Another Use of Integral Experiments for Nuclear Data Validation: Bias Factor Methods

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Motivation for Using Bias Factor Method

- **Currently in advanced reactor design cross section adjustments are falling out of favor for many different reasons. This has been the case for projects under development in France, USA, and Japan.**
- **Among the reasons that are mentioned we find:**
  - **The adjustment methodology is difficult to understand by the designers and they want something simpler (Japan).**
  - **The adjustment methodology is cumbersome and difficult to implement for practical applications and require a significant effort (France, USA).**
  - **The adjustment methodology relies on modification of multigroup infinite dilution cross sections, if other approaches are used (e. g. Monte Carlo, ultra-fine groups) it is not clear how to apply.**
- **The bias factor methodology is a relatively easier and useful alternative for taking into account the uncertainties of cross sections, especially in a preliminary design stage.**

Bias Factor Methods and Their Application

- We will illustrate various bias factor methods including:
  - Standard Bias Factor Method
  - The Representativity Weighted Bias Factor Method
  - The Generalized Bias Factor Method
  - The Product of Exponentials Bias Factor Method
- These methods will be applied to a practical case: the critical mass of a typical fast test reactor with metallic fuel and enriched U and Pu.

The Standard Bias Factor Methodology

- **The bias factor methodology exploits the information, i. e. the discrepancy between experimental and calculated values, of "pertinent" integral experiments. This allows to correct the calculated values of the target reactor, and to attach a reasonable uncertainty estimate to it.**
- **The standard bias factor method actually uses only one "mock up" experiment.**
- **The calculated value on the target reactor** R_c^i **is multiplied by a bias factor** f_E^i **for the corresponding integral parameter i defined as the ratio between the experimental** E_E^i **and calculated** E_c^i **value of the mock up experiment.**

$$R'^i_c = R_c^i f_E^i$$

$$f_E^i = \frac{E_E^i}{E_c^i}$$

The Representativity Factor

- In order to adopt standard bias factor methodology, one has to have in principle a real mock up experiment, i. e. a correspondence one to one between the target reactor and the integral experiment.
- One way to evaluate if the experiment is a real mock up one is to use the "representativity" factor.
- We start from the definition of the square of the uncertainty associated to neutron cross section data ΔR_i^2 for the integral parameter i end reactor r and characterized by the covariance matrix D.
- If we compute the sensitivity coefficient array S_r^i and the corresponding transposed one S_r^{i+}, we use the sandwich formula:

$$\Delta R_i^2 = S_r^{i+} D S_r^i$$

- Then, if we compute the corresponding sensitivity array S_E^i for the integral experiment, we can express the representativity factor r_{re} as:

$$r_{re} = \frac{\left(S_r^{i+} D S_E^i\right)}{\sqrt{\left(S_r^{i+} D S_r^i\right)\left(S_E^{i+} D S_E^i\right)}}$$

The Bias Factor Uncertainty

- **If the representativity factor is equal to 1, then we have a perfect mock up experiment. What it tells is that, if there is a change to the value of integral parameter i due to a change of a cross section and the representativity factor is equal to one, the two systems, reactor and experiment, react in exactly the same way.**
- **Moreover, it can be shown that, when the representativity factor is known, the uncertainty on the reactor calculated value can be reduced by the information coming from the integral experiment as:**

$$\Delta R_i'^2 = \Delta R_i^2 \left(1 - r_{re}^2\right)$$

- **Experimental and calculational uncertainties of the integral experiment have to be accounted for, as well as the technological uncertainties (e.g. dimensions and densities) impacting the reactor parameters of interest. For simplicity, here we include the technological uncertainties in the experimental ones, and, in particular, for the relative uncertainty on the bias factor we have**

$$\frac{\Delta f_E^i}{f_E^i} = \sqrt{\left(\frac{\Delta E_E^i}{E_E^i}\right)^2 + \left(\frac{\Delta E_c^i}{E_c^i}\right)^2}$$

The Representativity Weighted Bias Factor Method

- **We will use a combination of bias factors, where the uncertainty due to cross sections is quantified through the standard deviation of the dispersion of bias factors for a series of integral experiments.**
- **Having a number N of integral experiments for the integral parameter of interest, we define the weighted bias factor as:**

$$\widetilde{f^i} = \sum_{j=1}^{N} \omega_j f_j^i$$

$$\sum_{j=1}^{N} \omega_j = 1$$

- **The relative standard deviation of the weighted bias factor is calculated as:**

$$\frac{\Delta \widetilde{f^i}}{\widetilde{f^i}} = \frac{\sqrt{\frac{\sum_{j=1}^{N}\left(\omega_j\left(f_j^i - \widetilde{f^i}\right)\right)^2}{(N-1)/N}}}{\widetilde{f^i}}$$

The Representativity Weighted Bias Factor Method

- **To this term we have to add the weighted experimental and calculational uncertainty of the N experiments so that the final formula becomes:**

$$\frac{\Delta\widetilde{f^i}}{\widetilde{f^i}} = \left[\sum_{j=1}^{N}\omega_j\left(\left(\frac{\Delta E_{Ej}^i}{E_{Ej}^i}\right)^2+\left(\frac{\Delta E_{cj}^i}{E_{cj}^i}\right)^2\right)+\frac{\frac{\sum_{j=1}^{N}\left(\omega_j\left(f_j^i-\widetilde{f^i}\right)\right)^2}{(N-1)/N}}{\widetilde{f^i}}\right]^{1/2}$$

- **The calculational uncertainty of the reactor needs to be added to that of the weights bias factor as well as the technological ones $\frac{\Delta R_t^i}{R_t^i}$, so that the final uncertainty is computed as:**

$$\frac{\Delta R_i'}{R_i'} = \sqrt{\left(\frac{\Delta\widetilde{f^i}}{\widetilde{f^i}}\right)^2+\left(\frac{\Delta R_c^i}{R_c^i}\right)^2+\left(\frac{\Delta R_t^i}{R_t^i}\right)^2}$$

The Representativity Weighted Bias Factor Method

- In order to define the weights, we use the representativity factors formulas for discriminating among all the available experiments. We can use different definitions for the weighting factors (always normalized to 1):
  - The representativity factor: $\omega_j = \dfrac{\left(S_r^{i+} D S_j^i\right)}{\sqrt{\left(S_r^{i+} D S_r^i\right)\left(S_j^{i+} D S_j^i\right)}}$
  - A stronger "representative" weight as the inverse of the reduction factor: $\omega_j = 1/\left(1 - r_{re}^{j2}\right)$. This is the one adopted for the application.
  - A combination of the representative weight and one derived by the uncertainty of the experiments: $\omega_j^u = \dfrac{\frac{\Delta f_{jE}^i}{f_{jE}^i}}{\sum_{j=1}^{N} \frac{\Delta f_{jE}^i}{f_{jE}^i}}$. The two weights ω_j^{rep}, the representativity ones, and ω_j^u, the uncertainty ones, can be summed up, because they are normalized to one, and then renormalized.

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The Generalized Bias Factor Method

$$f_i = \frac{E_i}{C_i} \qquad f_{GB} = \sum_{i=1}^{N} D_i f_i. \qquad R_R = R_C \cdot f_{GB} = R_C \cdot \sum_{i=1}^{N} D_i \frac{E_i}{C_i}$$

If negative!

?

$$V\left(\frac{R_R}{R_t}\right) = \left(S_R - \sum_{i=1}^{N} D_i S_i\right) V_\sigma \left(S_R - \sum_{i=1}^{N} D_i S_i\right)^t + V\left(\Delta M_R - \sum_{i=1}^{N} D_i \Delta M_i\right) + V\left(\sum_{i=1}^{N} D_i \Delta E_i\right).$$

Variance is minimized with respect to the weights (first derivative equal to zero).

$$\sum_{j=2}^{N} D_j \{\Delta S_{i1} V_\sigma \Delta S_{j1} + \mathrm{cov}(\Delta M_{i1}, \Delta M_{j1}) + \mathrm{cov}(\Delta E_{i1}, \Delta E_{j1})\}$$
$$- \{\Delta S_{i1} V_\sigma \Delta S_{R1} + \mathrm{cov}(\Delta M_{i1}, \Delta M_{R1}) + \mathrm{cov}(\Delta E_{i1}, -\Delta E_1)\} = 0, \quad (i = 2, 3, \ldots, N)$$

and

$$\sum_{j=1}^{N} D_j = 1.$$

The Product of Exponentials Bias Factor Method

$$f_{PE} = \frac{E_{PE}}{C_{PE}} = \frac{\prod_{i=1}^{N} E_i^{F_i}}{\prod_{i=1}^{N} C_i^{F_i}}.$$

$$R_R = R_C \cdot f_{PE} = R_C \cdot \frac{\prod_{i=1}^{N} E_i^{F_i}}{\prod_{i=1}^{N} C_i^{F_i}}$$

$$V\left(\frac{R_R}{R_t}\right) = \left(S_R - \sum_{i=1}^{N} F_i S_i\right) V_\sigma \left(S_R - \sum_{i=1}^{N} F_i S_i\right)^t + V\left(\Delta M_R - \sum_{i=1}^{N} F_i \Delta M_i\right) + V\left(\sum_{i=1}^{N} F_i \Delta E_i\right).$$

?

If negative!

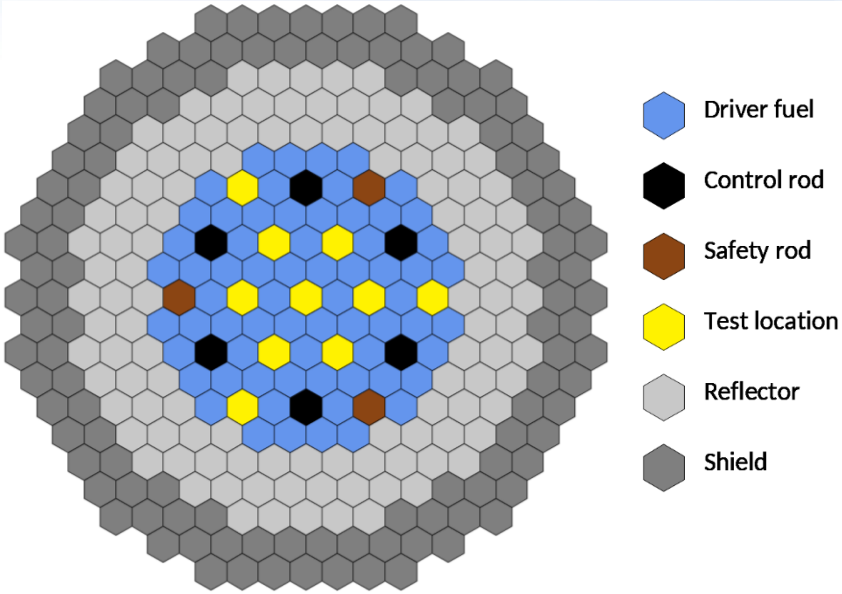
Variance is minimized with respect to the weights (first derivative equal to zero).

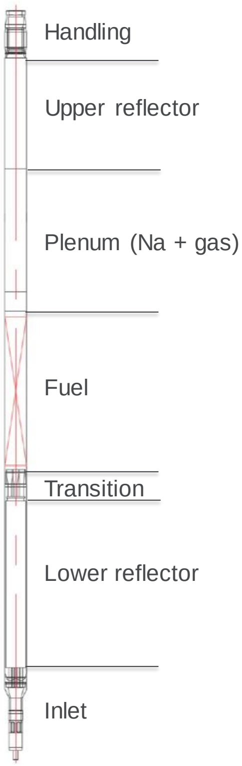
$$\sum_{j=1}^{N} F_j \{S_i V_\sigma S_j^t + \mathrm{cov}(\Delta M_i, \Delta M_j) + \mathrm{cov}(\Delta E_i, \Delta E_j)\} - \{S_R V_\sigma S_i^t + \mathrm{cov}(\Delta M_R, \Delta M_i)\} = 0. \quad (i = 1, 2, 3, \ldots, N)$$

Application to a Fast Test Reactor

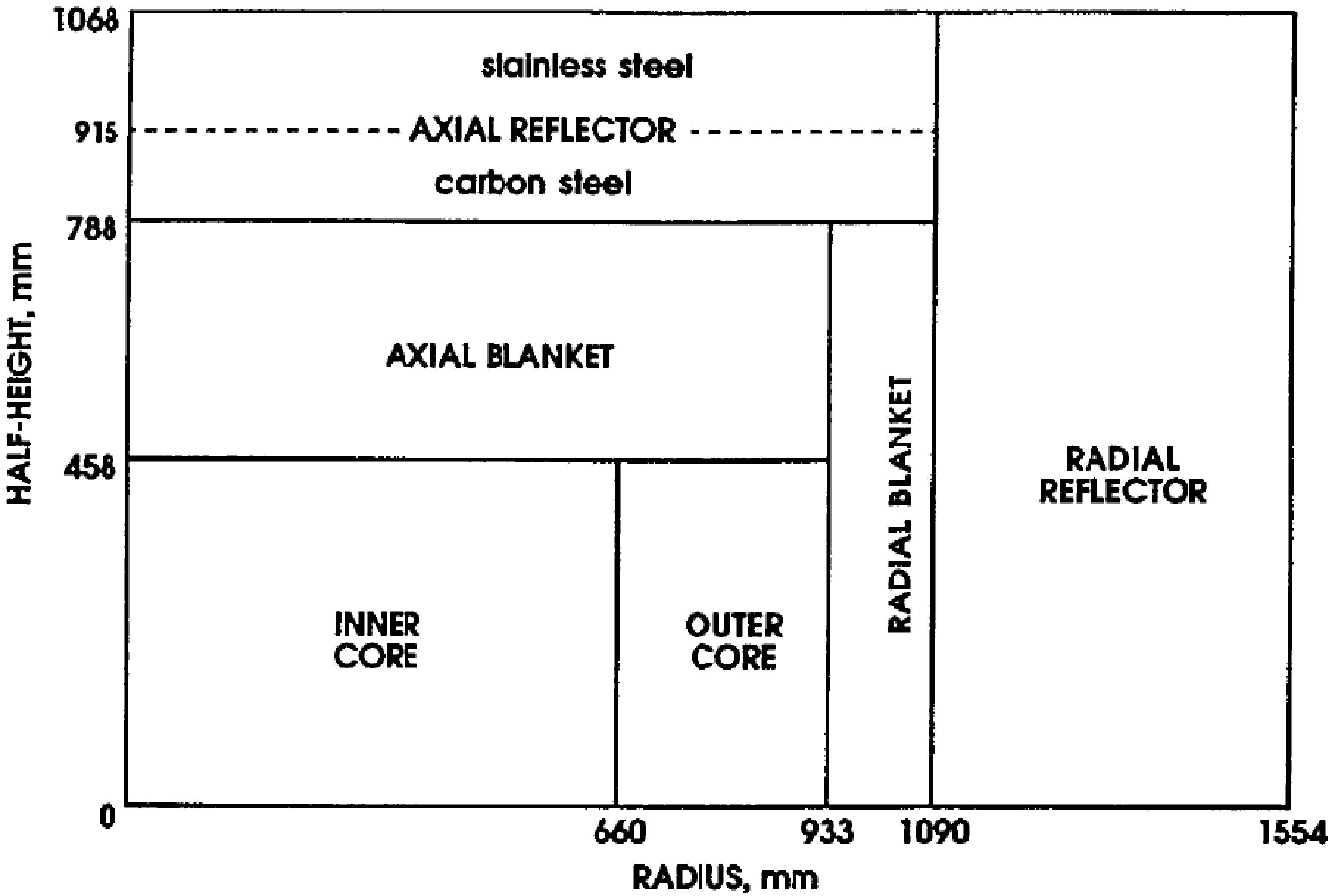
- The application is the critical mass (K_{eff}) of a typical fast test reactor with metallic fuel and enriched U and Pu.
- A threshold of 0.75 was used for the representativity factor in order to select the relevant experiments.
- Four uncorrelated experiments were selected:
  - ZPPR-15 A
  - CIRANO 2B
  - FFTF start up configuration
  - ZPR3-56B
- No correlation among experiments exist, and calculational uncertainty was put equal to zero (with Monte Carlo is in the pcm range).
- Besides bias factor methods, also adjustments was performed using the same experiments in order to compare results.

FTR

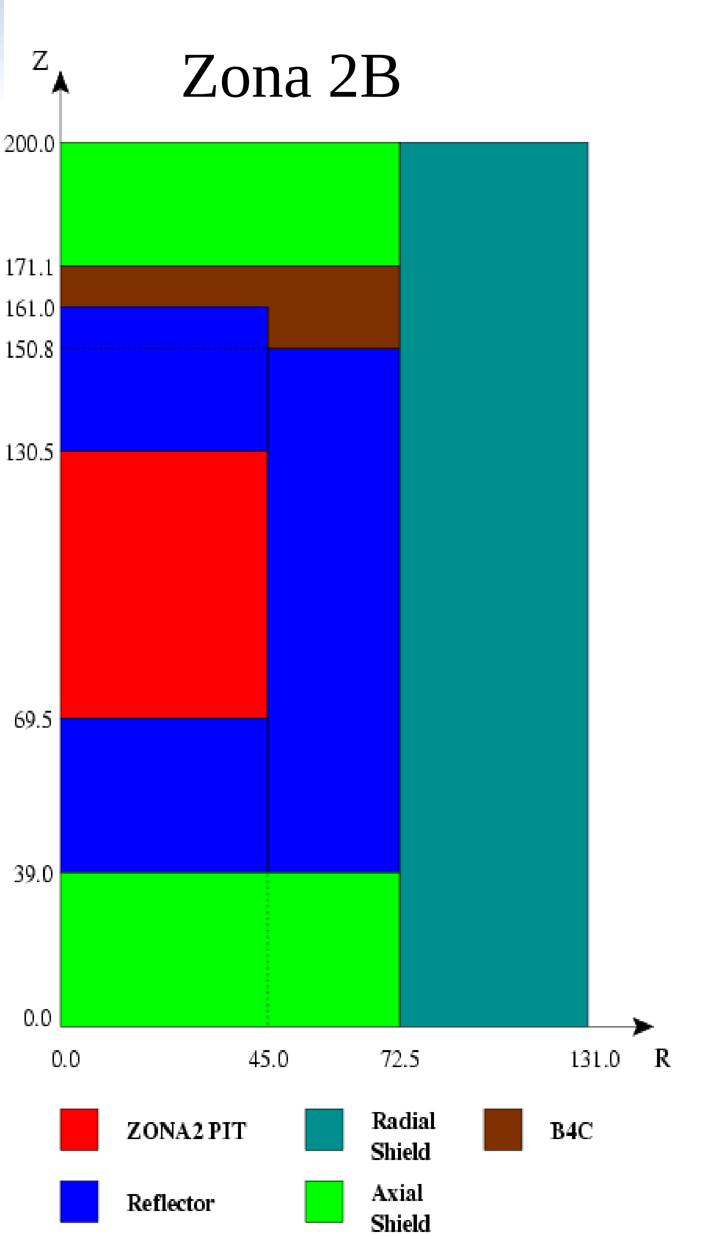

Driver fuel
Control rod
Safety rod
Test location
Reflector
Shield


Handling
Upper reflector
Plenum (Na + gas)
Fuel
Transition
Lower reflector
Inlet

ZPPR-15


1068
915
788
458
0
stainless steel
AXIAL REFLECTOR
carbon steel
AXIAL BLANKET
INNER CORE
OUTER CORE
RADIAL BLANKET
RADIAL REFLECTOR
HALF-HEIGHT, mm
660
933
1090
1554
RADIUS, mm

CIRANO


Zona 2B
Z
200.0
171.1
161.0
150.8
130.5
69.5
39.0
0.0
0.0
45.0
72.5
131.0
R
ZONA2 PIT
Radial Shield
B4C
Reflector
Axial Shield

FFTF First Critical Core

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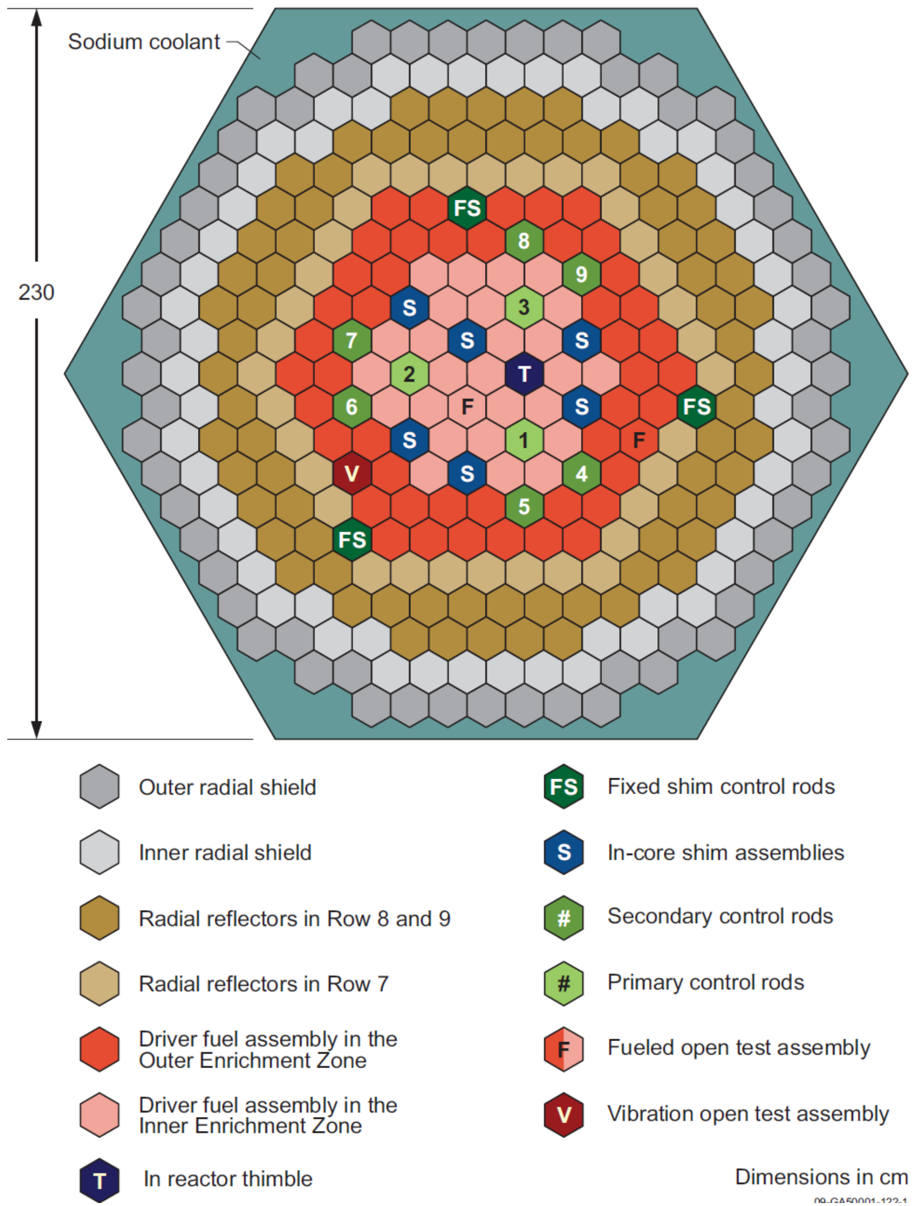

ZPR3-56B

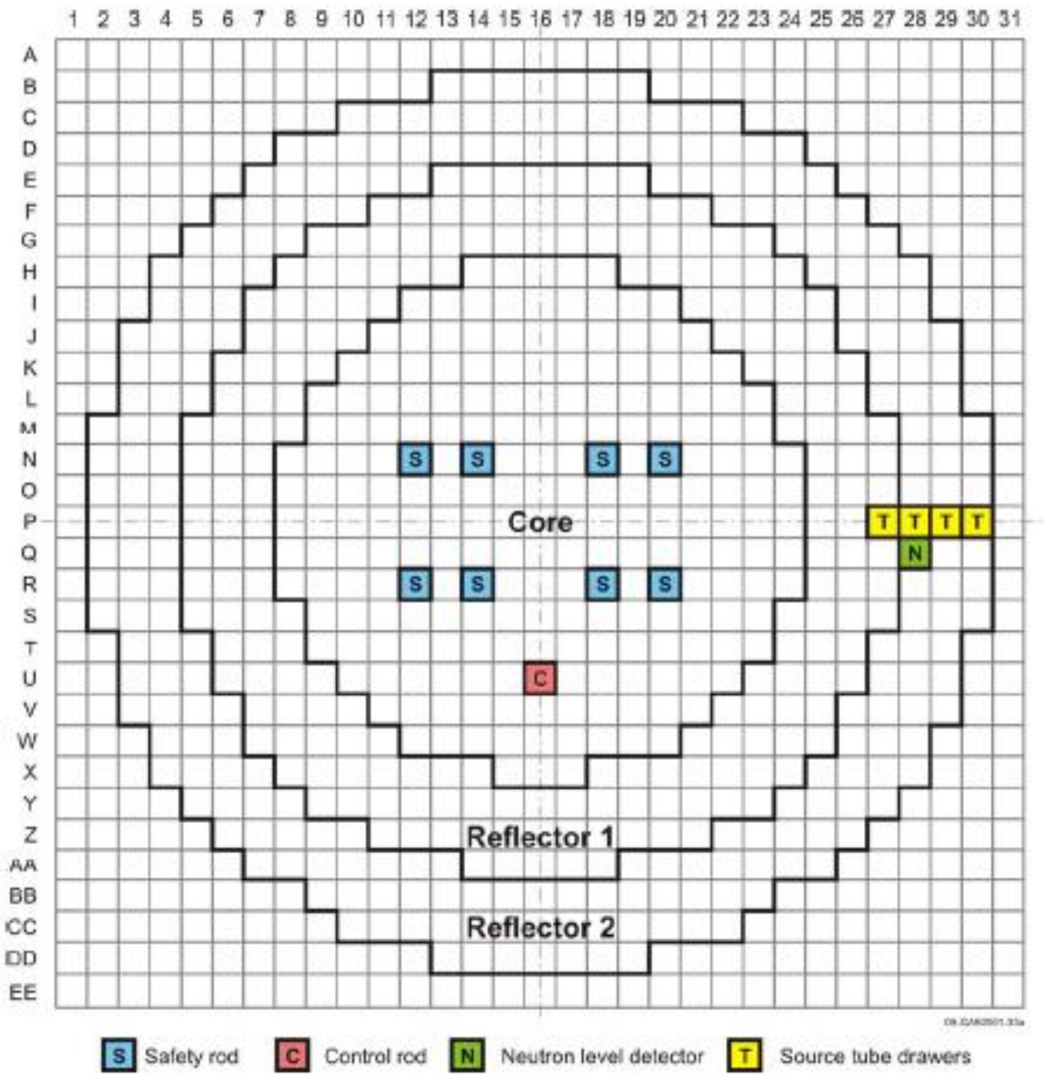

Figure 4. ZPR-3/56B Loading 17 Core Layout – Half 1 (Stationary Half).

Characteristics of the Experiments

Estimated

Experiment	ZPPR-15 A	CIRANO 2B	FFTF	ZPR3-56B
Bias Factor	1.0018	0.9936	0.9962	0.9969
Exper. Uncert. pcm	89	200	211	150
Represent. Factor	0.7819	0.9485	0.8618	0.8366
Reduced Uncert.[a)] pcm	535	272	436	470
Total Uncert. pcm	542	338	484	493

List of K_{eff} experiments, bias factors, and uncertainty with respect to the FTR (Fast Test Reactor).

Starting FTR Uncertainty due to nuclear data: 859 pcm.

[a)] Using Standard Bias Factor Methodology

3 Experiments Weights

Experiment	ZPPR-15 A	FFTF	ZPR3-56B
Represent. Weighted BF	0.2628	0.3970	0.3403
Generalized BF	0.1663	0.4553	0.3783
Product of Expon. BF [a)]	0.1807	0.4109	0.3666

Table -. Bias Factor weights for different methods. 3 experiments used.

[a)] Exponents of the product

3 Experiments Results

Method	Bias Factor	Nucl. Data Unc. pcm	Exper. Unc. pcm	Total Unc. pcm
Represent. Weighted BF	0.9979	287	166	331
Generalized BF	0.9974	433	173	466
Product of Expon. BF [a)]	0.9976	455	167	485
Adjustment	0.9977 [a)]	470		470

Bias Factor and uncertainty with respect to the FTR for different methods. 3 experiments used.

[a)] Derived as the ratio between the calculated K_{eff} with adjusted cross section and unadjusted ones.

4 Experiments Weights

Experiment	ZPPR-15 A	CIRANO 2B	FFTF	ZPR3-56B
Represent. Weighted BF	0.1303	0.5042	0.1968	0.1687
Generalized BF	0.1659	0.7324	0.1017	0.0 [b)]
Product of Expon. BF [a)]	0.2066	0.8034	0.0241	-0.1113

Bias Factor weights for different methods. 4 experiments used.

[a)] Exponents of the product

[b)] Negative value of -0.084 was obtained, replaced with zero, and renormalized the weights to 1.

4 Experiments Results

Method	Bias Factor	Nucl. Data Unc. pcm	Exper. Unc. pcm	Total Unc. pcm
Represent. Weighted BF	0.9957	314	184	364
Generalized BF	0.9952	264	187	324
Product of Expon. BF [a)]	0.9955	307	180	356
Adjustment	0.9968 [a)]	317		317

Bias Factor and uncertainty with respect to the FTR for different methods. 4 experiments used.

[a)] Derived as the ratio between the calculated K_{eff} with adjusted cross section and unadjusted ones.

CONCLUSIONS

- **Bias factor methods have been compared in a practical case, the critical mas of fast test reactor, using four different experiments and compared against cross section adjustments using the same experiments.**
- **Results are quite comparable in terms of resulting bias factor and reduction of the initial uncertainty related to nuclear data; however, the generalized and product of exponential methods can produce negative weights, and potentially this could be a problem.**
- **The standard bias factor method is preferable in presence of significantly high representativity factors (e.g. 0.95).**
- **In bias factor methods is crucial to use experiments that capture the physics of the integral parameter under consideration. For the FTR is very important to capture the reflector effect.**

REFERENCES

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