. This is reflected in the two cases of Fig. 3. In general, some strain hardening is required to justify the use of a single parameter such as $J$ or $\delta_c$ to correlate fracture of different cracked configurations under large scale yielding conditions.

A quantitative assessment of the limitations of a single parameter approach as related to strain-hardening and configuration dependence is just beginning to emerge. The edge-cracked strip in bending seems to be reasonably well in hand. The standard compact
tension specimen can be regarded as a bend-type configuration for the purposes of this discussion.

McMeeking and Parks [11] employed the same finite strain, finite element procedure referred to earlier. They showed that the near-tip fields of the small scale yielding problem were essentially identical to the near-tip fields in the fully plastic edge-cracked strip in bending at corresponding values of \( J \) as long as

\[
b > 25.1/\sigma_0
\]

This condition does not seem to be strongly dependent on the initial yield strain. Of greater importance is the fact that the correspondence held up even without strain hardening. Condition (13) had been suggested earlier and has been indirectly verified experimentally [2, 14]. Since \( \delta_{\text{cr}} = 0.63/\sigma_0 \) in plane strain for moderate to low strain hardening, (13) states that the uncracked ligament \( b \) must satisfy (approximately)

\[
b > 40\delta_{\text{cr}}
\]

Under fully plastic conditions the zone of dominance \( R \) discussed earlier is necessarily some fraction of the uncracked ligament \( b \), assuming yielding is confined to the ligament. The functional connection is of the form

\[
R = g(n, \sigma_0) b
\]

The above discussion suggests that for the edge-cracked strip in bending \( R \) is not strongly dependent on either \( n \) or \( \sigma_0 \).

A comparison of (12) and (14), noting (15), gives the estimate \( g \approx 0.07 \). The more fundamental expression of the condition for \( J \)-dominance, i.e. \( R = 0.7b \) with \( R > 3\delta_{\text{cr}} \), translates into the better known expressions (14) or (13) when \( R \) is eliminated. Work of Shih and German [12] lends additional support to the value \( g \approx 0.07 \) in (15). Using a small strain finite element procedure, they compared calculated stress and strain fields in the fully plastic edge-cracked strip in bending with the dominant singularity fields (9) and (10) at corresponding values of \( J \). The agreement between the two predictions was reasonably good within a distance \( R \) of the tip less than about \( R \approx 0.07b \) for the two levels of hardening exponent considered, \( n = 3 \) and \( n = 10 \).

At the other extreme is the center-cracked plane strain strip in tension. As already discussed, the radius \( R \) of the zone of dominance must vanish as \( n \to \infty \) since the near-tip fields in the elastic-perfectly plastic limit are inherently different from the corresponding limit of (9) and (10). Studies along the lines of those described above [11, 12] suggest that the counterpart to (13) for the center-cracked strip in tension is (tentatively)

\[
b > 200/\sigma_0
\]

for fully yielded conditions with moderately low strain hardening \( (n = 10) \). At this level of strain hardening, Eqs. (12), (14) and (15) imply \( g \approx 0.01 \). That is, the singularity fields dominate a region of only about one percent of the uncracked ligament when \( n = 10 \).

Condition (16) places a severe limitation on
the applicability of a single parameter characterization for fully plastic center-cracked tensile configurations, as will be discussed further below.

INITIATION OF CRACK GROWTH

The potential of $J$ for extending engineering fracture mechanics into the large scale yielding range was appreciated immediately after it was first introduced [6, 15, 16]. But it was the innovative experimental work of Begley and Landes [15, 14] that established the feasibility of using $J$ and that provided the initial impetus for much of the work of the last five years, including some of that just described in the previous section.

Begley and Landes showed that it was possible to determine the fracture toughness under large scale yielding conditions using various types of test specimens. With $K_{IC}$ denoting the fracture toughness (i.e., the stress intensity factor at initiation as determined by a plane strain small scale yielding test), the corresponding value of $J$ at initiation should be [6]

$$J_{IC} = (1-\nu^2)K_{IC}^2/E$$  \hspace{1cm} (17)

where $\nu$ is Poisson's ratio. The test series of Begley and Landes verified this connection.

Subsequent work in a number of laboratories has refined and improved upon these first studies (see the discussion in [2] and various references in [17]). There now appears to be a consensus that bend-type test specimens can be employed under large scale yielding conditions to determine fracture toughness.

For testing purposes alone this is a major accomplishment since it eliminates the necessity of employing the huge test specimens required in small scale yielding testing of relatively high toughness metals with intermediate yield strength.

There has been some success in relating measured values of the crack tip opening displacement at initiation, $\delta^C_0$, to $J_{IC}$ through (11) -- see [18] and the discussion in [10]. A difficulty involved in making this comparison is the apparent relatively strong dependence of $\delta^C_0$ in (11) on strain hardening. Typical values of $\delta^C_0$ range from less than .01 mm for high strength low toughness metals to several tenths of a millimeter for intermediate strength high toughness metals. For a bend-type specimen where $S_0 = .2$ mm, say, (14) implies that the uncracked ligament must be at least 8 mm. For a center-cracked tension specimen of the same material, (16) requires a ligament about eight times as large. If either specimen were sized such that initiation occurred under small scale yielding conditions (i.e., under valid $K_{IC}$ testing conditions) a ligament of at least about 250 mm would be required. The advantage of the fully plastic bend-type specimen is obvious!

Shortly after Begley and Landes's preliminary work was finished, a very useful formula for $J$ for deeply edge-cracked bend-type specimens, such as that in Fig. 4, was obtained by Rice, Paris and Merkle [19]. For a deeply-cracked specimen they found a rigorous formula for $J$ in terms of load and
displacement quantities measurable in a test. With $\Delta_{c} - \Delta_{nc}$ denoting the load-point deflection of the specimen in Fig. 4 without a crack ($a = 0$) at load $P$, let

$$\Delta_c - \Delta - \Delta_{nc}$$

(18)

where $\Delta$ is the total deflection in the presence of the crack. (A spring has been inserted in series with the specimen in anticipation of the discussion of stability given later. The spring does not alter the relation between $J$ and $P$ or $\Delta$ as previously discussed.) The result of [19] is

$$J = \frac{2}{b} \int_0^{\Delta_c} Pd\Delta$$

(19)

The existence of simple formulas such as (19) tend to favor the use of $J$ as a crack-tip parameter over other potential candidates for which analogous simple formulas are not available.

![Fig. 4 Three-point bend specimen loaded in series with a linear spring](image)

**J-CONTROLLED CRACK GROWTH**

A relatively simple means of analyzing limited amounts of stable, quasi-static crack growth has been proposed by Paris, et al. [20] and Garwood, et al. [21] as a result of experimental findings which will now be described. In conducting tests to determine the critical value of $J$ associated with initiation ($J_{IC}$ in plane strain), experimentalists [17] used (19), or a formula like it, to measure the relation between $J$ and crack advance $\Delta a$ for small amounts of growth. A representative J-resistance curve, $J_R(\Delta a)$, for a typical intermediate strength, high toughness steel is depicted in Fig. 5. A small apparent growth due to crack-tip blunting prior to initiation has been subtracted off in Fig. 5. For such steels the advance $D$ needed to double $J$ above $J_{IC}$ is typically less than a few millimeters. These curves were used to extrapolate back to the initiation value $J_{IC}$. But it became evident that under certain restrictive conditions, called J-controlled growth, the J-resistance curve could be regarded as a material characterizing curve which was independent of geometry -- see, for example, the discussion in Rice's review [3].

![Fig. 5 J-resistance curve](image)
The J-integral is based on the deformation theory of plasticity which cannot model effects of elastic unloading or highly nonproportional plastic loading. Thus the argument for J-controlled growth relies on the conditions that the region of elastic unloading and nonproportional loading, which is a region on the order \( \Delta a \) in radius, be embedded within, and controlled by, the singularity fields (9) and (10), as depicted in Fig. 6. The two conditions for J-controlled growth [22] are

\[ \Delta a \ll R \]  

and

\[ D \ll R \]  

where \( D \) shown in Fig. 5 is

\[ D = \frac{J_{IC}}{(dJ/d\alpha)_c} \]  

The first condition is apparent. The second, (21), ensures that \( J \) increases sufficiently rapidly as the crack advances such that deformation theory is a good approximation within an annular region inside \( R \), as shown in Fig. 6.

Fig. 6 Schematic of near-tip conditions for J-controlled growth

For fully yielded configurations, such as those in Fig. 3, in which yielding is confined to an uncracked ligament, \( R \) is related to \( b \) by (15). In such cases the condition (21) can be stated nondimensionally as

\[ \frac{b}{r} \frac{dJ}{(d\alpha)_c} \gg 1 \]  

(23)

Judging from (20) and the discussion on the size of \( R \), the amount of crack growth possible under J-controlled conditions is small. But for many of the intermediate strength alloys relatively large increases of \( J \) above \( J_{IC} \) are nevertheless possible under J-controlled conditions since \( D \) is very small.

Efforts to refine the conditions (20) and (23) have only recently been made. For edge-cracked bend-type configurations Shih and Dean [23] have performed numerical calculations which have led to the tentative proposal that (20) and (23) should be (approximately)

\[ \Delta a < 0.06b \]  

(24)

\[ \omega > 10 \]  

(25)

For center-cracked tension configurations it is expected that these conditions will be much more restrictive, as has already been indicated by a few tests [24].

STABILITY OF J-CONTROLLED CRACK GROWTH

Paris and coworkers [20] have proposed a stability analysis based on the J-resistance curve which is similar in spirit to the resistance curve analysis of linear elastic fracture mechanics. If small amounts of crack growth are to be tolerated, with the attendant relatively large increase in \( J \), it becomes essential to be certain that such growth is stable.

To illustrate the approach of [20] and [22]
An approach with common features to that described above is also being developed by Garwood, Robinson and Turner [25 and unpublished work].

PROGRESS TOWARDS A UNIFIED NEAR-TIP FRACTURE CRITERION FOR INITIATION AND GROWTH

The approach described above is inherently empirical in that $J_{IC}$ and the resistance curve must be obtained experimentally for each material for every set of conditions. In addition the range of potential application, although important, is quite restricted, particularly in that it is limited to relatively small amounts of crack growth. Thus the basic problem of identifying a near-tip fracture criterion based on the fracture processes very close to the tip is of considerable practical importance as well as fundamental scientific interest. The problem is far from being "solved" but some significant first attempts have been made. Probably the most ambitious attempt to understand the mechanics of ductile crack initiation is that of Rice and Johnson [27] who carried out an approximate analysis of the linking-up process of a void with the crack-tip. Other approaches, one level removed from dealing with the micro-mechanical fracture processes, have been proposed for combined initiation and growth [28, 29, 30].

We will make use of McClintock's [28, 29] early results in anti-plane shear (mode III) to indicate the source of stable crack growth and to predict initiation and growth in terms of near-tip fracture criteria of the type used in [28, 29]. With $\gamma$ denoting the total shear strain ahead of the crack, the condition for growth is a critical strain criterion

$$\gamma = \gamma_c$$

at $r = r_c$ (32)

where $r_c$ is a material length characterizing the fracture process zone.

Small scale yielding is assumed. The material is elastic-perfectly plastic with initial yield stress in shear as $\tau_0$ and yield strain as $\gamma_o = \frac{T}{G}$ where $G$ is the elastic shear modulus. A Mises yield condition is used. Prior to initiation the strain ahead of the crack in the plastic zone ($r < r_p$, where $r_p$ is the plastic zone extent ahead of the crack)

$$\gamma = \gamma_o \frac{r_p}{r}$$

where $r_p = \frac{\pi}{2} \frac{r}{\gamma_o} \frac{1}{G}$ (33)

Imposition of (32) using (33) gives the value of $J$ at initiation

$$J = \frac{\pi}{2} \gamma_o \gamma_c \frac{r_c}{r_p}$$

and $r_p = \delta r_c$ (34)

where

$$\delta = \gamma_c / \gamma_0$$

(35)

Next consider steady-state growth where the crack has grown sufficiently far ahead such that it is able to progress at constant $J$. In this case the strain ahead of the crack is

$$\gamma = \gamma_o [1 + \ln(r_p/r) + \frac{1}{2} \ln^2(r_p/r)]$$

(36)

Chitailey and McClintock [31] have shown that $r_p$ is still given by (33), to a very good approximation. A comparison of (36) with (33) shows that the strain near the tip in a growing crack at steady-state is much less than the corresponding strain the same distance ahead of the crack in the stationary problem at the
consider the system in Fig. 4. We will assume that the total load-point displacement, $\Delta_T$, is imposed. The compliance of the linear spring $C_N$ can be regarded as the compliance of a test machine or as the compliance of the surrounding structure transmitting load to the cracked element. Assume the crack had advanced an amount $\Delta a$ and is currently loaded for further possible advance, i.e.,

$$J = J_R(\Delta a)$$

(26)

Stability at this state, with $\Delta_T$ prescribed, requires

$$\left(\frac{3J}{2a}\right) < \frac{d\Omega^R}{\partial a}$$

(27)

which simply ensures that any small "accidental" advance of the crack can be sustained by the tearing resistance of the material. Paris et al. [20] introduced a nondimensional tearing force and tearing resistance as

$$T = \frac{E}{\sigma_T^2} \left(\frac{3J}{2a}\right)$$

and

$$T_R = \frac{E}{\sigma_T^2} d\Omega^R$$

(28)

so that the stability condition becomes

$$T < T_R$$

(29)

A relatively simple formula for $T$ can be obtained for the system in Fig. 4 which is exact in the deeply-cracked limit [22]. For the case of a fully yielded, elastic-perfectly plastic cracked beam that result is

$$T = \frac{4EP^2}{\sigma_T^2 b^2} (C_{nc} + C_N) - \frac{EJ}{\sigma_T^2 b}$$

(30)

Here $P$ is the limit load of the cracked beam and $C_{nc}$ is the elastic compliance of the uncracked beam. As expected, the system compliance has a significant influence on the stability through $T$, whereas it does not affect $J$. Of course, under dead load ($C_M = \infty$) the fully yielded, perfectly plastic beam is unstable.

Paris et al. [25] conducted a test series in which a spring of adjustable compliance was inserted in series with just such a deeply-cracked bend specimen. By testing a sequence of identical specimens in series with springs of differing compliance, they were able to check the validity of the stability condition (29). Their material had a tearing resistance at initiation of $T_R = 36$, $D_w = 1.2$ mm, and their specimens met (25) with $\omega = 15$. Their tests did reveal a transition from stability to instability at $T$-values very close to $T_R = 36$.

A table of values of $T_R$ for a wide variety of steels at various temperatures has been compiled in [20]. For high strength, low toughness alloys $T_R$ is often as small as or below unity. On the other hand, many of the intermediate strength steels have $T_R$-values which exceed 30, some being as large as 200.

In many circumstances the $T$-values will be far smaller so that small amounts of crack growth can be safely sustained. As an illustration, consider a finite crack in an infinite body whose material behaves in simple tension as $\varepsilon = (\sigma/\sigma_T)^n$. If $\varepsilon_\infty$ is the remote strain due to a remote uniaxial stress normal to the crack face, then

$$T = h(n)(\varepsilon_\infty/\varepsilon_T)^{n+1}$$

(31)

where $h(n)$ is roughly 3 for $n$ less than 10 [26]. Only when the overall strain $\varepsilon_\infty$ exceeds approximately 10 times the effective yield strain will $T$ exceed 30.
same value of \( J \). The significantly weaker singularity in (36) is a consequence of the highly nonproportional plastic flow which occurs ahead of the crack in the growing crack. It is the substantial resistance of plastic flow to nonproportional stressing which is the primary source of stable crack growth.

Invoking the growth criterion (32) using (36), together with (33) for \( \tau_p \) in terms of \( J \), gives the value of \( J \) necessary to drive the crack under steady-state conditions

\[
J_{ss} = \frac{1}{2} J_c \Gamma_c \gamma_0 \tau_0 \exp(\sqrt{2\beta-1}-1) \tag{37}
\]

The ratio of \( J_{ss} \) to \( J_c \) is

\[
\frac{J_{ss}}{J_c} = \frac{1}{\beta} \exp(\sqrt{2\beta-1}-1) \tag{38}
\]

Large values of \( \beta = \gamma_c/\gamma_0 \) imply substantial potential stable crack growth. Approximate calculations of the full \( J \)-resistance curve in mode III based on the criterion (32) have been reported in [29, 6]. Here we will be content to report the result for the initial slope of the \( J \)-curve following initiation which has been obtained using McClintock's analysis for the transient case. In nondimensional form that result is

\[
T_R = \frac{J_{ss} \left( \frac{dJ_c}{da} \right)}{\gamma_c J_c} = \frac{1}{2} (\beta - 1 - \ln \beta) \tag{39}
\]

A "perfectly brittle" material with \( \beta = 1 \) corresponds to \( T_R = 0 \), while for large \( \beta \)

\[
T_R \approx \beta/2. \]

The material-based length quantity, \( D \), is

\[
D = \frac{J_c}{(dJ_c/da)} = \frac{\beta}{\gamma_c} \frac{\beta}{\beta - 1 - \ln \beta} \tag{40}
\]

It is interesting to note that for \( \beta \) larger than about 10, \( \beta \) is essentially the characteristic length associated with the fracture process zone, \( r_c \).

The model suggests certain implications relating macroscopic fracture resistance to features of the fracture process zone. In particular, note that the ratio, \( J_{ss}/J_c \), in (38) and the nondimensional tearing modulus \( T_R \) in (39) depend only on \( \beta = \gamma_c/\gamma_0 \).

Furthermore, for large \( \beta \), \( J_{ss}/J_c \) increases exponentially while \( T_R \) increases linearly in \( \beta \). Note that for \( \beta \approx 60 \), \( T_R \approx 1000 \) and \( J_{ss}/J_c \approx 10000 \). For larger values of \( \beta \) the small strain assumptions will certainly be violated for typical values of \( \gamma_0 \) at the point where \( r = r_c \). But the model does suggest the source of the large values of \( T_R \) which are observed. The very large values of \( J_{ss}/J_c \) for large \( \beta \) result from the considerable resistance an elastic-plastic material offers to nonproportional straining, as has already been noted. This effect is undoubtedly overestimated by the simple smooth yield surface of Mises (and Tresca in mode III) used in the present analysis. In this sense the values of \( J_{ss}/J_c \) for large \( \beta \) may be considerably in excess of observable values.

Rice and Sorensen [30] have considered the more difficult mode I, plane strain problem in small scale yielding. Qualitatively the findings are similar to mode III and several features of the analysis are closely analogous. While the criterion (32) is sensible in mode III, a critical strain condition cannot be taken to be met ahead of
the crack in plane strain mode I since the
strains are most intense above and below the
tip in the small strain solution. Instead,
Rice and Sorensen used an alternative criterion
which is essentially an integration of the near-
tip strains. They require the crack opening
displacement to reach a critical value at some
fixed small distance back behind the tip. By
making contact with numerical results they are
able to obtain an approximate integration of
the equations relating the crack opening
displacement, the crack advance and J.
Resistance curves are determined. Large
tearing resistance is found, typical of
observed values, with realistic choices for
the near-tip fracture criterion.

ACKNOWLEDGEMENT

Recent work by the author in fracture
mechanics has been supported in part by the
National Science Foundation under Grant
NSF ENG78-10756, and by the Division of
Applied Sciences, Harvard University. Parts
of the present survey were abstracted from a
set of notes "A Course on Nonlinear Fracture
Mechanics" which were prepared in the Fall
of 1978 at the Technical University of
Denmark.

REFERENCES

1. A. J. Carlsson, in Theoretical and Applied
Mechanics, Proc. 14th IUTAM Congress,
Delft, ed. W. T. Koiter, North-Holland,
3. J. R. Rice, in Mechanics of Fracture,
6. J. R. Rice in Fracture: An Advanced
Treatise, ed. H. Liebowitz, 2, Academic
16, 13, 1968.
9. C. F. Shih, "Relationships Between the
J-Integral and Crack Opening Displacement
for Stationary and Extending Cracks", General Electric Company Report, October
1978 (to be published).
11. R. N. McMeeking and D. M. Parks, "On
12. C. F. Shih and M. D. German, "Requirement
ments for a One Parameter Characterization of Crack Tip Fields by the BRR Singularity", General Electric Company Report, October
1978 (to be published).
13. F. A. McClintock, in Fracture: An
14. J. A. Begley and J. D. Landes, ASTM-STP560,
176, 1974.
15. J. A. Begley and J. D. Landes, ASTM-STP514,
1, 24, 1972.
17. Mechanics of Crack Growth, ASTM-STP590,
1976.
18. G. Green, R. F. Smith and J. F. Knott
in Mechanics and Mechanisms of Crack
Growth, ed. M. J. May, British Steel
19. J. R. Rice, P. C. Paris and J. G. Merkel,
20. P. C. Paris, H. Tada, A. Zahoor and
H. Ernst, in Elastic-Plastic Fracture,
21. S. J. Garwood, J. N. Robinson and
C. E. Turner, Int. J. Fracture, 11, 528,
1975.
22. J. W. Hutchinson and P. C. Paris, in
Elastic-Plastic Fracture, ASTM-STP668,
to be published in 1979.
23. C. F. Shih and R. H. Dean, "On J-Controlled
Crack Growth: Evidence, Requirements and
Applications" in process of preparation.
24. J. A. Begley and J. D. Landes, Int. J.
Fracture, 12, 1976.
25. P. C. Paris, H. Tada, A. Zahoor and
H. Ernst, "A Treatment of the Subject of
Tearing Instability", U.S. Nuclear Reg.


ABSTRACT

Plastic Tearing Instability — What Is It?

W H Irvine & A Quirk

The fracture locus approach is used to discuss the basic mechanism of tearing instability and to examine the evidence as to whether it should be regarded as a material parameter or whether it can be explained in terms of Newton's third law, as proposed by Orowan almost a quarter of a century ago. It is shown that the latter explanation is adequate.
CSNI SPECIALIST MEETING ON PLASTIC TEARING INSTABILITY

St Louis, Miss. United States
25-27 September 1979

Plastic Tearing Instability - What Is It?

W H Irvine & A Quirk
Safety and Reliability Directorate, UKAEA

1. INTRODUCTION

The term 'plastic tearing instability' apparently refers to the extension of a crack front in a solid in an unstable manner under conditions which are predominantly plastic or at least involve a considerable amount of plasticity. This Paper considers the circumstances under which such conditions might come about in a practical engineering structure or laboratory test piece and the criteria for unstable crack extension that apply.

2. BEHAVIOUR OF ENGINEERING STRUCTURES

Most engineering structures are designed to operate under elastic conditions, i.e. at stresses below the yield point. Occasionally, with materials of high toughness and overload conditions limited amounts of local yielding may be allowed to occur, e.g. in nuclear reactor pressure vessels. The interaction of material fracture parameters and structural characteristics for the case of a flat plate with a central crack fracturing under uniaxial tension has been described in Ref 1, and Fig 1 is reproduced from this reference, to summarise the conditions under which brittle and ductile fast fracture can occur. As pointed out by Orowan² the conditions for brittle and ductile fast fracture are fundamentally different. In the brittle case fast fracture occurs where the critical combination of stress level crack size and material toughness gives fracture and crack extension at the crack tip, whereas in the ductile case there is the additional requirement that the unloading path of the structure be sufficiently energetic. In practice, the unloading path may lie anywhere in the 90° quadrant in Fig 1 bordered on the low energy side by the fixed grip or constant strain condition, and on the high energy side by the constant load condition. With gross stresses below yield point it is clear that the incremental tearing occurs in materials of high fracture toughness with large crack sizes. In the limit, when the crack size plus its associated plastic zones approach the plate width the amount of plasticity involved in the fracture increases rapidly. These large crack sizes can occur in a number of ways, however, one mechanism which is of particular interest is that which was probably associated with the Cackenzie
Boiler Drum failure. Using the flat plate fracture locus as an analogue of the boiler drum behaviour this is depicted in Fig 2. The point A describes the condition relevant to a very small crack, possibly in low toughness heat affected material associated with the attachment welds of an internal bracket, with the as-yet unrelieved welding stresses being just below yield point level. At the commencement of thermal stress relief it is possible that the additional thermal stresses triggered off a fast brittle fracture which then, because of the constant strain nature of residual stresses caused unloading down the fixed grip line ABC with crack arrest at C and a large crack size corresponding to the line OCD. When the drum was loaded up a number of times in the hydraulic proof test to the neighbourhood of point D on the high toughness locus corresponding to the plate properties, failure occurred by sudden rupture of the drum. A simple calculation shows that the stress on the remaining ligament at the test condition corresponded to the ultimate tensile strength of the material. Although the conditions in this failure must have been quite close to those for plastic tearing there was no macroscopic evidence of this and the fracture surface was of a brittle appearance.

A form of tearing that has occasionally occurred is associated with the use of partial penetration attachment welds on steam drums. This is in the form of incremental extension of the unfused land between the two welds as in Fig 3(a) under the driving force of thermal stresses developed by operational transients. Provided that the material has sufficiently high fracture resistance to prevent the fracture locus having a positive slope in the small crack size region of the diagram, the extension of such cracks can be by small increments, since the unloading path under the predominantly displacement controlled stress is a vertical line on Fig 3 (b). However, the rates of crack extension occurring in this situation will be much higher than for simple fatigue crack growth. Peak stress levels may still be below the yield stress.

3. **Behaviour of Laboratory Test Pieces**

The load-displacement characteristic of a typical laboratory test piece is shown in Fig 4. An important feature is the small physical size and the effect of this in limiting the amount of strain energy available. Because the test piece can be loaded into a plastic condition redistribution of stresses can occur unless special precautions have been taken to ensure that the stress distribution is independent of the material condition.

These factors can combine to give a situation in which the crack can only extend in
a slow manner, i.e. in the region A to B in Fig 4. This is independent of the structural characteristics of the loading system. On reaching the maximum load at point B, fast fracture becomes possible under constant load conditions only. For harder, less energetic systems the point for the onset of fast fracture is further down the characteristic at greater deflections, the actual condition being defined by the point at which the unloading path is tangent to the characteristic. This pattern of behaviour is confirmed by a programme of tests on 6" thick tensile specimens containing partial thickness surface crack reported in references 8 & 9 and analysed in Ref 10. The essential data are reproduced in Table 1.

4. SUMMARY

(i) The type of tearing behaviour which has been observed in large engineering components such as boiler drums can be explained by means of the fracture locus concept. This concept only involves a knowledge of the load extension relationship for the component and a fracture mechanics criterion for fracture of the crack tip material.

(ii) The essential characteristics of this type of tearing are that it occurs under stresses which are predominantly displacement controlled and have peak values that may be less than the yield stress.

(iii) In contrast with the above, tearing in machine-loaded specimens in the laboratory is associated with overall strains that are considerably in excess of maximum elastic strain and with stresses in excess of the yield stress. Many specimens are forced into a widely distributed plastic state by the artificially favourable load condition.

(iv) The combination of circumstances which leads to tearing in laboratory specimens can be complex and usually involves such factors as inadequate size of specimen or unsuitable gripping arrangements.

(v) As the combination of crack and its crack tip process zone increases in size to occupy the whole section and then eventually leads to a so-called plastic collapse situation - so the associated strain increases. A case of steam drum failure during a pressure test in which it reached the condition described above failed to show any evidence of tearing.

Sept. 1979
### TABLE I

**6" THICK FLAWED TENSILE SPECIMENS**

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Strain @ Max Load %</th>
<th>Tearing Δa ins</th>
<th>$\sigma_g/\sigma_{ys}$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4.1</td>
<td>1.4</td>
<td>1.084</td>
<td>Tearing at stresses in excess of yield point</td>
</tr>
<tr>
<td>2</td>
<td>3.83</td>
<td>0.7</td>
<td>1.165</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.88</td>
<td>0.36</td>
<td>1.087</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.48</td>
<td>0.96</td>
<td>0.725</td>
<td>Tearing with too small a specimen. Gross stress level below yield point</td>
</tr>
<tr>
<td>5</td>
<td>0.35</td>
<td>0</td>
<td>0.815</td>
<td>No tearing. Gross stress level below yield point</td>
</tr>
<tr>
<td>3</td>
<td>0.24</td>
<td>0</td>
<td>0.792</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.09</td>
<td>0</td>
<td>0.394</td>
<td></td>
</tr>
</tbody>
</table>

Elastic strain at yield point, 0.25%

Elastic strain at UTS, 0.3%
REFERENCES


**Fig. 1. The effect of increasing fracture toughness**
Fig. 2. Diagrammatic description of the Cockenzie failure.
FIG. 3a. CRACKED STEAM DRUM NOZZLE.

FIG. 3b. DESCRIPTION OF CRACK EXTENSION MECHANISM IN NOZZLE WELDS.
Fig 4. The effect of the stiffness of the loading system on the failure point in a cracked tensile specimen.
Title: Study of instability of growing cracks using damage functions. Application to warm prestress effect.

Author: F. M. BEREMIN* (Presented by A. Pineau)

ABSTRACT

This paper deals with the approach which is developed for the study of crack initiation and crack propagation in a 508 class 3 steel. This approach is based on computational fracture models which use damage functions. The computational method involves the node release technique in finite element calculations whilst the damage functions are experimentally established and model the physical microfracture processes taking place either in ductile rupture or in cleavage fracture. Those functions are obtained from critical experiments on specimens calculated by the finite element method.

Some results illustrating this approach will be first given. These results are obtained using a simplified criterion for ductile rupture. It is shown that possible crack characterization parameters (such as J.R. curves) calculated with this method are in qualitative agreement with experimental results obtained on this type of steel.

Then the methodology developed for the determination of damage function is presented. This methodology mainly uses asymmetric notched tensile specimens which are calculated by finite element stress analysis. The physical damage introduced into those specimens by tensile loading in a temperature range between -196°C and +100°C is analysed using conventional metallographic observations. For ductile rupture the formation of cavities from inclusions and the growth of those cavities have been quantitatively studied. In the case of cleavage fracture the value of the critical cleavage stress ($\sigma_c$) and a characteristic distance, as well, have been experimentally determined at -196°C using various specimen geometries.

* F. M. BEREMIN regroups French specialists who belong to several organizations, including FRAMATOME, Bureau de Contrôle de la Construction Nucléaire and Centre des Matériaux EMM, Groupe Métallurgique Mécanique ERA CNRS 767. The main activities of this group are the development of computational fracture models based on microfracture processes and the development of new and improved methods dealing with the structural integrity of nuclear installations.
At last, the application of the damage function approach to the case of warm prestress effect is shown. Experimental results obtained on axisymmetric and CT specimens have confirmed that a prestressing at 100°C leads at an apparent increase in the fracture toughness at lower temperature (-196°C). Various loading sequences have been employed in order to assess the effect of crack blunting and the role of residual stresses. It is shown that the experimental results can be explained using two parameters (\(\sigma_c, \lambda\)) for the damage function added to the condition that plasticity is necessary for cleavage initiation.

**INTRODUCTION**

Continuum mechanics models for the prediction of crack initiation and stable or unstable crack propagation are required. The need for such models is related to the need to define accurately the effect of flaws on the burst resistance of nuclear pressure vessels, particularly the reactor vessel. Linear elastic fracture mechanics cannot accurately predict this behavior for several reasons. The most important is due to the fact that at operating conditions this analytical tool assumes that the materials are essentially elastic and brittle, which is not the case. Other approaches are currently developed for non linear elastic fracture mechanics (COD J.). These approaches are limited to the study of simple defects like planar flaws, submitted to a symmetrical mode I loading which is not the general case. Moreover, it is difficult to apply these approaches to loading histories which are not isothermal.

For these reasons, we prefer to attempt to use a damage function approach. This approach has already been partly presented (1, 2). Essentially this approach implies the use of accurate finite element calculation in addition to damage criteria which describe precisely the physical damage processes taking place at the tip of a crack submitted to the most general type of loading.

The general strategy for developing this approach is as follows.

1. Conduct appropriate experiments to obtain the proper damage functions. Ductile rupture and cleavage fracture are investigated. Several types of specimens are used in order to investigate the "size effect" and several "stress conditions", since it is well established that fracture occurs only when the damage function has attained a critical value over a characteristic distance. This is particularly well known in cleavage fracture. These damage functions are investigated in the frame of continuum mechanics since the results of the experiments are interpreted using the results of finite element calculations.

2. Develop numerical industrial tools which can be realistically used for modelling the crack tip stress strain field. For instance, in a certain number of problems the modelling of crack blunting is necessary whilst in others it is quite superfluous. This implies that the mesh size be chosen in conjunction with the physical analysis of the fracture processes which are investigated.

3. Conduct critical fracture experiments on appropriate specimen geometry to verify the predictive capability of the approach developed. This implies amongst other the choice of specimen geometries which can be easily calculated and to avoid most of the complications related to three dimensional loading like that encountered on CT or CCP specimens. In this area, critical experiments allowing the observation of crack bifurcation in complex loading are equally conducted. This is used in conjunction with the finite element calculation to verify the directional predictive capability of the damage
functions introduced.

In this paper the feasibility of this tentative approach will be first shown. The example of ductile tearing with the use of a simplified damage criterion is presented. Then the methodology employed for obtaining damage functions both for ductile rupture and cleavage fracture is shown. Experimental results on cleavage fracture in A 508 steel are given. Finally the application of the damage function valid for cleavage fracture to the case of a more or less complex thermomechanical loading, which is warm prestress effect, is shown.

In this study all the experiments were conducted on A 508 class 3 grade steel. The material was cut near the quarter of the thickness of a nozzle shell. The J-\Delta a curves were determined on another typical carbon steel.

FEASIBILITY STUDY OF THE DAMAGE FUNCTION APPROACH USING A SIMPLIFIED DAMAGE CRITERION FOR DUCTILE RUPTURE

1. Introduction

This study was performed using the most simplified form for the three stage damage function corresponding to ductile rupture since it was based only on cavity growth and cavity coalescence. The main aim of this investigation was to verify that this approach was able to predict the general trends observed in the experimental results and therefore to be a numerical industrial tool for predicting crack behavior.

2. Computations

We choose the two dimensional plane strain loading with symmetrical conditions (pure mode I loading) taking three point bend specimens of various widths (W) but of the same a/W = 0.475 ratio. The geometries of these specimens are shown in Fig. 1.

The incremental elastoplastic finite element computations were made with the TITUS program using the small geometry change approximation. This program uses the initial stress and tangent stiffness method with Mises yield criterion and Hill’s maximum work principle. A perfect plastic material with a yield stress \( \sigma^* \) equal to 520 MPa was assumed. This type of constitutive equation was chosen because it was not intended in this first part of the study to model necessarily a real material. For the same reason the most simplified form of the three stage damage function was selected. We assumed that inclusions either break or separate from the matrix as soon as plastic flow begins. Cavity growth was computed using the relationship proposed by RICE and TRACY for a single spherical void in an infinite rigid plastic material, i.e.

\[
\frac{d(\log R_0)}{R_0} = 0.28 \left( \frac{\sigma_m^*}{\sigma_f} \right) \cdot \frac{d \varepsilon_{eq}^*}{\varepsilon_{eq}^*} \cdot \exp \left( 1.5 \cdot \frac{\sigma_m^*}{\sigma_f} \right),
\]

where \( R_0 \) is the initial cavity radius, \( R \) its actual mean radius, and \( \sigma_m^* = (\sigma_{xx}^* + \sigma_{yy}^* + \sigma_{zz}^*) / 3 \) is the hydrostatic stress at infinity and

\[
d \varepsilon_{eq}^* = \frac{2/3 (d \varepsilon_{ij}) d \varepsilon_{ij})^{1/2}}{3}
\]

is the equivalent Mises strain at infinity with

\[
d \varepsilon_{eq}^* = \frac{d \varepsilon_{ij}^* - d \varepsilon_{lk}^* \delta_{ij}}{3}
\]

This formula does not take into account void interac-
tion nor work-hardening. Here the values of stresses and strains must be understood as the average values over a characteristic volume. In this volume which is given by the mesh size the rupture is assumed to take place as soon as a critical ratio \( \xi_c = (R/L_c) \) is obtained. In this ratio \( R \) is the actual mean cavity radius and \( L \) is the minimum of \( L \) with \( L_i = L_o \exp ( \xi_i ) \), \( i = xx, yy, zz \) where \( L_o \) is the initial distance between cavities.

It is worth mentioning that more sophisticated damage functions can be easily used. Most importantly to realize is that the damage function must incorporate mean values of stresses and strains over a characteristic volume or a characteristic distance which represents the scale of the physical processes of rupture as already proposed by Mc Clintock (4). In our approach we take the first elements of the mesh at the crack tip as the characteristic volume. Therefore in this simplified numerical model for ductile tearing there are only two parameters. The first one is the critical value of void growth whilst the second one is the mesh size. Most of the computations were made using \( \xi_c / \xi_o = 1, 6 \) and a mesh size of 0.2 mm. In these calculations 8 nodes isoparametric elements were used.

Calculations are conducted as follows. During loading the \( \xi / \xi_o \) ratio increases in all the elements in front of the crack. When the critical value \( \xi_c \) is reached in the first element, crack initiation takes place and the force on the 2 nodes of the first mesh element at the crack tip are released to zero in several small steps. During this process the \( \xi / \xi_c \) ratio still increases in the other elements so that it is necessary to verify if the criterion has been reached at the end of the relaxation process in this new crack tip element. If it has been the case, we proceed to relax the nodal forces of the next element. If not, the load or the displacement applied to the specimen are increased until the \( \xi_c \) criterion is reached again and so on.

3. Results

Two homothetic three point bend specimens were studied in displacement controlled loading, one with \( W = 60 \) mm, the other with \( W = 140 \) mm. One calculation has also been made in load control. Very similar results were obtained in both cases. The normalised load displacement curves (\( P/BW \gamma \), \( U_y/W \)) are the same until initiation for the two types of geometries since they are homothetic. However crack initiation and crack growth behavior are different. This of course arises from the characteristic volume of material over which ductile tearing occurs. As the ratio of the characteristic distance \( 0.2 \) mm to the width of the specimen is different, it is obvious that, at homologous loads, damage is less important in the smaller specimen. It results that crack initiation takes place latter in the smaller specimen as shown in Fig. 2.

Another set of interesting results is given by this numerical study (for more detail see Ref. 1). A first example is given on Fig. 3 which shows that the work done during relaxation, \( \gamma_p \), is remarkably constant. The importance of this parameter has already been discussed by several authors (eg 5). A second example is given in Fig. 4, where computed \( J - \Delta a \) curves are plotted. In this figure it is observed that crack initiation is almost the same for both types of specimens although the slope of the \( J - \Delta a \) curve is higher in the bigger specimen. In Fig. 4, some experimental results obtained on CT 30 and CT 50 specimens in a carbon steel are equally shown. It is observed that the trend of the calculated \( J - \Delta a \) curves is very similar to that of
experimental curves.

In summary for us this study has allowed the preparation of an industrial numerical program which can now be used in conjunction with more sophisticated damage functions. The methodology used for deriving these damage functions is presented hereafter.

**METHODOLOGY FOR DERIVING DAMAGE FUNCTIONS**

Ductile rupture and brittle fracture are examined successively. It is well known that the process of ductile rupture includes three elementary physical stages which are inclusion decohesion, cavity growth and cavity coalescence, respectively. The methodology used for deriving damage functions which describe quantitatively each of these three stages is presented. As far as brittle fracture is concerned, a certain number of experimental and theoretical investigations have shown that cleavage cracking obeys a simple damage function which can be stated as follows: cleavage fracture is consistent with the attainment of a maximum principle stress (σₘₚ) over a characteristic distance (λ). The methodology used for determining those two parameters and the experimental values obtained in A 508 steel, as well, will be presented.

1. Ductile rupture

1.1. Experimental procedure and calculations

The experimental work is conducted on circumferentially notched tensile specimens. The geometry of these specimens is shown in Fig. 5. The reason for the choice of this type of specimen is two fold. First a bidimensional calculation is straightforward. Difficulties associated with stress conditions lying in between plane stress and plane strain condition like those encountered with CT type specimens are avoided. Second the geometry of the specimens - especially the ratio between notch radius and specimen radius at minimum section can be changed very easily in order to produce different strain and stress conditions through the section of the specimen.

Those specimens are loaded at 100°C. The mean diametral deformation (Δ) defined as $\Delta = 2 \log \frac{d}{d_0}$ where $d_0$ is the initial diameter and $d$ is the actual diameter is continuously recorded. An example of a loading curve is shown in Fig. 6. This curve shows that it is possible to interrupt the test just before complete fracture of the specimen. The observation of the minimum section of the specimen using a longitudinal section indicates that the acceleration of the decrease of the load at the end of the test corresponds to the formation of a small crack of about 1 mm long in the middle of the specimen. This allows the definition of fracture strain (Δf) as a function of stress triaxiality using various specimen geometries. In order to investigate the influence of the anisotropy both longitudinal direction and transverse direction are studied.

Further specimens are given various amounts of overall strain (Δ) lower than Δf. These specimens are longitudinally sectioned and polished in order to observe damage inclusions using a technique similar to that already employed by Argon and al (6).
In the longitudinal direction it is observed that most of the inclusions are broken whilst in the transverse direction inclusion damage corresponds to the decohesion of the interface between Manganese sulfides and the matrix. These longitudinal sections allow the determination of a limiting curve for a given specimen geometry and a given applied strain (\( \bar{\varepsilon} \)). Inside this curve all the inclusions are damaged whilst outside this frontier all the inclusions are not yet damaged. Examples of such curves corresponding to different applied overall strains are shown in Fig. 7. Provided that the strains and the stresses are accurately known all along this limiting curve it is possible to derive a criterion for inclusion decohesion.

Each specimen geometry is analysed by finite element calculations. Calculations are made for overall strains up to about 10%. The comparison between experimental curves and calculated curves corresponding to various geometries is made on Fig. 8. A very good agreement between these two sets of curves is observed. This allows a good confidence in the values which are calculated at every points inside the specimen for strains and stresses. Fig. 9 shows the variation of the ratio (\( \sigma_m^v / \sigma_{eq}^v \)) calculated in the center of the specimen as a function of the overall strain. This ratio which measures stress triaxiality is reported here because it is well known that it is an important parameter in ductile rupture processes. The curves shown in Fig. 9 indicate that \( \sigma_m^v / \sigma_{eq}^v \) is practically constant for \( \bar{\varepsilon} \) larger than about 6%. This observation allows the extrapolation of the results obtained by finite element method in the range of overall strains between 0 and 10% to larger strains.

1.2. Results

1.2.1. Inclusion decohesion

The experimental results are interpreted in a diagram (\( \Sigma_1^v, \sigma_{eq}^v, \sigma^y \)) where \( \Sigma_1^v \) is the maximum principal stress, \( \sigma_{eq}^v \) is the Von Mises equivalent stress and \( \sigma^y \) is the yield stress. This diagram is shown in Fig. 10 both for the longitudinal and the transverse loading direction. Detailed analysis will be published elsewhere where (7). A linear relationship \( \Sigma_1^v = k (\sigma_{eq}^v - \sigma^y) \) is obtained. It has been shown that these results can be interpreted in term of a critical stress applied to the inclusions. The value of this critical stress was evaluated using an extension of the Eshelby inclusion theory (8), the mechanical parameters given by \( F, E, M \) being used as limit conditions (7).

It is worth mentioning that if these results are used for modelling the damage at the head of a crack tip they indicate that inclusion decohesion occurs very soon in ductile tearing. This is due to the attainment of high values for the ratio (\( \sigma_m^v / \sigma_{eq}^v \)) in the vicinity of the crack tip. This justifies the use of a damage function for ductile rupture taking place at crack tip based essentially on the physical processes of void growth and cavity coalescence.

1.2.2. Cavity growth and cavity coalescence

No experimental work has yet been undertaken to investigate the process of cavity growth although the geometry of our specimens allows the study of stress triaxiality on this stage. Therefore the differential equation proposed by Rice and Tracey (3) is used. This differential equation is integrated from \( \bar{\varepsilon} \) to \( \bar{\varepsilon}_c \), where \( \bar{\varepsilon}_d \) and \( \bar{\varepsilon}_v \) are the equivalent strains corresponding to inclusion decohesion and find fracture, respectively. These values for \( \bar{\varepsilon}_d \) and \( \bar{\varepsilon}_v \) are taken in the mesh located in the center of the specimens since it has been shown that rupture occurs at this place. This integration gives the variation of \( (R/R_c) \) and the variation of \( \alpha_c = (R/\bar{\varepsilon}_c) \) as a function...
of stress triaxiality. These two values are used in the numerical modelling of ductile tearing, as indicated previously.

Typically, it is found that in the longitudinal direction \( R/R_0 = 1.75 \), whilst in the transverse direction cavity coalescence occurs much easier for \( R/R_0 \geq 1.25 \). Moreover the experimental results indicate a small but significant decrease of \( R/R_0 \) as stress triaxiality \( (\sigma_m/\sigma_{eq}) \) is increased. The results are limited in the range of \( (\sigma_m/\sigma_{eq}) \) between 0.39 and about 1.5. Further work is needed to investigate the effect of stress triaxiality on cavity coalescence, especially in the range of higher values for \( (\sigma_m/\sigma_{eq}) \).

2. Cleavage fracture

The experimental study is equally carried out on circumferentially notched tensile specimens. Another geometry corresponding to \( \varphi = 20 \) mm was added to those already shown in Fig. 5. These specimens are broken at \(-196^{\circ}C\). Detailed results are given elsewhere (9, 10).

Fig. 11 shows the variation of the maximum principal tensile stress at fracture \( (\sigma_{c}) \) as a function of the overall strain at fracture. The results obtained on conventional tensile specimens are equally included on this figure. It is worth noting that in spite of the large variation of the deformation at fracture with specimen geometry it can be reasonably assumed that there is a well defined value for the cleavage stress \( \sigma_{c} \), equal to 1435 MPa in this steel.

This value of the cleavage stress was obtained using the results of finite element calculations. As an example, Fig. 12 and 13 show the variation of the maximum principal stress in the minimum section of two geometries \( (\varphi = 10 \) mm and \( \varphi = 2 \) mm) as a function of the applied load. Examination of these two figures suggests that fracture in \( \varphi = 10 \) mm geometry takes place in the center of the specimen where \( \sigma_{max} \) is maximum, whilst in \( \varphi = 2 \) mm geometry, cleavage is initiated close to the periphery where \( \sigma_{max} \) reaches its maximum value at fracture. Scanning electron microscopy observation are in agreement with this analysis (10).

These geometries cannot be used for the determination of the critical distance \( (\lambda) \) because they give a rather flat profile for the maximum principal tensile stress. Further tests were performed using \( V \) notched specimens similar to those shown in Fig. 14. The notch angle was studied between 45 and 90 degrees. In these specimens the radius of curvature at the notch tip was close to 0.2 mm in order to produce a steep gradient for the maximum principal stress. These supplementary specimens were equally tensile load \(-196^{\circ}C\). An example showing the case of a 45° specimen with \( \varphi = 0.26 \) mm is shown in Fig. 15, where the results of the slip line field solution are also included. The determination of the load at fracture added to the curves giving the variation of the maximum stress as a function of the applied load and the distance from the edge of the specimen as well allows the estimation of the critical value for the characteristic distance \( (\lambda) \). In the case of the experiments reported in Fig. 15, it is found that \( \lambda \simeq 80 \mu m \). This value corresponds to a distance equal to about three grains and is very close to that found by Ritchie, Knott and Rice (11).
Further tests are presently done in order to determine more precisely the value of $\lambda$.

APPLICATION TO WARM-PRESTRESS EFFECT

1. Introduction

The importance of warm-prestress effect for crack initiation and unstable fracture intervenes in one of the postulated events for a nuclear pressure vessel which is a sudden loss of coolant accident followed by operation of the emergency core cooling system. In the postulated accident high thermal stresses are introduced. If the wall contains a crack like defect, the stress intensity factor at the crack tip $K_1$ increases with time to a maximum and then decays as the temperature gradient throughout the wall becomes smaller. Moreover neutron bombardment causes a marked shift to higher temperatures of the temperature dependence of the critical fracture toughness $K_{1c}$. During the postulated event the stress intensity factor can attain the critical value for initiation. Because of the decrease of $K_1$ with time the prediction of crack initiation does not necessarily correspond to the original $K_{1c}$-T curve. The crack tip material has been subjected to "warm prestress".

A certain number of studies (12, 13, 14) suggest that warm prestress increases the "effective" fracture toughness of the crack tip material so that crack extension will not occur when the stress intensity factor reaches the $K_{1c}$ value of the virgin material at the lower temperature. A recent study by NRL showed the effect of various loading sequences on the apparent value of $K_{1c}$ (15).

This study was undertaken in order to confirm warm prestress effect. An attempt was principally made for using the damage function approach which has been explained previously to specimen geometries which can be completely calculated.

2. Experimental study and finite element calculations

Two types of specimens were used, CT 50 type specimen 50 mm thick and axisymmetric tensile specimens. The dimensions of these axisymmetric specimens are given in Fig. 16. In both cases these experiments were fatigue precracked at 25°C according to ASTM procedure. Preloading was applied at 100°C and the specimens were broken at -196°C after various loading sequences which are described here after.

Axisymmetric specimens were employed because they allowed easily bidimensional calculations contrary to the case of CT specimens. Various types of finite element meshes were used (4 nodes and 8 nodes). In addition in order to model the influence of crack blunting, an appropriate technique involving initially a very small radius of curvature at the crack tip (10 $\mu$m) was employed. It has been shown previously that this technique can account for crack blunting (16). The use of very mesh sizes at the crack tip was necessary because of the very steep gradient of the maximum stress as it is shown subsequently. Fig. 17, shows this refined mesh at the crack tip.
Kinematical hardening was used. The $G - \varepsilon$ constitutive equations employed were those experimentally determined at 100°C and -196°C for $\varepsilon$ lower than about 10%. Above this value, that is above the homogeneous elongation of the tensile specimens, linear strain hardening was used.

3. Results

The results of the experiments on CT 50 specimens are given in Fig. 18. All the specimens were preloaded at crack initiation. The corresponding load was previously determined. The fracture load and the apparent $K_I$ which can be calculated from this load are given. These results indicate that the apparent fracture toughness is increased from 34 MPa $\sqrt{m}$ to 63 MPa $\sqrt{m}$ when the specimen is unloaded after preloading at 100°C. Another sequence in which the specimen, after preloading at 100°C, is directly cooled down to -196°C whilst it is still loaded gives a much higher apparent $K_{IC}$ (118 MPa $\sqrt{m}$). These limited results confirm those obtained at NRL (15).

In order to investigate what is conventionally named "plane stress" and "plane strain" effect a supplementary test was carried out. In this test a CT 50 specimen was also loaded at crack initiation then unloaded. It was subsequently machined in order to reduce its thickness down to 10 mm. This reduction was obtained by machining each of the lateral surfaces of the specimen. Then the CT specimen whose thickness was reduced, was loaded -196°C. This further test gave a value for $K_{IC}$ equal to 54MPa$\sqrt{m}$. This value as compared to that determined on 50 mm thick specimen suggest that most of the beneficial influence of warm prestress on fracture toughness cannot be attributed to a plane stress effect.

The results corresponding to axisymmetric specimens are given in Fig. 19. Several tests showed that the fracture toughness of the virgin material at -196°C was close to 28 MPa $\sqrt{m}$ in this material which was slightly different from that used for CT 50 specimens. The axisymmetric specimens were given different preloading conditions at 100°C. In Fig. 19 the preloading conditions are indicated by the values of the longitudinal displacement (2 Uy) determined on a gage length of 50 mm which was applied to the specimens. It has been shown previously that in this material crack initiation occurred at 2 Uy = 300 μm. In addition to the loading sequences already used for CT specimens another sequence with a superimposed heat treatment at 620°C was used. This heat treatment was applied in order to anneal residual stresses.

These results confirm the tendency observed on CT 50 specimens. In addition the comparison of the results obtained in test 3 and in test 4 shows directly the effect of compressive residual stress on the apparent value of $K_{IC}$ for a given preloading condition. In Fig. 19 it is also worth noting that for a given loading sequence the beneficial warm prestress effect is higher, the higher is the preload.

4. Discussion

Two factors can qualitatively account for the effect of warm prestress. First the influence of residual stresses is clearly illustrated in the experiments which include an annealing heat-treatment. Second crack tip blunting which changes the stress-strain field at the crack tip is also an important parameter. Finite element calculations added
to the introduction of damage parameters specific to cleavage fracture ($\sigma_c, \lambda$) allows the quantitative assessment of these two factors.

The first step in this approach necessitates the calculation of residual stresses after unloading. Fig. 20 shows the variation of the maximum principal stress in front of the crack tip during unloading. These results indicate that high compressive residual stresses are introduced at the crack tip. It is worth mentioning that an isotropic model instead of a kinematical model should have given higher compressive stresses.

The second step is similar to that already used by Ritchie, Knott and Rice (11). It consists with the attainment of stress intensification close to the crack tip such that $\sigma_y$ exceeds $\sigma_c$ over a characteristic distance ($\lambda$).

The calculated curves corresponding to three experiments on axisymmetric specimens (test 1, 5 and 6) are shown as an example in Fig. 21. The curves are plotted for the value of the stress intensity factor equal or very close to that corresponding to the apparent fracture toughness measured experimentally. These curves show that the results of tests 1 and 6 are consistent with $\sigma_c = 1435$ MPa over a distance of the order of 80 $\mu$m. However test 5 shows that a value of $\sigma_c$ equal to 1435 MPa gives a distance which is much higher and is of the order of 250 $\mu$m. This value is much higher than that obtained previously on V notched specimens. This relative poor agreement can be explained as follows. First the linear strain hardening model is not necessarily the most appropriate. In particular the slope of the $\sigma$-$\delta$ curve corresponding to small strains has been extrapolated to higher strains in a range where a better experimental definition of the constitutive equation is not available. Second, as mentioned earlier, an isotropic model should have given higher compressive residual stresses which, in turn, should have strongly influenced the stress field after reloading at -196°C. In spite of these difficulties it is believed that this approach accounts reasonably well for the effect of warm prestress.

This is particularly well illustrated in the case of test 2 which is shown in Fig. 22. In this case the calculations are made using a simple square mesh with a mesh size of 0.2 mm. This value is appropriate since in this example the zone corresponding to the maximum principal stress where cleavage initiation takes place is located at a distance larger than 300 $\mu$m. It was checked that at this distance this type of mesh size gave results in good agreement with those obtained with a more refined mesh. Stress profiles at 100°C and at -196°C are shown in this figure. At -196°C the crack tip stress-strain behavior is first completely elastic because of the increase of the yield stress with temperature. This elastic stress strain field occurs for applied loads lower than that corresponding to a stress intensity equal to 69.5 MPa $\sqrt{m}$. At this value the first point is plastically deformed. The examination of Fig. 22 indicates that at $K_0 = 69.5$ MPa $\sqrt{m}$ $\sigma_y$ is larger than 1435 MPa over a distance larger than 300 $\mu$m. However fracture does not take place because plasticity is not yet spread over a sufficient distance. Apparent values for the stress intensity factor close to 75 MPa $\sqrt{m}$ have to be reached before the plastic zone in front of the crack tip has a size of about 50 $\mu$m. This dimension is also of the order of 2 or 3 grain size. Then a cleavage microcracking can be initiated. It is immediately unstable since $\sigma_y$ is higher than $\sigma_c = 1435$ MPa over a distance larger than 600 $\mu$m. This clearly shows that plasticity is necessary for cleavage fracture as expected from all the microstructural models for cleavage.
CONCLUSIONS

A global method for deriving different damage functions which are used for modelling crack initiation and stable and unstable crack propagation has been presented.

1. Inclusion decohesion is described in terms of a critical value for the maximum principal stress which is a function of the equivalent strain.

2. Cavity growth and cavity coalescence are analysed using the Rice and Tracey formulation. The critical value for cavity growth at fracture can be determined from experimental observations.

3. Cleavage fracture is consistent with the attainment of a critical maximum principal stress \( (\sigma_c^\ast) \) over a characteristic distance \( (\lambda) \). \( \sigma_c^\ast \) and \( \lambda \) were determined experimentally.

The damage function for cleavage fracture \( (\sigma, \lambda) \) is employed to account for the effect of warm prestress on the apparent fracture toughness of A 508 class 3 grade 3 steel. It is shown that the computed values for the critical fracture toughness are in good agreement with the experimental results.

AKNOWLEDGMENTS

This study has been parity sponsored by D.G.R.S.T. (contract n° 78.7.26.92) and SCSIN (contract n° 296.686). Financial support for numerical computations from FRAMATOME is greatly acknowledged.
REFERENCES


[9] F.M. BEREMIN. Elastoplastic calculation of circumferentially notched specimens using the finite element method, submitted for publication to "Journal de Mécanique Appliquée".


\( \frac{a}{W} = 0.475 \)

<table>
<thead>
<tr>
<th>W</th>
<th>b</th>
<th>c</th>
<th>e</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>1.5</td>
<td>15</td>
<td>4.8</td>
<td>330</td>
</tr>
<tr>
<td>140</td>
<td>2.5</td>
<td>35</td>
<td>11.2</td>
<td>770</td>
</tr>
</tbody>
</table>

Fig. 1 - Diagram showing the dimensions of 3 points bend specimens used for numerical calculations (all dimensions are given in mm).
Fig. 2 - Results of finite element computations giving the loading curves \( F/B \), \( W \sigma_v \) for two specimen geometries \( W = 60 \text{ mm} \) and \( W = 140 \text{ mm} \). Crack initiation is indicated.
\[ \gamma_p = \frac{2}{\Delta a} \int_0^{V_1 \max} f_1 \, dv_1 + \frac{2}{\Delta a} \int_0^{V_2 \max} f_2 \, dv_2 \quad (8 \text{ node elements}) \]

\[ \Delta \mathbf{w} \ 140 \ (8n, U_p \text{ imp.}, \Delta a = 0.2 \mathbf{mm}) \]
\[ \bullet \ \mathbf{w} \ 60 \ (8n, U_p \text{ imp.}, \Delta a = 0.2 \mathbf{mm}) \]
\[ \Delta \ \mathbf{w} \ 140 \ \text{2}^\text{nd} \ \text{kind of relaxation.} \]

**Fig. 3** - Definition of \( \gamma_p \) (work corresponding to node relaxation). Variation of \( \gamma_p \) with crack extension (\( \Delta a \)) for two geometries (\( W = 60 \ \text{mm} \) and \( W = 140 \ \text{mm} \)).
Fig. 4 - Calculated and experimental J-Δa curves
Fig. 5 - Dimensions of the circumferentially notched tensile specimens used in this study (all the dimensions are given in mm).
Fig. 6 - Typical experimental loading curve of an axisymmetric notched tensile specimen at +100°C. The enclosed micrograph shows the formation of a small crack in the center of the specimen.

- Section polie de l'éprouvette arrêtée juste avant rupture.
- Polished section of the specimen, interrupted test just before rupture.
Fig. 7 - Limit curves determined experimentally on axisymmetric notched specimens ($\rho = 10$ mm) which were given different values of overall strain ($\bar{\varepsilon}$).
Fig. 8 - Calculated and experimental loading curves for various axisymmetric notched specimens geometries ($\rho = 2, 4, 10$ mm). The tensile ($\sigma$-$\varepsilon$) curve is also given. The position of the plastic zones corresponding to various applied strains is shown.
Fig. 9 - Variation of $\sigma_m/\sigma_{eq}$ as a function of the overall deformation $(\varepsilon)$ in the center of axisymmetric notched specimens $(\rho = 2$, 4 and 10 mm).
Fig. 10 - Experimental results showing the variation of the maximum principal stress ($\Sigma_1$) as a function of ($\sigma_{eq} - \sigma_o$). Longitudinal and transverse directions are indicated.

Material I

<table>
<thead>
<tr>
<th>Test Temp</th>
<th>100°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>▼ Long.</td>
<td></td>
</tr>
<tr>
<td>● Trans.</td>
<td></td>
</tr>
</tbody>
</table>

$\Sigma_1 + 1.30(\sigma_{eq} - \sigma_o) = 1060$

$\Sigma_1 + 0.5(\sigma_{eq} - \sigma_o) = 730$

$\sigma_{eq} - \sigma_o$ (MPa)
Fig. 11 - Cleavage fracture in A 508 steel at -196°C. Variation of the cleavage stress ($\Sigma_C$) as a function of the equivalent strain determined on axisymmetric notched specimens.
Fig. 12 - Variation of $\sigma_{yy}$ as a function of the applied load in axisymmetric notched specimens ($\rho = 10$ mm). At $-196^\circ$C, fracture takes place close to loading n°8. For this loading, it is observed that $\sigma_{yy}$ is maximum in the center of this type of specimen geometry.
Fig. 13 - Axisymmetric notched tensile specimen (p = 2 mm).
Variation of \(\sigma_{yy}\) as a function of loading. Fracture at -196°C occurs close to loading n°5. It is worth noting that \(\sigma_{yy}\) is maximum close to the edge of the specimen.
Fig. 14 - Dimensions of the axisymmetric 45 V notched tensile specimens.
Fig. 15 - 45 V notched axisymmetric specimen. Results of finite element calculations. The solution corresponding to slip line field theory is equally shown. In this case $\rho = 0.20$ mm.
Fig. 16 - Dimensions of the fatigue precracked axisymmetric tensile specimens used in the study of warm prestress effect.
Fig. 18 - Warm prestress effect. Experimental results on CT 50 specimens. Various loading sequences are indicated.

- Valid $K_{IC}$ test
- Preloading at 100°C up to crack initiation unloading at 100°C and fracture at -196°C.
- $K_{IC}$ apparently
- Preloading at 100°C up to crack initiation, the load is maintained, the specimen is broken at -196°C.
<table>
<thead>
<tr>
<th>Nb</th>
<th>Loading sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Valid K\textsubscript{1C} test at -196°C, K\textsubscript{1C} = 28</td>
</tr>
<tr>
<td>2</td>
<td>Preloading at 100°C up to crack initiation - The load is maintained and the specimen is fractured at -196°C. K\textsubscript{1C} = 73.3.</td>
</tr>
<tr>
<td>3</td>
<td>Preloading at 100°C up to crack initiation - Unloading + heat treat at 620°C-2hrs - Fracture at -196°C. K\textsubscript{1C} = 46.2.</td>
</tr>
<tr>
<td>4</td>
<td>Preloading at 100°C up to crack initiation - Unloading - Fracture at -196°C. K\textsubscript{1C} = 69.</td>
</tr>
<tr>
<td>5</td>
<td>Preloading at 100°C (2U\textsubscript{y} = 190 μm) - Unloading - Fracture at -196°C. K\textsubscript{1C} = 58</td>
</tr>
<tr>
<td>6</td>
<td>Preloading at 100°C (2U\textsubscript{y} = 100 μm) - Unloading - Fracture at -196°C. K\textsubscript{1C} = 30</td>
</tr>
</tbody>
</table>

Fig. 19 - Warm prestress effect. Experimental results on fatigue precracked axisymmetric specimens. Various loading sequences are shown. 2U\textsubscript{y} is the displacement applied to the specimens.
Fig. 20 - Variation of $\sigma_{yy}$ at the front of the crack tip during unloading. The results corresponding to various types of finite element mesh are shown.
Fig. 21 - Calculated fracture toughness of specimens which were given various loading sequences (see Fig. 19).
Axisymmetric specimens.

<table>
<thead>
<tr>
<th>Test</th>
<th>KIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>58</td>
</tr>
<tr>
<td>1</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Distance at the crack tip (μm)
Fig. 22 - Warm prestress effect (Test n°2 - See Fig. 19) - Variation of $\sigma_{yy}$ in front of the crack tip
Remarks on Unstable Ductile Crack Growth

C. E. Turner

SUMMARY

From a brief review of various theories of fracture, it is concluded that to describe initiation of fracture in plane strain, the J contour integral is the most embracing, though probably more relevant to circumstances of high constraint, than to the low constraint cases. An R-curve picture of stable ductile crack growth is seen to be a measure of total plastic dissipation of work rather than of toughness per se, and expression through J is seen as either a characterising description valid whilst a J field exists for \( \omega \gg 1 \), or as an attempt to define a normalised work rate independent of size and shape, at least for the flat fracture component, through use of the factor \( \eta/B_b \). Either argument gives results that are identical for small amounts of growth by defining a material tearing resistance, \( T_{\text{mat}} \), describable through \( (dJ/da)_{\text{mat}} \). The inequality that leads to instability can be expressed from a characterising term \( (dJ/da)_{\text{appl}} \), or through arguments of compatibility of displacement, or as a balance of energy rates. All reduce to the same dominant terms for small amounts of growth, \( \omega \gg 1 \), though subject to some numerical differences according to the various approximations made. For larger amounts of crack growth the methods diverge. The physical argument of a balance of energy rates is still valid in concept but the existence of \( T_{\text{mat}} \) unique for all configurations and sizes is not established and assessment of which terms for \( T_{\text{appl}} \) are truly of second order is not yet clear. Suggestions are made for assessing the magnitude of the term \( \omega \gg 1 \) to ensure the relevance of J theory to plasticity. The energy balance concept is also expressed as an effective toughness by combining the size and shape factors \( \eta \) and \( B_b \) with \( (dJ/da)_{\text{mat}} \).

* Mechanical Engineering Department,
Imperial College, London

August 1979
Introduction

Numerous theories have been introduced to describe fracture. Most concentrate on defining the initiation of growth of a crack from a pre-existing sharp defect, whilst some do not distinguish between initiation and final separation. A few recent studies concentrate particularly on the stability or otherwise of crack growth. The object of this paper is not to propose new theories but to discuss existing proposals with a view to relating them by distinguishing their common features and differences. If a theory for stable crack growth and final instability is to form part of an overall fracture control plan, it would appear sensible to phrase both initiation and slow growth in related terms, if technically feasible. Thus, before discussing slow growth, various elastic-plastic fracture models are examined to see which offers the most promising basis.

In examining the various descriptions of fracture, distinction is made between the underlying theory for each and any procedure that may be recommended by its protagonists. For example, if there is thought to be strong reason for using full thickness test pieces, then most if not all such arguments can be attached to any fracture theory as an issue separate from the merits of the theory. It is accepted here that a theory embodying certain empirical factors may more closely represent behaviour of a certain class of component or material than a wider theory. Similarly, a procedure that presents rules ready for practical application, such as treatment of residual stress or welding porosity, will have a more immediate relevance for ad hoc decision-making than a more fundamental theory. Nevertheless, the writer believes that the basis to an engineering fracture control plan should be the most embracing theory currently available, simplified for rule of thumb use where appropriate, or embroidered with empirical factors to accommodate its shortcomings, to ensure that its foundation is as sound as current knowledge allows.

Comparison of the J contour integral and other descriptions of fracture

Several current elastic-plastic fracture theories were summarised in (1), and related more specifically to J in (2). In briefest summary, it was suggested that these theories fell into two groups, one in which the concepts, however expressed originally, could now be seen to be compatible with J, and a second in which the final results could be related to J even though developed from a quite different basis.

The first group contained the proposals of

(a) Gross section yield, Soete
(b) Equivalent energy, Witt
(c) Non-linear energy, \(\tilde{E}\), Eftis & Liebowitz
(d) Tangent modulus, or gross strain, Merkle

(3) (4) (5) (6)
As but a single sentence statement of the arguments, * Soete's four regimes of 
behaviour appear to the writer to be identical with the regimes of lefm, contained 
(small scale) yield, yield to the lateral boundary (net section yield), yield 
to the end boundary (gross section yield), that emerge as distinct regimes in 
the usage of G and J for a design curve type approach (7) (8). Equivalent energy 
relates directly to J for those cases where the factor, \( \eta \) that relates G or J to 
work, \( w \),

\[
J = \frac{\eta w}{B(W-a)} \quad \text{Eqn.1}
\]
is the same for elasticity (i.e. \( \eta_{el} \) where \( J_{el} = G \)) as for the overall elastic-
plastic case, and such cases are now seen (9) to be more extensive, for at least 
an approximate equality in values, that at one time seemed likely. Under certain 
circumstances, the formulation of G is formally identical to J, and although not so 
expressed, some of the terms relating to work implying a relation similar to Eqn.1. 
Features in the tangent modulus method parallel the trends in \( \eta \) with net or gross 
section yield, albeit extended to the part through crack configuration. Thus, 
subject to a number of further complexities (such as allowance for variation 
of toughness with temperature and thickness in the gross strain method) the basic 
trends, be they derived by other arguments or observed as experimental fact, are 
describable in terms of J theory, though not necessarily by the deep notch small 
size testing procedures with which J is often identified.
The second group of engineering methodologies are 

e) COD, Wells (10), its developments by Dawes (11) and others, 
and derived proposals, such as by Heald, Spink & Worthington (12) 
f) Two criteria, Dowling and Townley (13) 
e) Two parameters, Newman (14) 
f) Stress concentration, Irvine and Quirk (15) 
The original COD method could perhaps be better classified in the first group 
since J parallels it completely, through the relation 

\[
J = \frac{mg}{\delta} \quad \text{Eqn.2}
\]
subject only to the variation in \( m \) from about \( 1 < m < 3 \) with extent of yielding,

* For brevity but one reference is offered for each theory. In most cases 
many more exist in the literature and several are instanced in (1) and (2). 
The single sentence statement here is obviously inadequate in respect of both 
the complexity of the various theories and the similarities seen with J, which 
in some cases are rather explicit, but in others rather indistinct. Each 
argument deserves a paper in its own right.
configuration and usage of first yield or some flow stress to account in
part for hardening and triaxiality, but the original COD theory was inappro-
priate beyond net section yield.

All these methods allow interpolation between lefm and collapse. Some express
collapse explicitly. In others collapse is implicit, as it is in J theory, in
that J or other parameters in question increase rapidly as collapse is approached.
One main difference is the combination of plasticity and shape effects. In
essence the above theories take an explicit function for the effect of plasticity,
such as the ln sec term in the Dugdale-COD model, and if effects of configuration
are considered at all, combine this with the lefm shape factor. A virtue is
simplicity since many shape factors are readily available. Computations of J,
and indeed of COD, permit the interaction of plasticity with configuration.
These points are illustrated, Fig.1, from (2). A non-work hardening law is used
for simplicity but that is not a restriction, since hardening can easily be in-
corporated. Various cases are shown for J calculated for different configurations,
to illustrate the way the increase of J over the elastic value G varies with shape
and loading of component. A disadvantage is the need for individual computation
for each configuration once contained yield is exceeded. Within contained yield,
J is closely predicted by lefm with plastic zone correction. A major difference
between some of the theories, and indeed between the engineering design use of
COD (8) a, (11), and the now quite separate HSW and Two-Criteria method (12), (13),
which all originated in the Dugdale model, is the expression of fracture
in terms of applied load or applied deformation. This was rationalised (8) b
according to whether a whole structure or sub-structure technique was used for
the overall analysis, the latter being essential to protect against fracture in
regions of stress concentration if a load-based method is used. If the basis of
the analysis is deformation, then the so-called J or COD design curve is found,
Fig.2, suitable for rule of thumb use, including appropriate simple treatments of
residual stress. The conclusion drawn is that J offers more flexibility than
other theories. Thus, the J and COD methods readily allow analysis in terms of
either load or deformation. Either can also be condensed to a simple design curve
approach where desired. J embraces all the features of the other theories, though
not readily providing explicit formulae for extensive yield of the more compli-
cated cases. Distinctive features of each case are brought out by virtue of the
interaction of configuration and degree of plasticity, and for contained yield lefm
corrected for size of plastic zone provides a good approximation that is compat-
bile with J theory for all configurations.
Stress state

Despite the foregoing conclusion that J is the most pertinent fracture theory currently available, it does not appear all embracing. The main restriction seen by the writer is that, considering plane strain only, for any given hardening exponent the HRR theory relates to a fixed degree of triaxality, whereas the degree of in-plane biaxality may well differ from case to case. When the J contour integral is used with incremental plasticity, even though it retains a fairly close degree of path independence (within about 5%, say) there is no assurance that the crack tip stress state that is being described is unique in its constituents and varying only in its intensity. This has been shown by finite element computations for a small geometry change formulation (16) and using a large geometry change model, (17). The lack of uniqueness also applies to COD. It reflects no more than Mc- Clintock's observation (18) that in the non-hardening limit the crack tip state, according to slip line solutions, depends upon the remote geometry. Thus, although the full force of that statement is ameliorated by the effect of elasticity, hardening and finite geometry change, it still appears that a single term, be it J or other, is inadequate to describe the crack tip state of stress as limit load is reached. Whether this effect is best described by triaxality ratio or through a higher order term in the crack tip stress field that is sensitive to hardening exponent and boundary conditions, is unclear. In lefm all plane strain solutions give \( \sigma_{yy}, \sigma_{zz}, \sigma_{xx}; 1:2:1 \) along the line of the crack, \( \theta = 0 \), just ahead of the tip. For small-scale yielding the buried Prandtl slip line field gives (2+\( \pi \)): (1+\( \pi \)); \( \pi \) (1.0 : 0.8 :0.6). Similar values occur in the limit state for deep DEN and moderate to deep 3NB, whereas for SEN and CCP the non-hardening slip line field solution gives 1 : 0.5 : 0. The stress terms in the J field of the HRR solution have a high ratio that is a function of the hardening exponent, falling between 1 : 1 : 1 for \( n = 1 \) and 1 : 0.8 : 0.6 for the non-hardening limit which reduces to the Prandtl case.

Since the lefm plane strain case is taken as the reference for all fracture purposes and neglecting the fairly small differences in triaxality between lefm and the HRR solutions in plane strain for a given hardening exponent, J appears as a logical measure of intensity of crack tip deformation whilst the stress ratios remain similar to the high constraint of the lefm case. It is unreasonable to expect a single parameter to describe the differences of both intensity and triaxality that occur when low hardening material passes from contained yield to uncontained yield in a configuration that induces low constraint. Thus, for plane strain, J should be relevant with high hardening which in the limit \( n \to 1 \) approaches lefm, for all circumstances. For a low hardening material J might be descriptive of lefm and contained yield for all configurations and also with uncontained yield for those configurations that experience

* My thanks to J.R.Rice for discussion of this point.
high constraint in the limit state. A detailed attempt to support these arguments in terms of published fracture data is not made here. The degree to which differences in constraint within plane strain affect fracture no doubt depends upon the micro-mechanism of separation, and in the present state of knowledge experiment is a necessary guide. When one parameter is insufficient to describe the crack stress state, by the same token, one parameter, be it J or other, is inadequate to serve as a universal criterion of fracture, unless the micro-mechanism is insensitive to triaxiality. It is difficult to distinguish in many experimental studies between the effects of material variability, degree of plane stress or plane strain outside lefm, uncertainty of pin-pointing initiation, use of inadequate formulae and the genuine effects of configuration. The point to be made is that if J is not completely adequate for characterising initiation in certain cases it can hardly be appropriate for characterising slow growth in those cases. However, a J-like term may yet serve even then, either if justified on other grounds or if used for comparing cases of similar in-plane constraint without reference to a datum that is high constraint or lefm.

**Slow Crack Growth**

A number of procedures have emerged recently for guarding against unstable fracture beyond the lefm regime. Proposals can be classified as extensions of the lefm R-curve concepts or as extensions of one or other of the various fracture procedures already mentioned. In accordance with the arguments of the previous section, only the fracture mechanics based concepts will be examined here, either J or COD being regarded as the extension of an lefm term G. The two major aspects of the problem are the model for slow stable growth and the criterion for instability. This latter must contain two terms, the applied severity and the material response, in the evaluation of which the simplification or approximations of the various theories are likely to differ. Although much early discussion of the mechanics of slow growth was given by Broberg (19), Wnuk (20), Rice (21), and others, a presentation of results more directly related to test procedures and structural analysis only emerged recently, e.g. Paris et al (22), Turner (23), Shih (24) and Garwood (25), and it is this last group of papers that is the focus of the present discussion. Unless stated otherwise, only growth by ductile micro-mechanism is considered.

a) The meaning of an R-curve

There seems general agreement (e.g. (20-27) that the crack tip opening angle is a measure of the characteristics of the tip of an advancing crack, albeit with some differences on interpretation and detail, perhaps reflecting the use of different test techniques and materials. For example, use of formulae from the conventional COD test technique (28) does not distinguish between monotonic loading and the advancing crack, whereas infiltration (29) allows local measurement, which for the ductile structural steel used in (29) also revealed a small COD at the advancing
tip (there called $\delta_a$ with a value about one quarter of the initiation value $\delta_i$) associated with the more or less constant flank angle. Computed data in (24) is also consistent with the crack opening angle picture. Use of a crack tip parameter, such as COD, is clearly an attempt to characterise the separation on a local scale. An alternative and not necessarily conflicting picture, arises naturally from the use of curves of nominal $J$-da obtained in conventional initiation testing. Many such data show sensibly linear slopes so that $dJ/da$ appears as a constant, at least for small amounts of growth. This term, evaluated just after initiation using conventional techniques and formulae is here called $dJ_0/da$ (i.e. the original value) to distinguish it from other $J$-derived terms. Use of $J$ and $dJ/da$ after some slow growth is rationalised in two ways. Hutchinson and Paris (30) argue that a $J$-dominated field will exist for small amounts of growth such that

$$\omega \equiv \frac{b}{(b/J)(dJ/da)} > 1$$

Eqn. 3

where $b$ is the ligament ($W-a$) and $dJ/da$ implies $dJ_0/da$. McMeeking (17) has shown that with growth the $J$ contour value in incremental theory is not path-independent but can be characterised by two values, $J_{\text{tip}}, J_{\text{far field}}, J_{\text{ff}}$ with the latter substantially path-independent. From similar studies Shih (24) concludes that experimental values of $J$ derived (for compact tension) from Eqn.1 relate to $J_{\text{ff}}$. For growth rather less than $a/b < 0.1$ a mean value of $J$ found on contours in the near field, $J_{nf}$, not immediately adjacent to the tip, was in reasonable agreement with $J_{\text{ff}}$. A schematic picture of $dJ/da$ evaluated by Shih for $J_{\text{ff}}$ is shown Fig.3(a) and translated to a $J-R$ curve, Fig.3(b). To the extent that Fig.3(a) is idealised, it implies a near constant initial slope, $dJ_0/da$, and a different near-constant, but non-zero final slope (steady state) $dJ_{ss}/da$. For structural usage steady state crack growth would usually imply contained yield. Even though evaluated in the far field these arguments interpret $dJ/da$ as a parameter that characterises growth in a manner analogous to that accepted for $J$ prior to initiation. It thus appears as the measure of some local event, namely, the $J$ field close to the tip that is dominating the actual separation. A physical meaning other than stress field intensity, is not offered. Indeed, expressed in terms of the tearing modulus $T$, as either

$$T_J = \frac{E}{\sigma} \frac{dJ_0}{da} \quad \text{or} \quad T_\delta = \frac{E}{\sigma} \frac{d\delta}{da}$$

Eqn.4a,b

Shih (24) concludes that provided $\omega > 1$ is satisfied for $J$ or crack opening angle to control crack growth, $T_\delta$ is "the more appealing because it is a more fundamental quantity and is relatively more constant for a given material", again emphasising the local nature of the characterisation being sought. Willoughby et al (31) have recently stated a term corresponding to $\omega > 1$ expressed through COD for the deep notch bend piece as
where \( \frac{d\delta}{da} \) is the slope of the COD curve with crack growth measured at the original crack tip. \( \delta_i \) is the COD value at initiation and \( r \) is the rotational constant in the COD formula \((r = 0.45 \) for deep notch bending). They deduce a value for \( m \) (relating \( J \) to COD in Eqn.2) as \( L/2r \) where \( L \) is the plastic constraint factor. In so far as \( L \) and \( r \) are nearly invariant for deep notch bending at the limit state then by direct differentiation of Eqn.2

\[
\frac{dJ}{da} = m\sigma_Y \frac{d\delta}{da}
\]

Eqn.6

In other cases a term in \( dm/da \) would be expected, relating to variation in constraint and position of the effective centre of rotation with crack growth.

Following crack growth studies using the infiltration technique that in all essentials are consistent with the above picture in terms of COD, Garwood et al (32) offered a rather different interpretation that subsequently led to Turner's model of instability (23) in terms of overall energy rate. It was observed (32) that a \( J-R \) curve of the form of Fig.3b was obtained from tests in three-point bending with deep side grooves, with a C-Mn structural steel giving failure by micro-void coalescence. No increase of toughness on the micro scale was found (32), (33) so that, in the absence of shear lip, it was concluded that the \( J-R \) curve (derived from the increment of work as expressed (32) or the equivalent by differentiation of Eqn.1, and called \( J_r \)) must reflect the total rate of dissipation of work. Most of the absorption occurs as general plastic deformation remote from (or at least not immediately adjacent to) the crack face and is thus not strictly a toughness or notional surface energy effect. It was therefore argued that the definition of an \( R \)-curve should be the measure of total dissipation rate. This would be a logical extension of the left usage of G-R curves, although in slyy the dissipation in the plastic zone is so local that it is treated as an effective fracture toughness.

In (23) and (32) it was accepted that the \( J \) field might be lost for very small amounts of growth on the grounds that COD had changed by the first infiltration measurement \((\Delta a = 0.2 \text{mm})\) from the value of \( \delta_i \) at initiation to the lesser value \( \delta_a \) at the advancing tip. It could now be argued from Shih's remarks (24) that the \( \delta \) measured was in the tip region and that a surrounding zone characterised by \( J \) still existed. However, the \( J_r \) curves derived were rationalised in terms of work rate, not characterising role, by definition. The simplest derivation is by differentiation of Eqn.1 to give

\[
\frac{dJ_r}{da} = \frac{d}{da} (\text{Eqn.1}) = \frac{n}{Bb} \frac{dw}{da} + \frac{J}{b} \left[ 1 + \frac{b}{h} \frac{dn}{da} \right]
\]

Eqn.7

It was originally disclaimed that this term had the \( J \)-contour integral meaning, except through Eqn.1, prior to initiation. This disclaimer now seems unnecessary
for amounts of growth such that \( \omega >> 1 \). It is not clear whether \( J_{eff} \) remains a useful measure beyond this restriction, and if so, whether it correlates better with \( dJ_0 \) than \( dJ_0' \), as these two terms diverge. Neglect of \( dn/da \) may be accepted for simplicity for deep-notch three-point bend where \( h = 2 \) for a range of \( a/W \), and if \( \omega >> 1 \) then \( J/b \) can be neglected. However, neither of these steps is inherent to the definition of the R-curve in terms of \( dJ_0 \). The question of whether the resulting R-curve is a material characteristic or is still geometry-dependent is not yet resolved and has no simple answer.

Three separate approximations or restrictions must be noted:

i) a \( J \) field will exist after initiation in plasticity only if \( \omega >> 1 \) (30)

ii) the apparent \( J \) measured in the conventional initiation test procedure, \( J_{app} \), may need correction for use after initiation.

\[
J_{app} \equiv (J_0 + dJ) = \frac{n}{Bb} (w + du)
\]  
Eqn.8a

or equivalently

\[
\frac{dJ_0}{da} = \frac{n}{Bb} \frac{du}{da}
\]
Eqn.8b

where \( n = 2 \) for deep notch three-point bend or the Corten-Merkle value (which is roughly 2) for compact tension.

Referring to Fig.4 where crack growth starts at B, \( w = \) area OBC, \( w + dw = \) area ODE, \( du = \) area BDEC. In strict J theory for non-linear elastic material area OBDO is recoverable but this is neglected in Eqn.8. If the net area ODE is used to define a corrected \( J \), \( J_{corr} \), then as shown elsewhere:

\[
J_{corr} = J_{app} \left(1 + \frac{\Delta a}{b} \frac{(1 - \eta + \frac{b}{n} \frac{dn}{da})}{\eta} \right) = J_{app} \left[1 + \frac{\Delta a f_i(\eta - \eta_i)}{b}\right]
\]  
Eqn.9a

If \( \eta \) is near invariant with crack depth as for the conventional deep notch test pieces

\[
J_{corr} = J_{app} \left(1 - \frac{\Delta a (\eta - 1)}{b}\right)
\]
Eqn.9b

If \( n = 2 \), as is closely so for the standard test pieces

\[
J_{corr} = J_{app} \left(1 - \frac{(\Delta a/b)}{2}\right)
\]
Eqn.9c

However, use of \( \omega >> 1 \) to ensure the existence of a characterising \( J \) field requires the neglect of terms in \( \Delta a/b \) in passing from the differential of Eqn.1 (Eqn.7) to Eqn.9, so that if the correction in Eqn.9c is significant, it appears that the characterising \( J \) field is lost. Use of \( J_{corr} \) as in Eqn.9c, or more completely, in Eqn.9a, corresponds to the recovery of area OBDO and is in agreement with the definition of \( J \) for use with crack growth as proposed by Garwood et al (37) and used (23) (33) without the restriction \( \omega >> 1 \).

iii) A choice must be made between use of the non-linear elastic model for which the rigorous J treatment is available, or of a plasticity model with elastic unloading for which a rigorous treatment is not available. This is discussed in the following.

Unstable crack growth

The theories. Although the several pictures of unstable crack growth that are emerging are by no means identical, there is a measure of agreement between the implications of several of the theories.

In the following distinction is shown between the use of J to characterise crack deformation (consistent with lefm usage of K, albeit expressed in units of KE$^2$) and the energetic use of J (consistent with lefm usage of G). Whereas in lefm the two usages are synonymous, in plasticity they are not, J having a characterising role (subject to the previous discussion of effect of stress state) and an energy dissipation rate meaning, but not that of an energy rate available, unless the strict non-linear elasticity model is taken.

If the existence of a J dominated field is accepted for $\omega >> 1$ then presumably the relevant measure of it is J, or specifically $J_{}\tau$, following the earlier discussion of (31). Thus, a characterising description of slow growth can be expressed in terms of J and by analogy with the use of K-R curves in lefm growth will occur once $J_{\tau}$ is exceeded when $(\partial J/\partial a)_{appl} \geq (d J/da)_{mat}$. Note $(\partial J/\partial a)_{appl}$ can be taken at constant load or deflection as appropriate to the problem. This usage of J as a characterising term, with no specific comment on the balance of energy rate, appears to be the basis of Garwood's method (25), with no explicit restriction to $\omega$.

Paris et al (22) formulated instability in terms of displacement by saying that unstable crack growth would occur if the rate of elastic constraction of the component with crack growth was greater than the growth of plastic opening at the crack. The change in elastic displacement on reduction in load is a function of the overall configuration including compliance and was expressed at limit load in non-dimensional form as an applied tearing severity, $T_{appl}$. The crack opening was related to J by Eqn.2, with m taken as unity so that the change in ODD was expressed as $d J/da$ with no intermediate argument on the meaning of the R-curve. Values of $(d J/da)_{mat}$ were taken from conventional test data that has here been called $d J/da_{o}$. The restriction $\omega >> 1$ was introduced to ensure the existence of a J field and hence validity of $d J/da_{o}$, although the basic model appears to be in terms of ODD rather than J.

Turner's picture of instability (23) stated that unstable growth would occur, if, after crack initiation, the elastic energy release rate at constant deflection
exceeded the total plastic work dissipation rate. The dissipation rate was expressed in terms of $dJ_r/da$ using the shape and size factors $\eta/Bb$. The elastic release rate was termed $I$ and shown to be $G < I < J$ with $I+G$ for lefm. The "constant deflection" was taken over the whole machine or structure to include the effects of compliance. The restriction $\omega >> 1$ was not made in the basic formulation, although application in (23) was restricted to that case. The physical model was based on plastic-loading-elastic-unloading behaviour, not non-linear elasticity, and the R-curve was used in an energetic sense as a means of normalising the work done rate as between one configuration and another.

For the present purpose four features emerge. In the terminology of Paris et al (22)

i) the recognition of $(E/\sigma _Y^2)(dJ/da)$ as the material tearing modulus. $T_{mat'}$ by implication based on the initial slope $dJ_o/da$;

ii) the recognition of a term, $T_{appl}$, for the applied severity of tearing deformation where (e.g.) for tension $T_{appl} \propto D/W$ ($D$ is overall length) and for bending $T_{appl} \propto b^2S/W^2$ ($S$ is span);

iii) the restriction of the theory to $\omega >> 1$ in order that a $J$-dominated field still exists around the crack;

iv) the value of the multiplicative constants of proportionality in the expression for $T_{appl}$ for any given configuration.

The tearing resistance, $T_{mat}$

The theories agree on the major feature of tearing resistance. The group $(E/\sigma _Y^2)(dJ/da)$ arises as the measure of crack growth resistance. Based on initial slope the values here called $dJ_o/da$ and implying a characterisation of the crack tip field in (22) and $dJ_r/da$, implying work dissipation rate in (23), are identical. An uncertainty on how invariant the R-curve is, even for the initial slope, is common to all theories. If a shear lip is eliminated and the R-curve found for flat fracture alone, it can perhaps be used as a lower bound in any approach.

There is general agreement that the R-curve is expressible as a function of crack growth $\Delta a$, hopefully independent of geometry. The further question then arises of whether the R-curve is a function of the degree of deformation since tearing may originate at $J$ in ssy for a large component, or perhaps near limit state for a small piece. It is clearly argued that if in both cases $\omega >> 1$, so that a $J$ field exists, then the initial value $(dJ/da)$ will be a material property, at least for the restricted case of flat fracture. It is also clear that if the terms $\eta$ and $Bb$ are indeed adequate normalising factors then the whole $J$ curve will be unique, again for the flat fracture component, irrespective of the value of $\omega$. Intuition would suggest that this is most likely to be so where $\eta$ is invariant with degree of
deformation. Whilst the $J$ field exists a unique relationship is also implied between $\delta$ and $J$ and $d\delta$ and $dJ$. For a given material, but such a relationship must be restricted to the cases already discussed under effect of stress state for which $J$ or $\delta$ is a realistic term. In a general statement of $T_{\text{mat}}$ (when $\omega \rightarrow 1$) clearly other terms arise, according to the assumptions made. The term $OBDQ$, Fig.4, equals $-BJda$ so that if recoverable $dw = du - BJa$. Eqn.10

If elastic-plastic behaviour is followed it was argued (23) that the term defined as $dJ_{f}$ included an increase in $G$ since the $R$-curve is formulated at constant (slightly rising) load. This term is the linear elastic rate $BGda$ corresponding to the non-linear rate $BJda$ that distinguishes $dw$ from $du$. The logic for subtracting this term is that in an energy rate picture of instability at constant displacement only dissipative terms are required. In a characterising argument it could be reasoned that this term should not be subtracted. If the term $BJda$ or $BGda$ is negligible then $\frac{dw}{du}$ Eqn.11 and the characterising and energetic arguments are again the same. This is discussed below.

Of these various uncertainties the effect of anisotropy and of shear lip probably dominate, although the effect of stress field (bending or tension) could be significant for isotropic materials showing little shear lip. A better model thumbnail cracks growing into the thickness or the already full thickness crack spreading laterally, Garwood (25) recommends finding the $R$-curve in the relevant test situation, also using the stress field of interest, if possible. Clearly, if all features of size, configuration, loading etc., have to be modelled rather closely, much of the use of an analytical model is lost, although the need for close simulation is a common requirement of other sub-critical crack growth studies such as fatigue or corrosion and may have to be faced if directly quantitative rather than conservative qualitative assessments are sought for monotonic growth.

The tearing severity $T_{\text{appl}}$

To assess $T_{\text{appl}}$ characterisation by a $J$ stress field must be accepted or rejected. Expressions for $\frac{\partial J}{\partial a}$ in terms of $n$ were given (34) for a power law relationship between $Q$ and displacement $q$, i.e. $Q = q^n$, where $\psi$ is a compliance function, $\psi(a/W)$, independent of the degree of loading and $n$ $(0 < n < 1)$ is not a function of $a$,

$$\frac{\partial J}{\partial a} \bigg|_q = \frac{J}{b} \left(1 + n \frac{b}{n} \frac{dn}{da} \right) \equiv \frac{J}{b} \left(f_{\text{q}}(\eta) + \eta \right)$$ Eqn.12a

$$\frac{\partial J}{\partial a} \bigg|_q = \frac{J}{b} \left(1 - n \frac{b}{n} \frac{dn}{da} \right) \equiv \frac{J}{b} \left(f_{\text{q}}(\eta) - \eta \right)$$ Eqn.12b
The circumstance for the existence of \( \eta \) independent of degree of deformation were shown (38) to be that the variables of deformation and configuration were separable. When that is satisfied then clearly \( \partial \psi / \partial a \big|_q \) or \( \partial \psi / \partial a \big|_q \) are proportional to \( J / \delta \) with the same term that relates \( \partial G / \partial a \) to \( G \) in lefm since the result is independent of \( n \).

The physical picture on which an energetic assessment of \( T_{\text{appl}} \) is based must also distinguish between either \( J \) for loading and unloading, i.e., a true non-linear elastic behaviour (nle) or non-linear loading but linear unloading. In the latter \( J \) can be accepted for elastic-plastic loading on the arguments already discussed for use prior to initiation, together with the restriction \( \omega >> 1 \), but the linear unloading must be treated via lefm. For brevity this model is referred to as elastic-plastic-elastic (epe).

Considering first the ideal cases in which nle is described by the power-law load deflection diagram, and recalling the independence of \( \eta \) on \( a / W \) for \( \eta \) to exist independent of deformation, then using the terminology I for energy release rate for all cases, then in the limiting case of no plasticity

\[
I = J \quad \text{Eqn. 13a}
\]

The energy release rate for the epe model was stated (23) for the limit load case. For any general load \( Q \) it is

\[
I = G - q_{el} \left( (\eta_o - \eta_{el}) / \eta_o \right) \left( \partial G / \partial a \right) \quad \text{Eqn. 14}
\]

where \( \eta_o \) is the overall value of \( \eta \) that must depend on the model used, i.e., for limit state plasticity \( \eta_o = \eta_{pl} \) (as in (23)); for a nle model of loading \( \eta_o = \eta_{nl} \) as in J theory; for elastic-plastic behaviour \( \eta_o \) can be computed from overall values of \( J \) and work, or compounded from elastic and plastic components as discussed (34).

For the epe model with \( J \) accepted for loading, then \( \eta_{nl} \equiv \eta_{el} \) since the compliance function \( \psi (a / W) \) is in this ideal model independent of \( n \) and hence equal to the lefm compliance. Thus,

\[
I = G \quad \text{Eqn. 15}
\]

For "real material" several departures from these ideal cases can be seen. Total theory plasticity (following nle) may not be regarded as adequate even for loading although it is usually accepted for monotonic crack cases. \( J \) may be judged inadequate for certain cases as discussed under Stress State. Heuristically this might imply the variables were not separable in the load displacement model as plasticity became uncontained. A \( J \) field may be lost as crack growth exceeds \( \omega >> 1 \). For any such case \( \eta_o \neq \eta_{el} \) (specifically at limit state in tension \( \eta_{pl} \neq \eta_{el} \) although in bending there is close agreement) so that in general

\[
G \leq I \leq J \quad \text{Eqn. 16}
\]
For contained yield \( G \) corrected for size of plastic zone, here denoted \( G_p \), is a good estimate of \( J \). Thus,

\[
\text{Eqn.13b}
\]

However, for epe based on the above ideal mode, Eqn.15 holds any degree of yielding be it contained or extensive. The implication on lefm usage of R-curves is discussed elsewhere.

**Discussion**

**Non-linear elasticity or plasticity?**

If non-linear elasticity is to be a useful measure of plastic dissipation then the recovery or otherwise of area OBDO (Fig.4) must be irrelevant and this requires \( dw = du \). By differentiation of Eqn.1

\[
\frac{dw}{da} = BJ \frac{b \frac{dJ}{da} - b \frac{dn}{da}}{n} - 1 \text{ Eqn.17a}
\]

\[
= BJ \frac{\omega - f_1(n)}{n} \text{ Eqn.17b}
\]

Combining with Eqn.10

\[
\frac{du}{da} = BJ \left( \omega - f_1(n) + n \right) \text{ Eqn.18}
\]

If

\[
\omega - f_1(n) >> n \text{ Eqn.19}
\]

then

\[
dU = dw \text{ and } dw \text{ (nle)} = dw \text{ (epe)} \text{ Eqn.20}
\]

For all the data so far gathered on \( \eta \) (e.g. (34), \( \eta \leq 1 \) or 2 and \( dn/da \equiv 0 (\eta/a) \) for deep notch pieces and \( \eta \approx 1 \) for shallow notch pieces, so that \( f_1(\eta) = 0 (b/a) \) and satisfaction of \( \omega >> 1 \) to ensure a \( J \) field will ensure Eqn.20 unless \( b >> a \). Thus, for deep notches if

\[
\omega >> 1 \text{ and } \omega >> f_1(\eta) \text{ and } \omega >> \eta \text{ Eqn.21}
\]

then

\[
dU = dw = BJ \omega/\eta = (b \omega/\eta) (dJ/da) \text{ Eqn.22}
\]

For deep notches \( f_1(\eta) = 2 \) and \( \eta \) is typically 1 or 2 so that the requirement for Eqn.22 is, say, \( \omega >> 2 \).

For shallow notches even if \( \omega >> 1 \) it is possible that \( \omega \) is comparable to \( f_1(\eta) \) which is of order \( b/a \) (say 10 for \( a/W = 0.1 \)). In that case \( f_1(\eta) >> \eta \) so that

\[
dw = du \text{ as in Eqn.20, but Eqn.22 is not a good estimate unless } \omega >> b/a. \text{ For the power law load-deflection curve } dw \text{ and } du \text{ can be particularised to}
\]

\[
dw = BJ/n \quad du = BJ(n+1)/n \text{ Eqn.23a.b}
\]

so that if \( dw \) and \( du \) are to be nearly equal then \( 1/n >> 1 \). Note, since \( n \) here describes the overall load deflection diagram it is strongly influenced by configuration and type of loading as well as by hardening. For \( n \) small a limit state type behaviour is implied. Combining Eqs. 22 and 23
\[ \omega = \eta/n \quad \text{with} \quad n > 1 \quad \text{and} \quad \omega > 1 \quad \text{Eqn.24} \]

This expression combines the geometric factor \( \eta \) with the load-displacement index \( n \) and must be satisfied if both characterizing and energetic meanings of \( J \) are to be relevant to plasticity. This condition is easily satisfied for, say, deep notch bending near the limit state (e.g. \( n = 0.1, \eta = 2 \), so that \( \omega = 20 \)) but not for shallow notch cases and/or high hardening for which a representative value of \( \omega \) might be only 2 or even less than 1 (e.g. \( n = 0.3, \eta = 1 - n \) for limit state deep notch tension with hardening, so that \( \omega = 2.3 \), or \( \eta = 0.2 \) for shallow notch contained yield tension, so \( \omega = 0.7 \)).

Another circumstance when \( n \leq n \) is a good estimate of \( \varepsilon \) is when
\[ d\omega (n \leq n) = d\varepsilon U - B\delta a \neq d\varepsilon U - B\varepsilon a \neq d\omega (\varepsilon) \quad \text{Eqn.25} \]

This occurs either for \( n \geq 1 \) or, on the guidance of pre-initiation studies, when yield is well-contained, in which case \( G = J \), or better \( G_p = J \), even for non-hardening material. If the R-curve is indeed expressed for the net term \( d\omega \) as in the definition of \( J \) (35) or \( J \) _corr _ (Eqn.9) then the subtraction of a linear elastic term \( G \) as suggested (23) now seems appropriate. The further distinction between whether the recovered term is \( B\delta a \) (in \( n \leq n \)) or \( B\varepsilon a \) (i.e. \( \varepsilon \)) seems of little import whilst the terms are similar, within contained yield. Thus, near equality of \( d\omega (n \leq n) \) and \( d\omega (\varepsilon) \) via Eqn.25 rather than by Eqn.20, seems to imply that either theory is an adequate measure of release rate, even though the restriction \( \omega > 1 \) is not imposed to ensure the existence of a \( J \) field. In short, for \( n \leq n \), \( J \) is relevant irrespective of \( \omega \) and for \( \varepsilon \) the release rate is assessed adequately by \( n \leq n \) whilst Eqn.25 is satisfied by contained yield.

The heuristic view is therefore suggested that in contained yield, where \( J \approx G \), the restriction \( \omega > 1 \) is unnecessary since the \( n \leq n \) and \( \varepsilon \) models do not differ significantly (via Eqn.25) but for uncontained yield the restriction to \( \omega > 1 \) (or better, \( \omega > f_1(\eta) \)) ensures that even when \( J \) exceeds \( G \), perhaps by a large amount, the energetic meaning is retained (via Eqn.20). Moreover, as \( J \) exceeds \( G \), the idealised model suggests that \( I+G \) is required as the measure of energy rate even when \( \omega > 1 \), so that loss of \( J \) theory as a means of estimating \( \varepsilon \) appl can be accepted.

The existence of a unique R-curve (at least for flat fracture in a given direction of propagation) is still unproven but seems more likely where a unique stress state, (i.e. as defined by \( J \)) is known to exist. Clearly, \( \eta, B\delta \), and \( f_1(\eta) \) provide first order terms for normalising the work rate with respect to configuration and loading. In so far as they may not be adequate they reflect the doubts existing on relevance of \( J \) theory for initiation to cases that cause low constraint even in plane strain, which must first be resolved.
Instability

A bounding statement of instability during a test is that it must occur if dw becomes negative. This is tantamount to saying that for instability BI > dw/da and then allowing I to be neglected so that O > dw/da. If dw is to be negative, then \( \omega < f_1(\eta) \). Estimates have been given for \( f_1(\eta) \) as 2 or 3 for deep notches and b/a for shallow. Shallow notches thus appear the more demanding once initiation has occurred. Note \( \omega >> 1 \) automatically provides stability in this simple argument, except for \( b >> a \), if \( w_{app} \) is assessed during the test in question. If \( \omega_{\text{fract}} \) is evaluated from material property data then the driving force I or \( T_{\text{appl}} \) cannot be neglected, of course.

The use of the first derivative of energy, I, in a statement of instability may be questioned. It was restricted (23) by evaluation at constant displacement (i.e. a bounding statement in the sense that stable growth cannot be maintained beyond that point but may occur before it when there is a "follow up" load system) and made realistic rather than bounding by use of an effective length to allow for compliance. A convex R-curve was assumed and cases avoided where I might decrease with crack growth (such as crack line loading). The first derivative treatment in effect states that for unstable growth the total energy rate available (G in lefm or I in plasticity) is equal to or greater than an effective toughness, whereas an equality of the derivative of energy rate, \( dG/da \), is required when the first order term (corresponding to the initiation toughness) is cancelled out to leave a balance of increments (or rates) that controls growth. The niceties of calculating G or I to allow for crack growth and locating an effective R-curve at an origin \( a+\Delta a \) rather than just at \( a \) is at the discretion of the user in making an adequate estimate.

In short, writing \( G > G_{\text{eff}} \) or \( I > J_{\text{eff}} \) models the rising R-curve by a size related falling curve of effective crack growth resistance, Fig.5, to which a value \( G_{\text{eff}} \) or \( J_{\text{eff}} \) can be attached for comparison with G or I, just as at initiation.

The effective toughness \( J_{\text{eff}} \) is not, however, a real value of J but by definition a term that equals the actual dissipation rate \( dw/Bda \).

\[
J_{\text{eff}} = \frac{dw}{Bda} = \frac{J (b \frac{dJ}{da} - f_1(\eta))}{\eta \frac{J}{da}} \quad \text{Eqn.26a}
\]

\[
\text{for } \omega >> f_1(\eta) \quad \text{Eqn.26b}
\]

As already noted for deep notch cases \( \omega >> f_1(\eta) \) is often satisfied by \( \omega >> 1 \) and the definition of \( dJ \) in the relationship is strictly not \( dJ \) but \( dJ \). For high toughness materials \( J_{\text{eff}} \) starts much larger than the "real" or characterising J, since the total plastic dissipation is being simulated by an effective toughness term. \( J_{\text{eff}} \) is size-dependent, through b, and shape dependent, through \( \eta \), so that a family of curves
exist for $J_{\text{eff}}$. To predict the actual instability iteration is implied to match $dJ$ and $\Delta a$ values, because $dJ/da_{\text{mat}}$ reduces with crack growth, whereas $f dJ/da$ were in fact constant, an estimate could be based on this initial value. If all the relevant terms are estimated precisely, and if $J$ remained a characterising term for the amount of growth in question, the second derivative $dJ$ tangency method and the first derivative $J_{\text{eff}}$ method would predict the same answer, as at point $X$, Fig.5. As the $J$ field is lost when $\omega > 1$, it is open to discussion whether inclusion of $f_1(\eta)$ as in Turner's formulation (23) is adequate to allow a reasonable assessment of instability, or whether a lack of relevance to $J$ implies that a normalised $R$-curve does not exist for large amounts of crack growth and/or uncontained plasticity.

The term $b/\eta$ is seen as a measure of the "filter factor" in Broberg's arguments (19) or of the process zone of magnitude comparable to the maximum extent of plastic zone introduced in Rice's analysis (37).

Conclusions
1) The $J$ contour integral method is seem as the most embracing of current fracture theories for crack initiation in plane strain, even though probably not relevant to circumstances where uncontained yield leads to low constraint.

2) Stable ductile crack growth can be examined in terms of $J$ either by considering $J$ as a characterising parameter or by regarding a $J$-$R$ curve as a normalised measure of energy dissipation rate.

3) It is not yet clear which aspect of $J$ is most important:
   a) the existence of a unique $J$ field which (in so far as $J$ is valid for plasticity materials at all) allows $J$ to characterise the elastic-plastic case, implying that a unique material response will exist;
   b) the use of $J$ theory to evaluate terms such as energy dissipation or release rate, for elastic-plastic material with linear unloading ($e_{\text{pe}}$).

4) For $\omega > 1$ (or better $\omega > f_1(\eta)$) the characterising and energetic roles of $J$ are both satisfied for $e_{\text{pe}}$ material. The present study suggests that a $J$ description of non-linear elastic material ($e_{\text{ne}}$) is adequate for $e_{\text{pe}}$ in a) uncontained yield with $\omega > f_1(\eta)$; b) contained yield.

5) As a guide to the meaning of $\omega > 1$ the term $\omega > f_1(\eta)$ is offered where $f_1(\eta) = 2$ for deep notch bending and $f_1(\eta) = b/a$ for shallow notch cases.

6) The statement of instability in terms of energy release rate $I$ and total dissipation rate is equivalent to taking an effective toughness combining the material tearing resistance, $dJ/da$, and the size and shape factors $\eta$, $B_b$ and $f_1(\eta)$. 
As a direct guide to instability it is satisfactory in the sense that if
\( nEI/bq_0^2 \ll T_{\text{mat}} \) stability is assured for no crack growth, but since \( T_{\text{mat}} \) reduces with crack growth, this first assessment of instability could be misleading. An iterative procedure is required where \( I \) increases with load and \( T_{\text{mat}} \) decreases with crack growth until equality is reached. This equality is exactly equivalent to the R-curve tangency model of instability, in which \( R \) has an energetic meaning.

7) Subject to the chosen neglect or approximation of various terms, perhaps for simplicity, perhaps because of assessment as of second order for a particular application, the three J based descriptions of unstable, ductile crack growth (22), (23), (25) reduce to the same result for \( \omega \) small. For uncontained yield with \( \omega \) not small, considerable differences could arise between the characterising and energetic viewpoints and between the acceptance of a nle or epe model of material behaviour.

Acknowledgments

The writer gratefully acknowledges many helpful discussions with colleagues, notably Professor P.C.Paris, Dr.H.Ernst, (Washington University, St.Louis) and Dr.S.J.Garwood (Welding Institute, U.K) none of whom necessarily agrees with some parts of the views expressed.


Fig. 1  Comparison of various theories using the implied value of J/G as a measure of the relative severity of the plastic problem as a function of applied loading. After Ref.2.

Computed J results (plane strain): A, P, H₁, H₂
Other theories, D, DH, (1/φ)₂, SCT

Key:
A  Centre cracked plate in tension
P  Thick walled cylinder with radial crack
H₁  Crack buried beneath a hole in a plate in tension
H₂  Crack at the edge of a hole in a plate in tension
D  Dugdale ln sec formula as in the Two Criteria method
DH Dugdale ln sec formula applied to the local stress 3σ at a hole
(1/φ)² Newman Two Parameter method (schematic): n increasing ↑
     decreasing ↓
SCT  Irvine Elasto-plastic Stress Concentration Theory

Fig. 2  J and COD design curves for rule of thumb use. After Ref.8b

Fig. 3  dJ/da and implied J-R curve (schematic, after Ref.24)

Fig. 4  Increments of plastic and elastic work

Fig. 5  Assessment of instability by an effective toughness that decreases with crack growth (Flat fracture data from Ref. 32)
J_{eff}(1) is based on Eqn. 26a. J_{eff}(2) is based on Eqn.26b, for b=5mm.
For J_{apl} the slopes J/ a Q are estimated from Eqn. 12a neglecting crack growth and suggest instability at constant load at about a=0.5mm
by the tangent construction method in agreement with point X.
(Note curves of J_{eff} and hence point X are size dependent whereas the R-curve is conceptually size independent).
FIRST YIELD OF:
PLATE CYLINDER

REMOT StRESS $\sigma/\sigma_Y$ (P/\sigma_Y FOR CYLINDRER)
-97-

**COD METHOD**

**J METHOD WITH**

ALLOWANCE FOR

YIELD LEVEL

RESIDUAL STRESS

PRESENT \(J\)

DESIGN CURVE

COD DESIGN

REGION IN TERMS

OF \(J\) USING \(1 \leq m \leq 2\)

\[
\frac{J E}{\sigma Y^2 a}
\]

\(m = 2\)

\(m = 1\)

\(m = 2\)

\(m = 1\)

STRAIN IN THE UNCRACKED BODY \(e/e_Y\)

FIG. 2
A technique is presented which allows the evaluation of crack growth resistance data, $J_R$ or $K_R$, knowing only the initial and final crack sizes in a test, the load at the end of a test and the non-linear displacements recorded during the test. The technique is based upon the curve fitting procedures of Chell and Milne, 1976, and involves only a simple graphical interpolation for the analysis. Advantages over other techniques include obviating the need to identify when cracking initiates and having to measure the area under the load displacement curve. Thus many sources of error are eliminated. At the same time the effects on the resultant $J_R$ or $K_R$ of other influences, such as indentation due to loading pins, can be very easily evaluated.

Two examples are presented to illustrate the use of the technique.
1. INTRODUCTION

A number of procedures have recently been proposed to determine the instability conditions of structures containing cracks which under monotonic loading grow by ductile mechanisms. (For example, Paris, Tada, Zahoor and Ernst, 1979, Turner 1979, Milne 1979). In these methods the structural materials resistance to crack growth is characterised by a J-resistance curve (J_R -curve) where J is determined assuming non-linear-elastic behaviour i.e. its value is given by the instantaneous values of the applied load and crack length and is not dependent on load history. For the two most common specimen test geometries, compact tension (CT) and three point bend (3PB), the usual methods for obtaining J at the initiation of crack growth use the area under the load displacement curve (Witt 1970, Sumpter and Turner, 1976, Rice, Paris and Merkle, 1973, Merkle and Corten, 1974). If crack extension occurs during a fracture test, J_R is no longer simply related to the total area under the load displacement curve, and correctional procedures should be applied (Hutchinson and Paris, 1979, Garwood, Robinson and Turner, 1975, Burns and McMeeking 1978).

In this paper a simple alternative technique for obtaining J_R is presented based on an adaptation of a method for analysing elastic-plastic fracture data suggested by Chell and Milne(1976). This procedure does not require the area under the load displacement record, but uses instead the instantaneous value of the load, crack length and displacement. In the procedures the crack growth resistance parameter, K_R, is evaluated where K_R = (E'J_R)^2 and E' is Young's modulus, E, for plane stress and E/(1-v^2) in plane strain, where v is Poisson's ratio.
2. **THE METHOD**

The loading pin displacement $\delta$ can be written as (Chell and Milne, 1976)

$$\delta = \lambda_0 P + B \int_0^a \frac{\partial}{\partial P} \int_0^a J \, da$$  \hspace{1cm} (1)

where $P$ is the applied load, $\lambda_0$ the uncracked specimen compliance, $B$ the specimen thickness and $a$ the crack length. If a functional form for $J$ is known which contains one unknown parameter, equation (1) may be used to determine the parameter from experimentally measured displacements. This is the principle underlying the Chell and Milne method. They considered the following form for $J$

$$J = \frac{8}{\pi^2 E} a Y^2 \sigma_1^2 \ln \sec \left( \frac{\pi \sigma}{2 \sigma_1} \right)$$  \hspace{1cm} (2)

where $Y$ is a geometric term such that the stress intensity factor $K$ is given by

$$K = \sigma \, a^\frac{1}{2} \, Y,$$  \hspace{1cm} (3)

$\sigma$ is the applied stress and $\sigma_1$ is the plastic collapse stress.

For CT and 3PB specimens, $\sigma_1$ is approximately given by

$$\sigma_1 = \alpha \, \bar{\sigma} (1-a/w)^2$$  \hspace{1cm} (4)

where $\alpha$ is a dimensionless constant, which can be taken as 0.5 and 2 for CT and 3PB specimens respectively, and $\bar{\sigma}$ is a flow stress. Following Chell and Milne we take $\bar{\sigma}$ as the unknown parameter.

Substituting $J$ from equation (2) into equation (1) yields

$$\delta = \lambda_0 \frac{Bw^2}{A} + \frac{4Aa\bar{\sigma}}{\pi w E} \int_0^a a Y^2 (1-a/w)^2 \tan \left( \frac{\pi \sigma}{2 \sigma_1 (1-a/w)^2} \right) \, da$$  \hspace{1cm} (5)

where $\sigma = \frac{AP}{Bw}$.
and \( w \) is the specimen width. \( A \) is a dimensionless geometric constant equal to 1 for CT and 6\( S/4w \) for 3PB specimens where \( S \) is the span. The elastic component of displacement, \( \delta_e \), is given by

\[
\delta_e = \frac{\sigma}{E} w \int \frac{a}{w} F(a/w)
\]

where

\[
F\left(\frac{a}{w}\right) = \frac{B\lambda}{A} + \frac{2A}{w^2} \int_0^a a Y^2 \; da
\]

so that only the non-linear component of displacement need be measured for the evaluation of \( \bar{\sigma} \). This facilitates the evaluation of \( J \) in tests where the machine displacement is recorded in preference to the loading pin displacement. Here \( \bar{\sigma} \) can be obtained by equating the measured non-linear displacement to \( \delta - \delta_e \) with these determined from equations (5) and (6).

3. **THE PROCEDURE**

Normalised load displacement curves are plotted in Figs. 1 and 2 for CT and 3PB specimen geometries for a range of crack sizes. These were computed from equation (5). Also shown in these figures are values of \( K_R = K_R/\bar{\sigma}^{\frac{1}{2}} \) plotted against normalised displacement where

\[
K_R = \left( E'J \right)^{\frac{1}{2}}
\]

and \( J \) was evaluated from equation (2). In Figs. 3 and 4 the elastic compliance term, \( F(a/w) \), is shown for the two geometries. This information, together with experimentally measured load displacement curves and crack sizes before and after a test, provide a simple and quick means of evaluating the value of the crack growth resistance parameter \( J_R \) and hence \( K_R \) from equation (7). The procedure is as follows.

1. From the fracture surface of the specimen measure the initial crack size \( a_0 \) and the final crack size \( a_f \) using whichever recommended technique is preferred.
2. From the load displacement curve obtained in the test, measure the final load, \( P_{f'} \), and the non-linear displacement up to this point, \( \delta_{nl} \).

3. By means of Fig. 3 or 4 evaluate \( \delta^o_e \), the linear displacement of the specimen containing the original crack \( a_o \), at the final load.

4. Choose an arbitrary value of \( \bar{\sigma} \) and evaluate the normalised stress, \( \bar{\sigma}^* \) and displacement, \( \bar{\delta}^* \), at failure as
   \[
   \bar{\sigma}^* = \frac{\sigma_f}{\bar{\sigma}}
   \]
   \[
   \bar{\delta}^* = \frac{\Delta}{\omega \bar{\sigma}}
   \]
   where
   \[
   \Delta = \delta^o_e + \delta_{nl}
   \]
   and \( \sigma_f \) is the stress corresponding to \( P_{f'} \).

5. Plot this point as a coordinate \((\bar{\delta}^*, \bar{\sigma}^*) \) on Fig. 1 or 2 as appropriate and draw a straight line through this point to the origin.

6. The unknown parameter, \( \bar{\sigma} \), is then given by \( \sigma_f / \sigma_i^* \), where \( \sigma_i^* \) is the value of the normalised stress at which the straight line intersects the normalised load displacement curve relating to \( a_f / \omega \).

7. If \( \delta_{nl}^* \) is the value of the normalised displacement at the intersection, read off the relevant value of \( K_R^* \) from the curves in the upper half of Fig. 1 or 2 and evaluate \( K_R \) from
   \[
   K_R = \sigma_i^{1/2} K_R^*
   \]
   Alternatively, from equation (2),
   \[
   E' J = K_R^2 = K_f^2 \frac{8}{\pi^2 S_i^2} \ln \sec \left( \frac{\pi}{2} S_i \right)
   \]
   where
   \[
   S_i = \sigma_i^*/\omega (1-a_f/\omega)^2
   \]
   and \( K_f \) is the linear elastic stress intensity factor evaluated for \( \sigma_f \) and \( a_f \).
4. **EXAMPLES**

4.1 50mm CT specimen of A533B

\[
\begin{align*}
\frac{a_o}{w} &= 0.508 \\
\frac{a_f}{w} &= 0.519 \\
\delta_{nl} &= 2.2 \text{mm} \\
P_f &= 267.5 \text{kN} \\
E' &= 200 \text{GPa}
\end{align*}
\]

From Fig. 3, \( F(0.508) = 41 \) and hence

\[
\delta^0_e = \frac{267.5 \times 10^{-3}}{0.05 \times 200 \times 10^{-3}} \times 41 = 1.10 \text{mm}
\]

Therefore \( \Delta = 3.3 \text{mm} \)

and so taking \( \bar{\sigma} \) arbitrarily as 400 MPa

\[
\delta^* = \frac{3.3 \times 100}{0.1 \times 400} = 16.5
\]

and

\[
\sigma^* = \frac{267.5 \times 10^{-3}}{0.05 \times 0.1 \times 400} = 0.1338
\]

This point is marked as a cross on Fig. 5a. The straight line connecting this to the origin intersects the normalised load displacement curve for \( \frac{a}{w} = 0.52 \) at point A with coordinates \( \delta^*_1 = 14.1, \sigma^*_1 = 0.115 \).

Therefore

\[
\bar{\sigma} = \frac{0.1338}{0.115} \times 400 = 465 \text{MPa}
\]

The value of \( K_R^* \) at the point A" (Fig. 5a) corresponding to \( \delta^*_1 = 14.1 \) is 2.35 and hence \( K_R = 465 \times (0.1)^{\frac{1}{2}} \times 2.35 = 345 \text{MPa} \)

4.2 50mm square Section 3PB specimen of weld metal

\[
\begin{align*}
S &= 4w \\
\frac{a_o}{w} &= 0.529 \\
\frac{a_f}{w} &= 0.534 \\
\delta_{nl} &= 1.35 \text{mm} \\
P_f &= 111.9 \text{kN} \\
E' &= 200 \text{GPa}
\end{align*}
\]
From Fig. 4, \( F(0.529) = 10.7 \) and hence

\[
\delta^0_e = \frac{6 \times 111.9 \times 4 \times 0.05}{4 \times (0.05)^2 \times 200 \times 10^6} = 0.72 \text{mm}
\]

so \( \Delta = 2.07 \text{mm} \).

Taking \( \sigma = 600 \text{MPa} \) gives

\[
\delta^* = 13.8
\]

and \( \sigma^* = 0.448 \)

This point is marked by a cross on Fig. 5b. The coordinates of the intersection of the straight line through through this point and the origin with the normalised load displacement curve for \( a_e/N = 0.534 \) are obtained by simple interpolation to lie at A and are \( \delta^*_1 = 13.3 \), \( \sigma^*_1 = 0.43 \) and

\[
\sigma^* = \frac{0.448}{0.43} \times 600 = 625 \text{MPa}.
\]

The value of \( K^*_R \) at the point A' obtained as in the previous example is 1.75 and hence

\[
K^*_R = 625 \times (0.05)^{1/2} \times 1.75 = 245 \text{MPa m}^{1/2}.
\]

5. **DISCUSSION**

The method proposed has previously been shown to agree with results obtained from other frequently quoted methods (Chell, 1976). It has however the following advantages over these

1. The method is simpler and quicker to use than that proposed by Garwood, Robinson and Turner, 1975, for estimating \( J^*_R \). Their procedure corrects for crack growth by dividing this into a number of incremental steps and then successively applying a formula to each step to evaluate the instantaneous value of \( J^*_R \). It should be noted that whereas this method requires identification of the crack growth initiation point on the load displacement curve, the present technique does not.
(2) Since \( \sigma \) is determined using only non-linear displacements the method avoids errors resulting from unwanted or unquantifiable elastic displacements. These would increase the measured area under the load displacement curve and hence overestimate the predicted toughness based upon the techniques of Rice, Paris and Merkle, 1973, Sumpter and Turner 1976 and Merkle and Corten (1974) and underestimate the toughness if the equivalent energy method of Witt (1970) was used. The alternative ways of dealing with this problem are either by calibration, or measuring displacements directly across the loading pins. Neither of these are entirely satisfactory, especially at non-ambient temperatures.

(3) The effects of erroneous non-linear displacements can be readily evaluated. Such errors can result for example, from indentations at the loading pins and can be corrected for by calibrating the indentation against load or by measuring the indentation after the test, and subtracting the necessary quantity from the non-linear displacement measured in the test.

6. CONCLUSIONS

The curve fitting technique proposed by Chell and Milne (1976) for obtaining \( J \) from a load displacement record has been adapted into a form suitable for obtaining \( K_R \) or \( J_R \) values quickly and reliably from a knowledge of the initial and final crack lengths measured after testing. The method has the advantage over others in that neither the initiation of crack growth, nor the area under the load displacement curve, need be determined for its use.

ACKNOWLEDGEMENT

The work was carried out at the Central Electricity Research Laboratories and the paper is published by permission of the Central Electricity Generating Board.
REFERENCES


Carnegie-Mellon University.
FIG. 1 NORMALISED LOAD DISPLACEMENT CURVES FOR COMPACT TENSION SPECIMENS.
THE DASHED PART OF THE CURVES ARE EXTRAPOLATED.
FIG. 2  NORMALISED LOAD DISPLACEMENT CURVES FOR 3 PT BAND SPECIMENS WITH S = 4W
THE DASHED PART OF THE CURVES ARE EXTRAPOLATED.
TOTAL ELASTIC DISPLACEMENT OF SPECIMEN, $\delta_e$, AT LOAD P
IS GIVEN BY

$$\delta_e = \frac{P}{BE'} F \left( \frac{a}{w} \right)$$

FIG. 3 ELASTIC COMPLIANCE CALIBRATION FACTOR, $F(a/w)$, FOR COMPACT TENSION
TOTAL ELASTIC DISPLACEMENT OF SPECIMEN, $\delta_e$, AT LOAD $P$ IS GIVEN BY

$$\delta_e = \frac{6P}{BE'} F\left(\frac{2a}{w}\right)$$

FIG. 4 ELASTIC COMPLIANCE CALIBRATION FACTOR, $F(a/w)$.

3 PT BEND WITH $S = 4W$
\[ \delta^* = \frac{E\Delta}{\bar{\sigma}W} \]

FIG. 5b CONSTRUCTION REQUIRED TO EVALUATE \( K_R \) FOR EXAMPLE 4.2
STABLE CRACK GROWTH ESTIMATES BASED ON EFFECTIVE CRACK LENGTH AND CRACK-OPENING DISPLACEMENT*

J. G. Merkle
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37830

C. E. Hudson†
Auburn University
Auburn, Alabama

ABSTRACT

A method has been developed for estimating the amount of stable crack growth that has occurred in a fracture toughness specimen that has been loaded into the plastic range and for which only a monotonically increasing load-displacement curve has been measured. The method has been applied to data from several pressure vessel steels. The resulting \( J \) vs \( \Delta a \) values compare favorably with a resistance curve obtained by the multiple specimen heat-tinting technique for A533, Grade B, Class I steel. The method for estimating stable crack growth uses several existing concepts heretofore mainly used separately. These concepts include an approximate expression for \( J \) for the compact specimen proposed by Andrews, the effective crack length concept of McCabe and Landes, the UK representation of the crack profile as a pair of straight lines intersecting at a hinge point, and Wett's expression, \( J = m_0 \delta \), for relating the crack-opening displacement to the value of \( J \). The value of the constraint factor, \( m \), at the advancing crack tip is estimated by means of a relation between ductility and fracture toughness. When calculated with respect to the COD at the original fatigue crack tip, the constraint factor, \( m_0 \), is found to have a value consistently close to 2.0 for compact and


†Work performed during Co-op assignment in the Metals and Ceramics Division, Oak Ridge National Laboratory.

By acceptance of this article, the publisher or recipient acknowledges the U.S. Government's right to retain a nonexclusive, royalty-free license in and to any copyright covering the article.
precracked Charpy specimens. The method of estimation requires no auxiliary load-deflection measurements or calculations, and so permits single specimen estimates of stable crack growth to be made without the necessity of making high precision unloading compliance measurements.

INTRODUCTION

The useful application of fracture mechanics to the safety analysis of nuclear pressure vessels has been considerably enhanced by the development of elastic-plastic methods for measuring high values of fracture toughness with small specimens,¹⁻³ and by the high safety margins demonstrated by the intermediate pressure vessel tests conducted by the HSST Program.⁴ Nevertheless, two aspects of these results still require additional clarification before standard methods of elastic-plastic fracture toughness measurement and flaw evaluation can be considered appropriate. The first aspect is the tendency for maximum load fracture toughness values in the upper shelf temperature range to increase with increasing specimen size. The second aspect is the occurrence of relatively large amounts of stable crack growth before failure in the upper shelf intermediate pressure vessel tests. In addition, because of space limitations and other size-related problems, only small specimens can be used as irradiation surveillance specimens in reactor pressure vessels. Consequently, there is no practical alternative to the use of small specimens for measuring irradiated fracture toughness values. Furthermore, in order to justify the use of such values in fracture safety analyses, it is first necessary to explain their physical basis and also to develop procedures for using them analytically that do not become unconservative.

The physical basis for the occurrence of maximum load toughness values that increase with increasing specimen size was shown by Griffis⁵ to be the amount of stable crack growth that occurs prior to maximum load. Griffis'⁵ data showed that the absolute amount of stable crack growth occurring at maximum load, for notched bend specimens of HY-180 steel, increased with increasing
specimen size, although the fractional value decreased slightly, from about 6% for a 1.6-mm-thick specimen to about 4% for a 53-mm-thick specimen. On the other hand, the toughness at the onset of crack extension was about constant for specimens exceeding about 2 cm in thickness. If the toughness required to develop stable crack extension is indeed an increasing and a single valued function of the amount of stable crack growth, then it follows that to justify using values of fracture toughness higher than those corresponding to the onset of crack extension, the amount of stable crack growth as well as the toughness must be determined from specimen test data, and both values must be used for a correct safety analysis of a flawed structure.

Multiple direct or indirect measurements of stable crack growth in single specimens are not easily made, for a variety of reasons. One method for estimating stable crack extension that is considered potentially applicable to irradiated specimens, but that also illustrates the precision problems associated with auxiliary crack length measurements, is the unloading compliance method. By this method, the specimen is partially unloaded from the elastic-plastic range, and the crack length is calculated from the change in the elastic unloading compliance and the known elastic compliance of the specimen as a function of crack length. Because the change in compliance due to a given change in crack length is inversely proportional to the specimen size, it can be estimated that to prevent errors in the value of the toughness at the onset of crack extension from exceeding 10% for a 4T compact specimen, which is used in research for obtaining baseline data, the unloading displacement must be measured within 30 μm. Consequently, efforts are underway to improve the precision and the accuracy of unloading displacement measurements, and simultaneously to develop alternate means for estimating small amounts of stable crack extension without the requirement of unloading. Andrews et al. at the General Electric Company have proposed such a method, based on two crack opening displacement measurements made at different distances from the crack tip, from which the opening displacement at the original fatigue sharpened crack tip can be calculated. By correlation, the difference between this displacement and the displacement at the same location that would have occurred if there had been no crack extension are used to estimate the amount of stable crack extension. Another method has been proposed by Paris et al., based on the analytical or experimental determination of the family of load-displacement curves for specimens of constant crack length, covering the
range from initial to final crack sizes occurring in the specimen. Both of these methods are still under development and therefore their suitability for routine application cannot yet be judged. In the meantime, it appears worthwhile to continue exploring for other possible methods of estimating crack length changes without unloading, especially if these methods appear to be easily applied and their results promise to be reasonably accurate. This discussion describes one such possible method which, if it proves to be feasible, can be applied to any new or existing elastic-plastic fracture toughness data that include both load and either load point or front face clip gage displacement values.

EFFECTIVE CRACK LENGTH

In order to estimate the increase in crack length in a specimen without partially unloading the specimen or making other auxiliary additional measurements, the increase in crack length must be related to some characteristic of the measured load-deflection curve for continued loading. A relationship adaptable to this purpose was proposed by Bucci et al.,\textsuperscript{10} who suggested that the secant modulus of the load-deflection curve could be estimated by adding an $\gamma_y$ plastic zone size correction to the original crack length, and then estimating the secant modulus as the elastic stiffness corresponding to the resulting effective crack length. Recently, McCabe and Landes\textsuperscript{11} suggested reversing this procedure, so that the effective crack length is estimated from the measured secant modulus of the load-deflection curve. The latter procedure has the realistic advantage that the effective crack length cannot exceed the specimen width. Furthermore, McCabe and Landes\textsuperscript{11} showed that $J$ calculations based on the elastic formula for $K$, the actual load and the effective crack length were consistently close to the experimentally determined values of $J$ defined as minus the rate of change of area under the load-deflection curve with increasing crack area, at constant deflection. This latter result turns out to be of considerable practical importance, even though there is presently no complete theoretical explanation for the result itself.

SINGLE SPECIMEN EQUATIONS FOR $J$

Soon after the relationship between the effective crack length and the nonlinear load-deflection curve was suggested,\textsuperscript{10} equations began to be developed by
which \( J \) could be calculated from a single specimen nonlinear load-displacement test record. For the notched beam, Rice, Paris and Merkle\(^2\) derived the equation

\[
J = \frac{2A}{DB},
\]

where \( A \) is the area under the load-displacement curve, \( b \) is the ligament width, and \( B \) is the specimen thickness. Subsequently, for the compact specimen, Merkle and Corten\(^3\) derived the equation

\[
J = \frac{(1 + a)}{(1 + a^2)} \cdot \frac{2A + \alpha(1 - 2\alpha - a^2)}{bB} \cdot \frac{2(P - A)}{(1 + a^2)^2} \cdot \frac{2}{bB},
\]

where \( P \) is the load, \( \Delta \) the displacement, and \( \alpha \) is the fraction of the net section that carries the applied force at limit load, and is given by

\[
\alpha = \left[ \left( \frac{2a}{b} \right)^2 + 2 \left( \frac{2a}{b} \right) + 2 \right]^{1/2} - \left( \frac{2a}{b} + 1 \right)
\]

Note that Eq. (1), as well as Eq. (2), is written here in terms of the total displacement, in agreement with the subsequent findings of several investigators.\(^{12-14}\)

Also, in this discussion, the displacement \( \Delta \) will be the load line displacement and the area \( A \) will be the area under the load versus load line displacement, unless otherwise specified.

Recently, several investigators have proposed simplified approximations to Eq. (2) for the compact specimen. Andrews\(^8\) found that the expression

\[
J = \frac{3}{2 + (a/W)} \cdot \frac{2A}{bB}
\]

agrees with Eq. (2) within three to four percent, for \( a/W \geq 0.5 \). On the other hand, Landes, Walker and Clarke\(^14\) found that satisfactory agreement with experimentally based values of \( J \), for a series of blunt notched compact specimens of
HY-130 steel, could be obtained by using only the first term in Eq. (2), that is, by writing

\[ J = \frac{1 + \alpha}{1 + \alpha^2} \cdot \frac{2A}{bB} \]  

(5)

Based on the same comparison between experimentally determined and calculated values of \( J \), Clarke and Landes\(^5\) have since recommended Eq. (5) as the best expression to use for determining \( J \) for the compact specimen.

Both Eqs. (4) and (5) are single term equations for \( J \) reminiscent of the equivalent energy formula\(^1\) for the compact specimen, because they are both equations of the form

\[ J = \frac{\lambda A}{bB} \]  

(6)

The difference between Eqs. (4) and (5) can be investigated by considering a general approximation for \( \lambda \) to be of the form

\[ \lambda = \frac{2A_0}{B_0 + (a/W)} \]  

(7)

where

\[ B_0 = A_0 - 1 \]  

(8)

An appropriate value of \( A_0 \) can be determined graphically by noting that the reciprocal of \( \lambda \) is a linear function of \( a/W \), and that, for \( a/W = 0 \),

\[ \frac{2}{\lambda} = \frac{B_0}{A_0} \]  

(9)

Thus by plotting the values of \( 2/\lambda \) calculated from Eq. (5) versus \( a/W \), and then fitting a straight line to the results, as shown in Fig. 1, it can be determined
that within one percent accuracy, for $a/W \geq 0.3$, $B_0/A_0 = 7/9$, so that Eq. (5) can be represented by the expression

$$J = \frac{9}{7 + 2(a/W)} \cdot \frac{2A}{bB}.$$  \hspace{1cm} (10)

Figure 1 also shows a comparison between Eqs. (4) and (2), for the elastic case. In general, the accuracy of Eq. (4) will depend on the value of $a/W$ and the extent of yielding. However, it can be seen from Fig. 1 that Eq. (10) is a closer approximation to Eq. (5) than Eq. (4) is to Eq. (2). In addition, Eq. (10) always gives slightly smaller values of $J$ than Eq. (4). The difference is six percent at $a/W = 0.5$, and it decreases steadily as $a/W$ increases. Consequently, the value of $B_0 = 3.5$ will be used for subsequent calculations for the compact specimen, based on Eq. (10).

ENERGY AND COMPLIANCE EQUATIONS

One way of checking the accuracy of an approximate energy based expression for $J$ is to use it to estimate the area under the load-displacement curve as a function of crack length at constant displacement. For this purpose, combining Eqs. (6) and (7) with the basic definition of $J$ gives

$$\frac{A_0}{B_0 + (a/W)} \cdot \frac{2U}{bB} = - \frac{1}{\Delta} \frac{\partial U}{\partial a} \bigg|_{\Delta = \text{const}}.$$ \hspace{1cm} (11)

where, in accordance with common usage, $U$ and $A$ are synonymous. After using Eq. (8) and the substitutions

$$b = W - a$$ \hspace{1cm} (12)

and

$$\frac{b}{W} = \kappa,$$ \hspace{1cm} (13)
and noting that $U$ varies only with $b$ if $\Delta$ is held constant, Eq. (11) can be converted to the ordinary differential equation

$$
\frac{dU}{U} = \frac{2A_0}{A_0 - x} \cdot \frac{dx}{x}.
$$

(14)

By direct integration, the solution to Eq. (14) is

$$
\frac{U}{U_0} = \left(\frac{1 - (a/W)}{1 - (a_0/W)}\right)^2 \frac{(B_0 + (a_0/W))^2}{(B_0 + (a/W))}, \quad \Delta = \text{const.}
$$

(15)

where $U_0$ is the value of $U$ at any reference value of crack size, denoted by $a_0$.

The predictions of Eq. (15) can be compared with experiment by first noting that the ratio of energies for two specimens of different crack lengths is independent of displacement. Therefore, at any displacement,

$$
\frac{P}{P_0} = \frac{U}{U_0}.
$$

(16)

From Eq. (16) it follows that the variation of elastic compliance with crack size can be estimated from

$$
\frac{\sigma}{\sigma_0} = \frac{U_0}{U}, \quad \Delta = \text{const.}
$$

(17)

The predictions of Eq. (17) are compared with the boundary collocation values obtained by Newman,\textsuperscript{16} in Fig. 2, using Newman's elastic compliance values at $a/W = 0.6$ as reference values. From Fig. 2 it can be seen that using $B_0 = 3.5$ produces a very close estimate of Newman's front face crack mouth opening compliance curve, while $B_0 = 2.0$ produces a more accurate estimate of the load line compliance curve than does $B_0 = 3.5$. Equation (16) has also been applied in the elastic-plastic range to the series of load-displacement curves for ten blunt notched compact specimens of HY-130 steel obtained by Landes, Walker and Clarke\textsuperscript{14} shown in Fig. 3. Using $B_0 = 3.5$, the predicted loads at $\Delta = 2$ mm (0.08 in.) are slightly high for $a/W < 0.6$, the load for which was used as the reference
value, but they are extremely good for \(a/W > 0.6\). Since all of the comparisons made thus far indicate that Eqs. (7) and (15) are good approximations, the use of Eq. (7) in a method for estimating the extent of stable crack growth will now be examined.

**SOLUTIONS FOR THE EFFECTIVE CRACK LENGTH**

At first glance, a method for reconciling the effective crack length concept of McCabe and Landes\(^{11}\) with the area based \(J\) formulas of Andrews\(^8\) and Landes et al.\(^{14}\) is not obvious. Nevertheless, it is possible to equate the \(J\) values calculated by the two methods, and thus obtain a solution for the effective crack length. By this approach, for the compact specimen,

\[
\frac{J(BW)}{A_0} = \frac{2A}{[B_0 + (a/W)][1 - (a/W)]} = \frac{PA}{[B_0 + (a_e/W)][1 - (a_e/W)]},
\]

(18)

where \(a_e\) is the effective crack length. The expression for effective crack length obtained by rearranging Eq. (18) is

\[
\frac{a_e}{W} = \frac{1}{2} \left\{ \sqrt{(B_0 - 1)^2 - 4\left[\frac{PA}{2A} \left( B_0 \frac{a}{W} \right) \left( 1 - \frac{a}{W} \right) - B_0 \right]} - (B_0 - 1) \right\}.
\]

(19)

If it is assumed that the ratio \(PA/A\) is the same whether based on front face or load line displacement, then \(a_e/W\) can be determined before the ratio of the two displacements is known.

To be considered useful, Eq. (19) must give estimates of effective crack length that agree with certain logical restrictions. These are that (1) for a completely linear load-displacement curve, the effective crack length should be the actual crack length, \(a\); (2) for large displacements at limit load, the effective crack length should not exceed the distance to the point of stress reversal; and (3) the effective crack length should not exceed the width of the specimen under any conditions. From Eq. (18) it is easily seen that requirement (1) is satisfied. To determine if requirement (2) is satisfied, Eq. (19) needs
to be evaluated for the case of $A = P_0$, and the result compared with the distance to the point of stress reversal at limit load which, based on Fig. 4, is given by

$$
t = \frac{a}{W} + \frac{(1 + \frac{a}{W})}{2} \left( 1 - \frac{a}{W} \right),$$  \hspace{1cm} (20)

where $a$ is given by Eq. (3). The results are given in Table 1 which indicates that for an infinite displacement at limit load, the effective crack tip approaches the neutral axis, but remains within the tensile yielding zone. For either $a/W = 1$, which is the extreme limit for a very deep crack, or $P_0/2A = 0$, which is the extreme limit for a point on the load-displacement curve well past maximum load, Eq. (19) reduces to $a/W = 1$, indicating that requirement (3) is satisfied.

The solution for the effective crack length in a notched beam is even more straightforward. Combining the area based$^2$ and the effective crack length$^{11}$ equations for $J$ gives

$$J = \frac{2A}{bB} = \frac{PA}{b_e B},$$ \hspace{1cm} (21)

where $b_e$ is the effective ligament length. From Eq. (21) it follows that

$$b_e = \frac{PA}{2A}.$$ \hspace{1cm} (22)

For elastic behavior, $P_0/2A = 1$ and $b_e = b$. For an infinite displacement at limit load, $P_0/2A = 1/2$ and $b_e = 1/2 b$. At the end of the descending branch of the load-displacement curve, $P_0/2A = 0$ and $b_e = 0$. Thus requirements (1), (2), and (3) are all satisfied.

**DETERMINATION OF THE PHYSICAL INCREASE IN CRACK SIZE**

The effective crack length has two components, as shown in Fig. 5. The component of main interest is the physical crack length, denoted by $a_p$. The remainder of the effective crack length is the intensely yielded but still intact
zone of length \( \rho \). By utilizing the familiar although approximate assumption that the profile of each side of the effective crack surface remains a straight line, it follows from Fig. 5 that for a compact specimen

\[
\frac{\rho}{\delta} = \frac{a_e + z}{\Delta g}.
\]  

(23)

where \( \delta \) is the crack opening displacement at the actual physical crack tip. From Eq. (23) it follows that

\[
\frac{\rho}{\bar{W}} = \frac{\delta}{\Delta g} \left( \frac{a_e}{\bar{W}} + \frac{z}{\bar{W}} \right).
\]  

(24)

For a notched beam under three point bend loading, neglecting arm curvature,

\[
\theta = \frac{\delta}{\rho} = \frac{4A}{S}
\]  

(25)

where \( S \) is the span and \( A \) is the displacement of the load. For a standard specimen with \( S = 4\bar{W} \),

\[
\frac{\delta}{\rho} = \frac{A}{\bar{W}},
\]  

(26)

so that

\[
\rho = \frac{\delta}{A} \bar{W}.
\]  

(27)

The value of \( \delta \) in Eqs. (24) and (27) will be determined here by using the relation between \( J \) and \( \delta \) proposed by Wells,\(^{17}\) namely

\[
J = m \sigma_y \delta,
\]  

(28)

where \( m \) is the crack tip triaxial constraint factor. However, once stable crack growth has occurred, it is necessary to distinguish between the values of crack
opening displacement and constraint factor that exist at the actual advancing crack tip from those that pertain to the original fatigue crack tip (see Fig. 5). The former values will be denoted here by the symbols $\delta$ and $m$, and the latter values by $\delta_0$ and $m_0$. Values of $m_0$ have been estimated by several different investigators. Boyle and Wells$^{18}$ found that, for plane strain, $1.7 < m_0 < 2.1$, by analyzing cracked specimens of several different geometries in plane strain. Based on other analyses, Harrison$^{19}$ found that $m_0 = 1.3$ for plane stress and $m_0 = 1.7$ for plane strain. Finally, on the basis of experimental data, Dawes$^{20}$ found that $1.5 < m_0 < 2.1$. All of the above values of $m_0$ were determined from the equation

$$m_0 = \frac{J}{\sigma_Y \delta_0}.$$  \hspace{1cm} (29)

However, a different problem exists here because $m$ must be determined before $\delta$ is known. Consequently, $m$ must be determined from its basic definition as the ratio of the actual crack tip stress to the yield stress. Therefore, it is necessary to have a relationship between toughness and the maximum crack tip stress. The relationship to be used here is one that has already been shown to relate plane strain ductility to fracture toughness, for A533-B steel,$^{21}$ up to $K_{IC}$ values approaching 154 MN m$^{-3/2}$ (140 ksi \(\sqrt{\text{in.}}\)). According to this relationship,

$$K_{IC} = \sqrt{\pi \rho_0} \sqrt{E \sigma_Y} \left( \epsilon^{\theta/2} - 1 \right)$$  \hspace{1cm} (30)

where $\rho_0$ is the effective root radius, $H \sigma_Y$ is the slope of the strain hardening branch of the stress-strain curve, and $s$ is the maximum crack tip strain. For $\rho_0 = 0.05$ mm (0.002 in.), $H = 3$, $\sigma_Y = 483$ MPa (70 ksi) and $E = 20.68 \times 10^4$ MPa ($3 \times 10^7$ psi), Eq. (30) can be rearranged to read

$$s = 2 \ln \left( 1 + \frac{K_{IC}}{220} \right),$$  \hspace{1cm} (31)
where $K_{IC}$ is expressed in $\text{MN} \cdot \text{m}^{-3/2}$. For inelastic conditions, $K_{IC}$ can be calculated from the value of $J$, using

$$K_{IC} = \sqrt{EJ} .$$

(32)

Furthermore, for the assumed case of linear strain hardening,

$$m = 1 + Hs .$$

(33)

Finally, once $m$ is known, $\delta$ can be calculated from the equation

$$\delta = \frac{J}{m \sigma_Y} .$$

(34)

For the compact specimen, once the values of $\delta$ and $\rho$ are determined, the amount of stable crack extension can be calculated from

$$\Delta a = \left( \frac{\alpha_e}{W} \frac{\alpha}{W} - \frac{\rho}{W} \right).$$

(35)

In the case of compact specimen data consisting of load and crack mouth rather than load line displacement values, the ratio of the two displacements must be known in order to calculate $J$. If this ratio is assumed to be approximately constant, then it is given by

$$\Delta_L = \frac{(\alpha_e/W)}{(\alpha_e/W) + (\rho/W)} .$$

(36)

Consequently, the value of $J$ can be calculated from

$$J = \left( \frac{\Delta_L}{\Delta_G} \right) \cdot \frac{\lambda A_G^{1/2}}{EB} ,$$

(37)

where $A_G$ is the area under the load versus front face clip gage displacement curve.
For checking purposes, the value of $m_0$ can be calculated, once $\frac{\Delta L}{a_e}$ is known. From Fig. 5,

$$\frac{\Delta L}{a_e} = \frac{\delta_0}{a_e - a},$$

and

$$m_0 \delta_0 = m \delta.$$

Hence it follows, by using Eqs. (34), (38), and (39), that

$$m_0 = \frac{1}{\left[1 - \frac{(a/W)}{(a_e/W)}\right]} \cdot \frac{I}{\sigma_y \frac{\Delta L}{a_e}},$$

Correspondingly, for the notched beam,

$$(a_e - a) = \Delta a_e = b \left(1 - \frac{b_e}{b}\right),$$

and

$$\Delta a = \Delta a_e - \rho.$$

Also, since, from Fig. 5,

$$\delta = \delta_0,$$

$$\rho = \frac{\Delta a_e}{a},$$

combining Eqs. (39) and (43) gives

$$m_0 = m \left(\frac{\rho}{\Delta a_e}\right).$$
TRIAL CALCULATIONS

The foregoing equations were applied to several sets of experimental data for pressure vessel steels, and the results were compared for reasonableness with an available $J - \Delta a$ resistance curve for A533-B steel.\textsuperscript{22} The specimens chosen for analysis were those for which the middle region of the crack advanced in the plane of the original fatigue precrack, and for which the load-displacement diagram showed no sudden load drops. The calculations were each made for the maximum load point of a test record. Specimen types and sizes ranged from precracked Charpy V-Notch specimens to 1T compact specimens. All the specimens were loaded monotonically to a displacement past maximum load. The data analyzed are listed in Tables 2 and 3, and the results are plotted in Fig. 6.

Nine of the specimens analyzed were precracked Charpy V-Notch specimens. These data are listed in Table 2. Of these, three were from the V-7B weld repair region of HSST Program V-8 prolongation,\textsuperscript{23} and six were from base plate material in HSST weldment W57.\textsuperscript{24} Eleven of the specimens analyzed were compact specimens. These data are listed in Table 3. Of these, two were Charpy thickness compact specimens of A537, class 1 steel; seven were 1T specimens of the same material; one was a 1T specimen of A537, class 2 steel; and one was a 1T specimen of A508, class 1 steel.\textsuperscript{25}

As shown in Fig. 6, the calculated results lie quite close to the $J - \Delta a$ resistance curve for A533, grade B, class 1 steel.\textsuperscript{22} This indicates both the reasonableness of the analysis method despite its several approximations, and the probable similarity of the resistance curves for several different pressure vessel steels of similar yield strength. It is especially noteworthy that the values plotted in Fig. 6 were obtained from single specimen test data, without any auxiliary crack length measurements, experimental data or analyses being required. In fact, stable crack growth determinations were not even planned when the original tests were performed. Most of the compact specimens were tested with load line displacement gages, but two of them (the Charpy thickness compact specimens) had only front face clip gages.

All of the specimens analyzed here were loaded monotonically to displacements beyond their maximum load points. Preliminary analysis of data from a single specimen that underwent cyclic unloading and reloading for crack length measurements indicates that such cycling may affect the crack tip constraint factor,
because of reversed yielding near the crack tip. While this may create a problem in comparing calculated results for unloading compliance specimens, it would not be involved in the analysis of monotonically loaded specimens.

DISCUSSION

Although the method of data analysis developed here contains several approximations, it is important to note that most of the approximations were developed by others, and have been in use separately for some time. The equation for $J$ for the compact specimen has the same form as the equation proposed by Andrews, and it agrees numerically with the recent proposal of Clarke and Landes. The geometric treatment of the crack profile as a pair of straight lines intersecting at a hinge point, and the relation between $J$ and the crack opening displacement, both agree with accepted practice in the UK. The estimation of an effective crack length is taken from the recent work of McCabe and Landes, and the assumption of a relation between ductility and fracture toughness agrees in principle with the recent EPRI sponsored work of Lawrence Livermore Laboratory and Fracture Control Corporation. Therefore, although it contains several different approximations, the method proposed here for estimating stable crack growth from upper shelf toughness data is consistent with several other accepted approaches and, perhaps most important of all, it is simple, both analytically and experimentally.
REFERENCES


18. C. F. Boyle and A. A. Wells, A Finite Element Study of Plane Strain Fracture Criteria Under Elastic-Plastic Conditions, Department of Civil Engineering, Queen's University, Belfast, Ireland (June 1973).


Table 1. Asymptotic values of effective crack length at infinite displacement for compact specimens at limit load

<table>
<thead>
<tr>
<th>$\alpha/W$</th>
<th>$a_e/W$</th>
<th>$t/W$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B_0 = 3.5$</td>
<td>$B_0 = 2.0$</td>
</tr>
<tr>
<td>0.3</td>
<td>0.682</td>
<td>0.702</td>
</tr>
<tr>
<td>0.5</td>
<td>0.766</td>
<td>0.775</td>
</tr>
<tr>
<td>0.7</td>
<td>0.855</td>
<td>0.858</td>
</tr>
</tbody>
</table>
Table 2. Precracked Charpy V-notch specimen data used for stable crack growth estimates

<table>
<thead>
<tr>
<th>Material</th>
<th>V-7B weld</th>
<th>V-7B weld</th>
<th>V-7B weld</th>
<th>W57-B.P.</th>
<th>W57-B.P.</th>
<th>W57-B.P.</th>
<th>W57-B.P.</th>
<th>W57-B.P.</th>
<th>W57-B.P.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test temp., °F</td>
<td>150</td>
<td>200</td>
<td>200</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>0</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>(\sigma_y), ksi</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>66</td>
<td>63</td>
<td>63</td>
<td>70</td>
<td>63</td>
<td>70</td>
</tr>
<tr>
<td>(K_{\text{IC}}), ksi√in.</td>
<td>2270.5</td>
<td>2908.5</td>
<td>1994.2</td>
<td>2300.3</td>
<td>1830.9</td>
<td>1534.4</td>
<td>2546.9</td>
<td>1994.7</td>
<td>1912.6</td>
</tr>
<tr>
<td>(\sigma_y), ksi√in.</td>
<td>260.99</td>
<td>295.39</td>
<td>244.59</td>
<td>262.69</td>
<td>234.37</td>
<td>214.55</td>
<td>276.42</td>
<td>244.62</td>
<td>239.54</td>
</tr>
<tr>
<td>(\rho), in.</td>
<td>6.0103</td>
<td>6.4422</td>
<td>5.7931</td>
<td>6.0325</td>
<td>5.6534</td>
<td>5.3732</td>
<td>6.2079</td>
<td>5.7934</td>
<td>5.7245</td>
</tr>
<tr>
<td>(\delta), in.</td>
<td>0.0054</td>
<td>0.0064</td>
<td>0.0049</td>
<td>0.0038</td>
<td>0.0051</td>
<td>0.0025</td>
<td>0.0059</td>
<td>0.0055</td>
<td>0.0048</td>
</tr>
<tr>
<td>(\rho), in.</td>
<td>0.0271</td>
<td>0.0306</td>
<td>0.0261</td>
<td>0.0287</td>
<td>0.0303</td>
<td>0.0288</td>
<td>0.0292</td>
<td>0.0299</td>
<td>0.0289</td>
</tr>
<tr>
<td>(\beta_0/b)</td>
<td>0.6481</td>
<td>0.5782</td>
<td>0.5868</td>
<td>0.5923</td>
<td>0.6004</td>
<td>0.6011</td>
<td>0.5882</td>
<td>0.5818</td>
<td>0.5937</td>
</tr>
<tr>
<td>(\Delta K_I), in.</td>
<td>0.0657</td>
<td>0.0825</td>
<td>0.0731</td>
<td>0.0734</td>
<td>0.0722</td>
<td>0.0663</td>
<td>0.0777</td>
<td>0.0757</td>
<td>0.0714</td>
</tr>
<tr>
<td>(\Delta K_I), in.</td>
<td>0.0385</td>
<td>0.0520</td>
<td>0.0470</td>
<td>0.0446</td>
<td>0.0420</td>
<td>0.0375</td>
<td>0.0485</td>
<td>0.0458</td>
<td>0.0425</td>
</tr>
<tr>
<td>(m_0)</td>
<td>2.4826</td>
<td>2.3860</td>
<td>2.0687</td>
<td>2.3626</td>
<td>2.3680</td>
<td>2.3339</td>
<td>2.3355</td>
<td>2.2900</td>
<td>2.3136</td>
</tr>
</tbody>
</table>

*Conversions:
\( ^\circ \text{C} = 5/9 (^\circ \text{F} - 32) \);
1 ksi = 6.8948 MPa;
1 in. = 25.4 mm;
1 lb = 4.4482 N;
1 in.-lb = 0.1130 J;
1 in.-lb/in.\(^2\) = 0.17513 KJ·m\(^{-2}\);
1 ksi√in. = 1.0988 MN·m\(^{-3}/2\).*
Table 3. Compact specimen data used for stable crack growth estimates ($E_0 = 3.5$)*

| Material | Spec. No. | Test temp., °F | $\sigma_y$, ksi | $B$, in. | $W$, in. | $\alpha$, in. | $b$, in. | $z$, in. | $\alpha/W$ | $z/W$ | $P$, lb | $\Delta_2$, in. | $\Delta_2^*$, in.-lb | $A^*$, in.-lb | $\sigma_t/\sigma_y$ | $J$, in.-lb/in.$^2$ | $K_{IC}$, ksi $\sqrt{\text{in.}}$ | $s$ | $m$ | $\delta$, in. | $\rho/W$ | $\Delta_\alpha$, in. | $m_0$ |
|----------|-----------|----------------|-----------------|---------|----------|-------------|---------|---------|-------------|--------|--------|---------------|-------------------|-----------------|-------------------|-----------------|-----------------|-----------------|---------|-----------------|---------|-----------------|---------|-----------------|---------|
| A508-1   | 02T3P1    | 71.6           | 50.7            | 0.9972  | 1.9964  | 1.0764     | 0.9200  | 0.0         | 0.5392      | 0         | 12,525 | 0.2153        | 2339              | 1899.4          | 0.7474            | 5681            | 412.8           | 2.240            | 7.72   | 0.0145          | 0.0504            | 0.315             | 1.87  |
| A537-1   | 02C1P1    | 64.4           | 52.2            | 1.0003  | 2.0008  | 1.0839     | 0.9161  | 0         | 0.5420      | 0         | 13,000 | 0.1698        | 1937.2            | 2212.7          | 0.7467            | 4615            | 372.1           | 2.10             | 7.31   | 0.0121          | 0.0532            | 0.303             | 1.90  |
| A537-1   | 01C3P1    | 212            | 56.5            | 1.0004  | 2.0008  | 1.0819     | 0.9189  | 0         | 0.5407      | 0         | 12,000 | 0.1885        | 1937.2            | 2212.7          | 0.7447            | 4694            | 375.2           | 2.11             | 7.34   | 0.0113          | 0.0447            | 0.319             | 1.61  |
| A537-1   | 02C0P1    | 32             | 52.2            | 1.0005  | 2.0013  | 1.0668     | 0.9345  | 0         | 0.5331      | 0         | 13,350 | 0.1909        | 1937.2            | 2212.7          | 0.7329            | 5282            | 398.1           | 2.19             | 7.57   | 0.0134          | 0.0512            | 0.315             | 1.87  |
| A537-1   | 01C6P2    | 32             | 55.8            | 1.0005  | 2.0021  | 1.0468     | 0.9350  | 0         | 0.5240      | 0.0516    | 15,000 | 0.1813        | 1937.2            | 2212.7          | 0.7301            | 5403            | 396.2           | 2.18             | 7.55   | 0.0127          | 0.0514            | 0.317             | 1.78  |
| A537-1   | 01E7P2    | 77             | 58.7            | 1.0004  | 2.0021  | 1.0468     | 0.9530  | 0         | 0.5331      | 0.0517    | 15,500 | 0.1714        | 1937.2            | 2212.7          | 0.7301            | 5232            | 398.8           | 2.18             | 7.55   | 0.0127          | 0.0503            | 0.327             | 1.89  |
| A537-1   | 01C3P2    | 64.4           | 58.7            | 1.0004  | 2.0021  | 1.0468     | 0.9530  | 0         | 0.5240      | 0.0516    | 14,900 | 0.1783        | 1937.2            | 2212.7          | 0.7340            | 5301            | 406.7           | 2.19             | 7.55   | 0.0118          | 0.0506            | 0.327             | 1.89  |
| A537-1   | 01C4P2    | 212            | 54.8            | 1.0004  | 2.0021  | 1.0506     | 0.9545  | 0         | 0.5240      | 0.0516    | 16,400 | 0.1987        | 1937.2            | 2212.7          | 0.7340            | 5514            | 406.7           | 2.22             | 7.66   | 0.0128          | 0.0506            | 0.327             | 1.89  |
| A537-1   | 01E4P4    | 32             | 52.9            | 1.0003  | 2.0024  | 1.0477     | 0.9545  | 0         | 0.5240      | 0.0516    | 20,555 | 0.1987        | 1937.2            | 2212.7          | 0.7340            | 6388            | 437.8           | 2.22             | 7.96   | 0.0136          | 0.0506            | 0.327             | 1.89  |
| A537-1   | 02A6P1    | 167            | 60.0            | 1.0003  | 2.0045  | 1.0506     | 0.9545  | 0         | 0.5240      | 0.0516    | 20,555 | 0.1987        | 1937.2            | 2212.7          | 0.7340            | 6388            | 437.8           | 2.22             | 7.96   | 0.0136          | 0.0506            | 0.327             | 1.89  |
| A537-1   | 03A6P1    | 302            | 50              | 1.0003  | 2.0045  | 1.0506     | 0.9545  | 0         | 0.5240      | 0.0516    | 20,555 | 0.1987        | 1937.2            | 2212.7          | 0.7340            | 6388            | 437.8           | 2.22             | 7.96   | 0.0136          | 0.0506            | 0.327             | 1.89  |

*Conversions:

$°C = 5/9 (°F - 32)$;  
1 ksi = 6.8948 MPa;  
1 in. = 25.4 mm;  
1 lb = 4.4482 N;  
1 in.-lb = 0.1130 J;  
1 in.-lb/in.$^2 = 0.17513 \text{ KJ} \cdot \text{m}^{-2}$;  
1 ksi $\sqrt{\text{in.}} = 1.0988 \text{ MN} \cdot \text{m}^{-3/2}$.

†$A$ is the area under the measured load-displacement curve. If the measured displacement is $\Delta_y$, then the area listed is $A_y$. 
Fig. 1. Graphical determination of coefficients in the approximate expression for J for a compact specimen.
Fig. 2. Comparison between approximate and theoretical boundary collocation values of the non-dimensional compliances, at the crack mouth and load line, for a compact specimen.
Fig. 3. Estimated loads at a displacement of 2 mm (0.08 in.) for a series of 22.9 mm (0.90 in.) thick, blunt notched IT profile compact specimens of HY 130 steel with various \( a/W \) ratios (experimental data from Ref. 14).
Fig. 4. Geometric definitions for the compact specimen at limit load.
Fig. 5. Dimensions characterizing the crack profile model used for analysis.
Fig. 6. Comparison of calculated values of $J$ and $\Delta a$ at maximum load for several monotonically loaded precracked Charpy V-Notch and compact specimens of various pressure vessel steels, with a reference curve for A533, grade B, class 1 steel.
AN ENGINEERING APPROACH FOR EXAMINING CRACK GROWTH AND
STABILITY IN FLAWED STRUCTURES

C. F. Shih

GENERAL ELECTRIC COMPANY
Corporate Research and Development
Scheneectady, New York 12301

ABSTRACT

The paper summarizes progress made in two research programs sponsored by
the Electric Power Research Institute (EPRI), to identify viable parameters
for characterizing crack initiation and continued extension, and to develop an
engineering/design methodology, based on these parameters, for the assessment
of crack growth and instability in engineering structures which are stressed
beyond the regime of applicability of linear elastic fracture mechanics. The
ultimate goal in the development of such a methodology is to establish an im-
proved basis for analyzing the effect of flaws (postulated or detected) on the
safety margins of pressure boundary components of light water-cooled type
nuclear steam supply systems. The methodology can also be employed for struc-
tural integrity analyses of other engineering structures.

Extensive experimental and analytical investigations undertaken to evalu-
ate potential criteria for crack initiation and growth, and the selection of
the final criteria for analyzing crack growth and stability in flawed struc-
tures are summarized; the experimental and analytical results obtained to date
suggest that parameters based on the J-integral and the crack tip opening dis-
placement δ are the most promising. This is not surprising since from a theo-
retical basis, the two approaches are similar if certain conditions are met.

An engineering/design approach for the assessment of crack growth and in-
stability in flawed structures is outlined. The approach exploits the conse-
quences of J-controlled crack growth - the J resistance (J_{R'}) curve is a mate-
rial property, and crack driving forces can be determined from deformation
plasticity analyses. Crack driving forces for the complete range of elastic-

*Presented at the OECD-CSNI Specialist Meeting at Washington University,
plastic deformation are obtained from a simple estimation scheme. The basic elements of the estimation scheme are the linear elastic solutions and the fully plastic solutions for the relevant crack configuration - the latter solutions are cataloged in a plastic fracture handbook. Crack driving force diagrams together with the $J_R$ curve are employed to construct stability diagrams and predict the load-deformation and crack growth behavior of several crack geometries. The predictions are in good agreement with experimental data and full-blown numerical crack growth calculations.

Relative merits and difficulties associated with the $J$ or $\delta$ resistance curve approach for treating stable crack growth and fracture instability in structural components are touched upon. In applications involving relatively small amounts of crack extension, the $J$ resistance approach assuming $J$-controlled growth has significant advantages and the fracture predictions are in good agreement with actual test data. For larger amounts of crack extension, the situation is less certain. The limited studies carried out under non-$J$-controlled conditions suggest that the predictions of the engineering/design methodology will be conservative.
AN ENGINEERING APPROACH FOR EXAMINING CRACK GROWTH AND STABILITY IN FLAWED STRUCTURES

C. F. Shih

GENERAL ELECTRIC COMPANY
Corporate Research and Development
Schenectady, New York 12301

1. INTRODUCTION

Nuclear steam supply system structural integrity is presently assured by designs that adhere to the ASME Pressure Vessel Code and various regulatory guides. These requirements for structural integrity are based on linear elastic fracture mechanics, which assumes that the material behave for the most part elastically, with little ductility. Consequently, the designs are conservative. Recent work in elastic-plastic fracture mechanics, has demonstrated that more realistic measures of actual design margins can be obtained through the use of these elastic-plastic analyses [1]. So far, however, the methodology for plastic or ductile fracture mechanics has required rather elaborate analysis by finite element or finite difference techniques. These techniques, while potentially providing much accurate information regarding the fracture behavior, are also very expensive and difficult to use in routine design applications and safety margin analyses. Consequently, it is desirable to develop simplified engineering approaches that can be used in routine assessments of structural integrity and fracture behavior of flawed structural components.

The paper summarizes progress made in two research programs [2, 3] sponsored by the Electric Power Research Institute (EPRI), to identify viable parameters for characterizing crack initiation and continued extension, and to develop an engineering/design methodology, based on these parameters, for the assessment of crack growth and instability in engineering structures which are stressed beyond the regime of applicability of linear elastic fracture mechanics. The ultimate goal in the development of such a methodology is to establish an improved basis for analyzing the effect of flaws (postulated or detected) on the safety margins of pressure boundary components of light water-cooled type nuclear steam supply systems under postulated accident
conditions. The engineering/design methodology can also be employed for structural integrity analyses of other engineering structures.

2. THE BASIS FOR THE ENGINEERING APPROACH

The experimental results from the Heavy Section Steel Technology program, and tests on large plates at the Southwest Research Institute, showed that part-through cracks in A533B steels advanced in a flat fracture mode under essentially plane strain conditions for a considerable amount of crack extension; shear fracture developed only after the attainment of maximum load. In our investigation, attention has therefore been focused on flat fracture under essentially plane strain conditions and accompanied by extensive yielding.

Shih and co-workers carried out an integrated experimental and analytical investigation of stable crack extension under fully plastic conditions [1 - 7]. They employed side-grooved compact specimens of A533B steels (tested at 93°C) and observed that the leading edge of the crack remained straight with crack extension. The fracture surfaces were macroscopically flat, and there were no lateral contractions along the crack front, i.e., crack extension had occurred under plane-strain conditions. Crack extensions were measured by the unloading compliance technique, and these were generally in good agreement with the measurements made by heat tinting. The J-resistance (JR) curve thus measured is unambiguously defined.

In the numerical investigations, the crack growth rate in the finite element model (of the compact specimen) was prescribed to follow the experimentally measured load-line displacement (LLD) - crack extension (a - a0) data. This was accomplished by shifting the crack-tip node in the model so that the simulated crack growth followed the experimental LLD vs. a - a0 relationship. When the remaining element length (in the path of crack extension) reached a critical fraction of the overall element size, the crack-tip node was released and further crack extension was simulated by shifting the next crack-tip node. A number of parameters were computed at each increment of crack extension - these include the J-integral, the crack opening displacement (COD), crack opening angles (COA), and several variants of the Griffith energy release rate. An evaluation of the analytical and experimental results led to the conclusion that J and δ are the most viable parameters for characterizing crack initiation and growth. A detailed discussion of the potential
fracture criteria evaluated in the study, the strategy employed in the evaluation, procedures for measuring the J-integral, COD and crack extension, detailed examination of the analytical and experimental results, and finally an assessment of these fracture parameters that were found viable are given in an EPRI Special Report, and published papers [1 - 7].

The following main conclusions emerged from our investigations:

1. Macrocopically flat fracture surfaces with a straight leading edge can be produced by employing side-grooves on test specimens. In the case of A533B steels, side-groove depths of 25 percent of the specimen thickness are recommended, since they promote an essentially uniform plane strain constraint along the crack front while producing minimal effect on specimen compliance and stress intensity factor. Side-grooves also led to a sharper and unambiguous measurement of crack initiation and extension - features which are essential to the determination of $J_{IC}$ and J-resistance curves.

2. Experimental data support the characterization of some amount of crack growth by the J or $\delta$ resistance curve; these curves appear to be independent of specimen size and extent of plastic deformation, if plane-strain conditions prevail in the crack tip region and some other minimal requirements are met. In fact, the variation in the resistance curves between specimens can often be attributed to mixed mode crack extension. For the smaller sized specimens, the central portion of the crack front advances ahead and eventually the trailing edges fail in shear under conditions closer to plane stress. In side-grooved specimens or thicker specimens, the extending crack front remains straight, and the fracture surfaces are macroscopically flat. Correspondingly, the J-resistance curves appear to approach a limiting lower bound curve (or plane-strain curve) with increasing specimen dimensions. These observations suggest that a portion of the J or $\delta$ resistance curve is a material property, if plane-strain conditions and some other requirements are met. The lower bound plane strain $J_{R}$ curve can be obtained from smaller specimens when side-grooves are employed.

3. J-resistance data from tests with center-cracked panels (CCP) are generally higher than the corresponding data from similar sized compact specimens (CS). With increasing CCP dimensions, the slope of the J-resistance data decreases and the resistance curves seem to approach the plane strain curves
from compact specimen. Observations based on available test data suggest that significantly larger CCP are required to generate valid plane strain resistance curve. Thus the lower bound plane strain $J_R$ curve is best obtained from compact specimen or similar bend specimens.

4. Analytical investigations strongly suggest that $J$ and COD can be employed as characterizing parameters for crack initiation and growth. The numerical studies based on an incremental theory of plasticity revealed that $J$ is path independent everywhere except very near the crack tip for some amount of crack growth. For specimens subjected to remote bending, e.g. compact specimen, the path independence of $J$ is observed for growth up to 6% of the original uncracked ligament. It is noted that the slope of the $J$-resistance curve ($dJ_R/da$) is approximately constant for a relatively short interval of crack extension, while the slope of the COD-resistance curve ($d\delta_R/da$, also called the average or nominal crack opening angle) and the local crack opening angle (COA) remain practically constant over the range of crack extension explored in our experimental and analytical investigations. The $J$ characterizing parameter approach is valid for limited range of crack growth; the range of validity depends on strain hardening properties, crack configurations. In particular for tough materials like A533B steels at the upper shelf, the $J$-based criteria is valid for crack growth up to 6% of the original uncracked ligament in test specimens subjected primarily to bending, e.g. compact specimens. The COD-based criteria appear to be valid for larger amounts of crack growth.

5. The tearing modulus ($T_J = (E/\sigma_o^2)dJ/da$) proposed by Paris and co-workers [8] as a measure of material resistance to stable growth is constant over relatively short intervals of growth. Our investigations suggest that a tearing modulus based on the COA ($T_\delta = (E/\sigma_o)d\delta/da$) is an attractive alternative. The latter modulus is measurable or can be deduced directly and appears to be constant over a large range of stable growth. Fracture toughness associated with crack initiation is measured by $J_{IC}$ or $\delta_{IC}$ while the material resistance to crack growth is measured by $T_J$ or $T_\delta$. The two parameter characterization of fracture properties by $J_{IC}$ and $T_J$ or $\delta_{IC}$ and $T_\delta$ are analogous to the characterization of material deformation properties by the yield stress and the strain hardening exponent. These observations provide some of the basis for the treatment of crack growth and instability by an $R$ curve approach based on
J [8, 9]. Similarly, an R curve approach based on COD can be developed along the lines introduced in [8, 9].

6. Detailed finite element stationary crack calculations showed that the HRR singularity [10, 11] do dominate the crack tip fields in strain hardening materials. However the size R of the region governed by the HRR field is strongly dependent on crack configuration. At the same level of applied J, R is larger in bend configurations than in similar sized tensile configurations [12, 13]. These results imply that the size requirement for the validity of the J approach for crack initiation and growth will depend on crack configuration or type of remote loading. In fact the size requirements for the tensile configuration are much more stringent than those for bend configurations. This conclusion is in accord with the experimental and numerical crack growth results discussed above.

7. Further analytical and numerical studies clarified the requirements for J as a characterizing parameter for crack growth or J-controlled growth. In addition to the size requirement described in the preceding paragraph, the \( \omega \) parameter (defined by \( \frac{C}{J} \frac{dJ}{da} \)) introduced by Hutchinson and Paris [9] must be sufficiently large. Studies by Shih, Dean and German [14, 15] suggest \( \omega \) values of about 10 and 80 will suffice to ensure J-controlled growth in bend and tensile configurations respectively. Under large scale yielding conditions, large \( T_{mat} \) or \( \psi \) (defined by \( \frac{1}{\sigma_0} \frac{d\gamma}{da} \)) will also promote J-controlled growth.

8. A unique relationship between J and an appropriately defined \( \delta_t \) can be obtained directly from the asymptotic crack tip fields [10, 11]. Results from detailed finite element studies of strain hardening materials for several crack configurations support the relation between J and \( \delta_t \). The slope of the J-resistance curve and the crack opening angle appear to be similarly related. These results suggest that an R curve approach, based on COD, for treating crack growth and instability is consistent with the J-controlled growth approach promulgated by Paris, et al [16].

**J-Controlled Crack Growth**

The theoretical justification and the conditions for J-controlled growth and the analyses of the stability of crack growth are adequately discussed in
[8, 9, 14, 15]. Here the consequences of J-controlled crack growth and its implication on the engineering fracture methodology based on J-controlled growth are summarized.

Under conditions of J-controlled crack growth, the \( J_R \) curve obtained from fully yielded specimens will be the same as the \( J_R \) curve obtained from specimens with limited yielding (or small-scale yielding) as long as plane strain or plane stress conditions prevail in both situations. The \( J_R \) curve will also be independent of crack configurations. In other words, the \( J_R \) curve is a material property.

Secondly, stable crack extension and crack instability under large-scale plasticity can be analyzed by the resistance curve approach based on J, which is a generalization of Irwin's resistance curve approach for small-scale yielding based on the elastic stress intensity factor \( K \) [17]. In fact, the \( R \)-curve approach based on J will be applicable to crack analyses for the complete range of elastic and elastic-plastic deformation. This approach has been employed to examine the stability of several crack configurations [3, 8, 9].

Lastly, the J-integral crack driving force can be determined from analyses using the deformation theory of plasticity. The formulas, obtained by Rice, Paris, and Merkle [18] for deeply cracked specimens and the estimation schemes developed by Shih and Shih and Hutchinson [19, 20] for more general situations, could be employed to estimate the J-driving force without recourse to detailed numerical calculations associated with incremental plasticity theory.

The above factors led to the evolvement of an engineering approach for examining crack growth and stability of flawed structures [3] which is discussed in greater detail in the subsequent section.

3. **ELEMENTS OF THE ENGINEERING APPROACH**

There are several essential elements to the engineering approach [3] and these are summarized below.

Fully plastic solutions of the type discussed in [19, 20] will be generated and catalogued in a Plastic Fracture Handbook. Solutions will be obtained for typical test specimen geometries and certain common structural
configurations (thick-walled cylinders with circumferential and longitudinal cracks). These specimens will be treated as either plane strain, plane stress, or axially symmetric models, as appropriate. Certain more difficult configurations concerning the nozzle corner flaw will also be studied and documented. The latter configurations are three-dimensional but will be approximated by "equivalent" two-dimensional models with the appropriate boundary conditions, and the equivalent fully plastic two-dimensional results will be documented in the handbook.

Estimation scheme discussed in [19, 20] will be further refined to ensure a smooth transition in the behavior of the estimated solutions from the contained plasticity to the fully plastic state. The fully plastic crack solutions will be employed together with linear elastic solutions [21] to produce relatively simple formulas for quantities such as the J-integral, crack opening displacement and other relevant parameters for the complete range of elastic-plastic deformation. The formulas, which have been shown to be very accurate in the problems thus far examined will be further developed for complex crack configurations.

An engineering methodology for predicting crack growth and fracture instability in flawed structures will be developed. The methodology is in essence a resistance curve approach based on the J-integral and/or the crack opening displacement and is similar to the procedure developed by Paris, Hutchinson and co-workers. The material $J_R$ curve will be given by the statistical mean of all crack growth data from specimens which meet the conditions for $J$-controlled growth, i.e. primarily from compact specimens. A lower bound $J_{RL}$ curve is given by the three standard deviation curves. The J-integral crack driving force diagrams determined from the estimation scheme together with the appropriate experimentally measured $J_R$ curve are the essential components of the stability analyses. The methodology assumes that J-controlled growth is applicable. If such is not the case, the methodology will give conservative estimates of the load carrying capacity and the fracture resistance of the structure.

In subsequent paragraphs, some of the key ideas in the engineering approach for treating crack growth and fracture instability are discussed. Details are given in [3, 19, 20].
Fully Plastic Solutions

In linear elasticity, crack parameters like the J-integral, the crack or mouth opening displacement, \( \delta \) and the load line displacement \( \Delta_c \) (due to crack) can be expressed in the following forms:

\[
\frac{J}{\sigma_o \varepsilon_o} = \left[ \frac{\sigma^\infty}{\sigma_o} \right]^2 \hat{J}(a/b)
\]

\[
\frac{\delta}{\varepsilon_o} = \left[ \frac{\sigma^\infty}{\sigma_o} \right] \hat{\delta}(a/b)
\]

\[
\frac{\Delta_c}{\varepsilon_o} = \left[ \frac{\sigma^\infty}{\sigma_o} \right] \hat{\Delta}_c(a/b)
\]

where \( \sigma^\infty \) is the remotely applied stress, \( \sigma_o \) and \( \varepsilon_o \) are some reference stress and strain (the connection \( \sigma_o = E \varepsilon_o \) can always be made, but is not necessary, E is the elastic modulus), \( \hat{J}, \hat{\delta}, \) and \( \hat{\Delta}_c \) are dimensionless functions of crack length-to-width ratio \( a/b \).

Consider now a completely incompressible nonlinear or fully plastic material where the strain is related to the stress in uniaxial tension by

\[
\varepsilon / \varepsilon_o = a (\sigma / \sigma_o)^n
\]

where \( a \) is a material constant. Generalization of Equation (2) to multiaxial stress states using J2 deformation theory gives

\[
\frac{\varepsilon_{ij}}{\varepsilon_o} = \frac{3}{2} a \left[ \frac{\sigma^\infty}{\sigma_o} \right]^{n-1} \frac{S_{ij}}{\sigma_o}
\]

where \( S_{ij} \) and \( \sigma^\infty = \frac{3}{2} S_{ij} S_{ij} \) are the stress deviator and effective stress, respectively. Ilyushin [22] noted that solution of the boundary value problem based on (3) and involving only a single load or displacement parameter which is increasing monotonically has two important properties.

First, the field quantities increase in direct proportion to the load or displacement parameter raised to some power dependent on \( n \). For example, if \( \sigma_o \) is generally associated with the yield stress or in some applications the flow stress.
the applied load or stress is \( \sigma^\infty \), then the field solution has the simple functional form

\[
\frac{\sigma_{ij}}{\sigma_0} = \left[ \frac{\sigma^\infty}{\sigma_0} \right] \hat{\sigma}_{ij}(\bar{x}, n)
\]
\[
\frac{\varepsilon_{ij}}{\varepsilon_0} = \alpha \left[ \frac{\sigma^\infty}{\sigma_0} \right]^n \hat{\varepsilon}_{ij}(\bar{x}, n) \tag{4}
\]
\[
\frac{u_i}{\varepsilon_0 \lambda} = \alpha \left[ \frac{\sigma^\infty}{\sigma_0} \right]^n \hat{u}_i(\bar{x}, n)
\]

where \( u_i \) is the displacement, \( \lambda \) is a length parameter, and \( \hat{\sigma}_{ij}, \hat{\varepsilon}_{ij}, \) and \( \hat{u}_i \) are dimensionless functions of spatial position \( \bar{x} \) and \( n \).

The second property follows from the first (4). Since the stress and strain at every point increase in exact proportions, the fully plastic solution (4) based on the deformation plasticity theory (3) is also the exact solution to the same problem posed for flow theory of plasticity.

The simple functional dependence of the field quantities on the applied load or displacement also mean that quantities such as the J-integral, the crack opening displacement \( \delta \), and other crack parameters have the following forms

\[
\frac{J}{\sigma^\infty \varepsilon_0 a} = \left[ \frac{\sigma^\infty}{\sigma_0} \right]^{n+1} \hat{J}(a/b, n)
\]
\[
\frac{\delta}{\varepsilon_0 a} = \left[ \frac{\sigma^\infty}{\sigma_0} \right]^n \hat{\delta}(a/b, n) \tag{5}
\]
\[
\frac{\Delta_c}{\varepsilon_0 a} = \left[ \frac{\sigma^\infty}{\sigma_0} \right]^n \hat{\Delta}_c(a/b, n)
\]

where the applied stress appears explicitly in the manner shown. The dimensionless quantities \( \hat{J}, \hat{\delta} \) and \( \hat{\Delta}_c \) are functions only of \( a/b \) and \( n \) and is independent of the applied stress \( \sigma^\infty \). The above expression (5) can also be written to depend on the remote displacement parameter \( \Delta \) (instead of \( \sigma^\infty \)), but the formula is a little more complex. The form as given by (5) can be readily employed in load or displacement controlled analyses.
The functional forms (5) are similar to those for linear elasticity (1), except the solutions depend additionally on $n$. Consequently, it is feasible to tabulate the fully plastic solutions $\hat{J}$, $\hat{\delta}$, and $\hat{\Delta}_c$ corresponding to specific values of $a/b$ and $n$ for the basic crack configurations as tabulated in elastic crack handbooks. In fact the solutions for $n = 1$ are the values tabulated in [21]. The fully plastic solutions in plane stress are readily obtained from conventional finite element techniques as discussed in [19, 20]. In plane strain, the incompressible deformation introduces constraint/constraints on the displacement gradients, and special techniques are required to handle fully plastic problems. An efficient technique for solving incompressible nonlinear problems is presented in [23] - a brief discussion of other techniques is also included. Fully plastic plane strain solutions have been obtained for several crack configurations including the compact specimen and these are discussed in [3, 24, 25, 26].

**Estimation Scheme**

Fully plastic crack solutions and analyses are applicable to situations where the cracked configurations are completely yielded, i.e., the plastic strains are large compared to elastic strains everywhere in the body. Most crack problems of practical interest are in the elastic-plastic regime; in this range an estimation scheme may be employed.

By exploiting the functional forms of the fully plastic solutions (5) and the linear elastic solutions (1), simple approximate formulas have been obtained for quantities such as $J$, $\delta$, and $\Delta_c$ which interpolates over the range from small-scale yielding to fully plastic conditions. In essence, the interpolation formulas combine the linear elastic and the fully plastic contributions and are of the form

\[
J = J(a_{\text{eff}}) + J(a, n)
\]

\[
\delta = \delta(a_{\text{eff}}) + \delta(a, n)
\]

\[
\Delta_c = \Delta_c(a_{\text{eff}}) + \Delta_c(a, n)
\]

(6)

$J(a_{\text{eff}})$, $\delta(a_{\text{eff}})$ and $\Delta_c(a_{\text{eff}})$ are the elastic contributions based on an adjusted crack length $a_{\text{eff}}$; the latter is the Irwin's effective crack length modified to account for the strain hardening [19]. $J(a, n)$, $\delta(a, n)$ and
Δ(a, n) are the plastic contribution based on material hardening exponent n*.

In small scale yielding, the plastic contribution is small compared to the elastic contribution and hence (6) reduces to the well-known elastic solutions adjusted by Irwin's effective crack length. At the other extreme, i.e. in the fully plastic range, the plastic contribution is the dominant term in (6) - thus the crack parameters can be computed by substituting (5) into (6) and ignoring the elastic terms. Formulas (6) have been found to be in good agreement with finite element calculations for the complete range of elastic-plastic deformation and material hardening properties for a number of crack configurations [19, 20].

The fully plastic solutions (5), and the estimation scheme formulas (6) in conjunction with the experimentally determined $J_R$ curve are the essential elements of the engineering approach for treating stability of crack growth under $J$-controlled growth conditions. Analyses of the extent of stable crack growth and the onset of instability parallels exactly the resistance curve approach for small scale plasticity with $J$ replacing $K$ as the basic parameter in the analyses [17].

4. DETAILS OF THE ENGINEERING APPROACH

The material in this section is largely taken from two recent articles [3, 26]. The approach will be detailed for the compact specimen and application to the single-edge crack panel is also discussed. Applications to other crack configurations are fairly obvious and straightforward.

Fully Plastic Solutions for Compact Specimens

Plane strain fully plastic solutions for pure power law materials (3) have been obtained for the compact specimen for a wide range of crack length to width ratios $a/b$ and hardening exponent $n$ using a special finite element technique developed in [23]. As discussed in the previous section, these solutions are scalable -- the $J$-integral, the mouth opening displacement $δ$ and the load line displacement $Δ_L$ are tabulated in the following forms:

---

*The material hardening exponent can be determined from matching the plastic stress-strain curve with (2).
\[ J = \alpha \sigma_o \varepsilon_o c h_1(a/b, n)(P/P_o)^{n+1} \]  
\[ \Delta = \alpha \varepsilon_o a h_2(a/b, n)(P/P_o)^n \]  
\[ \Delta_L = \alpha \varepsilon_o a h_3(a/b, n)(P/P_o)^n \]

where \( P \) is the load per unit thickness and \( c = b - a \) is the uncracked ligament. It is noted that \( h_1, h_2 \) and \( h_3 \) in the above equations are functions of \( a/b \) and \( n \) alone. \( P_o \) is the limit load per unit thickness and is given by

\[ P_o = 1.455 \eta c \sigma_o \]

where \( \eta \) is defined as

\[ \eta = \left[ \frac{(2a)^2}{c} + 2 \frac{2a}{c} + 2 \right]^{1/2} - \frac{2a}{c} + 1 \]

A detailed discussion of the above choice of the limit load expression is given in [3, 26]. Values of \( h_1, h_2 \) and \( h_3 \) are tabulated in Table 1. Fully plastic plane stress solutions are also available and these are given in [26].

**Estimation Scheme for Elastic-Plastic Compact Specimens**

Following the procedure developed in [19, 20], let us consider the Ramberg and Osgood stress-strain law which in uniaxial tension has the form

\[ \frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} + \alpha \left( \frac{\sigma}{\sigma_o} \right)^n \]

For materials which may be approximated by (12), the \( J \)-integral, mouth opening displacement \( \Delta \) and the load line displacement is given by the sum of the adjusted linear elastic and fully plastic contributions. In abbreviated notation, the elastic-plastic formulas are of the form

\[ J = J(a_e, n = 1) + J(a, n) \]

\[ \Delta = \Delta(a_e, n = 1) + \Delta(a, n) \]

\[ \Delta_L = \Delta_L(a_e, n = 1) + \Delta_L(a, n) \]

where \( a_e \) is the adjusted or effective crack length which will be defined later. Elastic-plastic formulas for another representation of stress-strain laws are discussed in [19].
The elastic terms in (13) are available from several elastic crack handbooks and are generally available in the following format:

\[ J = \frac{a \frac{F^2}{E'} (a/b)}{E' b^2} P^2 \] \hspace{1cm} (14)

\[ \delta = \frac{1}{E'} V_1 (a/b) P \] \hspace{1cm} (15)

\[ \Delta_L = \frac{1}{E'} V_2 (a/b) P \] \hspace{1cm} (16)

where \( P \) is the load per unit thickness, \( E' \) is the appropriate elastic modulus and \( F_1', V_1 \) and \( V_2 \) are tabulated in [21]. These expressions are more conveniently rewritten as

\[ J = f_1 (a) \frac{P^2}{E'} \] \hspace{1cm} (17)

\[ \delta = f_2 (a) \frac{P}{E'} \] \hspace{1cm} (18)

\[ \Delta_L = f_3 (a) \frac{P}{E'} \] \hspace{1cm} (19)

where \( f_1 = \frac{a F^2}{b^2}, f_2 = V_1 \) and \( f_3 = V_2 \).

Using the quantities defined in (7) - (9) and (17) - (19), the estimation formulas (13) for the entire range of elastic-plastic deformation have the specific forms:

\[ J = f_1 (a) \epsilon^e \frac{P^2}{E'} + \alpha \sigma_0 \epsilon_0 c h_1 (a/b, n) (P/P_0)^{n+1} \] \hspace{1cm} (20)

\[ \delta = f_2 (a) \epsilon^e \frac{P}{E'} + \alpha \epsilon_0 a h_2 (a/b, n) (P/P_0)^n \] \hspace{1cm} (21)

\[ \Delta_L = f_3 (a) \epsilon^e \frac{P}{E'} + \alpha \epsilon_0 a h_3 (a/b, n) (P/P_0)^n \] \hspace{1cm} (22)

The crack length adjustment employed in this paper is slightly different from the effective crack length of the earlier papers. To ensure continuity of the partial derivatives of \( J \) and \( \Delta_L \) with respect to applied load at \( P = P_0 \),

\[ ^* \text{The crack driving force or the tearing modulus involve such partial derivatives.} \]
the adjusted crack length is defined by

$$a_e = a + \phi r_y$$  \hspace{1cm} (23)

where

$$r_y = \frac{1}{\beta \pi} \left[ \frac{n - 1}{n + 1} \right] \left( \frac{K}{\sigma_o} \right)^2$$  \hspace{1cm} (24)

and

$$\phi = \frac{1}{1 + \left( \frac{P}{P_c} \right)^2}$$

For plane stress $\beta$ equals 2, and for plane strain $\beta$ equals 6. The length $r_y$ is based on Irwin's idea of a plastically adjusted crack length, but modified to account for strain hardening; as an additional simplification, $r_y$ is taken as an explicit function of the elastic stress intensity factor $K$ [19, 26].

**Crack Growth Stability Analyses**

The following discussion follows the treatment given in [9] and assumes that $J$-controlled growth is applicable.

At any applied load $P$ and crack length $a$, the condition for continued crack growth is

$$J(a, P) = J_R(a - a_o)$$  \hspace{1cm} (25)

where $J(a, P)$ is the crack driving force and $J_R$ is the material resistance to crack growth which is a function of the amount of crack growth only. Crack growth is unstable if

$$\left( \frac{\partial J}{\partial a} \right)_{\Delta T} > \frac{d J_R}{d a}$$  \hspace{1cm} (26)

The subscript in (26) denotes a partial derivative with the total displacement $\Delta T$ held fixed; $\Delta T$ is defined by

$$\Delta T = \Delta + C_M P$$  \hspace{1cm} (27)

where $C_M$ is the compliance of a linear spring placed in series with the cracked body. In this and subsequent section $\Delta$ and $\Delta_L$ are used interchangeably. The
crack driving force can be expressed to reveal the role of the system compliance $C_M$ through

$$\left(\frac{2J}{3a}\right)_{\Delta_T} = \left(\frac{2J}{3a}\right)_P - \frac{2J}{3a} \left[ \frac{8A}{3a} P \right]_P \left[ C_M + \left(\frac{8A}{3a} P \right)_P \right]^{-1}$$

(28)

Paris, et al. [8] introduced the nondimensional quantities

$$T_J = \frac{E}{\sigma_o} \left(\frac{3J}{2a}\right)_{\Delta_T} \quad \text{and} \quad T_{JR} = \frac{E}{\sigma_o} \frac{dJ}{da}$$

(29)

The instability criterion (26) can be phrased in terms of the tearing modulus

$$T_J > T_{JR}$$

(30)

As discussed previously, the stability of crack growth can also be analyzed through the use of the crack opening displacement parameter [16]. As in the $J$-controlled growth situation, equilibrium of crack growth requires

$$\delta(a, F) = \delta_R(a - a_o)$$

(31)

and instability develops if

$$\frac{3\delta}{3a} \left(\frac{\delta}{\Delta_T}\right) > \frac{\delta_R}{da}$$

(32)

The quantity $\frac{\delta_R}{da}$ is sometimes called the crack opening angle and is a measure of the material resistance to crack growth. The crack driving force and the crack growth resistance can be expressed again in terms of respective dimensionless parameters

$$T_\delta = \frac{E}{\sigma_o} \left(\frac{3\delta}{3a}\right)_{\Delta_T} \quad \text{and} \quad T_{\delta R} = \frac{E}{\sigma_o} \frac{d\delta_R}{da}$$

(33)

Condition (32) can be restated as

$$T_\delta > T_{\delta R}$$

(34)

An instability analyses based on COD is appealing (see Section 2) but has several difficulties. The readily measured or calculated mouth opening displacement is not a measure of the strength of the crack tip fields and is therefore not appropriate for analyses of the type discussed in the preceding
A physically meaningful characterizing parameter is the crack opening displacement at the tip of the crack. However there is as yet no general agreement or consensus on a precise definition for a near tip COD. An unambiguous definition of \( \delta_t \) is given by the separation between the intersection of 45° lines with the crack faces as shown in Figure 1. One attractive feature of this definition is the consequently explicit relation between \( \delta_t \) and \( J \), namely

\[
\delta_t = d_n \frac{J}{\sigma_o} \tag{35}\*
\]

where \( d_n \) is strongly dependent on \( n \) and weakly dependent on \( \sigma_o/E \). In plane strain values of \( d_n \) range from about 0.3 for \( n = 3 \) to about 0.8 for \( n \to \infty \). In plane stress \( d_n \) varies between 0.4 and 1 [13, 16].

Under somewhat more restrictive conditions, the slope of the \( \delta_R \) resistance curve of the crack opening \( \frac{d\delta}{da} \) is related to the slope of the \( J_R \) curve by [16]

\[
\frac{d\delta}{da} = \left( \frac{n}{n+1} \right) \frac{d_n}{\sigma_o} \frac{dJ_R}{da} \tag{36}
\]

Expressions (35) and (36) suggest that the very conditions that validate \( J \)-controlled growth will also validate COD-controlled growth.

The seemingly unchanging value of the crack opening angle or \( T_\delta R \) over an interval of crack extension, which is large compared to the size \( R \) of the HRR singularity, is an attractive feature of the COD approach. The observation is based on limited numerical and experimental results but has a fundamental appeal**. However the experimental measurement of \( \delta_t \) and \( \delta_R \) to generate the data base for the COD \( R \) curve approach remain rather difficult. Also, numerical calculations of \( \delta_t \) will be somewhat sensitive to the near tip element type and size [2, 4].

---

*Expression (35) assumes that the HRR singularity dominates a distance of several \( \delta_t \)'s in the vicinity of the tip [16].

**Rice, et al [27] obtained from theoretical studies, \( J \) resistance curves based on the attainment of a critical opening displacement \( \delta_c \) at a fixed distance \( r_m \) (comparable to the size of the fracture process zone) behind the current crack tip. The \( \delta_c-r_m \) criterion is similar to the constant crack opening angle criterion for continuing crack extension and practically equivalent when \( \psi \) is large [4, 16].
Determination of Material $J_R$ Curve

The material $J_R$ curve is an essential element of the crack growth stability analyses. Experimentally measured $J$ vs. $a - a_o$ data for test specimens which satisfy the conditions for $J$-controlled growth are employed in a statistical determination of the $J_R$ curve. Most center-cracked panel or single edge cracked panel (loaded in tension) did not meet the conditions and were excluded from the data base. The $J$ vs. $a - a_o$ data from these specimens are consistently higher than similar data from compact specimens or bend bars. Consequently, the data base is built mainly from compact specimen and bend bar data [2, 4]. Some of the crack growth data, the mean $J_R$ curve and the lower bound curve $J_{RL}$ (based on three standard deviations) are shown in Figure 2.

The straight line $J_{RL}$ is based on crack extension data of up to 0.8 inch. In the subsequent text, $J_R$ will in general denote the mean curve and $J_{RL}$ the lower bound curve. A linear curve based on limited amounts of crack extension (corresponding to about 6% of the ligament of a 4T specimen) is also indicated.

5. APPLICATIONS OF THE ENGINEERING APPROACH

The predictions of the engineering approach are compared with test measurements obtained from carefully controlled experiments and full numerical calculations based on a $J_2$ incremental plasticity theory and observed crack growth behavior. Two radically different crack configurations, namely the compact specimen and the single-edge crack panel (SECP), are selected for this study. These specimens have fundamentally different slip line fields in the fully yielded condition as shown in Figure 3.

Predictions, Experimental Measurements and Full Numerical Calculations - Compact Specimens

Four A533B test specimens are examined here. These are 4T compact specimens (25% side-grooved) and have remaining ligaments of 3.39, 3.08, 2.73 and 1.59 inches respectively. The coefficients of the Ramberg-Osgood law (12) appropriate to A533B steel at 93°C were obtained by least squares fitting of the uniaxial stress-strain data given in [2]. Values of the coefficients are $a = 1.115$ and $n = 9.7$, and $\nu$, $E$ and $\sigma_o$ are taken to be $0.3$, $29 \times 10^6$ lb./in.$^2$ and $60 \times 10^3$ lb./in.$^2$ respectively.
The J-integral crack driving force diagrams are computed for a typical 4T compact specimen using (20) - (22) for the material properties given above. The values of the f and h functions are taken from Table 1 and [21]. Figure 4 shows the crack driving force for two limiting situations - the solid line indicates the variation of J with crack length with the applied load held fixed (i.e. $C_M = \infty$, the value of the load is indicated in figure), and the dashed line corresponds to displacement $\Delta_L$ being held fixed (i.e. $C_M = 0$, the value of the displacement is also indicated). With load held fixed, the driving force increases with crack length while the $\Delta_L$ held fixed, the driving force decreases as they should. However it may be noted that for a sufficiently compliant loading system (i.e. $C_M$ is large), the crack driving force, for $\Delta_T$ held fixed, will in fact increase with crack length.

To determine the load-deformation behavior of a 4T compact specimen with initial crack length of 4.615 inches, the experimentally measured J-resistance curve denoted by $J_R$ (from Figure 2) and indicated by the heavy solid line, is superimposed on the diagram at an initial crack length of 4.615 inches. Equilibrium of crack growth require that the applied J equal the material resistance $J_R$. Thus the solid line (load) and dashed line (displacement) that intersect at a particular point on the $J_R$ curve yield the respective values of $P$ and $\Delta_L$ which are in equilibrium at that crack length. By repeating the process at different points along the $J_R$ curve, the complete load-deformation behavior is obtained.

The load-deflection behavior obtained by the above procedure, the measured load-deflection record for the A533B steel 4T compact specimen with an initial crack length of 4.615 inches, and the finite element crack growth calculations for the same configuration based on $J_2$ flow theory of plasticity, are shown in Figure 5. The agreement between all three results is very good, in fact the estimated curve follows completely the trend of the experimental data and the full numerical calculations. Details concerning the experimental data and the crack growth calculations based on flow theory are given in [2, 4, 5].

The same procedure is employed to determine the deformation behavior of the other three compact specimens. Load-displacement behavior associated with each of the three resistance curves (indicated in Figure 2) are displayed in Figures 6 through 8. The predicted behavior based on the mean $J_R$ curve are in
good agreement with the measured load-displacement record. Predictions based on $J_{RL}$ underestimates the maximum load by 10 to 15%. The predictions using a lower bound linear extrapolation of the $J_R$ data (based on a restricted range of crack growth data) lie between the other two predictions at the earlier stages of crack growth but rises above them at large amounts of growth. The behavior of the specimen (again obtained from the estimation scheme) where the crack is not allowed to grow, is included to indicate the possible errors associated with analyses which do not allow for crack growth.

The following features of the predictions from the estimation scheme are noted. At the earlier stages of crack growth the agreement with test data is rather good. With further growth, discrepancies begin to show up. Also the predictions for the larger specimens appear to be in better agreement with test data. These observations are consistent with the conditions or limitations placed on $J$-controlled growth.

**Predictions, Measurements and Full Numerical Calculations - Single-Edge Cracked Panel**

Experiments were carried out on single-edge cracked panel (SECP) loaded remotely in tension to ensure no bending across the ligament as illustrated in Figure 3 [2]. Such a configuration circumvents the ambiguity of the crack growth data associated with two crack fronts - the single crack front allows a more precise measurement of crack extension. In addition, the applied load to reach the fully yielded state is half that of a similarly sized center-cracked panel. Thus for a given test machine capacity, larger SECP specimens can be tested.

Two specimens of A533B steel were analyzed - one has a width of 12 inches and the other 6 inches. The respective ligaments are 4.95 and 1.59 inches; both specimen are 1 inch thick. Since the specimen is better approximated by plane stress assumption, the $h$ functions to be employed in the analyses are taken from the plane stress center-cracked panel solutions [19] which are reproduced in Table 2. The crack driving force for this configuration is again given by formulas (20) - (22) where the appropriate values of the $f$ and $h$ functions are taken from [21] and Table 2.

Following the procedure discussed in the previous section, the deformation
behavior of the specimens were obtained. The predictions based on the J-
resistance curves indicated in Figure 2 are shown in Figures 9 and 10. For
the larger SECP, the predictions are in good agreement with measured load-
deformation record. The predictions underestimate the load level by anywhere
from 5 to 15% in the smaller specimen.

*Stability Diagrams for Compact Specimen*

The estimation scheme (20) - (22) together with the expression (28) are
employed to compute the tearing modulus for a typical A533B steel 4T compact
specimen for a range of values of $C_M$.

In Figure 4, the crack driving force for two limiting situations are de-
picted; the solid lines correspond to the variation of the driving forces with
crack length with load held fixed (i.e. $C_M = \infty$) while the dashed line are
driving forces with displacement $\Delta_L$ held fixed (i.e. $C_M = 0$). Such diagrams
are useful for estimating the load carrying capacity and the deformation be-
behavior of the structure. In the analyses of the extent of stable crack ex-
tension and the onset of instability, it is only necessary to construct one
set of crack driving force diagrams corresponding to the actual loading applied
by the system. For example, Figure 11 shows the crack driving force diagram
for an A533B 4T compact specimen subjected to fixed grip loading (i.e. $C_M = 0$
or $\Delta_L = \Delta_T$ the value of $\Delta_L$ is indicated in figure) - the experimentally deter-
mined $J_R$ curve is superimposed on the diagram at an initial crack length of
4.615 inches. Crack growth in this configuration will clearly be stable since
the driving force decreases with increase of crack length while the $J_R$ curve
rises rapidly with crack growth. In other words, the instability condition
posed by (26) or (30) cannot be met in this configuration. This can be ob-
served by sliding the $J_R$ curve along the crack length axes to correspond to
different initial crack sizes.

The crack driving force in a soft loading system say $C_M = 1000$ ($C_M = E C_N$)
acting on the compact specimen is displayed in Figure 12. The driving force
corresponds to $\Delta_T$ held fixed, and the value of $\Delta_T$ is indicated. From this
diagram, it is clear that a 4T A533B compact specimen with initial crack
length of 4.615 inch will withstand about 0.4 inch of stable growth (reaching
a $J$ value of about 12,000 in. lb./in.$^2$). At the onset of instability the
tearing modulus $T_J$ (applied) and $T_{JR}$ (material) are equal and the value is
given by the slope at the tangential contact. The applied load at instability is about 42 kips (unit thickness) while the maximum load is 47 kips (attained at $\Delta a = 0.12$ in., see Figure 4).

The crack driving force diagram for an infinitely soft system (i.e. $\bar{C}_M = \infty$ or dead load system) is shown in Figure 13 (the value of the applied load is indicated in figure). For the dead load situation, the crack will grow stably for about 0.12 inch - instability occurs at an applied load of 47 kips (per unit thickness) at a $J$ value of about 6,000 in. lb./in.$^2$. The tearing modulus at instability is again given by the slope at the tangential contact.

The three examples illustrate that the extent of stable growth is strongly dependent on the loading system and the material properties. The amount of stable growth decreases with increasing compliance - the value of $J$, $T_J$ and the applied load at instability also depend on the compliance of the loading system. The crack driving force diagram is really a procedure to obtain a graphical solution to crack growth stability as posed by (25) and (26). In this approach, the slope of the $J_R$ can vary with crack extension; the strain hardening properties of the material and the system compliance are accounted for in the driving force. Furthermore the extent of stable growth prior to instability, the value of $J$ and $T_J$ at instability and the load carrying capacity of the configuration at various stages of growth are explicitly given by the graphical solution.

In some applications, it is convenient to explicitly display $T_J$ as a function of relevant crack parameters and there are many possible combinations [3]. As an example, the tearing modulus 'applied' $T_J$ for plane strain conditions is shown in Figure 14 as a function of $J$ normalized by $c_0^2/E$ for a typical structural steel. The specimen has an $a/b$ ratio of 0.6 and the wide range of $\bar{C}_M$ ($\bar{C}_M = E C_M$) is intended to correspond to a typical rigid grip test machine and a soft loading machine. For a typical test machine with $C_M$ ranging from 10 to 100, $T_J$ will always be less than 5. Now $T_{JR}$ for most structural steels ranges from 20 to 200 [2, 4, 8]. Thus it is apparent that stable crack extension will almost always be observed in compact specimens tested in a typical rigid or displacement controlled machine. However it can be seen that the driving force $T_J$ increases fairly rapidly with compliance.

The plane stress $T_J$ based on the $h$ functions for plane stress are shown
in Figure 15 [26]. A consistent difference between plane strain and plane stress driving force is noted. With all quantities being equal, i.e., the in-plane dimensions, material deformation properties, etc., $T_J$ associated with plane strain condition is larger than that for plane stress condition. This was also noted in [9]. The treatment of crack growth and instability as given here can be extended to other cracked structural configurations.

### Predicting $J_R$ Curve from Load-Displacement Record

There are materials (fracture tests carried out before the J-integral approach became widely known and tests with irradiated specimens) where the $J_R$ curve were not or could not be measured, but the load-displacement record for an extended interval of stable crack growth are available. For these materials it is possible to construct the $J_R$ curve from the load-displacement record.

Using the appropriate deformation properties, the J-integral crack driving force are computed according to (20) - (22) for the specific crack configurations under consideration. On the J-integral diagram identify the solid line and the dash line (e.g. Figure 4) corresponding to the respective measured pair of load and load-line displacement. The intersection of these two lines yield the value of J and crack length a that satisfy the prescribed $P$ and $A_L$ for this particular crack configuration. By repeating the process for other measured pairs of load and load-line displacement, the $J_R$ curve may be constructed.

In Figure 16, the $J_R$ curve determined from the J-integral driving force diagrams in conjunction with the load-displacement record are compared with actual experimental data from two specimens. We note the good agreement between the predicted and actual curve. The comparison clearly suggest that the $J_R$ curve can be estimated to a fair degree of accuracy using (20) - (22) and the load-displacement record.

A note of caution must be made. The $J_R$ curve obtained from the specimen load-displacement record using the above procedure or similar procedures will be the proper material curve only if the conditions for $J$-controlled growth are satisfied by the specimen in concern. Otherwise, the estimated $J_R$ curve will be nonconservative as evident in the lower figure in Figure 13 - this
aspect will become clear in the next section.

6. DISCUSSION

The estimation scheme (20) - (22) coupled with a crack initiation criteria, based either on \( J_{IC} \) or \( \delta_{IC} \), could be employed to assess the combination of load and crack length which will cause the onset of crack growth in a structure. Such analyses assume that the HRR singularity \([10, 11]\) dominates over a microstructurally meaningful distance at the crack tip. The conditions and the minimum size requirements essential to the validity of a one parameter crack initiation criterion based on \( J \) or \( \delta_t \) (\( J \) and \( \delta_t \) are the amplitudes of the HRR singularity) have been discussed in \([12, 13]\). In essence, the uncracked ligament must be larger than some multiple of \( J_{IC}/\sigma_o \) or \( \delta_{IC} \) - the multiple being strongly dependent on crack configuration.

Similarly continued crack growth and growth instability can be examined using (20) - (22) coupled with the growth and stability criteria (25) and (26) (or with (31) and (32)). The amount of stable crack extension and the onset of crack growth instability is clearly dependent on the system compliance \( C_M \) and the material resistance to fracture as characterized by \( J_{IC} \) and \( T_{JR} \). The conditions for \( J \)-controlled growth are discussed in \([9, 14, 15]\) and are summarized below:

\[
\Delta a < 0.06 \, c \\
\omega > \frac{0.6}{g_n} \\
c > M J/\sigma_o \\
B > c
\]

(37)

For a specimen subjected to bending growth of up to 6% of the uncracked ligament could be termed \( J \)-controlled. From numerical studies on moderately hardening materials \( g_n \) is about 0.06 for bend specimen and less than 0.01 for tensile specimen*. Thus \( \omega \) should be larger than about 10 for bend configurations and larger than about 80 for tensile configurations. Similarly \( M \) is about 25 and 200 for the former and latter configurations respectively. The

*\( g_n \) is the ratio of the characteristic radius \( R \) of the HRR singularity to the remaining ligament, i.e. \( g_n = R/c \).
condition on the relevant thickness dimension B is intended to maintain plane
strain constraint along the crack front and in the yielded ligament. Numerical
studies also suggest that large values of \( \psi (\psi = \frac{1}{\sigma_0} \frac{dJ}{da}) \) promote J controlled
growth*. The \( \psi \) parameter is proportional to the crack opening angle \( \delta_R/da \).

The conditions (37) for J-controlled growth limit the applicability of
the method to small amounts of crack growth. Nevertheless the method could
be cautiously employed for larger amounts of growth (say 20% of the ligament)
for the following reasons. The deformation theory solutions assures that the
estimated J crack driving force (20) for the stability analyses is always
larger than the actual force made available by the configuration and system in
the non-J-controlled growth regime. Furthermore experimental data show that
the crack growth resistance curve rises above the valid \( J_R \) curve as conditions
(37) are increasingly violated. Thus the J-controlled growth approach will
lead to conservative predictions when conditions (37) are violated.

We return to the analyses carried out in Section 5. The material \( J_R \)
curve for the specific heat of A533B steel employed in the analyses were deter-
mined using valid J-resistance data, i.e., mainly test data from compact speci-
mens. SECP test data did not meet requirements for J-controlled growth and
were excluded. The predicted load-deformation behavior for the compact speci-
mens using the engineering approach to fracture are in close agreement with
the test data for the larger specimens. As the test specimens become smaller,
the predictions underestimate the load carrying capacity of the specimen.
Similar evaluations of this type are given in [3, 26]. In other words, as the
conditions for J-controlled growth are increasingly violated, the predictions
of the load carrying capacity of the flawed structure and the final fracture
event become increasingly conservative.

The latter observation is confirmed by the analyses of the SECP (loaded
in tension) using the J-controlled growth methodology. For the larger speci-
men (\( \omega = 80 \)), the predicted loads are slightly below the measured values. In
the smaller specimen (\( \omega = 30 \)), the predictions are rather conservative. Thus

*This is certainly true of unirradiated A533B steel where \( \psi \) is about 0.3
to 0.5. For material with \( \psi \) less than about 0.1 but has the same value of \( J_{IC} \)
the relevant configuration dimension must increase by a factor of 3 to 5 for J-controlled
growth. It should also be noted that the deformation and flow theory definition of \( dJ \) (see [27]) differ by second order times when \( \psi \) is
large compared to the rotation of the bend specimen.
in the crack growth regime which is not well understood and not readily characterizable, the method is conservative.

The load-deformation behavior, with no crack growth allowed, are indicated by the monotonically rising curves in Figures 6 to 10 - the difference between the stationary and growing crack is substantial. Crack growth greatly lowers the load carrying capacity of specimens. Thus limit load analyses which do not account for stable crack growth may not be conservative at all.

In one application related to the leak rate analyses of pipes, the mouth opening area (MOA) of a through crack in a pipe subjected to internal fluid pressure (i.e. axial load only) were determined by the engineering fracture approach. The estimated MOA were 10% to 30% larger than the measurements made by Battelle [28] - values obtained from the Dugdale or Goodier and Field models significantly underestimated the test data. The results are summarized in Figure 17. Details of the analyses are given in [2].

It may be noted that with a slightly different interpretation of (2), the fully plastic results (e.g. Tables 1 and 2) are directly applicable to a stationary crack in a material undergoing steady state power law creep [24]. Landes and Begley [29] successfully correlated creep crack growth data by the $C^*$ parameter. They employed a rather complex experimental technique to obtain $C^*$. If the steady state creep properties are known, then $C^*$ is readily determined from (5) (replace $J$ by $C^*$ in (5)). Haigh [30] proposed to use $\delta_t$ as a creep crack growth parameter. The $C^*$ or $\delta_t$ approach are equivalent (as $J$ and $\delta_t$ are both equivalent in time independent plastic fields [16]) and both may be computed from the fully plastic results which are catalogued in the plastic fracture handbook. The regimes of applicability of the $K$ parameter and the $C^*$ or $\delta_t$ parameters have been discussed by Riedel and Rice [31].

7. **CONCLUSIONS**

An engineering approach for treating crack growth and the onset of instability in flawed structures has been discussed in some detail. The essential elements of the approach are:
1. A Plastic Fracture Handbook containing fully plastic solutions of fracture mechanics specimens and flawed structural components. The handbook will be oriented toward easy use by fracture analysts and designers.

2. Estimation schemes which will enable the construction of elastic-plastic solutions and the crack driving force diagrams for cracked structural configurations through the combination of results from the Plastic Fracture Handbook and existing elastic fracture handbooks.

3. A methodology for predicting the extent of stable crack growth and the onset of instability based on the resistance curve concept and J-controlled crack growth. The material J_\infty curve for the configuration being evaluated is also an essential element.

The approach is simple and can be readily employed in design applications and safety margin analyses. The limited investigations carried forth show that the method describes accurately the fracture behavior of simple crack configurations.

Some tentative conclusions may be reached from the crack growth analyses of the two radically different crack configurations - one satisfying the conditions of J-controlled growth while the other clearly do not. Under J-controlled growth conditions, the engineering approach presented in the paper will give slightly conservative estimates of the load carrying capacity of the flawed structure. In non-J-controlled growth situations, the engineering approach will be rather conservative. In the latter situations, the crack driving force determined from the deformation plasticity solutions will always be larger than the "actual" force made available by the configuration and the system. Furthermore, the resistance of the configuration to growth is actually larger than that characterized by the so-called material J_\infty curve. These two features of the approach based J-controlled growth are quite desirable in view of the fact that there is no other simple fracture mechanics based approach for treating crack growth under large-scale plasticity. In applications where the material tearing modulus is extremely high and the structure fails by plastic collapse, the above approach will still yield a conservative estimate of the collapse load. Contrary to some current thinking, limit load analyses may not give a lower bound to the collapse load because stable crack growth proceeding collapse
is not accounted for.

ACKNOWLEDGMENT

The author acknowledges the assistance rendered by Dr. V. Kumar and Mrs. M. D. German without which this work could not be accomplished. This work was sponsored by the Electric Power Research Institute (EPRI), Palo Alto, California. The encouragements given by Dr. D. F. Mowbray of General Electric Company and Drs. R. L. Jones and T. U. Marston of EPRI is gratefully acknowledged.

REFERENCES


<table>
<thead>
<tr>
<th>a/b = 1/4</th>
<th>h₁</th>
<th>h₂</th>
<th>h₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 1</td>
<td>2.227</td>
<td>2.048</td>
<td>1.783</td>
</tr>
<tr>
<td>n = 2</td>
<td>2.048</td>
<td>1.783</td>
<td>1.475</td>
</tr>
<tr>
<td>n = 3</td>
<td>1.783</td>
<td>1.475</td>
<td>1.334</td>
</tr>
<tr>
<td>n = 5</td>
<td>1.475</td>
<td>1.334</td>
<td>1.248</td>
</tr>
<tr>
<td>n = 7</td>
<td>1.334</td>
<td>1.248</td>
<td>1.258</td>
</tr>
<tr>
<td>n = 10</td>
<td>1.248</td>
<td>1.258</td>
<td>1.325</td>
</tr>
<tr>
<td>n = 13</td>
<td>1.258</td>
<td>1.325</td>
<td>1.566</td>
</tr>
<tr>
<td>n = 16</td>
<td>1.325</td>
<td>1.566</td>
<td>14.563</td>
</tr>
<tr>
<td>n = 20</td>
<td>14.563</td>
<td>10.887</td>
<td>9.371</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>a/b = 3/8</th>
<th>h₁</th>
<th>h₂</th>
<th>h₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 1</td>
<td>2.148</td>
<td>1.716</td>
<td>1.392</td>
</tr>
<tr>
<td>n = 2</td>
<td>1.716</td>
<td>1.392</td>
<td>0.970</td>
</tr>
<tr>
<td>n = 3</td>
<td>1.392</td>
<td>0.970</td>
<td>0.693</td>
</tr>
<tr>
<td>n = 5</td>
<td>0.970</td>
<td>0.693</td>
<td>0.443</td>
</tr>
<tr>
<td>n = 7</td>
<td>0.693</td>
<td>0.443</td>
<td>0.276</td>
</tr>
<tr>
<td>n = 10</td>
<td>0.443</td>
<td>0.276</td>
<td>0.176</td>
</tr>
<tr>
<td>n = 13</td>
<td>0.276</td>
<td>0.176</td>
<td>0.098</td>
</tr>
<tr>
<td>n = 16</td>
<td>0.176</td>
<td>0.098</td>
<td>0.370</td>
</tr>
<tr>
<td>n = 20</td>
<td>0.098</td>
<td>0.370</td>
<td>10.887</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>a/b = 1/2</th>
<th>h₁</th>
<th>h₂</th>
<th>h₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 1</td>
<td>1.935</td>
<td>1.509</td>
<td>1.242</td>
</tr>
<tr>
<td>n = 2</td>
<td>1.509</td>
<td>1.242</td>
<td>0.919</td>
</tr>
<tr>
<td>n = 3</td>
<td>1.242</td>
<td>0.919</td>
<td>0.685</td>
</tr>
<tr>
<td>n = 5</td>
<td>0.919</td>
<td>0.685</td>
<td>0.461</td>
</tr>
<tr>
<td>n = 7</td>
<td>0.685</td>
<td>0.461</td>
<td>0.314</td>
</tr>
<tr>
<td>n = 10</td>
<td>0.461</td>
<td>0.314</td>
<td>0.216</td>
</tr>
<tr>
<td>n = 13</td>
<td>0.314</td>
<td>0.216</td>
<td>0.132</td>
</tr>
<tr>
<td>n = 16</td>
<td>0.216</td>
<td>0.132</td>
<td>0.317</td>
</tr>
<tr>
<td>n = 20</td>
<td>0.132</td>
<td>0.317</td>
<td>0.236</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>a/b = 5/8</th>
<th>h₁</th>
<th>h₂</th>
<th>h₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 1</td>
<td>1.763</td>
<td>1.449</td>
<td>1.237</td>
</tr>
<tr>
<td>n = 2</td>
<td>1.449</td>
<td>1.237</td>
<td>0.974</td>
</tr>
<tr>
<td>n = 3</td>
<td>1.237</td>
<td>0.974</td>
<td>0.752</td>
</tr>
<tr>
<td>n = 5</td>
<td>0.974</td>
<td>0.752</td>
<td>0.602</td>
</tr>
<tr>
<td>n = 7</td>
<td>0.752</td>
<td>0.602</td>
<td>0.459</td>
</tr>
<tr>
<td>n = 10</td>
<td>0.602</td>
<td>0.459</td>
<td>0.347</td>
</tr>
<tr>
<td>n = 13</td>
<td>0.459</td>
<td>0.347</td>
<td>0.248</td>
</tr>
<tr>
<td>n = 16</td>
<td>0.347</td>
<td>0.248</td>
<td>0.368</td>
</tr>
<tr>
<td>n = 20</td>
<td>0.248</td>
<td>0.368</td>
<td>9.371</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>a/b = 3/4</th>
<th>h₁</th>
<th>h₂</th>
<th>h₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 1</td>
<td>1.709</td>
<td>1.424</td>
<td>1.263</td>
</tr>
<tr>
<td>n = 2</td>
<td>1.424</td>
<td>1.263</td>
<td>1.003</td>
</tr>
<tr>
<td>n = 3</td>
<td>1.263</td>
<td>1.003</td>
<td>0.864</td>
</tr>
<tr>
<td>n = 5</td>
<td>1.003</td>
<td>0.864</td>
<td>0.717</td>
</tr>
<tr>
<td>n = 7</td>
<td>0.864</td>
<td>0.717</td>
<td>0.575</td>
</tr>
<tr>
<td>n = 10</td>
<td>0.717</td>
<td>0.575</td>
<td>0.448</td>
</tr>
<tr>
<td>n = 13</td>
<td>0.575</td>
<td>0.448</td>
<td>0.345</td>
</tr>
<tr>
<td>n = 16</td>
<td>0.448</td>
<td>0.345</td>
<td>0.665</td>
</tr>
<tr>
<td>n = 20</td>
<td>0.345</td>
<td>0.665</td>
<td>10.887</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>a/b + 1</th>
<th>h₁</th>
<th>h₂</th>
<th>h₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 1</td>
<td>1.568</td>
<td>1.45</td>
<td>1.35</td>
</tr>
<tr>
<td>n = 2</td>
<td>1.45</td>
<td>1.35</td>
<td>1.18</td>
</tr>
<tr>
<td>n = 3</td>
<td>1.35</td>
<td>1.18</td>
<td>1.08</td>
</tr>
<tr>
<td>n = 5</td>
<td>1.18</td>
<td>1.08</td>
<td>0.95</td>
</tr>
<tr>
<td>n = 7</td>
<td>1.08</td>
<td>0.95</td>
<td>0.85</td>
</tr>
<tr>
<td>n = 10</td>
<td>0.95</td>
<td>0.85</td>
<td>0.73</td>
</tr>
<tr>
<td>n = 13</td>
<td>0.85</td>
<td>0.73</td>
<td>0.63</td>
</tr>
<tr>
<td>n = 16</td>
<td>0.73</td>
<td>0.63</td>
<td>1.136</td>
</tr>
<tr>
<td>n = 20</td>
<td>0.63</td>
<td>1.136</td>
<td>10.887</td>
</tr>
</tbody>
</table>
TABLE 2. $h_1$, $h_2$ and $h_3$ for Plane Stress Center Cracked Panel in Tension.

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>$n = 1$</th>
<th>$n = 1.5$</th>
<th>$n = 2$</th>
<th>$n = 3$</th>
<th>$n = 5$</th>
<th>$n = 7$</th>
<th>$n = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/8</td>
<td>2.800</td>
<td>3.231</td>
<td>3.543</td>
<td>4.00</td>
<td>4.518</td>
<td>4.761</td>
<td>4.861</td>
</tr>
<tr>
<td></td>
<td>$h_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.347</td>
<td>0.491</td>
<td>0.640</td>
<td>0.949</td>
<td>1.537</td>
<td>2.048</td>
<td>2.630</td>
</tr>
<tr>
<td>1/4</td>
<td>$h_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.544</td>
<td>2.820</td>
<td>2.972</td>
<td>3.140</td>
<td>3.195</td>
<td>3.106</td>
<td>2.896</td>
</tr>
<tr>
<td></td>
<td>$h_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.611</td>
<td>0.820</td>
<td>1.010</td>
<td>1.352</td>
<td>1.830</td>
<td>2.083</td>
<td>2.191</td>
</tr>
<tr>
<td>1/2</td>
<td>$h_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.211</td>
<td>2.242</td>
<td>2.195</td>
<td>2.056</td>
<td>1.811</td>
<td>1.643</td>
<td>1.465</td>
</tr>
<tr>
<td></td>
<td>$h_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.382</td>
<td>2.182</td>
<td>2.003</td>
<td>1.703</td>
<td>1.307</td>
<td>1.084</td>
<td>0.892</td>
</tr>
<tr>
<td></td>
<td>$h_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.924</td>
<td>1.085</td>
<td>1.180</td>
<td>1.254</td>
<td>1.183</td>
<td>1.051</td>
<td>0.888</td>
</tr>
<tr>
<td>3/4</td>
<td>$h_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.073</td>
<td>1.893</td>
<td>1.708</td>
<td>1.458</td>
<td>1.208</td>
<td>1.082</td>
<td>0.956</td>
</tr>
<tr>
<td></td>
<td>$h_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.611</td>
<td>1.215</td>
<td>0.970</td>
<td>0.685</td>
<td>0.452</td>
<td>0.361</td>
<td>0.292</td>
</tr>
<tr>
<td></td>
<td>$h_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.933</td>
<td>0.886</td>
<td>0.802</td>
<td>0.642</td>
<td>0.450</td>
<td>0.361</td>
<td>0.292</td>
</tr>
</tbody>
</table>
FIGURE 1. Sharp crack and deformed crack profile illustrating the 45° procedure for defining crack tip opening displacement.
FIGURE 2. Portion of J-resistance data for A533B steel at 93°C [2, 4, 5]. The mean $J_R$ curve, $J_{RL}$, based on crack growth data of up to an inch, and a lower bound $J$ based on about 6% growth limit in 4-inch ligament are also indicated.
FIGURE 3. Slip line field for fully yielded compact specimen and single-edge cracked panel.
FIGURE 4. A typical J-integral crack driving force diagram for 4T compact specimen - A533B steel at 200°F, $E = 29 \times 10^3$ ksi, $\nu = 0.3$, $\sigma = 60.1$ ksi, $\alpha = 1.115$, $n = 9.708$. The maximum load is given by the tangential contact of the constant load curve with the $J_R$ curve as indicated. Under load-controlled condition, the maximum load is the point of fracture instability.
FIGURE 5. Comparison of predicted and experimentally measured load-displacement relationship for 4T A533B steel compact specimen with C = 3.38 in., \( \Psi_M = 0.40 \), \( \omega = 60 \). Results from full numerical calculations based on \( J_2 \) flow theory of plasticity are also included.
FIGURE 6. Comparison of predicted load-deformation behavior with test data for 4T A533B steel compact specimen, C = 3.08 in., $\Psi_M = 0.40$, $\omega = 50$. 
FIGURE 7. Comparison of predicted load-deformation behavior with test data for 4T A533B steel compact specimen, $C = 2.73$ in., $\psi_m = 0.40$, $\omega = 45$. 

LOAD VERSUS LOAD LINE DISPLACEMENT

- Experiment
- Stationary Crack
- Mean $J_R$
- Lower Bound $J_{RL}$
- Lower Bound $J_R$ (6% Crack Growth)
FIGURE 8. Comparison of predicted load-deformation behavior with test data for 4T A533B steel compact specimen, $C = 1.59$ in., $\psi_M = 0.40$, $\omega = 25$. 
FIGURE 9. Comparison of predicted load-deformation behavior with test data for single-edge cracked panel loaded in tension, b = 12 in., c = 4.95 in., ψ_M = 0.40, ω = 82.
FIGURE 10. Comparison of predicted load-deformation behavior with test data for single-edge cracked panel loaded in tension, $b = 6$ in., $C = 1.59$ in., $\psi_M = 0.40$, $\omega = 25$. 

- LOAD VERSUS MOUTH OPENING DISPLACEMENT

- Experiment
- Stationary Crack
- Mean $J_R$
- Lower Bound $J_{RL}$
- Lower Bound $J_R$ (6% Crack Growth)
FIGURE 12. J Crack Driving Force for Soft Loading System, i.e. $\bar{C}_M = 1000 - 4T$ A533B Steel Compact Specimen.
Figure 14. Plane strain numerical results for $T_J$ vs. $EJ/(\sigma_0^2 c)$ for A533B steel compact specimen with $a_0/b = 0.75$. Typical test machine compliance $C_M = 10$. 
FIGURE 15. Plane stress numerical results for $T_J$ vs. $EJ/(\sigma_0^2 c)$ for A533B steel compact specimen with $a_o/b = 0.75$. Typical test machine compliance $C_M = 10$. 
FIGURE 16. $J_R$ curve predicted from the measured load-displacement records for compact specimen T52 and T32 in conjunction with the crack driving force diagram. The experimentally measured data for T52 and T32 are indicated.
FIGURE 17. Predicted mouth opening areas for two \( a/b \) ratios and two definitions of \( \sigma_{\text{net}} \) - Battelle test data for \( a/b = 0.25 \) is indicated [28]. Results from Goodier and Field solutions are also included.
SIZE AND GEOMETRY EFFECTS ON ELASTIC-PLASTIC
FRACTURE CHARACTERIZATION

J. D. Landes
Structural Behavior of Materials Department
Westinghouse R&D Center
Pittsburgh, PA 15235

ABSTRACT

The ultimate goal of the elastic-plastic fracture technology development is the use of laboratory specimen test results to determine the fracture behavior of large structures. Incorporated in this is the development of analysis techniques as well as laboratory test methods and basic material fracture properties.

One of the most attractive methods for developing this fracture methodology is the use of a characterizing parameter. This allows the ductile fracture process to be broken into four steps: crack tip blunting, the initiation of a tearing crack, the stable propagation of this crack and ultimate ductile instability. Each step can be characterized separately. Many parameters have been proposed for characterizing these fracture steps, several of them have attractive features. In this presentation J is proposed as a single characterizing parameter. J is not a universal choice for a characterizing parameter, however, it has many attractive features. It is easy to measure in the laboratory using loading parameters taken remotely from the crack tip and can be calculated analytically for a structure without a need for a detailed crack tip analysis. In a recent paper Turner concluded that J may have limitations, however, no other approach offers a more fundamental understanding of fracture behavior. In fact other theories can often be expressed as simplifications of the J approach implying that they are less widely applicable. The J approach will be used to discuss each of the four steps in the fracture process, to illustrate
remaining problems with fracture characterization and to indicate directions for future work.

The crack tip blunting process is characterized by J through an inference to the crack tip opening displacement and may not be represented by a single expression as assumed in the \( J_{IC} \) test method. However, it is used in this test as a reference for taking the initiation point of stable cracking. For the lower toughness, higher strength materials the choice of a blunting line makes little difference on the \( J_{IC} \) value. For higher toughness, lower strength materials this choice may make a difference; however, for these materials the toughness is often better represented by the slope of the R curve (J versus stable crack extension, \( \Delta a \)) than by \( J_{IC} \).

The point of initiation of stable crack growth is labeled \( J_{IC} \) and is often taken as the primary fracture toughness criterion. A standard ASTM \( J_{IC} \) test method is in the advanced stages of development. Tests have been conducted on many metal alloys which show that the proposed \( J_{IC} \) method works very well for characterizing the initiation point of the fracture process. These tests include two round robin series in which laboratories from all over the world have successfully used this test method.

Although the \( J_{IC} \) criterion works very well for characterizing fracture toughness, these values are sometimes too conservative for given applications. The stable crack extension in the fracture process often represents a region of safe fracture behavior which may extend very far beyond the \( J_{IC} \) point. Since the region of stable crack extension may terminate with a ductile instability, the characterization of this instability event is a necessary step in verifying a region of safe stable crack extension. This instability event has been characterized in terms of J by Paris, et al., through use of the tearing modulus approach by Shih, et al., by ductile R curve characterization. These methods of instability prediction assume that, a given region of stable crack extension can be characterized by J. Several analyses have shown that this is possible
and have developed criteria for establishing regions of "J controlled crack growth". These criteria essentially give the conditions for which the stress and strain field at the crack tip are characterized by J. They include a criterion for a stationary crack tip which relates the value of J and flow stress to the size of the remaining uncracked ligament and a criterion for an advancing crack tip which relates the value of J and the R curve slope to the size of the remaining ligament. An additional criterion limits the maximum allowable crack extension. These criteria are dependent on the type of loading applied to a structure, being much more severe when the loading is primarily tension as opposed to primarily bending loads. The data available for most structural steels show that the criterion based on a stationary crack is usually more severe than the one for a growing crack.

Results have shown that both the initiation of cracking and the stable crack extension can be characterized by J independently of specimen size when the specimen type is primarily a bend one. No results are available to show that this behavior can be characterized by J when the specimen type is primarily a tension one. These data show no consistency between stable cracking as characterized by J when the bend and tension specimen results are compared. All of the results from tension specimens suffer from the fact that the conditions for J controlled growth are not met and perhaps this inconsistency should be expected. Also, the tension specimen sizes needed to meet these conditions seem to be prohibitively large for structural steels and a positive test for J controlled growth may be difficult. A major concern is that many large structures have primarily tension loading while the test specimens are primarily of the bend type. If a consistency in stable crack characterization cannot be demonstrated between tension and bending loading many applications of laboratory test data to structural analysis may be doubtful. This area of concern will receive more attention in the future.

A question of interest for the development of laboratory test results is whether or not these specimens should be side grooved. Side
grooved specimens should develop more crack tip constraint and may be more representative of a crack in a large structure. Results from tests of side grooved specimens have shown that the slope of the stable crack growth region, $dJ/da$, is always lower than that for smooth specimens. However, these results do not show that the side grooved specimen results are more representative of the behavior of a crack in a large structure. The side grooved specimens are often easier to test especially when the stable crack growth region is measured by a single specimen technique. This result plus the observation that the side grooved specimen data appears to always be conservative makes side grooving of specimens attractive for many applications.

An additional problem with the four step characterization of the ductile fracture process arises for steels near the fracture mode transition temperature. Very often the stable crack advance process is interrupted by a sudden unstable cleavage fracture. When this happens, the ductile instability predictions cannot be used and the point of cleavage fracture must be taken as the toughness value. The problem is further complicated by the observation that the cleavage fracture point shows a good deal of scatter from one specimen to another. Large specimens often show a lower cleavage fracture point than smaller specimens. Sometimes the smaller specimen fractures in a completely ductile mode while at the same temperature the larger specimen exhibits cleavage fracture implying that the smaller specimen test results will not adequately characterize the fracture behavior of a large structure. This problem should be addressed in future work. At present, a statistical method proposed for using small specimen toughness results to predict the fracture behavior of the large structure shows promise.

A final concern is how to handle fracture prediction when the conditions for J controlled crack growth cannot be met. Certainly the need to guarantee safe fracture behavior beyond the $J_{IC}$ initiation point is unnecessary for many applications and a $J_{IC}$ fracture criterion, which does not depend on J controlled growth, can be used. When the absence of
J controlled growth is due to a very low slope for the dJ/da curve, a $J_{IC}$ criterion is likely to be the only safe one. For the very tough materials where the absence of J controlled growth is due to the very high values of J attained, there appears to be little need for concern. The results obtained to date, although not completely verified, indicate that the absence of J controlled growth makes the toughness appear to be greater than that for J controlled growth. Results from tension specimens which do not meet these requirements always show higher values of J required for a given crack extension than those for the bend specimens which do meet the J controlled growth requirements. If the laboratory test results do meet this requirement and the structure does not, then an assumption of J controlled growth would appear to give a conservative fracture prediction. Future work should give more insight on how to deal with this problem.

In summary, J can be used as a single characterizing parameter to correlate four steps in the ductile fracture process. The use of $J_{IC}$ to characterize the initiation of stable cracking provides a primary fracture criterion. The resistance to stable cracking taken from an R curve can be used to predict ductile instability when the cracking is J controlled. Much of the future work will deal with the verification of conditions for J controlled cracking and demonstration of the applicability of ductile instability criteria for the prediction of fracture behavior in structures.
Start: Sharp Crack Tip (Fatigue Precrack)

Step 1: Crack Tip Blunting

Step 2: Initiation of Stable Crack Growth

Step 3: Continued Stable Crack Growth

Step 4: Ductile Instability

Fig. 10—Four steps in ductile fracture process
Fig. 1 – Four steps of ductile fracture process on an R-curve
Fig. 2—J vs. Δa R-curve for A508 Cl 2 steel from IT compact specimens (297°K)
BLUNTING LINE INTERPRETATION

$$CTOD = d \frac{J}{\sigma_y}$$

$$\Delta a_{blunting} \approx \frac{CTOD}{2}$$

If $$d = 1$$

$$\Delta a_{blunting} = \frac{1}{2} \frac{J}{\sigma_y}$$

Experimental: $$0.35 < d < 1$$

General Observation:

1) For Low $$\frac{dj}{da}$$, Blunting Line Position is Not Important in $$J_{lc}$$ Determination

2) For High $$\frac{dj}{da}$$, $$J_{lc}$$ is Somewhat Ambiguous and $$\frac{dj}{da}$$ is More Important
Fig. 3 - Schematic of method used to determine J_Q point.

- Blunting Line
- 0.15 mm Offset Line
- R-Line (Regression Line)

J Integral M {Joules/m}^2

Crack Extension - Δa mm

Δa (min)

Δa (max)

J_Q
Steel Alloys
Ductile (Upper Shelf) Fracture
Compact Specimens

○ $J_{IC}$ Point

Fig.  - $J$ vs $\Delta a$ for several steel alloys—upper shelf temperatures
Fig. 15—Plastic instability criterion

\[ T = \frac{dJ}{da} \frac{E}{\sigma_0^2} \]

Paris Instability

\[ T_{\text{matl}} \leq T_{\text{appl}} \]
Fig. 64 - Experimental results from instability studies on a 3 point bend bar, $T_{MAT}$ vs $T_{appl}$
J Integral crack driving force diagram, Shih, et al.
1. Crack Growth Limit
   \[ \Delta a < 0.06 b \]

2. R Curve Slope
   \[ \omega = \frac{b}{J} \frac{dJ}{da} > \begin{cases} 
   2.5 & \text{Bend Specimen} \\
   20 & \text{Tension Specimen} 
\end{cases} \]

3. Specimen Size Requirement
   \[ b > M \frac{J}{\sigma_Y} \]
   \[ M = \begin{cases} 
   25 & \text{Bend Specimen} \\
   200 & \text{Tension Specimen} 
\end{cases} \]

4. Plane Strain
   \[ B > b \]

Fig. — Conditions for J controlled stable crack growth
<table>
<thead>
<tr>
<th>Mat'l</th>
<th>J_{lc}</th>
<th>dJ/da</th>
<th>J_{lc}/dJ</th>
<th>a_y</th>
<th>b &gt; M \frac{J_{lc}}{a_y}</th>
<th>b &gt; \frac{S_{lc}}{dJ/da}</th>
</tr>
</thead>
<tbody>
<tr>
<td>6061 Al</td>
<td>100 in-lb/in^2</td>
<td>500 psi</td>
<td>0.2 in</td>
<td>50 ksi</td>
<td>0.05 in</td>
<td>0.4 in</td>
</tr>
<tr>
<td>Steels</td>
<td>600</td>
<td>$1 \times 10^4$</td>
<td>0.06</td>
<td>140</td>
<td>0.10</td>
<td>0.85</td>
</tr>
<tr>
<td>NiCrMoV</td>
<td>900</td>
<td>$1.5 \times 10^4$</td>
<td>0.06</td>
<td>150</td>
<td>0.15</td>
<td>1.2</td>
</tr>
<tr>
<td>HY-130</td>
<td>1500</td>
<td>$2.5 \times 10^4$</td>
<td>0.06</td>
<td>75</td>
<td>0.50</td>
<td>4.0</td>
</tr>
<tr>
<td>A533B</td>
<td>2200</td>
<td>$3.4 \times 10^4$</td>
<td>0.065</td>
<td>85</td>
<td>0.65</td>
<td>5.2</td>
</tr>
<tr>
<td>2 1/4 Cr-1 Mo</td>
<td>4000</td>
<td>$3 \times 10^4$</td>
<td>0.013</td>
<td>50</td>
<td>2.0</td>
<td>16</td>
</tr>
<tr>
<td>316 SS</td>
<td>500</td>
<td>$8 \times 10^3$</td>
<td>0.063</td>
<td>110</td>
<td>0.11</td>
<td>0.9</td>
</tr>
<tr>
<td>A533B (Quenched) Irradiation Simulation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
LIGAMENT REQUIREMENTS BEYOND $J_{IC}$

1. $b > M \frac{J}{\sigma_Y}$ for a given $\Delta a$

$$b > M \left[ \frac{J_{IC} + \frac{dJ}{da} \Delta a}{\sigma_Y} \right]; \text{ Depends on } \frac{dJ}{da}$$

2. $b > \frac{\omega J}{dj/da}$ for a given $\Delta a$

$$b > \omega \left[ \frac{J_{IC} + \frac{dJ}{da} \Delta a}{dj/da} \right] = \omega \left[ \frac{J_{IC}}{dj/da} + \Delta a \right]$$

Independent of $dj/da$

3. At $\Delta a = 0.06 b$

Bend: $b > 2.5 \left[ \frac{J_{IC}}{dj/da} + 0.06 b \right]$

or $b > 2.9 \frac{J_{IC}}{dj/da}$

Tension: $b > 20 \left[ \frac{J_{IC}}{dj/da} + 0.06 b \right]$

$$= 20 \frac{J_{IC}}{dj/da} + 1.2 b \text{ Impossible Requirement}$$
Fig. 17 – J vs Δa R curve for all A533B CL.1 material heat treated to obtain a lower value of upper shelf energy, test temperature, 24°C.
Fig. 15—J vs. Δa R-curve for A508 Cl 2 steel for all compact specimen sizes tested. Test temperature, 24°C (for 1/2T, 1T and 3T) and 52°C (for 4T).
Fig. — $J$ vs $\Delta a$ for HY-130 steel comparing a large planar specimen ($w = 24$ in) with a small planar specimen ($w = 2$ in)
Fig. 1—J vs crack extension for a CCP specimen and a IT-CT specimen - Ni Cr MoV at 250°F
Fig. J vs Δa for 304 SS, compact and center cracked tension specimens (3/8 inch thickness)
Fig. — J vs Δa for CT and CCT specimens of A533B class 1 steel at 200°F (data from Andrews)
Fig. 26 - Results of a series of multispecimen heat tint tests on 20% side grooved IT compact specimens of A533B Cl.1 02 baseplate material tested at 149°C (300°F) with a/W = 0.6
Fig. — \( J \) vs \( \Delta a \) for A533 B steel showing the effect of side grooves (300°F) (data from Gudas and Joyce)
Fig. 20 – J vs ∆a R-curve for A533B Cl. 1 HSST baseplate 02 with a/W = 0.6 tested at 149°C (300°F)
Fig. 28 — J vs Δa values for A533B Cl. 1 02 baseplate pressure vessel steel tested at 149°C (300°F)
Fig. 5 — J vs Δa for crack extension beyond the $J_{IC}$ bounds, ASTM A471 Steel
Fig. 3 — Fracture toughness (J) for cleavage versus temperature for an ASTM A471 steel.
Fig. 10 — Fracture toughness for cleavage, $J_c$, versus temperature comparing IT-CT mean and Weibull estimates with 4T-CT values.
PROBLEM OF NO J CONTROLLED GROWTH

A. Observations:

1. The growth size requirement "w" may be less than the static analysis size requirement "M"
2. For low $\frac{dJ}{da}$ the margin against instability may be too small to rely on
3. R curve slopes for J controlled growth appear to be lower than for no J controlled growth
4. J controlled growth is easier to achieve on a bend type test specimen than on a structure in tension

B. Suggestions:

1. Use an initiation criterion for design, R curve instability analysis to determine safety margin
2. When there is no J controlled growth, use only the initiation criterion
3. As a last resort, assume the J controlled growth prediction is conservative
Specimen: 1.6 T Compact (a/W = .5)  
Temperature: 149 °C  
Material: A533B

- ▲ Using No Δa Correction For J  
- △ Heat Tint Value Of Δa  
- ○ Using Δa Correction For J

Fig. 31 – J vs Δa plot of A533B 02 baseplate with and without correction for crack extension
CRACK GROWTH RESISTANCE

GEOMETRY EFFECTS

&

STRUCTURAL PREDICTIONS

IN A533B CLASS I STEEL

S.J. GARWOOD

The Welding Institute,
Abington Hall,
Abington,
Cambridge,
CB1 6AL
U.K.

Tel: 0223 - 891162
ABSTRACT

Nine 500mm wide 110mm thick A533B Class 1 wide plates were tested at +70°C. Three crack types were used, semi-elliptical, surface notched and through thickness centre cracked. Various notch depths were employed. The progress of the stable propagating ductile tear resulting in the specimen from the use of the relatively stiff wide plate rig was marked periodically using an unloading technique.

Estimates of the crack propagation resistance ($J_R$) curves for these tests were made and compared to resistance curves obtained from small scale laboratory bend specimens.

The crack morphology exhibited by the surface notched wide plate specimens which propagate in the through thickness direction was essentially normal ductile rupture in contrast to the through thickness crack which showed mostly shear rupture.

Semi-elliptical notched specimens initially behaved in a similar fashion to the surface notched and after penetration of the plate thickness continuing propagation behaved as a centre cracked plate.

Derivations of a simple estimation formulae for the $J_R$ curves for the surface notched and semi-elliptical geometries have been made. Using these formulae comparisons of all tests with the crack propagating in the L-T and L-S orientations were made.

The results of large scale tension and large and small scale bend tests indicate that laboratory tests give reasonable lower bound estimates of the more structurally relevant tension situation in the L-T orientation where full plate thickness bend specimens are employed.
In the L-S orientation much more restrictive thickness requirements are necessary to achieve conservative estimates from laboratory bend tests and as an initial guideline the requirement $B > 25 \frac{J_R}{\sigma_Y}$ is suggested for surface notched geometries although this may well be not restrictive enough for very ductile materials.

This guideline is assessed on oversquare and sidegrooved bend specimens in the L-S orientation and the requirements for the production of conservative resistance curves, for the description of structural situations, in the laboratory are outlined.

The relevance of the use of maximum load toughness from the standard laboratory tests as a conservative estimate of the structural ductile instability value in assessment techniques is also discussed.
**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$</td>
<td>Aspect ratio ($a_{\text{max}}/2c$)</td>
</tr>
<tr>
<td>$a$</td>
<td>Crack length</td>
</tr>
<tr>
<td>$a_{\text{max}}$</td>
<td>Maximum length, through thickness, of semi-elliptical notch.</td>
</tr>
<tr>
<td>$a_o$</td>
<td>Initial crack length</td>
</tr>
<tr>
<td>$B$</td>
<td>Specimen thickness: $B_{\text{nom}} = \text{nominial thickness} &amp; B_{\text{act}} = \text{actual thickness}$.</td>
</tr>
<tr>
<td>$c$</td>
<td>Half length of surface of semi-elliptical notch</td>
</tr>
<tr>
<td>CCT</td>
<td>Centre-cracked tension</td>
</tr>
<tr>
<td>CT</td>
<td>Compact tension</td>
</tr>
<tr>
<td>$D$</td>
<td>Specimen height</td>
</tr>
<tr>
<td>$G_R$</td>
<td>G resistance (the value of crack extension force after some crack growth)</td>
</tr>
<tr>
<td>$J$</td>
<td>The J integral</td>
</tr>
<tr>
<td>$J_R$</td>
<td>J resistance</td>
</tr>
<tr>
<td>$L$</td>
<td>Constraint factor</td>
</tr>
<tr>
<td>L-T</td>
<td>Specimen extracted in the rolling direction, notched in the transverse plane</td>
</tr>
<tr>
<td>L-S</td>
<td>Specimen extracted in the rolling direction, notch through the thickness</td>
</tr>
<tr>
<td>$P$</td>
<td>Load</td>
</tr>
<tr>
<td>$q$</td>
<td>Displacement</td>
</tr>
<tr>
<td>$R$</td>
<td>Resistance to fracture</td>
</tr>
<tr>
<td>$S$</td>
<td>Span</td>
</tr>
<tr>
<td>SENT</td>
<td>Single edge notch specimens</td>
</tr>
<tr>
<td>SESNT</td>
<td>Semi-elliptical surface notch tension</td>
</tr>
<tr>
<td>$U$</td>
<td>Energy under load displacement record</td>
</tr>
<tr>
<td>$W$</td>
<td>Specimen width</td>
</tr>
<tr>
<td>$Z$</td>
<td>Knife edge thickness</td>
</tr>
<tr>
<td>$\Delta a$</td>
<td>Incremental crack extension</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Multiplication factor relating $J$ and $U$</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>Ultimate tensile stress</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>Yield stress</td>
</tr>
</tbody>
</table>
SUBSCRIPTS

c  Elastic
i  Initiation
L  Limit load value
n  Current value
p  Plastic
Y  Yield value
INTRODUCTION

The measurement and analysis of the crack growth resistance of structural steels has received much recent attention\(^{(1-6)}\). Ongoing studies have involved the adaptation of existing yielding fracture mechanics to attempt to characterise the behaviour of a crack in structural steel extending by a ductile mechanism in the presence of large scale plasticity\(^{(5,6)}\). The intention of this research is to solve the current problem concerning the determination of allowable flaw sizes in particular for the nuclear industry.

Current assessment techniques tend to use the initiation of tearing as a measure of the material toughness. However, this seems an unduly conservative approach when viewed in the light of subsequent large toughness increases in crack growth resistance with crack extension often exhibited by these ductile materials.

The behaviour of the material with crack extension can be measured in terms of the crack growth resistance or R curve\(^{(1,2,6)}\). If a move is to be made away from the use of initiation of tearing as the characteristic toughness value, then the measure of toughness to be employed must be obtained from a consideration of the resistance curve of the material and the relevant driving force curve, which describes the applied loading conditions.
It would appear that the resistance curve is not a material parameter being dependent on test piece geometry\(^{(6 - 8)}\). The object of this test programme is to evaluate the possibility of using laboratory test piece resistance curves to describe the behaviour of the more structurally significant large scale tension tests with through thickness, surface and semi-elliptical cracks.

**ANALYSIS**

Recent work has concentrated on the use of the J integral for describing the resistance of a stable tear. Initially any crack extension was considered by many to cause an invalidation of the theoretical justification of the use of the J integral for elastic-plastic materials\(^{(9, 10)}\). This argument was based on the fact that the non-linear elastic analogue, which is the premise upon which the J integral is applied to elastic plastic materials, can only be relevant to monotonic loading. Hence crack growth, with its accompanying relaxation of the original crack tip, invalidates this premise.

Recently, theoretical justifications have been made allowing the use of J integral procedures for the description of crack growth provided a "J dominant field" is maintained\(^{(11)}\). This condition is dependent on the parameter \(\omega = \frac{dJ}{da} \frac{(W-a)}{J}\) which has to be \(>> 1\). Other investigations have intimated that crack growth should not exceed 6% of the remaining ligament for steels such as A533B, and for the centre cracked tension geometry the size requirement of 25 to 50 \(J/\sigma_{flow}\) should be increased to 200 \(J/\sigma_{flow}\)\(^{(12)}\).

Prior to these theoretical justifications, resistance curve measurements have been made with cracking extending beyond the now prescribed limits for J dominance\(^{(2, 6, 8)}\). This has been done on the basis that no theoretical claims have been made for \(J_R\) measurements. Instead the \(J_R\) techniques developed have been assumed to provide comparative estimates.
of the toughness which would be experienced in contained yielding structural situations i.e. the $G_R$ resistance curve.

Evaluation of $J_R$ for the standard test geometries is based on measurement of the area under the load displacement diagram.

For extensive amounts of tearing corrections must be made to these area measurements to take account of crack extension which has occurred.

Initially, a method known as the 'three parameter technique' based on the J-U relationship developed by Rice, Paris and Merkle(13) and Sumpter(14) was employed. However, a simpler estimation method has been developed based on the final load and displacement measurements(15). The comparison of these methods was carried out in ref. 6 and a good correlation obtained between the simple estimation and three parameter techniques with the former technique providing conservative estimates.

This simple estimation method was originally developed for the three point bend and CCT geometries in ref. 6. However, the method is extended to CT, SENT and semi-elliptical geometries in the appendix.

To summarise, the formulae used for J determinations in this paper were as follows:-

For bend with $0.4 < \frac{a_0}{W} < 0.7$ (ref. 16 sets these limits of applicability)

$$J_n = \frac{P_n q_n}{B(W-a_n)} + \frac{L \sigma_Y (W-a_n)}{S} q_p \ldots (1)$$

$L$ was assumed to be 1.3 for the geometry chosen.

For CCT

$$J_n = \frac{P_n q_p (1 - 2n_e) + 2n_e P_n q_n}{2B(W-2a_n)} + \frac{L \sigma_Y q_p}{2} \ldots (2)$$

$L$ was assumed to be 1.155 for this geometry. (14)
For single edge notch tension (see appendix)

\[ J_n = p_n \left( 1 - \frac{\eta}{\eta_n} \right) + \eta \left( \frac{n}{n_n} \right) + \frac{L \sigma_Y q_p}{2} \]

... (3)

L was also taken as 1.155

For semi-elliptical surface notched tension (see appendix)

\[ J_n = \frac{p_n q}{2B(\pi c_n - a_{\text{max}} n)} + \frac{L \pi c_n}{2B} - \frac{a_{\text{max}} n}{2} \sigma_Y q_p \]

... (4)

For a conservative estimate L was assumed to be unity.

**EXPERIMENTATION**

**Specimen Design**

Nine 508mm wide, 110mm thick, \(915\)mm long tension pieces were tested in the wide plate rig. Three types of notch were employed; surface, semi-elliptical and centre cracked, see fig. 1. Hence, crack propagation is through thickness for the surface notched (SENT) i.e. the L-S orientation, whilst the CCT geometry has a crack in the L-T orientation. The semi-elliptical notch behaves as a combination of SENT - CCT with propagation both through the plate thickness and along the width of the specimen. Three of each notch type were used with different \(a/W\) ratios.

(See Table 2.)

Seven types of bend specimen were employed. \(A\) Types 1, 4, 5 and 6 were extracted in the L-T orientation with 2, 3 and 7 in the L-S orientation. Full thickness (110mm) specimens were used for Types 1, 2 and 6 with W's of 250, 125 and 500 respectively. Types 3, 4, 5 and 7 were half thickness (53mm) with W's of 65, 250, 125 and 125 respectively. All specimens had an initial \(a_{\circ}/W\) of 0.4.

**Experimental Details**

All tests were carried out at +70°C to ensure fully ductile behaviour of the A533B employed.
All bend tests were conducted using a conventional 1800KN hydraulic universal machine at a crosshead speed of 2mm/min. Load was monitored against crosshead displacement which was corrected for extraneous displacement by indenting the rollers into the broken halves of the specimen after the test as described in ref. 17.

For the wide plate tension tests load was monitored against the displacement of the clip gauge mounted across the mouth of the notch in the centre of the specimen with a knife edge spacing of 17.5mm. Moveable knife edges were employed since periodic adjustments of the knife edge spacing was necessary due to the large displacement encountered in the tests. It was unnecessary to correct for extraneous displacement in these tests because of the use of a mouth opening mounted clip gauge.

To remount the clip gauge, periodic unloading to zero of the wide place specimens were necessary. All tests in this programme behaved in an entirely stable manner achieving a maximum load with stable tearing, via microvoid coalescence, continuing thereafter under a decreasing load. In the majority of the bend tests initiation of tearing occurred after net section yield of the specimen but well before maximum load.

Type 6 bend and the wide plate tests however experienced net section yield and initiation of tearing at very similar displacements.

Prior to testing, all specimens were fatigue pre-cracked. The bend specimens were pre-cracked according to BSI DD19 specifications (18). The surface notched and semi-elliptical wide plates were fatigued in bending. Some difficulty was experienced in obtaining a fatigue crack in the latter specimens. Specimen 1006 with \( \frac{D}{W} = 0.21 \) had little or no fatigue propagation and hence the initial notch diameter would approximate to the thickness of the abrasive slitting wheel used to cut the initial notch of 0.15mm. Specimen 1007 also had a fatigue crack length of < 1mm in places along the crack front. The
centre cracked specimens were fatigued in two halves as three point bend specimens and the halves electron beam welded to form the wide plate specimens.

The type 6 bend specimens had to be tested using a clamp and beam arrangement to double the effective load of the test machine since these specimens required a centre point load of \( \sim 3,000 \text{ KN} \).

To obtain crack growth resistance curves for the smaller bend specimens it was necessary to load a series of specimens to various displacements to obtain different amounts of crack extension, unload, cool to below the transition temperature and fracture the specimens to reveal the amount of stable tearing. This 'break open' procedure was also used for the wide plate specimens, however, it was found that the periodic unloadings during the test had left marks on the fracture surface presumably caused by the fracture faces coming together at the crack tip when complete unloading occurred (see fig. 2). Hence it was possible to plot a number of points for these specimens and obtain an approximate resistance curve from each individual specimen. This 'unloading marking' method was also found to work on the larger bend tests (types 1, 4 and 6). Tests results involving repeated unloadings were compared with monotonically loaded specimens and very similar values obtained indicating little or no effect of the unloadings on the load, displacements or amounts of stable crack growth experienced.

After final fracture of each specimen, average measurements of initial crack length and the amounts of stable tearing were taken.

Alternating current potential difference monitoring of crack extension was carried out on all tests, however, determination of initiation and the accuracy of stable crack determinations were found to be inconsistent. Where potential measurements could be calibrated from unloading crack marks however, it was found possible to evaluate the shape of the resistance
curve between these points by using crack extension measurements derived from the pd readings.

RESULTS

i) Initiation determination

The aim of this test programme was to evaluate the geometric and orientation dependence of the R curve in A533B. Since the entire resistance curves were being monitored very few specimens were stopped with small amounts of crack extension, therefore, the initiation of tearing value for the different geometries was not obtained directly. Values could be derived from the pd traces but these results must be treated with extreme caution. In view of this, four type 5 and type 7 (with W = 100m, B = 50mm) specimens in the L-T and L-S orientations were tested following ASTM E24:01:09 Task Group recommendations\(^{(19)}\) to evaluate initiation values in the two test orientations. The results of these tests are given in fig. 3. Unlike ref. 19 specifications, this figure plots values of J against crack extension excluding stretch zone. This obviates the need for a blunting line construction and initiation values can be read off the figure at Δa = 0. These are 0.4 MJ/m\(^2\) and respectively 0.3 MJ/m\(^2\) for the L-T and L-S orientations. The slope of the resistance curve was obtained employing a linear regression analysis.

Although, as stated previously, accurate initiation values were not obtained for the other specimen types the results obtained would indicate very similar values to those obtained in fig. 3 for the relevant crack orientation.

ii) Bend Resistance Curves

In all, seven specimen types were tested. Results for types 1, 4 - 6 in the L-T orientation are given in Fig. 4 whilst types 2, 3 and 7 in the L-S orientation are depicted in Fig. 5.
In all cases \( J_R \) values were calculated using the formula given in equation (1) based on the final load, corrected total and plastic displacements and final crack length.

iii) Wide Plate Tension Tests

Three centre cracked tests were carried out, the results from which are shown in Fig. 6. Three different \( \frac{2a}{W} \) ratios were used (0.27, 0.42 and 0.52). Each point marked on the figure indicates an unloading position. \( J_R \) estimates were made using equation (2) with the incorporation of the correct \( \eta_e \) factor based on the gauge length of the measurement point to the width of specimen ratio \( \frac{D}{W} \), and the \( \frac{2a}{W} \) ratio. In this instance \( \frac{D}{W} \) was the knife edge spacing divided by 508mm which was approximately 0.03.

Although the knife edge spacing was altered during the test the value of \( \eta_e \) was found to be insensitive to this change in the \( \frac{D}{W} \) when compared to the dependence on \( \frac{2a}{W} \). Values of \( \eta_e \) were found to be in the range 2.0 to 0.5 for \( \frac{2a}{W} \) between 0.3 and 0.8.

Crack propagation in these specimens was in the L-T orientation.

Single edge notch tension specimens were tested with three \( \frac{a_o}{W} \) ratios (0.3, 0.34 and 0.47).

These specimens were surface notched along the 508mm edge (see fig. 1) hence these tests had an effective width of 110mm and thickness of 508mm with crack propagation in the L-S orientation.

Results of these tests are given in fig. 7. \( J_R \) calculations were carried out using equation (3) which is derived in the appendix. \( \eta_e \) values in this case were found to vary between 5 and 1.9 for \( \frac{a}{W} \) in the range 0.3 - 0.8.
The final three wide plate specimens had semi-elliptical surface defects as shown in fig. 1. All specimens had an \( \frac{a_{\text{max}}}{2c} \) ratio of 0.16 with \( \frac{a_{\text{max}}}{W} \) ratios of 0.21, 0.34 and 0.5. As in the simple edge notch tests \( W \) was taken as 110mm with the specimen thickness \( B = 508 \text{mm} \). Crack propagation occurred in both through thickness (a) and width (2c) directions (see fig. 2).

\( J_R \) calculations were made using equation (4), as determined in the appendix, for initial propagation of the crack which is assumed to occur at a constant aspect ratio with \( a_{\text{max}} \) and 2c being defined by actual measurements at each unloading point. Once the crack had propagated through thickness the CCT analysis was assumed to apply as given by equation (2).

Values of \( J_R \) for this geometry are plotted against average crack extension in fig. 8.

For the initial part of the curve this incremental crack extension is the average through thickness extension whilst for the final section which behaves as a CCT specimen with two crack fronts, \( \Delta a \) refers to propagation along one crack front only.

DISCUSSION

i) Bend Tests

The initiation of tearing toughness in the L-T orientation would appear higher than in the L-S direction as evident from fig. 3 (which gives \( J_I \)'s of 0.4 and 0.3 MJ/m² respectively). Subsequent propagation would indicate a steeper slope for the L-S orientation with the same value of toughness being obtained for both orientations after 1mm of crack extension. When viewed in the light of the scatter which is normally experienced with \( J_I \) determinations using the multi-specimen technique, however, the results in Fig. 3 show very similar behaviour between the orientations.

The effect of different specimen configurations in the L-T orientation can be seen in fig. 4. A distinct geometric dependence is evident but there is a complicated relationship between \( W \) and \( B \) and their effect on the resistance curve. Generally the larger the \( \frac{B}{W} \) ratio the lower the resistance
curve. In this orientation, however, there appears little effect of width on the full thickness plate results. Type 6 with \( W = 500 \text{mm} \) does give a higher resistance curve than the Type 1 with \( W = 250 \text{mm} \), but the difference is small compared to the effect of crack orientation in the wide plate tests.

Figure 5 depicts the geometric effect in the L-S orientation. Here a much larger effect is exhibited than in fig. 4. Types 2 and 3, with similar \( \frac{B}{W} \) ratios, follow the same resistance curve, whilst Type 7 with a smaller ratio has a much steeper \( R \) curve.

ii) Wide Plate Tests

The behaviour of the centre cracked wide plates is shown in fig. 6. Similar curves are obtained for all three specimens, there would appear to be little effect of the \( \frac{2a}{W} \) ratio.

A typical fracture face for the CCT geometry is shown in fig. 2. Crack extension in this geometry starts with normal ductile rupture tunnelling in the centre of the specimen with \( 45^\circ \) shear lips forming near the surfaces. As extension proceeds so the shear lip width increases until after \( 40 \text{mm} \) of crack extension full \( 45^\circ \) shear occurs. The knee in the resistance curve would appear to coincide with the point on the fracture surface when the shear lips are almost fully formed.

The crack growth resistance curves relating to the surface (or single edge) notched specimen behaviour is given in fig. 7. As in the CCT case, no absolute effect of the \( \frac{a}{W} \) ratio can be ascertained. Since the crack is propagating through thickness these curves relate to the L-S orientation. A typical fracture surface is shown in fig. 2, the major portion of the fracture surface being normal rupture, small shear lips forming near the surface.

From fig. 7 it can be seen that a peaking of the resistance curve appears to occur after \( 35 \text{mm} \) of crack extension. This phenomenon could be
attributed to the reduction in shear fracture present due to the increasing bending component and hence constraint factor, as the crack approaches the back surface of the specimen.

The semi-elliptical surface notched specimens behaved in a similar fashion to the SENT test pieces with the majority of the fracture surface being formed of normal ductile rupture. In this case however the shear lips tended to form on the front surface of the specimen inhibiting crack extension in the thickness (2c) direction (see fig. 2). In the centre of the specimen however the crack appears to propagate with an almost constant \( \frac{a_{\text{max}}}{2c} \) ratio until the crack penetrates the back surface after which further extension occurs as if the specimen were centre cracked.

This behaviour makes analysis particularly difficult. As stated in the appendix a \( J_R \) analysis of the semi-elliptical crack can be approached in three ways. The conservative approach is to assume propagation through thickness only. The least conservative assumes propagation with a constant \( \frac{a_{\text{max}}}{2c} \) ratio. The most realistic assumption would seem to be assuming a constant \( \frac{a_{\text{max}}}{2c} \) ratio but impose the actual value of \( a_{\text{max}} \) (which is incidentally the maximum through thickness value rather than the average as in the other geometries) and 2c which exist at each unloading position. This should still provide a conservative estimate since the resulting crack front geometry assumed will leave a larger remaining ligament than actually present in the test geometry. Once the crack becomes through thickness the CCT formulations apply.

Values of \( J_R \) using the constant aspect ratio with test \( a_{\text{max}} \) and 2c values are plotted in fig. 8 for the three specimens tested. If any trend due to \( \frac{a_{\text{max}}}{W} \) ratio is to be interpreted from this figure it would be that the larger the ratio, and thus the nearer the specimen is to becoming a CCT geometry, the steeper the \( R \) curve.
The resistance curve for specimen 1046 propagating as a through thickness crack (L-T orientation) is also plotted in fig. 8 which would appear to be remarkably consistent with prior propagation in the semi-elliptical mode (L-S orientation).

A comparison of results from the bend and tensile tests is made in fig. 9. The entire bend result scatter band falls below the CCT scatter band enhancing confidence in the use of the bend geometry tests for the conservative prediction of structural resistance curves using full thickness specimens.

In contrast, non-conservative estimates of more structurally relevant result geometries in the L-S orientation, particularly for the initial part of the curve, as can be seen from inspection of fig. 10. Good correlation is obtained between semi-elliptical and surface notched defects, however, which is consistent with their similar fracture appearance.

The major reason for the steeper bend resistance curves in the L-S orientation is due to thickness effects. The wide plates had an effective thickness of 508mm whilst the maximum thickness of the bend specimen tested was 110mm. Hence the fracture surface of the bend tests contained a much larger shear lip proportion than did their tensile counterparts.

Therefore, to provide conservative estimates of through thickness propagation (L-S orientation) some thickness criteria are obviously necessary. As a first approximation the plane strain requirements specified in BS 5447(20) could be applied, i.e.

\[ B > 2.5 \left( \frac{K}{\sigma} \right)^2 \]

in terms of J this converts to

\[ B > 2.5 \frac{EJ}{\sigma^2} \]
For the specimens tested herein where \( J_i = 0.3 \text{ MJ/m}^2 \), \( B \) would have to be greater than 720mm and even thicker if more representative values of toughness after some crack extension had occurred were used.

A more reasonable criterion would be to adopt the ASTM E24:01:09 Task Group recommendations\(^{(19)}\) over the entire range of the resistance curve for plane strain validity, thus

\[
B > \frac{J_R}{\sigma_Y} \quad \ldots (7)
\]

Hence, on the basis of the results from the SENT geometry at initiation \((J_i = 0.3 \text{ MJ/m}^2)\) a thickness \( B > 16\text{mm} \) would be required, after 10mm crack extension \((J_i = 6.5 \text{ MJ/m}^2)\) \( B > 355\text{mm} \) and after 20mm extension \((J_i = 10\text{MJ/m}^2)\) \( B \) would have to be \( > 547\text{mm} \) for continuation of plane strain conditions.

These requirements would not be necessary of course if full thickness plate specimens are employed for the description of crack propagation in the L-T orientation, since although they would not necessarily be plane strain curves they should still provide conservative estimates of the structural curve.

iii) Further Tests

To investigate further the effects of thickness on bend specimens in the L-S orientation two additional series of tests were planned. The first set were identical to the Type 2 tests carried out in the initial programme with \( W = B = 110\text{mm} \), however, in this instance 6mm wide 275mm deep side grooves were machined reducing the specimen thickness to 55mm. This gives a side groove ratio \( (SR = \frac{B - B_{\text{act}}}{B}) \) of 0.5 which was found in Ref. 23 to give the lowest bound resistance curve. The second series of tests were plain sided specimens in the L-S orientation but with oversquare dimensions \( W = 110\text{mm}, B = 250\text{mm} \). These tests were aimed at establishing a thickness criteria for the description of through thickness crack growth on laboratory bend tests.
The side grooved tests have now been completed the results of which are given in Figs. 11 - 13. The effect of the side grooving is to reduce the initiation toughness exhibited by the Type 7 specimens (Fig. 3), which incidentally met the thickness requirement of $B > 25 \frac{J_{IC}}{\sigma_Y}$, from 0.29 to 0.19 MJ/m$^2$. Whether this result is due to the side grooving producing more severe conditions than normally experienced or is a reflection on the inadequacy of the thickness criterion should be highlighted by the over-square specimens' test results when they are available.

The reduction in initiation toughness is also accompanied by a slight shallowing of the initial resistance curve slope as shown in Fig. 7. This effect is even more evident after continued crack extension (see Fig. 8) the side grooved results falling well below the Type 2 specimens which had equivalent nominal dimensions (i.e. $W = 110\text{mm}$, $B_{\text{nom}} = 110\text{mm}$) and well below Type 7 which had a similar actual specimen thickness.

The side grooved specimens do, however, provide a conservative estimate of the resistance curves exhibited by the wide plate tests as shown in Fig. 13. Thus, it is possible to determine safe predictions for surface and semi-elliptical surface defects from laboratory bend specimens. The thickness criterion for this requirement is still to be established however.

Indications from the side grooved series are that $B > 25 \frac{J_R}{\sigma_Y}$ may not be severe enough.
MAXIMUM LOAD TOUGHNESS

It is well known that the toughness value derived from the maximum load point in laboratory bend tests is a geometry dependent parameter. However, it has recently been suggested that values of COD determined at the point of first attainment of the maximum load ($\delta_{\text{max}}$) determined using the COD test procedures on the preferred three-point bend specimens described in BS.5762:1979\textsuperscript{(18)} will provide conservative estimates of the value of toughness at which ductile instability would occur in structural situations for the majority of structures. This concept would apply equally well to the use of $J_{\text{max}}$ determined from bend or CT specimens in a J design curve approach.

The philosophy behind this suggestion is fully described in Ref. 24. In brief, it is argued that the maximum load value derived from deeply notched bend geometries, such as the compact tension or three point bend specimens give conservative estimates of the maximum load or instability toughness of predominantly tensile loaded structural situations provided certain conditions are met.

The loading system energies in both the laboratory test and structural situation are not required for the analysis as maximum load toughness corresponds to instability in load controlled situations provided the material does not experience time dependent crack growth or is strain rate sensitive. Therefore, provided the resistance curve measured in the laboratory is shallower than the structural curve, and driving force curves are steeper for the test geometries than the structure, then the derived maximum load toughness must give conservative estimates of structural instability even under load controlled conditions. A greater degree of safety will be evident in the majority of structural situations which tend to be displacement controlled.
Geometry dependence of driving force curves has recently been investigated\(^{(22)}\) for the elastic situation as shown in Fig. 14. This figure indicates that driving force curves tend to be steeper for the bend geometries than for the more structurally relevant tension configurations under elastic conditions. This has yet to be verified for elastic-plastic conditions.

As discussed above the crack growth resistance curves are geometry dependent for a given thickness. However, as demonstrated in Fig. 9 the bend tests on full thickness specimens gives a conservative estimate of the through thickness tension (L-T orientation) resistance curve.

For surface notched situations (L-S orientations) a thickness criterion must be applied to ensure conservative estimates however. It was suggested above that \(B > 25 \frac{J_R}{\sigma_Y}\) could be an initial guideline. However, the two further tests series suggest that this criterion may not be restrictive enough.

Figure 15 indicates the variation of \(J_{\text{max}}\) with thickness. Values of \(J_{\text{max}}\) for the 50 and 100mm thick specimens with \(J_{\text{max}} = 2.5 \div 2.6\) MJ/m\(^2\) fall outside the thickness criterion suggested above. The oversquare geometry, however, with \(B = 248\)mm falls within the thickness criterion and has a \(J_{\text{max}} = 1.8\) MJ/m\(^2\). This point falls on the \(B = 65\) \(\frac{J_{\text{max}}}{\sigma_Y}\) requirement line. However, for the side grooved specimens which arguably tend to induce plane strain conditions \(J_{\text{max}} = 0.8\) MJ/m\(^2\).

Hence if side grooving does produce an accurate estimate of the most severe constraint conditions experienced in practice (it is conceivable that the conditions produced are too severe) then the thickness requirements may well need to be very much more restrictive for maximum load as well as complete resistance curve determinations for this ductile steel.
CONCLUSIONS

1. The initiation toughness of A533B plate tested in the L-T and L-S orientations was determined as 0.4 and 0.3 MJ/m$^2$ respectively. Although the initiation toughness was slightly lower in the L-S orientation the initial resistance curve slope was steeper. Hence generally similar behaviour was exhibited by the two orientations. Side grooving to remove shear lips reduced the initiation toughness in the L-S orientation to 0.2 MJ/m$^2$.

2. Three point bend specimen tests in the L-T and L-S orientations with variable width and thickness indicated distinct geometric effects on the crack propagation resistance curves. Generally the larger the thickness to width ratio the lower the resistance curve. However, the indications are that once a certain thickness is reached there is little effect of width on the resistance curve. The geometric effect which does exist indicates the smaller the width the more conservative the result.

3. Centre cracked wide plates form much larger shear lips than their equivalent thickness bend counterparts and exhibit much steeper resistance curves. Hence, provided full thickness bend specimens are employed conservative resistance curves should always be obtained for most structural applications with cracking in the L-T orientation.

4. Surface and semi-elliptical notched wide plate specimens having crack propagation in the L-S orientation behave in a very similar fashion for the initial period of crack propagation. However, once the semi-elliptical crack breaks through to the back surface of the specimen the test piece behaves as a CCT with propagation in the L-T orientation.
5. In the L-S orientation conservative resistance curves are not necessarily obtained from bend specimens of thickness equivalent to the plate thickness. In this orientation the thickness must be governed by plane strain thickness requirements and as an initial guide it is suggested that the ASTM E24:01:09 Task Group guidelines are extended to R curve testing in this orientation. This gives the requirement:

\[ B > 25 \frac{J_R}{\sigma_Y} \]

to provide conservative estimates of tensile resistance curves in the L-S orientation. There is no evidence to suggest that such severe and restrictions are necessary for a or \( W \) in fact it would appear that the smaller the \( W \) the lower the \( R \) curve.

6. On the basis that conservative resistance curves are obtained from laboratory test pieces it is argued that values of toughness relating to the maximum load point in test specimens can be used to predict flaw sizes below which ductile instability will not occur in structural situations.

ACKNOWLEDGEMENTS

The views expressed in this paper are those of the author and do not necessarily represent the policy of the Nuclear Installations Inspectorate whose sponsorship of this work is gratefully acknowledged. The author wishes to thank B.A. Wakefield and the staff of the Brittle Fracture Laboratories of The Welding Institute for their assistance with the experimental work reported in this paper.
REFERENCES


20. "Methods of test for Plane strain fracture toughness ($K_{1C}$) of metallic materials BS 5447, 1977.


23. GARWOOD S.J. and TURNER C.E. "The use of the J integral to measure the resistance of mild steel to slow stable crack growth". Fracture 3, 1977, 279-84.

TABLE 1. Properties of steel to A533B employed.

a) Chemical compositions wt %

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>S</th>
<th>P</th>
<th>Si</th>
<th>Mn</th>
<th>Ni</th>
<th>Cr</th>
<th>Mo</th>
<th>V</th>
<th>Cu</th>
<th>Nb</th>
<th>Ti</th>
<th>Al</th>
<th>B</th>
<th>Pb</th>
<th>Sn</th>
<th>Co</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.20</td>
<td>&lt;.005</td>
<td>.006</td>
<td>0.20</td>
<td>1.39</td>
<td>0.59</td>
<td>0.12</td>
<td>0.50</td>
<td>&lt;.01</td>
<td>0.10</td>
<td>&lt;.005</td>
<td>&lt;.01</td>
<td>0.023</td>
<td>&lt;.001</td>
<td>&lt;.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

b) Mechanical properties at +70°C of A533B

<table>
<thead>
<tr>
<th>YIELD STRESS</th>
<th>TENSILE STRENGTH</th>
<th>ELONGATION</th>
<th>REDUCTION OF AREA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_Y )</td>
<td>( \sigma_u )</td>
<td>( % )</td>
<td>( % )</td>
</tr>
<tr>
<td>465</td>
<td>600</td>
<td>30</td>
<td>65</td>
</tr>
<tr>
<td>TYPE</td>
<td>a/W</td>
<td>B</td>
<td>W</td>
</tr>
<tr>
<td>------</td>
<td>-----</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>1</td>
<td>0.4</td>
<td>110</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
<td>110</td>
<td>125</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
<td>55</td>
<td>63</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>55</td>
<td>250</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>55</td>
<td>125</td>
</tr>
<tr>
<td>6</td>
<td>0.4</td>
<td>110</td>
<td>500</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
<td>55</td>
<td>125</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>55</td>
<td>125</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
<td>55</td>
<td>125</td>
</tr>
</tbody>
</table>
Fig. 1. Crack types - wide plate specimens

Fig. 2. Wide plate fracture surfaces: a) Surface notch; b) semi-elliptical; c) centre cracked.
Fig. 3. Initiation determination from bend specimens in the L-T and L-S orientations.
Fig. 4. Bend resistance curves L-T orientation

Fig. 5. Bend resistance curves L-S orientation
Fig. 6. Crack propagation resistance - centre crack tension.

Fig. 7. Crack propagation resistance - single edge notch tension.
Fig. 8. Crack propagation resistance - semi-elliptical notch tension

Fig. 9. Resistance curve comparison - L-T orientation.
Fig. 10. Resistance curve comparison - L-S orientation
Fig. 11  Side Grooved Initiation Determination

Fig. 12  Side Grooved resistance curve L's orientation
Fig. 13  
Resistance curve comparison L-S orientation
Fig 14. Typical elastic driving force curves ($G_P$ vs $a$) for various geometries.

Fig 15. Maximum load toughness variation versus specimen thickness.
APPENDIX

The determination of $J_R$ (the value of $J$ for a propagating crack) from a single specimen for SENT and semi-elliptical surface notch tension (SESNT) geometries.

The calculation of $J_R$ for the three point bend and centre cracked tension geometries was derived in ref. 6 and the relevant formulae stated in equations (1) and (2).

The method is based on a postulated 'loading curve of the crack length of interest (see fig. A.1).

The area under this curve (U total) is given by

$$U_{\text{Total}} = U_p + U_e \quad \ldots \quad \text{A.1}$$

where

$$U_p = \frac{P_n + P_L}{2} q_p \quad \ldots \quad \text{A.2}$$

$$U_e = \frac{P_n}{2} (q_n - q_p) \quad \ldots \quad \text{A.3}$$

The plastic component of $J$ can be derived from the equation

$$J_p = -\frac{d(U_p)}{B \, da} \bigg|_{q_p} \quad \ldots \quad \text{A.4}$$

assuming rigid plastic behaviour (i.e. $P_n = P_L$ in equation A.2)

then

$$J_p = \frac{1}{B} \frac{d(P_L)q_p}{da} \bigg|_{q_p} \quad \ldots \quad \text{A.5}$$

for the SENT geometry

$$P_L = L \, \sigma_Y \, B (W-a) \quad \ldots \quad \text{A.6}$$

then

$$J_p = L \, \sigma_Y q_p \quad \ldots \quad \text{A.7}$$

substituting for

$$q_p = \frac{U_p}{P_L} \quad \ldots \quad \text{A.8}$$

then

$$J_p = \frac{U_p}{B(W-a_n)} \quad \ldots \quad \text{A.9}$$
The value of \( J \) after crack initiation can be related to the elastic and plastic areas under the postulated load displacement diagram by:

\[
J = \frac{\eta_e U_e}{B,W_n} + \frac{\eta_p U_p}{B,W_n} \quad \ldots \text{A.10}
\]

From equation A.9 \( \eta_p = 1 \)

hence by combining equations A.2, A.3 and A.10

\[
J = \frac{\eta_e P_n (q_n - q_p)}{2B(B-a_n)} + \frac{(P_n + P_L)q_p}{2B(W-a_n)} \quad \ldots \text{A.11}
\]

and re-arranging and substituting for \( P_L \) from A.6 gives, for the SENT geometry

\[
J = \frac{P_n q_p (1 - \eta_e)}{2B(W-a_n)} + \frac{\eta_e P_n q_n}{2B(W-a_n)} + \frac{L \sigma_Y q_p}{2} \quad \ldots \text{A.12}
\]

The semi-elliptical surface notched tension geometry is very difficult to analyse because of the curved crack front and propagation in both width and thickness directions.

To obviate the need for the lengthy and difficult calculation of \( \eta_e \), two approaches could be applied.

i) For large plastic displacements the elastic contribution \((U_e)\) could be ignored.

or ii) For large plastic displacements since \( U_e < U_p \), and generally for the tension geometries \( \eta_e > \eta_p \), a less conservative assumption than i) is to assume that \( \eta_e = \eta_p \) hence

\[
J = \frac{\eta_p U_{\text{total}}}{\text{remaining ligament area}} \quad \ldots \text{A.13}
\]
If the crack is assumed to be semi-elliptical and limit load is based on the remaining ligament area then

\[ P_L = L \sigma_Y \left( WB - \pi \frac{a_{\text{max}}}{2} c \right) \]

... A.14)

To calculate J values in this configuration three different assumptions can be applied:

i) Assume propagation through thickness only, i.e. in the direction using actual \( a_{\text{max}} \) measurements.

ii) Assume propagation at a constant aspect ratio \( (A = \frac{a_{\text{max}}}{2c}) \) measurements using actual \( a_{\text{max}} \).

iii) Assume ii) with actual measurements of both \( a_{\text{max}} \) and \( c \).

i) Assuming propagation through thickness only, i.e. constant \( c \).

from equation A.5 and A.14

\[ J = \frac{\sigma_Y \pi c}{2B} q_p \]

... A.15)

then substituting for \( q \) from equation A.8 and for \( P_L \) from A.14

\[ J = \frac{\pi c}{2B} \left( \frac{U_p}{WB - \pi \frac{a_{\text{max}}}{2} c} \right) \]

... A.16)

\[ \therefore \eta_p = \frac{\pi c}{2B} \]

... A.17)

Hence assuming \( \eta_e = \eta_p \) and since

\[ U \text{ total} = \frac{P_n q_n}{2} + \frac{P_L q_p}{2} \]

... A.18)

\[ \therefore J(c) = \frac{\pi c}{2B} \left( \frac{P_n q_n}{2(WB - \pi \frac{a_{\text{max}}}{2} c)} \right) \]

... A.19)

substitute for \( P_L \) from A.14 in equation A.19 gives

\[ J = \frac{\pi c}{4B} \left( \frac{P_n q_n}{WB - \pi \frac{a_{\text{max}}}{2} c} \right) + \frac{\pi c}{4B} L \sigma_Y q_p \]

... A.20)
ii) Assuming propagation at constant \( A = \frac{a_{\text{max}}}{2c} \) 

re-arranging equation A.14

\[
P_L = \sigma_Y BW(1 - \frac{\pi a_{\text{max}}}{4WBA})^2 \quad \ldots \text{A.21}
\]

then from A.5 and A.14

\[
J = \frac{2\pi a_{\text{max}}}{4WBA} \sigma_Y Wq_p \quad \ldots \text{A.22}
\]

then substituting for \( q_p \) from equation A.8 and for \( P_L \) from A.21

\[
J = \frac{2\pi a_{\text{max}}}{4WBA} \frac{U}{\sigma_Y Wq_p} B(1 - \frac{\pi a_{\text{max}}}{4WBA})^2 \quad \ldots \text{A.23}
\]

then assuming \( \eta_e = \eta_p \) from equation A.18 and substituting for \( P_L \) from A.21

\[
J(A) = \frac{\frac{P_n q_n}{2WBA}}{2B(\pi a_{\text{max}} - \frac{a_{\text{max}}}{2})} + \frac{\pi a_{\text{max}}}{4AB} \sigma_Y q_p \quad \ldots \text{A.24}
\]

iii) Assuming propagation at constant \( A \) with test values of \( c \) and \( a_{\text{max}} \).

If equation A.24 is used with current values of \( c \) and \( a_{\text{max}} \) it can be re-arranged by substitution of \( A = \frac{a_{\text{max}}}{2c} \) into equation A.24 giving

\[
J = \frac{\frac{P_n q_n}{2B(WB/\pi c_n - a_{\text{max}}/2)}}{2B(WB/\pi c_n - a_{\text{max}}/2)} + \frac{\pi c_n}{2B} L \sigma_Y q_p \quad \ldots \text{A.25}
\]
For the CT geometry

Merkle and Corten\(^{(21)}\) give the \(\eta_p\) factor as

\[
\eta_p = \frac{2(1 + \alpha)}{(1 + \alpha^2)^2}
\]

where

\[
\alpha = \sqrt{\frac{2a_n}{W-a_n}} + \frac{2a_n}{W-a_n} + 2 \left( \frac{2a_n}{W-a_n} + 1 \right)
\]

therefore equation A.10 becomes for the CT geometry

\[
J = \frac{\eta_p U_e}{B(W-a_n)} + \frac{2(1+\alpha)U_p}{(1 + \alpha^2)B(W-a_n)}
\]

combining equations A.2, A.3 and A.28 gives

\[
J = \eta_p \frac{P_n (q_n - q_p)}{2B(W-a_n)} + \frac{(P_n + P_L)q_p(1+\alpha)}{B(W-a_n)(1+\alpha^2)}
\]

where \(P_L\) in equation A.29 is given by

\[
P_L = L\sigma_Ba(W-a_n)
\]

A simpler form of equation A.29 may be obtained using the analysis of ref. 22, which is similar to the Merkle and Corten approach with the exception of an assumption that the CT specimen has a moment arm of \((a_n + 0.6(W-a_n))\). Using this expression the limit load equation becomes

\[
P_L = \frac{L\sigma_B(W-a_n)^2}{4(0.4a_n + 0.6W)}
\]

using A.31 in the analysis described above (see SESNT section) gives

\[
\eta_p = 1 + \frac{1}{0.6 + \frac{0.4a_n}{W}}
\]
and the J estimation formula becomes

\[ J = \frac{p_n q_p (1 + \frac{1}{0.6 + 0.4a_n} - n_e) + p_n q_r n_e}{2B(W-n_n)} \]

\[ \frac{L}{2}[1 + \frac{1}{0.6 + 0.4 a_n} \frac{a_n}{W} \frac{(1 - \frac{a_n}{W})}{(0.6 + 0.4 a_n) \frac{\sigma_q q_p}{W}} \]

\[ \ldots \quad A.33 \]
Fig. A1. Postulated load-displacement diagrams: a) Actual load-displacement record and postulated non-linear elastic behaviour; b) an approximation to the non-linear elastic curve.
ISSUES IN DEVELOPING A PLANE STRAIN J₁-CURVE TEST PROCEDURE

J.P. GUDAS
DAVID W. TAYLOR NAVAL SHIP R&D CENTER
ANNAPOLIS, MARYLAND

J.A. JOYCE
U.S. NAVAL ACADEMY
ANNAPOLIS, MARYLAND

P. ALBRECHT
U.S. NUCLEAR REGULATORY COMMISSION
WASHINGTON, DC

CSNI SPECIALIST MEETING ON "PLASTIC TEARING INSTABILITY"
WASHINGTON UNIVERSITY
ST. LOUIS, MISSOURI
SEPT 25-27, 1979
ABSTRACT

Currently, a working group of ASTM Committee E-24.08 is developing a test procedure for determining the plane strain $J_I$-$R$ curve. A review of technical issues impacting this effort has been developed with emphasis on recent data which relates to these issues. The method of computation of $J_I$, measurement of crack extension and influences of slip mode and specimen constraint are discussed. The progress in test and analysis procedures for developing the complete $R$-curve is discussed. Requirements for additional experimentation, particularly in the areas of minimum $\omega$ requirement and limits of allowable crack extension are identified.
INTRODUCTION

Currently, a working group of ASTM Committee E-24.08 is developing a test procedure for determining the plane strain $J_I$-$R$ curve. This effort is in its early stages, and the emphasis is on the entire regime of crack growth beyond $J_{IC}$. The attempt to formulate a test procedure has pointed up several issues which should be addressed, not only for testing purposes, but as a sound basis for application of $J_I$-$R$ curve information. These issues include:

- Method of computation of $J_I$;
- Measurement of crack extension;
- Influence of slip mode;
- Influence of specimen constraint.

The purpose of this paper is to discuss these issues and review recent research data which is directed toward their resolution.

COMPUTATION OF $J_I$

The computation $J_I$ in specimen tests after crack initiation is a complicated process. Certain specimen types require correction for rotation and bending effects [1], and all $J_I$ values must be corrected for crack growth. Further, minimum $\omega$ requirements as set forth by Hutchinson and Paris [2] must be evaluated experimentally and related to theoretical considerations to establish limits on $J$-controlled growth.

Crack Growth Correction

Progress in the formulation of crack growth corrections for the $J_I$ calculation has been substantial over the past two years. The successful experimental development of the key curve analysis
by Ernst [3] and Joyce and co-workers [4] has provided methods
to directly develop the $J_I$-$R$ curve from load-displacement re-
cords with all corrections incorporated. Data from such
computations can then be used to evaluate expressions for
approximating crack growth corrections. Ernst developed one such
analyses of $J_I$ for compact specimens which is expressed in the
following form [5]:

$$J = J_0 \frac{b}{\Delta a \left( \eta - 1 + \frac{\eta'}{\eta} \frac{b}{W} \right)} + J_0$$  \hspace{1cm} (1)

where:

- $J = J_I$ corrected for crack growth;
- $J_0$ = Merkle-Corten Corrected $J$
- $b$ = remaining ligment;
- $\Delta a$ = crack extension;
- $W$ = crack length;
- $\eta$ = non dimensional numerical calculation parameter which is function of $a/W$ only;
- $\eta' = \frac{\partial \eta}{\partial a}$

This expression was evaluated by comparison of $J_I$-$R$ curves for ASTM
A 533-B steel developed with the key curve formulation using pre-
cracked subsize compact specimens. Figures 1 and 2 show that in the
case of planar IT compact specimens, and 20% side-grooved specimens
of the same geometry, the Ernst correction adequately modelled the
$J_I$-$R$ curves from the key curve analysis. As seen previously, both
methods produce $J_I$-$R$ curves with slopes reduced from unloading com-
pliance data previously reported by Vassilaros and co-workers [6]
without crack growth correction.
\section*{Requirements}

Hutchinson and Paris [2] introduced a non-dimensional parameter, \( \omega \), which was used to evaluate the condition for \( J \)-controlled crack growth such that:

\[
\omega = \frac{b}{J} \frac{dJ}{da}
\]

The minimum \( \omega \) requirement for a fully yielded specimen has been suggested to be:

\[
\omega >> 1
\]

and is seen as a sufficient requirement for \( J \)-controlled crack growth. In addressing the question of the minimum \( \omega \) requirement, Hutchinson and Paris pointed up the dependence upon specimen configuration and strain hardening, and suggested systematic studies of the amount of crack growth allowable.

Vassilaros and co-workers [6] carried out an experimental investigation of the \( J_I \)-R curve with ASTM A 533-B steel using 2T plan compact specimens. Figure 3 presents the test matrix for this study for which the thickness/ligament ratio varied from 0.4 to 3.9. The results of these tests presented in Figure 4 showed that the tearing modulus values fell into a relatively narrow band and were insensitive to the \( \omega \) values calculated at 6\% crack growth which ranged from 1.3 to 7.9. This lack of \( J_I \)-R curve dependence on \( \omega \) was interpreted as indicating that the minimum \( \omega \) requirement may be of the other of 1.

It was also suggested that this point should be explored with additional materials and specimen configurations, particularly those developing less crack growth resistance than ASTM A 533-B.
steel. These tests also pointed to the need of side grooving to keep the crack front straight, and allow comparison of specimens of various scales. This topic is discussed further in the following section.

MEASUREMENT OF CRACK EXTENSION

Side Groove Effects

The unloading compliance test procedure initially developed by Clarke and co-workers [7] has emerged as the most common $J_{I}$-R curve test procedure in the United States. During the development of the computer interactive unloading compliance test procedure, it was found that crack tunneling with certain alloys resulted in d$J$/da curves with slopes elevated from those related to physical crack growth [8]. One method to eliminate tunneling was the use of side grooves with various specimen geometries as first evaluated by Andrews and Shih [9]. Studies of effects of side groove geometry on the tearing modulus of HY 130 and ASTM A 533 B steels using the computer interactive unloading compliance procedure have shown that moderate side grooves cause planar crack extension which correlates well with prediction, and which results in reduced values of tearing modulus at various crack length ratios [10, 6]. Additional tests have been performed with HY 80 and A516 Gr. 70 steels at room temperature and 170°C respectively. These results from computer interactive unloading compliance tests were plotted in Figures 5 and 6. For HY80 steel, it can be seen that 15% side grooves are sufficient to saturate the tearing modulus measurement, whereas there is some indication that deeper side grooves are required with A 516
Gr. 70 steel. Figures 7 and 8 are photographs of HY 80 and A 516 Gr. 70 specimens respectively with various side groove depths. Both figures show the change from tunneled cracks to planar crack extension with increasing depth of side grooves.

**Limits of Crack Extension**

The second issue in dealing with the measurement of crack extension is that of the limits of crack growth. Shih and co-workers [11] have suggested the the J-Integral based R curve criteria appear to be valid for limited crack growth, estimated to be on the order of 6% of the original remaining ligament in bend test specimen. The developing $J_{IC}$ test procedure currently limits the consideration of crack extension to this value. With the advent of routine single specimens $J_{I}$-R curve test procedures and key curve analyses which describe the R-curve in detail, it is now desirable to extend crack growth measurements to the physical limits of instrument capability. Data gathered in this manner can then be employed to evaluate the actual limits of crack extension allowable in $J_{I}$-R curve testing.

**INFLUENCE OF SLIP MODE**

The investigation of thickness/lignment ratio effects with ASTM A 533-B steel summarized earlier [6] was directed to evaluating $J_{IC}$ and $dJ/da$ as the slip mode was changed from planar to cross slip. The results of these tests presented in Figure 4 showed that there was no clear dependence of the tearing modulus on the B/b ratio in the range 0.6 to 3.9. Figure 9 shows the type of crack growth developed in 25 mm thick specimens and Figure 10 shows the type of crack growth in 12. 25 and 50 mm thick specimens with a a/W ratios in the order of 0.6. It can be seen with sub-
stantial cross slip was developed, but this did not apparently effect the tearing modulus of this steel.

**INFLUENCE OF CONSTRAINT**

Much of the investigation of specimen constraint has been directed to the validity of $J_{IC}$ measurement. $J_{IC}$ validity criteria have been suggested by Paris [12] such that:

$$B, b > 25-40 \frac{J}{a_0}$$  \hspace{1cm} (4)

Hutchinson and Paris have developed the following condition:

$$\frac{b}{\frac{a_0}{J}} \gg 1$$  \hspace{1cm} (5)

Results of unloading compliance tests with 17-4 PH steel using 1T compact specimens with thickness ranging from 5 to 25 mm have shown that $J_{IC}$ measurements can be raised, and the $J_R$ curve slope lowered when the thinnest specimen was tested [8]. Results of tests with stainless steel piping alloys have shown instances where the slope of the $J$-$\Delta$ curve is similar to that of the blunting line [13]. This makes the determination of $J_{IC}$ quite difficult, particularly when the degree of apparent crack extension due to blunting is on the order of 2-3 mm.

**SUMMARY**

The purpose of this paper was to provide a brief discussion of technical issues related to the development of a $J_R$ curve test standard. These include the computations of $J_R$, the measurement of crack extension, influence of slip mode and the influence of specimen constraint. Over the past three years, substantial progress has been made in developing methods to accurately determine $J_R$ and crack extension over the whole range of the fracture
mechanics test record. Accurate methods have been established
to correct \( J_I \) for crack growth. The \( \omega \) requirements for \( J \)-controlled
-crack growth has been minimally explored from an experimental stand-
point, although the results indicated less restriction than
originally suggested. The role of side grooves has been thoroughly
explored with various steels. Results have shown that the crack
extension front can be straightened and the tearing modulus can
be saturated with moderate size grooves. In the one experiment
reported, the influence of widely different slip modes did not
affect the tearing modulus measurement with ASTM A 533-B steel.
Finally, the degree of specimen constraint was seen to affect
both \( J_{IC} \) and \( dJ_I/da \) as expected.

Additional experimental effort remains to allow further
evaluation of minimum \( \omega \) requirements for \( J \)-controlled crack
growth, and maximum amount of crack growth under which the \( J \)
singularity dominates. Further, the design of test specimens may
have to be modified to accommodate minimum and maximum geometry
requirements addressing both small scale yielding and fully plastic
deformation.

It is anticipated that many of the remaining issues related
to the development of a \( J_I-R \) curve test procedure will be addressed
to two fronts. The role of individual experiments will likely
increase the number and diversity of results to evaluate such factors
as the minimum \( \omega \) requirement. On the other hand, the ASTM Round
robin efforts planned for the near future will likely serve to
address such questions as the amount of crack growth to be con-
sidered in developing \( J_I-R \) curve calculations. If the progress
of the last three years in sustained, the development of a $J_I^{-R}$ curve test procedure is seen as a likely occurrence, and its merging with the $J_{IC}$ standard is anticipated.
REFERENCES


5. Ernst, H. Informal Communication, April 1979


Figure 1 - $J_\text{l}$ versus Crack Extension for ASTM A 533-B Steel from Key Curve Analysis, Unloading Compliance Tests and Ernst Correction of Unloading Compliance Data; a/W 0.7, no side grooves
Figure 2 - $J_T$ versus Crack Extension for ASTM A 533-B Steel from Key Curve Analysis, Unloading Compliance Tests and Ernst Correction of Unloading Compliance Data; $a/W = 0.7$, 20% side grooves.
Figure 3 - Test Matrix for Investigation of Slip Mode Effects with ASTM A 533-B Steel.
Figure 4 - Tearing Modulus versus Thickness/Ligament Ratio for ASTM A 533-B Steel where \( \omega \) values calculated at 6% Crack Growth ranged from 1.3 to 7.9.
Figure 5 - Tearing Modulus versus Side Groove Depth for HY 80 Steel from Unloading Compliance Tests.
Figure 6 - Tearing Modulus versus Side Curve Depth for A 516 Gr. 70 Steel from Unloading Compliance Tests.
Figure 7 - Photographs of the Fracture Surfaces of HY80 Specimens with Various Side Groove Depths
Figure 8 - Photograph of the Fracture Surfaces of A 516 Gr. 70 Specimens with Various Side Groove Depths
Figure 9 - Photograph of the Fracture Surfaces of 25 mm Thick Compact Specimens of ASTM A 533-B Steel with Various Thickness/ Ligament Ratios
Figure 10 - Photographs of the Fracture Surfaces of 12, 25 and 50 mm Thick ASTM A 533-B Steel Compact Specimen with a/W ratios on the order of 0.6.
J-R CURVE CHARACTERIZATION OF IRRADIATED NUCLEAR PRESSURE VESSEL STEELS

F. J. Loss, B. H. Menke, R. A. Gray, Jr. and J. R. Hawthorne

Naval Research Laboratory
Washington, D.C. 20375

ABSTRACT

The J-R curve trends of an A533-B plate and a submerged arc weld deposit having a high copper impurity content have been characterized by means of the single specimen compliance technique. The program involved IT compact specimens with 20% side grooves which were irradiated to a nominal fluence of $1 \times 10^{17}$ n/cm$^2$ >1 MeV. The R curve for these steels exhibits a power law behavior. This curvature precludes both an exact determination of $J_{\infty}$ by the proposed ASTM multispecimen standard as well as the calculation of a single value of tearing modulus to represent the tearing resistance in the region of J-controlled growth. An alternative measurement procedure for $J_{\infty}$ is proposed whereby this quantity is taken as the J value corresponding to a fixed level of crack extension which exceeds that due to crack-tip blunting. This procedure provides a straightforward means for evaluating $J_{\infty}$ for cases of cleavage failure for which the maximum ductile crack extension exceeds that due to blunting by a relatively small amount. In addition, this method provides an unambiguous determination of upper shelf behavior for the R curves measured under quasi-static loading. Within this "static" upper shelf these steels, nevertheless, will exhibit a cleavage failure preceded by a small amount of ductile crack extension. This phenomenon poses a potential problem to the application of the tearing instability concept. However, these mixed mode fractures were no longer observed within the upper shelf temperature regime as defined by a dynamic test such as the Charpy V-notch.

After irradiation the plate material exhibited only a slight decrease in $J_{\infty}$ for tests in the upper shelf region. On the other hand, the $J_{\infty}$ for the irradiated weld metal was reduced to half of its preirradiation toughness. For both materials the tearing modulus exhibited a greater change with irradiation than did $J_{\infty}$. The former was reduced to one-half and one-third of the preirradiation values for the plate and weld deposit, respectively.
1. INTRODUCTION

At the Naval Research Laboratory (NRL) an extensive program is underway to characterize the fracture toughness of structural steels. Under sponsorship of the Electric Power Research Institute (EPRI) a data base is being developed for steels comprising USA reactor pressure vessels which includes A302-B and A533-B plate, A508-2 forging and submerged arc welds of these steels. Under Nuclear Regulatory Commission (NRC) sponsorship we are investigating low upper shelf behavior and a means for irradiation embrittlement relief through periodic heat treatment (annealing). Both of these efforts involve J-R curve characterizations of irradiated materials. This research includes Charpy-V notch (CV) characterizations as well to permit comparisons to be made between the two test procedures.

Current emphasis is on the fracture toughness trends in the upper shelf regime. Impetus for these investigations comes from the use of weld metal in commercial USA reactor vessels which is sensitive to irradiation embrittlement. This sensitivity is due primarily to a high copper impurity level in the steel. After less than 10 years of service, the initiation toughness of these steels is projected to be below that desired to provide an acceptable safety margin for certain accident conditions. Consequently, research which was previously directed to initiation toughness may not provide an acceptable solution for this situation. For upper shelf conditions, however, the concept of tearing instability [1] provides a means to assure structural reliability on the basis of an R curve behavior which exhibits an increase in apparent fracture toughness with crack extension. The tearing instability concept is therefore being actively pursued in relation to the integrity of nuclear structures.

Because of the limited space for irradiation, both in test reactors as well as power reactors, it is required to minimize the size and number of the fracture test specimens. The gamma heating during accelerated irradiation in a test reactor precludes exposure of specimens having a thickness greater than approximately 100 mm. Unfortunately, irradiations of this size are difficult to perform and less than 20 specimens as large as this have been irradiated in the USA. A more practical limit for test reactor irradiations involves 25 to 50 mm thick specimens (e.g., 1T to 2T CT specimens). From a surveillance viewpoint, however, an even smaller specimen size (e.g., ¼ T-CT) is considered necessary. In keeping with the requirement to minimize the numbers of irradiated specimens, a single specimen technique to assess J-R curve behavior is mandated. Our work has emphasized the single specimen compliancance technique (SSC) for this purpose. After years of development work we believe this technique is practical for the testing of irradiated material and that it can provide highly accurate results for the conditions of straight crack front and small specimen size.
Work reported here describes the first application of the SSC technique for the J-R curve characterization of irradiated steels. We have chosen to highlight the results from an A533-B steel plate and an A533-B submerged arc weld deposit, the latter having a high impurity copper content. The material properties, including $C_v$ trends, are given in Ref. 2 and 3.

2. EXPERIMENTAL APPROACH

A 25.4 mm thick compact specimen (IT-CT) was selected because it can be irradiated easily and yet its relatively large size provides sufficient mechanical constraint to infer high levels of plane strain initiation toughness when the specimen itself exhibits elastic plastic behavior. For example, the criterion of $b > 25 J_{IC}/\sigma_f$ will permit measurement of $J_{IC}$ close to 500 kJ/m$^2$ for irradiated material where $b$ is the unbroken ligament and $\sigma_f$ is the flow stress. From this value of $J_{IC}$ it is possible to infer a plane strain toughness $K_{IC}$ of approximately 350 MPa$\sqrt{\text{m}}$.

The experimental procedure utilizes a CT specimen of E-399 proportions but having a modified notch geometry to permit the mounting of razor knife edges for the measurement of load line deflection with a clip gage (Fig. 1). The notch modification has required a small increase in the distance between hole centers of the E-399 standard specimen. In addition, certain specimens were side grooved using a $C_v$ notch cutter ($45^\circ$ angle, 0.25 mm root radius) in order to produce a straight crack front extension. Relative crack lengths ($a/W$) of 0.5 and 0.6 have been used for this study.

With the SSC technique, the specimen is periodically unloaded by approximately 10%. These load vs load-line deflection ($p$ vs $\delta$) records are highly amplified (Fig. 2) and are used to infer the crack extension $\Delta a$ via a compliance calibration relationship, that is, $EB\delta/p$ vs $a/W$ where $E$ is Young's modulus and $B$ is the thickness. In this case the theoretical relationship of Hudak and others [4] was employed.

Successful application of the SSC technique relies upon accurate crack length predictions. This, in turn, requires the minimization of both friction and electronic noise in the experimental apparatus so that the specimen compliance can be established with precision. The experimental record of Fig. 2 is typical of a highly amplified unload-reload test sequence from a IT-CT specimen to determine compliance changes. From a repeatability viewpoint, the initial four unloadings in this record are sufficiently accurate to define the crack length to a precision of ± 0.05 mm (2 mils) with a 95% confidence limit. However, repeatability is not to be confused with accuracy. The latter is difficult to define exactly because of its dependence on the compounding of errors due to load cell and clip gage calibration factors, modulus determination, validity of the compliance expression and variations caused by the electronic noise level. Nevertheless,
our development of the SSC method has resulted in test records which exhibit a scatter of only ± 0.08 mm (± 3 mils) in the predicted crack length in a IT-CT specimen. As a test of this predictive capability a correspondence between the predicted and the optically-measured final crack lengths, when these crack fronts are straight, is typically within ± 0.13 mm (± 5 mils). On the other hand, large discrepancies between predicted and measured crack extensions have been detected for smooth specimens having curved or tunneled crack fronts; the same is true of specimens which have been machined from an A302-B steel plate having a high non-metallic inclusion content [5] even though straight crack fronts were exhibited.

The magnitude of $J$ is computed from the relationship:

$$J = \frac{1 + \alpha}{1 + \alpha^2} \frac{2A}{b \cdot B_n}$$

where $A$ is the specimen energy absorbed based on total deflection, $b_n$ is the original unbroken ligament, $B_n$ is the net thickness and the term $(1 + \alpha)/(1 + \alpha^2)$ is a modified Merkle-Corten correction to account for the tension component of the loading [6]. A correction [5] is also applied to account for minor specimen rotation which occurs during loading. This rotation reduces the moment arm of the load and also results in a deviation of the clip gage loading points from the true load line. A computer is used to provide on-line determinations of $J$ and crack extension, $\Delta a$. The computer code was originally developed by Joyce and Gudas [7].

3. **$J_{lc}$ MEASUREMENT PROCEDURE**

Figure 3 illustrates a typical $J$-$R$ curve for the unirradiated weld metal (Code V86) at a temperature within the upper shelf regime as determined by the static tests. The $J_{lc}$ value is currently defined by the proposed ASTM multispecimen standard [7] as the intersection of the blunting line ($\Delta a = J/2 \sigma_f$) with the least squares fit of the points lying between lines 1 and 2 illustrated in the figure. Line No. 1 is drawn parallel to the blunting line at a crack extension of 0.15 mm. This is termed an "exclusion line" and is believed to be an indicator of the first real crack extension. In other words, any crack extension less than the exclusion line may be difficult to distinguish from blunting. The 0.15 mm exclusion line criterion is also within the projected accuracy of the SSC technique as applied to IT-CT specimens; however, this accuracy may be more difficult to obtain with larger specimens.
For two reasons an alternative definition of \( J_{IC} \) to that of Ref. 8 is proposed here. First, the \( R \) curve for nuclear pressure vessel steels appears to exhibit a power law behavior for which a \( J_{IC} \) measurement procedure based upon a least squares fit of the data may be inappropriate. Second, in cases of cleavage failure (Fig. 4) the maximum crack extension could exceed that due to blunting by only a small amount. For this situation insufficient data points may exist so as to preclude a least squares fit. Consequently, an alternative definition of \( J_{IC} \) is proposed: \( J_{IC} \) will be defined here as the \( J \)-level corresponding to the intersection of the 0.15 mm exclusion line with a smooth curve through the SSC data points, as illustrated in Fig. 3. A smooth curve through the data is felt to be an accurate representation of the nonlinear characteristic of the \( R \) curve. For the case of specimen failure at a crack extension less than that defined by the exclusion line (e.g., some types of cleavage failure) \( J_{IC} \) would be computed at the level of failure.

The advantage of the alternative method for \( J_{IC} \) is that it will define the same value of \( J_{IC} \) irrespective of how much or how little crack extension is developed beyond the exclusion line prior to cleavage failure. Secondly, the method deals with the nonlinear \( R \) curve so as not to force a straight line fit to the data points for purposes of \( J_{IC} \) measurement. Note, however, that both the alternative method and the proposed ASTM method can yield comparable values of \( J_{IC} \) for a large crack extension as illustrated in Fig. 3. Thus, it is felt that the two measurement procedures for \( J_{IC} \) are not incompatible.

Figure 4 illustrates the \( R \) curve for an A533-B steel specimen which exhibited a small crack extension beyond the 0.15 mm exclusion line before failure in the cleavage mode. This expanded plot highlights the \( \pm 0.08 \) mm (\( \pm 3 \) mill) scatter which can be produced with IT-CT specimens. (This figure is typical of the largest scatter which has been observed with the current experimental technique. This specimen was cut from another A533-B steel plate, Code CBB, whose properties are given in Ref. 2.) For this case, \( J_{IC} \) is given as 214 kJ/m\(^2\) (1225 lb/in.) by the alternative procedure. Since the crack extension is less than twice the extension to the blunting line, the proposed ASTM procedure would define \( J_{IC} \) at the level of fracture. The latter corresponds to a value of 308 kJ/m\(^2\) (1765 lb/in.) which is 44% greater than that given by the alternative definition.

Note that the \( R \) curve of Fig. 4 exhibits a sharp change in slope prior to cleavage fracture. This is not a result of data scatter and has been observed with other specimens cut from A533-B steel plate (see Fig. 5). Possibly this is a new phenomenon associated with a "hardening" of the steel prior to cleavage fracture; this behavior corresponds with a flattening of the load versus deflection trace just prior to failure when the failure
occurs after the maximum load. For a similar-type behavior but at a somewhat greater crack extension, the proposed ASTM procedure requires a least squares fit of the data in order to define $J_{IC}$. The hardening phenomenon observed for cleavage failure would appear to make such a least squares fit inappropriate.

4. R-CURVE COMPARISONS

R curves from the A533-B plate (Code CAB) in the $C_y$ upper shelf temperature regime are compared in Fig. 6. Fracture toughness properties for these and other tests are listed in Table 1. The specimens were side grooved by 20 and 25% in order to produce a relatively straight front extension. While 25% side grooves appear to be sufficient for this purpose, the improvement over a 20% groove depth is minimal as illustrated in Fig. 7. A groove depth less than 20%, however, was insufficient to produce a straight crack front in both the unirradiated and irradiated conditions for this steel.

The postirradiation R curve behavior for plate CAB is shown in Fig. 8. The initial position of the R curve is within the scatterband for unirradiated tests in the upper shelf regime. Thus we conclude that the upper shelf toughness is changed very little from the unirradiated condition. In actuality a small (~15%) decrease in $J_{IC}$ has been determined for the irradiated material because its higher flow stress results in a steeper exclusion line and hence a lower $J_{IC}$ value. On the other hand, the tearing modulus T [1] for the irradiated material is approximately half of that for the unirradiated condition, thereby suggesting a larger effect of irradiation in this quantity than in $J_{IC}$.

A comparison of R curves from the weld deposit (Code V86) in the pre- and postirradiated conditions is presented in Fig. 9 for specimens without side grooves. After irradiation, $J_{IC}$ is approximately two-thirds of the value found with unirradiated material. The tearing modulus, however, exhibits a greater reduction with irradiation than does $J_{IC}$ and is less than half the level measured with unirradiated material. This behavior is similar to that of the plate.

The specimens in Fig. 9 without side grooves exhibited a tunneled crack extension, as expected; the final crack-extension increment length was underpredicted by 20% and 30% for the pre- and postirradiated specimens, respectively. From our experience, the SSC technique will always underestimate the measured crack length when tunneling occurs. (We have determined this to be a mechanical phenomenon by comparing the compliance from two specimens; one with a straight, machined crack and the other with a machined-tunneled crack front). Consequently, the R curves in Fig. 9 are not physically correct and must be rotated clockwise to agree with the actual crack extension which
occurred during the test. On this basis, the actual R curves for the tests in Fig. 9 are approximated (dash lines) by linearly proportioning the crack-extension estimation error when the test was halted. While this adjustment has little effect on $J_{iC}$, it has resulted in a drop of about 15% in the $J$ and $T$ values at a crack extension of 1.5 mm.

As with the A533-B plate, the crack tunneling in weld metal specimens can be eliminated with 20% side grooves (Fig. 10). It was interesting to note that a side-groove depth of only 10% was sufficient to straighten the crack for the irradiated weld deposit. However, a 10% depth was not sufficient to straighten the crack for the irradiated plate material. This behavior suggests that the optimum choice of side grooves may be alloy-dependent.

In general, it was found that a tunneled crack front resulted in a steeper R curve than was associated with a straight crack front. An example of this behavior is illustrated in Fig. 11 for irradiated specimens having 0, 10 and 20% side grooves. The specimens with 10 and 20% side grooves produced a straight crack front and exhibited R curves of similar slope. It should be noted, however, that the adjustment procedure to account for the underprediction of crack extension in the case of a tunneled crack front (i.e., the specimen with 0% side grooves) will decrease the R curve slope so as to diminish the apparent effect of side groove depth.

R curves for the weld deposit in the pre- and postirradiated condition are illustrated in Fig. 12 for specimens with 20% side grooves. Here, $J_{iC}$ for the irradiated weld deposit is 65 kJ/m². This value is approximately one-half of that from unirradiated material (see Table 1). The tearing modulus from the irradiated tests in Fig. 12 is 30. This quantity was computed on the basis of a least squares fit through the data for a crack extension less than 1.5 mm. This value of $T$ is one-third of that exhibited by the unirradiated specimens having 20% side grooves.

Since the R curve exhibits a nonlinear behavior, the value of $T$ decreases with crack extension. For example, at a real crack extension of 1.5 mm (dashed line no. 1 in Fig. 12), $T$ for the irradiated material is approximately 15. The variation in this quantity is similar for all materials tested, both irradiated and unirradiated, and amounts to a factor of three decrease over a crack extension increment of 1.5 mm beginning with the $J_{iC}$ point (Table 1). Also tabulated is the value of $\omega$ as computed at the $J_{iC}$ point and at a crack extension increment of 1.5 mm. This parameter decreases by almost an order of magnitude over the crack extension increment. However, there is little difference in $\omega$ between the irradiated and unirradiated material. In other words, the lower R curve slope for the irradiated material is offset by a corresponding drop in $J$. 
5. UPPER SHELF BEHAVIOR

A comparison of Figs. 5 and 6 suggests that the R-curve slope is influenced very little by temperatures within a 100°C (180°F) interval. Consequently, once this curve crosses the exclusion line, essentially the same value of \( J_{IC} \) will be defined regardless of whether the specimen exhibits fibrous tearing or cleavage instability as a result of further crack extension beyond this point. It is proposed, therefore, that an upper "shelf initiation toughness" be defined as that level where 0.15 mm crack extension is first achieved. As a result of this approach the trend in \( J_{IC} \) or \( K_{JC} \) vs temperature will exhibit a sharp break when the upper shelf temperature is reached as illustrated in Fig. 13. (In this figure \( K_{JC} \) has been computed from \( K_{JC} = [EJ_{IC}/(1 - v^2)]^{1/2} \) where \( v \) is Poisson's ratio).

It can be observed in Fig. 13 that some specimens exhibit a brittle (cleavage) fracture at temperatures in the "static" upper shelf regime even though this has been preceded by a small amount of ductile crack extension. Only 0.2 mm of ductile crack extension was observed prior to cleavage failure for the test at \(-11^\circ C\). In the irradiated condition, however, the subject steels show a similar upper shelf behavior but the measured ductile crack extension increment prior to cleavage on the upper shelf averages 1 mm (seven specimens). For example, the specimen in Fig. 8 exhibited 1.5 mm of crack extension followed by a cleavage failure.

Confusion can arise on the application of the term upper shelf toughness to the \( J_{IC} \) trends. Traditionally, the shelf toughness has been associated with the \( C_v \) test and defines a region of completely ductile fracture for which the specimen energy absorption is relatively temperature insensitive. In other words, one does not normally associate cleavage fracture with upper shelf behavior. With the R curves it is also possible to define a region of ductile initiation toughness which is relatively insensitive to temperature. The only difference is that the ductile crack extension may switch to a cleavage mode. (This is a phenomenon which many years ago we have termed strain-induced cleavage.)

The preceding phenomenon carries with it certain structural implications regarding application of the tearing instability concept. Chief among these is that it may be unconservative to apply this approach in certain regions of the upper shelf temperature regime when the latter has been determined on the basis of a quasi-static test. On the other hand, cleavage fractures have not been observed in the upper shelf regime as defined by a dynamic test. For certain structural steels the \( C_v \) test can be used to define this region. (However, this is not universally true for all carbon and low-alloy steels). For comparison purposes, the \( C_v \) and \( K_{JC} \) trends are illustrated in Fig. 13. It
can be seen that cleavage breaks in the R-curve specimens occurred up to a temperature level equivalent with approximately one-third of the $C_V$ upper shelf energy. The same trend appears to hold for the V86 weld deposit in the irradiated condition and for the plate CAB in the pre- and postirradiated conditions.

6. SUMMARY

We have presented R-curve trends from two irradiated pressure vessel steels, an A533-B plate and an A533-B submerged arc weld deposit. These represent the first results from irradiated material by means of the single specimen compliance (SSC) technique. For a fluence of $\sim 1 \times 10^{19}$ n/cm$^2 > 1$ MeV the plate exhibited a relatively small reduction in $J_{IC}$ but this quantity for the weld deposit was reduced to one-half of its preirradiation value. The tearing modulus for both plate and weld exhibited a greater reduction with irradiation than did $J_{IC}$. The former was reduced to one-half and one-third of the preirradiation values for the plate and weld, respectively.

The R-curves measured by the SSC technique exhibit a power law behavior. This is somewhat inconsistent with the linear behavior established by investigators using the multispecimen (heat tint) technique and this point remains unresolved. Nevertheless, the accuracy obtained with the SSC technique permits one to conclude that a power law relationship exists. The result of this behavior is a factor of two increase in $J$, a factor of three reduction in $T$, and almost an order of magnitude variation in $\omega$ over a 5.5 mm crack extension increment beginning with $J_{IC}$. No apparent differences in the preceding trends between pre- and postirradiation material were observed.

As a consequence of the R-curve slope an alternative procedure is proposed for the measurement of $J_{IC}$ which differs from that currently proposed by ASTM for the multispecimen procedure. Both procedures, nevertheless, yield comparable $J_{IC}$ values for R curves which exhibit a crack extension of at least 5.5 mm (an average difference of only 2% between the two methods was shown for the examples in Table I). However, the alternative procedure permits a straightforward measurement of $J_{IC}$ for a small amount of stable crack extension which is terminated by cleavage (brittle) fracture. In connection with the latter, an apparent new phenomenon of hardening prior to cleavage fracture was shown.

A definition of upper shelf toughness for quasi-static R curve tests has also been proposed. The result shows that cleavage fracture occurs within the "static" upper shelf at temperatures below the onset of an upper shelf as defined by a dynamic test procedure. This phenomenon must be weighed carefully when applying the concept of tearing instability.
7. ACKNOWLEDGEMENT

The authors gratefully acknowledge the sponsorship of the Electric Power Research Institute, T. U. Marston, project manager, and of the Nuclear Regulatory Commission, Office of Nuclear Regulatory Research, Pedro Albrecht, project manager, for their support of this research.

3. REFERENCES


### Table 1 - Fracture Toughness Properties

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Side</th>
<th>Flouence</th>
<th>Test</th>
<th>Temp</th>
<th>( J_{IC} )</th>
<th>( \Delta J_{IC} )</th>
<th>25 ( J_{IC} / \sigma_f )</th>
<th>25 ( J_{IC} / \sigma_f )</th>
<th>( \beta )</th>
<th>( \tau )</th>
<th>( \sigma )</th>
<th>( \omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAB(^a)</td>
<td>1</td>
<td>25</td>
<td>0</td>
<td>121</td>
<td>182</td>
<td>171</td>
<td>9.6</td>
<td>24.2</td>
<td>24.5</td>
<td>225</td>
<td>148</td>
<td>79</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>20</td>
<td>1</td>
<td>1</td>
<td>223</td>
<td>221</td>
<td>11.7</td>
<td>25.4</td>
<td>24.5</td>
<td>285</td>
<td>138</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>20</td>
<td>1.1</td>
<td>50</td>
<td>156</td>
<td>141</td>
<td>6.1</td>
<td>24.7</td>
<td>82</td>
<td>66</td>
<td>26</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>1</td>
<td>60</td>
<td>162</td>
<td>153</td>
<td>6.4</td>
<td>11.8</td>
<td>24.5</td>
<td>144</td>
<td>60</td>
<td>39</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>1.5</td>
<td>82</td>
<td>177</td>
<td>184</td>
<td>7.0</td>
<td>13.6</td>
<td>25.1</td>
<td>133</td>
<td>72</td>
<td>24</td>
<td>37</td>
</tr>
<tr>
<td>V86(^b)</td>
<td>24</td>
<td>-100</td>
<td>0</td>
<td>-110</td>
<td>23</td>
<td>-</td>
<td>1.1</td>
<td>-</td>
<td>19.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>-40</td>
<td>0</td>
<td>-72</td>
<td>38</td>
<td>-</td>
<td>1.0</td>
<td>-</td>
<td>19.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>31</td>
<td>-60</td>
<td>0</td>
<td>-60</td>
<td>62</td>
<td>-</td>
<td>3.0</td>
<td>-</td>
<td>19.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>-50</td>
<td>0</td>
<td>-50</td>
<td>89</td>
<td>-</td>
<td>4.3</td>
<td>-</td>
<td>19.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>-40</td>
<td>0</td>
<td>-119</td>
<td>-</td>
<td>5.7</td>
<td>-</td>
<td>19.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-119</td>
<td>0</td>
<td>-119</td>
<td>-</td>
<td>5.8</td>
<td>-</td>
<td>18.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>20</td>
<td>124</td>
<td>119</td>
<td>6.1</td>
<td>15.9</td>
<td>19.3</td>
<td>202</td>
<td>103</td>
<td>68</td>
<td>40</td>
<td>4.7</td>
</tr>
<tr>
<td></td>
<td>-3</td>
<td>0</td>
<td>200</td>
<td>123</td>
<td>6.4</td>
<td>13.0</td>
<td>19.3</td>
<td>128</td>
<td>70</td>
<td>49</td>
<td>24</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>-26</td>
<td>0</td>
<td>132</td>
<td>120</td>
<td>6.8</td>
<td>18.3</td>
<td>19.8</td>
<td>196</td>
<td>116</td>
<td>87</td>
<td>35</td>
<td>5.3</td>
</tr>
<tr>
<td></td>
<td>-15</td>
<td>10</td>
<td>144</td>
<td>138</td>
<td>7.4</td>
<td>17.6</td>
<td>19.8</td>
<td>174</td>
<td>102</td>
<td>81</td>
<td>29</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>-18</td>
<td>0</td>
<td>79</td>
<td>76</td>
<td>3.2</td>
<td>8.2</td>
<td>20.3</td>
<td>91</td>
<td>43</td>
<td>31</td>
<td>44</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>-21</td>
<td>10</td>
<td>80</td>
<td>92</td>
<td>3.3</td>
<td>5.2</td>
<td>19.3</td>
<td>74</td>
<td>21</td>
<td>15</td>
<td>35</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>-27</td>
<td>20</td>
<td>66</td>
<td>77</td>
<td>2.7</td>
<td>6.4</td>
<td>19.8</td>
<td>89</td>
<td>28</td>
<td>15</td>
<td>50</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>-13</td>
<td>20</td>
<td>65</td>
<td>63</td>
<td>2.7</td>
<td>5.4</td>
<td>19.8</td>
<td>54</td>
<td>24</td>
<td>15</td>
<td>31</td>
<td>3.8</td>
</tr>
</tbody>
</table>

\(^a\)A533-B plate  
\(^b\)A533-B, B/A weld  
\(^c\)\(10^{15}\) n/cm² > 1 MeV  
\(^d\)Alternate method

\(^e\)Proposed ASTM definition  
\(^f\)Evaluated at \( \Delta a = 1.5 \) mm plus blunting  
\(^g\)Original unbroken ligament  
\(^h\)Evaluated at \( J_{IC} \)  

\(^i\)Using ASTM least squares fit
Fig. 1. CT compact specimen geometry for an e/W of 0.6. A precrack of 2.5 mm (0.100 in.) is added prior to testing. All dimensions are in inches (1 in. = 25.4 mm).
Fig. 2  Highly amplified unload-reload traces taken periodically during the loading of a TT-CT specimen. Note the absence of hysteresis. Also shown is an example of the material relaxation behavior (upper left) for unloading No. 16.
Fig. 3 Illustration of the typical curved behavior of the R curve. An alternative measurement of $J_{IC}$ is indicated by the intersection of the smoothly drawn R curve with Line No. 1 drawn at 0.15 mm crack extension. Also shown is the proposed ASTM definition of $J_{IC}$ which uses a least squares fit to the R curve.
Fig. 4  K curve for an A533-B steel which exhibits cleavage fracture at a crack extension slightly in excess of the 0.15 mm exclusion line. The scales have been expanded to highlight the data scatter which represents the extremes using current experimental techniques.
Fig. 5  Illustrating the R curve for a specimen which exhibited a limited stable crack extension prior to brittle fracture. Note the sharp increase in R-curve slope prior to fracture. The trend of upper shelf behavior for this steel is also illustrated.
Fig. 7  Comparison of crack front curvature resulting from 20 and 25% side grooves in IT-CT specimens cut from A533-B plate. While the 25% grooves appear to result in a straight crack front, the improvement over the 20% grooves is minimal.
Fig. 8 Illustrating the J-R curve behavior of irradiated material. The trend of unirradiated material (Fig. 6) is shown for comparison.
Fig. 9 Comparison of R curves from weld metal in the pre- and postirradiated conditions for specimens without side grooves. The dashed lines approximate the R curves to eliminate the crack-length prediction error associated with a tunneled crack front.
Fig. 10  The influence of 0, 10 and 20% side grooves (left to right) on crack-front shape. No apparent difference exists between the specimens having 0 and 10% side grooves. However, a slight overcorrection has occurred as illustrated by the reverse tunneling exhibited by the specimen with 20% side grooves.
Fig. II. Comparison of R curves from irradiated specimens of weld deposit having 0, 10 and 20% side grooves.
Fig. 12. Comparison of R curves for the weld deposit in the pre- and postirradiated conditions for specimens having 20% side grooves.
Fig. 13  Comparison of preirradiation J integral and $C_v$ energy trends for an AS33-B weld deposit. All J integral specimens had 20% side grooves unless otherwise indicated.
CNSI - Specialist Meeting on Tear Instabilities

Saint Louis Missouri 25 - 27 septembre 1979

CORRELATION BETWEEN GENERAL YIELDING
AND TEAR INITIATION IN CT - SPECIMENS

B. MARANDET*, J.C. DEVAUX**, A. PELLISSIER TANON***

* IRSID 185, rue du Président Roosevelt, 78104 Saint Germain en Laye
** Framatome Chalon BP 13, 71380 Saint Marcel
*** Framatome Tour Fiat cedex 16, 92084 Paris la Défense

Introduction

We present several results on the influence of size and temperature on the value of $J$ at tear initiation on CT specimens, and discuss them in keeping with the results of a plane strain elastoplastic finite element calculation of a CT specimen having a similar a/w ratio.

In our nomenclature, the thickness $B$ of the specimen is given in mm. Ex. : CT 50 means 50 mm thick.

Experimental results

The ferritic steels used in these experiments are presented at table I.

The CT specimens are cut in accordance with the ASTM E 24.08 recommendation on $J_{IC}$, with a/w close to 0.62.

The crack initiation is determined either by the multiple specimen technique, with a micrographical measurement of the crack advance, or by the alternative current potential variation method described in [1].
For all but the largest specimens, it has been checked that the range of values of J corresponding to the minimum of the potential-displacement trace (Figure 1), coincides with the onset of crack growth seen by the micrographic examination, as illustrated by the J -Δa resistance curve of the CT25 and CT40 of the 28NCD 8-5 steel (Figures 2 and 3).

Figure 1 presents the general pattern of the fracture process in the transition of ferritic steels: below Ttf, the fracture occurs by cleavage; between Ttf and Ttd, some amount of stable crack growth by tear precedes the cleavage fracture; above Ttd, the cleavage fracture can no longer take place. The value of J at tear initiation increases between Ttf and Ttd, then levels off at Ttd.

The whole set of values of J measured at onset of cleavage or at tear initiation, on the steel 28NCD 8-5 using specimens CT15, CT25, CT40 and CT75 is presented at Figure 4.

The values of J at the temperature Ttd obtained in these experiments and in tests with other steels are given at table II with the corresponding temperatures and yield stresses Re. The table shows that the values of KJC defined from the values of J at the cleavage-tear initiation transition coincides remarkably with the value of KJC defined from J* = Reb/75.

Figure 5 presents the measurements made at 20°C on the 28NCD 8-5. Figure 6 presents the ranges of values of J at the potential drop minimum and the values of JIC, according to the ASTM E 24.08 recommended regression procedure, measured at 20°C on the 28NCD 8-5 steel, as a function of the ligament width w - a (or b):

For the CT15, CT25 and CT40 specimens, the onset of crack propagation coincides with the range of the potential drop minima. But for the CT75, the crack starts propagating below the range of the potential drop minima. Here again, it can be seen that, for the first three specimen sizes the values of J* fall in the range of the values of J at the potential drop minimum. However the value of J at the onset of tear propagation appears to tend towards an upper limiting value as the size of the CT specimens increases up to the CT75.

Additionally, Figure 3 which presents results of specimens having subscale thicknesses (B) shows no influence of the thickness on the potential minimum and on the J-Δa a resistance curve.

Discussion

For a set of similar specimens (same shape, proportional in size) of the same steel, an equal extent of the plastic deformation is marked by a fixed position in the reduced load-displacement record F/B2 versus d/B (F load, d relative displacement of the load points). From J = \( \frac{\Delta U}{Bb} \) (U Work of deformation in
the $F - d$ loading curve, shape factor), it is seen that a constant value of $J/b$ marks the same state of deformation. To compare different steels below the maximum load, a same state of deformation corresponds approximately to a same value of $J/\sigma_y b$.

The state of deformation of the CT specimens corresponding to $J^* = \sigma_y b/75$ can be determined through a finite element calculation.

We consider the results of the following plane strain calculation on a CT50 specimen having $a/w = 0.62$ :

The finite element mesh with 8 node isoparametric finite elements is given at Figures 7 and 8. The stress strain curve is bilinear with:

$E = 206,160$ MPa, $\sigma_y = 276$ MPa, $m = 2180$ MPa (plastic tangent modulus).

The load displacement curve calculated is pictured at Figure 9 with the states of development of the plastic zone in the specimen. The values of $J$ are calculated from the load displacement curve of figure 9 according to the ASTM E24 08 recommandation.

With $\sigma_y = 276$ MPa, $b = .38$mm, $J^* = 0.13$ MJ/m2 :

This value coincides with the loading point N° 28 of Figure 9. It corresponds to a level of load slightly higher than the onset of generalized plastic deformation through the ligament. (general yield)

This result clearly tends to show that specimens with increasing sizes behave as if the onset of general yield is stirring the onset of tear propagation (tear initiation), until, finally, the onset of tear propagation takes place below general yield, and then become no longer dependant on the size of the specimen.

Practical Conclusions on tearing toughness characterisation

These results suggest that an intrinsic measure of the plane strain tear initiation toughness should require a sizing of the specimens enabling to obtain initiation of tearing below general yield.

At least for the stability analysis of large cracks in small scale yielding or contained plasticity, the use of $J - \Delta a$ resistance curves or of a conventional $J_{IC}$ measured below general yield should be the more relevant. The above experiments show that measurements on smaller specimens above general yield should give a lower bound to the contained plasticity resistance curve (lower value of $J$ for a given $\Delta a$). These conclusions which are supported only by experiments with limited crack growth ($\gamma < 1$ mm) do not necessarily apply to the measurement of the tearing modulus.

Références

<table>
<thead>
<tr>
<th>Nuance d'acier</th>
<th>Composition chimique (%)</th>
<th>Caractéristiques mécaniques de traction (T = + 20°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>Mn</td>
</tr>
<tr>
<td>28 NCD 8-5</td>
<td>0,29</td>
<td>0,63</td>
</tr>
<tr>
<td>20 CND 8</td>
<td>0,22</td>
<td>0,66</td>
</tr>
<tr>
<td>A 508 Cl. 3</td>
<td>0,14</td>
<td>1,37</td>
</tr>
<tr>
<td>Nuance d'acier</td>
<td>Type d'éprouvette</td>
<td>$T_t$ (°C)</td>
</tr>
<tr>
<td>----------------</td>
<td>------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>28 NCD 8-5</td>
<td>CT 40</td>
<td>+20</td>
</tr>
<tr>
<td></td>
<td>CT 25</td>
<td>-40</td>
</tr>
<tr>
<td></td>
<td>CT 15</td>
<td>-70</td>
</tr>
<tr>
<td></td>
<td>20 x 20</td>
<td>-60</td>
</tr>
<tr>
<td>20 CND 8</td>
<td>CT 25</td>
<td>+20</td>
</tr>
<tr>
<td></td>
<td>CT 15</td>
<td>-20</td>
</tr>
<tr>
<td></td>
<td>20 x 20</td>
<td>+20</td>
</tr>
<tr>
<td>A 508 Cl. 3</td>
<td>CT 25</td>
<td>+3</td>
</tr>
</tbody>
</table>

*TABLE 2*
\[ K_{jc} = \sqrt{\frac{E J_{lc}}{(1 - v^2)}} \]

- ductile tear initiation
- cleavage instability
- maximum load

\[ J^* = \frac{R_e b}{75} \]

\[ P, N \]

\[ P_{\text{max}}, P_{\text{fracture}} \]

\[ d, d_c, d_e \]

\[ T_{td}, T_{tf} \]
$J_{ut}=0,174 \text{ MJ/m}^2$ (moyenne des valeurs mesurées par la méthode électrique)
FIGURE 3

Acier 28 NCD 8-5
T = 20°C

offset

 Valeurs de Jc mesurées par la méthode électrique

\( Jc = \text{Re}(\text{Im}) \)

\( (W-a) = 30\text{mm} \)
\( B = 40\text{mm} \)
\( B = 25\text{mm} \)
\( B = 20\text{mm} \)
\( B = 15\text{mm} \)

\( \Delta \delta, \text{mm} \)

0.7
0.6
0.5
0.4
0.3
0.2
0.1

0
1.0
1.5
2.0

offset
Tenacité à rupture
$K_{Ic}$, MPa\sqrt{m}

Acier 28 CND 8-5

- Valeurs mesurées directement AFNOR A03-180
  (CT 150, CT 100, CT 75)

- Valeurs calculées à partir de $J_{Ic}$:
  - CT 15
  - CT 25
  - CT 40
  - 20x20

Niveau estimé d'après la formule proposée par:
Roll, Novak, Barsam (ref.)

Courbe estimée d'après la méthode IRSID (ref.)

Température, °C

FIGURE 4
FIGURE 8

ÉPREUVE CT 50 - FISSURE À 62 mm

DISCRETISATION ADOPTÉE
Figure 9

EProuvette CT50
Deformation Plane

Courbe force en fonction du déplacement $U_y$
THE MEASUREMENT OF DUCTILE CRACK INITIATION:
A COMPARISON OF DATA FROM MULTIPLE AND SINGLE SPECIMEN
METHODS AND SOME CONSIDERATIONS OF SIZE EFFECTS

T Ingham and E Morland

Risley Nuclear Power Development Laboratories
UKAEA, Northern Division
England
ABSTRACT

R curves have been obtained, using the multiple specimen method, from A533B Class 1 steel of both US and UK origin, and also from a Si killed Al grain refined C-Mn steel. The specimen geometries studied have included square and rectangular section SENB specimens 25 and 12.5mm thick, 25mm thick CT specimens and, in the case of the C-Mn steel, 20mm CT specimens. Values of initiation toughness, measured in terms of J or COD, were defined from R curves constructed using data from tests terminated at, or before, the maximum load instability condition and extrapolating the toughness/Δa plots to zero crack growth. The specimen sizes and ranges of crack growth examined were too small to permit valid application of the method of R curve construction proposed by ASTM but US and UK procedures are compared and discussed in terms of small scale testing.

A comparison of individual J/Δa plots suggests a small but consistent size effect on both initiation toughness and resistance to growth. Tests on both samples of A533B indicate that 25mm thick square cross-section three point bend specimens notched to an a/W ratio = 0.33 yield higher values of initiation toughness and lower resistance to growth than 25mm thick deep notched (a/W 0.5-0.6) rectangular cross-section three point bend or compact specimens. There were no significant differences in the R curve data from 12.5mm and 25mm thick deep notched specimens. The main restriction to using 12.5mm thick specimens is the limited range of crack growth available for R curve construction.

DC and AC potential drop techniques have been used to provide information on the suitability of these techniques for single specimen monitoring of ductile crack initiation. DC pd indicated initiation toughness values were obtained from 25 x 25mm and 12.5 x 25mm three point bend tests on A533B steel. AC pd indicated toughness values were obtained from the tests on C-Mn steel and from a limited number of three point bend tests on A533B steel. In general, the DC pd method under-estimated initiation toughness values derived from multiple specimen tests. AC pd measurements were less easy to interpret than DC pd measurements and gave an over-estimate of initiation toughness compared with that predicted from an R curve.
NOMENCLATURE

- \( a \): crack depth
- \( a_0, a_i \): initial crack depth
- \( a_f \): final crack depth
- \( A \): area under load-load line displacement curve
- \( B \): specimen thickness
- \( E \): Young's modulus
- \( J \): contour integral
- \( J_{IC}, J_{IC} \): toughness ductile crack initiation
- \( J_A \): J calculated using area under load/displacement curve
- \( J_{VG} \): J calculated using load, displacement values taken from load/displacement curve
- \( K \): stress intensity factor
- \( m \): a proportionality factor
- \( P_f \): final load
- \( P_L \): limit load
- \( r \): a rotational constant
- \( s \): loading span
- \( V_g \): clip gauge displacement
- \( V_{ge} \): elastic component of clip gauge displacement
- \( V_{gp} \): plastic component of clip gauge displacement
- \( V_{gf} \): total clip gauge displacement
- \( W \): specimen width
- \( z \): knife-edge height
- \( \alpha \): a correction term for tensile loading
- \( \alpha' \): a correction term for crack growth in three point bend tests
- \( \beta \): a correction term for crack growth in compact specimen tests
- \( \delta \): crack opening displacement
- \( \delta_i \): crack opening displacement at ductile crack initiation
- \( \Delta a \): ductile crack growth
- \( \Delta a_{VII} \): a seven-point average measurement of ductile crack extension (tearing)
- \( \Delta a^1 - \Delta a^7 \): measurements of ductile crack extension
- \( \Delta a_{1} - \Delta a_{9} \): measurements of ductile crack growth (stretch zone growth and tearing)
- \( \Delta a_{IX} \): a nine point average measurement of ductile crack growth (stretch zone growth and tearing)
- \( \sigma_f \): flow stress
- \( \sigma_u \): ultimate tensile stress
- \( \sigma_y \): yield stress
- \( \theta \): crack opening angle
- \( v \): Poisson's ratio
INTRODUCTION

1. Recent interest in the upper shelf behaviour of reactor pressure vessel steels has highlighted the need to include in surveillance/accelerated dose monitoring programmes a specimen type capable of interpretation using elastic/plastic fracture mechanics. Space/cost limitations in irradiation embrittlement monitoring programmes require that an optimum specimen size is selected which is just sufficient to provide valid measurements of both the fracture toughness at ductile crack initiation and resistance to the ductile crack growth which precedes specimen instability. It is unlikely that specimens exceeding 25mm thickness will be suitable for standard surveillance programmes and there is considerable incentive to use even smaller specimens. The paper examines the applicability of 25mm thick (or less) specimens for measuring upper shelf toughness.

2. The information reported herein complements data given in a paper(1) presented at the CSNI Specialists Meeting on Elasto-Plastic Fracture Mechanics held in 1978 at Daresbury, England. Ref (1) compared methods for measuring the upper shelf toughness of a 150mm thick A533 Grade B Class 1 steel produced in the UK and HY130 steel. Sub-size three point bend (3PB) specimens were subsequently machined from the broken halves of the 25 x 50mm 3PB specimens used in Ref (1) to examine the influence of specimen size on upper shelf toughness. Upper shelf toughness tests have also been done using 25mm and 12.5mm thick specimens taken from a sample of an A533 Grade B Class 1 reference plate produced in the US (plate HSST-01) and 20mm thick specimens of a low strength C-Mn structural steel. Tests on C-Mn steel provided additional information on the effect of material strength on "blunting line" relationships for constructional steels. Tensile strengths of the materials tested encompass the range 895-466 MPa.

3. The work reported has been done as a preliminary exercise to establish multiple specimen and single specimen test methods which will be used at RNL in a comprehensive test programme to examine the upper shelf toughness of a sample of 150mm thick A533 Grade B Class 1 steel typical of that currently used in LWR fabrication.

MATERIAL DETAILS

4. The 150mm thick as-rolled A533B plate was manufactured in the UK and heat treated to simulate a fully fabricated reactor pressure vessel, ie

- Austenitise 920°C for 6 hours
- Spray water quench from 820°C minimum
- Temper 600°C for 6 hours
- Stress relieve - 600°C for 36 hours
- 650°C for 6 hours

Manufacture and heat treatment of the 295mm thick HSST plate is fully documented in Ref (2). The 25mm thick, fully killed, Al-grain refined C-Mn steel (BS1501-224-28) was tested in the as-rolled condition.

5. Chemical analyses for the UK A533B steel and C-Mn steel and relevant tensile properties used in R curve analyses are given in Tables 1 and 2 respectively.
6. The specimen geometries tested were as follows:

<table>
<thead>
<tr>
<th>UK A533B</th>
<th>25 x 25 x 110mm 3PB a/W = 0.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 x 25 x 110mm 3PB a/W = 0.6</td>
<td></td>
</tr>
<tr>
<td>12.5 x 25 x 110mm 3PB a/W = 0.6</td>
<td></td>
</tr>
<tr>
<td>(tests on 25 x 50 x 235mm 3PB and 25mm CJ specimens have been reported in Ref (1)).</td>
<td></td>
</tr>
<tr>
<td>HSST-01</td>
<td>25mm CJ a/W = 0.5</td>
</tr>
<tr>
<td>25 x 25 x 110mm 3PB a/W = 0.33</td>
<td></td>
</tr>
<tr>
<td>12.5 x 25 x 110mm 3PB a/W = 0.6</td>
<td></td>
</tr>
<tr>
<td>C-Mn</td>
<td>20mm CJ a/W = 0.5</td>
</tr>
</tbody>
</table>

All 3PB specimens were tested using a loading span (S) = 4W.

7. Values of initiation toughness were obtained for each geometry using multiple specimen (interrupted) testing. Specimens were heat tinted for \( \frac{1}{2} \)hr at 250°C, after unloading, and were broken open after soaking in liquid nitrogen. Specimen displacements between successive interrupted loadings were selected to provide sufficient results to allow the definition of R curves using pre-maximum load instability J (or COD)/Crack growth data. Values of J were calculated using the area under the load/displacement trace as recommended by ASTM E24 01 09(3) and a method developed by Sumpter(4) which uses force and displacement data directly from the force/displacement record and includes a correction term to allow for slow crack growth. The appropriate expressions used in J calculations were:

### A. **CJ tests**

J from Area:

\[
J_A = \frac{2A}{B(W-a_f)} \frac{1 + \alpha}{1 + \alpha^2}
\]

where

\[
\alpha = \left(\frac{2a_0}{W-a_f}\right)^2 + 2\left(\frac{2a_0}{W-a_f}\right) + 2\left(\frac{2a_0}{W-a_f}\right) + 1
\]

J from the force/clip gauge displacement:

\[
J_{Vg} = J_{\text{elastic}} + J_{\text{plastic}}
\]

\[
= \frac{K^2(1-v^2)}{E} + \frac{(2W+a_f)(W-a_f)(5W+a_f)}{(W+a_f)^2(2W+a_f)^2} \frac{P_f}{B} (V_{gf} - \delta V_{ge})
\]

where

\[
\beta = \frac{h (a_f/W)}{h (a_f/W)}
\]

*The nomenclature 'CJ' is used to distinguish the specimens tested - compact specimens modified for load-line displacement measurement - from conventional compact specimens where displacement is measured at the notch mouth opening.*
and \( h(a/W) = -245.51 + 1659.5(a/W) - 3582.1(a/W)^2 + 2788.9(a/W)^3 \)

**B. 3PB tests**

J from force/clip gauge displacement:

\[
J_{Vg} = \frac{K^2(1-v^2)}{E} + 2PL \left[ \frac{W - a_f}{B(W-a_i)^2} \right] \left[ \frac{W}{a_f + r(W-a_f)+z} \right] (Vgf - \alpha'Vge)
\]

where \( r = 0.45 \) for \( a/W < 0.45 \)
\( r = 0.4 \) for \( a/W > 0.45 \)

\[ \alpha' = \frac{h(a_f/W)}{h(a_i/W)} \]

\( a(W) = -43.0 + 403.0(a/W) - 1073.2(a/W)^2 + 1162.8(a/W)^3 \)

COD values were calculated using the expression given in the recently re-drafted method for crack opening displacement testing (due to be issued as a British Standard in Summer 1979).

\[
\text{COD} (\delta) = \frac{K^2(1-v^2)}{2G_y E} + \frac{0.4(W-a)Vp}{0.4W + 0.6a + z}
\]

This expression was developed for standard 3PB specimens \( (W = 2B, a/W = 0.5) \) for CJ tests \( z = 0 \).

8. The amount of ductile crack growth (\( \Delta a \)) was measured in two ways to permit comparison of US and UK methods of R curve construction. For those deep notched geometries where the J formulae were applicable, \( \Delta a \) was measured using the method recommended in Ref (3); measurements were taken at nine positions across the full specimens thickness \( (\Delta a1 \) to \( \Delta a9 \)), each measurement being the total growth from the fatigue crack tip and therefore includes the growth contribution due to crack opening displacement (stretch zone growth) as well as that due to ductile crack extension (tearing). The growth, given the nomenclature \( \Delta a_{IX} \), was taken to be:

\[
\frac{\Delta a1 + \Delta a9}{2} + \Delta a2 + \Delta a3 + \Delta a4 + \Delta a5 + \Delta a6 + \Delta a7 + \Delta a8
\]

where \( \Delta a1 \) and \( \Delta a9 \) are crack growth measurements at each specimen surface.

For all geometries examined, crack growth was recorded as an average value of ductile crack extension, measured from the leading edge of the stretch zone. Crack growth, \( \Delta a_{VII} \), was taken as the average of this ductile crack extension at the seven internal measurement positions used to define \( \Delta a_{IX} \):

\[
\frac{\Delta a1' + \Delta a3' + \Delta a4' + \Delta a5' + \Delta a6' + \Delta a7' + \Delta a8'}{7}
\]

9. Estimates of the onset of ductile crack extension from single specimen tests were obtained using both DC and AC potential drop systems. The 3PB tests on UK A533B steel were monitored using the same DC pd system used in the work reported in Ref (1). In this case, the constant current input was reduced from 50 amps to 30 amps to prevent overheating of the smaller specimens. Crack initiation was identified with a positive change in slope
on a continuously increasing DC potential drop/clip gauge displacement trace. The AC pd system was evaluated during tests on C-Mn steel after making a few preliminary tests on spare samples of A533B steel. These experiments were done to examine the accuracy of an improved version of AC pd equipment used in an earlier fracture toughness programme(6). The system operates at a frequency of 2KHz and provides a current of 1 amp which flows only in the "skin" of the specimen. The skin depth (d) can be estimated from:

\[ d = \frac{1}{(\pi f \sigma \mu)^{\frac{1}{2}}} \]

where \( f \) is the frequency, \( \sigma \) the electrical conductivity, and \( \mu \) the permeability of the material. For C-Mn steel, \( d \) was taken to be 0.3mm.

10. AC pd trials using A533B steel were done on a conventional universal testing machine. These tests produced potential output/clip gauge displacement traces similar in shape to those obtained using the DC pd system, ie a continuous rise in potential output once general yield was exceeded. Rate of loading (manual control) influenced the precise shape of the potential output traces particularly in the vicinity of general yield. The tests on C-Mn steel were done on a servo-hydraulic machine operated at a constant ram displacement rate of 0.05mm\(^{-1}\). These tests produced 'typical' AC pd traces as seen by other workers(7) with an initial rise in pd, followed by the characteristic fall-off in potential to a minimum value after which the potential again rises. Crack initiation was assumed to correspond to the minimum of the potential/clip gauge (load-line) displacement curves. Typical test records from the preliminary tests on A533B steel and the tests on C-Mn steel are shown in Figure 1.

RESULTS

C-Mn Steel

11. J \( \Delta a \) \( R \) curve data and initiation toughness values derived using the AC pd/clip gauge displacement records are given in Table 3. J-\( \Delta a \) J \( R \) curves and initiation values derived using J calculated by the area (J\(_A\)) and clip gauge displacement (J\(_V\)g) methods are compared in Fig 2 and the COD-\( R \) curve is shown in Fig 3. Figure 4 compares AC pd indicated initiation toughness values with J\(_A\)-\( R \) curve data; also shown on this figure is the mathematical error (\( \pm \Delta J_i \)) on the intercept of the regression line of J\(_A\) on \( \Delta a \). The error was calculated using:

\[ \pm \Delta J_i = \left( \frac{1}{n} + \frac{(\Delta a)^2}{\sum (\Delta a-\bar{a})^2} \right)^{\frac{1}{2}} \left( \frac{\sum (J - m\Delta a-J_i)^2}{n-2} \right) \]

for n data points (J, \( \Delta a \)) and the R-curve, J = m\( \Delta a \) + Ji.

12. R curves can be constructed according to the E24 01 09 recommendations provided that all data points used to define the J\(_A\)-\( \Delta a \) regresion line satisfy the requirement:

\[ a, B, (W-a) > \frac{25J}{\sigma_f} \]
For the C-Mn steel, 25J/ae exceeds 20mm for J > 0.30 Mm⁻¹ and the specimen size tested is clearly too small to provide a valid interpretation of JIC, the initiation toughness defined by the intersection of the J = 4σfαa blunting line and a J = 2σfαa blunting line. Seven of the tests were terminated at J < 0.3 Mm⁻¹ and the J/Δa data from these tests can be used to examine the validity of the J = 2σfαa blunting line for C-Mn steel. J-R curves plotted applying the concepts of the E24 01 09 method are compared in Fig 5. Figure 5(a) includes the ASTM J = 2σfαa blunting line and Figure 5(b) the blunting line, J = 4σfαa, defined by experimental data. The J/Δa regression lines shown in Fig 5 were defined using all data to the right of the 0.15mm offset lines and are not valid according to the 25J/ae size requirement.

**UK A533B Steel**

13. J-R curve data for 25 x 25mm and 12.5 x 25mm 3PB tests and estimates of initiation toughness from the DC pd records are presented in Table 4(a). Table 4(b) lists the J/Δa data for 25 x 50mm 3PB and 25mm CJ tests which were reported in Ref (1) together with corresponding Jyg calculations to permit comparison with smaller scale tests. Individual Jyg-ΔaII R-curves for 25 x 25mm and 12.5 x 25mm 3PB tests are compared with DC pd data in Fig 6. Figure 7 shows the effect of temperature on R-curves for deep notched 25 x 50mm 3PB and 25mm CJ tests. The influence of specimen size on crack initiation and resistance to growth can be seen from Fig 8 which compares all R-curve data shown in Fig 6 with the maximum scatter band for ambient temperature tests on deep notched 25 x 50mm specimens plotted in Fig 7.

**HSST-01 Steel**

14. R-curve data for tests on HSST-01 steel are given in Table 5. Areas under force/load-line displacement test records were measured only for the "standard" 25mm CJ specimens and specimen geometry/temperature effects have been examined using the Jyg formulae. Support for this approach can be seen in Fig 9 which shows good correspondence between initiation values, for 25mm CJ specimens derived using both Jα and Jyg. The COD-R curve for 25mm CJ specimens is shown in Fig 10, and individual Jyg-ΔaII R-curves for the different geometries/test temperatures examined are compared in Fig 11.

15. The Jα-ΔaII curve for 25mm CJ specimens is presented in Fig 12. J values exceeding 0.547 Mm⁻¹ do not meet the proposed size requirement (a, b, w-a > 25J/ae) and the results cannot be used to define a valid JIC value. An initiation toughness has been defined by the intersection of a J = 4σfαa blunting line with the J/Δa regression line on all pre-maximum load data. In this case, no pre-initiation interrupted test data were available to define an experimental blunting line and the 4σfαa blunting line had to be selected arbitrarily. This is not considered to be too unreasonable, since both the specimen failing by cleavage at +20°C (BG1) and that providing the lowest J value from the interrupted tests at +60°C (BG8) both gave a measured amount of ductile crack extension using a seven point Δa measurement. Photographs of the fracture surfaces from these tests are shown in Fig 13. Evidence of actual crack growth (as opposed to crack opening) in specimen BG1 must be debatable but a small amount of ductile crack extension was measured on specimen BG8. The exceptionally low scatter in J and COD-ΔaII data plotted in Figs 9 and 10 supports the observation that ductile crack extension has occurred in specimen BG8.
DISCUSSION

16. The use of a multiple specimen R curve/blunting line procedure to evaluate a fracture toughness corresponding to the initiation of ductile crack extension requires definition of an appropriate expression to represent the blunting characteristics of the material under examination. Test results for both A533B and C-Mn steels indicate that the $J = 2\sigma_f \Delta a$ blunting line is not applicable to these materials. The expression, $J = 4\sigma_f \Delta a$, appears to provide a better approximation to the measured blunting behaviour of the C-Mn steel and has also been used to estimate crack initiation toughness for the A533B steel. This interpretation of the blunting line is supported by earlier work on deep notched 3PB/CJ specimens which indicated that a $J = 4\sigma_f \Delta a$ blunting line would apply to tests on A533B steel.[1]. Using $\sigma_Y$ or $\sigma_{flow}$ in the blunting line relationship does not significantly influence the measurement of initiation toughness, for example, the $J_I$ value derived using $J = 4\sigma_f \Delta a$ is 0.175 MMm$^{-1}$ (Fig 12) whereas that derived using a $J = 4\sigma_f \Delta a$ blunting line is 0.18 MMm$^{-1}$. Precise interpretation of blunting line behaviour can only be achieved from accurate measurements of both stretch zone size and COD to define $'m'$ and $'e'$ in the relationship $J = m_{ef} \delta$ where $\delta = f(e, \Delta a)$ and $e$ can be approximated by $e = \sigma_{flow} \Delta a/2\sigma_f$. Unfortunately, such measurements are particularly difficult to make because of the problem in defining the exact location at which the COD should be measured. Experimentally determined $J$ and COD initiation data cannot be used to define an accurate value of $m$ because the expression used to calculate COD is itself a simple approximation to real behaviour which already incorporates the relationship $J = 2\sigma_f \delta$ for calculating a pseudo-elastic component of COD. The relationships between experimentally derived $J$ and COD values at initiation are: -

- C-Mn steel: $J_A = 1.4\sigma_f \delta$
- HSST-01 steel: $J_A = 1.5\sigma_f \delta$

The factor 'm' is thought to be influenced by constraint and degree of strain hardening, and, if this is so, each material should possess its own characteristic blunting curve. For practical test purposes however, it seems reasonable to group materials on the basis of strength/strain hardening capacity and specify simple blunting line equations for each group. $J_A-\Delta a_{IV}$ data for A533B and C-Mn steels indicate that a $J = 4\sigma_f \Delta a$ blunting line will be applicable to low to moderate strength structural steels used in nuclear applications. Results from the ASTM round robin test programme which were used to develop the recommended procedure for $J_{IC}$ testing[3] can be cited to support the validity of $J = 2\sigma_f \Delta a$ for higher strength (high work hardening) materials similar to HY130 steel.

17. A comparison of $J_A-\Delta a_{IV}$ and $J_A-\Delta a_{VII}$ R-curve procedures was dealt with in detail in Ref [1]. Valid $J_{IC}$ values could not be determined using available $J_A-\Delta a_{IV}$ data from tests on either C-Mn or HSST-01 steel, nevertheless, consistent initiation values ($'J_I'$) were determined from both construction procedures when $J - \Delta a$ lines were derived using pre-maximum load instability data. This observation has important implications regarding small scale tests on irradiated samples and will be discussed later (para 20). Plots of $J$- pre-maximum load-$\Delta a_{VII}$ produced acceptably close values of initiation toughness using either the $J_A$ or $J_{VII}$ estimation procedure (Figs 2 and 9). Values of $J_A$ and $J_{VII}$ correspond almost exactly for $\Delta a_{VII} < 0.5mm$. As $\Delta a_{VII}$ increases beyond 0.5mm, the degree of correspondence decreases due to the increasing
significance of the crack growth correction factor incorporated in the \( J_{vg} \) calculation (\( J_{vg} < J_A \)). An advantage of the \( J_{vg} \) method is that initiation toughness can be determined without measuring the area under the force/load-line displacement curve. It must be noted, however, that the \( J_{vg} \) equation for 3PB specimens includes a limit load term (\( P_L \)) which often has to be determined by an equal area construction (8) on the force/displacement curve. (The load point indentation problem also complicates J-\( \Delta a \) measurements on 3PB specimens). An obvious advantage of either J-\( \Delta a \) method is that the initiation value is obtained directly from a "baseline" regression line and the R-curve construction is redundant. A real problem with the \( \Delta a \) measurement is the difficulty in identifying the transition from crack tip opening (stretch zone) to actual ductile crack extension (tearing). The ability to measure tearing only (\( \Delta a \)) depends on differences in heat tint colours between the stretch zone and tearing portions of the fracture surface. In order to assess the error associated with identifying this interface, stretch zone sizes were re-measured by different operators and it was concluded that average stretch zone sizes could be underestimated by up to 0.05mm. Since all \( \Delta a \) measurements were made in one operation, \( \Delta a \) could have been over-estimated by up to 0.05mm and, therefore, the resultant J-\( \Delta a \) regression lines will underestimate initiation values. The maximum error in initiation toughness will be associated with tests on material showing the highest resistance to growth, i.e. HSST-01 steel. Re-analysis of \( J_{vg} \)-\( \Delta a \) data for HSST-01 steel after subtracting 0.05mm from \( \Delta a \) measurements results in a JI value of 0.205 MNNm\(^{-1}\) which is 15% greater than the value shown in Fig 9. The measurement of all growth from the tip of the pre-crack (\( \Delta a \)) or some variant of this is the least ambiguous method for measuring \( \Delta a \). However, until blunting line relationships are more clearly defined for a range of materials, the authors believe that measurements of ductile crack extension (\( \Delta a \)) and extrapolation to zero crack growth, provides greater assurance of identifying initiation fracture toughness.

19. The influence of specimen size on initiation toughness and resistance to growth can be seen from Figs 6 and 8 (UK A533B) and Fig 11 (HSST-01). Individual \( J_{vg} \)-R curves for 25 x 25mm 3PB specimens show an apparent effect of notch depth on J. Specimens notched to \( a/W = 0.33 \) provide higher J values, at any given \( \Delta a \), than deep notched (\( a/W = 0.6 \)) specimens and the J-\( \Delta a \) regression lines indicate a higher value of Ji (0.199 MNNm\(^{-1}\)) than that given by tests on deep notched specimens (0.148 MNNm\(^{-1}\)). Deep notched 12.5 x 25mm and 25 x 25mm specimens gave the same initiation toughness although resistance to growth was higher in the smaller rectangular cross-section specimens. Comparison of these data with the scatter band for deep notched 25 x 50mm 3PB and 25mm CJ tests (Fig 8) shows no overlap in scatter from the two sets of tests on 25 x 25mm specimens. At low \( \Delta a \), the shallower notched 25 x 25mm data fall consistently at the top end of the scatter band. With the exception of one result for a 12.5 x 25mm specimen, all other results fall within the scatter band indicating that initiation toughness measured from deep notched specimens is independent of the range of specimen geometries examined. Individual R curves for HSST-01 steel (Fig 11) also show the effect of notch depth on J. It appears therefore, that the initiation value derived at ambient temperature from shallow notched specimens does not fulfill a plane strain size requirement. For all materials/geometries examined, the condition \( a, (W-a) > 50J_i/c_f \) is fulfilled only by data from tests on 25mm CJ/25 x 50mm 3PB specimens tested at ambient temperature and by the initiation measurement from 12.5 x 25mm 3PB specimens tested at +250°C. A minimum size requirement which covers all of the deep notched test data would be \( a, (W-a) > 35J_i/c_f \). Initiation toughness decreases with increasing upper shelf temperature.
(Fig 7). For A533B steel, the relative decrease in toughness exceeds the relative decrease in flow stress, and, consequently, the initiation toughness determined for shallow notched 25 x 25mm 3PB specimens tested at +250°C also meets a 35J/\sigma_f criterion.

20. Neutron irradiation of steels results in an increase in tensile strength and a reduction in ductility, and any size requirement developed for unirradiated materials should be, at least, equally applicable to materials tested in the irradiated state. For example, if the yield stress and tensile strength of HSST-01 increase by typically 25% and 15% respectively after irradiation, to say, 2 x 10^9 N/cm^2 > 1 MeV and the fracture toughness after irradiation remained constant at the unirradiated value of 0.18 MNm^{-1}, (measured from 25mm CJ tests) then a minimum specimen dimension (a, b, W-a) of 10mm would be required to fulfill the 35J/\sigma_f requirement.

Irradiation is expected to reduce the upper shelf toughness and this calculation serves to illustrate the potential for obtaining valid upper shelf initiation fracture toughness data from relatively small specimens which can be more easily accommodated in surveillance capsules. A 12.5mm compact specimen should provide equivalent results to those obtained from 12.5 x 25mm 3PB specimens and has obvious advantages in terms of economy/available irradiation space. The 35J/\sigma_f size requirement applies only to initiation values derived from multiple specimen tests and an analysis of J- pre-maximum load-\Delta a_{\gamma II} data. A limitation of this method is the small range of crack extension which is available to develop J-R curve data; for example, a maximum \Delta a_{\gamma II} of only 0.38mm was used to develop R curves for UK A533B steel and HSST-01 steel (Figs 6 and 11).

21. The use of a single specimen technique to predict the onset of ductile crack extension and hence initiation toughness is particularly attractive where tests are to be made on irradiated specimens. Two of the more commonly used single specimen methods are DC and AC potential drop techniques and pd indicated toughness values obtained from both these methods have been compared with initiation values from multiple specimen tests. The AC pd system provided estimates of initiation toughness which, in general, over-estimated the corresponding R curve value (Fig 4). However, taking into account the mathematical error associated with the R curve itself (+9%), there is reasonable agreement between single and multiple specimen initiation data. The troughs on AC pd/Vg traces for C-Mn steel were relatively shallow and the point of tangency taken to correspond with crack initiation was less easy to identify than the corresponding prediction point for a DC trace which was taken as a positive change in gradient. The preliminary AC tests on A533B steel illustrate a possible disadvantage in the general applicability of the AC technique. Reasons for the different pd traces (Fig 1) obtained from specimens tested on conventional or servo-hydraulic machines have still to be established. Further development of the AC system is worthwhile only if these differences are shown to arise from variations in loading rates experienced in the two types of test rather than from differences in material behaviour/specimen geometry.

22. Although interpretation of the DC pd traces was more straightforward, the DC pd predicted values of initiation toughness showed a larger scatter than that from AC tests and tended to under-estimate the initiation value predicted by the R curve (Fig 6). Comparison of the results from 25 x 25mm (Fig 6(a), (b)) and 12.5 x 25mm 3PB (Fig 6(c)) tests shows that the smaller specimens consistently under-estimate crack initiation. Average values of pd indicated toughness still slightly under-estimate the R curve value and some off-set from the point where the pd/Vg trace deviates from linearity would have to be selected to obtain better correspondence between single and multiple specimen initiation data.
CONCLUSIONS

1. Tests on A533B steel and C-Mn steel have shown that consistent values of initiation toughness can be obtained from $J-\Delta a$/$\Delta R$ curve constructions when the blunting line is defined by $J = 4 \sigma_f \Delta a$.

2. Until blunting line relationships have been identified for a wider range of materials, the $J-\Delta a$/$\Delta R$ construction method has more general applicability for defining a value of upper shelf fracture toughness corresponding to the onset of ductile crack initiation.

3. Tests on square cross-section 3PB specimens notched to $a/W = 0.33$ and 0.6 show an apparent effect of notch depth on $J$ estimated by a method based on measurements of load and displacement taken directly from force/clip gauge displacement record ($J_{yg}$).

4. Geometry independent upper-shelf initiation toughness values have been obtained from multiple specimen tests on 25mm to 12.5mm thick specimens when results were analysed using a $J_{yg}-\Delta a$/$\Delta R$ curve construction and when $a, (W-a) \geq 35 \gamma_f / \sigma_f$. If this size requirement is substantiated, a 12.5mm thick compact specimen would be acceptable for examining the effects of irradiation on upper shelf toughness.

5. Single specimen tests using AC and DC pd systems to predict crack initiation have shown that whilst there is merit in pursuing the development of the AC system, the DC system is unlikely to provide a viable alternative to multiple specimen testing.
REFERENCES


2. Childress C E: "Fabrication of the first two 12 inch thick ASTM A533 Grade B Class 1 steel plates of the heavy section steel technology program". Documentary Report ORNL-4813, February 1969.


### TABLE 1

**CHEMICAL COMPOSITION OF UK STEELS (Wt %)**

<table>
<thead>
<tr>
<th>Steel Type</th>
<th>C</th>
<th>Si</th>
<th>S</th>
<th>P</th>
<th>Mn</th>
<th>Ni</th>
<th>Cr</th>
<th>Mo</th>
<th>Cu</th>
<th>Co</th>
<th>Sn</th>
</tr>
</thead>
<tbody>
<tr>
<td>A533B</td>
<td>0.19</td>
<td>0.205</td>
<td>0.013</td>
<td>0.017</td>
<td>1.25</td>
<td>0.68</td>
<td>0.13</td>
<td>0.49</td>
<td>0.07</td>
<td>0.009</td>
<td>0.011</td>
</tr>
<tr>
<td>C-Mn</td>
<td>0.15</td>
<td>0.22</td>
<td>0.024</td>
<td>0.014</td>
<td>1.10</td>
<td>0.05</td>
<td>&lt; 0.05</td>
<td>0.02</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### TABLE 2

**TENSILE DATA USED IN ANALYSES**

<table>
<thead>
<tr>
<th>Steel</th>
<th>Temp °C</th>
<th>MPa</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>σ_y</td>
<td>σ_u</td>
<td></td>
</tr>
<tr>
<td>UK A533B</td>
<td>+ 20</td>
<td>450</td>
<td>580</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 250</td>
<td>407</td>
<td>580</td>
<td></td>
</tr>
<tr>
<td>HSST-01</td>
<td>+ 20/+ 60</td>
<td>476</td>
<td>617</td>
<td></td>
</tr>
<tr>
<td>(Ref 9)</td>
<td>+ 250</td>
<td>409</td>
<td>599</td>
<td></td>
</tr>
<tr>
<td>C-Mn</td>
<td>+ 20</td>
<td>284</td>
<td>466</td>
<td></td>
</tr>
</tbody>
</table>

Flow stress, \( \sigma_f = \frac{\sigma_y + \sigma_u}{2} \)
## TABLE 3

R CURVE DATA C-Mn STEEL 20mm C3 a/W ~ 0.5 TESTED AT 20°C

<table>
<thead>
<tr>
<th>Spec Ident</th>
<th>JAREA MmN⁻¹</th>
<th>Jvg MmN⁻¹</th>
<th>COD(δ)</th>
<th>Δa₁X mm</th>
<th>ΔaⅡVII mm</th>
<th>R Curve Regression lines + Specimens used in Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF46</td>
<td>0.656</td>
<td>0.573</td>
<td>1.05</td>
<td>1.34</td>
<td>1.42</td>
<td>(i) BLUNTING LINE METHOD: Jₐ</td>
</tr>
<tr>
<td></td>
<td>(0.174)</td>
<td></td>
<td>(0.34)</td>
<td></td>
<td></td>
<td>BF46, 45, 49, 47 Jₐ = 0.42X δa₁X + 0.105</td>
</tr>
<tr>
<td>BF45</td>
<td>0.543</td>
<td>0.483</td>
<td>0.86</td>
<td>1.11</td>
<td>1.16</td>
<td>(ii) All specimens excluding</td>
</tr>
<tr>
<td></td>
<td>(0.203)</td>
<td></td>
<td>(0.37)</td>
<td></td>
<td></td>
<td>BF2, 15, 3, 5 Jₐ = 0.391 δa₁X + 0.135</td>
</tr>
<tr>
<td>BF49</td>
<td>0.497</td>
<td>0.468</td>
<td>0.81</td>
<td>0.82</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.145)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BF47</td>
<td>0.421</td>
<td>0.393</td>
<td>0.71</td>
<td>0.77</td>
<td>0.78</td>
<td>(i) UK METHOD Jₐ</td>
</tr>
<tr>
<td></td>
<td>(0.145)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>ATT specimens excluding BF2,15 Jₐ = 0.363 δaⅡVII + 0.15</td>
</tr>
<tr>
<td>BF42</td>
<td>0.370</td>
<td>0.372</td>
<td>0.64</td>
<td>0.56</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.217)</td>
<td></td>
<td>(0.41)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BF48</td>
<td>0.300</td>
<td>0.299</td>
<td>0.52</td>
<td>0.41</td>
<td>0.36</td>
<td>(i) Jvg METHOD</td>
</tr>
<tr>
<td></td>
<td>(0.146)</td>
<td></td>
<td>(0.27)</td>
<td></td>
<td></td>
<td>ATT specimens excluding BF2,15 Jvg = 0.297 δaⅡVII + 0.171</td>
</tr>
<tr>
<td>BF36</td>
<td>0.236</td>
<td>0.256</td>
<td>0.44</td>
<td>0.31</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.214)</td>
<td></td>
<td>(0.41)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BF41</td>
<td>0.254</td>
<td>0.254</td>
<td>0.44</td>
<td>0.32</td>
<td>0.28</td>
<td>(i) COD</td>
</tr>
<tr>
<td></td>
<td>(0.187)</td>
<td></td>
<td>(0.34)</td>
<td></td>
<td></td>
<td>ATT specimens excluding BF2,15 δ = 0.541 δaⅡVII + 0.291</td>
</tr>
<tr>
<td>BF5</td>
<td>0.178</td>
<td>0.180</td>
<td>0.32</td>
<td>0.16</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>BF3</td>
<td>0.134</td>
<td>0.139</td>
<td>0.25</td>
<td>0.12</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>BF2</td>
<td>0.095</td>
<td>0.104</td>
<td>0.19</td>
<td>0.06</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>BF15</td>
<td>0.038</td>
<td>0.034</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

(*) AC pd indicated initiation toughness
<table>
<thead>
<tr>
<th>Specimen Type</th>
<th>Ident.</th>
<th>( J_{vg} ) MNm(^{-1} )</th>
<th>( \Delta ) VII mm</th>
<th>J-( \Delta )a Regression Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 x 25mm 3PB a/W = 0.33 + 20°C</td>
<td>BC7/1</td>
<td>0.45* (0.17)</td>
<td>1.27*</td>
<td>( J_{vg} = 0.306 \Delta ) VII + 0.199 ( J_I = 0.199 ) MNm(^{-1} )</td>
</tr>
<tr>
<td></td>
<td>BC3/1</td>
<td>0.41 (0.14)</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC3/2</td>
<td>0.39 (0.24)</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC8/1</td>
<td>0.37 (0.21)</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC8/2</td>
<td>0.35 (0.16)</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC6/1</td>
<td>0.30 (0.21)</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC6/2</td>
<td>0.25 (0.16)</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC7/2</td>
<td>0.22 (0.20)</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>25 x 25mm 3PB a/W = 0.6 + 20°C</td>
<td>BC11</td>
<td>0.37* (0.10)</td>
<td>0.74*</td>
<td>( J_{vg} = 0.342 \Delta ) VII + 0.148 ( J_I = 0.148 ) MNm(^{-1} )</td>
</tr>
<tr>
<td></td>
<td>BC1/1</td>
<td>0.32 (0.11)</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC1/2</td>
<td>0.28 (0.11)</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC2/2</td>
<td>0.25 (0.15)</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC10</td>
<td>0.25 (0.11)</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC12</td>
<td>0.23 (0.16)</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC2/1</td>
<td>0.20 (0.13)</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>12.5 x 25mm 3PB a/W = 0.6 + 20°C</td>
<td>BD3</td>
<td>0.28</td>
<td>0.34</td>
<td>( J_{vg} = 0.420 \Delta ) VII + 0.150 ( J_I = 0.150 ) MNm(^{-1} )</td>
</tr>
<tr>
<td></td>
<td>BD4</td>
<td>0.27 (0.16)</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BD6</td>
<td>0.21 (0.10)</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BD5</td>
<td>0.21 (0.11)</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BD8</td>
<td>0.18 (0.10)</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BD7</td>
<td>0.16 (0.15)</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Ac pd trials</td>
<td>BD2</td>
<td>0.21 (0.11)</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BD9</td>
<td>0.23 (0.15)</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BD1</td>
<td>0.30 (0.18)</td>
<td>0.35</td>
<td></td>
</tr>
</tbody>
</table>

*Post maximum load data (not used in regression analysis)
( ) Dc pd indicated initiation toughness ( ) Ac pd indicated initiation toughness.
### Table 4(b)

**J-△a Values and Initiation Data, UK A533B Steel**

<table>
<thead>
<tr>
<th>Specimen Type</th>
<th>Ident.</th>
<th>$J_{Area}$ MNm⁻¹</th>
<th>$J_{Vg}$ MNm⁻¹</th>
<th>$\Delta a_{IX}$ mm</th>
<th>$\Delta a_{VII}$ mm</th>
<th>$J_0 - \Delta a$ Regression Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 x 50mm 3PB + 20°C</td>
<td>BC7*</td>
<td>0.515*</td>
<td>0.447*</td>
<td>1.05*</td>
<td>1.13*</td>
<td>$J_A = 0.447 \Delta a_{IX} + 0.135$</td>
</tr>
<tr>
<td></td>
<td>BC1</td>
<td>0.420</td>
<td>0.364</td>
<td>0.64</td>
<td>0.64</td>
<td>$J_0 = 0.24$ MNm⁻¹(*)</td>
</tr>
<tr>
<td></td>
<td>AY1</td>
<td>0.438</td>
<td>0.432</td>
<td>0.59</td>
<td>0.60</td>
<td>$J_0 = 0.399 \Delta a_{VII} + 0.163$</td>
</tr>
<tr>
<td></td>
<td>AY2</td>
<td>0.333</td>
<td>0.349</td>
<td>0.47</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AY4</td>
<td>0.281</td>
<td>0.267</td>
<td>0.32</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AY3</td>
<td>0.194</td>
<td>0.186</td>
<td>0.19</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC3</td>
<td>0.195</td>
<td>0.186</td>
<td>0.13</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>25mm CJ + 20°C</td>
<td>BB1*</td>
<td>0.552*</td>
<td>0.470*</td>
<td>1.28*</td>
<td>1.37*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB7</td>
<td>0.402</td>
<td>0.370</td>
<td>0.66</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB5</td>
<td>0.341</td>
<td>0.330</td>
<td>0.40</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB8</td>
<td>0.290</td>
<td>0.274</td>
<td>0.39</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB2</td>
<td>0.232</td>
<td>0.227</td>
<td>0.22</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB3</td>
<td>0.219</td>
<td>0.193</td>
<td>0.15</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>25 x 50mm 3PB + 250°C</td>
<td>BC8*</td>
<td>0.267*</td>
<td>0.219*</td>
<td>0.85*</td>
<td>0.88*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AY8</td>
<td>0.239</td>
<td>0.201</td>
<td>0.54</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC2</td>
<td>0.195</td>
<td>0.168</td>
<td>0.47</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AY5</td>
<td>0.160</td>
<td>0.153</td>
<td>0.35</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AY7</td>
<td>0.114</td>
<td>0.114</td>
<td>0.17</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC6</td>
<td>0.121</td>
<td>0.108</td>
<td>0.11</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AY6</td>
<td>0.067</td>
<td>0.071</td>
<td>0.05</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

*Post maximum load data (not used in regression analyses)

(*) $J_1$ determining using $2\sigma_0\Delta a$ blunting line$^{(1)}$
### TABLE 5
R-CURVE DATA. A533B STEEL HSST-01

<table>
<thead>
<tr>
<th>Specimen Type</th>
<th>Test Temp °C</th>
<th>$J_A$ MINm⁻¹</th>
<th>$J_{Vg}$ MINm⁻¹</th>
<th>COD(mm)</th>
<th>$\Delta a_{IX}$ mm</th>
<th>$\Delta a_{VII}$ mm</th>
<th>R-Curve Regression Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>25mm CJ a/W = 0.5</td>
<td>+20</td>
<td>0.168⁺</td>
<td>0.163⁺</td>
<td>0.20⁺</td>
<td>0.11⁺</td>
<td>0.01⁺</td>
<td>$J_A = 0.702 \Delta a_{IX} + 0.115$</td>
</tr>
<tr>
<td></td>
<td>+60</td>
<td>0.887⁺</td>
<td>0.756⁺</td>
<td>0.99⁺</td>
<td>1.48⁺</td>
<td>1.60⁺</td>
<td>$J_A = 0.633 \Delta a_{VII} + 0.171$</td>
</tr>
<tr>
<td></td>
<td>+60</td>
<td>0.608</td>
<td>0.560</td>
<td>0.68</td>
<td>0.72</td>
<td>0.71</td>
<td>$J_{Vg} = 0.558 \Delta a_{VII} + 0.179$</td>
</tr>
<tr>
<td></td>
<td>+60</td>
<td>0.520</td>
<td>0.487</td>
<td>0.58</td>
<td>0.56</td>
<td>0.54</td>
<td>$\delta = 0.670 \Delta a_{VII} + 0.215$</td>
</tr>
<tr>
<td></td>
<td>+60</td>
<td>0.422</td>
<td>0.408</td>
<td>0.48</td>
<td>0.42</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+60</td>
<td>0.296</td>
<td>0.287</td>
<td>0.35</td>
<td>0.28</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+60</td>
<td>0.213</td>
<td>0.212</td>
<td>0.26</td>
<td>0.14</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>25 x 25mm 3PB a/W = 0.33</td>
<td>+20</td>
<td>0.391</td>
<td>0.391</td>
<td>0.391</td>
<td>0.391</td>
<td>0.391</td>
<td>$J_{Vg} = 0.33 \Delta a_{VII} + 0.215$</td>
</tr>
<tr>
<td></td>
<td>+20</td>
<td>0.348</td>
<td>0.348</td>
<td>0.348</td>
<td>0.348</td>
<td>0.348</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+20</td>
<td>0.307</td>
<td>0.307</td>
<td>0.307</td>
<td>0.307</td>
<td>0.307</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+20</td>
<td>0.274⁺</td>
<td>0.274⁺</td>
<td>0.274⁺</td>
<td>0.274⁺</td>
<td>0.274⁺</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+20</td>
<td>0.257</td>
<td>0.257</td>
<td>0.257</td>
<td>0.257</td>
<td>0.257</td>
<td></td>
</tr>
<tr>
<td>25 x 25mm 3PB a/W = 0.33</td>
<td>+250</td>
<td>0.478⁺</td>
<td>0.478⁺</td>
<td>0.478⁺</td>
<td>0.478⁺</td>
<td>0.478⁺</td>
<td>$J_{Vg} = 0.315 \Delta a_{VII} + 0.133$</td>
</tr>
<tr>
<td></td>
<td>+250</td>
<td>0.378</td>
<td>0.378</td>
<td>0.378</td>
<td>0.378</td>
<td>0.378</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+250</td>
<td>0.305</td>
<td>0.305</td>
<td>0.305</td>
<td>0.305</td>
<td>0.305</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+250</td>
<td>0.238</td>
<td>0.238</td>
<td>0.238</td>
<td>0.238</td>
<td>0.238</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+250</td>
<td>0.189</td>
<td>0.189</td>
<td>0.189</td>
<td>0.189</td>
<td>0.189</td>
<td></td>
</tr>
<tr>
<td>12.5 x 25mm 3PB a/W = 0.6</td>
<td>+250</td>
<td>0.225</td>
<td>0.225</td>
<td>0.225</td>
<td>0.225</td>
<td>0.225</td>
<td>$J_{Vg} = 0.363 \Delta a_{VII} + 0.097$</td>
</tr>
<tr>
<td></td>
<td>+250</td>
<td>0.202</td>
<td>0.202</td>
<td>0.202</td>
<td>0.202</td>
<td>0.202</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+250</td>
<td>0.156</td>
<td>0.156</td>
<td>0.156</td>
<td>0.156</td>
<td>0.156</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+250</td>
<td>0.130</td>
<td>0.130</td>
<td>0.130</td>
<td>0.130</td>
<td>0.130</td>
<td></td>
</tr>
</tbody>
</table>

*Post maximum load data (not used in regression analyses)

⁺ Cleavage (not used in regression analysis)
(a) Test on a universal test machine. Manual control. Duration of test ~ 15 mins

(b) Test on a servo-hydraulic machine. Constant ram displacement rate = 0.05 mm sec⁻¹. Duration of test ~ 4 mins

FIG. 1 ACPD / CLIP GAUGE DISPLACEMENT TEST RECORDS
FIG. 2 COMPARISON OF J-R CURVES DERIVED USING J CALCULATED BY THE AREA (J_{A}) AND CLIP GAUGE DISPLACEMENT (J_{VG}) METHODS. C-Mn STEEL. 20mm CJ, \( \sigma/\omega = 0.5 \) LT ORIENTATION.

\[ J_{A} = 0.363 \Delta a + 0.150 \]

\[ J_{VG} = 0.297 \Delta a + 0.171 \]

+ 20°C

J_{A} •

J_{VG} \( \Delta \)

FIG. 3 COD-R CURVE. C-Mn STEEL. 20mm CJ, \( \sigma/\omega = 0.5 \) LT.

\[ \delta = 0.541 \Delta a + 0.291 \]

+ 20°C
FIG. 4 COMPARISON OF $\Delta A$ INDICATED INITIATION TOUGHNESS VALUES WITH $J_A$-R CURVE DATA. C-Mn STEEL. 20mm CJ, $a/w = 0.5$, LT.
FIG. 5 COMPARISON OF J-R CURVES CONSTRUCTED USING J AREA AND (a) $J = 2 \sigma f \Delta a$, (b) $J = 4 \sigma f \Delta a$ BLUNTING LINES C-Mn.STEEL, 20mm CJ SPECIMENS $\gamma/w = 0.5$, LT ORIENTATION.
Fig. 6 Comparison of DCpd predicted initiation toughness with $J_{vg}$-R curve data. A533B steel, $+20^\circ C$.
FIG. 7 J-R CURVES FOR 25×50 mm 3PB AND 25 mm CJ TESTS. δ/w = 0.6. U.K. A533 B. LT ORIENTATION
FIG. 8 EFFECT OF SPECIMEN SIZE ON J-R CURVES.
3 PB TESTS ON UK A533B STEEL, LT ORIENTATION,
TESTED AT +20°C.
FIG. 9 COMPARISON OF J-R CURVES DERIVED USING J CALCULATED BY THE AREA (J_A) AND CLIP GAUGE DISPLACEMENT (J_vg) METHODS. HSST-01 STEEL. 25 mm C J. \( \theta/w = 0.5 \). LT ORIENTATION

\[ J_A = 0.635 \Delta a + 0.171 \]
Error in \( J_i \) = ± 7.7%

\[ J_{vg} = 0.558 \Delta a + 0.179 \]
Error in \( J_i \) = ± 9.4%

**Post maximum load**

\[ \Delta c \]

\[ \Delta c \]

**FIG. 10 COD-R CURVE. HSST-01 STEEL. 25 mm C J. \( \theta/w = 0.5 \). LT ORIENTATION**

\[ \delta = 0.670 \Delta a + 0.215 \]

**+60°C**

**+20°C**

\[ c = cleavage \]
FIG. 12 $J_A$ - R DATA FOR HSST-01 STEEL

$J = 4Gf\Delta a$

$J = 2G_{Flow} \Delta a$

Regression line on all pre-maximum load + 60°C data

$J = 0.702 \Delta a + 0.115$

$\frac{25J}{G_f} > 25$ mm

$J_i = 0.175$ MNm$^{-1}$

($K_I = 199$ MNm$^{-3/2}$)

HSST-01

25mm CJ

+ 60°C ●

+ 20°C ○

c = cleavage

Crack opening + Crack growth $\Delta a_{ix}$ (mm)
25mm C.J. SPECIMENS

+20°C  +60°C

BG 1  BG 2  BG 3  BG 4  BG 5  BG 6  BG 8

FIG 13 FRACTURE SURFACES FOR INTERRUPTED TESTS ON HSST-01 STEEL
Formula Giving the J-R curve from the Results of one Experimental Test

R.L. ROCHE

DEMT/CEN SACLAY BP N°2 91190 GIF FRANCE

To be presented to the CSNI Specialist Meeting on Plastic Tearing Instability, St-Louis, Miss. 25-27 Sept. 1979
At the onset of crack propagation, the $J$ value can be obtained from the load-deflection curve given by testing one sample. The formula used for the computation of this $J$ value are not valid when stable propagation occurs. This paper proposes modified formula applicable for the determination of $J$ values during plastic tearing.
1 - Introduction

When only the onset of crack propagation is studied, the use of the J integral method of analysis implies the knowledge of the J critical value of the material. This knowledge is generally obtained from the results of experimental tests on samples. As the direct measure of J value is not possible in experiments, the test results are translated in J value with the help of an analytical method.

The first methods were not very easy to use\(^{(1)}\). Several tests on several specimens were needed and graphical (or numerical) derivation has to be performed. A great improvement was obtained by other methods\(^{(2)}\) \(^{(3)}\), for only one test on one specimen was needed and the computation of J value was easy.

In ductile tearing studies, it has been proposed to use the J value. As the crack length is growing, it is necessary to know the J value of the material as a function of the crack growth -the so called "R curve" \(^{(4)}\). Therefore the experimental determination of the J value is required for practical application. This is also true for other theories like this using the tearing modulus\(^{(5)}\).

As for the preceding problem (onset of crack propagation), the experimental results are translated to J value by analytical formulae. The formulas used for crack initiation are not valid when crack propagation occurs. Special method has been proposed\(^{(6)}\), but it seems that more general methods are possible. The aim of the paper is to propose such methods.
2 - Theoretical Considerations

2-1 - Principle of virtual work

The principle of virtual work is well known. Its expression is

\[ \int_S F \delta u \, d\alpha + \int_V F \delta u \, dv = \int_V \delta \bar{W} \, dv \]

with the following notations:

- \( F \) surface external forces
- \( F \) volume external forces (which will be neglected)
- \( \bar{W} \) strain working density
- \( u \) displacement

Two very important points must be emphasized about this principle.

2-2 - Work and Potential Energy

Generally it is not possible to integrate this expression. The work between two different states is depending of the used way. In other words \( \bar{W} \) is not a potential energy and is not given by the strain state of the material.

Only in the case where the material is elastic \( \bar{W} \) is a function of the state of the material, no depending of the way used to reach this state. If the external forces are conservative, it is then possible to integrate the expression (1). The result is the principle of minimum energy.

When only an external force \( F \) is applied -it can be a generalized force- there are the above equations

\[ \delta \bar{W} = F \delta u \quad F = \frac{\partial \bar{W}}{\partial u} \quad (2) \]

\( u \) being the displacement corresponding to \( F \), and \( \bar{W} = \int \bar{W} \, dv \) the elastic energy of the solide.
2-3 - Virtual displacement

The virtual displacement $\delta u$ used in equations (1) and (2) must satisfy the kinematic conditions -equations of compatibility and boundary conditions. The continuity of the material must be kept by virtual displacement (no voids).

In Fracture Mechanics, where crack propagation is considered, these kinematic conditions are not met. Therefore, the conventional form of the principle of virtual work cannot be applied. A more general form of this principle must be considered, taking into account the material displacements and the defect forces (and defect couples)\(^7\)(8).

Material displacements can be defined as the displacement of the material properties through the solid. As an example displacement of a dried fruit through the dough of an unbacked cake is a material displacement. Crack propagation is a material displacement. In each point of the solid the material displacement is defined by the translation $\delta a$ and the rotation $\delta \Omega$ of the material properties.

Defect forces $j$ and defect couple $l$ are associated to the material displacement in the same fashion as conventional forces are associated to virtual (spac'a:) displacement:

In neglecting the volume forces, the generalized principle of virtual work can be written

$$\int_S (F \delta u) \, d\rho - \int_S \left( j \delta a + l \delta \Omega \right) \, d\rho = \int_V \delta W \, dV \quad (3)$$

2-4 - Used definition of J integral

From now, the assumption will be made that the material is elastic. As only simple samples are considered, the material displacement will be taken as the crack propagation, that is to say that the properties
of the material are subject to an \textit{uniform translation} $\delta a$ (without rotation). The $J$ integral is the defect force applied to the crack tip (resultant of all the defect forces includes in a slice of the solid). Hence it is possible to write

\[
\begin{align*}
\delta W &= F \delta u - J B \delta a \\
F &= \frac{\partial W}{\partial u} \\
JB &= -\frac{\partial W}{\partial a} \\
B &= \text{thickness of the sample}
\end{align*}
\]  

(4)
3 - Bases of the method

3-1 - What is available

The equations (4) are the safe base for the methods of determination of J value from experimental tests.

In order to apply these equations, some comments can be useful:
- The assumption of material elasticity means that

\[ W \text{ is only a function of } \]
\[ \text{the deflection } u \text{ and of } \]
\[ \text{the crack length } a \]

the same property is true for the applied force F and J
- The applied force F, the deflection u and the crack length can be measured during experimental tests on samples. The J value can only be computed from these experimental values. The practical problem is to choose a good method of computation.
- Obviously, the complementary form of (4) can be employed

\[ \delta (F - W) = u \delta F + J B \delta a \]

3-2 - General scheme of the methods

From these considerations, one general scheme of the methods can be extracted. This method is a two steps method.

The first step is the determination of the elastic energy W by integration of equation (4) when the length of the crack remain constant

\[ W(u, a) = \int_0^u F \, d \, u \]

with \( a = \text{cte} \)

\[ (5) \]

![Diagram](image-url)
The second step is the derivation of the obtained expression of $W$

$$J = - \frac{1}{B} \frac{\partial W(a, u)}{\partial a}$$  \hspace{1cm} (6)

Such a method had been straightly used by BEGLEY and LANDES, but serious drawbacks are met with it:

- Experimental tests must be made on several samples with different crack lengths.
- Derivation of experimental results is not a suitable way of computation and large error can be added during this process.
- Last, but not the least, when stable crack propagation occurs (it is the case for ductile tearing) it is not possible to obtain the wanted results.

3-3 - Need for an additional hypothesis

In order to avoid these drawbacks, the current practice is to make an supplementary assumption. This assumption is concerning the features of expression giving the applied force $F$ as a function of the crack length $a$. 
In other words, it is assumed that the knowledge of $F$ versus $u$ for a given $a$, is enough to know $F(u)$ for all the other values of the crack length $a$.

This is the base of known methods devoted to the determination of $J$ at the onset of propagation. Such is the base of the methods applicable in case of stable propagation.
4 - Practical formula

4-1 - Scaling on the force axis

An often convenient hypothesis is the following: the applied force \( F \) is the product of a function of displacement \( u \) by a function of the crack length \( a \)

\[
\begin{align*}
\text{hypothesis} & \quad F = A(a) \varphi(u) \\
A & \quad \text{known function of } a \text{ only} \\
\varphi & \quad \text{unknown function of } u \text{ only}
\end{align*}
\]

(7)

The function \( A(a) \) is a known function of the \( a \), but \( \varphi(x) \) is unknown (depending on the material) and must be obtained from experiments.

This hypothesis can be translated in a graphical language. There is an uninc curve giving \( \varphi \) as a function of \( u \).

\[
\varphi = \frac{F}{A(a)}
\]

With such an hypothesis equations (5) and (6) become

\[
\begin{align*}
W(u,a) &= A \int_0^u \varphi \, du \\
J &= \frac{1}{B} \frac{dA}{da} \int_0^u \varphi \, du
\end{align*}
\]
as \( \varphi \) is not depending of the crack length, there is no restriction on the path of the integral. Hence the result is

\[
J = - \frac{1}{B} \frac{dA}{da} \int_0^u \frac{F}{A} \, du
\]

\( B = \) sample thickness

\( F = \) applied force

\( u = \) deflexion

\( A = \) known function of the crack length (scaling of \( F \))

(see equation (7))

4-2 - Application to the deep cracked specimens

The current practice is to admit that the elastic plastic bending is the main cause of the deflection.

This lead to put

\[
F = k b^n \varphi(u)
\]

\( b \) being the uncracked ligament length equal to \( W - a \).

The value of the exponent \( n \) being near two \((k \text{ is a constant})\) (2)

The expression of the function \( A \) is

\[
A = k b^n = k (W-a)^n
\]

\[
\frac{dA}{da} = -n k (W-a)^{n-1} = -n k b^{n-1}
\]

and the equation (8) can be written

\[
J = \frac{n b^{n-1}}{B} \int_0^u \frac{F \, du}{b^n}
\]

If there is no propagation, \( b \) is constant and we have the well known expression

\[
J = \frac{n}{B b} \int_0^u F \, du
\]
\[ J = - \frac{1}{B} \frac{d}{da} \left( \frac{d}{AC} \right) \int_0^u F du + \frac{1}{B} \frac{dC}{da} \frac{Fu}{C} \]

\( B \) sample thickness
\( F \) applied force
\( u \) deflection
\( A \) known function of the \( \text{sealing of } F \)
\( C \) crack length \( a \) \( \text{sealing of } u \)

(see equation (10))

4-4 - Application to notched specimens

If the width of the uncracked part is called \( b \), a good formula is

\[ \frac{F}{b^n} = \varphi \left( \frac{u}{b^s} \right) \]

\( b = W - a \)

hence, it can be written

\[ A = b^n \]
\[ C = b^s \]

\[ \frac{d AC}{da} = -(n+s) b^{n+s-1} \]

\[ \frac{dC}{C da} = \frac{s}{b} \]

and the equation (11) becomes

\[ J = \frac{(n+s) b^{n+s-1}}{b} \int_0^u \frac{F du}{b^{n+s}} - \frac{s}{Bb} \frac{Fu}{C} \]

If there is no propagation, \( b \) is constant and (12) lead to

\[ J = \frac{1}{B} \frac{1}{b} \left\{ (n+s) \int_0^u F du - s Fu \right\} \]

or

\[ J = \frac{1}{Bb} \left\{ n \int_0^u F du - s \int_0^u udF \right\} \]

which is a well known expression (10)
4-3 - Scaling of the force and the deflection

Generalisation of the preceding hypothesis can be made, in introducing scaling functions of the crack length, for both the force and the deflection hypothesis

\[
\begin{align*}
F &= A(a) \varphi \\
u &= C(a) \xi \\
A &= \text{known function of } a \\
C &= \text{unknown function of } \xi
\end{align*}
\]

The functions \( A(a) \) and \( C(a) \) are known functions of the crack length \( a \), but \( \varphi(\xi) \) is unknown and must be obtained with the help of an experimental test.

The translation in graphical language is near the preceding one: there is an unique curve giving \( \varphi \) as a function of \( \xi \)

\[ \varphi = \frac{F}{A(a)} \]

\[ \xi = \frac{u}{C(a)} \]

With this hypothesis equations (5) and (6) become

\[ W(u,a) = A.C. \int_0^{u/c} \varphi \, d\xi \]

\[ J = - \frac{1}{B} \frac{d}{da} AC \int_0^{u/c} \varphi \, d\xi - \frac{AC}{B} \varphi \frac{\partial}{\partial a} \left( \frac{u}{C} \right) \]

as the derivation \( \frac{\partial}{\partial a} \) must be meant as \( u = \text{cte} \), this lead to
5 - Conclusion

Conventional formula are not convenient for the determination of J values when the crack growth is noticeable. Other formula must be used in that case, their general forms are given by equations (8) or equations (11).

For practical use, it is sufficient to know that

- the formulae \( J = \frac{n}{B} \int \frac{F_{\text{du}}}{b^n} \)

must be used in lieu of \( J = \frac{n}{B \beta} \int F_{\text{du}} \)

- the formulae \( J = \frac{(n+s)\beta^{n+s-1}}{B} \int_0^u \frac{F_{\text{du}}}{\beta^{n+s}} - \frac{s}{\beta} F_{\text{u}} \)

must be used in lieu of

\[
J = \frac{1}{B \beta} \left\{ n \int_0^u F_{\text{du}} - s \int_0^u u dF \right\}
\]

or \( J = \frac{1}{B \beta} \left\{ (n+s) \int_0^u F_{\text{du}} - sF_{\text{u}} \right\} \)
(1) BEGLEY, J.A., and LANDES, J.D., in "Fracture Toughness"
ASTM - STP 514, ASTM 1972

(2) RICE, J.R., PARIS, P.C. and MERKLE, J.G., "Some Further Results
of J Integral Analysis and Estimates" ASTM - STP 536 - ASTM 1973

(3) SUMPTER, J.D. and TURNER, C.E. "A method for Laboratory dete-
mination of Jc" - ASTM - STP 601, ASTM 1976

(4) TURNER, C.E., "Description of stable and unstable crack growth in the
elastic plastic regime interns of Jr resistance curve" - 11th Nat.
Symp. Fracture - Virginia 1978

(5) PARIS, P.C. et al. "A treatment of the subject of tearing instabi-
ility" USNRC Report NUREG-0311, August 1977

of crack growth resistance curves (R-curves) using the J integral"
Int. J. of Fracture, 11 (1975) p.528

(7) ROCHE, R.L., "Defect Vectors and Path Integrals in Fracture Mechanics"
in "Transactions of 5th Int. Conf. on Structural Mechanics in Reactor
Technology" - North Holland Pb. Cy 1979

(8) ROCHE, R.L. "Defect Forces, Defect Couples and Path Integrals in
Fracture Mechanics" (in French) - Rapport CEA, CEN Saclay July 1979

(9) HICKERSON, J.P., "Comparaison of Compliance and Estimation Procedure
for calculating J integral values" ASTM STP 631 ASTM 1977

(10) MERKLE, J.G. and CORTEN, H.T., "A J integral analysis for the compact
specimen, considering axial forces as well as bending effects"
Annex

Extension of the rule given by HERKLESD CORTEN

1. The work is based on the formula:

$$\Theta = \text{function of } \frac{F}{F_k}$$

$$F_k = 2ac \overline{y} B$$
$$u = (a + (1 + d)c) \Theta$$
$$\alpha^2 + 2 \left( \frac{W}{c} - 1 \right) \Theta - 1 = 0$$

2. They can be written as:

$$u = C(a, \Theta)$$
$$F = A(a, \Theta)$$
$$\Theta = \text{function of } \Theta$$

$$C(a) = a + (1 + d) \frac{b}{2} \quad (1)$$
$$A(a) = \alpha b \overline{y} B \quad (2)$$
$$b = W - a \quad (3)$$

and

$$\alpha^2 + 2 \left( \frac{2a + 1}{b} \right) \alpha - 1 = 0 \quad (4)$$

3. Derivation of (4), elimination of $W$

$$\alpha' = -\frac{1}{b} \left( \frac{1 + 2d - \alpha^2}{1 + d^2} \right) \alpha$$

Elimination of $a$ between (1) and (4)

$$C = \frac{b}{2} \left( \alpha + 1 + \frac{2a}{b} \right) = \frac{b}{2} \left( \alpha + \frac{1 - d^2}{2d} \right)$$

$$C = \frac{b}{2} \left( 1 + d^2 \right) \quad (5)$$

$$AC = \frac{b^2}{2} (1 + d^2) \overline{y} B \quad (5)$$
\[ C = \frac{1}{\bar{v}} \left( \frac{1}{2} \right) \left( \frac{1}{(\alpha + 1)^2} \right) \]  
\[ C' = \frac{1}{\bar{v}} \left( \frac{1}{2} \right) \left( \frac{1}{(\alpha + 1)^2} \right) \alpha' = \frac{1}{\bar{v}} \left( \frac{1}{2} \right) \left( \frac{1}{(\alpha + 1)^2} \right) \]  
\[ C' = \frac{1}{2} \frac{\alpha' \bar{v}^2 + 1}{\alpha \bar{v}} \]  
(7)

\[ (AC)' = \frac{1}{\bar{v}} \left( \frac{1}{2} \right) \left( \frac{b^2 (1 + d^2)}{\bar{v} \gamma B} \right) \]
\[ = -b (1 + d^2) \bar{v} \gamma B \frac{b \alpha^2 \bar{v}^2 + 2 d^2 \bar{v} \gamma B}{2} \]
\[ = -b \bar{v} \gamma B \left[ \alpha + d^2 + \frac{(1 + 2d - d^2) \bar{v}^2}{4d^2} \right] \]
(8)

4) Equation (11) page 11

\[
J = \frac{2b}{B} \left[ \frac{4 + 3d^2 + 2d^3}{2(1 + d^2)} \sum \frac{F \mu}{(1 + d^2)} + \frac{1 - 2d - d^2}{(1 + d^2)^2} \sum \frac{F \mu}{(1 + d^2)^2} \right]
\]
(9)

\[ \alpha \text{ given by } \alpha^2 + 2 \left( \frac{2b}{b} + 1 \right) \alpha - 1 = 0 \]

5) If There is no propagation across both \( d = \lambda \)

\[
J = \frac{2b}{B} \left[ \frac{4 + 3d^2 + 2d^3}{(1 + d^2)^2} \sum F \mu + \frac{1 - 2d - d^2}{(1 + d^2)^2} \sum F \mu \right]
\]
(10)

6) Conclusion

When propagation occurs use (9) and not (10)
A SIMPLE PRACTICAL METHOD FOR DETERMINING THE
DUCTILE INSTABILITY OF CRACKED STRUCTURES

by

G.G. Chell and I. Milne
Materials Division
Central Electricity Research Laboratories
Kelvin Avenue
Leatherhead
Surrey, U.K.

SUMMARY

A method proposed by Milne for determining the maximum load bearing capacity of a structure during ductile stable crack growth is shown to be equivalent to an R-curve instability analysis. This equivalence is demonstrated using as an example a centre cracked plate.

The method is applied to two cases pertinent to pressure vessels, namely internally pressurized thick and thin walled cylinders. The results of these examples show that for values of the tearing modulus, $T_{\text{mat}}$, typical of pressure vessel steels, the instability pressure is approximately determined by the plastic collapse pressure obtained using the instantaneous crack length. This may be considerably below the collapse pressure corresponding to the initial defect size.

Practical aspects of instability analyses are discussed, such as the relevance of $J_R$ ($\Delta a$) curves obtained in the laboratory to practical problems and the different treatments required for part and full penetration cracks. Finally the suggested procedures for performing an instability analysis based on a failure assessment diagram are summarized.
CONTENTS

1. INTRODUCTION
2. MILNE’S PROPOSAL
3. RELATIONSHIP TO R-CURVE INSTABILITY ANALYSIS
4. APPLICATIONS TO PRESSURE VESSELS
   4.1 Thick Walled Cylinder
   4.2 Thin Walled Cylinder
5. PRACTICAL ASPECTS OF ANALYSIS
6. DISCUSSION
7. SUMMARY OF PROCEDURES
   7.1 Input Data Needed
   7.2 Analysis
8. CONCLUSIONS
9. REFERENCES

  Table 1

APPENDIX: EQUIVALENCE OF MILNE’S PROPOSAL TO R-CURVE ANALYSIS
Stable crack growth prior to structural instability is a common feature in the failure of cracked thin metal sheets. To take this into account failure analyses based upon a crack growth resistance curve (R-curve) have been proposed (see, for example, ASTM STP 527 (1973) which contains many examples and also references to early development work). These concepts were originally applied in the small scale yielding regime using either the strain energy release rate, \( G_R \), or the stress intensity factor, \( K_R \), as a measure of the materials' resistance to growth.

In recent years it has become widely recognised that ductile crack growth can also occur in a stable manner in thick section ferritic steels loaded above the ductile-cleavage transition temperature. In contrast to cleavage failure, the initiation of ductile cracking is in general not defined by a load instability and in moderately sized specimens often does not occur until substantial plastic displacements have been detected. Moreover, the load capacity of the structure may not be exceeded until large amounts of crack growth have occurred. Thus in thick section structures not only is it difficult to determine the initiation of ductile cracking, but the significance of the cracking is difficult to define. To overcome both these problems the R-curve concept has been extended to elastic-plastic failures by identifying the resistance parameter as \( J_R \), the value of the J-integral measured in a test using the instantaneous load and crack length (Paris, Tada, Zahoor and Ernst, 1977; Hutchinson and Paris, 1977). Some theoretical justification for using \( J \) has been attempted (Hutchinson and Paris, 1977; Rice and Sorensen, 1978) but for extensive stable crack growth the approach must be considered empirical.

To enable a \( J_R \)-curve analysis of real structures, Paris et al (1977) proposed an instability criterion based upon the normalized gradient, \( T_{\text{mat}} \), of the \( J_R(\Delta a) \) curve, where \( \Delta a \) is the increment of crack extension. When the applied value of \( T = \frac{E}{\sigma_0^2} \frac{dJ}{da} \), where \( E \) is Young's modulus, \( \sigma_0 \) the yield stress and \( \frac{dJ}{da} \) the derivative of \( J \) with respect to crack length) equals or exceeds \( T_{\text{mat}} \) instability is predicted, provided of course the applied \( J \) already exceeds the initiation value, \( J_i \). In the small scale yielding regime this is entirely consistent with an R-curve analysis based upon \( G_R \) given the assumption that \( J_R \) or \( G_R \) varies linearly with \( \Delta a \). The disadvantage of the method is that not only must an elastic-plastic analysis be performed to obtain \( J \) as a function of load and crack length, but also its derivative must be evaluated. This can be extremely involved as demonstrated by the few cases where solutions have been obtained (Hutchinson and Paris, 1977; Tada, Musicco and Paris, 1978).

Recently a simpler approach has been suggested by Milne (1978a) based upon the failure assessment diagram used in the analysis of the integrity of cracked structures (Harrison, Loosemore and Milne, 1977, hereafter referred to as HLH). This approach is easier to use as it obviates the need for detailed elastic-plastic analyses and requires only a linear elastic calculation and an estimate of the plastic limit load. In addition it does not require explicit evaluations of derivatives with respect to crack length. It is applicable to all problems involving ductile crack growth so long as time dependent effects are small enough to be negligible.
Instability is defined in terms of the maximum load a structure can withstand. Thereafter if the loading is displacement controlled, subsequent displacements reduce the load bearing capacity even though the structure is still stable; if the loading is load controlled, the attainment of maximum load coincides with structural instability (Milne, 1978a). Thus the stability/instability condition can be defined as the applied load equalling or exceeding the maximum load, regardless of the compliance of the loading system.

In this Note we demonstrate that the proposal of Milne (1978a) is equivalent to an R-curve instability analysis based on $J_R$, provided that the failure assessment curve which appears in the failure assessment diagram of HLM is a good approximation to the actual failure curve of the structure under analysis. The equivalence is demonstrated by analysing a centre cracked plate and showing that the $J_R$-curve method and Milne's proposal give identical instability conditions. The technique is then developed and applied to two examples relevant to pressure vessels, namely thick and thin walled internally pressurized pipes.

2. **MILNE'S PROPOSAL**

The failure assessment procedures developed by HLM are applicable for any stress level up to plastic collapse. Two parameters are evaluated:

$$ S_r = \frac{\sigma}{\sigma_1(a/t)} = \frac{L}{L_1(a/t)} $$

and

$$ K_r = \frac{K_1(\sigma, a/t)}{K_{1C}} $$

where $\sigma$ (L) is the applied stress (load), $\sigma_1(a/t)$ ($L_1(a/t)$) the plastic collapse stress (load), $K_1(\sigma, a/t)$ the linear elastic stress intensity factor for a crack of length $a$ in a structure of width $t$. Stress and load are simply related by a geometric factor. These two parameters are then entered as a coordinate point on the failure assessment diagram, Fig. 1(a) and judged against the failure assessment line,

$$ K_r = \frac{2S_r}{8 \ln \sec \left( \frac{\pi}{2} S_r \right) } $$

This failure assessment line interpolates between the two limits defined by Dowling and Townley (1972), the linear elastic limit, at $K_r = 1$ and the plastic collapse limit at $S_r = 1$. A more rigorous approach to elastic-plastic analysis is to use the $J$-integral. Chell (1977) has shown that the failure assessment line of Fig. 1 is equivalent to a $J$ analysis, taking the functional form for $J$ as

$$ J_p = J_1 \frac{8}{\pi^2} \frac{\ln \sec \left( \frac{\pi}{2} S_r \right) }{S_r^2} $$

... (1)
where $J_p$ and $J_1$ are determined using elastic-plastic and linear elastic analyses respectively. Chell pointed out that in general a failure curve may be derived from any $J$ analysis by plotting $\left(\frac{J_1}{J_p}\right)^{\frac{1}{2}}$ against $\frac{\sigma}{\sigma_1}$. Since failure is generally defined when $J_p = J_{1C}$, the failure curve can be interpreted as $\left(\frac{J_1}{J_{1C}}\right)^{\frac{1}{2}}$ plotted against $\frac{\sigma}{\sigma_1}$, where both these are evaluated as failure. Re-interpreting in terms of $K$ gives a failure curve in terms of $\frac{K_1}{K_{1C}}$ and $\frac{\sigma}{\sigma_1}$. Thus in principle each structure has a unique failure line similar to that in Fig. 1, which depends on the material properties as well as its geometry. The basic assumption behind the failure assessment line in Fig. 1(a) is that it provides a realistic lower bound representation of all practical failure curves. Given the uncertainties inherent in any failure assessment Chell (1977) has shown that this is certainly so within the approximations inevitably associated with analyses of real structures. Moreover, the constraints imposed on the failure line by the well defined linear elastic and plastic limits reduces the likelihood of error and avoids excessive reliance on elastic-plastic finite element calculations which at best can be considered uncertain at present (Wilson and Osias, 1978).

Any assessment point, e.g. A in Fig. 1(a), can be made to lie on the failure assessment line by multiplying the stress $\sigma$ by a factor, $F$, as shown. If $K_{1C}$ is defined as the initiation fracture toughness, either for cleavage or ductile cracking, the failure assessment diagram predicts the initiation of crack extension at a stress of $F\sigma$. The structure will be stable if it is loaded such that the assessment point lies within the boundary of the failure assessment line and unstable if its assessment point lies on or outside this line. Limiting the analysis to initiation thus infers that stable crack growth cannot be tolerated.

Milne (1978a) extended the concepts of this failure assessment diagram to the stable crack growth regime by proposing that the failure assessment line represents the boundary between stable and unstable conditions during stable crack growth as well as at initiation. This means that if the parameters $S_T$ and $K_T$ are calculated during slow crack growth the resulting locus of assessment points will follow the failure assessment line (Fig. 1(b)). Of course, in evaluating $S_T$ and $K_T$ allowance has to be made for the growing crack. This can be done by generalizing the parameters, so that after $\Delta a$ of growth

$$S_T = \frac{\sigma}{\sigma_1 \left(\frac{a+\Delta a}{t}\right)} \quad \ldots (2a)$$

and

$$K_T = \frac{K_1(\sigma, \frac{a+\Delta a}{t})}{K_{1C}(\Delta a)} \quad \ldots (2b)$$

The stress $\sigma$ is now variable and can be determined from the load displacement curve after $\Delta a$ of growth and the parameter $K_T(\Delta a)$ can be obtained from its $J$ equivalent, i.e. $K_T(\Delta a) = \sqrt{E'J_R(\Delta a)}$ where $E'$ is Youngs modulus, $E$ for plane stress and $E/(1-\nu^2)$ for plane strain,
\( v \) being Poisson's ratio. \( J_R(\Delta a) \) is the J-resistance curve. If \( \Delta a = 0 \), \( K_R = K_{JC} \) and the analysis relates to initiation criteria, if \( \Delta a \) is positive \( K_R \) defines the resistance to crack growth at \( \Delta a \), so the analysis relates to growth criteria.

To use the procedures it is necessary to plot a locus of assessment points as a function of \( \Delta a \) at a constant load \( \sigma_a \) say as indicated by ABC in Fig. 2. The factor on load \( F \), as defined for initiation in Fig. 1(a), is now variable, depending upon the value of \( \Delta a \). For the case shown in Fig. 2 it rises to a maximum at the point C on the growth locus. The maximum load is thus \( \sigma_m = F_C \sigma_a \) and it coincides with the amount of crack growth between A and C. In a similar way values for \( F \) can be determined for any amount of growth, so that the load and the loading path necessary to produce a given amount of growth can be determined.

The growth locus A'B'C'D' has been plotted for stress \( \sigma_a \) and it can be seen that this is tangential to the failure assessment line at C' (Fig. 2). Since the proposal requires that any point outside of this line is unstable we can follow the slow crack growth processes as the structure is loaded from the point A.

1. Applying \( F_A \sigma_a \) raises the assessment point to A", coincident with the failure assessment curve. The growth locus A"B" at this load falls within the failure assessment line so the structure is stable.

2. Increasing the load to \( F_B \sigma_a \) raises the assessment point to A"", outside the failure assessment line. However, the growth locus at this load re-enters the stable part of the failure assessment diagram at B"". In practice, of course, the point A"" is never reached, the crack growing along the path from A" to B"" as the load is increased to \( F_B \sigma_a \).

3. Further growth to C' is only possible by raising the load to \( F_C \sigma_a \). At this load the locus C'D' is followed. This is always outside the failure assessment line so the structure is unstable. Hence the point C' represents the limit of structural stability in that it defines the maximum load the structure can withstand given the original crack length at A. It also defines how much crack growth results from applying this maximum load. Under displacement control the load reduces on further crack extension and the growth may or may not remain stable, depending on the compliance of the structure. If stability is maintained then the assessment points will continue to fall on the failure line up to say D" (Fig. 2).

3. RELATIONSHIP TO R-CURVE INSTABILITY ANALYSIS

In the Appendix it is demonstrated that in principle the analysis proposed by Milne is equivalent to Jr-curve analysis and hence the method of Paris et al (1977). Here we demonstrate the factual equivalence of Milne's proposal and the R-curve analysis for the example of a centre cracked plate of width 80 mm containing an initial crack of length 20 mm. Let the initiation toughness \( K_I \) of the material be 100 MPa \( \sqrt{m} \) with a flow stress \( \sigma \) of 1000 MPa where \( \sigma = (\sigma_a + \sigma_u)/2 \) and \( \sigma_u \) is the ultimate tensile stress. If Young's Modulus E is 200 GPa, and Poisson's ratio \( \nu \) is 0.3, the initiation value \( J_{ij} = 45.5 \text{ kJm}^{-2} \).
On the failure assessment diagram (Fig. 3) the assessment point A corresponding to an applied stress of 200 MPa is given by

\[ K_r = \frac{\sigma \sqrt{a_0}}{Y/K_i} = 0.364 \]

\[ S_r = \frac{\sigma}{\sigma_i} = 0.267 \]

where \( \sigma_i = \frac{\sigma}{(1 - a_0/t)} \), where \( a_0 \) and \( t \) are half the crack length and plate width respectively, and \( Y \) is the centre cracked plate compliance function. The load at which initiation of crack growth occurs is thus

\( \sigma_i = 200(0A''/0A) \text{ MPa} = 492 \text{ MPa (Fig. 3).} \)

To calculate the instability load consider the \( K_R(\Delta a) \) curve shown in Fig. 4 as the crack growth resistance curve of the material. Using this data the curve ACB in Fig. 3 was constructed for \( \sigma = 200 \text{ MPa} \) and a varying between 10 and 18 mm by calculating the assessment points

\[ K_r = \frac{\sigma \sqrt{a}}{K_R(\Delta a)} \]

\[ S_r = \frac{\sigma}{\sigma_i(1-\frac{a}{t})} \]

(see Table 1). The stress and crack length needed for stable growth were then obtained by determining the factor, \( F \), defined in Fig. 1(a). The crack length corresponding to the maximum value of \( F \), and the stress \( F_m' \), then represents the instability condition. This is confirmed by increasing the distance of every point on the curve ACB by the maximum value of the factor \( F \) to give the curve A'C'B' (Fig. 3). The tangent point C' corresponds to instability, and is given by \( \sigma_m = 622 \text{ MPa} \) and \( a = 12.8 \text{ mm} \).

To show this is the instability point the curve \( K_p = \frac{EJ (\sigma, \sigma')}{p (m', \sigma')} \)

is plotted as a function of crack size in Fig. 4 where \( J_p \) is given by equation (1). This curve becomes tangent to the \( K_R(\Delta a) \) curve for \( \Delta a = 2.8 \text{ mm} \), i.e. \( a = 12.8 \text{ mm} \), as required.

4. APPLICATIONS TO PRESSURE VESSELS

4.1 Thick Walled Cylinder

This problem has recently been treated by Tada, Musicco and Paris (1978) who used an extended longitudinal crack in a thick walled cylinder to model a defect in the nozzle of a pressure vessel. Here we consider a similar problem using Milne's approach and demonstrate the sensitivity of the solution to different \( J_i \) and \( T_{mat} \) values.

The cylinder has an internal radius to wall thickness ratio \( r_i/t = 1 \) and is subject to an internal pressure \( P \). For this geometry the necessary \( K_1 \) solutions are given by Bowie and Freese (1972). For the collapse pressure we take a lower bound solution
\[ P_i = \frac{\bar{\sigma}}{t} \left( 1 - \frac{a}{t} \right) \left( \frac{r_i}{t + a} \right) \] ... (3)

obtained by equating forces on the half of the cylinder containing the defect. This differs from the solution used by Tada et al (1978) which they obtained by equating forces over the whole cylinder. Tada et al's solution is unsatisfactory because when \( a/t = 1 \), the crack has fully penetrated the cylinder wall but a pressure equal to \( 1/(r_i/t + 1) \) is still required for plastic collapse. This is formally correct, but clearly not the plastic collapse formula to be used in the failure assessment diagram when full penetration of the cylinder wall by the crack is, for practical purposes, failure of the vessel.

The cylinder dimensions were taken as \( r_i = t = 125 \text{ mm} \) with the flow stress of the material \( \bar{\sigma} = 500 \text{ MPa} \). Following similar procedures to those outlined for the center cracked panel example, the pressure to initiate cracking and the pressure and crack length at instability were determined for a series of initial crack lengths. Several calculations were performed to test the sensitivity of the instability pressure to different levels of the toughness parameters \( J_i \) and \( T_{\text{mat}} \). The results are shown in Fig. 5(a) for \( T_{\text{mat}} = 100^* \) and two values of \( J_i \), 71.1 \text{ kJ/m}^2 and 25.6 \text{ kJ/m}^2. The circled points on the instability locus define the final crack length and instability pressure as a function of the original crack lengths 10 mm, 20 mm etc. labelled 1, 2, 3 etc. Note that in this case, although the initiation pressures are quite sensitive to the different levels of \( J_i \), the value of \( J_i \) has no affect on either the instability pressures or the final crack length at instability.

Similar results are shown in Fig. 5(b) for \( J_i = 25.6 \text{ kJ/m}^2 \) and \( T_{\text{mat}} = 25, 100 \) and 200. This shows two features:

(1) The amount of crack growth prior to instability increases with decreasing \( T_{\text{mat}} \), even though the instability pressure decreases with decreasing \( T_{\text{mat}} \).

(2) For the two larger values of \( T_{\text{mat}} \) the instability pressure is very close to the plastic collapse pressure defined at the final crack length.

This second point, of course, can only be determined once detailed calculations have been performed to evaluate the expected crack growth. Moreover, for the lowest level of \( T_{\text{mat}} \) the instability pressure is considerably less than the collapse pressure. For this case the pressure versus crack extension locus is shown plotted in Fig. 5(b) for the 10 mm initial defect, labelled 1, which grows to 22 mm at the instability pressure.

\* Implicit in the specification of \( T_{\text{mat}} \) as a toughness parameter is the assumption that the gradient of the \( J_R(\Delta a) \) curve is constant and equal to \( \frac{1}{E} T_{\text{mat}} \). \( T_{\text{mat}} = 100 \) corresponds to \( \frac{dJ}{da} = 125 \text{ MPa} \), when \( E = 200 \text{ GPa} \), a not unreasonable value.
4.2 Thin Walled Cylinder

Calculations for this geometry containing an extended longitudinal crack are similar to those for the thick walled cylinder. This geometry may be used to simulate cracking in the beltline region of a pressure vessel. In this case we have taken \( r_i/t = 10 \) and used the stress intensity factor for an edge crack in a plate where the applied stress, \( \sigma \), is replaced by the stress \( (r_i/t + 1)P \). Equation (3) again represents the plastic collapse pressure with \( 500 \) MPa as before. This time the cylinder dimensions were taken as \( r_i = 2 \) m, \( t = 200 \) mm and the calculations were performed for one level of \( J_i \), \( 71.1 \) kJm\(^{-2}\), and two values for \( T_{\text{mat}} \), \( 100 \) and \( 25 \). The results are shown in Fig. 6 and again indicate that for the higher value of \( T_{\text{mat}} \), the instability pressure is determined by the collapse pressure for the crack length at instability. However, for the lower value of \( T_{\text{mat}} \), instability occurs at pressures increasingly below the collapse value as the original defect size is increased. A typical pressure versus crack extension curve for a \( 60 \) mm initial defect, labelled 6, which grows to \( 76 \) mm at the instability pressure is also shown in Fig. 6. This shows that most of the crack growth occurs in the last stages of pressurization prior to instability.

In summary, the results for the two sets of calculations show that in general the instability pressure decreases with decreasing \( T_{\text{mat}} \) but at the same time the allowable amount of defect growth increases.

5. PRACTICAL ASPECTS OF ANALYSIS

Of the input data required, the most important one is the resistance parameter \( J_R(\Delta a) \). Milne (1978a) has already discussed the limiting values of this parameter and the sensitivity of the results to the actual value of \( J_R(\Delta a) \) has been demonstrated above. For a safe assessment lower bound \( J_R(\Delta a) \) curves are required, for possibly extensive amounts of crack growth. Thus multi-specimen tests cannot be avoided and the test pieces should be designed so that reasonable amounts of growth can be obtained before collapse mechanisms dominate. It is also worth bearing in mind that cracks which grow non-uniformly are more difficult to analyse and can produce deceptive data because of the reduction of the stress intensity calibration factor at the tip of a thumbnailing defect (Neale, 1976). Consideration should thus be given to side grooved specimens since these tend to produce straight crack fronts and have the added advantage of simulating plane strain conditions especially for thin specimens. Moreover these cracks will grow without shear lips, so the \( J_R(\Delta a) \) curve will tend to be lower than in the presence of shear lips (Garwood and Turner, 1977; Willoughby, Pratt and Turner, 1978). This latter aspect is of particular importance for part-thickness defects which are propagating across the remaining ligament. For full thickness defects it may be more appropriate to generate these data from full section thickness specimens, as it can be argued that this will produce data more relevant to the real case. It is, of course, particularly important to get the crack growth directions in the specimens parallel to the expected growth direction in the structure, since ductile cracking is highly sensitive to inclusion size, spacing and orientation (Willoughby et al, 1978; Schwalbe, 1977).
The use of the parameter $T_{\text{mat}}$ to characterize the crack growth dependence of $J_R$ may be questioned in cases where appreciable stable crack extension occurs. For side-grooved specimens the $J_R(\Delta a)$ curve does not rise linearly with $\Delta a$ but rather tends towards a plateau value independent of $\Delta a$ (Rice and Sorensen, 1978). Here the use of $T_{\text{mat}}$ may result in optimistic instability predictions. Perhaps a more realistic representation of the $J_R(\Delta a)$ curve is to write

$$J_R(\Delta a) = J_1 + (J_R^{SS} - J_1) \left[ 1 - \exp \left\{ -\frac{\frac{dJ}{da} \Delta a}{(J_R^{SS} - J_1)} \right\} \right] \quad \ldots (4)$$

where $J_R^{SS}$ is the steady state value of $J_R$ (Rice and Sorensen, 1978) and represents a parameter obtained by the best fit of equation (4) to the experimental data. For small $\Delta a$ values this equation reduces to

$$J_R(\Delta a) = J_1 + \frac{dJ}{da} \Delta a = J_1 + \frac{\sigma^2}{E} T_{\text{mat}} \Delta a$$

in agreement with Paris et al (1977). For large values of $\Delta a$

$$J_R(\Delta a) = J_R^{SS}$$

This way of expressing the $J_R(\Delta a)$ curve should provide a reasonable fit to the $J_R(\Delta a)$ data and allow an instability assessment to be made if the crack growth in a real structure exceeds that measured in laboratory tests.

In performing an analysis there is considerable advantage to be gained in evaluating the sensitivity of the results to variations in the input data, in particular $J_1$ and $J_R(\Delta a)$. As in the examples in Fig. 5 and 6 it is often possible to demonstrate that large variations in these parameters have little influence on the instability loads, creating much needed confidence in an area which is frequently ridden with doubt.

6. **DISCUSSION**

In cases where the structural geometry is such that $\sigma_1$ depends only weakly on crack length, a precise determination of the defect size at instability using the failure assessment diagram is difficult. The load at instability is still definable to good accuracy however, because near instability the defect grows without appreciable change in load (compare the load-crack extension curves in Fig. 5 and Fig. 6).

The precise crack length at instability can be more accurately obtained by using the following equation for the load factor $F$

$$F = \frac{2}{\pi} \cos^{-1}\left( \exp \left( -\frac{2S}{K_r^2} \right) / \frac{S}{K_r^2} \right) \quad \ldots (5)$$

where $S_r$ and $K_r$ are evaluated as in equation 2(a) and (b) at the load of interest. For an arbitrary applied constant stress $\sigma_r$, $F$ depends on the crack length $a$, $F = F(a)$. If the original defect length is $a_o$,
F\(\sigma\) represents the stress level at which initiation occurs. During growth F\(\sigma\) represents the stress level to cause stable growth of \(\Delta a = a - a_0\). The crack length, \(a'\), which maximizes F\(\sigma\) is thus the crack length at instability and the corresponding stress level is \(\sigma_m = F(a')\sigma\). Thus by using equation (5) to determine the factor on load a failure assessment can be made, including stable growth, without recourse to the use of the diagram.

Although only primary loading has been considered the extension of the method to secondary loading e.g. thermal and residual stresses, is quite straightforward. Here the secondary loading modifies the origin with respect to which the load factor F is determined (Chell, 1977; Milne, 1978b). In the presence of ductile crack extension account must be taken of the change in this origin due to the change in \(K_R(\Delta a)\) as well as crack length \(a\), but otherwise the procedures for determining the instability conditions are the same (Milne and Chell, 1979).

7. SUMMARY OF PROCEDURES

7.1 Input Data Needed

Stress intensity factors as a function of load and crack size, \(K_1(L, a)\).

Crack growth resistance parameter in terms of \(K\), i.e. \(K_R(\Delta a) = \sqrt{E'J_R(\Delta a)}\), obtainable from \(J_R(\Delta a)\) resistance curves, which need not necessarily be linear.

Plastic Limit Loads \(L_c\) as a function of crack size.

7.2 Analysis

Evaluate \(S_r\) and \(K_r\) as a function of crack growth, \(\Delta a\), at any constant load \(L\), e.g. at normal operational load, and any appropriate initial crack size.

Note that for \(\Delta a = 0\)

\[
S_r(a) = \frac{L}{L_c(a)}
\]

\[
K_r(a) = \frac{K_1(L,a)}{K_{1C}} = \frac{K_1(L,a)}{\sqrt{E'J_{1C}}}
\]

for \(\Delta a > 0\)

\[
S_r(a+\Delta a) = \frac{L}{L_c(a+\Delta a)}
\]

\[
K_r(a+\Delta a) = \frac{K_1(L,a)}{\sqrt{E'J_R(\Delta a)}}
\]
Evaluate the load factor \( F \) from the equation

\[
F(a+\Delta a) = \frac{2}{\pi} \cos^{-1} \left\{ \exp \left( -\frac{\pi S_r^2}{8K_r} \right) \right\} / S_r
\]

Alternatively plot the coordinates \( S_r (a+\Delta a), K_r (a+\Delta a) \) on the failure assessment diagram to develop the growth locus at the constant load, \( L \), and evaluate \( F(a+\Delta a) \) as in Fig. 1.

The instability load \( L_m \) is given by

\[
L_m = F_m L
\]

where \( F_m \) is the maximum value of the load factor \( F \). And the crack extension at instability is given by \( \Delta a \) at \( F_m \).

Perform a sensitivity analysis varying the input data as required. Note that the effect of varying the initial crack size cannot be determined by varying \( \Delta a \) on any given constant load growth locus. This can only be evaluated by calculating new growth loci for each value of a considered (Milne 1978a).

8. CONCLUSIONS

The method proposed by Milne (1978a) for determining the maximum load bearing capacity of a structure during ductile crack growth is equivalent to an R-curve instability analysis based upon \( J \) as the resistance parameter. For values of the tearing modulus, \( T_{\text{mat}} \), typical of pressure vessel steels, the ductile instability pressure of pressure vessels is independent of the initiation value of \( J \), and is sensibly determined by the plastic collapse pressure corresponding to the instantaneous crack length at instability.

9. REFERENCES

ASTM STP 527, 1973, Fracture toughness evaluation by R-curve methods, American Society for Testing and Materials


Chell, G.G., 1977, A procedure for incorporating thermal and residual stresses into the concept of a failure assessment diagram, C.E.R.L. Note No. RD/L/N 49/77. See also ASTM STP 668, 581 (1979)


Milne, I., 1978b, Assessment of the defect tolerance of structures subjected to combined secondary and primary loads, C.E.R.L. Note No. RD/L/N 112/78, see also SMiRT 5 (1979) paper G1/3


Neale, B.K., 1976, C.E.G.B. Report No. RD/B/N3787


Wilson, W.K. and Osias, J.R., 1978, Int. J. Fract., 14, R95
Table 1

Construction of Curves ACR and A'CB" in Fig. 6 for $\sigma = 200$ MPa

<table>
<thead>
<tr>
<th>Half crack length mm</th>
<th>$K_1$</th>
<th>$K_{Rf}$</th>
<th>$K_r$</th>
<th>$\sigma_1(a)$</th>
<th>$S_r$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>36.4</td>
<td>100</td>
<td>0.364</td>
<td>750</td>
<td>0.267</td>
<td>2.46</td>
</tr>
<tr>
<td>11</td>
<td>38.4</td>
<td>141</td>
<td>0.272</td>
<td>725</td>
<td>0.276</td>
<td>2.97</td>
</tr>
<tr>
<td>12</td>
<td>40.4</td>
<td>165</td>
<td>0.245</td>
<td>700</td>
<td>0.286</td>
<td>3.08</td>
</tr>
<tr>
<td>13</td>
<td>42.3</td>
<td>185</td>
<td>0.229</td>
<td>675</td>
<td>0.296</td>
<td>3.10</td>
</tr>
<tr>
<td>14</td>
<td>44.3</td>
<td>200</td>
<td>0.221</td>
<td>650</td>
<td>0.308</td>
<td>3.06</td>
</tr>
<tr>
<td>15</td>
<td>46.2</td>
<td>210</td>
<td>0.220</td>
<td>625</td>
<td>0.320</td>
<td>2.98</td>
</tr>
<tr>
<td>16</td>
<td>48.2</td>
<td>220</td>
<td>0.219</td>
<td>600</td>
<td>0.333</td>
<td>2.89</td>
</tr>
<tr>
<td>17</td>
<td>50.2</td>
<td>228</td>
<td>0.220</td>
<td>575</td>
<td>0.348</td>
<td>2.79</td>
</tr>
<tr>
<td>18</td>
<td>52.3</td>
<td>233</td>
<td>0.224</td>
<td>550</td>
<td>0.364</td>
<td>2.68</td>
</tr>
</tbody>
</table>
APPENDIX: EQUIVALENCE OF MILNE'S PROPOSAL TO AN R-CURVE ANALYSIS

Milne's proposal and an R-curve analysis have similar features in that they both define instability as the point where two curves become tangential to each other. However, it is not immediately obvious that they provide equivalent descriptions, and it is not clear what, if any, is the relationship between the failure assessment curve and a crack growth resistance curve.

A typical $J_R(\Delta a)$ curve is shown in Fig. 7(a). This is obtained from load-displacement test data similar to that shown in Fig. 7(b). For each point on the curve, 1, 2, 3 etc. beyond the initiation point 1, the value of $J_p$ applied to the specimen is obtained for the instantaneous load and crack length. The resistance curve $J_R(\Delta a)$ is the locus of these points, plotted against $\Delta a$ (Fig. 7(a)). For any load level, $L_1$, $L_2$ etc., the applied value of $J_p$ when plotted as a function of crack length intersects the $J_R(\Delta a)$ curve at points 1, 2 etc. The value of $J_p$ at the intersection will be called $J_p^R$ and is, of course, numerically equal to $J_R(\Delta a)$ where $R$ signifies the value quantities calculated using data obtained from the R-curve. Instability is predicted when this intersection is a tangent point for the two curves (e.g. the point 3 in Fig. 7(a) corresponding to a load $L_3$) because the applied $J_p$ then rises more rapidly with crack growth than $J_R(\Delta a)$. The method of Paris et al. (1977) determines this instability point by solving the equation

$$\frac{dJ_p}{d\Delta a} = \frac{dJ}{d(\Delta a)}$$

when $J > J_1$

The $J_p$ curves for $L_1$ and $L_3$ and the $J_R$ curve from Fig. 7(a) are shown normalized by $J_1$ in Fig. 8(a). This normalization does not change the instability condition, which is now determined by the tangency point of the two curves $J_p (L_3, a)/J_1 (L_3, a)$ and $J_R(\Delta a)/J_1^R (L, a)$.

In Fig. 8(b) these curves have been replotted with the ordinate inverted. At $L_1$, $J_1 (L_1, a)/J_p (L_1, a)$ falls below the projection of the curve $J_1^R (L, a)/J_R(\Delta a)$ for $a < a_0$ and above it for $a > a_0$, consistent with $a_0$ being the initiation crack length. At the load $L_3$, $J_1 (L_3, a)/J_p (L_3, a)$ falls below the $J_1^R (L, a)/J_R(\Delta a)$ curve everywhere except at the tangency point, consistent with instability when the crack has grown to $a_3$.

From Fig. 7(a) for any load less than $L_3$, say $L_1$, $J_1 (L_1, a)$ is equal to $J_R(\Delta a)$ only at $a = a_1$, being less than it for $a < a_1$ and greater than it when $a > a_1$. Thus dividing $J_1 (L, a)$ by $J_R(\Delta a)$ rather than $J_p (L, a)$ will invert the effect shown in Fig. 8(b) and produce the relationships shown in Fig. 8(c). The intersection and tangency points with the curve $J_1^R (L, a)/J_R(\Delta a)$ remain unchanged.

To relate the diagram in Fig. 8(c) to the failure assessment diagram of Fig. 2 a further change of axes is required. The ordinate must be redefined as $(J_1/J_p)^{1/4}$ and the abcissa as $\sigma/\sigma_1$. The former change is trivial, the latter needs some explanation. Since the instability point is determined by the tangency of two curves, the ordinate may be redefined as any monotonically increasing function of crack length without affecting this condition. For purposes of relating the curves to the failure curve a suitable function is $\sigma/\sigma_1$ which, for a given stress $\sigma$, increases with a due to the crack length dependence of the plastic collapse stress $\sigma_1$. 

Transforming Fig. 8(c) in this way leads to curves in Fig. 9. The locus of points \((J_1^R (L,a)/J_R(\Delta a))^\frac{1}{2}\) as \(L\) goes from \(L_1\) to \(L_3\) and \(a\) goes from \(a_0\) to \(a_3\) forms the curve \(A'C'\), and represents a renormalized and transformed version of the R-curve. The fundamental assumption in the failure assessment diagram is that to a good approximation all the curves of \((J_1^R)^{\frac{1}{2}}\) against \(\sigma/\sigma_1\) fall on one universal curve, the failure curve shown in Fig. 1. Accepting this then it is clear that the curve \((J_1^R (L,a)/J_R(\Delta a))^{\frac{1}{2}}\) falls on part of the failure curve since \(J_R(\Delta a) = J_p^R(L,a)\). Thus in Fig. 9 crack initiation occurs where the curve, \(A"C"B"\) = \(J_1(L_1,a)/J_R(\Delta a)\), first intersects the failure line given by \((J_1^R(L,a)/J_R(\Delta a))^\frac{1}{2}\) that is at \(A"\) where the value of \(\sigma/\sigma_1\) is given by \(\sigma_1(a_0)/\sigma_1(a_0)\).

Similarly the instability point labelled \(C'\) is coincident with the tangency of the two curves \((J_1(L_3,a)/J_R(\Delta a))^\frac{1}{2}\) and \((J_1^R(L_1,a)/J_R(\Delta a))^\frac{1}{2}\), which corresponds to the value of \(\sigma/\sigma_1\) given by \(\sigma_3(a_3)/\sigma_1(a_3)\). Furthermore since \(J_1^{\frac{1}{2}}\) varies linearly with load \(L\) or stress \(\sigma\), as does \(\sigma/\sigma_1\), then the two curves \((J_1(L_1,a)/J_R(\Delta a))^\frac{1}{2}\) and \((J_1(L_3,a)/J_R(\Delta a))^\frac{1}{2}\) are geometrically similar along rays emanating from the origin 0 (see Fig. 9). Hence the maximum load, \((\sigma_3 = \sigma_m)\), the structure can tolerate allowing for stable crack growth is simply \(OC'/OC"\) times the initiation load \(\sigma_1\). A geometrically similar curve to \((J_1(L_3,a)/J_R(\Delta a))^\frac{1}{2}\) (signified as \(A'C'B'\) in Fig. 9) can be constructed for any stress \(\sigma\)' say \(\frac{1}{F}\times \sigma_m\) merely by plotting points corresponding to distances along the rays 'OAA', 'OCC', 'OBB' etc. given by \(\frac{1}{F}\times OA'$, \(\frac{1}{F}\times OC'$, \(\frac{1}{F}\times OB'$ etc. Such a curve, ABC, is shown in Fig. 9.

On consideration it is clear that the curves, ACB and A'C'B' in Fig. 9 are the same as the growth loci, ABCD and A'B'C'D' plotted on Fig. 2. Thus in principle the instability proposal of Milne (1978a) is consistent with and equivalent to an R-curve instability analysis provided equation (1) is accepted as a reasonable, lower bound estimate of a universal failure curve.
AT FAILURE, \( K_r = S_r \left( \frac{8}{\pi^2} \ln \sec \left( \frac{\pi}{2} S_r \right) \right)^{1/2} \)

LOAD FACTOR \( F = \frac{OF}{OA} \)

(a)

FIG. 1  FAILURE ASSESSMENT DIAGRAM SHOWING (a) THE LOAD FACTOR \( F \) AND (b) THE ADAPTATION FOR STABLE CRACK GROWTH

AFTER INITIATION AND DURING GROWTH, \( \Delta a \), THE LOCUS OF ASSESSMENT POINTS IS COINCIDENT WITH THE FAILURE LINE

INITIATION POINT

INCREASING LOAD

CONSTANT CRACK LENGTH, \( a \)
FIG. 2 ADAPTATION OF THE FAILURE ASSESSMENT DIAGRAM TO PREDICTING INSTABILITY AFTER STABLE CRACK EXTENSION
FIG. 3  INSTABILITY OF A CENTRE CRACKED PLATE
PREDICTED USING MILNE'S PROPOSAL
\[ K_p = \sqrt{\frac{EJ_p(\sigma_m - \sigma)}{1 - \nu^2}} \]

\( \sigma_m = 622 \text{ MPa} \)

**INSTABILITY POINT PREDICTED USING MILNE'S PROPOSAL COINCIDENT WITH TANGENCY OF \( K_p \) AND \( K_R \) CURVES**

**FIG. 4 INSTABILITY OF A CENTRE CRACKED PLATE PREDICTED USING AN R-CURVE ANALYSIS**
FIG. 5  INSTABILITY OF A CRACKED THICK WALL CYLINDER, $R/A = 1$, $t = 125$ mm. PRESSURES AND CRACK SIZES AT INITIATION OF STABLE GROWTH AND INSTABILITY.

(a) SENSITIVITY TO $J_i$
FIG. 5  INSTABILITY OF A CRACKED THICK WALL CYLINDER, \( R/t = 1, \)
\( t = 125\) mm. PRESSURES AND CRACK SIZES AT INITIATION OF
STABLE GROWTH AND INSTABILITY.

(b) SENSITIVITY TO \( T_{\text{mat}} \)
FIG. 6 INSTABILITY OF A CRACKED THIN WALLED CYLINDER R/L = 10

\( t = 200 \text{ mm} \). THE PRESSURES AND CRACK SIZES AT INITIATION AND

INSTABILITY ARE SHOWN FOR DIFFERENT VALUES OF \( T_{\text{mat}} \).
FIG. 7 (a) A TYPICAL $J_R - \Delta a$ CURVE AND (b) ITS DERIVATION FROM A LOAD-DISPLACEMENT CURVE, SHOWING THE CONSTRUCTION OF $J_p(L,a)$ CURVES AND THEIR USE IN DETERMINING THE INSTABILITY POINT,

$$\frac{dJ_p(L,a)}{da} = \frac{dJ_R(\Delta a)}{d(\Delta a)} \text{ for } J_p > J_i$$
FIG. 9  TRANSFORMATION OF AXIS TO $(J_1/J_p)^{\frac{1}{2}}$ AND $\sigma/\sigma_1$

THE CURVE $(J_1^{R}(L,a)/J_p^{R}(\Delta a))^{\frac{1}{2}} = (J_1^R(L_a)/J_p^R(L,a))$

BECOMES COINCIDENT WITH THE FAILURE CURVE, AND

THE INSTABILITY POINT IS THE TANGENCY OF THIS TO

THE CURVE $(J_1(L_3,a)/J_{\Delta a})^{\frac{1}{2}}$
FRACTURE PROOF DESIGN

by Paul C. Paris*, H. Tada*, and S.E. Baldini*
Washington University, St. Louis, Missouri

ABSTRACT

The J-Integral based, so-called, "Tearing Instability Theory"[1], has established a crack stability criterion under elastic to fully plastic net section conditions with certain restrictions. The restrictions[2] are Charpy upper shelf material behavior (no appreciable cleavage instabilities), small amounts of crack growth compared to other dimensions, and a condition for proportionality of increments of strains in the field surrounding the crack tip. These conditions are met for many practical structural situations where avoiding crack instability will ensure safe structural behavior.

Further, it is demonstrated that statically indeterminate structures, with cracks meeting the conditions for tearing instability analysis, may often be proportioned in such a manner that they are "fracture proof". Indeed, by adjusting structural member proportions in combination with material properties, it is possible to produce "fracture proof designs" or more specifically a structure where no crack can become unstable.

In the current discussion, structural redundancies of the parallel tension member and closed frame type are used as the principal illustrations, in order to demonstrate the generality of the approach. However, a companion paper [3]
aimed specifically at nuclear piping, more clearly shows the practical nature of "Fracture Proof Design" in a specific application.


*Professor of Mechanics and Director, Senior Research Associate, and Research Assistant, respectively, The Center for Fracture Mechanics, Washington University, St. Louis, Missouri 63130.
The Concept of:

Fracture-Proof Design

(1) Statically Indeterminant Structure with a Cracked Member.

(2) Cracked Section Becomes Fully Plastic.

(3) Do Analysis which shows Sudden Crack Growth (unstable) Cannot Take Place!

(4) Or Show How to Redesign to Prevent Sudden Crack Growth!
Applications to Date

(1) Tension Structure
   (a) Multiple I-Bars
   (b) Cables
   (c) Heat-Exchanger Tubes

   —See Analysis of Tension—

(2) Bending Structure (Beams)
   (a) Piping
   (b) Nozzles (Pressure Vessels)
   (c) Welded Frames
   (d) Rails (Railroads)
   (e) Grillages of Beams

   —See Bending Analysis—

   (Possibilities seem quite unlimited!)
Review: Tearing Instability Concepts

(a) The J R-curve (Common Practice in F.M. Comunity):

\[ J = \frac{N}{b} \int P \, d\Delta \]
\[ \Delta a = \text{Change in Crack Length} \]

- Slopes: \( \frac{dJ}{da} \)
- Growing Crack
- Blunting (No Growth)

Define:

\[ T_{\text{MAT.}} = \frac{dJ}{da} \frac{E}{\sigma_0^2} \] (Resistance To Moving The Crack)

\( E = \text{Modules of Elasticity} \)
\( \sigma_0 = \text{Flow Stress} \)

(b) Also Note From J-Integral Field Theory:

\[ \sigma_T = \alpha \frac{J}{\sigma_0} \approx \frac{J}{\sigma_0} \]
\( (0.7 \leq \alpha \leq 1) \)
The Tension Member in a Rigid Test Machine

If Fully Plastic (Net Section)

\[ P = \sigma_0 (w - 2a) t \]

\[ \Delta_{EL} = \frac{PL}{AE} \]

\[ \Delta_{PL} = \delta_T \approx \frac{J}{\sigma_0} \]

\[ \Delta_{TOTAL} = \Delta_{EL} + \Delta_{PL} \]

During Crack Growth (da)

\[ dP = -2 \sigma_0 t \, da \]

\[ d\Delta_{EL} = \frac{dPL}{AE} = -2 \sigma_0 tL \, da \]

\[ d\Delta_{PL} = d\delta_T \approx \frac{dJ}{\sigma_0} \]

But Test Machine Grips Do Not Move

\[ d\Delta_{TOTAL} = 0 = -\frac{2 \sigma_0 tL \, da}{AE} + \frac{dJ}{\sigma_0} \]

or

\[ T_{APPL} = \frac{dJ}{da} \frac{E}{\sigma_0^2} = \frac{2L}{W} \]

\[ \leq T_{MAT} \]
Analysis of Tension

\[ \Delta_1 = \Delta_2 = \Delta_3 = \ldots = \Delta_m \]
\[ P = P_1 + P_2 + P_3 + \ldots + P_m \]
But \( P_2 = P_3 = P_4 \ldots = P_m \neq P_1 \)

During Crack Growth:

\[ P_i = \sigma_0 (w-2a) t \]

or

\[ dP_i = -2 \sigma_0 t \, da \]

\[ \Delta_t = \Delta_{tl} + \Delta_{pl} \]

\[ \Delta_{pl} = \delta_T = \frac{J}{\sigma_0} \]

\[ d\Delta = \left( -2\sigma_0 + da \right) \frac{L}{AE} + \frac{dJ}{\sigma_0} \]

\[ dP = 0 = dP + (m-1) \, dP_2 \]

or

\[ d\Delta_2 = \frac{dP_2L}{AE} = -\frac{dP_1}{(m-1) \, AE} \frac{L}{AE} = +2 \sigma_0 t da \frac{L}{AE} \]

(Let \( L, A, E \) of all bars be the same)
**GENERALIZED TENSION ANALYSIS**

\[ T_{\text{MAT}} \geq T_{\text{APPL}} = \left( \frac{m-m}{m-1-m} \right) \phi \]

**STABLE**

**m** = Number of Bars in Tension (Parallel with one Cracked)

**m** = Uncracked Bars Which Have Become Plastic

**SECTION**

\[
\begin{array}{c|c|c}
\text{Crack} & \phi & (\text{Number of Crack Tips Moving}) \\
\hline
\text{W} & 1 & \frac{2L}{W} \\
\text{D} & \frac{4 L d}{D^2} & \geq 6 \\
\text{t} & > 1 & \frac{2L d}{t d_m} \\
\end{array}
\]
Analysis of Bending of Beams

Rigid Plastic Analysis Report:

\[ J = - \frac{\Delta M_p}{t \Delta \alpha} \phi \]
or (Better)

\[ dJ = - \frac{\Delta M_p}{t \Delta \alpha} d\phi \]

If The Crack Grows:

\[ dM = - \sigma_o t h \, da \]

LET \[ h = \frac{b}{N} \]
\( N = \text{Section property} \quad (1 < N < 2) \)

or

\[ dM = - \frac{\sigma_o t b \, da}{N} \]
Structure Gives:

\[-d\phi = \frac{k}{E} dM\]

Redundant Elastic Structure \((k \sim \text{compliance})\)

Combining Results:

\[T_{\text{APPL}} = \frac{dJ}{da} \frac{E}{\sigma_0^2} = \frac{k b^2 t}{N^2}\]

Thus

\[T_{\text{MAT.}} > T_{\text{APPL}} = \frac{b^2 t}{N^2} \frac{L}{I}\]
\[ I \approx 2 \left[ B f \left( \frac{D}{2} \right)^2 \right] \]

Thus

\[ T_{\text{APPL}} = \psi \frac{b^2 t}{N^2} \frac{L}{I} = \psi \frac{2}{N^2} \frac{t}{f} \frac{b^2}{D^2} \frac{L}{B} \]

\[ \text{THE ORDER OF} \frac{1}{1} \]

Pipes (See TADA'S Curves) But
Note Similarity Here

\[ T_{\text{APPL}} = O(1) \cdot \frac{L}{D} \]
\[ d\phi = \psi \frac{dM_L}{EI} \]

\[ \psi = 1 \]

With Additional Hinges

\[ \psi = \frac{L^2}{3a^2} \quad \text{or} \quad \frac{a}{L} = \frac{1}{2} \rightarrow \psi = 1.33 \]

\[ \frac{a}{L} = \frac{1}{4} \rightarrow \psi = 5.33 \]

\[ \psi = \frac{1}{3} \left[ \frac{b}{a} - \frac{a}{L} - \frac{1}{2} \right] \]

or \( a/L = \frac{1}{2} \rightarrow \psi = \frac{1}{3} \)

\( a/L = \frac{1}{4} \rightarrow \psi = \frac{13}{12} \)
Application to Frames

\[ d\phi = \left[1 + \frac{\alpha + \beta}{3}\right] \frac{L}{I} \cdot \frac{dm}{E} \]

\[ k = \left[1 + \frac{\alpha + \beta}{3}\right] \frac{L}{I} = \psi \frac{L}{I} \]

or

\[ T_{MAT} > T_{APPL} = \left[1 + \frac{\alpha + \beta}{3}\right] \frac{b^2 t L}{N^2 I} = \psi \frac{b^2 t L}{I} \]

- \( \alpha = \beta = 1 \)  
- \( \psi \approx 1.67 \)

- \( \alpha = \beta \approx 0.8 \)  
- \( \psi \approx 1.55 \)

- \( \alpha = \beta = 0.7 \)  
- \( \psi \approx 1.59 \)

- \( \alpha = \beta = \frac{1}{3} \)  
- \( \psi = 1.22 \)
Finally

\[ \alpha = \beta = \frac{9}{3} \]

\[ \psi = 1.20 \]

**Conclusion**

\[ 1 \leq \psi \leq 2 \quad (\text{Normally}) \]

\[ T_{\text{MAT}} > T_{\text{APPL}} = \psi \frac{b^2 t L}{N^2 I} \]
\[
\frac{d^4N}{dx^4} + \rho^4 N = 0 \quad \rho^4 = \frac{k}{EI}
\]

\[N = e^{\mu x} \rightarrow m = \left( \pm \frac{i}{\sqrt{2}}, \pm \frac{i}{\sqrt{2}} \right) \rho\]

Let \( \mu = \rho \sqrt{2} \)

\[N = Ae^{-\mu x} \sin \mu x + Be^{-\mu x} \cos \mu x + 0\]

Other B.C. \( N \) is bounded at \( \infty \)

\[\frac{d^3N}{dx^3} \bigg|_{x=0} = 0 \quad \mu = \frac{\pi}{L'} \quad \rho = \frac{\sqrt{2}}{L'}
\]

\[\frac{EI \frac{d^2N}{dx^2}}{x=0} = M \]

Then Find

\[\Phi = 2 \bar{\Theta} = \frac{2\sqrt{2} M}{EI \left( \frac{k}{EI} \right)} \frac{1}{4} = \frac{2\sqrt{2} M}{EI \rho} = \frac{2ML'}{\pi EI}\]
Beam on Elastic Foundation

\[ \text{Foundation} \quad w = \Delta N \]

\[ EI \frac{d^4N}{dx^4} + LN = 0 \]

\[ \rho^2 = \frac{8}{EI} \]

\[ d \phi = \frac{2\sqrt{2}}{EI} \frac{dM}{(k \frac{\rho}{EI})^{1/4}} \]

\[ k = \frac{2\sqrt{2} M}{I \rho} \]

\[ T_{\text{APPL}} = \frac{2\sqrt{2}}{N^2 I} \frac{b^2 t}{\rho} \]

\[ T_{\text{MAT.}} > T_{\text{APPL}} = \frac{2}{\pi} \frac{b^2 t L'}{N^2 I} \]

\[ L' = \text{measure experimental peak to peak distance} \]
\[ T_{\text{Appl.}} = F_1 \cdot \left( \frac{1}{R} \right) + F_2 \cdot \left( \frac{JE}{R\sigma_o^2} \right) \]

Duane Arnold

I. R. = 5 \(3/8\)"  \( (R = 5.725\)"

\( t = 0.7\)"

\( t/R \sim 0.12 \)

\( a/t \sim 0.5 - 0.75 \)

\( 2\theta \sim 90^\circ \)

\( P/\sigma_o \sim 0.02 \quad (\sigma_o = 50 \text{ ksi}) \)

No Crack Closure

\[ \rightarrow F_1 \sim 0.4, \quad F_2 \sim -0.1 \]

\( J = 6,000 - 8,000 \quad \text{LB} \cdot \text{in}/\text{in}^2 \)

\( \frac{JE}{\sigma_o^2 R} \sim 13 - 17 \)

\[ T_{\text{Appl.}} \sim 0.4 \cdot \frac{1}{R} + (-0.1) \cdot (15) \]

\( \Phi = \frac{J}{\sigma_o RF_J} = (0.017 - 0.023) \text{ Rad.} \)
PROGRESS IN ELASTIC-PLASTIC FRACTURE
MECHANICS AND ITS APPLICATIONS

By Paul C. Paris and George I. Zahalak
Professor and Associate Professor, respectively,
Center for Fracture Mechanics, Washington University
St. Louis, Missouri 63130

ABSTRACT

This paper surveys recent developments in the application of J-integral methods to problems of elastic-plastic fracture. The analytical and experimental development of the J-integral concept over the last ten years is reviewed briefly. Tearing instability theory is presented in general terms, and specific applications of the theory are discussed. Principles of fracture-proof design are shown to follow naturally from the tearing instability theory. These principles are illustrated first for simple structures, and then generalized to more complex configurations and loading conditions. Examples include multiple member tension structures, beams, frames, nuclear reactor pressure vessel nozzles and piping, and beams on elastic foundations. It is concluded that J-integral based methods offer the best immediate opportunity for the development of sound analytical techniques for treating important practical problems of elastic-plastic fracture.

NOMENCLATURE

a  crack length
b  uncracked ligament length
d  crack diameter
D  bar diameter
E  modulus of elasticity
F_1, F_2  numerical coefficients
h  distance from the crack tip to the neutral axis
I  moment of inertia of a beam
J  Rice's J-integral
L  length of a member
L_0  half wavelength of a beam on an elastic foundation
M  number of fully plastic tension members (uncracked)
N  total number of tension numbers
R  radius
T  thickness of a member
t  tearing modulus, (E/\sigma_0^2) (dJ/da)
U  potential energy
W  width of a member
\Delta a  change in crack length
\omega  Hutchinson's criterion parameter, (b/J) (dJ/da)
\sigma_0  flow stress

"Submitted to the ASME National Meeting, December 1979, and for publication in ASME"
INTRODUCTION

It is now a little more than ten years since Rice [1] drew attention to the J-integral, which with deformation theory he showed to be path independent for circuits encompassing a crack tip, as well as possessing other useful analytical interpretations. At that time other attempts at formulating elastic-plastic fracture mechanics, such as the C.O.D. concept, did not enjoy widespread acceptance. Moreover, for several years Rice's J-integral was also not recognized as a significant analytical basis for elastic-plastic fracture criteria.

It was about four years later that Begley and Landes [2,3] published their first papers using J as a fracture criterion parameter. Initially the broader significance of the Begley-Landes results was not recognized and they cautiously drew attention to deformation theory assumptions themselves. Indeed, even with the apparent limitations cautiously overstated, their results were a landmark discovery. Later their results were more widely interpreted, understood, and accepted using interceding analyses of Rice [4] and Hutchinson [5], by regarding J as the intensity of the asymptotic crack tip stress-strain field.

The J-integral R-curve was then introduced by Begley and Landes [6] as a refinement of their original results. They measured a material's cracking response to monotonic loading by plotting J vs. crack length change, \( \Delta a \), in order to observe the J-value, \( J_{\text{RC}} \), at which crack extension begins. However, they left doubts on the interpretation which might be made about the crack growth implied unloading which raised questions about the relevance of deformation theory. Again, interceding analytical discoveries by Rice [7] significantly assisted in simplifying their approach, and alternative experimental methods have been developed [8].

Even though some doubt existed at the time on the significance of the crack growth portion of the J-R curve, Landes, Begley and co-workers [9, 10, etc.] produced and published a large amount of data demonstrating its quality for characterizing a material's fracture resistance, even for the crack growth portion of the J-R curve. Further, they applied and showed the J-integral useful in correlating fatigue crack growth data [11, 12] and creep crack growth data [13], which are conditions for which severe violations of deformation theory are expected.

During this period from seven years ago to about three years ago the J-integral based elastic-plastic fracture mechanics gained prominence as the only approach available with a sound and flexible analytical basis. This point and time period is more fully discussed in [14]. Other similar but more empirically based methods were developed principally by Professor C.E. Turner and his co-workers and students at Imperial College of London. Other less analytic methods and others with even more severe shortcomings have more or less lost interest. Attempts to produce a more fundamental basis than the J-based approach are yet to be recognized on the horizon. Consequently, since about three years ago, the J-basis approach has remained most ripe for development.

Moreover, until recently the J-integral methods had been principally employed in testing methods, rather than structural analysis, in the United States (although Turner's attention with similar methods was more balanced). Indeed, for ductile fracture (Charpy upper shelf) no one had developed a practical crack stability criterion for elastic-plastic conditions.
TEARING INSTABILITY THEORY

Approximately two years ago an approach to crack stability under ductile (not cleavage) plane strain (but also plane stress) conditions, called "tearing instability theory", was developed [5]. Though the initial approach was simplified by adopting elastic-perfectly-plastic material and fully plastic uncracked ligaments, it can be explained in most general terms as follows.

The J-integral and its deformation theory assumptions virtually presume that a non-linear elastic pseudo-system exists for the real elastic-plastic system, where the monotonic loading stress strain curves are identical, and providing proportional straining occurs. Now J for a crack tip in both systems are identical, but for the non-linear elastic pseudo-system, Rice has shown

\[ J = \frac{dU}{da} \]  

where U is the potential energy of the system. Thus J may be thought of as the applied J or imposed field of stresses and strains in the crack tip region. On the other hand, the J-integral R-curve gives the J value a material may sustain at a crack for a given amount of crack growth. With this interpretation, equilibrium at a crack tip is expressed by

\[ J_{\text{applied}} = J_{\text{material}} \quad \text{(equilibrium)} \]  

The stability of that equilibrium with regard to crack growth can be phrased in terms of the second derivative of potential energy, U, for the pseudo-system, thus

\[ \frac{d^2U}{da^2} = \frac{dJ_{\text{applied}}}{da} > \frac{dJ_{\text{material}}}{da} \quad \text{(unstable)} \]  

Defining for other convenience a non-dimensional "tearing modulus" T by

\[ T = \frac{E}{\sigma_0} \frac{dJ}{da} \]  

where E is elastic modulus and \( \sigma_0 \) is flow stress. Then the stability criterion, Equation (3) can be re-expressed as:

\[ T_{\text{applied}} \geq T_{\text{material}} \quad \text{(unstable)} \]  

Hutchinson [16] observed that for this analysis to be valid the asymptotic stress-strain field surrounding the crack tip must be J-controlled. He noted that J-control with its deformation theory basis will appropriately exist if essentially proportional straining exists in the asymptotic field and outside it (not necessarily right at the crack tip) as crack extension occurs. By obtaining the differential of strains implied due to simultaneous increments of J/loading and crack growth, \( \Delta a \), he observed that sufficiently proportional straining occurs if

\[ \omega = \frac{b}{J} \frac{dJ}{da} \gg 1 \]  

where b is the width of the uncracked ligament. This condition, Equation (6), not only justifies the use of Equations (3) or (5) as stability criteria but also qualifies the crack growth portion of the J-R curve as relevant. J_{material} vs. \( \Delta a \), material characterization. This is to say that \( dJ_{\text{mat}}/da \) is the slope of a
proper J-integral R-curve. Hence, the formal basis of the "tearing instability theory" is fully rigorous and amounts to strict application of deformation theory in cases where it gives results identical to incremental theory.

FIRST APPLICATIONS OF TEARING INSTABILITY THEORY

It is informative to apply tearing instability theory on an approximate basis in order to understand its broad structural implications in simplest possible terms. For that reason, a first order approach is adopted here.

Consider the example of a center cracked strip of width \( W \) and length \( L \), which is long compared to the width or thickness. Presume that the strip is stretched in displacement control in a rigid test machine until the uncracked ligaments adjacent to the crack tips are fully plastic, i.e. at limit load, with non-hardening material. Instability can be described in the following scenario, as in [15]:

1. If the crack should grow by an increment, the fully plastic net section is reduced in area so the load must drop.
2. If the load drops, the elastic stretch in the strip diminishes.
3. Since the test machine is rigid with fixed grips, the plastic stretch in the strip must increase to make the net stretch zero.
4. An increase in the plastic stretch implies an increase in \( J \) at the crack tip which implies further crack growth.
5. If the originally applied crack growth causing the induced increase in \( J \) is not exceeded by that for the material's J-R curve, instability ensues.

In other words, it is the global elastic relaxation of deformation (including the loading system if it is not rigid) that drives ductile fracture instability under fully plastic conditions. Putting simple equations to the scenario for the center cracked strip, then as in [15]:

\[
T_{\text{applied}} = \frac{E}{\rho_0^2} \frac{dJ_{\text{applied}}}{da} = \frac{2L}{W} T_{\text{material (unstable)}} \tag{7}
\]

Zahoor [17] has developed a full elastic-plastic power hardening tearing instability analysis of the center cracked strip which reduces to Equation (7) under appropriate assumptions. Hence, Equation (7) has been shown to give the dominant features leading to instability under fully plastic conditions. Notice that in \( T_{\text{applied}} \), the crack length is absent and the gross geometric proportion, \( L/W \) dominates. This is a clue that fully-plastic or ductile fracture instability is very different than linear-elastic fracture mechanics, and indeed it is actually much simpler when familiarity is gained.

Next, consider a first example of tearing instability theory in fully plastic bending; that is, using the same scenario as above, but for flexure instead of extension. In particular, take the example of a 3-point bend test specimen, i.e. a simple span of length \( L \), cracked at the center section from one edge leaving a ligament, \( b \), and having a rectangular section of depth \( W \). Presume flexural (transverse) displacement is imposed at the center (opposite the crack) until the cracked section is fully plastic. A simple flexural analysis similar to the scenario, as in [15 or 16], for
the 3-point bend specimen leads to:

\[ T_{\text{applied}} = \frac{2b^2L}{W^2} - \frac{J}{b_0^2} \geq T_{\text{material}} \quad \text{(unstable)} \quad (8) \]

The first term in \( T_{\text{applied}} \) dominates for 3-point bending where conditions for J-controlled growth apply, i.e. \( J \gg 1 \). (Note that from equation (6) and Equation (4) that the second term \( T_{\text{applied}} \) must be small compared to \( T_{\text{material}} \).) Considering that \( 2b^2/W^2 \) in \( T_{\text{applied}} \) is of the order of 1, then again it is the gross proportion, \( L/W \), which drives instability. However, here as the crack grows, or \( b \) diminishes, the situation tends toward stability. Again, the situation is unlike linear-elastic fracture mechanics.

A COMMENT ON THE INFLUENCE OF TYPICAL MATERIAL PROPERTIES

Typical pressure vessels, nuclear reactor systems and civil engineering materials examined to date have \( T_{\text{material}} \)-numbers, as computed from the initial crack growth slopes of J-R curves, ranging normally from 100 to well over 200. Limited tests indicate some low Charpy upper shelf energy turbine rotor steels and severely irradiation damaged high copper steels might have \( T_{\text{material}} \)-numbers below 30. On the other hand, some very tough stainless steels exhibit \( T_{\text{material}} \) numbers in excess of 400. Moreover, some high strength steels and relatively brittle non-ferrous alloys may have \( T_{\text{material}} \) numbers as low as 1. Reference [15] contains an early table of \( T_{\text{material}} \)-numbers for some typical materials.

Re-examination of Equations (7) and (8) for normally proportioned test specimens it is readily seen why tearing instability is observed in low toughness materials, but that for low to medium strength steel in the ductile range well above transition temperature tearing instability normally never occurs unless very severe damage is done to the material (like a very bad weld, etc.).

Typical fracture mechanics tests use compact specimens, which are principally of the bending type with the equivalent of small \( L/W \) (of the order of 1). If tested in a stiff machine in displacement control, it can be noted from Equation (8) that there is a strong tendency to remain stable even for very low \( T_{\text{material}} \). This is desirable in a test where the J-R curve is sought, so that instability does not intervene and complicate the test. However, the lack of instability in such a test, or even in a 3-point bend test, does not imply necessarily that stability will be the case in a structural application of that material. The actual structural proportions, loading system constraint and structural redundancies (multiple load paths) must be considered, and tearing instability theory is a quantitative method of making such assessments.

THE BEGINNINGS OF FRACTURE PROOF DESIGN

For structural application of tearing instability theory it has been noted in some earlier work as in Equations (7) and (8) that gross structural proportions play an important role in inhibiting crack instability under fully plastic conditions. Moreover, more recently the internal or external statically indeterminate nature of most structures and components also have been seen to play a key role.

The first approach was made in analysis of the tearing instability possibilities in cracked pressure vessel nozzles [18]. It was assumed that the pressure would be limited by various controls and safety systems, and therefore the failure mode was regarded to
be a nozzle crack from the interior surface growing due to stress corrosion or fatigue to a point where full plasticity ensues on the remaining ligament. At that point, and for further growth, crack stability was examined using tearing instability theory. The initial analysis was made removing the nozzle from the constraining effects of the vessel to treat it as an elastic ring, except at the fully plastic cracked section with the crack propagation radially throughout the length of the nozzle and with pressure on the crack surfaces as well as the interior surface. The results for \( T_{\text{applied}} \) turn out to be most dominantly dependent on the radius ratio of the nozzle [18, Figure 15], analogous to the L/W influence for tension and bending cited earlier herein. Therefore, for nozzles with a low ratio of inside radius to outside radius, and for typical pressure vessel steel material properties, it was shown that sudden rupture of the nozzle could not occur even with cracks extending almost through the wall. However, the radius ratios were below the typical or practical range, so the next step was to reintroduce only the partial elastic constraining effect of the vessel wall by a simplified but conservative finite element model. For typical nuclear vessel nozzle this analysis showed that with a crack almost through the nozzle all along its length, \( T_{\text{applied}} \) was significantly less than 100 whereas nuclear nozzle materials exhibit material typically in excess of 150. Under these conditions the nozzle must leak substantially long before any rapid rupture could possibly occur. It was realized that nozzle and vessel wall combinations could be more deliberately proportioned along with material property selection to produce a fracture proof design.

A second example arose with concern about main nuclear piping systems of stainless steel prone to the development of circumferential stress corrosion cracks. In one case, a through crack about 90° around the circumference was sustained. Tearing instability theory was applied [19] to the generic circumstances in order to assess the potential danger of the possibility of sudden failure due to extreme loads, such as very severe earthquake.

A difficulty in performing such an analysis was encountered because no simple method was found for realistically addressing the possible magnitude of the loads. However, this was resolved by assuming the loads were large enough to bring the piping to the verge of plastic collapse without a crack present. Since A.S.M.E. codes are supposed to be adequate to insure against such severe conditions, if a crack is stable under such conditions, then a plastic collapse problem, not a crack problem, controls.

The results of the study showed that for the worst loading and end fixity conditions that, as in [19]:

\[
T_{\text{applied}} = F_1 \frac{L}{R} + F_2 \frac{JE}{\sigma_0} \frac{z}{R} \quad (9)
\]

where \( F_1 \) and \( F_2 \) are numerical coefficients whose maximum values are 1.3 and 0.5, respectively, but where \( F_2 \) becomes negative for a through crack of more than about 60° around the circumference of the pipe. As with Equation (8), the dominant influence on \( T_{\text{applied}} \) for the pipe in Equation (9)

For the particular generic problem examined, the usual L/R was about 40, and the maximum relevant \( T_{\text{applied}} \) was about 50, whereas typical of stainless steel are material values well over 200, so stability is assured by a wide margin. Thus it was directly shown that plastic collapse of the pipe would necessarily ensue before
crack instability, hence proportions and material properties again assured a fracture proof design.

These two examples led to consideration of a more general approach to fracture proof design conditions for structures. Some of these considerations were presented in an earlier discussion[20].

THE FRACTURE PROOF DESIGN CONCEPT

A fracture proof design analysis procedure is outlined as follows:

(1) Consider a statically indeterminate structure with a cracked member.

(2) Assume the structure is loaded so that the cracked section becomes fully plastic (other sections may become fully plastic just short of forming a collapse mechanism).

(3a) Perform a tearing instability analysis to show that sudden unstable cracking cannot take place.

or

(3b) Reproportion the design, add statically indeterminate constraints or stiffness, and/or select materials with higher material behavior in order to prevent the possibility of sudden unstable cracking.

If this procedure is followed for all potentially worst crack locations, the resulting design will be fracture proof.

It is noted that analyzing loading the structure to levels where the cracked section becomes fully plastic (and especially where other sections become fully plastic) may be very conservative. That may be mitigated by assuming lesser loads short of fully plastic conditions but with a substantial loss in both safety margin and analysis simplicity, if required. However, it is presumed that the fracture proof design approach is mainly applicable to situations where safety is the largest concern and the use of lower-strength high toughness materials (and Charpy upper shelf conditions) are already required. For such cases, the fracture proof design concept is called upon to quantitatively assure safety against crack instability.

SOME EXAMPLES OF ANALYSIS RESULTS FOR $T_{applied}$

To date, the principal general analysis results are parallel tension configurations or for statically indeterminate bending [10]. The piping analysis mentioned previously is in the bending category, but the nozzle analysis herein and surface flaw analysis in [15] are other types of examples of more complicated indeterminate conditions.

It is of interest first to cite results for parallel tension members constrained to deform to equal extension over some characteristic length, $L$, and subjected together to a total dead load. Cables, multiple I-bars, some arrangements of heat exchanger tubes and some composite materials behave in this fashion. If there are $n$ members of equal elastic stiffness and one is cracked and fully plastic at the cracked section, and if $m$ other members have become fully plastic, simple tearing instability analysis gives:

$$T_{applied} = \frac{(n-m)}{(n-1-m)} \frac{L}{W}$$  \hspace{1cm} (10)

This result assumes a single edge crack in a rectangular section. An additional factor 2 appears for a center cracked strip, so that as $n-m \rightarrow 0$, Equation (7) is recovered. A deep double edge notched strip tends toward an additional factor of 6 due to constraint.
elevating the apparent yield strength by a factor of 3 and a velocity field which magnifies the effect of plastic displacement at the crack tip by a factor of 2. For an internally cracked round bar L/W is replaced by 2d/D² where d is the crack diameter and D is the bar diameter, and again for an external rather than internal crack all around, a factor of 6 ensues for deep cracks. For a partial thickness crack around the inside or outside of a tube, L/D is replaced by L/t for thin tubes where t is the thickness. Moreover, n-m can be regarded as the effective elastic stiffness of uncracked elastic bars or alternate load paths compared to the cracked bar. The relative loading system stiffness, if any, can be included in this factor as additional equivalent load path.

Therefore, Equation (10) with appropriate simple adjustments can handle many tension problems. Bending is a case of equally sweeping simplifications.

Consider first a fixed ended beam of length L; moment of inertia I, with a fully plastic cracked section where t is the section thickness at the crack tip (crack front length) and h is the distance from the crack tip to the neutral axis at the cracked section. It is presumed that dead loads still applied cause the cracked section to be fully plastic. Now, by assuming an increment of crack growth which changes the limit moment at the cracked section, a tearing instability analysis leads to

\[ T_{\text{applied}} = \frac{h^2 t L}{I} \quad (11) \]

If the cracked section is in the central portion of the beam (between quarter points) loading to full form of an additional plastic hinge at the nearest end (most likely) magnifies this result by a factor of 1.3 to 5, depending on the crack location. However, if the cracked section is near the end of the beam, additional hinge formation actually reduces the result in Equation (11).

Further, if instead of fixed ends, the beam is in a two legged frame with equal column to beam stiffness and with pinned or fixed bases (and no side sway), Equation (11) should be increased by a factor of 1.67 or about 1.55 respectively. A full (four member) square frame is between these results. Adding other members into the frame increasing its stiffness drives the factor back toward 1. Grillages of beams, 3-D frames with torsion, etc. are more complicated examples but similarly can be handled and simplified.

The combination of h, t, and I in Equation (11) are a cross section property, which upon selecting a particular cross section, reduces to a numerical factor of the order of 1 divided by a larger characteristic length of the section. Thus, for a pipe, \( T_{\text{applied}} \) tends to the first or dominant term in Equation (9). The similarity with (8) is even more directly observed.

The beam on elastic foundation leads to a similar result. In this case

\[ T_{\text{applied}} = \frac{2}{\pi} \frac{h^2 t L'}{I} \quad (12) \]

where \( L' \) is the peak to trough distance in the waves of beam at sections free of loads other than the elastic support. That is to say

\[ L' = \pi \frac{4E\varepsilon}{k} \]

where \( k \) is the elastic support stiffness in force per unit length per unit deflection. Rails and long pipes on closely spaced
elastic supports seem to be among relevant applications of this result.

FINAL COMMENTS ON PROGRESS

J-integral based fracture mechanics, because it is an analytically based method, has led to tearing instability theory and fracture proof design approaches all within the past two years. It remains the most appealing approach for rapid progress in the near future, especially in the area of practical application to quantitative fracture safety analysis of structures with ductile materials.

ACKNOWLEDGEMENTS

This manuscript as well as much of the work on fracture proof design concepts was supported by a National Science Foundation Grant ENG-77-20937.

Other portions of the work were supported under contracts with the U.S. Nuclear Regulatory Commission. The NRC contracts include NRC-03-77-029, NRC-04-78-227, NRC-03-78-135, and NRC-03-79-134, with Washington University and Washington University Technology Associates, Inc. The support is gratefully acknowledged. The encouragement and many useful discussions with colleagues at the U.S.N.R.C. and the Center for Fracture Mechanics at Washington University are also acknowledged with thanks.
REFERENCES


SUMMARY

Experimental results of burst tests performed in Cadarache Research Centre on EDITH facility, through an agreement between CEA and FRAMATOME, are presented; it enables us to determine critical flaw lengths for zero power conditions ($p=7\text{MPa}$) and operating conditions ($p=5.3\text{MPa}$). Break is created in a very short time with a pyrotechnical system on the pressurized pipe in ferritic steel.

A semi-empirical fracture mechanics analysis based on a two-criteria approach was applied to our experimental results. Foundations and limitations of this approach which has already been used in industrial applications, are discussed.

The two following criteria have been studied:

1. a toughness criterion for brittle behaviour

   $$K_c = M \sigma \sqrt{a} \quad (1)$$

2. a plastic flow stress criterion for ductile behaviour

   $$\bar{\sigma} = M \sigma$$

They correspond to the limiting behaviours of the cracked structure. Shape factor $M$ is related to curved shell geometry, $\sigma$ is the hoop stress and $a$ the crack half-length. When applied to our results, criteria (1) and (2) permit us to determine $K_c$ and $\bar{\sigma}$ values; consistency of $\bar{\sigma}$ values confirms validity of ductile tear criterion; predictions are made for other pressure levels, by using criterion (2).

Conclusions are drawn on this simplified approach and scaling effects are discussed.
1. INTRODUCTION

The experimental programme performed in Cadarache Research Centre on EDITH facility through an agreement between CEA and FRAMATOME is directed to studying mechanical behaviour of P.W.R. secondary piping in the reduced scale (1/10), after a break opening corresponding to ANSI Standards [1]. Some of these experiments were carried out with axial splits created by a pyrotechnical system. Levels of load correspond to zero power (p = 7 MPa) or normal operating (p = 5.3 MPa) conditions of secondary piping.

The major aim of this programme is to get a better knowledge of global behaviour of piping and associated structures from a technological point of view.

However we thought it was interesting to give an explanation of stability of longitudinal breaks based on a very simple fracture mechanics analysis:

Results of the whole EDITH programme were recently presented in the fifth SMIRT Conference (Berlin, August 79) [2].

2. EXPERIMENTAL PROCEDURE AND RESULTS

A longitudinal break is created with a pyrotechnical system in a very short time (lower than 0.5 ms) on the pressurized pipe. Flaw behaviour depends on two parameters : internal pressure (p) and flaw length (l = 2a).

Two situations are observed :

a) break remains stationary, opening area is small and pressure decrease is slow,

b) break propagates, opening surface is large and pressure drops rapidly.

Pipe geometry is as follows :

<table>
<thead>
<tr>
<th>Thickness</th>
<th>e = 3 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean radius</td>
<td>R = 39.15 mm</td>
</tr>
</tbody>
</table>

Large value of ratio R/e = 13 enables us to make thin shells assumptions in the analysis.
Very little information on mechanical properties and metallurgical structure of material is presently available.

All the experimental data we have on this ferritic steel, come from a uniaxial tensile test performed on a traction specimen (at T = 300 °C) taken in the longitudinal sense of the tube. We obtained the following estimations for yield stress ($\sigma_Y$), ultimate stress ($\sigma_U$) and strain ($\epsilon_U$):

\[
\begin{align*}
\sigma_Y &= 315 \text{ MPa} \\
\sigma_U &= 695 \text{ MPa} \\
\epsilon_U &= 12 \% 
\end{align*}
\]

We cannot account for material anisotropy which certainly exists because data on mechanical properties in the transverse sense are lacking.

Tests were conducted at $T = 280$ °C and gave the following results on flaw behaviour.

<table>
<thead>
<tr>
<th>$p$ (MPa)</th>
<th>$a$ (mm)</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>55</td>
<td>NO</td>
</tr>
<tr>
<td>7.0</td>
<td>45</td>
<td>YES</td>
</tr>
<tr>
<td>5.3</td>
<td>75</td>
<td>NO</td>
</tr>
<tr>
<td>5.3</td>
<td>65</td>
<td>YES</td>
</tr>
</tbody>
</table>

Approximate values of critical flaw length can be calculated by taking an average value in each case and it gives:

<table>
<thead>
<tr>
<th>$p$ (Mpa)</th>
<th>$a_c$ (mm)</th>
<th>$a_c/R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>50</td>
<td>1.3</td>
</tr>
<tr>
<td>5.3</td>
<td>70</td>
<td>1.8</td>
</tr>
</tbody>
</table>

When flaw propagates, fracture surfaces are slanted at 45 ° to the shell surface: such an appearance characterizes a plane stress situation.
We decided to use a very simple approach to interpret the previous results. We think it is not worth performing expensive finite elements calculations and using elaborate criteria coming from elastic-plastic fracture mechanics theories, for several reasons:

a) we have insufficient data on material properties,

b) experimental results are global (flaw is stable or not) and do not permit us to study stable crack growth which occurs between initiation and instability,

c) specific effects (due to experimental procedure) cannot be accounted for, as thermal and mechanical shock of pyrotechnical opening,

d) created flaw has a rough geometry: width (~2 mm), ends of the notch,

e) critical lengths are not known with a good precision (~10 %)

We will use a semi-empirical two-criteria method which has already given good results for predicting crack initiation on slowly pressurized pipes [3,4]. We think this method can be applied to our tests although experimental procedure is quite different.

3. PRESENTATION OF CRITERIA

The idea is to consider two limiting behaviours of the cracked structure:

a) brittle fracture governed by crack tip events which can be modelled with linear elastic fracture mechanics theories,

b) ductile tearing governed by plastic flow and ultimate collapse.

These two limiting cases can be connected by an empirical transition curve [5].

This approach has the advantage of by-passing EPFM theories.
3.1 - Brittle fracture

LEFM theories, applied to thin shell geometries, were developed by Folias, Erdogan and Kibler and more recently by Krenk [4].

Stress intensity factor K (which characterizes singular field at the crack tip in the opening mode) is given by:

\[ K = M \frac{PR}{e} \sqrt{\pi a} \]

\( \sigma = \frac{PR}{e} \) is the membrane stress of the pipe.

\( M \) is a shape factor introduced to account for magnification of stresses with respect to the infinite flat plate (usually taken as a reference case); in thin cylinders under pressure with an axial crack, shape factor is due to bulging effects.

Tabulations of \( M \) (versus the shell parameter \( \lambda \)) can be found in the literature:

\[ \lambda = \left[ \frac{12(1-\nu^2)}{12(1-\nu^2)} \right] 0.25 \frac{a}{\sqrt{Re}} \]

For applications, we used Krenk’s tabulation [4] (see also figure 1) and obtained the following values of \( M \), at critical conditions:

<table>
<thead>
<tr>
<th>a_c (mm)</th>
<th>( \lambda )</th>
<th>( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>8.34</td>
<td>4.63</td>
</tr>
<tr>
<td>70</td>
<td>11.67</td>
<td>6.15</td>
</tr>
</tbody>
</table>

Failure occurs when factor \( K \) reaches a critical value (\( K_c \)) called toughness. Criterion of brittle fracture can be written:

\[ K_c = M \frac{PR}{e} \sqrt{\pi a} \quad (1) \]
Equation (1) gives a relation between flaw length and pressure at critical conditions, depending on Kc value.

3.2 - Ductile tearing

In order to extend applicability of the LEFM formulation to cases in which large scale yielding is present at the crack tip, real crack length a is replaced by a fictitious crack length \( \psi a \) where \( \psi > 1 \) is a plasticity correction factor.

In the reference case (infinite flat plate) various simplified plasticity models have been developed which lead to expressions of \( \psi \) versus \( \sigma / \bar{\sigma} \), where \( \bar{\sigma} \) is the mean plastic flow stress (its value lies between \( \sigma_y \) and \( \sigma_u \)); but all these models are based on physically incorrect assumptions and discrimination of their respective merits must be based on experimental correlations involving various geometries and materials [4].

Plastic instability condition corresponds to an infinite value of \( \psi \), i.e. \( \sigma = \bar{\sigma} \). For other geometries, generally adopted procedure is to consider an equivalent flat plate of same material, thickness and crack length but with a nominal stress \( \sigma_{eq} \) equal to \( M \sigma \), where \( M \) is the LEFM shape factor. For a thin cylinder under pressure, plastic instability condition (taken as ductile tearing criterion) becomes:

\[
\bar{\sigma} = M \frac{DR}{e} \tag{2}
\]

Equation (2) gives a relation between flaw length and pressure at critical conditions, depending on \( \bar{\sigma} \) value.

3.3 - Transition regime

Various formulas, which involve simultaneously toughness (Kc) and flow stress (\( \bar{\sigma} \)) have been proposed to describe transition between brittle fracture and ultimate collapse [4].
The usually accepted conditions of validity of

\[ \frac{MP}{e} < 0.6 \bar{\sigma} \]

a toughness based criterion is:

\[ Kc > 1.25 \bar{\sigma} \sqrt{\pi} a \]

a flow stress based criterion is:

3. APPLICATION TO EXPERIMENTAL RESULTS

Criteria (1) and (2) when applied to experimental critical conditions lead to following values of \( Kc \) and \( \bar{\sigma} \).

<table>
<thead>
<tr>
<th>( p ) (MPa)</th>
<th>( ac ) (mm)</th>
<th>( Kc ) (MPa(\sqrt{\pi} )) equation (1)</th>
<th>( \bar{\sigma} ) (MPa) equation (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>50</td>
<td>168</td>
<td>423</td>
</tr>
<tr>
<td>5.3</td>
<td>70</td>
<td>199</td>
<td>425</td>
</tr>
</tbody>
</table>

Criterion (1) gives two different values of toughness, whereas criterion (2) leads to a single value for the mean plastic flow stress:

\[ \bar{\sigma} \approx 424 \text{ Mpa} \]

An empirical correlation formula between flow stress (\( \bar{\sigma} \)) deduced from experiments and tensile properties (\( \sigma_Y, \sigma_U \)) in the longitudinal sense, can be used:

\[ \sigma = \alpha \sigma_Y + (1 - \alpha) \sigma_U \]

and leads to \( \alpha = 0.71 \)

Same result was obtained by Larsson and Bernard [4], from tests performed on AISI 304 stainless steel tubes at room temperature.
Condition of validity of ductile tear criterion (2) gives a lower bound of $K_c$

\[ K_c > 250 \text{ MPa}\sqrt{m} \]

Such a high toughness can be attributed to the small thickness of the pipe ($e = 3 \text{ mm}$); one could probably determine this quantity with tests on flat specimens (same thickness) based on equivalent energy method proposed by Witt [6].

Predictions made with criterion (2) ($\tilde{\sigma} = 424 \text{ MPa}$) have been plotted on figure 2. It must be noticed that ductile tear criterion obeys the laws of geometric similitude whereas criterion (1) does not.

It is not sure that failure in full-scale experiments is uniquely governed by plastic flow properties: toughness decreases with increasing thickness and brittleness could play a role.

4. CONCLUSIONS

In spite of specific test conditions (pyrotechnical system,...) and insufficient data on material properties, it is possible to use a two-criteria approach to predict condition of stability for a longitudinal break suddenly appearing on a pressurized pipe. Application of ductile tear criterion leads to a consistent value of mean plastic flow stress $\tilde{\sigma}$, which can be correlated to tensile properties in a simple way. This criterion can be used to predict critical conditions for various pressure levels.

Present safety assumptions and design criteria assume that a long stationary crack ($l = 4R$) may exist [1], resulting in prediction of large opening areas and fluid forces on piping; our test results and analysis show that such a long crack is unstable and propagates.

Similar conclusions have been drawn from studies on primary piping [7]. Ayres, with a finite elements dynamic elastoplastic analysis and a criterion based on $J$-integral, shows that $R < 1_c < 1.5 \text{ R}$ for normal operating conditions.
REFERENCES

"Design basis for protection of nuclear power plants against effects of pos- 
tulated pipe rupture".

J.L. CHEISSOUX - J.L. GARCIA - A. MARTIN 
5th International Conference on SMIRT (Berlin. August 1979) Vol. F n° F6/3

[3] "Ductile fracture initiation, propagation and arrest in cylindrical vessels" 
J.F. KIEFNER, W.A. MAXEY, R.J. EIBER, A.R. DUFFY 
ASTM STP 514 (1972)

[4] "Fracture of longitudinally cracked ductile tubes". 
H. LARSSON, J. BERNARD 
4th International Conference on SMIRT (San Francisco, August 1977) 
vol. F n° F7/3

A.R. DOWLING, C.H.A. TOWNLEY 
International Journal of Pressure vessels and Piping (3) (1975)

[6] "Experimental verification of lower bound K_{1c} values utilizing the equivalent 
energy concept" 
C. BUCHALET, T.R. MAGER 
ASTM STP 536

[7] "Determination of the largest stable suddenly appearing axial and circumferen-
tial through cracks in ductile pressurized pipe" 
D.J. AYRES 
4th international Conference on SMIRT (San Francisco, August 1977)vol.F n° F7/1
CURVATURE CORRECTION FACTOR FOR A CYLINDRICAL SHELL WITH AN AXIAL CRACK \((v = \frac{1}{3})\)

\[
\lambda = \left[ \frac{a}{2(1-v^2)} \right]^{0.25} \frac{a}{\sqrt{Re}}
\]

- \(R\) Mean radius
- \(e\) Thickness
- \(2a\) Crack length
CRITICAL FLAW LENGTH VERSUS PRESSURE

Criterion of ductile tear \[ \bar{\sigma} = 424 \text{ MPa} \]

Experimental results

\[ \bar{\sigma} = M \frac{R}{\varepsilon} \]

Longitudinal crack
AN ANALYSIS OF DUCTILE CRACK EXTENSION
IN BWR FEEDWATER NOZZLES

Research Project 1241-1

October 1978

Prepared by
B.A. Szabo
G.G. Musicco
M.P. Rossow

Center for Computational Mechanics
Washington University
St. Louis, Missouri 63130

Prepared for
Electric Power Research Institute
3412 Hillview Avenue
Palo Alto, California 94304
T.U. Marston, Project Manager
ABSTRACT

The feedwater nozzles of most operating boiling water reactor (BWR) systems have experienced thermal fatigue cracking due to feedwater bypassing the thermal sleeves of the nozzles. Generally these cracks are small and seldom penetrate the clad. However, fatigue analyses have indicated that if the bypass flow is not checked, then the thermal stresses due to startup and shutdown are sufficient to grow the crack to significant depths. There is some concern that if an overpressurization event occurs in a reactor, with a deeply cracked nozzle, unstable fracture might occur.

In this report the stability of ductile crack extension in deeply cracked BWR feedwater nozzles is examined through analysis of a conservatively idealized two-dimensional elastic-plastic model. The assumed crack length to nozzle thickness ratio was 0.95 and bilinear stress-strain relationships were used. The results indicate that Paris' stability criterion predicts stable crack growth for certain ranges of pressure and strain hardening parameter values. The model was not sufficiently detailed to permit establishment of close bounds on parameter values that would guarantee stable crack growth on the basis of the criterion applied.
1.0 INTRODUCTION

Current code provisions for flaw tolerance analyses of nuclear pressure vessels are contained in the ASME Boiler and Pressure Vessel Code in Sections III and XI. These provisions of the code are based on the concepts of linear elastic fracture mechanics and therefore imply the assumption that material yielding is confined to a small region near the crack tip and the yielded region is completely surrounded by unyielded (elastic) material. In a number of important practical situations, however, which include the case of crack extension in deeply cracked BWR feedwater nozzles, the size of the plastic region can be quite large in relation to the crack size and may not be contained entirely within a region of elastic material. Consequently, the methods of linear elastic fracture mechanics cannot be realistically applied.

Very substantial programs of research, concerned with ductile fracture, are currently underway. A survey cannot be attempted here; it is merely noted that progress is being made in three general areas:

(i) development of engineering theories of ductile fracture;
(ii) development of analytical methods required for modeling ductile fracture processes;
(iii) experimental determination of material properties that characterize ductile fracture.

A review document, summarizing ongoing projects sponsored by the Electric Power Research Institute in these areas, was released early in 1978 [1].

The work described in the present report was concerned with the application of a stability theory of ductile fracture, developed by
Paris and his associates [2,3], to a conservatively idealized, deeply cracked BWR feedwater nozzle.

The rationale in deciding on the scope and methods of this analysis was as follows: the present state of art in numerical stress analysis is such that three-dimensional elastic-plastic analyses are extremely expensive and time consuming. Thus it would not be feasible to carry out exploratory parametric studies on the basis of three-dimensional models. A conservative two-dimensional idealization on the other hand can be represented by a manageable model, which is suitable for parametric studies and, furthermore, if stability were to be indicated by such a model within the entire range of parameter values of interest and within the range of validity of the stability criterion employed, then it could be reasonably concluded that the prototype is stable and more detailed analyses would not be necessary. Of course, given the conservative nature of our model, indications of instability for any set of parameter values cannot be interpreted as conclusive evidence of instability in the prototype.

The main objectives of the project were as follows:

(i) to investigate the accuracy of strip yield models as applied to stability analyses of deeply cracked rings. Such models were employed by Tada, Musicco and Paris in their preliminary analysis of the stability of deeply cracked BWR feedwater nozzles [4].

(ii) to study the effects of internal pressure and material properties of the nozzle on the stability of ductile crack extension.
It was not within the scope of this project to carry out detailed analyses of the structural interaction between cracked nozzles and pressure vessels. In lieu of such analyses, the magnitude of membrane forces imposed by the pressure vessel on the nozzle was consistently overestimated. This point is discussed in some detail in the Appendix.

2.0 STABILITY CRITERION

The stability criterion proposed by Paris and his associates [2] for J-controlled crack growth is based on two observations: first, the resistance of ductile materials to small amounts of crack growth, induced by monotonically increasing loads, is adequately characterized by material dependent "J-resistance curves" or "J-integral R-curves". Second, the response to small amounts of crack extension, under constant load, is a change in J.

An idealized J-resistance curve is shown in Fig. 1. Let us assume that a crack length of \( a_0 \) exists prior to loading. Then, as the loading (and therefore J) increases, this crack first blunts which results in "some apparent growth from local opening or slipping-off processes" [5]. When a critical value of J, \( J_{IC} \), is exceeded then crack propagation takes place. We note that for any given J value, \( J_B \), there will be an equilibrium crack length, \( a_B \), corresponding to crack growth \( a_B - a_0 \).

Let us now assume that the structural system under consideration (which contains crack \( a_B \)), is subjected to constant load. Some external agency, such as corrosion or local stress transient, extends the crack by a small amount \( \Delta a_B \). Then, there will be a corresponding change in J.
Specifically, the change in J will be:

\[ J(a_b + \Delta a_b) - J(a_b) = \left( \frac{dJ}{da} \right)_{L} \cdot \Delta a_b \]

The subscript L indicates constant load conditions. The significant fact is that \( \frac{dJ}{da} \) under constant load is a computable quantity which depends on the loading, the geometric parameters and the material properties (constitutive laws) of the system under consideration. Paris' stability criterion states that a crack is stable, neutrally stable or unstable, depending on whether \( dJ/da \) under constant load is less than, equal to or greater than \( dJ/da \) under monotonically increasing load, which is a measurable material parameter.

The criterion is applied as follows. A dimensionless quantity, called the "tearing modulus" is defined:

\[ T = \frac{E}{\sigma_0^2} \left( \frac{dJ}{da} \right) \]

In this equation E is the modulus of elasticity, \( \sigma_0 \) is the flow stress of the material. The values of T under monotonically increasing load are material parameters which turn out to be substantially temperature-independent as well [2]. These are designated as \( T_{\text{mat}} \). The value of T under constant load is a parameter of the structural system and is variously designated either as \( T_{\text{syst}} \) or \( T_{\text{appl}} \). Stability analyses consist of computing \( T_{\text{appl}} \) and comparing it with \( T_{\text{mat}} \). If \( T_{\text{appl}} < T_{\text{mat}} \) then the structural system is considered to be stable. The method provides a rational basis for assessing the susceptibility of cracked structural components to unstable crack growth in the class of problems in
which large scale yielding occurs at the crack front. The validity of the method has been established for those cases in which the yield zone can be closely approximated by the deformation theory of plasticity everywhere except within a small, contained zone near the crack tip [3]. Establishing precise limits for the range of applicability of the method is the subject of current research.

From the computational point of view, obtaining accurate estimates of \( T_{\text{appl}} \) poses a more difficult problem than estimating \( J \). The reason is that \( dJ/da \) can be a very sensitive function of the relative compliances of interacting structural components. In the present study, for example, it was observed that \( T_{\text{appl}} \) depends strongly on the relative compliances of the cracked feedwater nozzle and the pressure vessel. Thus, realistic modeling of structural compliances is required in computing \( T_{\text{appl}} \). A similar observation was made by Zahoor in studying the stability of ductile crack extension in center-cracked panels [6].

3.0 PREVIOUS WORK

The first and only previous application of Paris' stability criterion to nozzle cracks in nuclear pressure vessels was by Tada, Musicco and Paris [4]. The objective of their analysis was to identify key parameters, establish trends and lay groundwork for the construction of more detailed nozzle stability models.

The cross-section of a deeply cracked BWR feedwater nozzle is shown in Fig. 2. The two-dimensional idealization of the nozzle, considered in reference [4] and in the present report, is shown in Fig. 3. Specifically, the nozzle was idealized as a cracked ring, subjected to internal
pressure $p$. The structural interaction between the nozzle and pressure vessel was simulated by a radial array of springs whose parameters were chosen to represent the corresponding stiffness of the pressure vessel. An initial displacement was imposed on the springs to simulate the membrane forces on the nozzle. The details are presented in the Appendix. The method of analysis neglects any constraining effect from the tangential stresses between the cracked nozzle and pressure vessel and only crudely approximates the effect of differences in the stiffness parameters of the cracked and uncracked nozzles on the distribution of membrane forces in the pressure vessel.

Because the cracked ring has an axis of symmetry, only one-half of the ring had to be modeled. Eight-node elastic, plane strain isoparametric finite elements of the STRUDL II computer program were used. The boundary conditions were: nodal forces, simulating the internal pressure $p$, at node points along the inside surface of the ring and the crack face; nodal forces, simulating the flow stress $\sigma_0$ along the ligament; zero displacement at the uncracked section of the ring in the direction normal to the symmetry line; and truss elements, representing the pressure vessel, connected to the node points along the outside surface of the ring.

The value of $J$ was estimated from the crack opening stretch at the crack front, $\delta_T$, using the relationship $J = \alpha \sigma_0 \delta_T$.

The main result presented in reference [4] was that the applied $T$-values range between 50 and 100 for deeply cracked nozzles ($a/t_n = 0.93, 0.86$) for a wide range of $t_v/l_n$ ratios (Fig. 4). The $p/\sigma_0$ ratio was
0.043, which corresponds to $p = 3,000$ psi; $\sigma_0 = 70,000$ psi. Since typical $T_{mat}$ values of pressure vessel steels range between 150 and 200 [2], reference 4 indicated that even highly idealized conservative models of deeply cracked BWR feedwater nozzles that account for the constraining effect of the pressure vessel, predict stable ductile crack growth according to Paris' stability criterion. The essential question left open by reference 4 was whether the strip yield model was sufficient for obtaining reasonably close approximation to the applied $T$ values. This question was investigated in the present project, by performing analyses, similar to those presented in reference 4, but using displacement compatible models and bilinear constitutive relationships.

4.0 COMPUTATIONAL PROCEDURE

The computations in this project were performed with the computer program NONSAP. Although the NONSAP version available to the writers did not have a capability for computing $J$, NONSAP is structured such that distinct program functions are clearly isolated into distinct subroutines, which made it feasible to introduce the necessary modifications.

Parks' virtual crack extension method [7] for calculating $J$ for elastic-plastic materials was implemented. In order to reduce the time and effort invested in computer code modifications, the implementation was in less than full generality - in particular the contribution to $J$ due to the work done by load acting on the crack face is not calculated by the program, but must be calculated by hand, upon completion of a computer run. Fortunately, this calculation is relatively straightforward
and brief. The tearing modulus, $T_{appl}$, is calculated by solving for $J$
for two crack lengths $a$ and $a + \Delta a$ ($\Delta a$ small) under constant pressure
and then using the forward finite difference formula: $T_{appl} = \frac{J_{a+\Delta a} - J_{a}}{\Delta a}$.
Since elastic-plastic finite element analyses for computing $J$ are quite
expensive, the computation of $T$ in this manner is doubly so.

Given the restriction described above, plus some minor limitations
on the generality of the modified program, the required program changes
consisted of adding one new subroutine and modifying four existing sub-
routines in NONSAP.

4.1 Test Problem

Several test problems were solved for the purpose of checking the
computer program. One of the test problems will now be described in
order to indicate the accuracy obtained from the program and also to
serve as documentation for other investigators who might be engaged in
similar studies. The problem consists of the center-cracked panel shown
in Fig. 5. For this simple geometry, analytical estimates of $J$ are
available. In the present study, the problem was solved numerically
using a coarse, uniform mesh of thirty-six bilinear isoparametric ele-
ments shown in the figure. (Only one-quarter of the domain had to be
modeled, because of symmetry). The virtual crack extension method used
in the program to calculate $J$ requires the computation of the energy
stored in a ring of finite elements enclosing the crack tip. The ring
chosen for the present problem is shown cross-hatched in Fig. 5.
Three cases were studied. First, the yield stress was chosen high enough so that no yielding occurred, even near the crack tip. Other parameters for this case were $h = b = 3a$, $\sigma = 3$ psi, and $E = 100$ psi. Plane stress was assumed. The value of $J$ computed by the program, 0.297 lb/in., compared well, at least for such a coarse mesh, with the value of 0.374 obtained from reference 8. In the second test case, the yield stress was chosen to be 5 psi, a value which led to small-scale (contained) yielding near the crack tip. A bilinear stress-strain curve was used and the slope of the curve above the yield point was $E/10$ (E, h, b, a, and $\sigma$ were the same as in the first case. Plane stress was assumed.) The computed value of $J$ was 0.316, which may be compared with the value of 0.334 estimated from the work of Hutchinson and Goldman [9]. In the third test case, the yield stress was chosen sufficiently low, so that large scale yielding occurred across the remaining ligament. A bilinear stress-strain curve was used in this test case, with a slope of $E/10.7$ above the yield point. Other parameters for this case were $E = 30,000$ ksi, $b = h/4 = 3a/2$, $\sigma = 37$ ksi., and the yield stress was 70 ksi. Plane stress was assumed. The computed value of $J$ was 354 lb/in.; an approximate value of 997 lb/in. was estimated from the analytical formulas found in Zahoor's dissertation [6] who employed Ramberg-Osgood stress-strain laws. A power hardening exponent of $n = 7$ was used for comparison since this choice best matched the bilinear stress-strain law used in our computer program. Thus the two values of $J$ are of the same order of magnitude; closer agreement probably could not be expected, since different types of approximation were involved in the two calculations.
In addition to validating the program, the test problems provided means for estimating an approximate size of the crack length increment, \( \Delta a \). It was found that the calculated values of J were approximately consistent (less than one percent variation occurred for \( \Delta a \) ranging between 0.01 inches and 0.000001 inches).

4.2 Analysis of a Deeply Cracked Nozzle Ring

In contrast to the analysis presented in reference 4, where entirely elastic material behavior was assumed (the yield stress across the remaining ligament was represented by a distributed load, Fig. 3), the present analysis assumed elastic-plastic material behavior with linear strain-hardening. The bilinear stress-strain laws were chosen to bracket the experimentally-determined stress-strain law of A508 Class 2 forging steel (Fig. 6), in order to establish the effect of strain hardening on the applied value of T.

Fig. 7 shows the overall layout of the finite element mesh, the truss elements representing the vessel, and the quadratic isoparametric elements representing the nozzle. For clarity, many of the truss elements near the crack have not been shown. The ratio of crack depth to nozzle thickness is \( a/t_n = 0.95 \), and the ratio of inside to outside radius is \( r_i/r_o = 0.5 \). Because the crack in the nozzle is deep, plastic yielding was found to be confined to the small shaded region indicated in the figure. Fig. 8 is an enlargement of the region ABCD of Fig. 7 and shows the distribution of finite elements in this region. Again, for clarity, many truss elements have not been shown. The extremely fine mesh near the crack tip, as shown in Fig. 8, was chosen
on the basis of the mesh used by Parks in reference 7. Parks' example problem, however, involved significant bending over the remaining ligament - an effect which is not present here. Thus the mesh of Fig. 8 is perhaps more refined than necessary, but it was thought prudent to err on the side of over-refinement rather than under-refinement. In all, a total of 60 elements (including 23 truss elements representing the effect of the vessel on the nozzle) and 403 degrees of freedom were used. NONSAP lacks a mesh generator and does not calculate the nodal loads corresponding to a uniform pressure load, so that for the mesh shown in the figure (and for several slightly altered versions of the mesh used in the course of this project) special purpose mesh generating and nodal load calculating programs had to be written.

The boundary conditions for the analysis consist of a uniform pressure acting over the crack face and the inside of the nozzle. The symmetry displacement conditions are schematically indicated by roller supports in the figure. In addition, as in reference 4, the effect of the vessel expansion due to internal pressure was simulated by radial displacements imposed at the ends of the truss elements. NONSAP has the restriction that nonhomogeneous displacement conditions cannot be imposed directly, but must be introduced through the artifice of applying at a boundary node a large nodal load and also truss elements which are very stiff compared to the stiffness of the adjoining structure. Unfortunately, implementing this artifice for the present mesh complicated the input data; an additional special purpose program was written to simplify the preparation of this data. The parameter values, listed in Table 1, were chosen to permit ready comparison of our results with those presented in reference 4.
TABLE I

Summary of Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>External nozzle radius</td>
<td>( r_0 )</td>
<td>10 inches</td>
</tr>
<tr>
<td>Internal nozzle radius</td>
<td>( r_i )</td>
<td>5 inches</td>
</tr>
<tr>
<td>Nozzle length</td>
<td>( l_n )</td>
<td>14 inches</td>
</tr>
<tr>
<td>Pressure vessel thickness</td>
<td>( t_v )</td>
<td>7 inches</td>
</tr>
<tr>
<td>Mean radius of pressure vessel</td>
<td>( r_v )</td>
<td>147 inches</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>( E )</td>
<td>30,000 ksi</td>
</tr>
<tr>
<td>Tangent modulus</td>
<td>( E'_T )</td>
<td>545 ksi</td>
</tr>
<tr>
<td>Tangent modulus</td>
<td>( E''_T )</td>
<td>300 ksi</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>( \nu )</td>
<td>0.3 inches</td>
</tr>
<tr>
<td>Truss stiffness*</td>
<td>( k_v )</td>
<td>16,154 ksi</td>
</tr>
<tr>
<td>Imposed displacement on trusses**</td>
<td>( u )</td>
<td>( 10.5 \times 10^{-6} ) p</td>
</tr>
<tr>
<td>Crack length</td>
<td>( a )</td>
<td>4.75 inches</td>
</tr>
<tr>
<td>Crack length increment</td>
<td>( \Delta a )</td>
<td>0.01 inches</td>
</tr>
</tbody>
</table>

* The truss stiffness given is per unit length of the circumference of the nozzle.

** p represents the pressure in the pressure vessel. It is to be specified in psi units.
5. RESULTS

In considering the results of our analysis with reference to the reported internal pressure $p$, it is important to recognize that the internal pressure was consistently underestimated by a factor of 1.52. This is discussed in the Appendix. Briefly, the reason is that the effect of internal pressure on the (uncracked) nozzle was simulated by imposing displacements on the truss members (in effect, preloading the truss members) which represent the pressure vessel wall. The correct truss displacements depend on the relative compliances of the nozzle and pressure vessel wall. Since one of the objectives of our investigation was to make comparisons with the data presented in reference 4, the truss displacements imposed in our model were identical to those on which the data in reference 4 were based. In reference 4, however, the membrane forces imposed on the nozzle by the pressure vessel wall (and the corresponding truss displacements) were intentionally overestimated in order to ensure that all assumptions were conservative. Specifically, the nozzle was assumed to be rigid.

The results of our analysis are shown in Figs. 9 through 12. The boundaries of the yielded region in the vicinity of the crack tip are shown for internal pressures ranging from 200 to 1500 psi and $E_T = 545$ ksi in Fig. 9. It is seen that the condition of fully yielded ligament, postulated for the strip yield model in reference 4, is reached at just above 500 psi internal pressure when the bilinear stress-strain relationship of Fig. 6 is used. Above 500 psi, the boundary of the yielded region spreads to substantial distances away from the ligament and moves along the crack tip. Of course, the effects of this phenomenon on the
applied values of \( J \) and \( T \) could not be investigated with the strip yield model of reference 4. It should be noted, however, that neither our model nor the strip yield model had provisions for evaluating the effect of the yield zone spreading into the pressure vessel.

The variation of equivalent stress along the ligament for various \( p \) and \( E_T \) values is shown in Fig. 10. The stresses were evaluated by means of the formula given for bilinear stress-strain laws by Hutchinson [10]. The plotted values were computed at those quadrature points in the finite element mesh which were nearest to the ligament.

\( J_{\text{appl}} \) is plotted as a function of internal pressure in Fig. 11 for various conditions: Model 1 is a strip-yield model, similar to the model described in reference 4, but differing in the finite element mesh division used and the computer code employed. Specifically, the finite element mesh of Figs. 7 and 8 and the NONSAP computer program were used to obtain all results plotted in Fig. 11. Values of \( J_{\text{appl}} \) were calculated using the relationship \( J = \alpha_o \sigma_o \delta \), in which \( \alpha = \alpha' = 1.0 \) for plane stress conditions and \( \alpha = \alpha'' = 0.7 \) for plane strain conditions. \( \sigma_o \) is the flow stress (in this case \( \sigma_o = \sigma_o' = 75 \) ksi was used). Computations performed with various \( \sigma_o \) values indicated that \( J_{\text{appl}} \) is not very sensitive to \( \sigma_o \).

Model 2 is the displacement compatible model. Two tangent moduli \( (E_T' = 545 \) ksi and \( E_T'' = 300 \) ksi) were used. For each tangent modulus, \( J_{\text{appl}} \) was evaluated twice: first with the crack length of 4.75 inches, and second with the crack length of 4.76 inches, in order to permit computation of the corresponding \( T \) values, as explained in Section 4.0. (The higher \( J \) values correspond to the longer crack lengths.)
Fig. 11 indicates that the $J_{\text{appl}}$ values computed with the strip yield model do not differ greatly from those computed with the displacement compatible model. It is also evident from Fig. 11 that $J_{\text{appl}}$ is not a sensitive function of the tangent modulus values. Thus the indication is that $J_{\text{appl}}$ can be estimated with good accuracy even with highly idealized models.

$T_{\text{appl}}^\prime$ on the other hand, was found to be very sensitive to both the modeling technique and the material stress-strain law used for high values of the internal pressure. $T_{\text{appl}}$ is plotted on Fig. 12 as a function of the internal pressure for two tangent moduli $E_T'$ and $E_T^"$ in the displacement compatible model (Model 2) and for the strip yield model (Model 1). The strip yield model indicates that $T_{\text{appl}}$ is nearly constant for the range of $p$ values plotted. This is consistent with the results presented in reference 4 where low $T_{\text{appl}}$ values were indicated even for $p$ values as high as 3,000 psi (Figure 4).

Model 2, on the other hand, indicates great sensitivity of $T_{\text{appl}}$ to the strain hardening characteristics of the material above 500 psi which is the internal pressure that causes the ligament to yield completely. It is seen that $T_{\text{appl}}$ rapidly increases with internal pressure for the lower tangent modulus value $E_T^" = 300$ ksi.

Figures 11 and 12 provide a basis for examining the question of whether Paris' stability criterion is applicable to the very deeply cracked feedwater nozzle considered in our analysis, in which the ligament to crack length ratio is only 1:19. The stability criterion has been established for those cases in which the parameter $\omega$, defined as:

$$\omega = \frac{b}{J} \cdot \frac{dJ}{da}$$

(3)
is much greater than 1 (reference 3). In eq. 3, b represents the length of the ligament. The lower limit of \( \omega \) is not presently known but is estimated to be between 5 and 10. For the reported internal pressure of 1000 psi, \( \omega = 6.3 \) for \( E_T = E_T '' = 300 \text{ ksi} \) and \( \omega = 1.4 \) for \( E_T = E_T ' = 545 \text{ ksi} \). Since our model incorporated some highly conservative assumptions, these \( \omega \) values probably represent conservative lower bound estimates. Also, \( \omega \) is expected to increase rapidly for increasing ligament to crack length ratios.

6.0 CONCLUSIONS

Even with the conservative assumptions employed in our analysis, it was found that Paris' stability criterion predicts stable ductile crack growth for deeply cracked BWR feedwater nozzles for certain ranges of pressure and material parameter values. Realistic stability evaluation of cracked feedwater nozzles, in which the ligament is in fully yielded condition, requires modeling of the structural interaction between the cracked nozzle and the pressure vessel in greater detail than could be attempted in the present project. Therefore, our model did not permit establishing close bounds on the parameter values that would guarantee stable crack growth on the basis of Paris' stability criterion.

The accuracy of the model could be greatly improved by considering the effects of tangential constraints on the nozzle and the spreading of the yield zone into the pressure vessel wall ahead of the crack tip. Such models are feasible but much more costly than the present model. For this reason, preliminary mesh optimization and the use of substructuring techniques are recommended for detailed parametric studies.
The $J_{appl}$ values appear to be insensitive to the strain hardening parameters of the material for a wide range of internal pressures. In fact, the $J_{appl}$ values computed with the strip yield model were remarkably close to the $J_{appl}$ values computed with the displacement compatible model. The $T_{appl}$ values, on the other hand, are evidently insensitive to the strain hardening parameters of the material only below the internal pressure at which the ligament becomes fully yielded. In our model this corresponded to an indicated internal pressure of 500 to 600 psi.

Application of nonlinear stress analysis procedures in engineering practice requires very substantial computational resources. Even for the highly idealized two-dimensional feedwater nozzle employed in our analysis, time and cost considerations imposed severe limitations on the number of parameter values that could be examined. In the opinion of the writers, there are substantial incentives for the development of stress analysis procedures that would efficiently meet the computational requirements posed by $J$-controlled crack growth theory.

7.0 ACKNOWLEDGEMENTS

The writers wish to thank Professors Paul C. Paris, Mario P. Gomez and Dr. Hiroshi Tada of the Center for Fracture Mechanics of Washington University for advice and assistance received in the course investigation.
8.0 APPENDIX: Simulation of the Structural Interaction Between the Pressure Vessel and the Nozzle

The structural interaction between the pressure vessel and the nozzle was approximated in reference 4 and in the present analysis by considering average membrane forces only. In particular, the vessel was idealized as an infinite plate, subjected to an axially symmetric stress distribution about the nozzle axis. The assumed radial stress (remote from the nozzle axis) was:

\[ \sigma = \frac{3}{4} \frac{r_v}{t_v} p \]  
(A-1)

where \( p \) is the pressure in the vessel, \( r_v \) and \( t_v \) are the mean radius and wall thickness of the vessel. The structural interaction between the vessel and the nozzle was determined by two parameters: the membrane force acting on the uncracked nozzle, \( F_n \), and the spring constant of the pressure vessel wall, \( k_v \). The spring constant depends only on the parameters of the vessel and the outside radius of the nozzle, \( r_o \), whereas \( F_n \) depends on the ratio of compliances of the nozzle and pressure vessel, which is not precisely known. The procedures followed for determining \( k \) and \( F_n \) are outlined below:

The starting point is the classical solution for the stress distribution in a thick-walled hollow cylinder subjected to uniform pressures on the inner and outer surfaces* (Fig. A1). The radial stress is:

\[ \sigma_r = \frac{A}{r^2} + 2C \]

*See, for example, Timoshenko and Goodier, "Theory of Elasticity", 2nd Edition, pp. 58-60 and p. 66.
in which

\[ A = \frac{a^2 b^2 (p_i - p_0)}{b^2 - a^2} \]

\[ 2C = \frac{b^2 p_0 - a^2 p_i}{b^2 - a^2} \]

We shall assume \( b \gg a \), which results in

\[ A = a^2 (p_i - p_0) \]

\[ 2C = p_0 \]

The radial displacement \( u \) for plane stress conditions is given by:

\[ u = \frac{1}{E} \left[ -\frac{1+v}{r} A + 2C(1-v)r \right] \]  \( (A-2) \)

To determine the spring constant \( k_v \), we let \( r = a = r_0; \ p_0 = 0 \) and obtain:

\[ u = -\frac{1+v}{E} r_0 p_i \]

Letting \( F_n = p_i \cdot t_v \) and \( k_v = \frac{F_n}{u} \) (our sign convention is: tensile membrane forces and outward radial displacements are positive, hence
the negative sign in the expression for $k_v$); we obtain:

$$k_v = \frac{E}{1+\nu} \cdot \frac{t_v}{r_o} \quad (A-3)$$

To determine the membrane force acting on the nozzle, $F_n$, we first express the displacement $u(r_o)$ from eq. A-2 for the following conditions:

$a = r_o$; $p_1 = 0$; $p_o = \bar{p}$ and obtain:

$$u(r_o) = \frac{2 \bar{p} r_o}{E}$$

Letting $\bar{F} = \bar{p} t_v$, the expression for $u(r_o)$ becomes:

$$u(r_o) = \frac{2}{1+\nu} \frac{\bar{F}}{k_v}$$

Finally, we match displacements between the nozzle and the pressure vessel. Denoting the spring constant for the nozzle by $k_n$, we have:

$$\frac{2}{1+\nu} \frac{\bar{F}}{k_v} = \frac{F_n}{k_n} + \frac{F_n}{k_v}$$

from which:

$$F_n = \frac{2 k_n \bar{F}}{(1+\nu) (k_v + k_n)} \quad (A-4)$$

The ratio $\frac{F_n}{\bar{F}}$ is plotted as a function of $\frac{k_n}{k_v}$ in Fig. A2 for $\nu = 0.3$. 

The displacement imposed on the truss members, assuming plane stress conditions, is:

\[ u = \frac{F_n}{k_v} = \frac{3r_{o}\cdot r_{n}\cdot k_{p}}{2Et_v(k_v+k_{n})} \]  \hspace{1cm} (A-5)

In reference 4 (and therefore in the present report) it was assumed that \( k_n \gg k_v \), which resulted in maximal loading of the nozzle \( F_n = 1.54 \bar{F} \) (Fig. A2). The displacement imposed on the trusses was:

\[ u = \frac{3r_{o}}{2Et_v} p \]  \hspace{1cm} (A-6)

The numerical value of \( u \) for the present analysis (Table I) was determined from eq. A-6.

We can estimate \( k_n \) from our idealization of the nozzle as a tube, characterized by length, \( l_n \), inside radius \( r_i \) and outside radius \( r_o \) (Fig. 3):
\[ k_n = \frac{E}{r_o^2 - r_i^2} \]
\[ r_o [ (1 + v) r_i^2 + (1 - v)r_o^2 ] \]

(A-7)

The radial expansion of the nozzle due to internal pressure \( p \) is:

\[ u_n = \frac{2pr_o^2}{E(r_o^2 - r_i^2)} \]

(A-8)

Equivalently, \( u_n \) can be written in terms of \( \tilde{F} \):

\[ u_n = \frac{\tilde{F}}{k_1} \]

where:

\[ k_1 = \frac{3E r_i^2}{8r_o^2} \]

(A-9)

Matching displacements between nozzle and pressure vessel, we have:

\[ \frac{2}{1 + v} \frac{\tilde{F}}{k_v} = \frac{F_n}{k_n} + \frac{F_n}{k_v} + \frac{\tilde{F}}{k_1} \]

From which:

\[ \tilde{F} = \frac{2k_n (k_1 - k_v)}{(1 + v)k_1 (k_v + k_n)} \]

(A-10)

It is seen that eq. A-4 is the special case of eq. A-10, in which \( k_1 = \infty \). For the parameter values listed in Table I: \( k_n = 33,770 \) ksi;
\[ k_i = 545,200 \text{ ksi.} \]  (In computing \( k_n \) and \( k_i \), it was taken into account that the nozzle ring was modeled assuming plane strain conditions. Thus, \( E = 32.97 \times 10^3 \text{ ksi} \), rather than \( E = 30 \times 10^3 \text{ ksi} \), was substituted into eqs. A-7 and A-9). With these values, we obtain from eq. A-10:

\[
\frac{F_n}{F} = 1.01
\]

On comparing this ratio with the limiting value of 1.54 used in reference 4 and in the present analysis, we conclude that \( F_n \) was overestimated by a factor of 1.52.
9.0 REFERENCES


Figure 1

Idealized material J-resistance curve for small amounts of crack growth
Figure 2

Deeply cracked BWR feedwater nozzle
The shaded area represents the crack surface.

Figure 3

Two-dimensional model of a deeply cracked BWR feedwater nozzle
Figure 4
Range of applied T values for deeply cracked nozzles. Strip yield model

\[ \frac{r_i}{r_o} = 0.65; \quad \frac{a}{t_n} = 0.93 \]

\[ \frac{r_i}{r_o} = 0.50; \quad \frac{a}{t_n} = 0.86 \]

\[ p = 3,000 \text{ psi}, \quad \sigma_o = 70,000 \text{ psi} \] (from reference 4)
Figure 5

Test problem: Center-cracked panel

Poisson's ratio: 0.3
Figure 6

Engineering tensile stress-strain curve for A508 Class Z Forging Steel (from Oak Ridge National Laboratory Drawing 73-6147). The stress-strain curves used in the present analysis are characterized by the elastic moduli $E$, $E'_{T}$, and $E''_{T}$. 
Figure 7

Finite element model and truss elements used for the displacement compatible model (Model 2). Detailed mesh for the region ABCD is shown in Fig. 9.
Figure 8
Finite element mesh in the crack tip region

Figure 9
Boundaries of the yielded region in the vicinity of the crack tip for reported internal pressures ranging from 200 to 1500 psi. \( E_T = 345 \text{ ksi} \). The finite element mesh for region EFGCDG is shown in Figure 8.
Figure 10

Variation of equivalent stress along the ligament. ($\sigma_y = 65$ ksi)
Figure 11

$J_{\text{appl}}$ vs. reported internal pressure $p$. Model 1: strip yield model ($\sigma_0 = 75$ ksi). Model 2: displacement compatible model, based on the bilinear stress-strain relationship of Figure 6 ($\sigma_0 = 75$ ksi).
Figure 12
$T_{appl}$ vs. reported internal pressure $p$. Model 1: strip yield model ($\sigma_0 = 75$ ksi); Model 2: displacement compatible model based on the bilinear stress-strain relationship of Figure 6 ($\sigma_0 = 75$ ksi).
A PRELIMINARY FRACTURE ANALYSIS ON THE INTEGRITY 
OF HSST INTERMEDIATE TEST VESSELS

BY

Akram Zahoor* and Paul C. Paris**

1. INTRODUCTION

Structural integrity of reactor pressure vessels requires 
consideration of flaws which may exist in the vessel wall 
and may escape any NDT inspection. Under the operating 
conditions these undetected flaws may grow causing serious 
problems to the integrity of the vessel and safety. The 
primary objective of this report is to utilize the elastic-
plastic fracture mechanics analysis methods. Specifically, 
the concept of J-integral and the Tearing Modulus approach 
I1,21 is utilized to assess the stability of the flaw growth. 
In particular, intermediate pressure vessels [3] tested 
under the Heavy Section Steel Technology (HSST) program are 
considered. Of them analysis for only two intermediate test 
vessels (V-7 and V-1) [4,5] has been carried out because of 
their distinct modes of fracture. Both the vessels had 
external surface flaws located longitudinally in the base 
metal. The mode of failure in the vessel V-1 was mixed mode 
and crack growth was unstable, whereas the vessel V-7 leaked, 
when subjected to internal pressure. First an analysis for 
the vessel V-7 is presented, then vessel V-1 is analyzed.

*Research Engineer, Washington University, St. Louis, Missouri 63130; 
Now with Battelle Columbus Laboratories, Columbus, Ohio, 43201

**Professor of Mechanics, Washington University.
2. **ANALYSIS OF THE VESSEL V-7**

The location of the external surface flaw in the vessel V-7 is shown in Figure 1, where various dimensions of the vessel are also given. The details of the flaw design are shown in Figure 2. The test vessel was made of A533B Class 1 pressure vessel steel and the surface flaw was located in the base metal. Table 1 summarizes the information available from the test which are needed to pursue the analysis.

**TABLE 1. Data for Fracture Analysis of the Vessel V-7**

<table>
<thead>
<tr>
<th>Material Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Temperature</td>
<td>196°F</td>
</tr>
<tr>
<td>Yield Stress, $\sigma_y$</td>
<td>65 ksi</td>
</tr>
<tr>
<td>Ultimate Stress, $\sigma_{ult}$</td>
<td>87 ksi</td>
</tr>
<tr>
<td>Materials Strain Hardening Index, $n$</td>
<td>5</td>
</tr>
<tr>
<td>Strain at the Initial Yielding, $\varepsilon_o$ (Fig. 3)</td>
<td>0.217%</td>
</tr>
<tr>
<td>Strain at the Onset of Strain Hardening, $\varepsilon_H$ (Fig. 3)</td>
<td>1.2%</td>
</tr>
<tr>
<td>Internal Radius of the Vessel, $R_i$</td>
<td>13.5 in.</td>
</tr>
<tr>
<td>Wall Thickness of the Vessel, $t$</td>
<td>6.0 in.</td>
</tr>
<tr>
<td>External Trapezoidal Surface Flaw</td>
<td></td>
</tr>
<tr>
<td>Surface Length, $2l$</td>
<td>18.0 in.</td>
</tr>
<tr>
<td>Depth, $a$</td>
<td>5.31 in.</td>
</tr>
<tr>
<td>Bottom Length</td>
<td>8.0 in.</td>
</tr>
<tr>
<td>Maximum Pressure, $p_f$</td>
<td>21.4 ksi</td>
</tr>
<tr>
<td>Circumferential Strain at the Inside Surface, 180° from the Flaw</td>
<td>0.289%</td>
</tr>
<tr>
<td>Circumferential Strain at the Outside Surface, 180° from the Flaw</td>
<td>0.12%</td>
</tr>
<tr>
<td>Mode of Failure</td>
<td>Leak (Stable Tearing)</td>
</tr>
</tbody>
</table>
Since the surface flaw is located longitudinally, the remote stress which controls the crack tip conditions is the circumferential stress. The first step is then to calculate the circumferential stress distribution in the vessel wall at the pressure when leak occurred. This stress distribution can be obtained by considering a thick-walled cylinder under internal pressure, i.e., excluding effect due to caps of the vessel. At the leak pressure, $p_f$, part of the thickness of the cylinder has yielded. Taking an elastic-perfectly plastic material behavior (See Figure 3 for the stress-strain curve and Table 1 for doing so) and assuming the Tresca yield criterion, the radius of the elastic-plastic boundary, $R_c$, of the thick-walled cylinder is given by [6]

$$\frac{p_f}{\sigma_y} = \ln \left( \frac{R_c}{R_i} \right) + \frac{1}{2} \left( 1 - \frac{R_c^2}{R_o^2} \right)$$ \hspace{1cm} (1)

where,

$\sigma_y$ = yield stress in simple tension,
$R_i$ = internal radius of the cylindrical shell
$R_o$ = outer radius of the cylindrical shell

Taking $p_f = 21.4$ ksi and other data from Table 1, we get

$$R_c = 15.8''$$ \hspace{1cm} (2)

This means that the vessel wall has yielded by about 2.3 in. deep. The circumferential stress distribution can now be
found [6], However, our interest is to calculate through-thickness average circumferential stress, which can be obtained directly from the equilibrium consideration. For a thick-walled cylinder, the through-thickness average stress, \( \bar{\sigma}_\theta \), is

\[
\bar{\sigma}_\theta = p_f \cdot \frac{R_i/R_o}{1 - (R_i/R_o)} = 48.2 \text{ ksi}
\]

In the discussions to follow \( \bar{\sigma}_\theta \) will be denoted by \( \sigma \) and for the sake of simplicity a value of 50 ksi will be used, i.e.,

\[
\sigma = 50 \text{ ksi}
\]

2.1 THE SURFACE FLAW MODEL

In order to calculate the crack tip field at the deepest point of the surface flaw, an analysis similar to that of [1] will be utilized here. Let us consider that the remaining material ahead of the surface flaw has yielded, which is the case with the vessel under consideration, as shown in Figure 4. The surface flaw may then, for the purpose of the approximate but conservative analysis, be regarded as an equivalent elastic through-the-thickness crack of length \( 2\ell \) with an average (through the thickness) closure stress \( \sigma' \) holding back the opening displacement, \( \delta \), at the center of the crack.
The closure stress, $\sigma'$, is then given by

$$\sigma' \cdot t = \sigma_o \left( t - \frac{a}{t} \right)$$

or

$$\sigma' = \sigma_o \left( 1 - \frac{a}{t} \right) \quad (4)$$

where,

$$\sigma_o = \frac{1}{2} (\sigma_y + \sigma_{ult})$$

The opening displacement, $\delta$, from the elastic analysis for a flat strip is

$$\delta = \frac{4L}{E} (\sigma - \sigma') \quad (5)$$

There are, however, two important features which are to be included in the above estimate for $\delta$ so that it can be used for the vessel under consideration. First, the flaw is located on a cylindrical surface so that there will be more opening of the crack surfaces than for a crack in a flat plate. For a thin elastic cylinder having a longitudinal through-the-thickness crack of length, $2\ell$, subjected to the internal pressure, the stress intensity factor, $K_I$, is given by

$$K_I = M \cdot \sigma \sqrt{\pi \ell} \quad (6)$$

where,

*M = The stress magnification factor which accounts for effects of shell curvature on the crack tip fields.*

*For the case of an internal surface flaw where crack surfaces are subjected to pressure, a subtracting term $p \frac{a}{t}$ will appear on the right side of equation (4).*
This means that the through-thickness crack tip field parameter \( J \) will contain an \( M^2 \) term. However, the opening displacement at the center, \( \delta \), will vary approximately linearly with \( M \). The second thing is to introduce effective modulus, \( E_T \), in place of \( E \) to account for yielding in the vessel wall.

Incorporating these, the approximate opening displacement, \( \delta \), for the case of vessel is given by

\[
\delta = M \cdot \frac{4L}{E_T} (\sigma - \sigma')
\]  

(7)

For a deep surface flaw, the opening displacement, \( \delta \), may be regarded as a conservative estimate of \( \delta_c \), the crack opening stretch. Then

\[
\delta_c = \delta = M \cdot \frac{4L}{E_T} [\sigma - \sigma_o (1 - \frac{a}{t})]
\]

Moreover, \( \delta_c \) is related to \( J \) by

\[
\delta_c = \frac{J}{\sigma_o}
\]

Hence

\[
\delta = M \cdot \frac{4L}{E_T} [\sigma - \sigma_o (1 - \frac{a}{t})] = \frac{J}{\sigma_o}
\]  

(8)
For a crack extension \( da \),

\[
\frac{dJ}{J_0} = M \cdot \left( \frac{4L}{t} \right) \cdot \frac{J_0}{E_T} \left[ 1 + \frac{t}{J_0} \left( \frac{d\sigma}{da} \right) \right] da
\]

or

\[
\left( \frac{dJ}{da} \cdot \frac{E}{J_0^2} \right) = T_{\text{applied}} = M \cdot \left( \frac{4L}{t} \right) \left[ 1 + \frac{t}{J_0} \left( \frac{d\sigma}{da} \right) \right] \cdot \frac{E}{E_T}
\]

If \( J_{\text{matl}} \) is the value on the material's \( J \)-integral \( R \)-curve, then for the stability of crack growth \( da \), [1],

\[
\frac{dJ}{da} < \frac{dJ_{\text{matl}}}{da}
\]

or

\[
\left( \frac{dJ_{\text{matl}}}{da} \cdot \frac{E}{J_0^2} \right) = T_{\text{matl}} > M \cdot \left( \frac{4L}{t} \right) \left[ 1 + \frac{t}{J_0} \left( \frac{d\sigma}{da} \right) \right] \cdot \frac{E}{E_T}
\]

(9)

If pressure in the vessel is held constant at any stage of crack growth, then \( d\sigma/da = 0 \); or in a situation where pressure drops due to crack growth, the term \( \frac{d\sigma}{da} \) will be negative. Hence, the second term in the right side of equation (9) will either be a subtracting term or absent completely. For conservative values of \( T_{\text{applied}} \) then, stability of crack growth requires that

\[
T_{\text{matl}} > M \cdot \left( \frac{4L}{t} \right) \cdot \frac{E}{E_T}
\]

(10)
where, $T_{mat}$ is obtained by taking slope of the $J$-integral $R$-curve at the calculated value of $J$ (equation (8)).

Equation (10) is the condition for the stability of crack growth at the deepest point of the surface flaw. If for the applied loading (pressure) and dimensions of the flaw and vessel, $T_{applied}$ is always less than $T_{mat}$, then the surface flaw will grow in a stable manner until it becomes a through-crack and leak will occur. Under this circumstance, it also becomes necessary to see if the flaw grows stably in the longitudinal direction of the vessel. This point will be discussed in a later section.

2.2 EFFECTIVE MODULUS AND THE STRESS MAGNIFICATION FACTOR FOR THE VESSEL

2.2.1 THE EFFECTIVE MODULUS

In order to be able to use equation (3), an estimation of the effective modulus, $E_T$, is needed which takes account of the part thickness yielding of the vessel. Let us suppose we are interested in estimating $E_T$ for the region where the circumferential strain, $\varepsilon$, lies between the initial yield and the onset of strain hardening, i.e.,

$$\varepsilon_0 \leq \varepsilon \leq \varepsilon_H$$

where,

$\varepsilon_0 = \text{strain at the initial yielding}$

$\varepsilon_H = \text{strain at the onset of strain hardening}$
Now, let

$$\varepsilon = A\varepsilon_o + B\varepsilon_H$$  \hspace{1cm} (11)

If $\varepsilon = \varepsilon_o$ then $A = 1, B = 0$

and if $\varepsilon = \varepsilon_H$ then $A = 0, B = 1$

so that in the range $\varepsilon_o \leq \varepsilon \leq \varepsilon_H$

$$A + B = 1$$  \hspace{1cm} (12-a)

For a given $\varepsilon$, equation (11) and (12) can be solved for $A$

and $B$. We then have

$$A = \frac{\varepsilon_H - \varepsilon}{\varepsilon_H - \varepsilon_o}$$  \hspace{1cm} (12-b)

For $\varepsilon = 0.289\%$ (circumferential strain at the inside surface, 180° from the flaw, measured at the leak pressure), and taking $\varepsilon_o = 0.217\%$ and $\varepsilon_H = 1.2\%$ (Refer Table 1), we get

$$A = 0.93$$

and $B = 0.07$  \hspace{1cm} (13)

Now,

when $\varepsilon \leq \varepsilon_o$ we have $E_T = E$, the linear elastic modulus

and when $\varepsilon \geq \varepsilon_H$, $E_T = E_H$, the slope of the stress-strain curve at the onset of strain hardening
So that for \( \varepsilon_0 \leq \varepsilon \leq \varepsilon_H \), an effective modulus may be approximated by
\[
E_T = AE = BE_H
\]
or in view of equations (12-a) and (12-b)
\[
E_T = E - (E - E_H) \frac{(\varepsilon - \varepsilon_0)}{(\varepsilon_H - \varepsilon_0)}
\]
substituting
\[
E = 30 \times 10^3 \text{ ksi,}
\]
\[
E_H = 6.42 \times 10^2 \text{ ksi,}
\]
and using values of \( A \) and \( B \) from equation (13), gives
\[
E_T = 27.85 \times 10^6 \text{ Lbs/in}^2 \quad (14)
\]
Another possible estimated value of \( E_T \) may be obtained by using the secant modulus, i.e.,
\[
E_T = E_S = \frac{65 \times 10^3}{0.289 \times 10^{-2}} = 22.5 \times 10^6 \text{ Lbs/in}^2 \quad (15)
\]
But since we know that the secant modulus somewhat overestimates the effective modulus, the actual modulus is expected somewhere between the two modulii. In the calculations to follow both the modulii will be used to get bounds on quantities of interest.
2.2.2 THE STRESS MAGNIFICATION FACTOR FOR THE VESSEL

For an elastic cylinder having a longitudinal through-the-thickness crack of length \(2\ell\), subjected to internal pressure, the elastic stress intensity factor, \(K_I\), is given by

\[ K_I = M \cdot \sigma \sqrt{\pi \cdot \ell} \]

where, \(\sigma\) is the mean circumferential stress and \(M\) is the stress magnification factor which accounts for effects of shell curvature on the crack tip fields. A plot of \(M\) vs. shell parameter, \(\lambda\), is given in [7]. The parameter \(\lambda\) is calculated in terms of the actual crack length, \(2\ell\), mean radius, \(R\), and thickness, \(t\), of the shell, i.e.,

\[ \lambda = \frac{\ell}{\sqrt{\frac{R}{t}}} \left[12(1 - \mu^2)\right]^{1/4} \]  \hspace{1cm} (16)

Recalling the values of \(\ell\), \(R\), and \(t\) from Table 1, i.e.,
\[ \ell = 9'' \]
\[ R = \frac{1}{2} (R_i + R_o) = 16.5'' \]
\[ t = 6'' \]
and \(\mu = 0.3\)

which gives \(\lambda = 1.70\)

and the corresponding value of the stress magnification factor, \(M\), [7] is

\[ M = 1.45 \]  \hspace{1cm} (17)
2.3 STABILITY OF CRACK GROWTH AT THE DEEPEST POINT OF THE SURFACE FLAW

We will now proceed to calculate the value of $J$ and $T$ applied at the deepest point of the surface flaw. Before making any computations, we would like to check for conditions of $J$-controlled crack growth, which require (i) the amount of crack growth to be small compared to the remaining ligament or other relevant dimensions and (ii) the parameter $\omega$ to satisfy

$$\omega = \frac{dJ}{da} \cdot \frac{b}{J} \gg 1$$

where, $b$ is the remaining ligament.

For the deep surface flaw of the vessel V-7,

$$b = 0.69 \text{ in.}$$

Since we are interested in knowing how large is $\omega$ at any stage of stable tearing, we will evaluate it for conditions just after $J_{IC}$. In which case, from the $J$-$R$ curve, Figure 5

$$J = J_{IC} = 1475.0 \text{ in-Lb/in}^2$$

and $$\frac{dJ}{da} = 1.61 \times 10^4 \text{ Lb/in}^2$$

With these values, we then get

$$\omega = 7.6$$
which is not as large as the $\omega$ requirement suggested in [2]. However, it is large enough to conclude, for the purpose of approximate analysis, that conditions of $J$-controlled crack growth are applicable for the V-7 vessel.

Recalling equations (8) and (10)

$$\delta = M \cdot \frac{4L}{E_T} \left[ \sigma - \sigma_o \left(1 - \frac{a}{t}\right) \right] = \frac{J}{\sigma_o}$$

and $T_{\text{applied}} = M \cdot \left(\frac{4L}{t}\right) \cdot \frac{E}{E_T}$.

Now, from the preceding discussion

$$\sigma = 50 \text{ ksi},$$
$$\sigma_o = \frac{1}{2} (\sigma_y + \sigma_{\text{ult}}) = 75 \text{ ksi},$$
$$a = 5.31 \text{ in.},$$
$$t = 6 \text{ in.},$$
$$L = 9 \text{ in.},$$
$$M = 1.45,$$

and the two modulii are

$$E_T = 27.85 \times 10^6 \text{ Lb/in}^2$$
and $E_S = 22.5 \times 10^6 \text{ Lb/in}^2$

Substituting these values in the equation for $\delta$ or $J$ with $L_{\text{eff}}$ (L modified by the plane strain plastic zone connection), we get values as shown in Table 2 below.
<table>
<thead>
<tr>
<th>$E_T$ (Lb/in²)</th>
<th>$\delta$ (in.)</th>
<th>$J$ (in-Lb/in²)</th>
<th>$\Delta a$ (in)</th>
<th>$T_{applied}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$27.85 \times 10^6$</td>
<td>0.084</td>
<td>6327</td>
<td>0.31</td>
<td>10.2</td>
</tr>
<tr>
<td>$22.5 \times 10^6$</td>
<td>0.105</td>
<td>7906</td>
<td>0.41</td>
<td>12.71</td>
</tr>
</tbody>
</table>

In the above $\Delta a$ is obtained from the J-integral R-curve at the point corresponding to the calculated value of $J$. It is to be noted that the estimated $\delta$ or $J$ above are obtained by taking the original flaw dimensions. Although at the leak pressure, the flaw had certainly grown both in the thickness and longitudinal directions. One way to account for this may be, as a first step, to modify the initial values of $a$ and $\ell$ by adding $\Delta a$ obtained in Table 2 and recalculating $J$ and the corresponding new value of $\Delta a$. For example this one step correction modifies $J$ from 6327 in-Lb/in² to 6913 in-Lb/in² and corresponding $\Delta a$ from 0.31" to 0.35" as shown in Table 3 below. For such a small amount of crack growth compared to the original crack length, one step correction will give good estimate for $J$ and $\Delta a$. Indeed, the predicted crack growth is in good agreement with the fractographic information available from the test (Figure 6).
TABLE 3

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\delta$</th>
<th>$J$</th>
<th>$\Delta a$</th>
<th>$T_{\text{applied}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lb/in$^2$</td>
<td>(in.)</td>
<td>(in-Lb/in$^2$)</td>
<td>(in)</td>
<td></td>
</tr>
<tr>
<td>27.85x10$^6$</td>
<td>0.092</td>
<td>6913</td>
<td>0.35</td>
<td>10.20</td>
</tr>
<tr>
<td>22.5x10$^6$</td>
<td>0.118</td>
<td>8883</td>
<td>0.47</td>
<td>12.79</td>
</tr>
</tbody>
</table>

The $T_{\text{applied}}$ values in Table 2 and 3 were obtained by using modified value of $\delta$, i.e., by adding $\Delta a$. It is evident that the $T_{\text{applied}}$ values, in both cases, are much smaller than the $T_{\text{material}}$ which for A533B class 1 material is about 100 [See Fig. 5]. So the surface flaw grew by stable tearing until the remaining ligament leaked under the applied internal pressure.

2.4 STABILITY OF CRACK GROWTH IN THE LONGITUDINAL DIRECTION

We now examine stability of crack growth in the longitudinal direction. Neglecting effects of the end caps, the problem can be analyzed by assuming a through-thickness longitudinal crack of length $2\ell = 18''$ in a cylindrical shell subjected to the uniform applied circumferential stress of $\sigma = 50.0\text{ ksi}$. If stability of crack growth is ensured without the end caps then crack growth in the vessel is
certainly stable. Moreover, for further conservativeness the length of the cylinder will be assumed finite. For plane strain crack tip conditions, the remaining ligament, b, and thickness, t, should be

\[ t, b \geq 25 \frac{J}{\sigma_o} \]

using \( J = J_{IC} = 1450 \text{ in-Lb/in}^2 \) and \( \sigma_o = 75 \text{ ksi} \), we get \( t, b \geq 0.5 \text{ in} \)

Hence the plane strain crack tip conditions are indeed operative. The crack tip J-integral value may be estimated well from the G (or K) formula when crack size is corrected for the plasticity. An alternate elastic-plastic method of analysis can also be used, and is discussed in the Appendix I.

Now, the elastic stress intensity factor, K, for a longitudinal crack of length \( 2\ell \) is

\[ K = M \cdot \sigma \sqrt{\ell} \cdot f(\ell/L) \]  \hspace{1cm} (18)

where,

- \( \sigma \) = uniform applied circumferential stress,
- \( M \) = stress magnification factor for the shell,
- \( 2L \) = Length of the cylinder excluding caps,

and

\[ f(\ell/L) = \sqrt{\pi} \cdot \text{Sec} \left( \frac{\pi \ell}{2L} \right) \]
Adjusting the crack size to account for plasticity, we have

\[ \ell_{\text{eff}} = \ell + r_y = \ell + \frac{K^2}{6\pi \cdot \sigma_0^2} \]

Accounting for plasticity, \( K \) is then given by

\[ K = M \cdot \frac{\sigma \sqrt{\ell_{\text{eff}}} \cdot f(\ell_{\text{eff}}/L)}{\sqrt{1 - \frac{M^2}{6\pi} \cdot \ell^2(\ell_{\text{eff}}/L) \cdot \left(\frac{\sigma}{\sigma_0}\right)^2}} \] (19)

and \( J \) is

\[ J = \frac{K^2}{E' \ell}. \] (20)

Now,

\[ \ell = 9 \text{ in.} \]
\[ L = 27 \text{ in.} \]
\[ \ell/L = 1/3 \]

For a center-cracked strip of finite width (\( \ell/L = 1/3 \)) subjected to the uniform applied stress \( \sigma = 50 \text{ ksi} \), the plastic zone corrected value from [8] is

\[ \frac{\ell_{\text{eff}}}{L} = 0.36 \]

and so

\[ f(\ell_{\text{eff}}/L) = 1.8016 \]
Substituting these values in (19), we get

\[ K = 442.76 \text{ ksi} \sqrt{\text{in}} \]

Substituting this in (20) and using values for the two modulii, we get the following for \( J \) and corresponding \( \Delta a \).

<table>
<thead>
<tr>
<th></th>
<th>( E_T = 27.85 \times 10^6 \text{Lb/in}^2 )</th>
<th>( E_T = E_S = 22.5 \times 10^6 \text{Lb/in}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J, \text{in-Lb/in}^2 )</td>
<td>6405.5</td>
<td>7929.0</td>
</tr>
<tr>
<td>( \Delta a, \text{in.} )</td>
<td>0.32</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Here again the estimated crack growth is in good agreement with the fractographic information (see Figure 6). The agreement is even better for the elastic-plastic analysis of Appendix I.

We will now calculate the applied value of \( T \) at the leak pressure to examine the stability of crack growing longitudinally in the vessel. In the presence of confined yielding, the applied value of \( T \) at the leak pressure (uniform circumferential stress, \( \sigma = 50.0 \text{ ksi} \)), assuming that the pressure was held constant for conservativeness, is given by, [1],
\[ T_{\text{applied}} = \frac{\{M^2 \cdot H(\ell_{\text{eff}}/L)\} \cdot (\sigma/\sigma_o)^2}{\left[1 - \frac{M^2}{6\pi} \{H(\ell_{\text{eff}}/L)\} \cdot (\sigma/\sigma_o)^2\right]} \cdot \left(\frac{E}{E_T}\right) \]

(21)

where, \( E_T \) has been used to account for the part thickness yielding of the vessel. \( M \) is the magnification factor for the shell. Recalling again the values from previous calculations

\[
\begin{align*}
\sigma &= 50.0 \text{ ksi}, \\
\sigma_o &= 75.0 \text{ ksi}, \\
M &= 1.45,
\end{align*}
\]

and the value of \( \ell_{\text{eff}} \) should now be corrected to include both the plastic zone correction as well as the actual crack growth predicted above, i.e.,

\[ \ell_{\text{eff}} = \ell + r_Y + \Delta a \]

Substituting these in equation (21), we get

\[ T_{\text{applied}} = 7.45 \text{ when } E_T = 27.85 \times 10^6 \text{ Lb/in}^2 \]

\[ = 9.30 \text{ when } E_T = 22.5 \times 10^6 \text{ Lb/in}^2 \]
For stability of crack growth, we should have

\[ T_{\text{applied}} < T_{\text{matl}} \]

Recall that \( T_{\text{matl}} \) is about 100 and so it is concluded that the crack should grow by stable tearing in the longitudinal direction at the leak pressure. Indeed, the vessel V-7 had stable tearing until it leaked.

3. ANALYSIS OF THE VESSEL V-1

The location of the external surface flaw in the vessel V-1 was the same as in the vessel V-7. The shape of the flaw, in the case of the vessel V-1, was a part-circular crack. The test vessel was made of A508, class 2 forging steel and the surface flaw was located in the base metal. Table 4 summarizes the information available from the test. In contrast to the deep surface flaw of the vessel V-7, the flaw in the vessel V-1 was shallow, and so the vessel V-1 sustained higher pressure causing gross yielding before fracture. The procedure for analyzing this vessel is similar to the analysis for the vessel V-7.
TABLE 4. Data for Fracture Analysis of the Vessel V-1

<table>
<thead>
<tr>
<th>Material</th>
<th>A508, Class 2 Forging Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Temperature</td>
<td>130°F</td>
</tr>
<tr>
<td>Yield Stress, $\sigma_y$</td>
<td>72.0 ksi</td>
</tr>
<tr>
<td>Ultimate Stress, $\sigma_{ult}$</td>
<td>93.0 ksi</td>
</tr>
<tr>
<td>Hardening Parameter, n</td>
<td>5</td>
</tr>
<tr>
<td>External Surface Flaw</td>
<td></td>
</tr>
<tr>
<td>Outside Surface Length, 2a</td>
<td>8.25 in.</td>
</tr>
<tr>
<td>Depth of the Surface Flaw, a</td>
<td>2.56 in.</td>
</tr>
<tr>
<td>Maximum Pressure, $p_f$</td>
<td>28.8 ksi</td>
</tr>
<tr>
<td>Circumferential Strain at the</td>
<td></td>
</tr>
<tr>
<td>Outside Surface, 180° from the</td>
<td>0.92%</td>
</tr>
<tr>
<td>Surface Flaw</td>
<td></td>
</tr>
<tr>
<td>Mode of Fracture</td>
<td>Unstable Crack Growth</td>
</tr>
<tr>
<td>Crack Opening Displacement</td>
<td></td>
</tr>
<tr>
<td>At Rupture</td>
<td>&gt; 0.88 in.</td>
</tr>
</tbody>
</table>

Since the flaw is located longitudinally, the stress which controls the crack tip conditions is the circumferential stress. Our interest is to calculate through-thickness average circumferential stress.
The average circumferential stress is then

$$\sigma = \sigma_0 = \frac{p_f}{1 - (R_1/R_0)} = 64.8 \text{ ksi}$$ \hspace{1cm} (22)

Now, at the maximum pressure, $p_f = 28.8 \text{ ksi}$, the vessel V-1 had yielded completely. So the tangent modulus appropriate for this situation is the slope of stress-strain curve in the strain hardening range. The stress-strain curve for A508 material is shown in Figure 7. Taking an average slope of the initial strain hardening curve, we get

$$E_T = 5.34 \times 10^5 \text{ Lb/in}^2$$ \hspace{1cm} (23)

and therefore, the ratio $(E/E_T)$ is

$$E/E_T = 56.0$$ \hspace{1cm} (24)

It may be somewhat misleading to use values of the stress magnification factor, $M$, based on the linear elastic fracture mechanics for the vessel under consideration. However, the stress magnification factor $M$ evaluated from a model vessel tested under similar loading conditions is expected to give reasonably good estimate. Taking the value of $M$ obtained from model vessels [5], we have

$$M = 1.6$$ \hspace{1cm} (25)

We will now proceed to examine the stability of crack growth in the vessel.
3.1 STABILITY OF CRACK GROWTH AT THE DEEPEST POINT OF THE SURFACE FLAW

We will first calculate the parameter \( \omega \) to check for J-controlled crack growth. For this case

\[
b = 3.44 \text{ in.}
\]

and from Figure 8 at 150°F

\[
J = J_{IC} = 1460 \text{ in-Lb/in}^2
\]

\[
\frac{dJ}{da} = 7.17 \times 10^4 \text{ Lb/in}^2
\]

\[
\therefore \quad \omega = \frac{dJ}{da} \frac{b}{J} = 169
\]

Hence soon after \( J_{IC} \) in the stable tearing regime, for small amounts of crack growth, conditions of J-controlled crack growth are satisfied. However, during stable crack growth, \( J \) may increase to several times the \( J_{IC} \) while \( b \) decreases and so does the \( \frac{dJ}{da} \), causing \( \omega \) value to decrease significantly. Equations (8) and (10) for \( J \) and \( T_{\text{applied}} \) are

\[
\delta = M \cdot \frac{4\xi}{ET} [\sigma - \sigma_o (1 - \frac{a}{t})] = \frac{J}{\sigma_o}
\]

\[
T_{\text{applied}} = M \cdot \left( \frac{4\xi}{E} \right) \frac{E}{ET}
\]
Now,

$$\sigma_o = \frac{1}{2} (\sigma_y + \sigma_{ult}) = 80 \text{ ksi}$$

$$\ell = 4.25 \text{ in.}$$

$$a = 2.56 \text{ in.}$$

Substituting these values in above, we get

$$\delta = 0.964 \text{ in.},$$

$$J = 7.714 \times 10^4 \text{ in-Lb/in}^2,$$

and $$\Delta a = 1.81 \text{ in.}$$

Where, $$\Delta a$$ is obtained from the J-integral R-curve for A508 material (Figure 8). Note that this $$\Delta a$$ is the estimated value because the J-R curve is available for very small amounts of crack growth. It can be seen that the predicted $$\Delta a$$ is in good agreement with fractographic information (Figure 9). We now calculate $$T_{\text{applied}}$$ at the deepest point of the surface flaw. Using new value of $$\ell$$ by accounting for crack growth, i.e.,

$$\ell = \ell + \Delta a = 4.125 + 1.81 = 5.94 \text{ in.}$$
With this then

$$T_{\text{applied}} = 355$$

which is greater than the $T_{\text{mat}} = 335$ (a very conservative number, based on the initial tearing slope of the J-R curve using the data from the nine-point average measurement of crack growth at 150 F, see Figure 8. Hence the flaw went unstable in the thickness direction. The initial tearing slope based on 3-point average of crack front measurement gives $T_{\text{mat}} = 200$. Actually $T_{\text{mat}}$ for A508 is not expected to be very much different than that of A533B cl 1. In that case, for A508 class 2 steel the $T_{\text{mat}}$ is probably between 100 and 200.

3.2 STABILITY OF CRACK GROWTH IN THE LONGITUDINAL DIRECTION

We now examine stability of crack growth in the longitudinal direction. Since the vessel has yielded completely, fully plastic hardening analysis [9], for the purpose of approximate analysis, is most appropriate. The tensile stress-strain curve for A508 material in the plastic range can be fitted very well by the following

$$(\varepsilon/\varepsilon_0) = a(\sigma/\sigma_0)^n$$

where,

$$a = 7.5 ,$$
$$n = 5 ,$$
$$\sigma_0 = 70 \text{ ksi} ,$$
and $$\varepsilon_0 = \sigma_0/E .$$
Taking the result for the fully plastic center-cracked strip [9] and modifying it by the shape factor to incorporate effects of the shell curvature on crack tip fields, the $J$ and $T_{\text{applied}}$ can be estimated by

\[
J = \alpha LM^2 \sigma_o \varepsilon_o \cdot \hat{f}_5 \left( \frac{\ell}{L}, n \right) \cdot \left( \frac{2}{\sqrt{3}} \cdot \frac{P}{P_o} \right)^{n+1}
\]

\[
T_{\text{applied}} = \frac{2an \cdot M^2}{(1-\ell/L)\phi_2} \left[ 1 + \frac{\phi_1 (1-\ell/L)}{(n+1)} \right] \cdot P^* (\ell/L, n) \cdot \left( \frac{2}{\sqrt{3}} \cdot \frac{P}{P_o} \right)^{n+1}
\]

where,

\[
\left( \frac{2}{\sqrt{3}} \frac{P}{P_o} \right) = \frac{\sigma}{\sigma_o (1-\ell/L)}
\]

$\sigma =$ average circumferential stress,

$\ell =$ half length of the through-thickness longitudinal crack,

$L =$ half length of the cylindrical shell,

and $\phi_1$, $\phi_2$ and $P^*$ are non-dimensional functions of the hardening index $n$ and $\ell/L$.

The above equation for $T_{\text{applied}}$ has been written for the dead load (pressure held constant) situation, which is the case for the vessel under consideration (Figure 10).

We will now calculate numerical values of $T_{\text{applied}}$ by trying various lengths, $2\ell$, of through-thickness longitudinal crack to see if crack instability is possible. Let
us first try \( \ell = 7 \text{ in.} \) For this then

\[
\frac{\ell}{L} = \frac{7}{27} = 0.26
\]

\[
\frac{2}{\sqrt{3}} \cdot \frac{P}{P_o} \cdot \frac{\sigma}{\sigma_o(1-\ell/L)} = \frac{64.80}{70(1-0.26)} = 1.25
\]

The values of the functions \( \phi_1, \phi_2, \) and \( P^* \) can now be read from the curves given in [9]. Substituting all these quantities in (26), we get

\[
T_{\text{applied}} = 180
\]

Similarly assuming crack lengths, \( 2\ell = 16" , 18" , 20.5" \), we get the following result

<table>
<thead>
<tr>
<th>( 2\ell )</th>
<th>( T_{\text{applied}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16&quot;</td>
<td>260</td>
</tr>
<tr>
<td>18&quot;</td>
<td>343</td>
</tr>
<tr>
<td>20.5&quot;</td>
<td>615</td>
</tr>
</tbody>
</table>

Now in the ductile tearing region, the instability of crack growth requires that \( T_{\text{applied}} > T_{\text{material}} \). If we assume that \( T_{\text{matl}} = 335 \) (a very conservative number, based on the initial tearing slope of J-R curve), then unstable crack growth would have occurred at approximately \( 2\ell = 18 \text{ in.} \). Referring back to Figure 9, we find that the ductile tearing occurred until about \( 2\ell = 20.5 \text{ in.} \) before going to cleavage.
This may be explained by arguing that actually crack went unstable by ductile tearing at $2\ell = 18$ in. and then the fast crack growth triggered the cleavage mode of fracture.

4. **CONCLUDING REMARKS**

From the preceding discussions it becomes clear that whenever pressure in the vessel is such that the vessel has not yielded, the $T_{\text{applied}}$ numbers are small indicating tendency for stable crack growth. However, for higher pressures which cause full thickness yielding in the vessel wall, the $T_{\text{applied}}$ numbers increase drastically and become comparable to the $T_{\text{mat}}$ values. Had the flaw in the vessel V-1 been subjected only to the leak pressure of the vessel V-7 ($P_f = 21.4$ ksi), the $T_{\text{applied}}$ numbers would have been of same order of magnitude as were obtained in V-7 and the flaw growth would have been stable.

A point is to be made here about the J-integral method of analysis for such large amounts of crack growth as occurred in the V-1 vessel. From the strict requirements of J-controlled crack growth [2], only those cases can be considered where (i) crack extension is small compared to the region dominated by the deformation theory singular field and (ii) $\omega >> 1$. Hence whenever above conditions are not met, the results should be considered as approximate.

Finally analyses similar to that of V-1 can be carried out for the vessels V-3 and V-6 [10]. Since the flaw sizes and
pressures in these two vessels were very much the same as in V-1, it can be easily shown that both the vessels had unstable crack growth.

As a concluding remark to the analyses presented here, all the quantities were estimates at the known leak or fracture pressure. In general, this method of analysis can be carried out in the parametric form by choosing a set of arbitrary values of pressures and crack lengths and then calculating the quantities of interest, i.e., J and $T_{applied}$. The resulting parametric plots, e.g., of J and $T_{applied}$ vs. pressure with crack length as a parameter along with the material's resistance curve may then be used as design curves for predicting stable or unstable crack growth.

Acknowledgements
The authors acknowledge the support of this work by the United States Nuclear Regulatory Commission, Contract Number NRC-03-77-029 with the Center for Fracture Mechanics, Washington University. The final phases of this work were also in part supported by Contract Number NRC-04-78-227. Many stimulating discussions with Dr. H. Tada of Washington University and the Heavy Section Steel Technology Program staff of Oak Ridge National Laboratory, especially John Merkle are acknowledged with thanks. The continued encouragement of Mr. R. Gamble of Nuclear Regulatory Commission is most gratefully acknowledged.
APPENDIX I

J-INTEGRAL ESTIMATE FOR THE V-7 VESSEL FROM
THE ELASTIC-PLASTIC FRACTURE MECHANICS CONSIDERATIONS

Consider a piecewise power law hardening material for
which the stress-strain relation is of the form

\[ \frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} \quad \text{for } \sigma \leq \sigma_o \]
\[ \frac{\varepsilon}{\varepsilon_o} = \left(\frac{\sigma}{\sigma_o}\right)^n \quad \text{for } \sigma > \sigma_o \]

where, \(\sigma_o\) and \(\varepsilon_o\) are the yield stress and yield strain in
simple tension respectively and \(n\) is the hardening index.
Such a relationship will fit very well the \(\sigma-\varepsilon\) curve for
A533B class 1 (Figure 3) except in the flat region of the
initial yielding. Let us consider now a through thickness
longitudinal crack of length \(2\ell\) in the cylindrical shell
subjected to the internal pressure. Our interest is to
estimate the crack tip field parameter \(J\) for this case.
When applied pressure is such that the average stress
(circumferential) in the remaining uncracked length of the
cylinder is less than the flow stress, the \(J\)-integral can be
estimated as [9]

\[ J = \sigma_o \varepsilon_o L(M^2) \left[ \psi_1(\ell/L, P/P_o) \cdot (P/P_o)^2 \right] \]

\[ = \frac{\sigma_o}{E_T} \cdot L(M^2) \left[ \psi_1(\ell/L, P/P_o) \cdot (P/P_o)^2 \right] \quad \text{for } P \leq P_o \]
where,

\[ 2L = \text{Length of the cylindrical shell}, \]
\[ P = \text{Load per unit thickness} = \sigma \cdot (2L), \]
\[ P_O = \text{Limit load, which can be expressed in terms of flow stress}, \]
\[ = 2(L-\ell)\sigma_0, \]
\[ \sigma_0 = \text{flow stress} \]

so that \( P/P_O = \sigma/\sigma_0 (1 - \frac{\ell}{L}) \)

The function \( \psi_1 \) is non-dimensional function of \( \ell/L \) and \( P/P_O \), and is plotted in [9] for various values of the hardening index, \( n \). For the present case, plane strain crack tip conditions are appropriate. Recall that

\[ \sigma = 50 \text{ ksi} \]
\[ \sigma_0 = 75 \text{ ksi} \]
\[ \ell/L = 1/3 \]
\[ n = 5 \]

\[ \therefore (P/P_O) = 1 \]

and from [9]

\[ \psi_1(\ell/L, P/P_O) = 0.6 \]
Substituting these values in above equation for $J$, we get

<table>
<thead>
<tr>
<th>$E_T$, Lb/in$^2$</th>
<th>$J$, in-Lb/in$^2$</th>
<th>$\Delta a$, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$27.85 \times 10^6$</td>
<td>6871</td>
<td>0.34</td>
</tr>
<tr>
<td>$22.5 \times 10^6$</td>
<td>8505</td>
<td>0.45</td>
</tr>
</tbody>
</table>
REFERENCES:


FIGURE 1. FLAW DESIGN FOR INTERMEDIATE TEST VESSEL V-7.
FIGURE 2. FLAW DESIGN FOR THE HSST PROGRAM INTERMEDIATE TEST VESSEL V-7.

FIGURE 3. TRUE STRESS-STRAIN DIAGRAM FOR A533 GRADE B CLASS 1 PRESSURE VESSEL STEEL.
FIGURE 4. THE SURFACE FLAW.
FIGURE 5. J-INTEGRAL R-CURVE FOR ASTM A533, GRADE B, CLASS 1 PRESSURE VESSEL STEEL.
FIGURE 7. TENSILE STRESS-STRAIN CURVE FOR A509, CLASS 2 FORGING STEEL (VESSEL V-1 MATERIAL).

FIGURE 10. PRESSURE AND CRACK-OPENING DISPLACEMENT VS. TIME PLOT FOR THE VESSEL V-1.
Some Considerations in the Application of J-Integral Analysis to Structures Subjected to Multiple Loads

by

George I. Zahalak and Paul C. Paris

Center for Fracture Mechanics
Washington University
St. Louis, Missouri 63130

Abstract

The general theory underlying Rice's J-Integral is reviewed briefly with a view toward applications to structures subjected to multiple loads. The theory is specialized to the case of a cracked member of arbitrary (symmetric) cross-section and arbitrary material behavior subjected to bending and axial load. For this case it is verified that under appropriate conditions a pseudo strain energy indeed exists which is expressible as a function of the generalized forces or displacements and crack length, and expressions are given for the pseudo strain energy and J. As a specific example the case of a cracked member of rectangular cross section is considered in detail under the assumption that the material is elastic-perfectly plastic. Generalized force-displacement relations are derived under the constraint of monotonic straining and explicit formulas for the pseudo strain energy and J associated with these relations are stated. The behaviors of the force-displacement relations, pseudo strain energy, and J are examined in the rigid-plastic limit, and it is shown that a pseudo strain energy persists in this limit and yields the limiting value of J. Finally constraints on the loading paths are derived which insure that there is
no unloading of plastically yielded material (except possibly some local unloading in the wake of an advancing crack); these constraints are stated both in terms of generalized displacements and forces. The constraints are interpreted geometrically in displacement space and examples of acceptable (no plastic unloading*) and unacceptable (plastic unloading) loading paths are given.

*In this paper the term "plastic unloading" will be used as a shorthand expression to signify "unloading of material which has previously undergone plastic yielding".
Introduction

Most of the fruitful applications of elastic-plastic fracture mechanics to structural analysis are currently based on Rice's J-integral [1]. These applications include the tearing instability theory developed by Paris and his associates [2] and have broad implications for fracture proof design. While it is a tool of great analytical power, the theory of the J-integral requires the existence of a pseudo strain energy function. This in turn requires that the stress-strain relations in the loaded body be modelled adequately by the deformation theory of plasticity. Thus loading histories which result in non-proportional straining or plastic unloading over substantial regions of the specimen would grossly violate the assumptions of J-theory and predictions of structural behavior based on this theory would not be expected to remain valid.

Until recently most of the applications of J-integral analysis were concerned with materials testing which normally involved test specimens loaded by a single generalized force. However in the structural applications of the tearing instability theory one is immediately faced with the need to evaluate J (and its derivative with respect to crack length) for bodies of complicated shape which are subjected to several independent forces. While J-theory certainly applies in these cases (under appropriate conditions), some complications arise in systems of multiple loads which are examined briefly in this report. In particular the problem of plastic unloading and its relation to the loading history is investigated. Some local unloading is unavoidable in situations where the crack advances, but this problem has been addressed and resolved satisfactorily by Hutchison and Paris [3] who showed that J could still be assumed to characterize the crack-tip stress-strain
field for limited amounts of crack growth. But even with limited
growth, or no growth at all, certain loading histories can produce
large-scale unloading which would destroy the uniqueness of the stress-
strain relations, the existence of the pseudo strain energy function,
and the J-theory connection between the crack-tip stress-strain inten-
sity parameter and the overall loads and displacements applied to the
body. In the following discussion we describe, for one simple example,
constraints on the load and deformation history which will preclude
plastic unloading ahead of the crack tip and insure the existence of
the required pseudo strain energy.

J-Theory

The theory of the J-integral is developed, for example, in [4] and
will be reviewed briefly below. For an elastic body in a two dimensional
state of stress, with geometry, stress and deformation invariant in the
x₃ - direction (see Fig. 1) Rice defined the J-integral as

\[
J = \int_{\Gamma} [\tilde{W}(\epsilon_{ij})dx_2 - T_i \frac{\partial u_i}{\partial x_i} ds]
\]

(1)

where \(\tilde{W}(\epsilon_{ij})\) is the strain energy density of the (generally nonlinear)
elastic body, \(\Gamma\) is any arbitrary curve enclosing the crack tip starting
at the lower surface and ending on the upper surface, \(u_i\) is the displace-
ment vector, \(s\) is the arclength along \(\Gamma\), \(T_i\) is the traction vector
\(\sigma_{ij} q_{ij}\), and \(n_i\) is the unit outward normal vector on \(\Gamma\) (\(\sigma_{ij}\) and \(\epsilon_{ij}\) are
respectively the stress and strain tensors). For elastic-plastic
bodies satisfying the assumptions of deformation theory of plasticity
there exist unique relations between stresses and strains and it can be
shown [4] that there exists a pseudo strain energy density \(\tilde{W}(\epsilon_{ij})\) satisfying
\[ \hat{W} = \int_0^{\epsilon_{ij}} \sigma_{pq} d\epsilon_{pq} \quad \text{and} \quad \sigma_{ij} = \frac{\partial \hat{W}}{\partial \epsilon_{ij}} \quad (2) \]

The integral above may be computed along any arbitrary path in strain space.

Thus deformation theory is the bridge between J-theory, which strictly speaking is defined for nonlinear elastic materials, and the behavior of real elastic-plastic bodies. Indeed with no substantial unloading and with sufficiently proportional straining, deformation theory and incremental plasticity theory give identical results, and both are physically realistic. In the following sections attention is focused on the avoidance of substantial unloading.

For a body of nonlinear elastic material undergoing a quasi-static process the principle of virtual work requires that an increment in the body's strain energy, \( W \), be equal to the work increment of the surface forces; that is

\[ dW = d \int_V \hat{W}(\epsilon_{ij}) \, dV = \int_S T_i du_i ds \quad (3) \]

Now if the work of the applied surface forces can be written as a sum of \( n \) products of generalized forces \( Q_i \) and conjugate generalized displacements \( q_i \), then

\[ dW = \sum_{i=1}^{n} Q_i dq_i \quad \text{and} \quad W = \int_0^{Q_i} \sum_{j=1}^{n} Q_j d\hat{q}_j \quad (4) \]
The integral above is computed along a path in displacement space, and it follows from the independence of the strain energy density on path in strain space that the integral for $W$ is independent of the particular displacement path chosen. The total strain energy of the body, $W$, may be regarded as a function of the $q_i$ and the crack length, $a$. Since Eq. (4) implies that

$$\left(\frac{\delta W}{\delta q_i}\right)_{q_j, a} = Q_i \quad (5)$$

$W$ can also be regarded as a function of the $Q_i$ and $a$. Note that a necessary and sufficient condition for the existence of a strain energy function is that the generalized forces satisfy the irrationality conditions

$$\frac{\delta Q_i}{\delta q_j} = \frac{\delta Q_j}{\delta q_i} \quad i, j = 1, 2, \ldots n \quad (6)$$

The potential energy of the body is defined as

$$U = W - \sum_{i=1}^{n} Q_i q_i \quad (7)$$

and it can be shown [4] that $J$ is the rate of decrease of the potential energy per unit increase of crack (half) area under conditions where the applied forces are held fixed. That is

$$-BJ = \left(\frac{\delta U}{\delta a}\right)_{Q_i} = \left(\frac{\delta W}{\delta a}\right)_{q_i} \quad (8)$$
where $B$ is the thickness of the body. $J$ can also be interpreted as the energy flowing into the crack tip per unit area of crack extension. Thus if an explicit expression is available for the strain energy as a function of the $q_i$ and $a$, $J$ can be computed directly from Eq. (8).

Alternately one can use Eq. (4) to express $J$ as a line integral

$$-BJ = \int_0^{q_i} \sum_{j=1}^{n} \left( \frac{\partial Q_j}{\partial a} \right)_{q_i} dq_j$$

(9)

where the integral is evaluated along an arbitrary path in displacement space.

**Bending and axial force with arbitrary cross-section and material properties**

Consider the segment of a cracked beam illustrated in Fig. 2. The beam is of constant cross-sectional shape, and this shape is arbitrary except that it is assumed to be symmetrical about the vertical axis. Further the material is assumed to be elastic-plastic with a uniaxial stress-strain curve of arbitrary shape. The length of the beam segment is $2h$, which is assumed to be small compared to the depth of the segment $w$. A crack of length $a$, perpendicular to the $y$-axis, extends partially across the cross-section. The segment is analyzed using the assumptions of simple beam theory (initially plane cross-sections remain plane after deformation).* The two generalized forces are chosen as $P$,

*This structure can be regarded as a thin segment embedded in an extended beam, or as a laboratory specimen provided with rigid plates at the ends. Although in the former case the assumption of plane sections may not be physically realistic, this model is appropriate for the exploratory investigation presented in this paper.
the total axial force acting on the cross-section, and \( M \), the moment about the centroid of the whole cross section (without the crack). The conjugate generalized displacements are the displacement at the centroid \( \delta \), and the rotation of the cross-section \( \phi \).

If \( a \gg h \) then the material above the crack may be assume stress-free. Let the material have a general uniaxial stress-strain relation

\[
\sigma = f(\varepsilon) \tag{10}
\]

The plane-sections assumption gives the displacement perpendicular to the cross section \( u \), and the strain \( \varepsilon \), respectively as

\[
u = \delta + \phi y \quad \text{and} \quad \varepsilon = \frac{1}{h} (\delta + \phi y) \tag{11}
\]

As we are interested primarily in problems of crack extension, we will restrict consideration to loadings which produce tensile strain at the crack tip: that is, we rule out loadings that result in crack closure. The condition for this is that

\[
\frac{\delta}{\phi} \geq (a-d) \tag{12}
\]

and we assume that this condition is satisfied throughout the loading history.
If each fiber is strained monotonically we can integrate the stresses over the cross-section to obtain expressions for the force and moment.

\[ P(\delta, \phi, a) = \int_{d-w}^{d-a} B(y)f[\frac{1}{h}(\delta + \phi y)]dy \]

\[ M(\delta, \phi, a) = \int_{d-w}^{d-a} yB(y)f[\frac{1}{h}(\delta + \phi y)]dy \]

In the above equations \( B(y) \) is the varying width of the section. It is clear that Eqs. (13) satisfy the irrationality condition

\[ \left( \frac{\partial P}{\partial \phi} \right)_{\delta, a} = \left( \frac{\partial M}{\partial \delta} \right)_{\phi, a} \]

so that a pseudo strain energy indeed exists, under the assumptions of simple beam theory and locally monotonic straining, for arbitrary cross-sectional shape and material behavior. In fact we can immediately express the strain energy as

\[ W(\delta, \phi, a) = \int_{d-w}^{d-a} hB(y)dy \int_0^{\frac{1}{h}(\delta + \phi y)} f(\xi)d\xi \]

Finally, using either Eq (15) or Eqs (13) together with Equation (9), we can express the J-integral as
\[ J(\delta, \phi, a) = 2h \int_0^1 \frac{1}{n} [\delta + \phi (d-a)] f(\xi) d\xi \]  

(16)

(The factor of 2 enters Eq.(16) because the beam segment is assumed to be loaded symmetrically about the cracked section). In principle Eqs.(13) could be inverted to express \( W \) and \( J \) as function of \( P \), \( M \), and \( a \).

The elastic - perfectly plastic cracked beam of rectangular cross-section

In order to obtain more explicit results we specialize the discussion of the preceding section to the example of an elastic - perfectly plastic material and a rectangular cross-section. The geometry is illustrated in Fig.3, and the material is characterized by a Young's modulus \( E \), and a flow stress \( \sigma_0 \). The uniaxial stress-strain behavior in monotonic loading is assumed to be symmetric in tension and compression. Even in this simple example the response of the member for arbitrary loading histories can be quite complicated, involving partial or complete crack closure as well as plastic flow and unloading. To limit somewhat this wide range of behavior we require that \( \phi \geq 0 \), in addition to Eq.(12), which simply states that

\[ (\delta/\phi) \geq a-w/2 \]  

(17)

Four cases must be distinguished in this example: no yielding, only the bottom fibers yielded, both the top and bottom fibers yielded, and only the top fibers yielded. These four cases define four regions in the \((\delta, \phi)\) displacement plane which are shown in Fig. 4; the boundaries of these regions depend on the crack length, \( a \).
Assuming monotonic loading of each fiber, Eqs.(13) can be evaluated explicitly. The result is

Region 1: \[ P = \frac{EB}{h} \left[ (w-a)\delta - \frac{a}{2} (w-a)\phi \right] \] (18)

\[ M = \frac{EB}{h} \left[ - \frac{a}{2} (w-a)\delta + \frac{1}{12} (w-a)\{3a^2+(w-a)^2\}\phi \right] \]

Region 2: \[ P = \frac{EB}{2h} \phi \left[ \left( \frac{w}{2} - a + \frac{\delta}{\phi} \right)^2 - \left( \frac{\sigma_o h}{E} \right)^2 \right] - \sigma_o B \left( \frac{w}{2} - \frac{\delta}{\phi} - \frac{\sigma_o h}{E} \right) \]

\[ M = \frac{EB}{12h} \phi \left( \frac{w}{2} - a + \frac{\delta}{\phi} + \frac{\sigma_o h}{E} \right) \left[ \frac{w}{2} - a + \frac{\delta}{\phi} + \frac{\sigma_o h}{E} \right] \]

\[ + 3 \left( \frac{w}{2} - a - \frac{\sigma_o h}{E} \right) - 3 \frac{\delta^2}{\phi^2} \] + \frac{1}{2} \sigma_o B \left[ \frac{w^2}{4} - \frac{\delta^2}{\phi^2} \right] \] (19)

Region 3: \[ P = \sigma_o B \left( 2 \frac{\delta}{\phi} - a \right) \]

\[ M = - \frac{B\sigma_o h^2}{3E\phi^2} + \frac{1}{2} \sigma_o B \left( \frac{w^2}{2} + a^2 - aw - 2 \frac{\delta^2}{\phi^2} \right) \] (20)

Region 4: \[ P = \frac{EB}{2h} \phi \left[ \left( \frac{\sigma_o h}{E} \right) - \left( \frac{w}{2} - \frac{\delta}{\phi} \right)^2 \right] + \sigma_o B \left( \frac{w}{2} - a + \frac{\delta}{\phi} - \frac{\sigma_o h}{E} \right) \]

\[ M = \frac{EB}{12h} \phi \left( \frac{w}{2} - \frac{\delta}{\phi} + \frac{\sigma_o h}{E} \right) \left[ \left( \frac{w}{2} - \frac{\delta}{\phi} + \frac{\sigma_o h}{E} \right) + 3 \left( \frac{w}{2} - \frac{\sigma_o h}{E} \right) - 3 \frac{\delta^2}{\phi^2} \right] \]

\[ + \frac{1}{2} \sigma_o B \left[ \left( \frac{w}{2} - a \right)^2 - \left( \frac{\sigma_o h}{E} - \frac{\delta}{\phi} \right)^2 \right] \] (21)

It can be verified directly in this example that the irrationality condition, Eq(14), is satisfied in each region, and therefore we can construct a pseudo strain energy function throughout the portion of the
displacement plane under consideration. To construct the strain energy we can use Eq.(15), or integrate the force-displacement relations directly. In region 1 we have the usual quadratic form elastic strain energy

\[ W = \frac{EB}{2h} \left[ (w-a)\delta^2 - a(w-a)\delta \phi + \frac{1}{12} (w-a)(3a^2 + (w-a)^2)\phi^2 \right] \quad (22) \]

whereas in region 3 we have

\[ W = \frac{B\sigma_0^2 h^2}{3E^2 \phi} + \frac{1}{2} \sigma_0 B \left[ (\frac{w}{2} + a^2 - aw)\phi + 2\delta (\frac{\delta}{\phi} - a) \right] - \frac{\sigma_0^2 hB}{2E} (w-a) \quad (23) \]

Note that \( W \) is continuous at the boundary point of regions 1 and 3. Although most loading paths would reach region 3 by traversing either region 2 or region 4, we omit writing expressions for \( W \) in the two latter regions, as in applications we are interested primarily in loadings approaching fully plastic conditions.

It remains to compute \( J \), which can be done most easily from the expressions for the pseudo strain energy. Thus if both top and bottom fibers are yielded (region 3) then

\[ J(\delta, \phi, a) = 2\sigma_0 \left[ (\frac{w}{2} - a)\phi + \delta \right] - \frac{\sigma_0^2 h}{E} \quad (24) \]

Differentiation of Eq.(22) with respect to \( a \) shows that \( J \) is a homogeneous quadratic function of \( \delta \) and \( Q \) in the elastic region 1, but becomes a linear function of these variables in region 4.
The Rigid-Plastic Limit

Recent applications of tearing instability theory have investigated the stability of crack extension under conditions of imminent plastic collapse. Thus it is of interest to examine the results of the last section as \( \sigma_0/E \rightarrow 0 \). In this (rigid-plastic) limit regions 1, 2 and 4 disappear and region 3 becomes the entire half-plane \( \phi > 0 \) (limited by Eq.(17)). Further Eqs.(20) reduce to

\[
P = \sigma_0 B (2 \delta - \phi) \quad \text{and} \quad M = \frac{1}{2} \sigma_0 B \left( \frac{M^2}{2} + a^2 - aw - 2 \frac{\delta^2}{\phi^2} \right)
\]

(25)

which could have been written directly from a rigid-plastic analysis. As expected \( P \) and \( M \) are no longer independent variables as the load point must lie on the limit surface. It should be noted, however, that Eqs.(25) still satisfy the irrationality condition, which means that a pseudo strain energy persists in this limit. In fact

\[
W = \frac{1}{2} \sigma_0 B \left[ (\frac{M^2}{2} + a^2 - aw) \phi + 2\delta (\frac{\delta}{\phi} - a) \right]
\]

(26)

and

\[
J = 2\sigma_0 \left[ (\frac{M}{2} - a) \phi + \delta \right]
\]

(27)

These are, of course, the rigid-plastic limits of the elastic-plastic \( W \) and \( J \). Thus the value of \( J \) can be computed unambiguously in terms of \( \delta \), \( \phi \) and \( a \) in this limit. If \( P=0 \), then \( \delta = a\phi/2 \) and
\[ J = \sigma_0 (w-a) \]

which is the value given by Rice et. al [5] for pure bending with a small remaining ligament, whereas if \( \phi = 0 \) one recovers the result, also given in [5], that

\[ J = 2\sigma_0 \delta \]

(Note again that factors of 2 appear because the beam segment is assumed to be symmetrically loaded).

**Unloading: "Acceptable" Loading Paths**

The existence of the pseudo strain energy function, and the expressions for \( J \) derived in the previous sections depend on the existence of unique relations between forces and deformations satisfying irrationality conditions, such as Eqs.18 through (21). These relations will cease to hold if any portion of the section ahead of the crack tip experiences plastic unloading, which in this case means simply that a fiber strained beyond the yield point undergoes a decrease in strain. It is therefore of considerable interest and importance to characterize those loading histories which involve no plastic unloading. For such loading paths \( J \) can be computed unambiguously and they will be termed "acceptable" loading paths; loading paths which are not acceptable will be called "unacceptable".
For the rectangular elastic-plastic beam the acceptable loading paths may be characterized very simply in the displacement plane. Thus referring to Fig. 3 it is apparent that a loading path is acceptable if during every increment of loading the dimension $c$ does not decrease and the dimension $(c+2e)$ does not increase. These are the conditions which must be satisfied in region 3, and similar considerations apply in regions 2 and 4. Of course, every path is acceptable in the elastic region 1. Note that a crack extension at fixed $\delta$ and $\phi$ does not invalidate the force-displacement relations, Eqs(18) through (21), and therefore the requirements for an acceptable path. Crack extension alone does not constitute "plastic unloading" in the context of this simplified model, as the stresses behind the crack tip have already been neglected. Based on the foregoing considerations the acceptable loading paths can be characterized as those which satisfy the following inequalities on every path segment.

$$
\left(-\frac{\sigma_0 h}{E}\right) \frac{d\phi}{\delta} + d(\frac{\delta}{\phi}) \geq 0
$$

where both signs apply in region 3, the plus sign applies in region 4, and the minus signs apply in region 2. These conditions have a simple geometrical interpretation in the displacement plane. For example consider a point on the displacement path $(\delta, \phi)$ and a path increment for which $d\delta$ and $d\phi$ are both positive. Suppose further that $\delta > \sigma_0 h/E$. Then Eqs.(28) may be written in the form
\[ \frac{d\phi}{d\delta} \geq \frac{\phi}{\frac{\sigma_0 h}{E} + \delta} \quad \text{and} \quad \frac{d\phi}{d\delta} \leq \frac{\phi}{\frac{\sigma_0 h}{E} - \delta} \] (29)

In region 4 this simply states that the local tangent to the displacement path must lie between the two rays passing through \((\delta,\phi)\) and emanating from the two points on the \(\delta\)-axis \((\sigma_0 h/E)\) and \(-\sigma_0 h/E\).

Thus acceptable and unacceptable paths can be differentiated easily by the criterion of Eq.(28) in the displacement plane, where, it should be noted, the criterion does not involve the crack length \(a\). Examples of acceptable and unacceptable loading paths are given in Fig. 4. Two observations which follow immediately from an examination of Eq.(28) are

1. all proportional displacement paths (straight line paths through the origin in the displacement plane) are acceptable, and,

2. as the plastic deformations become large (more specifically, as the displacement point moves away from the origin in region 4) the acceptable loading paths approach proportional displacement paths.

In the rigid plastic limit, \((\sigma_0/E) \to 0\), and the acceptability criterion becomes

\[ \pm \frac{d(\delta/\phi)}{d\delta} \geq 0 \]

This can be satisfied only if \((\delta/\phi)\) is constant, which means that the displacement path is a straight line through the origin, and the neutral axis does not move.
Acceptable loading histories cannot be characterized so simply in terms of the generalized forces. Except in the rigid-plastic limit, any proposed loading path in the generalized force plane could in principle be translated into an equivalent displacement path by using Eqs(18) through (21), and the image of this path in the displacement plane could be checked for acceptability by Eq(28). In region 3 Eqs.(20) can be inverted and Eq.(28) can be expressed directly as a constraint on the loading path. The resulting criterion is

\[
\frac{dM}{\sigma_0} = \frac{1}{2} \sigma_0 B \left[(2a-w)da - (a + \frac{P}{\sigma_0 B})(da + \frac{dP}{\sigma_0 B})\right] \tag{30}
\]

\[
+ \frac{1}{3} \sigma_0 B (da + \frac{dP}{\sigma_0 B}) \sqrt{3 \frac{w^2}{2L^2} + a^2 - aw - \frac{1}{2}(a + \frac{P}{\sigma_0 B})^2 \right] - \frac{3M}{\sigma_0 B} \geq 0
\]

for an acceptable loading path increment. This condition is considerably more complicated than the corresponding displacement condition, and in contrast to the latter it involves the crack length. Proportional loading paths (straight line paths in generalized force space) do not correspond to proportional displacement paths. However, for constant crack length representative calculations show that proportional loading paths rapidly approach proportional displacement paths in region 3, which indicates that proportional loading is acceptable under these circumstances.

Discussion

It is clear that the general theory of the J-integral formally applies to structures loaded by several generalized forces under conditions where a pseudo strain energy exists. A pseudo strain energy is expected
to exist in situations where the material behavior is modelled satisfactorily by the deformation theory of plasticity, and indeed we have explicitly verified this expectation for the combined bending and axial loading of a cracked beam segment with arbitrary cross-section and material properties. The detailed analysis of the cracked elastic-plastic beam of rectangular cross-section has further clarified this problem. Assuming the absence of plastic unloading an explicit expression was derived for the pseudo strain energy and J was computed directly therefrom. Significantly, the analysis shows that a rigid-plastic analysis, which is much simpler than the full elastic-plastic analysis, yields directly the correct limiting value of J. That is, with no unloading the pseudo strain energy persists in the rigid-plastic limit. However, in this limit the conditions necessary to strictly insure no plastic unloading ahead of the crack are very stringent: the neutral axis must not move.

Actually, the criteria for an acceptable loading path are quite subtle. For example, if a limited amount of unloading occurs at some point on an acceptable path but this unloading is insufficient to produce reversed plastic yielding of any fiber, and if the structure is subsequently re-loaded past the original point of unloading, the resulting path may be considered acceptable in that the force-displacement relations associated with monotonic straining hold in the final state. On the other hand, if the limited unloading is not followed by re-loading the resulting path would be, strictly speaking, unacceptable. Considerations of this nature and the related topics of structural behavior under cyclic loading are not treated in this initial investigation.
It is also significant that the expressions for \( W \) and \( J \) are each separable into two terms: one term is independent of the segment (half) length \( h \), while the other term is proportional to \( (h \sigma_0/E) \). Now the former term represents the rigid-plastic contribution, and the latter term vanishes in the rigid-plastic limit. But this second term also vanishes for fixed \( (\sigma_0/E) \) as \( h \rightarrow 0 \), that is, as the length of the segment is made very small compared to the cross sectional dimensions. This suggests that in computing \( W \) and \( J \) one can, at least approximately, separate the problem into two parts: a region near the cracked section where plasticity dominates and the elastic contribution is unimportant, and the remainder of the structure where, presumably, elasticity dominates. This point of view has been adopted by Tada et al [6] in their applications of tearing instability theory to piping problems.

The expressions derived for \( W \) and \( J \) in terms of the generalized displacements and crack length assume that the loading history follows an acceptable path. It should be emphasized that the word "acceptable" is used here in a restricted technical sense to mean paths which preclude plastic unloading on the uncracked ligament, according to the approximations of simple beam theory. Criteria have been presented by which any expected loading path may be checked for acceptability. If a loading path turns out to be unacceptable then the accuracy of the expressions for \( J \), and indeed the whole concept, becomes questionable. Nevertheless, it may still be true that if the path is not too far from acceptable then the expressions derived for \( J \) herein may be sufficiently accurate: this is a question to be decided by experiment and further analysis. In the case of large plastic deformations strictly acceptable paths must be close to paths of proportional displacement.
Conclusions

1) The theory of the J-integral, which establishes the relation between the crack-tip field intensity and the overall loads and deformations of a structure, can be applied to situations involving multiple independent loads providing that a pseudo strain energy can be shown to exist (at least approximately).

2) For the case of combined bending and axial force acting on a beam segment of arbitrary cross-section and material properties, explicit formulas have been given for the pseudo strain energy and J, for loading histories which involve no plastic unloading on the uncracked ligament.

3) For the special case of an elastic-perfectly plastic beam segment of rectangular cross-section explicit formulas for the pseudo strain energy and J were derived. The pseudo strain energy was shown to persist in the rigid-plastic limit, and the expression for J obtained directly by rigid-plastic analysis was shown to be the correct limiting value of J.

4) Criteria were derived to characterize acceptable loading paths which involve no plastic unloading on the uncracked ligament. For large plastic deformations acceptable loading paths must approach proportional displacement paths, which (for large deformations) coincide approximately with proportional loading paths.

Acknowledgement

The work reported in this paper was supported by funds from National Science Foundation Grant ENG77-20937 and Nuclear Regulatory Commission Contract No. 03-79-134. This support is gratefully acknowledged.
References

\[ b = \frac{\delta}{\phi} \quad \quad e = \frac{\sigma_0 h}{E \phi} \]

**FIGURE 3**
1. INTRODUCTION

The J-integral was originally proposed just over a decade ago by Rice [1]* as the contour integral

\[ J = \oint_{\Gamma} (Wdy - \mathbf{T} \cdot \frac{\partial \mathbf{u}}{\partial \mathbf{x}} \, ds) \]  

(1)

where \( W \) is the unique strain energy density

\[ W = \int_0^\varepsilon \sigma_{ij} \, d\varepsilon_{ij} \]  

(2)

assuming small strains, where \( \mathbf{T} \) is the traction normal to the contour, \( \Gamma \), and \( \mathbf{u} \) is the displacement vector. The term in equation (1) involving \( Wdy \) represents the strain energy over the contour and the term \( \mathbf{T} \cdot \frac{\partial \mathbf{u}}{\partial \mathbf{x}} \, ds \) is the work done on the contour by the tractions for a forward extension of the crack tip in a nonlinear elastic material. Based on deformation theory, the J-integral is path independent when integrated around a crack tip.

*The numbers in parentheses in the text indicate references in the Bibliography.
Alternate forms [2] represent $J$ equally well as

$$J = \int_0^P \left( \frac{\Delta}{\partial a} \right)_P \cdot dP = -\int_0^\Delta \left( \frac{\partial P}{\partial a} \right)_a \cdot d\Delta$$ (3)

where $\Delta$ is the work producing component of the loadpoint displacement for the load $P$, and $a$ is the crack length. The forms of equation (3) imply that either $P$ is a function of $\Delta$ and $a$ or $\Delta$ is a function of $P$ and $a$.

It is helpful to divide $J$ into the elastic and plastic parts by noting that total displacement is the sum of linear elastic and plastic displacements. Specifically,

$$\Delta_{TOTAL} = \Delta_{EL} + \Delta_{PL}$$ (4)

Then from the first form of (3)

$$J_{TOTAL} = \int_0^P \left( \frac{\partial \Delta_{EL}}{\partial a} \right)_P \cdot dP + \int_0^P \left( \frac{\partial \Delta_{PL}}{\partial a} \right)_P \cdot dP$$ (5)

Thus $J$ is the sum of the linearly elastic portion, which is precisely the Griffith $G$,

$$J_{EL} = G = \int_0^P \left( \frac{\partial \Delta_{EL}}{\partial a} \right)_P \cdot dP$$ (6)

and the plastic portion, $J_{PL},$

$$J_{PL} = \int_0^P \left( \frac{\partial \Delta_{PL}}{\partial a} \right)_P \cdot dP$$ (7)
However, \( J_{PL} \) can also be expressed using the second form of (3) and the total expression of \( J \) can be written

\[
J_{\text{TOTAL}} = G + J_{PL}
\]

\[
= G - \int_{0}^{\Delta PL} \frac{dP}{da} \Delta PL \ d\Delta PL \tag{8}
\]

This convenient form of \( J \) can be used to analyze a load-displacement diagram.

To reduce equation (8) to a form that is suitable for numerical computation, the considerations of dimensional analysis are helpful. It has been argued [3] that, assuming plasticity is confined to the remaining ligament in the specimen, the functional form of the relationship between load, crack length and plastic displacement must be compatible with units of stress (force/length\(^2\)) or strain (dimensionless). Pursuing this dimensional analysis argument leads to useful functional forms of the load, \( P \). One form especially useful in the present analysis is

\[
P = \frac{b^2}{W} F \left( \frac{PL}{W}, \frac{a}{W}, \frac{L}{W}, \frac{B}{W}, \ldots \right) \tag{9}
\]

where \( P \) is assumed per unit thickness, \( W \) is the depth of a bend specimen, \( a \) is the crack length, \( L \) is the span, \( B \) is the width, and \( b \) is the remaining (uncracked) ligament, \( W-a \). Substituting (9) into (8) gives the following expression for \( J \):
\[ J = G + \frac{2b}{W} \int_{0}^{\Delta PL} F \, d\Delta PL - \frac{b^2}{W^2} \int_{0}^{\Delta PL} \frac{3F}{3(a/W)} \, d\Delta PL \]  

(10)

One final simplification is the resubstitution of (9) into (10) to get

\[ J = G + \frac{2}{D} \int_{0}^{\Delta PL} P \, d\Delta PL - \frac{b^2}{W^2} \int_{0}^{\Delta PL} \frac{3F}{3(a/W)} \, d\Delta PL \]  

(11)

where \( G \) is \( J_{EL} \), the second term is attributed to Rice and the third term is the Merkle-Corten [4] correction type term. Equation (11) has lost none of the precision for \( J \) compared to equation (3). Rather, it is more easily computed from a load-displacement record and is exact only for the cases where no crack extension has occurred.

It has been proposed in recent literature [5] that equation (11) can be applied to the growing crack case. Consider the load-displacement record for a hypothetical test in Figure 1, with an initial crack length, \( a \). Equation (11) is perfectly valid up to point 1, the point of crack growth initiation, if one is following curve A. Likewise, equation (11) is equally valid up to point 2 following curve B. Since \( J \) is a unique function of \( a \) and \( J \), it is independent of the path taken to arrive at a given point. Thus one may use equation (11) for small amounts of crack growth to integrate a series of scaled curves, which shall be called calibration functions, up to their point of intersection with the actual test record.
This would give the value of $J$ at various points after the onset of crack growth and offer a way to evaluate $J$ for the growing crack.

If the dependence of $P$ on the ratio $a/W$ is small in equation (9), the final term in (11) will be negligible compared to the first two. This would suggest that in the plastic range, the load can be scaled down by the ratio $(b_{\text{new}}/b_0)^2$. Specifically,

$$P_{\text{scaled down}} = P \cdot \frac{b_{\text{new}}^2}{b_0^2} = \frac{b_{\text{new}}}{W} \cdot P \quad (12)$$

By plotting $PW/Bb^2$ versus $\Delta_{PL}/W$ for a series of specimens with various $a/W$ ratios it is possible to see the resulting band of scatter of the $F$-function of equation (9). Curve-fitting the data with a functional form dependent only on $\Delta_{PL}/W$ (nondimensional plastic displacement) allows a calibration function to be obtained which projects the plastic deformation and corresponding value of $F$ that would take place if no crack growth occurred. It is then possible to use the relationship (12) to obtain a set of scaled curves, representing small amounts of crack growth, which cross the actual load-displacement record at distinct points as depicted by the curve B and point 2 in Figure 1. Thus, the value of $J$ at point 2 can be obtained by applying equation (11) along curve B up to point 2. It should be noted that the compliance, hence
the initial elastic slope of the P-Δ record, changes with crack growth. A shift in displacement will occur accordingly in the respective calibration curves.
2. FRACTURE TESTING AND EVALUATION
OF THE J-INTEGRAL-R CURVE

2.1 TEST SETUP AND PROCEDURE

A set of ten specimens were tested in three point bending with a/W ratios ranging from 0.375 to 0.880. Table 1 lists the specimens along with corresponding values for a₀/W, a₀, and b₀. The specimens were cut from the same plate of A36 structural steel with the span parallel to the direction of rolling. Average yield point for the steel was 41.7 ksi while the average ultimate tensile strength was 69.9 ksi. The dimensions of the specimens were 1 inch by 2 inches by 9 inches (S,L = 8 inches, W = 2 inches, B = 1 inch). Five specimens were fatigued to achieve a smooth sharp crack. The remaining five were blunt notched, that is, a small hole was drilled at the end of a sawcut to represent the crack. Blunt notches allow greater plastic deformations in the specimen without crack growth than do the fatigue cracks, thereby affording greater confidence in obtaining the calibration function. Fatigue cracking followed the guidelines set is ASTM E399-74 [6] with the value of ΔK (range in stress intensity factor) at no time exceeding 25 ksi \( \sqrt{\text{in}} \).

All ten specimens were tested at 70°F under displacement controlled conditions. A very slow, constant displacement rate was imposed to achieve monotonic quasi-static
loading. The test record was in the form of a load-displacement plot on an X-Y plotter. This plot was in turn digitized for purpose of numerical analysis. Figure 2 shows the test setup for three point bending. After testing, the yet intact specimens were heat tinted to color the fracture surfaces. Afterward they were chilled below transition temperature with liquid nitrogen and severed completely with a hammer blow. Fractured specimens appear in Figure 3. A nine point average was taken for the values of initial fatigue crack length according to the formula [6]

$$a_{ave} = \left( \frac{\sum a_e}{2} + \sum a_i \right) / 8$$  \hspace{1cm} (13)

where $a_e$ is the fatigue crack length at the edge of the specimen, and $a_i$ is the fatigue crack length inside the specimen.

2.2 REDUCTION OF DATA TO OBTAIN J-R CURVES AND THE TEARING MODULUS

Digitized load-displacement data from the ten specimens were used to obtain desired results. Rearranging equation (9) we get

$$\frac{PW}{Bb^2} = F \left( \frac{\Delta P L}{W}, \frac{a}{W}, \ldots \right)$$  \hspace{1cm} (14)

To determine the effect of $a/W$ in equation (14) the values of $F$ (or $PW/Bb^2$) were plotted against $\Delta P L / W$ in Figure
4. The maximum scatter of the envelope bounding the data is seen to be ± 3%. This was thought to be sufficiently small to neglect the dependence of the $F$ function on $a/W$. An account of the error introduced by neglecting this effect will be discussed in the following section. The data in Figure 4 were fit with the exponential function

$$F = A_1 + A_2 e^{(A_3 \cdot \frac{a_{PL}}{W})}$$

(15)

This functional form was chosen because it gave the closest correlation to the data presented here. Values for the three constants were found to be

$$A_1 = 23000$$
$$A_2 = -12352$$
$$A_3 = -11.801$$

The plastic portion of the calibration function was scaled by a multiplicative constant to match the individual records and later scaled consistent with equation (12) to obtain $J$-values.

As crack growth occurs in the specimen, the elastic compliance increases, rendering necessary a corresponding shift in the displacement of the calibration function. Since the only change in compliance occurring with crack growth is that in the specimen, only the difference in theoretical compliance [7] need be added to the original.
\[
\frac{\Delta}{P} = C_{\text{TOTAL}} = \left(\frac{\Delta}{P}\right)_{\text{TESTING APPARATUS}} + \left(\frac{\Delta}{P}\right)_{\text{SPECIMEN}}
\] (16)

Then

\[
\frac{3}{3a} \left(\frac{\Delta}{P}\right)_{\text{TOTAL}} = \frac{3}{3a} \left(\frac{\Delta}{P}\right)_{\text{SPECIMEN}}
\] (17)

Figure 5 shows a typical load-displacement diagram fitted with the calibration function (dashed extension). This function conforms with the plastic region of the \( P-\Delta \) curve up to the point of crack growth initiation, then continues upward. For a crack growth of, say, 4\% the scaled curve represents a scaling of the calibration function in the plastic region by the square of the ratio of remaining ligaments as suggested by equation (12). A corresponding shift in the elastic portion due to the change in compliance is also shown as are the scaled curves up to \( \Delta a \) of 20\% of \( a \).

To evaluate \( J \) from equation (11) we need to recognize that the last term, involving the integral of \( \phi F/\phi (a/W) \), will be very small compared to the first two. An assessment of the relative size of this term will follow later. This leaves the remaining expression of \( J \)

\[
J = G + \frac{2}{b} \int_0^{\Delta PL} P \, d\Delta PL
\] (18)
\( J_{PL} \), the second term in the above equation, is simply the area under the load-plastic displacement record, or

\[
J_{PL} = \frac{2}{B} \left( A_T - \frac{P^2 C_e}{2} \right)
\]

where \( A_T \) is the total area under the \( P-\Delta \) record up to the final displacement and load, \( P \) is that final value of the load and \( C_e \) is the effective compliance for the current crack length. Figure 6 shows graphically the expression in parentheses in equation (19).

\( G \), the first term in equation (18), is \( J_{EL} \),

\[
G = J_{EL} = \frac{K^2}{E}
\]

where \( K \) is the apparent elastic stress intensity factor at the current crack length and value of \( P \) [7], and \( E \) is the effective elastic modulus.

The values of \( J \) can then be found for various small changes in crack growth by evaluating equation (19) up to the corresponding intersections of the scaled calibration curves and the actual test record. Figure 7 shows the plot of the \( J \)-integral versus \( \Delta a \) for four fatigue cracked specimens. The fifth specimen was discarded due to a large \( a/W \) value (0.88) and the interference of load point plasticity with the plasticity developed at the crack tip was suspect.
Slopes from the $J-\Delta a$ curves in Figure 7, $dJ/da$, were multiplied by the factor $E/\sigma_o^2$, where $\sigma_o$ is the flow stress from deformation theory, to yield the nondimensional tearing modulus [8],

$$T = \frac{dJ}{da} \cdot \frac{E}{\sigma_o^2}$$  \hspace{1cm} (21)

A plot of the tearing modulus versus the $J$-integral appears in Figure 8. These data illustrate that although $T$ is not constant for all values of $J$ past $J_{IC}$, as will be idealized and discussed later, there is an obvious lower bound on $T$ for a given value of $J$. Since $T$ is proportional to the slope of the $J-\Delta a$ curve, it represents the capacity a material possesses to resist further crack growth. Thus for a given amount of energy at the crack tip as described by a value of $J$, the question of crack growth stability is answered by the corresponding value of the tearing modulus. Reference 8 gives $J_{IC}$ and $T$ values for several other steels. Comparison with these data show that A36 structural steel has a $J_{IC}$ value comparable to ASTM-A471 (Ni - Cr - Mo - V) rotor steel and $T$ values similar to ASTM-A217 cast steel. Some of the applications of this stability concept will be discussed in a following section.
2.3 ANALYSIS OF ERRORS INTRODUCED BY NEGLECTING THE DEPENDENCE OF THE F-FUNCTION ON a/W

2.3.1 Error in $J_{IC}$

As was stated previously, neglecting the $a/W$ dependence in the plastic $F$-function of equation (9) results in the truncation of the final term, the Merkle-Corten term, in equations (10) and (11). It is possible to evaluate this term based on the data presented here. By taking the difference in the data values of $F$ and dividing by the difference in $a/W$, the resulting $\frac{\partial F}{\partial(a/W)}$ can be estimated and plotted against plastic displacement. This assumes that the partial derivative, $\frac{\partial F}{\partial(a/W)}$, is constant over the interval of $a/W$ in question. The integral in the error term

$$E_{M-C} = -\frac{b^2}{W^2} \int_0^{\Delta_{PL}} \frac{\partial F}{\partial(a/W)} d\Delta_{PL}$$

is then evaluated as the area under the plot times the leading coefficient

$$E_{M-C} = -\frac{b^2}{W^2} \cdot (Area \ under \ \frac{\partial F}{\partial(a/W)} \ vs. \ \Delta_{PL})$$

Figure 9 shows graphically $\Delta F/\Delta a/W$ versus $\Delta_{PL}$ for the worst case in specimen A36EB2. Area under the curve is 172 inch-pounds per square inch and the resulting error term from equation (23) is 62 in-lbs/in², an error of 8.8%
in the value of $J_{lc}$ for the specimen. Errors for the other specimens considered are listed in Table 2. To illustrate the sensitivity of this error analysis to material variability or other sources, the entire 3% envelope of $F$ suggested in Figure 5 is imposed in the $a/W$ region of the specimen A36EB2. By elevating the $F$ values for A36EB4 by 3% and depressing those for A36EB2 by the same amount, the error term is less than 64 in-lbs/in$^2$, still only a 9% error.

2.3.2 Error in $dJ_{pl}/da$

Consideration of the expression of $J_{pl}$ in equation (11) and the implicit functional form of $P$ in equation (9), an estimate of the error in the plastic slope of the $J$-$R$ curve $dJ_{pl}/da$, can be obtained. For convenience, the two equations are stated here:

$$J_{pl} = \frac{2}{b} \int_{0}^{\Delta_{pl}} P \cdot d\Delta_{pl} - \frac{b^2}{W^2} \int_{0}^{\Delta_{pl}} \frac{3F}{3a/W} \ d\Delta_{pl} \quad (11)$$

$$P = \frac{b^2}{W} \cdot F \left( \frac{\Delta_{pl}}{W}, \frac{a}{W}, \ldots \right) \quad (9)$$

By taking the differential of $P$ above

$$dP = \frac{b^2}{W^2} \frac{3F}{3(\Delta_{pl}/W)} \ d\Delta_{pl} + \frac{b^2}{W^2} \frac{3F}{3(a/W)} - 2b \ F \ da \quad (24)$$

it is possible to solve for $da$:
\[ da = \frac{b^2}{W^2} \frac{3F}{\delta(a/W)} \cdot d\Delta_{PL} - dP \]  

(25)

Similarly, by differentiating the expression for \( J_{PL} \) above, \( dJ_{PL} \) can be obtained

\[ dJ_{PL} = \left[ \frac{2b}{W} \cdot F - \frac{b^2}{W^2} \frac{3F}{\delta(a/W)} \right] d\Delta_{PL} \]

\[ + \left[ -\frac{2}{W} \int_0^{\Delta_{PL}} F \ d\Delta_{PL} + \frac{4b}{W^2} \int_0^{\Delta_{PL}} \frac{3F}{\delta(a/W)} \ d\Delta_{PL} \right] da \]

(26)

Equations (25) and (26) can be combined to give the expression for \( dJ_{PL} / da \) for the growing crack [3]

\[ \frac{dJ_{PL}}{da} = \left[ \frac{2b}{W} \cdot F - \frac{b^2}{W^2} \frac{3F}{\delta(a/W)} \right]^2 \left[ \frac{b^2}{W^2} \cdot \frac{3F}{\delta(\Delta_{PL})} - \frac{dP}{d\Delta_{PL}} \right] \]

\[ - \frac{2}{W} \int_0^{\Delta_{PL}} F \ d\Delta_{PL} + \frac{4b}{W^2} \int_0^{\Delta_{PL}} \frac{3F}{\delta(a/W)} \]

\[ - \frac{b^2}{W^3} \int_0^{\Delta_{PL}} \frac{3F}{\delta(a/W)^2} \ d\Delta_{PL} \]  

(27)
Computation of the four terms in equation (27) for specimen A36EB2 shows the first term to be about 26000, the second term to be -250, the third to be 150, and the fourth term about 13. Although the evaluation of the first term is numerically sensitive to choice of data points due to the uncertainty of the point of crack growth initiation, this effect vanishes with slight increases in computational step size. Comparison with the slope for A36EB2 found in Figure 7 shows the slope to be about 28000, the difference being made up of $dJ_{LE}/da$ from the relationship

$$
\frac{dJ}{da} = \frac{dJ_{PL}}{da} + \frac{dJ_{PL}}{da} \quad (28)
$$
3. STRUCTURAL APPLICATIONS OF J-INTEGRAL-R CURVES
AND THE TEARING MODULUS

3.1 INTRODUCTION

In review of the concepts behind elastic-plastic fracture mechanics, consider the schematic J-integral versus crack length curve in Figure 10. To a first approximation and for many practical purposes, the J-R curve can be represented by two linear regions, A and B. Region A is not shown in Figure 7 as only J versus crack growth after initiation is of interest. Although from Figure 7 it is seen that region B is not exactly linear, a conservative approximation can be assumed. The slope of this curve in region A characterizes the blunting of a sharp crack loaded monotonically in a quasi-static manner. The slope, dJ/da, in region B characterizes the behavior of a growing crack. Their intersection at the point $J_{IC}$ represents the transition between the two conditions and is used in the field as one of the parameters describing the fracture behavior of a particular material. The J-R curve in region B for A36 structural steel shown in Figure 7 suggests that the precise point for crack growth initiation (governing the area integral and value of $J_{IC}$) is not obvious. However, inspection of Figures 7 and 8 suggest that the slopes for the different specimens remain consistent for a given J value. It may be stated from Figure 8 that a reasonable lower bound on
T for A36 structural steel is 100, at least for J-values up to about 2400. As was stated previously, the tearing modulus for material, $T_{\text{MAT}}$, is comprised of material characteristics and governs the stability behavior of the crack. (Reference [7] presents experimental evidence to support this concept.) With this in mind, it is possible to approach the analysis of various indeterminate structures from a fracture mechanics point of view.

3.2 A GENERAL CASE: BEAM BENDING

Consider the simple case of a beam in bending. An analogous form of J presented in equation (3) is [2]

$$J = -\int_0^\phi \frac{3M}{3a} \, d\phi$$

(29)

where now M is the moment per unit thickness, and $\phi$ is the corresponding displacement, rotation. Taking the differential of J

$$dJ = -\frac{3M}{3a} \, d\phi$$

(30)

an exact expression for the increment of J per increment of rotation, $d\phi$, is obtained. Figure 11 shows the schematic representation of a general cracked cross section subject to bending. In Figure 11.b the fully plastic moment for the remaining ligament at the cracked section is shown along with the stress block acting on that ligament. The
expression for the increment of plastic moment can be written by inspection

\[ dM_p = -\sigma_o t \cdot h \cdot da + \text{second order effects} \quad (31) \]

Let a section property, \( N \), be the ratio of remaining ligament to the distance between crack tip and neutral axis,

\[ N = \frac{b}{h} \quad (32) \]

Note that \( N \) is always between 1 and 2. Then neglecting second order effects,

\[ dM_p = -\sigma_o \frac{tb}{N} \cdot da \quad (33) \]

Due to full plasticity in the uncracked ligament of a member in a redundant structural system, the cracked section acts as a hinge with regard to any loads placed on the rest of the structure. The rotation of the cracked section caused by an increment in moment at that section can be stated as

\[ d\phi = \frac{k}{E} dM \quad (34) \]

where \( k \) is proportional to the compliance, or inversely to the stiffness, and \( E \) is the elastic modulus. Combining equations (30), (33) and (34)

\[ \frac{dJ}{da} \cdot \frac{E}{c_o} = \frac{k b^2}{N} \cdot \frac{t}{2} = T_{APPL} \quad (35) \]
Note that the tearing modulus of (35) consists of parameters describing structural geometry and stiffness rather than material properties. Thus equation (35) is the tearing modulus "applied" to the crack from the remaining structure. By a comparison of \( T_{\text{APPL}} \) and \( T_{\text{MAT}} \) a crack growth stability criterion is established [9]

\[
\begin{align*}
T_{\text{APPL}} & \leq T_{\text{MAT}} & \text{stable} \\
T_{\text{APPL}} & > T_{\text{MAT}} & \text{unstable}
\end{align*}
\] (36)

A brief explanation of statement (36) is helpful. Given an indeterminate structure with dead or quasi-static loads on it and a fully plastic ligament at a cracked section, a small advance of the crack (a perturbation, in other words) causes a reduction of the remaining section area and a corresponding reduction of the plastic moment. Since the loads are assumed dead, this increment of moment must be absorbed elastically by the rest of the structure. This in turn imposes an increment of angle change at the cracked section. In short, if the material can withstand this angle change without advancing the crack further, the situation is stable, that is, rapid fracture does not occur. On the other hand, if the material cannot withstand this angle change without advancing the crack, a chain reaction will occur unstably. It is important to note from equation (35) that this behavior is independent of the dead load on the structure.
3.3 EXAMPLE CASES OF CRACK STABILITY ANALYSES ON STRUCTURAL SYSTEMS

3.3.1 The Fixed-Fixed Beam

Consider the simple case of a single span beam with fixed supports at both ends, a length, \( L \), and prismatic in cross section. With an arbitrary system of dead loads on the beam, a cracked section at midspan develops a fully plastic moment as shown in Figure 12.a. Allowing an increment of crack growth imposes an increment of moment and angle change

\[
d\phi = \frac{dM}{E} \cdot \frac{L}{I}
\]  

(37)

Comparing the above equation with (34) gives

\[
k = \frac{L}{I}
\]  

(38)

and from equation (35)

\[
T_{\text{APPL}} = b^2 t \cdot \frac{N^2}{I} \cdot \frac{L}{I}
\]  

(39)

Since \( N \) does not change as fast as \( b \), \( T_{\text{APPL}} \) for the fixed-fixed beam depends somewhat on \( b \) and \( T_{\text{APPL}} \) will decrease with increasing crack length. This implies that an unstable fracture might arrest itself as the crack grows and \( T_{\text{APPL}} \) drops below \( T_{\text{MAT}} \).

To illustrate this concept, numerical values are assumed for the fixed-fixed beam described above. With
a rectangular cross section identical to that for the
three-point bend specimens, the depth of the cross sec-
tion, \( W \), is 2 inches and the width, \( B \) (or \( t \) in equation
(39)), is 1 inch. For the rectangular section \( N \) remains
2 as the crack grows and the cross sectional moment of
inertia is \( 2/3 \text{ inch}^4 \). From Figure 8 a conservative value
of \( T_{\text{MAT}} \) is 100 for \( J \) values less than 2400 in \text{lbf/in}^2.
Thus, consideration of the stability criterion of (36)
and equation (39) yields

\[
T_{\text{APPL}} = \frac{3}{2} \cdot L
\]

for a remaining ligament of 2 inches (the worst case, i.e.,
a very small crack), and a span of 67 inches is required
to obtain instability. The slenderness ratio, \( L/r_y \),
where \( r_y \) is the radius of gyration about the weak axis,
is 230 for this span, i.e., the beam is very slender.
For purposes of design, the ratio of span to moment inter-
tia, \( L/I \), can be used to bound \( T_{\text{APPL}} \) for the worst case
(\( b=2 \)) and assure the stability criterion (36) is always
met for \( J \) below 2400.

This analysis can be carried a step further by first
generalizing equation (39)

\[
T_{\text{APPL}} = \psi \cdot \frac{b^2 t}{N^2} \cdot \frac{L}{I}
\]  

(40)
Now allow an additional plastic hinge to form at a non-cracked section, say, at the nearest fixed end, thereby changing the elastic characteristics of the beam for the incremental analysis. Figure 13 shows the double-fixed beam with a crack placed arbitrarily within the span and a dead load. From elementary structural analysis

\[ d\phi = \psi \cdot \frac{dM_P}{E} \cdot \frac{L}{I} \]  \hspace{1cm} (41)

where the coefficient \( \psi \) in equations (40) and (41) is the same. For the particular case in Figure 13, \( \psi \) is readily found to be

\[ \psi = \frac{L^2}{3\zeta^2} \]  \hspace{1cm} (42)

where \( \zeta \) is the distance from the cracked section to the location of the second plastic hinge. A worthwhile exercise in determining the range of \( \psi \) values in (42) can be performed by realizing that the limits on the ratio of distance from the crack to the near end to the total length, \( a \) are

\[ \frac{1}{4} \leq a \leq \frac{1}{2} \]  \hspace{1cm} (43)

where the upper bound of 1/2 is attributed to symmetry and the lower bound of about 1/4 is the point when the influence of shear as the cracked section nears the support
makes it unlikely to induce full plasticity at the cracked section. Thus, formation of the beam end plastic hinge increases $\psi$ from 1 to:

$$\frac{4}{3} \leq \psi \leq \frac{16}{3}$$  \hspace{1cm} (44)

if the effects of hardening are ignored and $\psi$ can be readily substituted into equation (40). Similar results are obtained if the crack is at the support and the additional plastic hinge is arbitrarily positioned in the span. In this case, if the effects of shear are small,

$$\psi = \frac{1}{6\xi} \left( 2\xi^3 - 6\xi^2 + 6\xi + 1 \right)$$  \hspace{1cm} (45)

where $\xi = L/a$. Considering the relationship (43)

$$\frac{5}{12} \leq \psi < \frac{57}{24}$$  \hspace{1cm} (46)

### 3.3.2 Simple Examples with Rigid Frames and Continuous Beams

Consider the continuous, three span beam of Figure 14. Again, arbitrary dead loading is assumed. It is a straightforward exercise to obtain the relationship between $d\psi$ and $dM$ at the cracked section using the moment area method

$$d\psi = \frac{dM \cdot L}{EI} \left( 1 + \frac{a+b}{3} \right)$$  \hspace{1cm} (47)

Comparing this result with equation (41)
\[ \psi = 1 + \frac{a + \beta}{3} \] (48)

An analogous system is the single bay rigid frame with pinned supports shown in Figure 15.a. With the prevention of sideways \( d\phi \) can be expressed from equation (47) with \( \alpha \) and \( \beta \) equal to \( \gamma \)

\[ d\phi = \frac{dM \cdot L}{EI} \cdot \left( 1 + \frac{2\gamma}{3} \right) \] (49)

and for \( \gamma = 1 \), \( \psi = 1.67 \). Note that care should be exercised when applying this relationship above to equation (40) as axial load in the columns will effect their apparent stiffness [10] and \( \alpha \) and \( \beta \) must be adjusted accordingly.

With the rectangular cross section from the bend test (2 inches by 1 inch), \( \alpha \) and \( \beta \) (scalars) can be set and the length of the span with the crack determined. First, let \( \alpha \) and \( \beta \) equal 1. Consideration of equation (40) and (36) with \( T_{\text{MAT}} \) equal to 100 yields a length of 40 inches in all three spans to induce instability of the crack. The slenderness ratio, \( L/r_y \), is 139, corresponding to a moderately slender beam. For \( \alpha \) and \( \beta \) equal to 2, \( L \) is 29 inches and the \( \alpha \)- and \( \beta \)-spans are 58 inches, quite slender sections with \( L/r_y \) of about 200. Now let \( \alpha \) equal 0, corresponding to a fixed support, and \( \beta \) equal 2. The value of \( \psi \) from equation (48) in this case is \( 5/3 \) and a cracked-section-span of 40 inches is required for instability. The 80 inch \( \beta \)-span, then, has a \( L/r_y \) ratio of 273, unacceptably slender.
The closed frame of Figure 15.b can also be analyzed quite easily using the method of consistent deformations. Here the expression for incremental angle change with incremental moment is found to be

$$d\psi = \frac{dM_o L}{EI} \cdot \frac{(3+\alpha)(1+\gamma)}{(3+2\alpha)}$$  \hspace{1cm} (50)$$

With this concept of apparent end stiffnesses in mind, the list of applications to structural problems seems quite long indeed and it is possible to estimate different length and/or stiffness parameters to obtain $\psi$. For instance, if the base supports for the single bay frame of Figure 15.a are fixed and $\gamma = 1$, the estimate for the lengths, $\alpha$ and $\beta$, of the analog, might be about 0.8 due to the inflection point in the columns. This would make $\psi$ about 1.6 to be put into equation (40). Analyses of substructures are also possible, such as the one shown in Figure 16. In this case, the $\alpha$ and $\beta$ values in equation (47) are 0.33 and $\psi$ is 1.22. There are many more examples which can be found by imposing cracks at various locations in almost any elementary structures problem, imposing a unit moment and determining the resulting angle change. For the more complex cases, it is possible to "trick" existing computer programs, such as STRUDL or NASTRAN, to give the angles induced at a joint release (the model of a fully plastic cracked section) by a unit moment at that joint.
Although these analyses are straightforward and require a minimum of effort to perform, there should be no attempt to use these methods without an understanding of the underlying assumptions of fracture mechanics. More rigorous analyses account for phenomena such as strain hardening, the coupling of axial load with bending torsion, and elastic buckling. Loading time effects as well as alternating plasticity at the crack tip also require more strenuous derivations. Yet experimental evidence [8] demonstrates very good agreement with this stability theory suggesting that more advanced considerations contribute little additional information at least for the case of three-point bending with various loading compliances. Although only the fracture analysis for the bending of frames is presented here, it should be noted that this type of stability analysis could be applied to other problems [11] such as those involving predominantly tension. As testing of these analyses for various configurations continues, confidence in the ability to design structures which resist unstable fracture will grow. It is important to realize that the need for the capability of locating cracks in structures will always exist. Yet in the light of the type of fracture stability analyses outlined here, a more realistic assessment of the remaining structural integrity of a damaged structure is possible.
4. SUMMARY AND CONCLUSIONS

A brief outline of the development of the J-integral into a form suitable for the analysis of load-displacement records with considerations of dimensional analysis, has been presented here. A series of three point bend specimens of A36 structural steel with varying a/W ratios have provided information necessary in assessing the size of errors in the Merkle-Corten type in J. Under the assumption that the data describe a definite trend in the dependence of the plastic F-function on crack size, the following conclusions are reached:

1) The methods of determining J-R (J-Δa) curves from load displacement records using scaled calibration functions is practical and sufficiently accurate.
   1a) Errors of less than 10% in the worst case for J were found.
   1b) Errors of less than 2% in the slope of the J-R curve, dJ/da, were also found.

2) Comparison of the J-R curve and T values of A36 structural steel with other steels shows that it is quite resistant to crack growth initiation as well as substantially resistant to unstable crack growth.

Applications of the tearing modulus concept for assessing fracture stability have also been presented.

It has been shown that the analysis of moment-curvature
relationships for various structural systems leads to a statement of stability involving the balance of T characteristic of the material, $T_{\text{MAT}}$, with the T applied from the structure, $T_{\text{APPL}}$. The following conclusions are reached:

3) For the cases presented, the T applied does not directly depend on the dead or quasi-static loads affecting the structure.

4) It is possible to discriminately choose system geometry, e.g., length and moment of inertia, such that for the worst conditions of cracked sections, $T_{\text{APPL}}$ is less than $T_{\text{MAT}}$ thereby allowing plastic collapse failure mechanisms rather than fracture mechanisms to govern.

5) In the light of conclusion (2), the application of A36 structural steel in some configurations makes unstable fracture under dead loading highly unlikely as other criteria such as slenderness govern design.
**TABLE 1**

A36 Three-Point Bend Specimens with Crack Type and Initial Crack Length

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Crack Type*</th>
<th>$a_o/W$</th>
<th>$a_o$</th>
<th>$b_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A36EB1</td>
<td>BN</td>
<td>0.375</td>
<td>0.75</td>
<td>1.25</td>
</tr>
<tr>
<td>A36EB2</td>
<td>FC</td>
<td>0.399</td>
<td>0.798</td>
<td>1.202</td>
</tr>
<tr>
<td>A36EB3</td>
<td>BN</td>
<td>0.550</td>
<td>1.100</td>
<td>0.90</td>
</tr>
<tr>
<td>A36EB4</td>
<td>FC</td>
<td>0.576</td>
<td>1.152</td>
<td>0.848</td>
</tr>
<tr>
<td>A36EB5</td>
<td>BN</td>
<td>0.700</td>
<td>1.400</td>
<td>0.600</td>
</tr>
<tr>
<td>A36EB6</td>
<td>FC</td>
<td>0.712</td>
<td>1.424</td>
<td>0.576</td>
</tr>
<tr>
<td>A36EB7</td>
<td>FC</td>
<td>0.810</td>
<td>1.620</td>
<td>0.380</td>
</tr>
<tr>
<td>A36EB8</td>
<td>BN</td>
<td>0.800</td>
<td>1.600</td>
<td>0.400</td>
</tr>
<tr>
<td>A36EB9</td>
<td>BN</td>
<td>0.875</td>
<td>1.750</td>
<td>0.250</td>
</tr>
<tr>
<td>A36EB10</td>
<td>FC</td>
<td>0.880</td>
<td>1.759</td>
<td>0.241</td>
</tr>
</tbody>
</table>

*BN : Blunt Notch  
FC : Fatigue Crack
<table>
<thead>
<tr>
<th>Specimen</th>
<th>Error Term in $J_{IC}$*</th>
</tr>
</thead>
<tbody>
<tr>
<td>A36EB2</td>
<td>- 62 $\frac{\text{in}-\text{lbs}}{\text{in}^2}$</td>
</tr>
<tr>
<td>A36EB4</td>
<td>0</td>
</tr>
<tr>
<td>A36EB6</td>
<td>- 19 $\frac{\text{in}-\text{lbs}}{\text{in}^2}$</td>
</tr>
<tr>
<td>A36EB7</td>
<td>- 13 $\frac{\text{in}-\text{lbs}}{\text{in}^2}$</td>
</tr>
</tbody>
</table>

* $(J_{IC} = 600)$
Figure 1. Hypothetical load versus displacement record for crack lengths \( a_0 \) and \( a_0 + da \).
Figure 2. Test setup for 3-point bending.
Figure 3. Fractured Specimens, a) A36EB1, blunt notched, b) A36EB2, fatigue cracked.
Figure 3, cont.  c) A36EB3, blunt notched, d) A36EB4, fatigue cracked,
Figure 3, cont.  e) A36EB5, blunt notched, f) A36EB6, fatigue cracked,
Figure 3, cont.  g) A36EB7, fatigue cracked, h) A36EB8, blunt notched,
Figure 3, cont.  i) A36EB9, blunt notched, j) A36EB10, fatigue cracked.
Figure 4. F-function versus $\Delta_{PL}/W$ for A36 Structural Steel 3-point bend specimens.

Envelope from blunt-notched specimens

$\frac{PW}{b^2} = 23000 - 12352 \cdot e^{(-11.8 \cdot \Delta_{PL}/W)}$

- A36EB2
- A36EB4
- A36EB6
- A36EB7

$\Delta_{PL}/W \quad (10^{-3})$
Figure 5. Load versus Displacement diagram for specimen A36EB2 with scaled calibration curves for up to 20% crack growth.
Figure 6. Schematic Load-Displacement diagram illustrating terms from equation (19).
Figure 7. J-Integral versus Crack Length Change (J-R curve) for A36 Structural Steel (3-point bend specimens).
Figure 8. Tearing Modulus versus J-Integral for A36 Structural Steel (3-point bend specimens).
Figure 9. Change in Plastic F-function with respect to crack length versus Plastic Displacement for a worst case.
Figure 10. Schematic J-Integral versus Crack Length (J-R curve) showing the regions of crack blunting and crack propagation.
Figure 11. A General Beam Subject to Bending, a) the fully plastic remaining ligament rotates through the angle, $\phi$, b) the general cracked section subject to a fully plastic stress block and incremental crack growth.
Figure 12. The Fixed-Fixed Beam With Crack Under Arbitrary Dead Loading, a) beam with fully plastic cracked section, b) increment of angle, $d\phi$, at the cracked section induced by increment of unloading moment, $dM$. 
Figure 13. The Fixed-Fixed beam of Figure 12 with an additional plastic hinge formed at a support.
Figure 14. A continuous three span beam under arbitrary loading develops a crack in the central span and a fully plastic remaining ligament.
Figure 15. Two Simple Frames, a) a single bay with pinned supports and a cracked cross beam, and b) a closed frame with cracked cross beam, both under arbitrary dead loading.
Figure 16. Substructure of a larger frame with a cracked beam.
6. ACKNOWLEDGEMENTS

The support of the Solid Mechanics Program of the National Science Foundation grant number ENG77-20937 is gratefully acknowledged. The author wishes to thank all of his colleagues at the Center for Fracture Mechanics at Washington University for the many stimulating conversations that have contributed so much to the author's understanding of Fracture Mechanics.
7. BIBLIOGRAPHY


STABILITY ANALYSIS OF CIRCUMFERENTIAL CRACKS 
IN REACTOR PIPING SYSTEMS *

by H. Tada, P.C. Paris, and R. Gamble

ABSTRACT

The high ductility and toughness of the stainless steel reactor piping system have made it virtually certain not to experience unstable crack extension. The present study has attempted to provide theoretical assurance that the piping system will not experience unstable crack extension even if severe cracking should occur.

The analysis is based on the tearing instability concept and the associated tearing modulus criterion. The method of analysis was discussed. Simplifications were conservatively made to facilitate the complicated analysis. The results were presented parametrically for convenience of general use. An application to a specific example was also discussed.

The results of this study indicate that the ratio $L/R$, is of major importance in consideration of crack stability, where $L$ is the length of the pipe between supports and $R$ is the radius of the pipe. This is readily seen from the general expression of $T_{appl}$, the applied value of $T$, which is given by

$$T_{appl} = F_1 \cdot \frac{L}{R} + F_2 \cdot \frac{JF}{\sigma_0^2 R}$$

The coefficients $F_1$ and $F_2$ are numerical values of the order of unity, depending on the geometry of the cracked section and the internal pressure. In the range of ordinary situations, the first term is large relative to the second term.

It is indicated that unstable crack extension would not occur in stainless steel piping systems designed in accordance with the ASME code even if severe stress corrosion cracking were present, provided that the values of $L/R$ are less than approximately 200. Since the values of $L/R$ for BWR stainless steel piping systems are generally much smaller, large margins against unstable fracture are assured for these systems. When $L/R$ is exceedingly large ($L/R > 200$), a more detailed analysis would be necessary to demonstrate crack stability.

*Published as NUREG/CR-0838, June 1979.
TECHNIQUES OF ANALYSIS OF LOAD-DISPACEMENT RECORDS BY J-INTEGRAL METHODS
by Hugo Ernst and Paul C. Paris

An outline was presented of the work done at the Center for Fracture Mechanics, Washington University, on the recently developed single test record analysis methodology.*

This analysis is based on recent work of Hutchinson [1] and Paris et al [2] which suggested that load-displacement records for pure bending could be analyzed to determine J-R curves and instability related material-structure properties, i.e. the material and applied tearing modulus, $T_{mat}$ and $T_{app}$. Although here, the analysis is generalized and shown to be applicable to all 2D configurations, and that it is especially useful for typical test configurations such as compact, 3-point bend, center cracked, etc. configurations.

Indeed, the analysis is "exact" from an analytical viewpoint, and is based on dimensional considerations in the spirit of Rice, Paris and Merkle analysis [3], based on Rice's J-integral concept [4]. The strict deformation theory of plasticity interpretation of J is used which have been shown to be exact under the size restrictions described by Hutchinson and Paris [1,5] for application to determining J-R curves and related material properties.

From the analysis, methods are developed to obtain from a load-displacement test record (without further instrumentation such as unloading compliance, for example), the following results:

1) Correct (exact)expressions for J with crack growth present.
2) Crack length change $\Delta a$.
3) The material characteristic J-R curve.
4) Material-structure tearing instability properties $T_{mat}$ and $T_{app}$.

*To be published by the U.S. Nuclear Regulatory Commission as a NUREG Report.
Furthermore, possible simplifications of the analysis are discussed via Turner's $\eta$-factor [6]. Conditions for the existence of $\eta$ are explored in terms of the separability of the load into multiplicative functions of displacement and crack length.

Finally, results of analysis of experimental work on H.Y. 130 steel and Ni-Cr-Mo-V rotor steel are presented showing the adequacy of the general methodology.

[1] Hutchinson J.W. and Paris P.C.,
"Stability Analysis of J-controlled Crack Growth"

"A Treatment of the Subject of Tearing Instability"NUREG Report 0311
Washington University, St.Louis 1977;

"Some Further Results of J-Integral Analysis and Estimates"
Fracture Toughness, ASTM, STP 514;

[4] Rice, J.R.
"A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks"

"Fracture Mechanics in the Elastic Plastic Regime"

"Method for Laboratory Determination of $J_{IC}$"
A PLASTIC ZONE INSTABILITY PHENOMENA
LEADING TO CRACK PROPAGATION

by
Jose A. Vazquez*
and
Paul C. Paris*

ABSTRACT
The stability of the static equilibrium between the elastic stress field and the plastic zone at the tip of a crack is analyzed.

The existence is shown of a situation in which the gradient with respect to the crack size of the elastic stress field at the tip of the crack becomes sufficiently large that the plastic zone cannot maintain stable static equilibrium and plastic zone instability occurs, followed by the propagation of the crack.

The theoretical results are verified experimentally using 2024-T3 and 7078-T6 Al alloys.

*Formerly Department of Mechanics, Lehigh University, Bethlehem, Pa. Now Professor of Mechanics, Washington University, St. Louis, Mo.

+Formerly Research Assistant, Lehigh University, then at Centro De Investigaciones Metalurgicas, Cordoba, Rep. Argentina; Now with the Research Laboratory of the Argentine Navy, Buenos Aires, Argentina.
I. INTRODUCTION

It is an experimental fact that crack propagation in a metallic body is always associated with plastic deformation. Orowan and Irwin [1][2] were the first to point out that the plastic deformation process could be considered as the principal energy sink during the crack propagation phenomenon. In this way, the toughness of a material, i.e., its capacity to absorb energy before the onset of fast crack propagation, is strongly conditioned by the plastic deformation processes taking place at the tip of the crack.

The mechanisms by which the material at the tip of the crack deforms plastically are quite complex. They take place at microscopic level by processes of which we have no quantitative satisfactory explanation. Therefore, an approach to the crack propagation problem based on the detailed processes (or energy exchanges) taking place along the leading edge of the crack would involve quite complex analytical work.

Linear Fracture Mechanics partially avoids this problem. This approach assumes that fast crack propagation takes place when a critical intensity of the distribution of stresses surrounding the tip of the crack occurs. The power of Linear Fracture Mechanics as an analytical tool is based principally on the fact that the distribution of stresses in the region near a crack tip does not depend on the component geometry or loading conditions but only on the manner in which the crack surfaces are displaced relative to one another [3]. With respect to the intensity of the stress field itself, it was found that it is controlled by a unique factor K which is a function of the crack size, specimen geometry and applied loads [3]. Since the stress
field equations describe the stress distribution, which for a given mode remain unchanged, it could be expected that complete similarity of the stress distribution is present for two cracks in a given material sustaining similar $K$ factors. With this observation, the statement of a fracture criterion is reasonably straightforward. The simplest view is that the fracture process should take place when the stress intensity factor reaches a critical value, $K_C$.

Based on the linear continuum theory, the predictions of Linear Fracture Mechanics are strictly valid for elastic–perfectly brittle materials and also are very good for small scale crack tip yielding situations. Physically, the inverse square root singularity term for the stresses appearing in the tip stress distribution equations would cause material to deform plastically at the very near tip region. This fact was explored as early as 1957 in a report of the Boeing Airplane Company [4]. In this report it was pointed out that plastic deformation should take place in a region of radius $r_y$ given by:

$$r_y = \frac{K^2}{2\pi \sigma_{ys}^2}$$

where $\sigma_{ys}$ was taken to be the tensile yield strength. In 1963 Irwin and Koskinen [5] pointed out that when $r_y$ is small relative to crack size and net section, the elastic stress field at and outside the elastic plastic boundary, is given by the elastic solution in which the leading edge of the crack is placed at the center of the circular plastic zone. This analytical result could be taken as the condition for existence of the so-called "small scale yielding situation" [6]. Actually, it would be exact for opening Mode III (out-of-plane shear) and usually it is assumed valid for Mode I (tensile).
Later plasticity analysis carried out on rather ideal plastic materials [6][7] shows that for situations other than small scale yielding, the plastic zone geometry is not a unique function of the factor K. Assuming that the material toughness is conditioned by the plastic deformation process taking place at the leading edge of the crack, the plasticity analysis results would invalidate the use of K as a fracture controlling parameter in situations other than small scale yielding. Therefore, it would be important to explore the applicability of the Linear Fracture Mechanics approach on non-small scale yielding failures [8].

II - ANALYSIS OF INSTABILITY OF MEMBERS WITH CRACKS

First of all, expression (1)

\[ r_y = \frac{k^2}{2\sigma_{ys}} = \frac{M E'}{2\sigma_{ys}} \]  

where \( r_y \) is the crack driving force [3] and \( E' = E \) for plane stress and \( E = \frac{E}{(1-\nu^2)} \) for plane strain, \( E \) being the Young's modulus and \( \nu \) the Poisson ratio, could be considered as the linear leading term of an analytical expression which accounts for the plastic deformation perturbation at the leading edge of the crack. From a physical point of view the most appropriate interpretation for \( r_y \) is not to consider it the physical region where plastic deformation takes place, even though this interpretation is valid in small scale yielding situations, but to merely consider it as a size parameter index of plasticity.

In considering the crack extension resistance curve, \( R \), it can be shown [9] that the condition for the crack equilibrium is:

\[ R = f \]  

(3)
and that the conditions for crack instability under a nominal critical stress $\sigma_c$ are:

$$ R \bigg|_{\sigma=\sigma_c} = \mathcal{B} \bigg|_{\sigma=\sigma_c} \quad \& \quad \frac{dR}{da} \bigg|_{\sigma=\sigma_c} = \frac{d\mathcal{B}}{da} \bigg|_{\sigma=\sigma_c} $$  \quad (4)

In order to analyze these equilibrium-stability relations, consider first, for simplicity, an infinite plate subjected to uniform tensile stress, $\sigma$, into which a transverse crack of length, $2a$, has been introduced.

Since we are dealing with plasticity effects, we have to introduce the crack plasticity correction, i.e., to consider the effective crack length, $a_{\text{eff}}$, given by

$$ a_{\text{eff}} = a_0 + \gamma_y $$  \quad (5)

where $a_0$ is the initial, non-plastically perturbed crack length.

For the configuration being discussed, the crack driving force $\mathcal{B}$ is given by (3):

$$ \mathcal{B}_{\text{eff}} = \frac{\gamma \sigma^2 a_{\text{eff}}}{E} $$  \quad (6)

or

$$ \mathcal{B}_{\text{eff}} = \frac{\gamma \sigma^2 a_0}{E \left(1 - \frac{\sigma^2}{2\sigma_y^2}\right)} $$  \quad (7)

Fig. 1 shows the hypothetical material crack resistance curve $B$, the plastic zone portion of extension resistance curve $\gamma_y$, and the $\mathcal{B}_{\text{eff}}$ applied curves correspondent to two nominal stresses $\sigma_0$ and $\sigma_c$. The stress $\sigma_c$ is the critical stress for which the crack propagates catastrophically.

Fig. 1 also shows a diagram of the accompanying plastic zone.

From Fig. 1, one can see that there always could exist an equilibrium plastic zone given by the intersection between the applied $\mathcal{B}_{\text{eff}}$ and the straight line $\gamma_y$ when the crack does not extend. However; the fact
is that in actual cases cracks exist in components of finite size and the finiteness of the cracked body causes the non-linearity of $B_{\text{eff}}$ as a function of the initial crack length $a_0$.

In this way, for a tensile through crack in an ideal brittle plate of width $W$, the crack driving force $B$ could be written:

$$B = \frac{a^2}{W} Y \left( \frac{a}{W} \right)$$

(8)

where $Y(a/W)$ is a function which represents the effects on the elastic stress field surrounding the tip of the crack, when the distance between the leading edge of the crack and the surface of the plate, becomes comparable to the crack length.

One of the characteristics of the finite width correction or calibration function $Y$ is that

$$Y(a/W) \rightarrow 1 \quad \text{when} \quad a \rightarrow W$$

(9)

The calibration function $Y$ introduces fundamental changes in considering the equilibrium of the plastic zone in the elastic stress field at the border of the crack.

Let us consider a finite, non-perfect brittle cracked body.

From (8) the effective driving force $B_{\text{eff}}$ is given by:

$$B_{\text{eff}} = \frac{a^2 (a_{\text{eff}}/E')}{W} Y^2 \left( \frac{a_{\text{eff}}}{W} \right)$$

(10)

where $a_{\text{eff}}$ is given by (5)

The $r_y$ curve is as always given by:

$$r_y = \frac{B_{\text{eff}} E'}{2\sigma_y p^2}$$

(11)

Fig. 2 shows the hypothetical crack extension resistance curve, $R$, of the material, the plastic zone extension, $r_y$, resistance curve
[given by (11)] and two applied \( J_{\text{eff}} \) curves corresponding to two applied nominal stresses, \( \sigma_0 \) and \( \sigma_{\text{Pl}} \).

For \( \sigma_0 \) the equilibrium plastic zone is \( r_y = r_{y_0} \), similar to \( r_{y_0} \) in Fig. 1.

However, for \( \sigma = \sigma_{\text{Pl}} \), due to the calibration function \( Y \), the \( J_{\text{eff}} \) curve is tangent to the \( r_y \) curve at the point 1.

At point 1 the rate of increase with increasing crack length of the elastic stress field surrounding the plastic zone becomes sufficient so that the plastic zone cannot maintain stable static equilibrium. From a theoretical point of view, point 1 in Fig. 2 would represent the condition for plastic zone "propagation," i.e., the plastic zone would become unstable and propagate across the specimen.

The necessity of being cautious in making interpretations involving the plastic zone parameter \( r_y \) has been pointed out. As a matter of fact, the ideal plastic zone "propagation" interpretation would be valid only for materials where simultaneously crack growth is negligible but might subsequently follow "plastic zone instability."

Consider now a real material, i.e., a material whose capacity to absorb plastic deformation energy is limited. When point 1 is reached, stable equilibrium cannot be maintained between the plastically deformed material and the elastic stress field, the external energy source, i.e., with plastic zone growth, the applied load has to decrease. Thus, the equilibrium is unstable with respect to plastic zone growth or plastic zone instability occurs, and perhaps crack growth almost simultaneously follows this plastic zone growth.

The instability occurring at point 1 is referred to as "plastic zone instability" and the corresponding stress \( \sigma_{\text{Pl}} \) is the plastic zone instability stress.
Consider now the quantitative condition for plastic zone instability. For a finite cracked member whose tensile yield stress is \( \sigma_{ys} \), and using \( 2 \) and \( 10 \), the effective crack driving force \( \mathcal{B}_{eff} \) could be written:

\[
\mathcal{B}_{eff} = \frac{P^2}{(BW)^2} (a_{eff})^2 (a_{eff}/W)
\]  

(12)

where \( P \) is the applied load and \( (BW) \) a generalized served gross section. In this way, \( B \) could be considered as a generalized thickness and \( W \) a generalized width.

As before, \( a_{eff} \) is the effective crack length:

\[
a_{eff} = a_0 + r_y = a_0 + \frac{\mathcal{B}_{eff} E^f}{2\sigma_{ys}^2}
\]  

(13)

The analytical conditions at point \( 1 \) in Fig. 2 could be written:

\[
\frac{\partial \mathcal{B}_{eff}}{\partial p} \Big|_{p=p_1} = \frac{\partial r_y}{\partial p} \Big|_{p=p_1} = R
\]  

(14)

\[
\frac{\partial \mathcal{B}_{eff}}{\partial a_{eff}} \Big|_{p=p_1} = \frac{\partial r_y}{\partial a_{eff}} \Big|_{p=p_1} = \frac{3R}{2a_{eff}}
\]  

(15)

Expression (14) gives the equilibrium plastic zone size in the stress field of intensity \( K \), whereas expression (15) gives the condition for incipient plastic zone instability.

From (2) one can get:

\[
\frac{\partial \mathcal{B}_{eff}}{\partial r_y} \Big|_{p=p_1} = \frac{2\sigma_{ys}^2}{E^f}
\]  

(16)

On the other hand, from (12)

\[
\frac{\partial \mathcal{B}_{eff}}{\partial a_{eff}} \Big|_{p=p_1} = \frac{p_1^2}{(BW)^2} \left[ (a_{eff}/W)^2 + 2a_{eff} \frac{3\gamma}{W} \left( \frac{a_{eff}}{W} \right)^3 \right]
\]  

(17)
Equating (16) and (17) one gets the condition for plastic zone instability:

\[
\frac{p_{pl}}{(8W)^{2}y_{p}} = \frac{\sigma_{pl}}{\sigma_{yp}} = \frac{2\pi}{\gamma + 2\frac{\alpha}{\gamma} Y'}
\]  

(18)

The plastic zone size for instability can be found substituting (18) in (14). The final result is:

\[
y_{p1} = \frac{\sigma_{pl}}{\sigma_{yp}} = \frac{\gamma}{1-\frac{2\pi}{\gamma}}\]

where

\[
Y = \frac{\gamma}{1-\frac{2\pi}{\gamma}}
\]

Crack Propagation Failure

On the other hand, unstable crack propagation can precede plastic collapse instability, in which case the analysis should proceed on a C or R curve basis in the well known sense. Thus, it will not be discussed in detail here.

However, the data to follow will contrast both types of failure mechanisms.

III - EXPERIMENTAL WORK

An experimental program has been conducted to verify the predictions of the Plastic Zone Instability theory in cracked specimen failures.

The principal objectives of such a program are to confirm experimentally the concept of "instability without crack growth" and to compare the experimental values with those given by (18).

Materials

The materials tested were 7075-T6 and 2024-T3 commercial aluminum alloys. Tensile data for these alloys are shown in Table 1. The compilation includes data reported by Kaufman et al. (10) for plane specimens.
Table 2 shows the values of toughness $K_c$ for these alloys determined with large center notched panels.

These alloys were used because of their different relative toughness performance ($K_c/\sigma_y$) for which one could expect different behavior, i.e. plastic zone instability vs critical fracture toughness instability.

**Procedure**

A sketch of the W.O.L. specimen used in this program is presented in Fig. 3. All the specimens tested were 0.125 in. thick. The specimens were pre-fatigued to different crack lengths at a low stress level. In all the tests the loading direction was transverse to the rolling direction. All the tests were conducted in air, at room temperature and on a closed loop machine.

Table 3 shows the geometrical characteristics of the tested samples. The same table shows the test load program for each specimen.

In all the tests the load was applied in such a way as to keep the relative velocity of the crack surfaces at the edge of the specimen constant (in displacement control), and this velocity was 0.002 in./sec.

The clip (compliance) gage technique was used to obtain the automatic load-displacement curves from which the effective crack length, $a_{eff}$, could be computed and monitored. The crack surface displacement was measured at the cracked edge of the specimen which simultaneously with the applied load was recorded on a standard X-Y plotter.

The displacement calibration curve for the specimen used is shown in Fig. 4.

The load displacement curve [(P,v) curve] was an effective experimental tool to study the plastic zone instability when used in conjunction with the displacement calibraton curve ($\frac{EvB}{P}$ curve).
From the \((P,v)\) curve, knowing the calibration function \(Y\), it is possible to compute the crack extension resistance curve, \(R\). The following diagram outlines the steps required to get the "R" curve:

\[
\begin{align*}
(P,v) \text{ curve} & \quad \xrightarrow{\frac{E v_0}{w}} \quad \text{curve} \\
\xrightarrow{\frac{a_{\text{eff}}}{w}} \quad Y(a_{\text{eff}}/W) \text{ curve} & \quad \xrightarrow{\frac{\sigma_{\text{eff}}}{w}} \quad \text{curve (equivalent to } R) \\
\end{align*}
\]

Experimental Results

Two typical load-displacement curves for 2024-T3 and 7075-T6 aluminum alloys are presented in Figs. 5 and 6 respectively. The resulting "R" curves for 2024-T3 and 7075-T6 alloys are shown in Figs. 9 and 10 respectively.

The estimated plastic zone effective crack extension, \(r_y\),
\[
\begin{align*}
R_y &= \frac{K_{\text{eff}}^2}{\pi^2} & \text{Plane Stress} \\
R_y &= \frac{K_{\text{eff}}^2}{\pi^2} & \text{Plane Strain} \\
\delta &= \frac{\sigma_{ys} + \sigma_{ul}}{2} \\
\end{align*}
\]

has been computed and plotted on each of the \(R\) diagrams.

Discussion

According to Fig. 7 examination of 2024-T3 \((P,v)\) curves indicate that no appreciable crack growth has occurred during the tests regardless of the maximum load level at instability. On the other hand examination of \((P,v)\) curves corresponding to 7075-T6 alloys indicate that some crack growth has always taken place during the tests.

Tests on specimens 2005 and 2008 are quite significant, Fig. 8.
The principal objective in testing these specimens was to demonstrate the existence of an "instability without crack growth" phenomenon, i.e. plastic zone instability. Since both of these specimens had the same geometry, the similarity in their $(P,v)$ curves was expected. Specimen 2006 was loaded to failure. The maximum load point appearing in the $(P,v)$ curve would indicate the point at which "crack growth" begins. However, when specimen 2008 (almost identical to specimen 2006) was unloaded from the maximum load point, the slope for unloading reveals that no crack growth occurred during the test.

Examination of the "R" curves corresponding to the tested alloys gives a better understanding of the plastic collapse failures. "R" curves for 2024-T3 Al. alloy, Fig. 9, shows that the "crack extension" resistance curves run within the plastic zone growth band. This means that if there were crack extension, it could not be distinguished from plastic zone growth. On the other hand, for the case of 7075-T6 alloy, Fig. 10, the crack extension resistance curves do not completely run within the plastic zone growth band. This implies that some crack growth has taken place.

The Plastic Collapse Failure Curve (P.C.F. Curve)

According to reference (11), (for $0.3 < e < 0.7$) the calibration function for the specimen is given by

$$Y = \left[0.2960 - 1.855\left(\frac{2}{w}\right) + 6.557\left(\frac{2}{w}\right)^2 - 10.17\left(\frac{2}{w}\right)^3 + 6.389\left(\frac{2}{w}\right)^4\right]10^2$$  \hspace{1cm} (21)

From (18) and (21) and using a "trial and error" method, curves for $P_{cr}$ were computed. Fig. 11 is the "plastic zone instability failure" curve (PZIF curve). It is noted that the PZIF curve is characteristic of the specimen geometry but independent of the material.
Fig. 12 shows the trend of experimental values of \( P_{\text{max}} \) for 2024-T3 and 7075-T6 alloys. It should be observed that the instability loads for 2024-T3 alloy are in very good agreement with those predicted by the PZIF curve.

For 7075-T6 alloy the experimental instability loads are lower than the plastic collapse loads. This is not unexpected since it was pointed out that some crack extension took place during 7075-T6 failures. Therefore, it was assumed that 7075-T6 failures were controlled by a crack growth instability process. Thus, assuming for 7075-T6 alloy a critical K factor \( K_c \) equal to 50 Ksi (in)\(^{1/2}\) one notices a very good agreement between experimental and calculated values.

Conclusions
1. This experimental program has demonstrated the existence of a plane stress type of plasticity failure, namely, the plastic zone instability failure, which is controlled by the instability of the plastic zone developed at the tip of a crack.
2. Plastic zone instability (PZIF) failures are controlled by the geometry aspects of the component as well as the mechanical properties of the material, in particular the yield strength.
3. The calibration function \( Y(a/W) \) is found to represent well the geometric parameter of strong influence in PZIF.
4. A quantitative analysis of PZIF has been carried out. The agreement between experimental and calculated values is quite satisfactory.

ACKNOWLEDGMENT
The authors wish to express appreciation to Mr. Jack Fitzgerald and Mr. David Jenkins for their assistance in obtaining the experimental
results. The financial support of this work by the General Electric Company through a contract with Lehigh University is also gratefully acknowledged. The testing work described was also supported by Del Research Corporation through free use of facilities for all of the experimental work.
REFERENCES

<table>
<thead>
<tr>
<th>Alloy and Temper</th>
<th>2024-T3</th>
<th>7075-T6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long. Ult. Strength (ksi)</td>
<td>Min. 64</td>
<td>Min. 77</td>
</tr>
<tr>
<td></td>
<td>72</td>
<td>86.0</td>
</tr>
<tr>
<td>Long. Yield Strength (ksi)</td>
<td>Min. 42</td>
<td>Min. 85</td>
</tr>
<tr>
<td></td>
<td>52</td>
<td>76.1</td>
</tr>
<tr>
<td>Long. Elongation %</td>
<td>Min. 15%</td>
<td>Min. 8%</td>
</tr>
<tr>
<td></td>
<td>18%</td>
<td>11%</td>
</tr>
</tbody>
</table>

Provided by supplier
Ref. (10)
Provided by supplier
Ref. (10)*
Provided by supplier
Ref. (10)

*Offset = 0.2%
Table 2. Average Results of Tension Tests of Large Center Notched Panels. Load Direction: Transverse (Ref. 10)

| Alloy and Temper | Kc  
|------------------|------
|                  | ksi (in.)\(^{1/2}\) |
| 2024-T3          | 89.7 |
| 7075-T6          | 36.2 |
|                  | 30.9 |
|                  | 34.4 |

<table>
<thead>
<tr>
<th></th>
<th>(\sigma_N/\sigma_{yp})</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024-T3</td>
<td>0.77</td>
</tr>
<tr>
<td>7075-T6</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Kc/(\tau_{ys}) (in.)(^{1/2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024-T3</td>
<td>1.72</td>
</tr>
<tr>
<td>7075-T6</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>-0.40</td>
</tr>
<tr>
<td></td>
<td>-0.45</td>
</tr>
<tr>
<td>Alloy and Temper</td>
<td>Sample Number</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>2024-T3</td>
<td>20-6</td>
</tr>
<tr>
<td>2024-T3</td>
<td>20-8</td>
</tr>
<tr>
<td>2024-T3</td>
<td>20-9</td>
</tr>
<tr>
<td>2024-T3</td>
<td>20-11</td>
</tr>
<tr>
<td>2024-T3</td>
<td>20-12</td>
</tr>
<tr>
<td>2024-T3</td>
<td>20-13</td>
</tr>
<tr>
<td>7075-T6</td>
<td>70-14</td>
</tr>
<tr>
<td>7075-T6</td>
<td>70-16</td>
</tr>
<tr>
<td>7075-T6</td>
<td>70-15</td>
</tr>
<tr>
<td>7075-T6</td>
<td>70-17</td>
</tr>
</tbody>
</table>

\(\ast\) \(a_0\) is the initial crack length. \(W\) is the width.
(1) Sample loaded to failure.
(2) Sample loaded to maximum load point and then unloaded.
(3) Sample loaded to maximum load point, then unloaded and finally reloaded.
LIST OF FIGURES

Fig. 1 Hypothetical "Kc" Failure.
Fig. 2 Hypothetical Plastic Zone Instability Failure.
Fig. 3 Specimen Used for Plastic Zone Instability Failure.
Fig. 4 Displacement Calibration Curve.
Fig. 5 Typical Load-Displacement Curve for 2024-T3 Al. Alloy.
Fig. 6 Typical Load-Displacement Curve for 7075-T6 Al. Alloy.
Fig. 7 Possible Parameters That Can Be Obtained From the Load-Displacement Curve.
Fig. 8 Load-Displacement Curves for Specimens 2024 and 7075 Showing the Existence of an "Instability Without Crack Growth" Phenomenon.
Fig. 9 Crack Extension Resistance for 2024-T3 Al. Alloys.
Fig. 10 Crack Extension Resistance for 7075-T6 Al. Alloy.
Fig. 11 PZIF Curve for W.O.L. (H/Hc=0.6) Specimen.
Fig. 12 Failure Loads for 2024-T3 and 7075-T6 Al. Alloys.
FIGURE 5

SPEC. 2013
$\frac{a^*}{W} = 0.30$

P(lb)

0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
14000
15000
16000
17000
18000
19000
20000
21000
22000
23000
24000
25000

displacement, in

0.02 in.
FIGURE 9

2024-T3
WOL
H/w = 0.6
t = 0.125"

(a₀/w)
○ 0.30
□ 039
△ 0465
△ 0.60
7075-T6 WOL
H/w = 0.6
t = 0.125"
(α/ω)

- FIGURE 10
Figure 12

- 2024-T3
- 7075-T6
- P.C.C.
- $k_c = 50\text{ksi (in)}^{1/2}$

Graph showing the relationship between $\frac{B}{tW}$ and $\frac{G_{fr}}{W}$.
THE APPLICATION OF THE PLASTIC ZONE INSTABILITY CRITERION TO PRESSURE VESSEL FAILURE

By

Jose Vazquez*
and
Paul C. Paris**

*Formerly Research Assistant, Lehigh University, now at Centro de Investigaciones Metalurgicas, Cordoba, Rep. Argentina; now with the Research Laboratory of the Argentine Navy, Buenos Aires, Argentina.

**Formerly Department of Mechanics, Lehigh University, Bethlehem, Pa. Now Professor of Mechanics, Washington University, St. Louis, Mo.
ABSTRACT

The recently developed theory on plastic zone instability failure is used to analyze the instability of a longitudinal through crack in a pressurized cylinder. The available experimental data and the predicted values are compared and analyzed.

I. INTRODUCTION

Several authors have used a Fracture Mechanics approach to analyze the onset of crack propagation in cylindrical pressure vessels. [1][2][3][4] In general, the agreement between their calculated values and the experimental ones is satisfactory. However, in order to get such an agreement it was necessary to introduce experimental parameters which have no clear physical or analytical basis. Moreover, the fracture criteria used, often depend on the values of some experimental parameters involved in each particular case. Under such circumstances, it is difficult to understand how fast crack propagation in pressure vessels, when considered as a physical problem, can be controlled by in some cases rather circumstantial criteria.

The principal aim of this paper is to show how Linear Fracture Mechanics (without using approximations other than those generally used in such an approach) in conjunction with the recently [5] developed theory on plastic zone instability failure can account for the available experimental data on pressure vessel failures.

II. THE ELASTIC STRESS FIELD ANALYSIS

Let us consider a longitudinal through crack of length 2a in a pressurized cylinder of inner radius R and wall thickness t. Two normal membrane stresses are associated with the internal pressure p.
These normal stresses are the hoop stress, \( \sigma_h = \frac{r}{R} \), acting in the direction normal to the crack and the longitudinal stress, \( \sigma_1 = \frac{a}{2} \).

In considering the elastic stress field at the tip of the crack, the contribution of the longitudinal stress \( \sigma_1 \) is considered negligible \([6]\). In this way, the through crack in a pressurized cylinder becomes a tensile crack under a uniform membrane stress \( \sigma_1 \) \(\ast\). According to Linear Fracture Mechanics analysis, the stress environment at the tip of the crack is described by a unique parameter, namely, the stress intensity factor \( K \). The continuum mechanics analysis shows \([7][8]\) that for a longitudinal through crack in a pressurized cylinder the expression for the \( K \) parameter is of the form:

\[
K = \sigma_h (2a)^{1/2} \sqrt{\frac{a}{R}}
\]

(1)

where \( Y \) is the so-called shell correction factor. Two analytical expressions for the shell correction are available in the literature. The first is due to Folias \([7]\) and the second was suggested by Erdogan and Kibler \([8]\). Due to the mathematical procedures employed, the Folias results as well as the Erdogan-Kibler results turn out to be valid only if the parameter \( \lambda \frac{a}{\sqrt{R}} \) is within a certain range. Moreover, this depends on the Poisson ratio, \( \nu \), of the material.

Assuming \( \nu = 1/3 \), the Folias result turns out to be valid if \( \lambda a \), while the Erdogan-Kibler analysis is sufficient for \( \lambda \geq 4.45 \). Their analytical expressions for the function \( Y \) can be written as follows:

Folias' analysis:

\[
Y = (1 + 1.6a)^{1/2} \quad 0 \leq a \leq 1
\]

\(\ast\) Note that this statement does not mean that a through crack in a pressurized cylinder is equivalent to a through crack in a flat panel.
Erdogan-Kibler analysis:

\[
Y = \begin{cases} 
(1 + 1.3\lambda^2)^{1/2} & 0 \leq \lambda \leq 1 \\
0.5 + 0.9\lambda & 1 < \lambda \leq 4.45 
\end{cases}
\]  \( (2) \)

Due to its wider validity range, this paper will use the Erdogan-Kibler expression for the function \( Y \). Fig. 1 shows the plots of the function \( Y \) obtained from both analyses. In such plots the function \( Y \) has been extrapolated to values of \( \lambda \) as large as 10. The justification of such an extrapolation will be given later by the comparison between calculated and experimental values. Using (1) with \( Y \) given by (2) one is able to describe the elastic stress field in the very near tip region of a through longitudinal crack in pressurized cylinders of interest here.

III. THE PLASTIC ZONE DISTURBANCE

When a material is capable of plastic deformation, a zone of plastically deformed material will develop at the leading edge of cracks. This plastic zone will disturb the elastic stress field considered in Section II. In a small scale yielding situation the plastic zone may be regarded roughly as a circle with radius \( r_y \) given by [9]:

\[
r_y = \frac{k^2}{2\theta^2}
\]  \( (3) \)

where \( \theta \) is a characteristic for plastic flow stress in the material. It has been shown [10] that for small scale yielding situations the stress field outside the elastic-plastic boundary is given by the elastic solution in which the leading edge of the crack is located approximately at the center of the plastic zone. This result would be exact for opening Mode III (anti-plane-shear) and it is usually assumed reasonable for Mode I (tensile) problems. In considering non-small scale yielding situations
the plastic zone would extend farther than \( r_y \) \([11]\). If one is primarily concerned with the physical aspects of the crack propagation phenomenon rather than its geometrical characteristics, it is possible to gain simplicity without loss of essential information by assuming \( r_y \) as a size parameter representing the plastic deformation features at the tip of the crack.

In the presence of plastic deformation at the leading edge of the crack, one should consider an effective value for the \( K \) parameter, \( K_{\text{eff}} \), given by \([11]\)

\[
K_{\text{eff}} = K(a_{\text{eff}})
\]  

(4)

where

\[
a_{\text{eff}} = a + r_y
\]  

(5)

and, as can be seen, the experimental results on plastic collapse failure \([5]\) seem to justify the use of (4), even for non-small scale yielding situations.

IV. PLASTIC ZONE INSTABILITY ANALYSIS

The basic idea in the plastic zone instability approach is to consider the stability of static equilibrium between the elastic stress field and the plastically deformed material at the tip of the crack. The analysis shows the existence of a situation in which the gradient with respect to effective crack size of the stress environment, \( K \), at the leading edge of the crack becomes sufficiently large that the plastic zone cannot maintain stable static equilibrium unless the applied load decreases with increasing effective crack size. In other words, plastic zone instability takes place. As is shown in \([5]\), on the basis of Linear Fracture Mechanics concepts, plastic zone instability would imply the "propagation" of the plastic zone across the cracked component.
However, since the capacity of the material to absorb plastic energy is limited, usually crack growth simultaneously follows unstable plastic zone "propagation." According to the analysis, the nominal generalized stress for plastic zone instability, $\sigma_{pl}$, using the proper (shell) correction function $\gamma$, turns out to be:

$$\sigma_{pl} = \beta \left[ \left( \gamma \frac{a_{eff}}{\sqrt{a}} \right)^{1/2} \left( \gamma \frac{a_{eff}}{\sqrt{a}} + 2 \frac{a_{eff}}{\sqrt{a}} \gamma \frac{a_{eff}}{L} \right)^{1/2} \right]$$

(6)

where for pressure vessels geometries the parameter $\beta$ is given by

$$\beta = \sqrt{\frac{E}{\gamma}}$$

In previous work [5] using W.O.L.specimens, (6) was verified for 2024-T3 Al. alloy under test conditions such that failures were essentially produced by a plastic zone instability mechanism. In the case of real cracked structures, they may also fail due to crack propagation. Therefore, (6) represents a maximum load or upper bound that such structures can support without plastic zone instability triggering failure, if other mechanisms do not cause earlier failure.

V. FAILURE CRITERIA

According to the Linear Fracture Mechanics approach, a crack will propagate when a critical stress field exists at its tip. (*) The intensity of such a critical stress field, $K_C$, would be characteristic of the material as well as of the state of stress (plane stress or plane strain) at the leading edge of the crack. This criterion is referred to as the "Griffith-Irwin criterion." [12] However, a cracked structure also can fail obeying a plastic zone instability criterion.

(*) The more elaborate instability criterion following the crack resistance curve concept could be alternately substituted here.
From (6) it should be noted that unlike the Irwin criterion, the condition for plastic collapse instability is strongly influenced by the geometrical configuration.

Let $K_c$ be the material toughness measured on a given specimen, for a given state of stress at the tip of the crack. From (6) one can show that the stress intensity factor for plastic zone instability, $K_{pl}$, is given by

$$K_{pl}^2 = 2\pi\sigma^2 \frac{a_{eff}}{1 + 2\frac{V}{Y}\frac{a_{eff}}{Y}}$$

(7)

therefore, the criterion for plastic zone instability failure (prior to crack propagation) turns out to be:

$$\frac{K_c^2}{2\pi\sigma^2} > \frac{a_{eff}/L}{1 + V\frac{a_{eff}}{Y}}$$

(8)

where, again, for pressure vessels geometries

$$L = \sqrt{rt}$$

It has been pointed out that the value for $K_c$ is associated with the state of stress at the crack tip. It is a known fact [13] that the material toughness increases as the stress state at the leading edge of the crack approaches plane stress conditions. So, in order to use (8) the values of $K_c$ should correspond to those at the crack tip in an identical stress state.

VI. THE PLASTIC ZONE INSTABILITY FAILURE FOR PRESSURE VESSELS

Fig. 2 shows the plastic zone instability curve (PZIC) for pressure vessels. It has been obtained from (6) using a "trial and error" method. It has been assumed that the $K$ parameter is given by (4) with the
correction function \( \gamma \) given by (2). Fig. 3 shows the curve representing the plastic zone instability criterion (8) for pressure vessels. In evaluating these curves the stress \( \bar{\sigma} \) appearing in (6) and (10) has been taken as the mean value between the 0.2% off-set tensile yield strength, \( \sigma_y \), and the ultimate tensile strength \( \sigma_u \)

\[
\bar{\sigma} = \frac{\sigma_y + \sigma_u}{2}
\]  

(9)

Figs. 4-12 show the available experimental data together with the failure curve predicted by the plastic zone instability theory. From the plastic zone instability criterion, (8) or from Fig. 2, one can see that the probability of having plastic zone instability decreases as the parameter \( \lambda = \frac{a}{2R} \) increases. Let us consider, for example, pressure vessels with the same radius \( R \) and the same wall thickness \( \xi \), but different crack lengths. In this way, depending on the material \( K_c \) value, pressure vessels with relatively long cracks (high \( \lambda \)) may fail according to the Griffith-Irwin criterion. However, if the crack length is shorter (lower \( \lambda \)'s) plastic zone instability failure may develop. This is apparently the situation for 7075-T6 Al. pressure vessels with a wall thickness of 0.016 inch (Fig. 4). On the other hand, if for a given stress state at the crack tip the material presents a relatively low crack toughness performance, \( K_c \), the pressure vessel failure may be entirely controlled by the Griffith-Irwin criterion. Such is, apparently, the case for 7075-T6 Al pressure vessels with a wall thickness of 0.025 inch shown in the same Fig. In fact, since the state of stress at the tip of the crack depends on the thickness, one could expect that if \( \sigma_c/\sigma \ll 1 \) as in this case, the \( K_c \) value for these pressure vessels to be lower than those corresponding to 0.016 inch wall thickness. Similarly, as shown in Fig. 5 the Griffith-Irwin criterion would control
the failure of 2014-T6 Al pressure vessels. For the remaining alloys and geometries the relationship between pressure failure and the parameter $\lambda$ seems to be that predicted by the plastic collapse theory, although the actual pressure values at failure are sometimes somewhat higher. One could expect such a behavior since usually, as it was pointed out before, crack propagation follows the onset of plastic collapse instability.

The location of the plastic collapse curve in the $\frac{\sigma_c}{\sigma}$ vs $\lambda$ diagram may shift due to the actual value of the characteristic plastic stress $\bar{\sigma}$. However, since plastic collapse theory, as presented here, is a very simple first order approach, the authors have preferred to avoid rather risky speculations using as the characteristic plastic stress the mean value given by (9).

CONCLUSIONS

(1) The instability of a longitudinal through crack in a pressurized cylinder is controlled, at least, by two criteria
   (a) The Griffith-Irwin criterion
   (b) The plastic zone instability criterion.

(2) The conditions for the existence of either one of the instability mechanisms depend on the material crack toughness performance, $K_C$, and on the geometric characteristic of the pressure vessel.

(3) A criterion [Eq. (8)] for plastic zone instability failure, prior to crack propagation has been suggested.

(4) The analyzed experimental data seem to indicate that when such a criterion is satisfied, the plastic zone instability theory (as a first order approach) provides a conservative estimate of failure conditions.
REFERENCES

LIST OF FIGURES

Fig. 1  Shell Correction Factor for a Longitudinal Through Crack in a Pressurized Cylinder.
Fig. 2  PZI Curve for Pressure Vessels.
Fig. 3  Plastic Zone Instability and Griffith-Irwin Failure Criteria.
Fig. 4  PZIF Curve and Failure Stress for 7075-T6 Al Cylinders [Ref. 1].
Fig. 5  PZIF Curve and Failure Stress for 2014-T6 Al Cylinders [Ref. 1].
Fig. 6  PZIF Curve and Failure Stress for Ti Alloy Cylinders [Ref. 2].
Fig. 7  PZIF Curve and Failure Stress for 2024-T3 Al Cylinders [Ref. 2].
Fig. 8  PZIF Curve and Failure Stress for X-50 and X-60 Grades Linepipe Steel Cylinders [Ref. 3].
Fig. 9  PZIF Curve and Failure Stress for Brass Cylinders [Ref. 14].
Fig. 10 PZIF Curve and Failure Stress for Ti-8 Al-1Mo-1V Cylinders [Ref. 15].
Fig. 11 PZIF Curve and Failure Stress for A106-B Pipeline Steel Cylinders [Ref. 16 and 17].
Fig. 12 PZIF Curve and Failure Stress for 0.36c and 0.13c Steels Cylinders [Ref. 18].
A106-B pipe line steel

\begin{tabular}{ccc}
R (in) & R/t & \bar{\sigma} (ksi) \\
111 & 6.5 & 55 \\
111 & 15.8 & 55 \\
312 & 7.25 & 55 \\
320 & 11.4 & 55 \\
417 & 12.9 & 55 \\
215 & 9.35 & 55 \\
\end{tabular}

\textbf{FIGURE 11}
PREDICTION OF DUCTILE TEARING
OF COMPACT SPECIMENS USING THE R-6 FAILURE ASSESSMENT DIAGRAM

J. M. Bloom
The Babcock & Wilcox Company
Research and Development Division
Alliance, Ohio (USA)

SUMMARY

The author has used recent work by Milne of CEGB on the reinterpretation of the R-6 failure assessment diagram to predict the ductile tearing behavior of compact fracture test specimens of three different materials:

1. HY130 steel,
2. SA533B steel, and
3. 7075-T651 aluminum.

The R-6 failure assessment diagram is based on the original CEGB Dowling and Townley two-criteria approach. The load-versus-slow crack growth (ductile tearing) was predicted for seven steel compact specimens. Maximum loads were predicted to within 7 percent for all seven specimens, with a standard error of 1.65 percent. Various combinations of material flow stress and limit load expressions were studied.

The conclusion based on initial predictions was that the most important input parameter is the expression for limit load. Concern with the drop-off of load after maximum load suggests that alternate failure assessment diagrams are needed to take into account the strain hardening behavior of the materials in order to accurately predict the complete load-versus-stable crack growth of ductile materials.

In addition, closed form expressions were developed which calculate load directly from the R-6 diagram and thereby eliminate time consuming graphical procedures. Lastly, maximum load for 14 compact 7075-T651
aluminum specimens were predicted using the R-6 diagram to within a standard error of 1 percent. These specimens were tested as part of a recent ASTM E24.06.02 committee's predictive round robin on fracture.

INTRODUCTION

Current ASME fracture analysis code practices consider only linear elastic fracture mechanics (LEFM) behavior, while most reactor coolant pressure boundary ferritic materials are fully ductile under normal operating conditions and most accident conditions. The Central Electricity Generating Board of the United Kingdom (CEGB-UK) has addressed this ductile phenomena through their failure assessment work of the past few years. While this work at CEGB in the area of failure assessment is well known in Europe, the implications and use of this work has not been fully studied in the United States. Past criticisms of their work in the United States have been directed at its use of overly conservative material crack initiation properties; i.e., $K_{IC}$ and overly conservative limit load expressions. The author has taken the approach that it is first necessary to remove these excessive conservatisms and try to predict actual specimen or structure behavior. This will then give us more confidence in the accuracy of the general techniques and procedures of their work, as well as a better understanding of the sensitivity of input data and limitations of the procedure.

With the above in mind, the author has used recent work by Milne\textsuperscript{(1)} of CEGB on his reinterpretation of the R-6 failure assessment diagram in order to predict the ductile tearing behavior of compact fracture test specimens.

BACKGROUND

The R-6 failure assessment approach is derived from the original CEGB Dowling and Townley two-criteria approach\textsuperscript{(2)}. This approach states that structures will fail by either of two mechanisms: brittle fracture or plastic collapse; and that these two mechanisms are connected by a transition curve which allows one to go directly from LEFM behavior to plastic instability. The CEGB transition curve is based upon the Dugdale\textsuperscript{(3)},
Bilby-Cottrell-Swinden\textsuperscript{(4)} model of strip yielding ahead of a crack. By viewing failure as occurring between the limits of two different distinct behaviors, the approach is one of interpolation. Previous work has focused on extrapolation procedures where LEFM is usually extended into the elastic-plastic fracture mechanics regime. The interpolation approach of the two-criteria method is quite similar to the estimation techniques of Shih\textsuperscript{(5)}.

For use in failure assessment, Harrison, Loosemore, and Milne\textsuperscript{(6)} of CEGB reformulated the two-criteria approach into a procedure now referred to as the R-6 failure assessment diagram which is illustrated in Figure 1. The coordinates of an assessment point on this diagram are calculated by

\[ K'_{r} = K_{I}/K_{IC} \]  \hspace{1cm} (1)

and

\[ S'_{r} = \sigma/\sigma_{I} \]  \hspace{1cm} (2)

where \( K_{I} \) is the stress intensity factor for the structure to be assessed and \( K_{IC} \) is the fracture toughness of the material; \( \sigma \) is the applied stress on the structure and \( \sigma_{I} \) is the plastic collapse stress. For a particular stress level and flaw size, the coordinates \( (S'_{r}, K'_{r}) \) can be calculated, and if this point lies on or outside of the failure assessment line given by

\[ K_{r} = S_{r} \left\{ \frac{8}{\pi^{2}} \ln \sec \left( \frac{\pi S_{r}}{2} \right) \right\}^{-1/2} \]  \hspace{1cm} (3)

the structure will fail. Points inside this curve dictate that the structure is safe from failure. Earlier work by CEGB has shown that this procedure works well for cleavage fracture, as well as for ductile fracture, up to the point of crack initiation. In fact, both Chell\textsuperscript{(7)} and Roche\textsuperscript{(8)} have shown this procedure to be equivalent to a J-integral analysis.

**DUCKILE TEARING**

Milne reinterpreted the R-6 diagram to account for stable crack growth beyond initiation. For initiation, \( K_{r}(a) \) can be interpreted to be
\[ K'^r(a) = \sqrt{J_{IE}(a)/J_{IC}} \]  

(4)

where \( J_{IE}(a) \) is the \( J \) value of the elastically calculated stress intensity factor given by

\[ J_{IE}(a) = \frac{K_{I}^2(a) \cdot (1 - \nu^2)}{E} \]  

(5)

for the case of plane strain where \( E, \nu \) are Young's modulus and Poisson's ratio, respectively. Now the position of the assessment point \((S'_r, K'_r)\) relative to the failure assessment curve determines how close the structure is to the initiation of ductile tearing, as shown in Figure 2 by the point \( L'_1 \). Now, since both \( K'_r, S'_r \) are directly proportional to applied load, \( L \), the load can be increased and the point \( L'_1 \) moves radially from the origin of the diagram to the point \( L'_1 \), which is the initiation point of ductile tearing. After initiation of ductile tearing, the locus of \((S_r, K_r)\) points will follow the failure assessment curve between the initiation load, \( L_1 \), and the maximum load point, \( L_m \); and thus define the region between stable and unstable crack growth as shown in Figure 2. For displacement controlled structures, the path follows the R-6 curve. For load controlled structures the path will go outside the assessment curve after the maximum load point \( L_m \) has been reached, indicating that the structure has become unstable.

In order to calculate the locus of points which follow the failure assessment line, \( K'_r \) and \( S'_r \) must be first redefined as follows:

\[ K'_r(a + \Delta a) = \sqrt{J_{IE}(a + \Delta a)/J_{R}(\Delta a)} \]  

(6)

and

\[ S'_r(a + \Delta a) = \sigma/\sigma_1(a + \Delta a) \]  

(7)

where now \( J_{IE}, J_{R}, \sigma_1 \) are functions of the amount of slow stable crack growth. \( J_{R} \) is the experimentally measured \( J \) resistance curve plotted as a function of slow stable crack growth, \( \Delta a \). \( J_{IE} \) is calculated as before from the elastic stress intensity factor for the current crack length, \( a + \Delta a \).
The mechanics of the calculations are explained briefly as follows. A load level is chosen for a particular structure of interest, such that $K'_r$, $S'_r$ calculated from equations (6) and (7) fall inside the failure assessment curve for a particular initial crack size, $a$, in the structure. The $K'_r$, $S'_r$ coordinates are then recalculated at the next crack growth interval, $a + \Delta a$, corresponding to a $J_R$ value at $\Delta a$ from the experimental $J$ resistance curve. An example of this curve for SA533B steel tested at 200°F from a 4-inch thick face-grooved compact specimen is shown in Figure 3. The results of the calculations are a locus of points as shown in Figure 4, where point 1 is calculated from the initial flaw size, $a$. The crack proceeds in a stable manner from point 1 to point 3. This locus of points is based on a constant load level. The actual load-versus-crack growth of the structure is determined by rationing the fixed relative distance $OB$ to $OA$; that is, the load level during which either initial or slow crack growth would place these calculated points ($S'_r$, $K'_r$) on the failure assessment curve. This is illustrated in Figure 5 where the author has derived the equations necessary to determine these load levels. The initial point is calculated at a chosen load level, such that the point is within the assessment curve as shown by the point $A$, ($S'_r$, $K'_r$) in Figure 5. The location of the final point which then determines the actual stable load level denoted by ($S'_r$, $K'_r$) is determined by calculating the angle $\phi$ where

$$\phi = \tan^{-1}\left(\frac{K'_r}{S'_r}\right)$$  \hspace{1cm} (8)

to obtain

$$S_r = \frac{2}{\pi} \cos^{-1}\left(\exp - \frac{\pi^2}{8} \cot^2 \phi\right)$$ \hspace{1cm} (9)

The value of $S_r$ from equation (9) is input into equation (3) (the assessment curve) to determine $K_r$. The structural load is then determined from

$$\text{LOAD} = \frac{\text{SCALE}}{\text{FACTOR}} \times \frac{OB}{OA} = \frac{\text{SCALE}}{\text{FACTOR}} \times \frac{\sqrt{K_r^2 + S_r^2}}{\sqrt{K'_r^2 + S'_r^2}}$$ \hspace{1cm} (10)

If the load level initially chosen was 10 kips, then this would be the scale factor that would be used in the above equation. The above closed
form expressions can be used to calculate the load directly from the R-6 diagram and thereby eliminate time consuming graphical procedures.

As will be seen in subsequently discussed failure assessment diagrams of the various analyzed compact specimens, the maximum load for these specimens corresponds to the point of tangency of the crack growth locus of points to the failure assessment curve. This is identical to the R-curve approach which is extensively documented in Reference 9. The validity and applicability of this R-curve approach has been demonstrated in Reference 5 in the prediction of ductile tearing of compact specimens.

DUCTILE TEARING OF COMPACT SPECIMENS

The author chose several compact test specimens to test out the extended R-6 failure assessment procedure by predicting the complete load versus slow crack growth behavior of seven compact specimens of two steels, SA533B and HY130. These steel specimens are listed in Table 1. Figure 6 illustrates the planar dimensions and configuration of the typical standard ASTM E399 compact specimen. The dimensions are given in inches and are for the 4T SA533B T52 compact specimen tested under the General Electric (GE)/Electric Power Research Institute (EPRI) contract RP601-2(10). All the specimens tested under this program (T52, T32, T22, T21, and T61) were 4T's with face grooving. This led to straight crack fronts and fairly flat ductile tearing fracture surfaces. These specimens were, therefore, treated as being in plane strain both for the calculation of $K_I$ and the plastic limit load, $P_0$. The expression used to calculate $K_I$ was taken from Tada(11) for the standard ASTM E399 compact specimen, while the limit load expression $P_0$ used was taken from Reference 5 and is attributed to Rice. For completeness, the expression for $P_0$ is given by

$$P_0 = \frac{2}{\sqrt{3}} \sigma_0 (w - a) b \left[ 0.3 + \sqrt{0.9 + 0.4\zeta} \right]$$  \hspace{1cm} (11)

where

$$\zeta = 1.1025 \left[ \frac{1 + a/w}{1 - a/w} \right]^2$$  \hspace{1cm} (12)

and $w, a$ are defined in Figure 6 and $b$ is the net thickness of the specimen. For all calculations made in this paper, the flow stress, $\sigma_0$, was taken
as the average of the uniaxial yield and ultimate tensile strengths of the materials. Various combinations of material flow stress definitions and limit load expressions were investigated. Based on results which gave the best overall prediction of the complete load-versus-stable crack growth for all the compact specimens, the combination of the upper bound Rice plane strain limit load along with the average of yield and ultimate strengths for the flow stress were chosen. Discussion of the implications of these choices will be left for another paper.

DISCUSSION OF RESULTS

The material toughness or resistance to fracture which was input into the calculations was the experimentally determined $J$ resistance curve. Specimen T52's $R$ curve is shown in Figure 3. Note that no attempt was made to smooth the test data, and only the General Electric dual gage measurements were used. Only those points plotted were used to determine the crack growth locus shown in Figure 4 for the T52 specimen. The locus of points shown are for a load level of 100 kips, with the first point denoted by 1. This point ratioed the load up to 128 kips for a crack growth of 10 mils. The maximum load was ratioed up from the point denoted by 2 and was found to be 148.7 kips for a stable crack advance of 75 mils. The last point of the locus denoted by 3 gave a load of 130 kips at 300 mils crack growth. Figure 7 shows the comparison of the predicted ductile tearing behavior of T52 with the experimentally determined load versus stable crack growth taken from Reference 12. Figure 8 illustrates the growth locus for T32 for a constant load level of 100 kips, with Figure 9 giving the results of the ratioing process in terms of load versus ductile tearing. Each prediction used the actual measured $J_R$ curve for that specimen taken from tables in Reference 12. Similar results are shown in Figures 11, 13, 15 for specimens T22, T21, and T61, respectively; with Figures 10, 12, 14 illustrating the respective crack growth locus-assessment plots with constant load levels of 50, 50, and 10 kips, respectively. Note that in all these five GE/EPRI SA533B 4T compact specimens that the R-6 failure assessment procedure predicts the load after the maximum load to follow the limit load behavior for a flow stress of $\sigma_0 = 73.75$ ksi, while the actual experimental data is higher. This effect is thought to be due to strain hardening.
effects of the SA533B materials which are not taken in effect by assuming a simple expression for the flow stress; namely, the average of the yield and ultimate tensile strengths of the material. This will be discussed in more detail later in this paper.

Figure 16 shows the growth locus calculated for an HY130 IT compact specimen tested at The Babcock & Wilcox Research and Development Division in Alliance, Ohio. The \( J_R \) curve input was taken from the actual measured single specimen \( J \) tests. The unloading compliance technique was used in the determination of \( \Delta a \). The resulting load versus slow stable crack growth curve is shown in Figure 17. Figure 18 illustrates the results of a comparison between prediction and experiment of a second HY130 IT compact specimen tested at Babcock & Wilcox. Note that both these specimens are predicted quite well by the R-6 failure assessment procedure, even after the maximum load point. For this material, HY130, the yield and ultimate tensile strengths are quite close together, being 136 and 139 ksi, respectively. This material has very little strain hardening, and therefore the specimen behavior after maximum load is predicted well by the limit load expression with a flow stress based on the average of the yield and ultimate tensile strengths of HY130.

Table 1 presents a comparison of the experimental and predicted maximum load values. The calculated standard error for the seven compact steel specimens was 1.65 percent.

As a further test of this R-6 procedure, 14 compact 7075-T651 aluminum specimens were analyzed and their maximum loads were predicted using the R-6 diagram to within a standard error of 1 percent. The results are given in Table 2. The test results were taken from a recently received ASTM E24.06.02 committee's predictive round robin on fracture\(^{13}\). It is to be noted, however, that similar excellent predictions could also have been made using a \( K_R \) approach, since all these 14 specimens failed in the LEFM region of the R-6 diagram.

Note from Figures 6, 8, 10, 12, and 14 showing the R-6 assessment of the SA533B compact specimens that their behaviors during test were in
the transition and fully plastic regions of the assessment diagram. The transition region of the assessment diagram is that region between the LEFM region and the fully plastic region. A rough definition of these regions can be made by referring to Figure 5 where the angle \( \phi \) is defined. For \( \phi \) greater than 0 degree but less than 30 degrees, this would define the fully plastic region of the diagram. Between \( \phi \) of 30 and 70 degrees defines the transition region, while for \( \phi \) greater than 70 degrees is the LEFM region. Both the HY130 compact specimen behaviors were entirely within the transition region of the R-6 diagram. Since the R-6 procedure predicted the behavior of both of these specimens quite well, this seems to indicate that the original choice of the transition model, the strip yield model of Bilby-Cottrell-Swinden, was indeed an excellent choice.

CONCLUSIONS

From the analysis of the three materials, it appears that the R-6 procedure can handle the prediction of ductile tearing in compact specimens very well. Limit load appears to govern the behavior of these specimens after the maximum load, while the actual determination of maximum load is influenced by the interaction of the \( J_R \) materials curve and limit load with \( J_{IC} \) having no effect whatsoever on the maximum load.

FUTURE WORK

The next step in this work is to investigate the validity of this procedure for other type specimens and structures. The important question is whether the application of \( J_R \) material curves (generated from compact specimens) to the prediction of ductile tearing of other type specimens is valid. In other words, are \( J_R \) curves generated from compact specimens material properties?

Another important question is whether the assumption of a simplistic flow stress is correct. For maximum load prediction, this assumption seems to be valid; but for predicting the complete load versus ductile tearing behavior, an interpretation of flow stress that would include strain hardening effects would seem appropriate. While the inclusion of strain
hardening effects will not significantly affect maximum load or instability calculations (also concluded by Hutchinson(14)); it would, however, give better assurance as to the validity and usefulness of this simplified procedure, especially in situations where an accurate determination of load levels after maximum load is desired.

ACKNOWLEDGMENTS

The author wishes to thank the Nuclear Power Generation and the Nuclear Equipment Divisions of The Babcock & Wilcox Company for their financial support of this work.

REFERENCES


## TABLE 1

PREDICTION OF FACE-GROOVED COMPACT SPECIMENS

<table>
<thead>
<tr>
<th>SPECIMEN NUMBER</th>
<th>a/W</th>
<th>MATERIAL</th>
<th>MAXIMUM EXPERIMENTAL LOAD (KIPS)</th>
<th>PREDICTED MAXIMUM LOAD (KIPS)</th>
<th>% DIFF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>T52</td>
<td>.58</td>
<td>SA533B</td>
<td>149.3</td>
<td>148.7</td>
<td>-0.7</td>
</tr>
<tr>
<td>T32</td>
<td>.62</td>
<td>SA533B</td>
<td>132.3</td>
<td>136.1</td>
<td>+2.9</td>
</tr>
<tr>
<td>T22</td>
<td>.72</td>
<td>SA533B</td>
<td>72.0</td>
<td>69.0</td>
<td>-4.2</td>
</tr>
<tr>
<td>T21</td>
<td>.66</td>
<td>SA533B</td>
<td>106.5</td>
<td>103.3</td>
<td>-3.0</td>
</tr>
<tr>
<td>T61</td>
<td>.80</td>
<td>SA533B</td>
<td>29.7</td>
<td>27.8</td>
<td>-6.4</td>
</tr>
<tr>
<td>130012</td>
<td>.58</td>
<td>HY130</td>
<td>17.5</td>
<td>16.9</td>
<td>-3.4</td>
</tr>
<tr>
<td>130017</td>
<td>.56</td>
<td>HY130</td>
<td>18.8</td>
<td>18.2</td>
<td>-3.2</td>
</tr>
</tbody>
</table>

STANDARD ERROR = 1.65%
### TABLE II

ASTM TASK GROUP E24.06.02  
PREDICTIVE ROUND ROBIN ON FRACTURE  
ALUMINUM ALLOY 7075-T651 COMPACT  
SPECIMEN PREDICTIONS

<table>
<thead>
<tr>
<th>SPECIMEN NUMBER</th>
<th>W (INCHES)</th>
<th>% DIFFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC 53</td>
<td>2</td>
<td>4.7</td>
</tr>
<tr>
<td>BC 73</td>
<td>2</td>
<td>7.6</td>
</tr>
<tr>
<td>BC 33</td>
<td>2</td>
<td>-0.2</td>
</tr>
<tr>
<td>BC 83</td>
<td>2</td>
<td>3.0</td>
</tr>
<tr>
<td>BC 82</td>
<td>4</td>
<td>2.0</td>
</tr>
<tr>
<td>BC 72</td>
<td>4</td>
<td>-2.4</td>
</tr>
<tr>
<td>BC 22</td>
<td>4</td>
<td>-2.9</td>
</tr>
<tr>
<td>BC 52</td>
<td>4</td>
<td>-3.4</td>
</tr>
<tr>
<td>BC 32</td>
<td>4</td>
<td>-1.0</td>
</tr>
<tr>
<td>BC 51</td>
<td>8</td>
<td>3.7</td>
</tr>
<tr>
<td>BC 21</td>
<td>8</td>
<td>5.2</td>
</tr>
<tr>
<td>BC 81</td>
<td>8</td>
<td>1.3</td>
</tr>
<tr>
<td>BC 31</td>
<td>8</td>
<td>-0.7</td>
</tr>
<tr>
<td>BC 71</td>
<td>8</td>
<td>-2.6</td>
</tr>
</tbody>
</table>

STANDARD ERROR = 1.00%
\[ K'_r(a) = \frac{K_1(a)}{K_{IC}} \quad \text{or} \quad \sqrt{\frac{J_{IE}(a)}{J_{IC}}} \]

\[ S'_r(a) = \frac{o}{o_1(a)} \]

\[ K_r = S_r \left\{ \frac{8}{\pi^2} \int_0^1 \sec (\pi S_r/2) \right\}^{1/2} \]

**Figure 1** R-6 Failure Assessment Diagram
\[ K'_r (a + \Delta a) = \sqrt{J_{1E} (a + \Delta a) / J_R (\Delta a)} \]

\[ S'_r (a + \Delta a) = \sigma / \sigma_1 (a + \Delta a) \]

**Figure 2** Failure Assessment Diagram in Terms of Stable Crack Growth
FIGURE 4 FAILURE ASSESSMENT DIAGRAM FOR SPECIMEN T-52
\[
\phi = \tan^{-1} \left( \frac{K_r}{S_r} \right)
\]
\[
S_r = \frac{2}{\pi} \cos^{-1} \left( \exp - \frac{\pi^2}{8} \cot^2 \phi \right)
\]
\[
K_r = S_r \left\{ \frac{8}{\pi^2} \ell \ln \sec \left( \frac{\pi}{2} S_r \right) \right\}^{-1/2}
\]

\[
\text{LOAD} = \text{SCALE FACTOR} \times \frac{\text{OB}}{\text{OA}} = \text{SCALE FACTOR} \times \frac{\sqrt{K_r^2 + S_r^2}}{\sqrt{K_r'}^2 + S_r^2}
\]

**FIGURE 5** LOAD DETERMINATION USING THE R-6 DIAGRAM
FIGURE 6  DETAILED GEOMETRY OF T-52 COMPACT SPECIMEN
FIGURE 7  LOAD VERSUS STABLE CRACK GROWTH FOR T-52 SPECIMEN
FIGURE 8  FAILURE ASSESSMENT DIAGRAM FOR SPECIMEN T-32
T-32 SPECIMEN (GE/EPRI DATA)

- EXPERIMENTAL DUAL GAGE MEASUREMENTS
- EXPERIMENTAL COMPLIANCE MEASUREMENTS
- ASSESSMENT CALCULATION (RICE LIMIT LOAD)

LIMIT LOAD, $a_o = (a_u + a_y)/2$

$a_o = 73.75$ KSI

FIGURE 9 LOAD VERSUS STABLE CRACK GROWTH FOR T-32 SPECIMEN
FIGURE 10 FAILURE ASSESSMENT DIAGRAM FOR SPECIMEN T-22
FIGURE 12 FAILURE ASSESSMENT DIAGRAM FOR SPECIMEN T-21
FIGURE 13 LOAD VERSUS STABLE CRACK GROWTH FOR T-21 SPECIMEN

T-21 SPECIMEN (GE/EPRI DATA)
- EXPERIMENTAL DUAL GAGE MEASUREMENTS
- ASSESSMENT CALCULATION (RICE LIMIT LOAD)
- LIMIT LOAD, $\sigma_0 = 73.75$ KSI
FIGURE 14 FAILURE ASSESSMENT DIAGRAM FOR SPECIMEN T-61
FIGURE 15 LOAD VERSUS STABLE CRACK GROWTH FOR T-61 SPECIMEN
FIGURE 16 FAILURE ASSESSMENT DIAGRAM FOR SPECIMEN HY130-130012
1T CT SPECIMEN HY130 - 130017
(BABCOCK & WILCOX DATA)

- EXPERIMENTAL COMPLIANCE MEASUREMENTS
- ASSESSMENT CALCULATIONS (RICE LIMIT LOAD)
- LIMIT LOAD, $\sigma_0 = 137.5$ KSI

FIGURE 18 LOAD VERSUS STABLE CRACK GROWTH FOR HY130 - 130017
List of Participants

AUSTRIA
Rieger, G. Visiting EG&G Idaho, Inc., Idaho Falls from OSGAE, Vienna

CANADA
Jarman, B. Atomic Energy Control Board, Ottawa
Panesar, J.C. Atomic Energy of Canada, Mississauga, Ontario

FINLAND
Ranta-Maunus, A. Visiting Northwestern University, Civil Eng. Dept., Evanston, Ind. from Institute of Radiation Prot., Helsinki

FRANCE
Bhandari, S. Novatome, Robinson
Faidy, C. Electricity of France, LaDefense
Pineau, J. Ecole Nat'l Super Des Mines, Cedex
Pellissier, A. Framatome, Paris
Roche, R.L. CEN Saclay, Gif Sur Yvette

NETHERLANDS
Bakker, A. Potter Damseweg, Al Delft

ITALY
Conti, M. CNEN, RAD, RSI, IN GIMP, Rome
Zampini, G. NIRA, Genova

SWEDEN
Carlsson, J. The Royal Institute of Technology, Stockholm

WESTERN GERMANY
Blauen, J.G. Institut fur Festkorper Mechanik, Freiburg
Azodi, D. Gesellschaft fur Reaktoricherheit, Koln

UNITED KINGDOM
Milne, I. Central Electricity Research, Leatherhead Surrey
Bevitt, E. UKAEA (SRD) Culcheth
Irvine, W.H. UKAEA (SRD) Culcheth
Ingham, T. UKAEA (RL) Risley
Fulford, I. Rolls Royce & Assoc.Ltd., Derby
Howard, I. University of Sheffield, Sheffield
Rose, R.T. Nuclear Power Company, Cheshire
Turner, C.E. Imperial College of Science & Technology, London
Garwood, S.J. The Welding Institute, Cambridge
USA

Albrecht, P. U.S. Nuclear Regulatory Commission, Washington DC
Johnson, R. U.S. Nuclear Regulatory Commission, Washington DC
Loss, F.J. Naval Research Laboratory, Washington, D.C.
Gudas, J. David W. Taylor Naval Research, Annapolis, Md.

Landes, J. Westinghouse R&D, Pittsburgh, Pa.
Hutchinson, J.W. Harvard University, Cambridge, Mass.
Irwin, G. University of Maryland, College Park, Md.
Shih, F. General Electric Co., Schenectady, N.Y.
Joyce, J. U.S. Naval Academy, Annapolis, Md.

Merkle, J.G. Union Carbide Corporation, Oakridge, Tenn.
Zahoor, A. Battelle Columbus Lab, Columbus, Ohio
Norris, D. Electric Power Research Institute, Palo Alto, Calif.
Bloom, J. Babcock & Wilcox, Alliance, Ohio
Chen, A. Sandia Laboratories, Albuquerque

Szabo, B. Washington University in St. Louis, Mo.
Paris, P.C. Washington University in St. Louis, Mo.

OTHERS

Lamain, L. Commission of the European Communities
ISPRA, Joint Research Center, Varese, Italy

Oliver, P. OECD Nuclear Energy Agency, Paris, France
PROGRAM (Tentative)

COMMITTEE ON THE SAFETY OF NUCLEAR INSTALLATIONS (CSNI) SPECIALIST MEETING
OECD NUCLEAR ENERGY AGENCY, Paris, France

on

Plastic Tearing Instability

Hosted by the Center for Fracture Mechanics, Washington University in St. Louis
September 25, 26, 27, 1979

TUESDAY MORNING  THEORY AND GENERAL CONSIDERATIONS, A. Pellissier Tanon, Chairman
September 25

(1) 9:00 - 9:45 AM  J.W. Hutchinson  "FOUNDATIONS OF TEARING INSTABILITY THEORY"
   9:45 -10:00 AM  Discussion and 5-Minute Break

(2) 10:00-10:25 AM  W.H. Irvine  "PLASTIC TEARING INSTABILITY-WHAT IS IT?"
   (Co-authored with A. Quirk)
   10:25-10:30 AM  Discussion
   10:30-11:00 AM  Break (with refreshments)

(3) 11:00-11:25 AM  A. Pineau  "STUDY OF INSTABILITY OF GROWING CRACKS USING
   DAMAGE FUNCTION: APPLICATION TO WARM PRE-STRESS EFFECT" (Authored by F.M.Beremin.)
   11:25-11:30 AM  Discussion

(4) 11:30-11:55 AM  R. Denys  "GROSS STRAIN CONCEPT"
   (Co-authored with W. Soete)
   11:55-12:00 N  Discussion
   12:00-12:15 PM  Summary Points, A. Pellissier Tanon, Chairman
   12:15-1:30 PM  Luncheon

TUESDAY AFTERNOON  THEORY AND GENERAL CONSIDERATIONS, J. Carlsson, Chairman
SEPTEMBER 25

(5) 1:30 -2:15 PM  C. E. Turner  "REMARKS ON UNSTABLE DUCTILE CRACK GROWTH"
   2:15 -2:30 PM  Discussion and 5-minute Break

(6) 2:30 -2:55 PM  I. Milne  "A TECHNIQUE FOR ANALYZING FRACTURE TOUGHNESS
   TEST DATA DURING SLOW CRACK GROWTH" (Co-authored with G.G. Chell)
   2:55 -3:00 PM  Discussion
   3:00 -3:30 PM  Break (with refreshments)
TUESDAY AFTERNOON
SEPTEMBER 25 (cont'd)  
3:30 - 3:55 PM  J.G. Merkle  "STABLE CRACK GROWTH ESTIMATES BASED ON EFFECTIVE CRACK LENGTH AND CRACK OPENING DISPLACEMENT"  (Co-authored with C.E. Hudson.)
3:55 - 4:00 PM  Discussion
(8)  4:00 - 4:25 PM  C.F. Shih  "AN ENGINEERING APPROACH FOR EXAMINING GROWTH AND STABILITY IN FLAWED STRUCTURES"
4:25 - 4:30 PM  Discussion
4:30 - 4:45 PM  Summary Points,  J. Carlsson, Chairman

WEDNESDAY MORNING
SEPTEMBER 26

(9)  9:00 - 9:45 AM  J. D. Landes  "SIZE AND GEOMETRY EFFORTS ON ELASTIC-PLASTIC FRACTURE CHARACTERIZATION"
9:45 -10:00 AM  Discussion and 5-minute break
(10)  10:00 -10:25 AM  S.J. Garwood  "CRACK GROWTH RESISTANCE - GEOMETRY EFFECTS AND STRUCTURAL PREDICTIONS IN A533B Class I STEEL"
10:25 -10:30 AM  Discussion
10:30 -11:00 AM  Break (with refreshments)
(11)  11:00 -11:25 AM  J. T. Gudas  "ISSUES IN DEVELOPING A PLANE STRAIN J C CURVE TEST PROCEDURE"  (Co-authored with J.A.Joyce and P.Albrecht)
11:25 -11:30 AM  Discussion
11:55 -12:00 N  Discussion
12:00 -12:15 PM  Summary Points,  C.E. Turner, Chairman
12:15 - 1:30 PM  Luncheon

WEDNESDAY AFTERNOON  "EXPERIMENTS AND APPLICATIONS" E.T. Wessel, Chairman
SEPTEMBER 26

(13)  1:30 - 2:15 PM  A. Pellissier Tanon  "TESTING AND APPLICATIONS"
2:15 - 2:30 PM  Discussion and 5 minute break
(14)  2:30 - 2:55 PM  T. Ingham  "THE MEASUREMENT OF DUCTILE CRACK INITIATION"
2:55 - 3:00 PM  Discussion
3:00 - 3:30 PM  Break (with refreshments)
(15)  3:30 - 3:55 PM  R.L. Roche  "FORMULAS GIVING THE J-R CURVE FROM RESULTS OF ONE EXPERIMENTAL TEST"
3:55 - 4:00 PM  Discussion

(more)
(16) 4:00 - 4:25 PM  G. G. Chell  "A SIMPLE METHOD FOR DETERMINING THE DUCTILE INSTABILITY OF CRACKED STRUCTURES" (Co-authored with I. Milne.)

4:25 - 4:30 PM  Discussion

4:30 - 4:45 PM  Summary Points,  E.T. Wessel, Chairman

THURSDAY MORNING  APPLICATIONS  G.R. Irwin, Chairman
SEPT. 27th

(17) 9:00 - 9:45 AM  P. C. Paris  FRACTURE PROOF DESIGN

9:45 - 10:00 AM  Discussion and 5-minute break

(18) 10:00 - 10:25 AM  J.L. Cheissoux  "CONDITION OF STABILITY OF AN AXIAL THROUGH CRACK APPEARING ON PWR SECONDARY PIPING IN REDUCED SCALE (1/10)

10:25 - 10:30 AM  Discussion

10:30 - 11:00 AM  Break (with refreshments)

(19) 11:00 - 11:25 AM  B. Szabo  "AN ANALYSIS OF DUCTILE CRACK EXTENSION IN BWR FEEDWATER NOZZLES" (Co-authored with G.G. Musicco, M.P. Rossow)

11:25 - 11:30 AM  Discussion

(20) 11:30 - 11:55 AM  A. Zahoor  "A PRELIMINARY FRACTURE ANALYSIS ON THE INTEGRITY OF HSST INTERMEDIATE TEST VESSELS" (Co-authored with P.C. Paris and M.P. Gomez)

11:55 - 12:00 N  Discussion

12:00 - 12:15 PM  Summary Points  G. R. Irwin, Chairman

12:15 - 1:30 PM  Luncheon

THURSDAY AFTERNOON  SEPT. 27th

1:30 PM  
- Conference Summary
- Recent work at Washington University
- Other Contributions
The Specialist Meeting on Plastic Tearing Instability was held at the Center for Fracture Mechanics, Washington University, St. Louis, USA, and was sponsored by the Committee on the Safety of Nuclear Installations (CSNI) of the OECD Nuclear Energy Agency. The meeting was hosted by the Center for Fracture Mechanics and by the United States Nuclear Regulatory Commission.

The Meeting constituted a forum for the 45 participants to make presentations and to thoroughly discuss details of the material presented. Copies of the material presented appear in this report.