



Thermal Neutron Scattering Law Research at North Carolina State University

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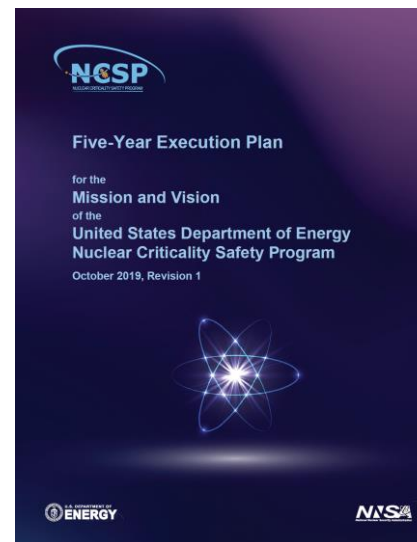
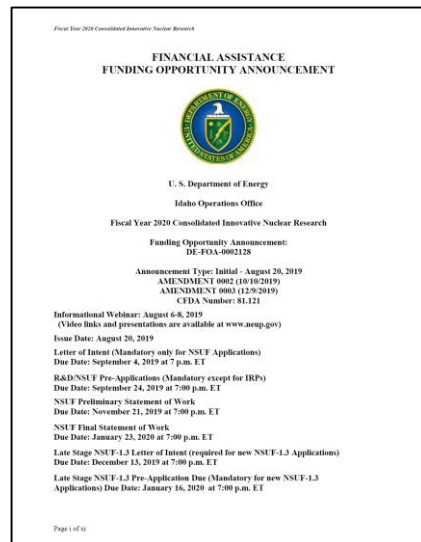
North Carolina State University

Raleigh, North Carolina, USA

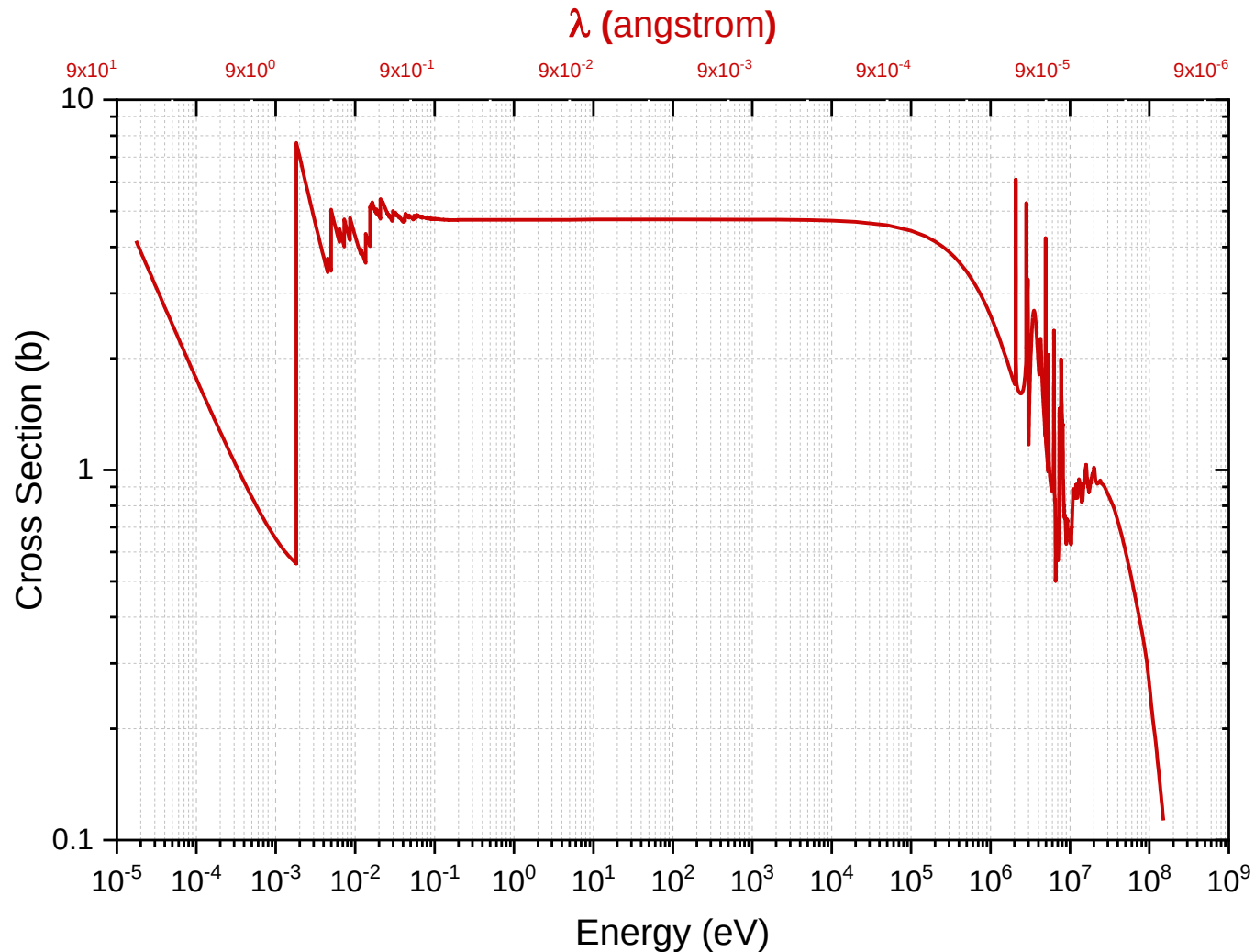
32nd Meeting of the
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Acknowledgment

- ❑ DOE NE through the Nuclear Energy University Program (NEUP)
- ❑ NNSA Nuclear Criticality Safety Program (NCSP)
 - in collaboration with LLNL
- ❑ Naval Nuclear Propulsion Program (NNPP)



Neutron Interactions – Carbon



Neutron Thermalization

Using first Born approximation combined with Fermi pseudopotential, it can be shown that the double differential scattering cross section has the form

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{1}{4\pi} \sqrt{\frac{E'}{E}} \left\{ \sigma_{coh} S(\vec{k}, \omega) + \sigma_{incoh} S_s(\vec{k}, \omega) \right\}$$

The dynamic structure factor $S(\vec{k}, \omega)$ is composed of two parts

$$S(\vec{k}, \omega) = S_s(\vec{k}, \omega) + S_d(\vec{k}, \omega)$$

Van Hove's space-time formulation

$$I(\vec{k}, t) = \int G(\vec{r}, t) \exp(i\vec{k} \cdot \vec{r}) d\vec{r}$$

$$S(\vec{k}, \omega) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(\vec{r}, t) e^{i(\vec{k} \cdot \vec{r} - \omega t)} d\vec{r} dt$$

where $G(\vec{r}, t)$ is the *dynamic pair correlation function* and can be expressed in terms of time dependent atomic positions.

$$S_s(\alpha, \beta) = k_B T \cdot S_s(\vec{k}, \omega)$$

$$\left. \frac{d^2 \sigma}{d\Omega dE'} \right|_{inelastic} = \frac{\sigma}{2k_B T} \sqrt{\frac{E'}{E}} S_s(\alpha, \beta)$$

Since 1960s
GASKET
NJOY/LEAPR
INCOHERENT
APPROXIMATION

$$\beta = \frac{E - E'}{k_B T} \quad \text{Energy transfer}$$

$$\alpha = \frac{(E + E' - 2\sqrt{EE'} \cos \theta)}{k_B T} \quad \text{Momentum transfer}$$

The scattering law (TSL) is the Fourier transform of a Gaussian correlation function

$$S_s(\alpha, \beta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\beta t} e^{-\gamma(t)} dt$$

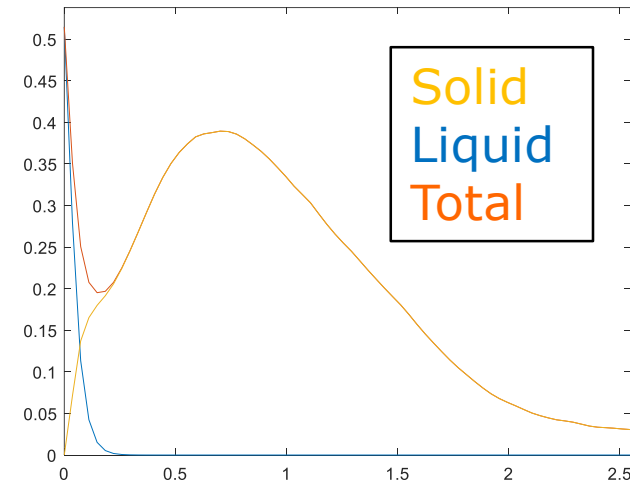
$$\gamma(t) = \frac{\alpha}{2} \int_{-\infty}^{\infty} \frac{\rho(\beta)}{\beta \sinh(\beta/2)} [1 - e^{-i\beta t}] e^{\beta/2} d\beta$$

$\rho(\beta)$ – density of states (e.g., phonon frequency distribution)

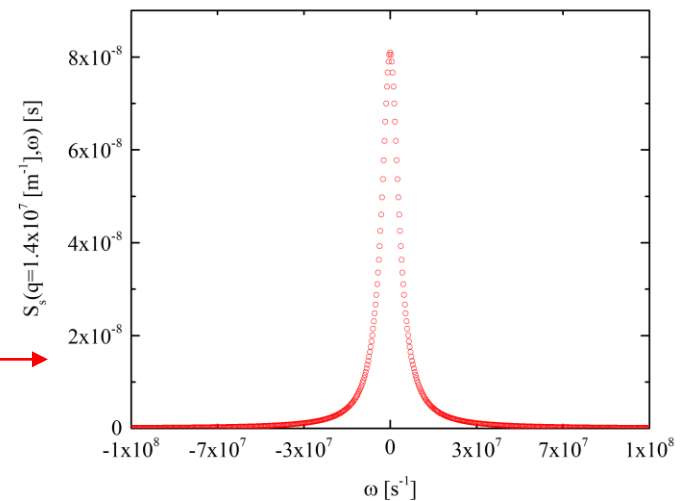
Thermalization in Liquids

□ **Separation** of the diffusive DOS from the continuous (solid) DOS

□ **Convolution** of the solid and liquid TSL components

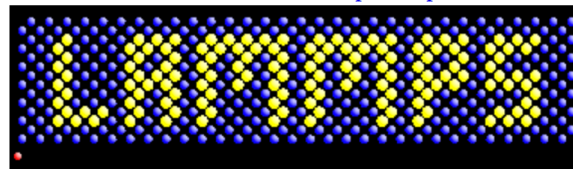


$$S_{total}(\alpha, \beta) = (S_{diff.}(\alpha, \beta) * S_{cont.}(\alpha, \beta))$$



Thermal Scattering Law Analysis

- ❑ Key development in the last 20 years is the use of atomistic simulations methods to support the evaluation process
 - Produce data necessary to calculate the TSL including
 - ❑ DOS for evaluation of TSL
 - ❑ Direct access to TSL using correlation analysis
 - ❑ Support computational and experimental analysis

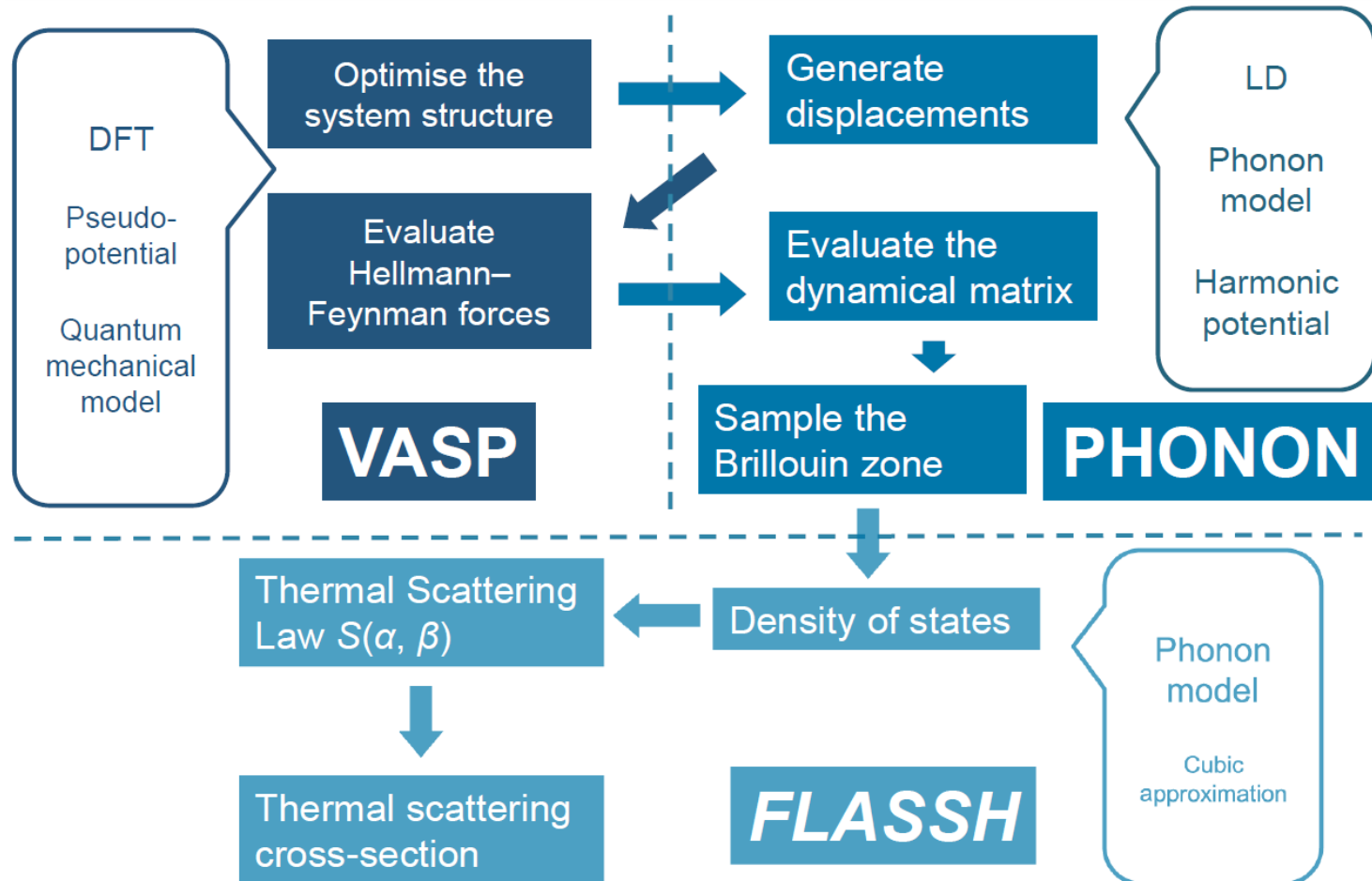


Analysis Approach

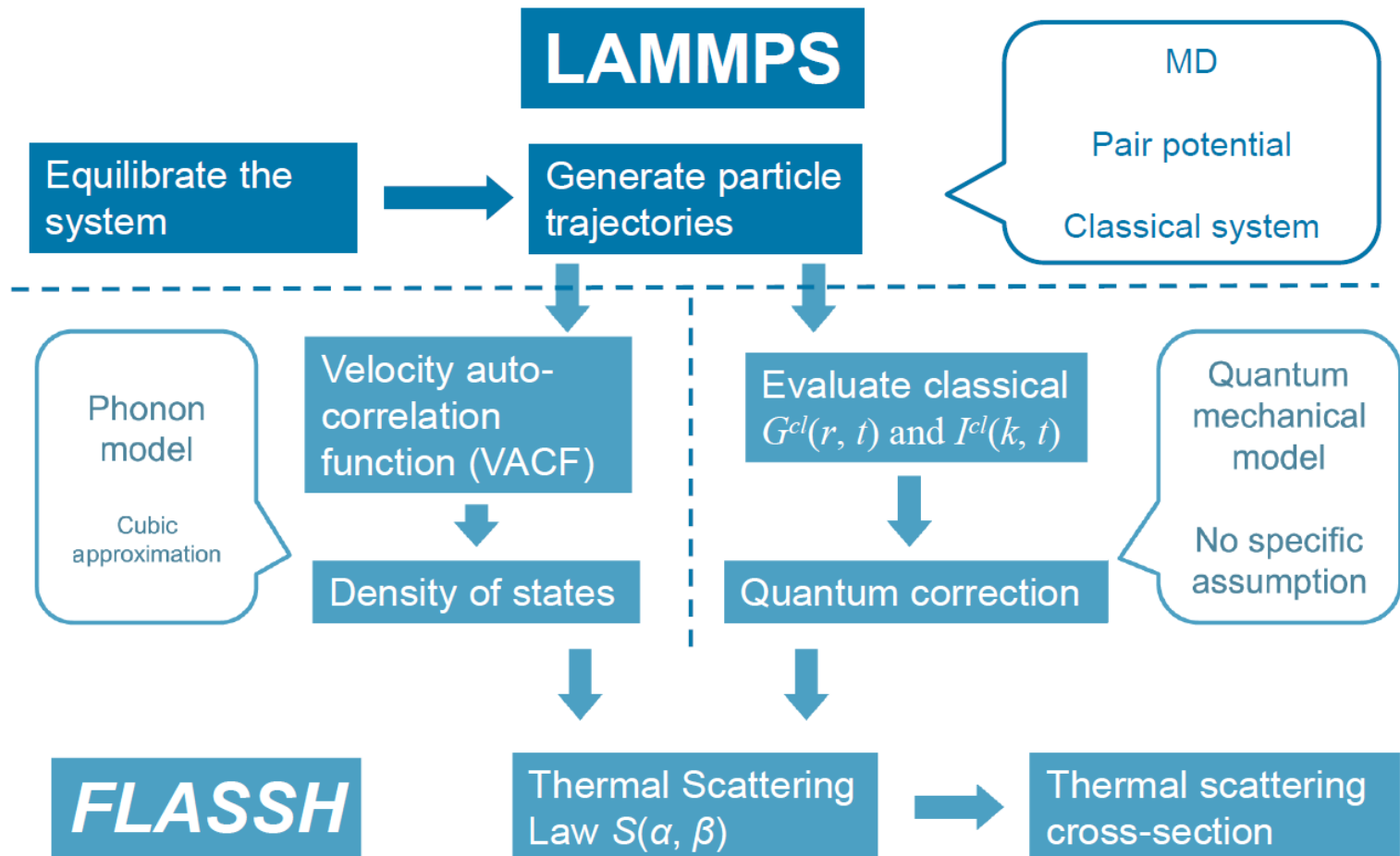
- ❑ Construct atomistic model of a material
- ❑ Verify ability of model to reproduce physical properties of the material (equilibrium conditions)
 - Density, thermal expansion, thermal conductivity,...
 - Ergodic behavior, correlations,...
- ❑ Generate input (DOS, ...) for TSL calculations
- ❑ Calculate TSL and produce thermal scattering cross sections
 - Check consistency of results with computational assumptions/models
 - Compare to experimental data

Thermal Scattering Law

DFT/LD



Thermal Scattering Law MD/QM

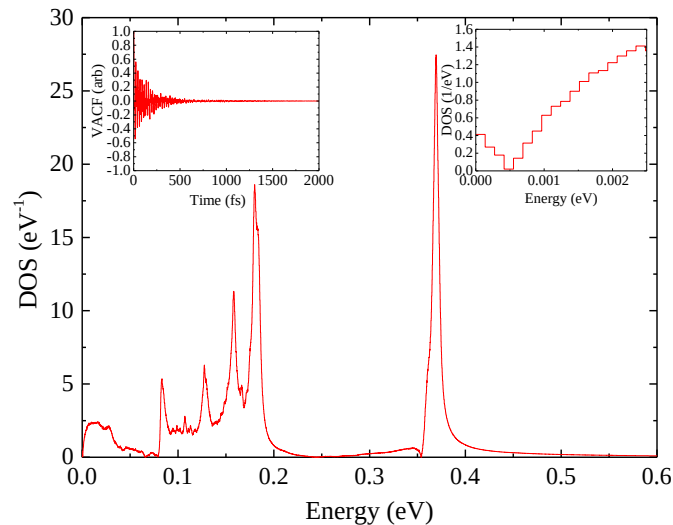
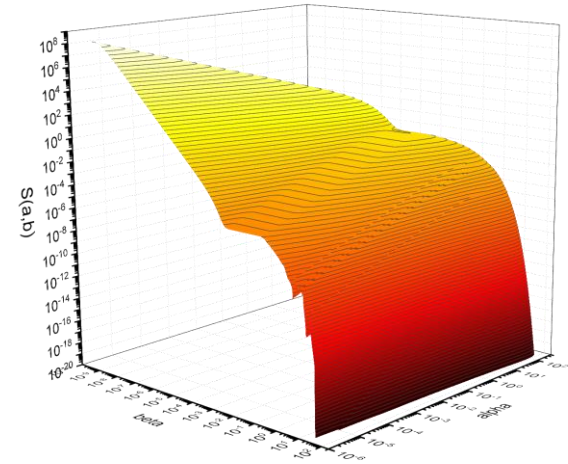
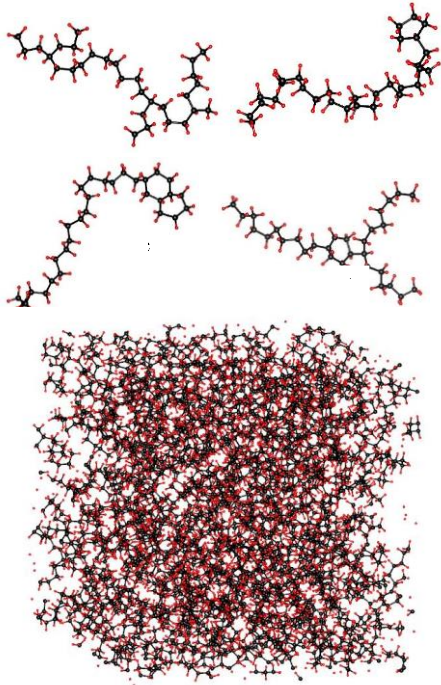


ENDF/B-VIII TSL Evaluations

Material	Status	Evaluation Basis	Institution
Beryllium metal	tsl-Be-metal.endf	DFT/LD	NCSU
Beryllium oxide (beryllium)	tsl-BeinBeO.endf	DFT/LD	NCSU
Beryllium oxide (oxygen)	tsl-OinBeO.endf	DFT/LD	NCSU
Light water (hydrogen)	tsl-HinH2O.endf	MD	CAB
Light water ice (hydrogen)	tsl-HinIceIh.endf	DFT/LD	BAPL
Light water ice (oxygen)	tsl-OinIceIh.endf	DFT/LD	BAPL
Heavy water (deuterium)	tsl-DinD2O.endf	MD	CAB
Heavy water (oxygen)	tsl-OinD2O.endf	MD	CAB
Polymethyl Methacrylate (Lucite)	tsl-HinC5O2H8.endf	MD	NCSU
Polyethylene	tsl-HinCH2.endf	MD	NCSU
Crystalline graphite	tsl-graphite.endf	MD	NCSU
Reactor graphite (10% porosity)	tsl-reactor-graphite-10P.endf	MD	NCSU
Reactor graphite (30% porosity)	tsl-reactor-graphite-30P.endf	MD	NCSU
Silicon carbide (silicon)	tsl-CinSiC.endf	DFT/LD	NCSU
Silicon carbide (carbon)	tsl-SiinSiC.endf	DFT/LD	NCSU
Silicon dioxide (alpha phase)	tsl-SiO2-alpha.endf	DFT/LD	NCSU
Silicon dioxide (beta phase)	tsl-SiO2-beta.endf	DFT/LD	NCSU
Yttrium hydride (hydrogen)	tsl-HinYH2.endf	DFT/LD	BAPL
Yttrium hydride (yttrium)	tsl-YinYH2.endf	DFT/LD	BAPL
Uranium dioxide (oxygen)	tsl-OinUO2.endf	DFT/LD	NCSU
Uranium dioxide (uranium)	tsl-UinUO2.endf	DFT/LD	NCSU
Uranium nitride (nitrogen)	tsl-NinUN.endf	DFT/LD	NCSU
Uranium nitride (uranium)	tsl-UinUN.endf	DFT/LD	NCSU
Paraffinic Oil	To NNDC	MD	NCSU
Sapphire (Al2O3)	To NNDC	DFT/LD	NCSU
Molten Salt (FLiBe)	Completed	MD	NCSU

New to NNDC

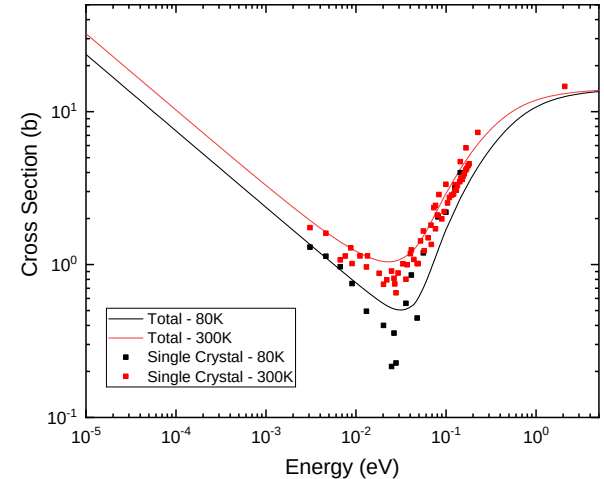
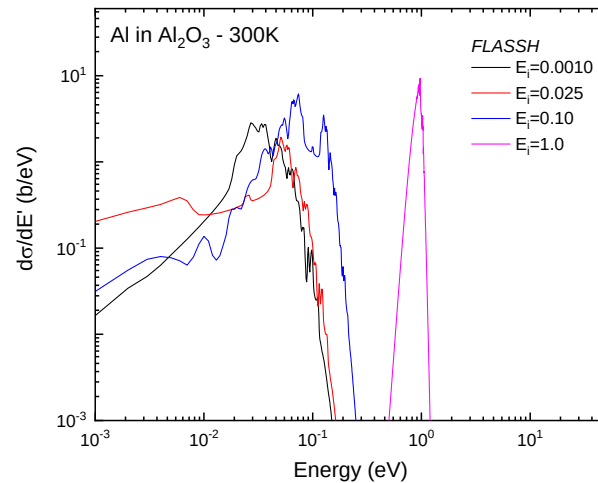
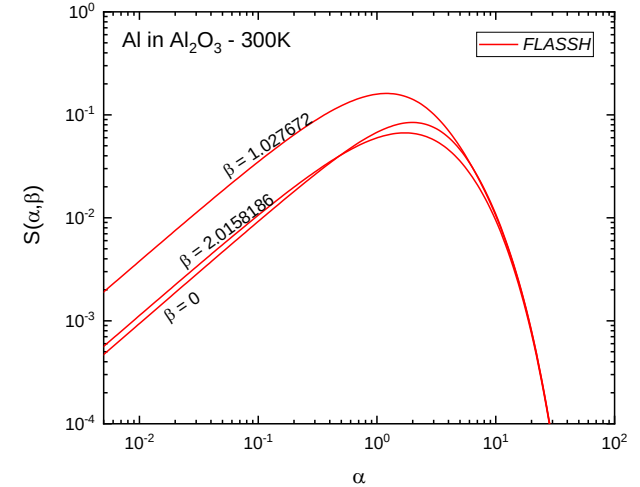
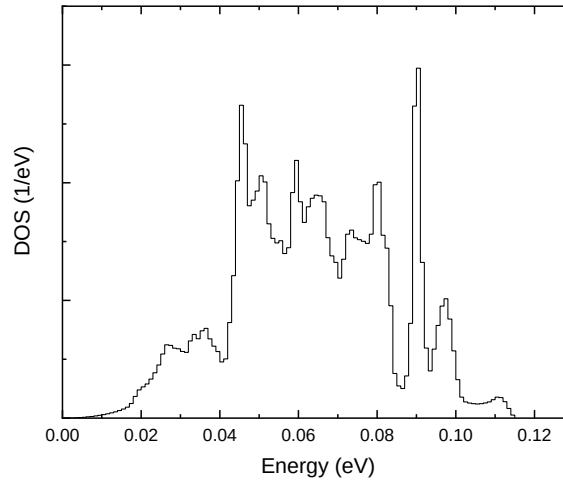
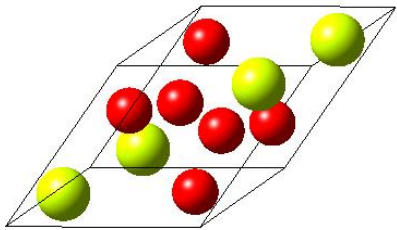
Heavy Paraffinic Oil



New to NNDC

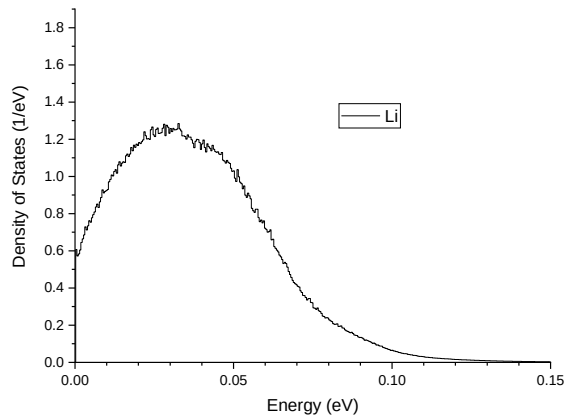
Single Crystal Sapphire TSL Data

- *Ab initio* lattice dynamics
- Predictive density of states (DOS)
 - Rhombohedral structure
 - 2x2x2 supercell
 - GGA-PBE

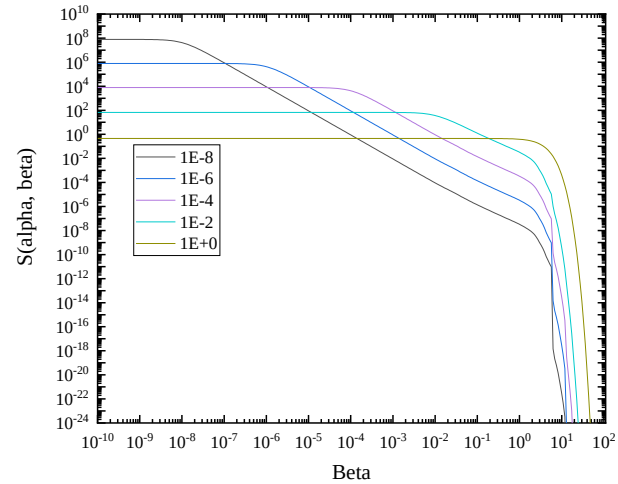


Completed FLiBe TSL Data

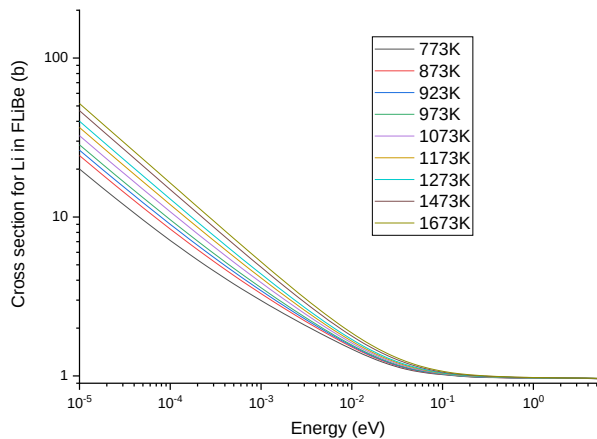
DOS using MD 773 K



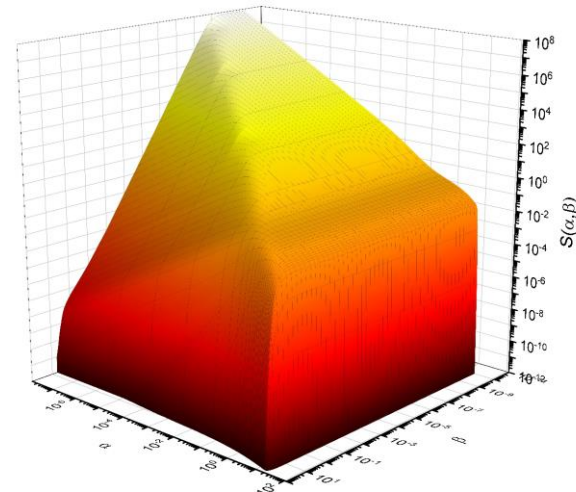
TSL using FLASSH 773 K



Cross section using FLASSH

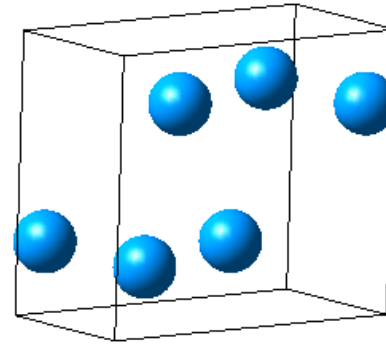


TSL using FLASSH 773 K

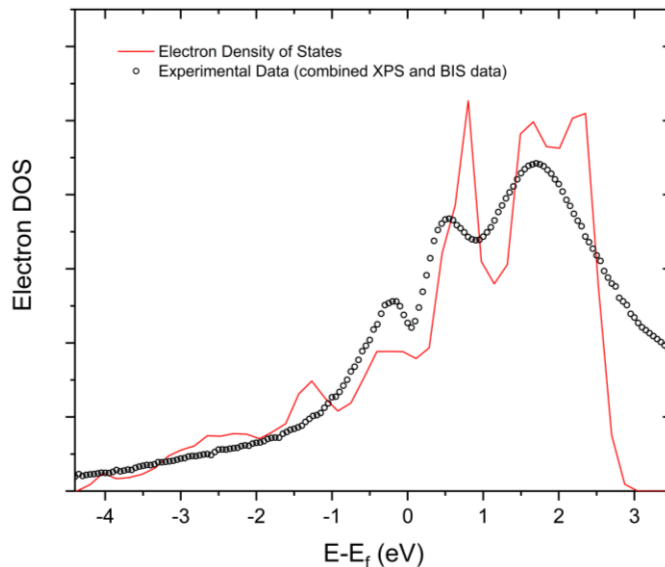


Uranium Metal

- ▶ α -Uranium Metal
 - ▶ Stable up to 668°C
 - ▶ *Ab initio* lattice dynamics
 - ▶ DFT – GGA-PBE plus an effective Coulomb term (+U) of 1eV for the *5f* electrons plus spin-orbit coupling



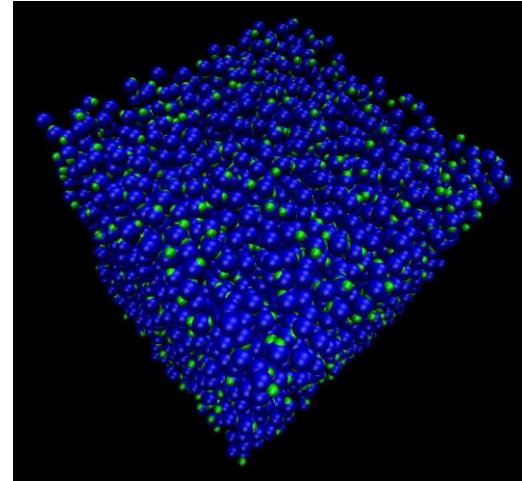
- Orthorhombic structure
- 550 eV plane wave cutoff
- 12x12x7 k-mesh



	Experiment (4.2 K)	Calculated	Diff. (%)
a (Å)	2.8444	2.8565	0.42
b (Å)	5.8689	5.8706	0.03
c (Å)	4.9316	4.9834	1.05

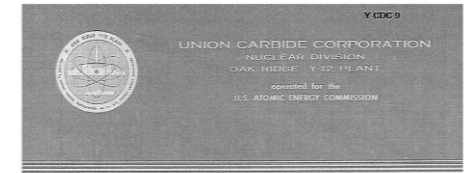
Hydrofluoric Acid MD

- ❑ Constructed potential function
- ❑ Molecular dynamics model
 - ❑ LAMMPS code
 - ❑ 4000 molecules



$$U^{total} = \sum_i \frac{1}{2} k_D |\mathbf{r}_{F,i} - \mathbf{d}_i|^2 + \frac{1}{2} k_{HF} (r_{HF} - d_{FH})^2 + \sum_{i < j} \sum_{s, s'} \frac{q_s q_{s'}}{|\mathbf{r}_{s,i} - \mathbf{r}_{s',i}|} \\ + \sum_{i < j} 4 \varepsilon_{FF} \left[\left(\frac{\sigma_{FF}}{|\mathbf{r}_{F,i} - \mathbf{r}_{F,j}|} \right)^{12} - \left(\frac{\sigma_{FF}}{|\mathbf{r}_{F,i} - \mathbf{r}_{F,j}|} \right)^6 \right]$$

- ❑ Testing is underway using experimental data derived from "CRITICALITY OF LIQUID MIXTURES OF HIGHLY ²³⁵U-ENRICHED URANIUM HEXAFLUORIDE AND HYDROFLUORIDE ACID"



CRITICALITY OF LIQUID MIXTURES OF HIGHLY ²³⁵U-ENRICHED URANIUM
HEXAFLUORIDE AND HYDROFLUORIDE ACID

Robert Giergans, Edouard Delort, Pierre Lelouché,
Louis Maubert, and Henri Revol

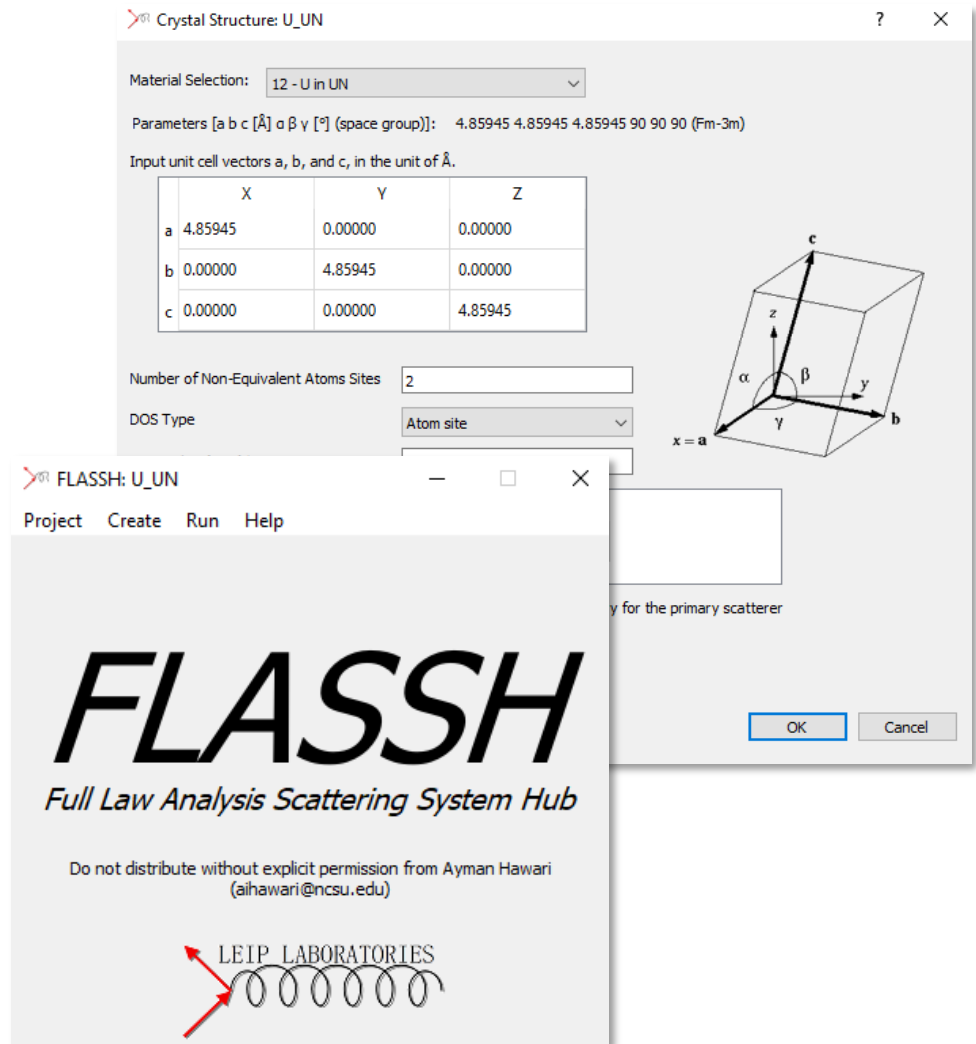
Commissariat à l'Énergie Atomique
Direction de la Protection et de la Sécurité Radiologique
Service D'Études de Criticité

CRITICALITY DATA CENTER

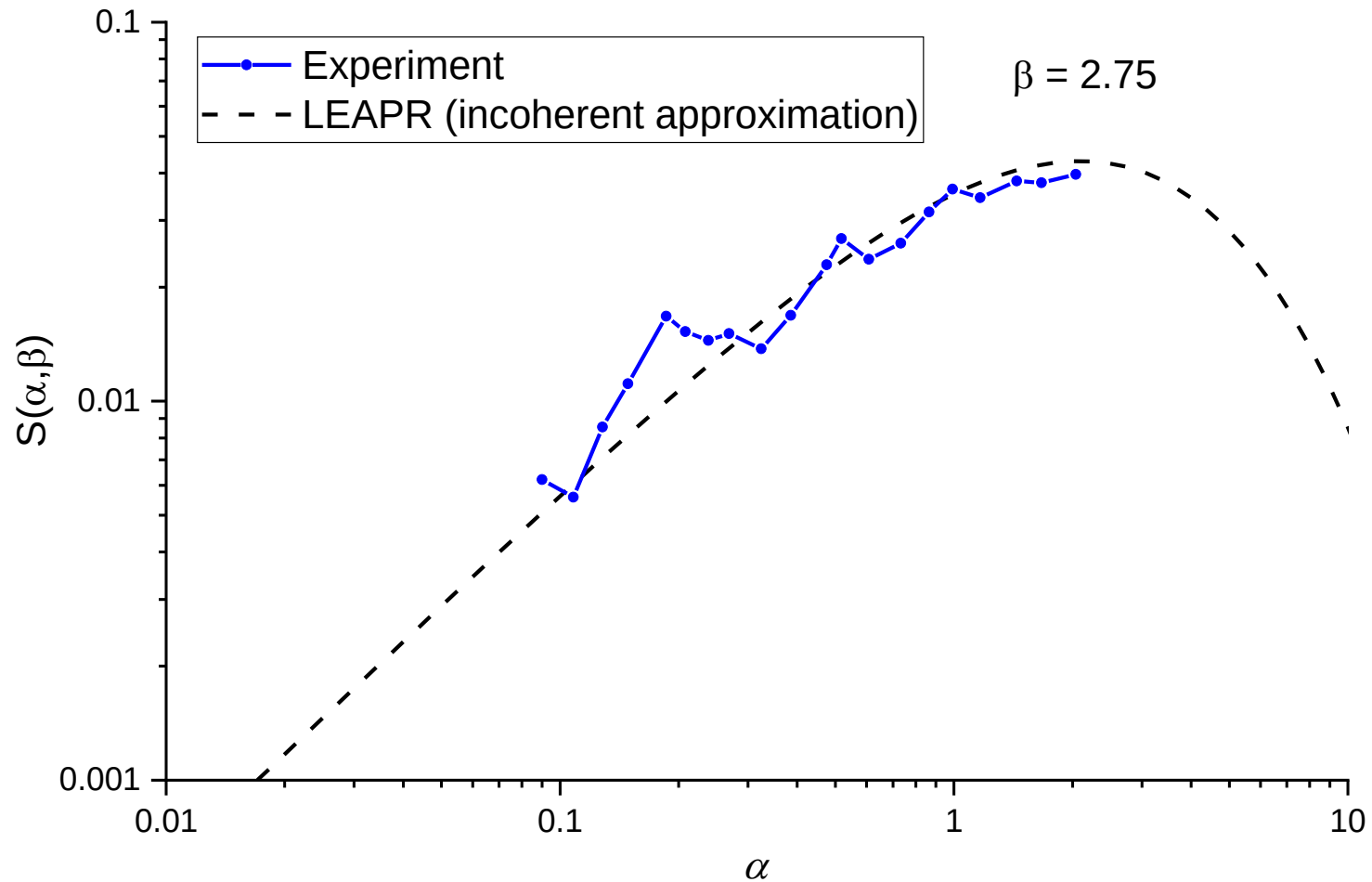


FLASSH Code

- **FLASSH Code**
 - Relaxed major approximations (incoherent, cubic, ect.)
 - Improved liquid physics
 - Improved output formatting
 - Warning Messages for the User
- **FLASSH GUI**
 - Error Checks
 - Crystal Structure Window/Inputs
 - ENDF Header Formatting

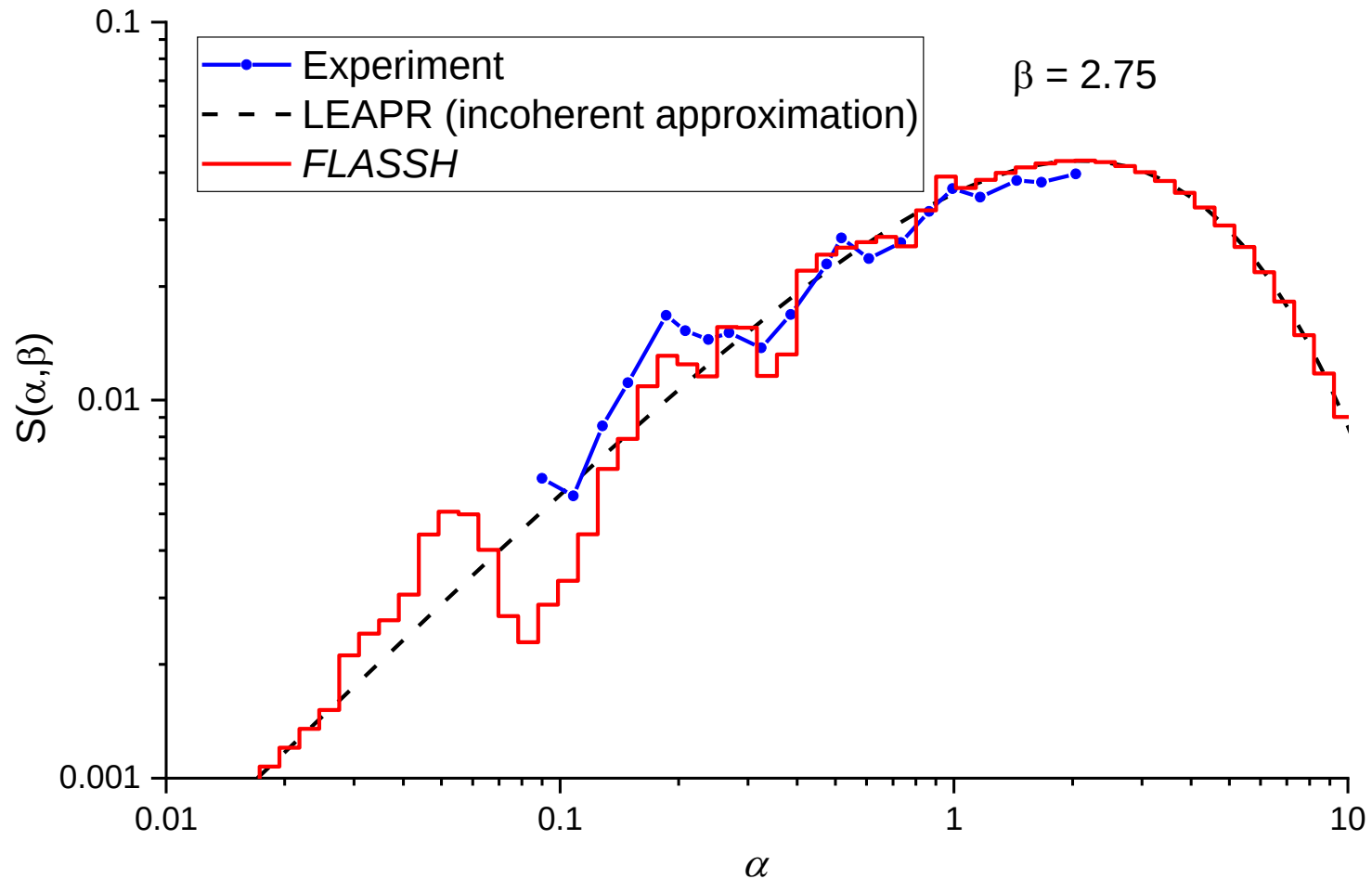


NJOY

Be Scattering Law $S(\alpha, \beta)$ 

FLASSH

Be Scattering Law $S(\alpha, \beta)$



FLASSH – Generalized TSL & Doppler Treatment

Generalized TSL

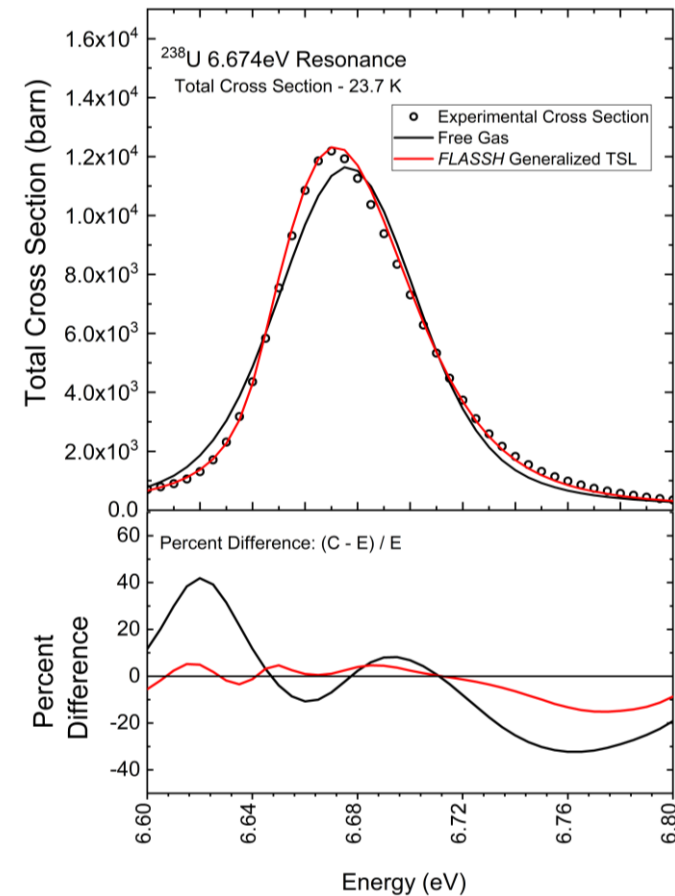
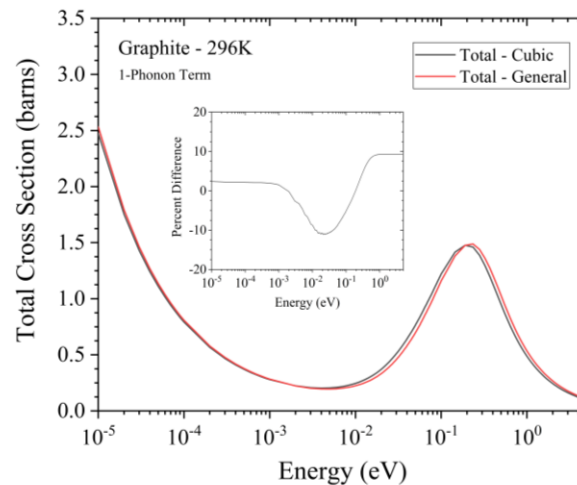
- Exact Structure
- Polarization vectors and frequencies directly input
- Removes cubic and atom site approximation

Doppler Broadening

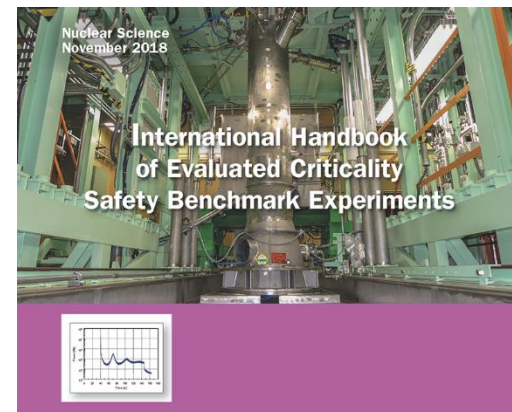
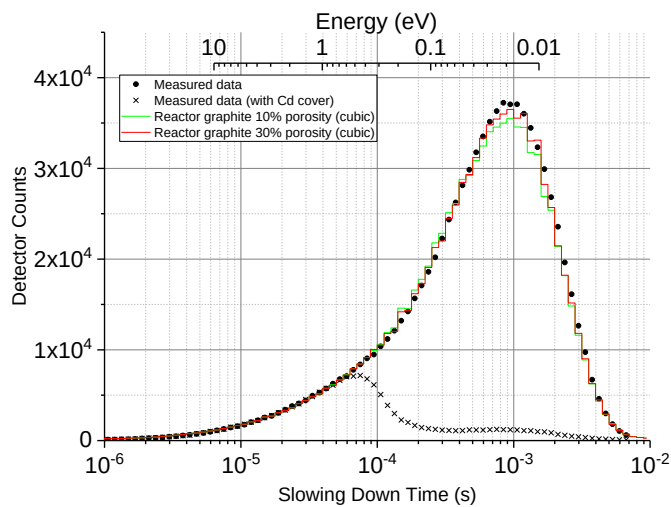
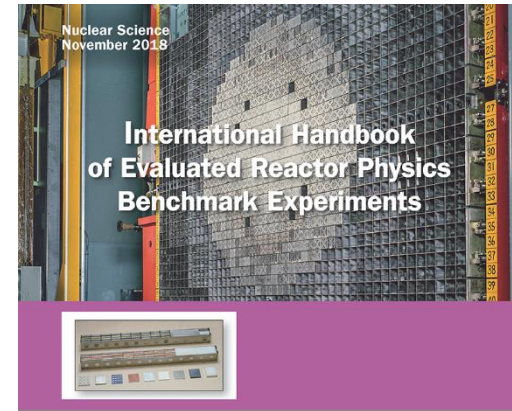
- FLASSH* TSL to determine velocity distribution for broadening
- Implements exact material structure
- Consistent evaluation for both thermal scattering and Doppler broadening

$$S_s^m(k, \omega) = \int e^{-i\omega t} I_s^m(k, t) dt$$

$$= \int e^{-i\omega t} e^{\langle U^2 \rangle_m + \langle UV_0 \rangle_m} dt$$



TSL Specific Benchmarks



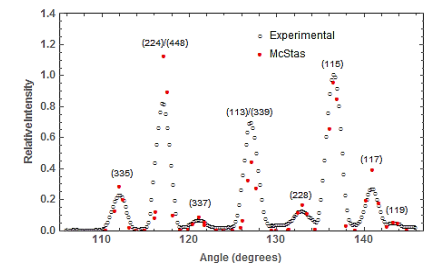
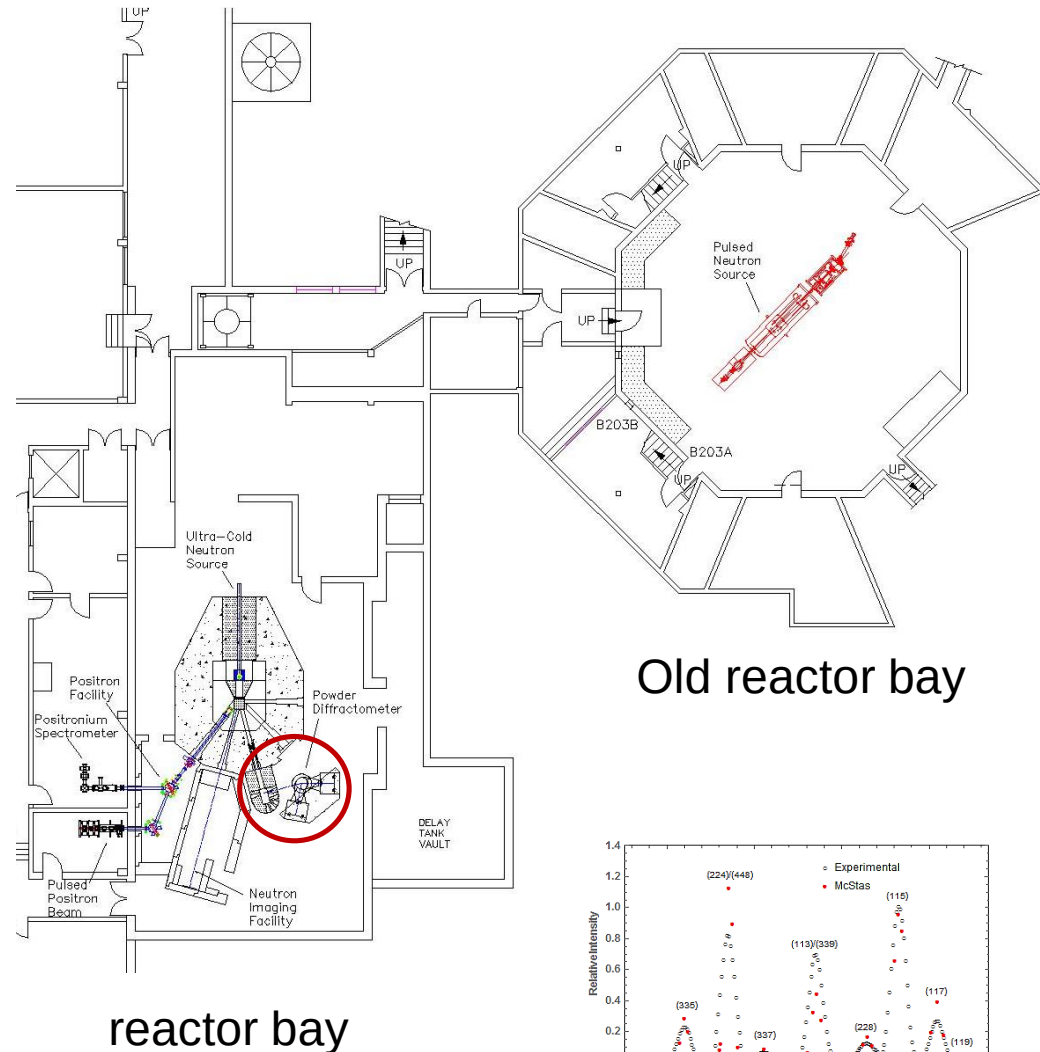
PULSTAR reactor

Current major facilities/capabilities

- **Neutron powder diffraction**
- Neutron imaging
- Intense positron beam
- Ultracold neutron source (under testing)
- Neutron activation analysis
- In-pool irradiation testing facilities

Current projects

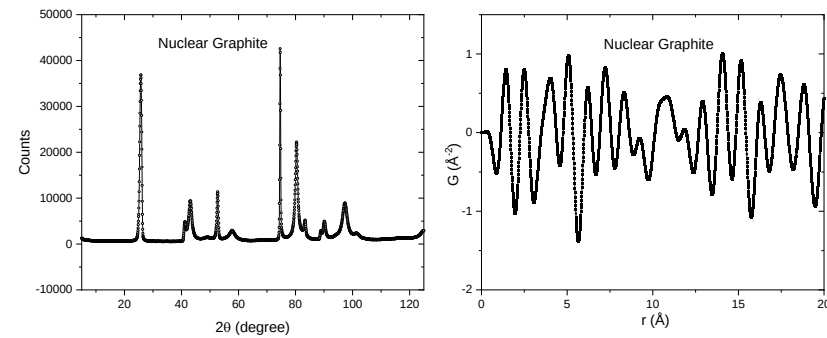
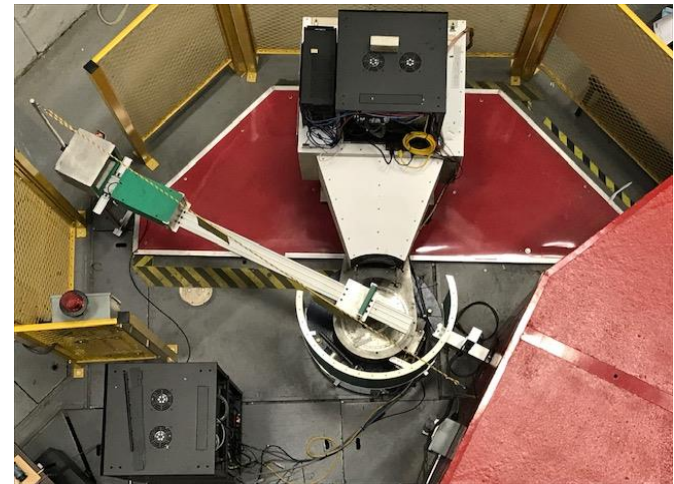
- PULSTAR power upgrade 1-2 MW (licensing stage)
- Various instrument and facility upgrades
- Pulsed accelerator neutron source (under testing)
- Fuel loop for fission gas release studies



Cross Section Measurement Project

NPDF Facility Upgrades – Dual Purpose:

- ❑ **Diffraction Measurements:**
 - 15 New Position Encoding Modules (PEM) – improved diffraction measurement resolution $\Delta d/d$ of 2.9×10^{-3} for 3mm holder
- ❑ **Transmission Measurement Capabilities:**
 - Monochromator capable of providing beam wavelengths of 1.085 Å, 1.180 Å, 1.479 Å, and 1.762 Å
 - Transmission Detection Apparatus with collimator.



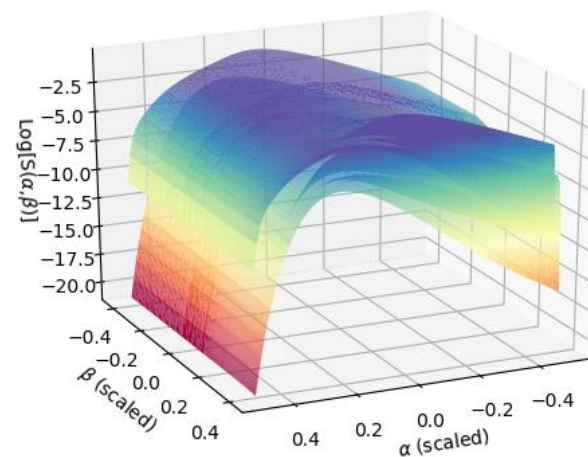
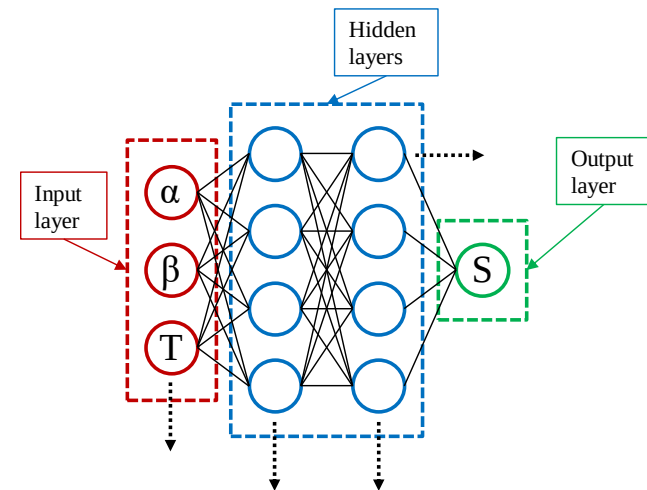
TSL NeTS

□ New TSL paradigm

- DL Neural Thermal Scattering (NeTS) modules

□ See papers and presentations

- ANS 2019 Winter Meeting, Washington, DC, USA
- PHYSOR 2020 Meeting, Cambridge, UK



Summary

- ❑ TSL Activities continue including evaluations and methods development
- ❑ Evaluations are contributed to NNDC
 - Paraffinic oil and single crystal sapphire
 - Molten salt FLiBe will be submitted soon
 - Hydrofluoric acid is initiated
 - Uranium metal is initiated
- ❑ *FLASSH* testing and several evaluations are underway
- ❑ Activities in data measurements and benchmark development are underway
- ❑ DL NeTS approach is under testing