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Summary of Derivations and Equivalence between Bias Factor Methods and Adjustment Methods

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Action Agreed at June 2019 Meeting*

- **K. Yokoyama** to provide papers to **G. Palmiotti** which illustrate the equivalence of different bias factors methods and their equivalence to the extended adjustment method

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- During the June 2019 meeting, eight papers listed in the last slide were delivered personally
- Through the mailing list of SG46, the list of the papers were distributed on September 19, 2019
- In this presentation, I would like to summarize the papers focusing on the equivalence between bias factor methods and adjustment methods

*: Summary Record, Meeting of WPEC SG46, 25-26 June 2019, NEA/NSC/WPEC/DOC(2019)4

Nomenclature in comparison with SG39's

- $P(\boldsymbol{A}|\boldsymbol{B})$: conditional probability of $\boldsymbol{A}$ given $\boldsymbol{B}$
- $E(\boldsymbol{A})$: expectation of $\boldsymbol{A}$
- $V(\boldsymbol{A})$: variance of $\boldsymbol{A}$
- $\boldsymbol{T}_0$: unadjusted cross sections (= $\boldsymbol{\sigma}$ in the SG39's common nomenclature)
- $\boldsymbol{T}_x$: adjusted cross sections by methodology x (= $\boldsymbol{\sigma}'$)
- $\boldsymbol{R}_e^{(1)}$: measured value of integral experiments (= $\boldsymbol{E}$)
- $\boldsymbol{R}_c^{(1)}(\boldsymbol{T})$: calculation value of integral experiments (= $\boldsymbol{C}$)
- $\boldsymbol{R}_x^{(2)}$: design value of the target system by methodology x
- $\mathbf{G}^{(1)}$: sensitivity matrix of integral experiments (= $\boldsymbol{S}$)
- $\mathbf{G}^{(2)}$: sensitivity matrix of integral parameters of the target system
- $\mathbf{M}$: covariance matrix of unadjusted cross sections (= $\mathbf{M}_{\boldsymbol{\sigma}}$)
- $\mathbf{V}_e^{(1)}$: covariance matrix of experimental error for the integral experiments (= $\mathbf{M}_{\boldsymbol{E}}$)
- $\mathbf{V}_m^{(1)}$: covariance matrix of analysis method error for the integral experiments (= $\mathbf{M}_{\boldsymbol{C}}$)
- $\mathbf{V}_{e+m}^{(1)} = \mathbf{V}_e^{(1)} + \mathbf{V}_m^{(1)}$ (= $\mathbf{M}_{\boldsymbol{EC}}$)
- $\mathbf{V}_m^{(2)}$: covariance matrix of analysis method error for the target system
- $\mathbf{V}_m^{(12)}$: cross-correlation between the integral experiments and the target system
- $\mathbf{F}_x$: combination factor matrix in the linear estimation by methodology x

CBCA (= Cross-section Adjustment, GLLS, etc.)

- Classical Bayesian Conventional XS Adjustment method

estimate

$$\boldsymbol{T}_{\mathrm{CBCA}} \equiv \underset{\hat{\boldsymbol{T}}}{\operatorname{argmax}} P\left(\hat{\boldsymbol{T}} | \boldsymbol{R}_e^{(1)}\right)$$

$$P \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$ Gaussian distribution

e.g. WPEC/SG33 intermediate report

$$\boldsymbol{T}_{\mathrm{CBCA}} = \boldsymbol{T}_0 + \mathbf{M}\mathbf{G}^{(1)\mathrm{T}}\,\mathbf{D}^{-1}\left(\boldsymbol{R}_e^{(1)} - \boldsymbol{R}_c^{(1)}(\boldsymbol{T}_{\mathbf{0}})\right)$$

$$\mathbf{D} \equiv \mathbf{G}^{(1)}\mathbf{M}\mathbf{G}^{(1)\mathrm{T}} + \mathbf{V}_{e+m}^{(1)}$$

- Well-known formula to members of SG46

EBPE & Matrix Form with Approximation

- Extended Bias factor method (Product of Exponentiated values)

$$R_{\mathrm{EBPE}}^{(2)} = R^{(2)}(\boldsymbol{T}_0)\left(\frac{\prod_i \left(R_{e,i}^{(1)}\right)^{F_{\mathrm{EBPE},i}}}{\prod_i \left(R_{c,i}^{(1)}\right)^{F_{\mathrm{EBPE},i}}}\right)$$

T. Kugo et al.
JNST 44[12]1509(2007)

$$\approx R^{(2)}(\boldsymbol{T}_0)\left(1 + \boldsymbol{F}_{\mathrm{EBPE}} \frac{\boldsymbol{R}_e^{(1)} - \boldsymbol{R}_c^{(1)}(\boldsymbol{T}_0)}{\boldsymbol{R}_c^{(1)}(\boldsymbol{T}_0)}\right)$$

K. Yokoyama et al.
JNST 49[12]1165(2012)
Eqs.(51)-(54)

$$\boldsymbol{F}_{\mathrm{EBPE}} \equiv \underset{\boldsymbol{F}}{\operatorname{argmin}}\, V\left(\frac{\hat{R}^{(2)}}{R_{\mathrm{true}}^{(2)}}\right)$$

1. Gaussian distribution is not assumed in EBPE
2. EBPE is equivalent to a linear estimation

NB: It is shown that a result of best representativity method by T. Umano et al. is completely equivalent to that of EBPE in Ref.2

$$\boldsymbol{F}_{\mathrm{EBPE}} = \left(\mathbf{G}^{(2)}\mathbf{M}\mathbf{G}^{(1)\mathrm{T}} + \mathbf{V}_m^{(12)\mathrm{T}}\right)\mathbf{D}^{-1}$$

- This formula is similar to CBCA but different

CBEA

- Classical Bayesian Extended XS Adjustment method

estimate

$$\boldsymbol{T}_{\mathrm{CBEA}} \equiv \underset{\hat{\boldsymbol{T}}}{\operatorname{argmax}}\, P\left(\hat{\boldsymbol{R}}^{(2)} \middle| \boldsymbol{R}_e^{(1)}\right)$$

$$P \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$ Gaussian distribution

$$\boldsymbol{T}_{\mathrm{CBEA}} = \boldsymbol{T}_0 + \left(\mathbf{M}\mathbf{G}^{(1)\mathrm{T}} + \mathbf{G}^{(2)+}\mathbf{V}_m^{(12)}\right)\mathbf{D}^{-1}\left(\boldsymbol{R}_e^{(1)} - \boldsymbol{R}_c^{(1)}(\boldsymbol{T}_\mathbf{0})\right)$$

$$\boldsymbol{T}_{\mathrm{CBEA}} \neq \boldsymbol{T}_{\mathrm{CBCA}}$$

$$\boldsymbol{R}_c^{(2)}(\boldsymbol{T}_{\mathrm{CBEA}}) \approx R_{\mathrm{EBPE}}^{(2)}$$

K. Yokoyama et al.
JNST 49[12]1165(2012)

- Design values of CBEA and EBPE are approximately the same

Contrast of Bayesian Inference & Linear Estimation

Bayesian inference

$$P\left(\hat{\boldsymbol{T}}|\boldsymbol{R}_e^{(1)}\right)$$

$$P\left(\hat{\boldsymbol{R}}^{(2)}|\boldsymbol{R}_e^{(1)}\right)$$

$$\mathbf{X}_x \equiv \underset{\hat{\mathbf{X}}}{\operatorname{argmax}} P\left(\hat{\mathbf{X}}|\boldsymbol{R}_e^{(1)}\right)$$

$$P \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

(assumption of Gaussian distribution)

Linear estimation

$$\hat{\boldsymbol{T}} - \boldsymbol{T}_0 = \mathbf{F}\left(\boldsymbol{R}_e^{(1)} - \boldsymbol{R}_c^{(1)}(\boldsymbol{T}_0)\right)$$

$$\hat{\boldsymbol{R}}^{(2)} - \boldsymbol{R}_c^{(2)}(\boldsymbol{T}_0) = \mathbf{F}\left(\boldsymbol{R}_e^{(1)} - \boldsymbol{R}_c^{(1)}(\boldsymbol{T}_0)\right)$$

$$\mathbf{X}_x = \mathbf{X}_0 + \mathbf{F}_x\left(\boldsymbol{R}_e^{(1)} - \boldsymbol{R}_c^{(1)}(\boldsymbol{T}_0)\right)$$

$$\mathbf{F}_x \equiv \underset{\boldsymbol{F}}{\operatorname{argmin}} \operatorname{tr}\left(V(\hat{\mathbf{X}})\right)$$

$$E(\mathbf{T}_0) = \mathbf{T}_{\text{true}}, \cdots$$

(assumption of unbiased estimation)

where $\mathbf{X}_x = \boldsymbol{T}_x$ or $\boldsymbol{R}_x^{(2)}$

K. Yokoyama et al.
JNST 56[1]87(2019)

BFRS (Bias Factor Method by T. Endo et al.)

- Bias Factor Method Using Random Sampling Technique

estimate

$$\boldsymbol{R}^{(2)}_{\mathrm{BFRS}} \equiv \underset{\boldsymbol{K}}{\operatorname{argmax}}\, P\left(\widehat{\boldsymbol{R}}^{(2)} \middle| \boldsymbol{R}^{(1)}_{e}\right)$$

$$P \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$ Gaussian distribution

$$\boldsymbol{R}^{(2)}_{\mathrm{BFRS}} = \boldsymbol{R}^{(2)}_{c}(\boldsymbol{T}_0) + \mathbf{K}\left(\boldsymbol{R}^{(1)}_{e} - \boldsymbol{R}^{(1)}_{c}(\boldsymbol{T}_{\mathbf{0}})\right)$$

$$\mathbf{K} \equiv \left(Cov\left(\boldsymbol{R}^{(2)}_{c}, \boldsymbol{R}^{(1)}_{c}\right) + \mathbf{V}^{(12)\mathrm{T}}_{m}\right)\left(Cov\left(\boldsymbol{R}^{(1)}_{c}, \boldsymbol{R}^{(1)}_{c}\right) + \mathbf{V}^{(1)}_{e+m}\right)^{-1}$$

$$Cov\left(\boldsymbol{R}^{(2)}_{c}, \boldsymbol{R}^{(1)}_{c}\right) \approx \mathbf{G}^{(2)}\mathbf{M}\mathbf{G}^{(1)\mathrm{T}}, \quad Cov\left(\boldsymbol{R}^{(1)}_{c}, \boldsymbol{R}^{(1)}_{c}\right) \approx \mathbf{G}^{(1)}\mathbf{M}\mathbf{G}^{(1)\mathrm{T}}$$

$$\boldsymbol{R}^{(2)}_{\mathrm{BFRS}} \approx R^{(2)}_{\mathrm{EBPE}}$$

T. Endo et al.
JNST 53[10]1494(2016)

- Design values of BRFS and EBPE are approximately the same

MLEA

- MVULE(Minimum Variance Unbiased Linear Estimation)-based Extended XS Adjustment method

$$\boldsymbol{T}_{\mathrm{MLEA}} = \boldsymbol{T}_0 + \mathbf{F}_{\mathrm{MLEA}}\left(\boldsymbol{R}_e^{(1)} - \boldsymbol{R}_c^{(1)}(\boldsymbol{T}_0)\right)$$

$$\mathbf{F}_{\mathrm{MLEA}} \equiv \underset{\boldsymbol{F}}{\operatorname{argmin}}\, \mathrm{tr}\left(V\left(\boldsymbol{R}_c^{(2)}(\widehat{\boldsymbol{T}})\right)\right) \cap \underset{\boldsymbol{F}}{\operatorname{argmin}}\, \mathrm{tr}\left(V(\widehat{\boldsymbol{T}})\right)$$

$$\boldsymbol{T}_{\mathrm{MLEA}} = \boldsymbol{T}_{\mathrm{CBEA}}$$

$$\boldsymbol{R}_c^{(2)}(\boldsymbol{T}_{\mathrm{MLEA}}) = \boldsymbol{R}_c^{(2)}(\boldsymbol{T}_{\mathrm{CBEA}}) \approx R_{\mathrm{EBPE}}^{(2)}$$

K. Yokoyama et al.
JNST 56[1]87(2019)

- EA (Extended Adjustment method) can be derived by a linear estimation

MLCA

- MVULE(Minimum Variance Unbiased Linear Estimation)–based Conventional XS Adjustment method

$$\boldsymbol{T}_{\mathrm{MLCA}} = \boldsymbol{T}_0 + \mathbf{F}_{\mathrm{MLCA}}\left(\boldsymbol{R}_e^{(1)} - \boldsymbol{R}_c^{(1)}(\boldsymbol{T}_0)\right)$$

$$\mathbf{F}_{\mathrm{MLCA}} \equiv \underset{\boldsymbol{F}}{\mathrm{argmin}}\,\mathrm{tr}\left(V(\widehat{\boldsymbol{T}})\right)$$

$$\boldsymbol{T}_{\mathrm{MLCA}} = \boldsymbol{T}_{\mathrm{CBCA}}$$

K. Yokoyama et al.
JNST 56[1]87(2019)

- The conventional cross-section adjustment method can be derived by a linear estimation
- Gaussian distribution is not assumed in MLCA although the assumption of unbiased estimation is required

Concluding Remarks

- The extended bias factor method (EBPE) is approximately equivalent to the extended adjustment method (CBEA) although EBPE does not adjust cross sections explicitly
  - EBPE is derived by a linear estimation
  - CBEA is derived by a Bayesian inference
- The (classical Bayesian) conventional cross-section adjustment method (CBCA) is different from CBEA
  - CBCA maximizes the probability of cross sections
  - CBEA maximizes the probability of design values
- The conventional and the extended cross-section adjustment methods can be derived by a linear estimation (MLCA and MLEA)
  - MLCA minimizes the variance of cross sections
  - MLEA minimizes the variance of design values

References (Provided Papers)

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2. T. Umano, K. Yoshioka, and T. Obara, "Development of 'best representativity' method for experimental data analysis and an application to the critical experiments at the Toshiba NCA facility," J.Nucl. Sci. Technol., Vol. 51, No. 5, pp.608-625 (2014). http://dx.doi.org/10.1080/00223131.2014.884953
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4. K. Yokoyama, M. Ishikawa, and T. Kugo, "Extended cross-section adjustment method to improve the prediction accuracy of core parameters," J. Nucl. Sci. Technol., Vol. No.12, pp.1165-1174 (2012). http://dx.doi.org/10.1080/00223131.2012.739020
5. K. Yokoyama, and A. Yamamoto, "Cross-section adjustment methods based on minimum variance unbiased estimation," J. Nucl. Sci. Technol., Vol.53, No.10, pp.1622-1638 (2016). http://dx.doi.org/10.1080/00223131.2016.1146637
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7. K. Yokoyama, and T. Kitada, "Comparative Study on Prediction Accuracy Improvement Methods with the Use of Integral Experiments for Neutronic Characteristics of Fast Reactors," Proc. Int. Conf. On ICAPP2018, Charlotte, North Carolina, April 8-11, 2018.
8. K. Yokoyama, and T. Kitada, "Generalized formulation of extended cross-section adjustment method based on minimum variance unbiased linear estimation," J. Nucl. Sci. Technol., Vol.56, No.1, pp.87-104(2018). https://doi.org/10.1080/00223131.2018.1531078

In this presentation, summarization of Refs. 5-7 are omitted for simplification