Update on Bias Factor Methods Experiments for Nuclear Data Validation

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Motivation for Using Bias Factor Method

- **Currently in advanced reactor design cross section adjustments are falling out of favor for many different reasons. This has been the case for projects under development in France, USA, and Japan.**
- **Among the reasons that are mentioned we find:**
  - **The adjustment methodology is difficult to understand by the designers and they want something simpler (Japan).**
  - **The adjustment methodology is cumbersome and difficult to implement for practical applications and require a significant effort (France, USA).**
  - **The adjustment methodology relies on modification of multigroup infinite dilution cross sections, if other approaches are used (e. g. Monte Carlo, ultra-fine groups) it is not clear how to apply.**
- **The bias factor methodology is a relatively easier and useful alternative for taking into account the uncertainties of cross sections, especially in a preliminary design stage.**

Bias Factor Methods and Their Application

- **We will illustrate various bias factor methods including:**
  - **Standard Bias Factor Method**
  - **The Representativity Weighted Bias Factor Method (RWBF)**
  - **The Generalized Bias Factor Method (GBF)**
  - **The Product of Exponentials Bias Factor Method (PEBF)**
  - **The Best Representativity Method (BRBF)**
  - **Extended Bias Factor Method (EBF)**
- **These methods will be applied to a practical case: the critical mass of a typical fast test reactor (FTR) with metallic fuel and enriched U and Pu.**

The Standard Bias Factor Methodology

- **The bias factor methodology exploits the information, i. e. the discrepancy between experimental and calculated values, of "pertinent" integral experiments. This allows to correct the calculated values of the target reactor, and to attach a reasonable uncertainty estimate to it.**
- **The standard bias factor method actually uses only one "mock up" experiment.**
- **The calculated value on the target reactor R_c^i is multiplied by a bias factor f_E^i for the corresponding integral parameter i defined as the ratio between the experimental E_E^i and calculated E_c^i value of the mock up experiment.**

$$R'^i_c = R_c^i f_E^i$$

$$f_E^i = \frac{E_E^i}{E_c^i}$$

The Representativity Factor

- In order to adopt standard bias factor methodology, one has to have in principle a real mock up experiment, i. e. a correspondence one to one between the target reactor and the integral experiment.
- One way to evaluate if the experiment is a real mock up one is to use the “representativity” factor.
- We start from the definition of the square of the uncertainty associated to neutron cross section data ΔR_i^2 for the integral parameter i end reactor r and characterized by the covariance matrix D.
- If we compute the sensitivity coefficient array S_r^i and the corresponding transposed one S_r^{i+}, we use the sandwich formula:

$$\Delta R_i^2 = S_r^{i+} D S_r^i$$

- Then, if we compute the corresponding sensitivity array S_E^i for the integral experiment, we can express the representativity factor r_{re} as:

$$r_{re} = \frac{\left(S_r^{i+} D S_E^i\right)}{\sqrt{\left(S_r^{i+} D S_r^i\right)\left(S_E^{i+} D S_E^i\right)}}$$

The Bias Factor Uncertainty

- **If the representativity factor is equal to 1, then we have a perfect mock up experiment. What it tells is that, if there is a change to the value of integral parameter i due to a change of a cross section and the representativity factor is equal to one, the two systems, reactor and experiment, react in exactly the same way.**
- **Moreover, it can be shown that, when the representativity factor is known, the uncertainty on the reactor calculated value can be reduced by the information coming from the integral experiment as:**

$$\Delta R_i'^2 = \Delta R_i^2 \left(1 - r_{re}^2\right)$$

- **Experimental and calculational uncertainties of the integral experiment have to be accounted for, as well as the technological uncertainties (e.g. dimensions and densities) impacting the reactor parameters of interest. For simplicity, here we include the technological uncertainties in the experimental ones, and, in particular, for the relative uncertainty on the bias factor we have**

$$\frac{\Delta f_E^i}{f_E^i} = \sqrt{\left(\frac{\Delta E_E^i}{E_E^i}\right)^2 + \left(\frac{\Delta E_c^i}{E_c^i}\right)^2}$$

The Representativity Weighted Bias Factor Method

- **We will use a combination of bias factors, where the uncertainty due to cross sections is quantified through the standard deviation of the dispersion of bias factors for a series of integral experiments.**
- **Having a number N of integral experiments for the integral parameter of interest, we define the weighted bias factor as:**

$$\widetilde{f^i} = \sum_{j=1}^{N} \omega_j f_j^i$$

$$\sum_{j=1}^{N} \omega_j = 1$$

- **The relative standard deviation of the weighted bias factor is calculated as:**

$$\frac{\Delta\widetilde{f^i}}{\widetilde{f^i}} = \frac{\sqrt{\frac{\sum_{j=1}^{N}\left(\omega_j\left(f_j^i - \widetilde{f^i}\right)\right)^2}{(N-1)/N}}}{\widetilde{f^i}}$$

The Representativity Weighted Bias Factor Method

- To this term we have to add the weighted experimental and calculational uncertainty of the N experiments so that the final formula becomes:

$$\frac{\Delta\widetilde{f^i}}{\widetilde{f^i}} = \left[\sum_{j=1}^{N}\omega_j\left(\left(\frac{\Delta E_{Ej}^i}{E_{Ej}^i}\right)^2 + \left(\frac{\Delta E_{cj}^i}{E_{cj}^i}\right)^2\right) + \frac{\frac{\sum_{j=1}^{N}\left(\omega_j\left(f_j^i - \widetilde{f^i}\right)\right)^2}{(N-1)/N}}{\widetilde{f^i}}\right]^{1/2}$$

- The calculational uncertainty of the reactor needs to be added to that of the weights bias factor as well as the technological ones $\frac{\Delta R_t^i}{R_t^i}$, so that the final uncertainty is computed as:

$$\frac{\Delta R_i'}{R_i'} = \sqrt{\left(\frac{\Delta\widetilde{f^i}}{\widetilde{f^i}}\right)^2 + \left(\frac{\Delta R_c^i}{R_c^i}\right)^2 + \left(\frac{\Delta R_t^i}{R_t^i}\right)^2}$$

The Representativity Weighted Bias Factor Method

- **In order to define the weights, we use the representativity factors formulas for discriminating among all the available experiments. We can use different definitions for the weighting factors (always normalized to 1):**
  - **The representativity factor:** $\omega_j = \frac{\left(S_r^{i+} D S_j^i\right)}{\sqrt{\left(S_r^{i+} D S_r^i\right)\left(S_j^{i+} D S_j^i\right)}}$
  - **A stronger "representative" weight as the inverse of the reduction factor:** $\omega_j = 1/\left(1 - r_{re}^{j2}\right)$ **.** **This is the one adopted for the application.**
  - **Successively a combination of the representative weight and one derived by the uncertainty of the experiments:** $\omega_j^u = \frac{\frac{\Delta f_{jE}^i}{f_{jE}^i}}{\sum_{j=1}^{N} \frac{\Delta f_{jE}^i}{f_{jE}^i}}$**. The two weights** ω_j^{rep}**, the representativity ones, and** ω_j^u**, the uncertainty ones, can be summed up, because they are normalized to one, and then renormalized.**

The Generalized Bias Factor Method

$$V(f_R) = R\left[\left\{S_R - \sum_{i=1}^{N} \omega_i S_i\right\} M_\sigma \left\{S_R - \sum_{i=1}^{N} \omega_i S_i\right\}^T + \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j M_E\right]$$

Variance is minimized with respect to the weights (first derivative equal to zero).

$$(S_R - S_1) M_\sigma (S_j - S_1)^T + V(\Delta E_1) = \sum_{j=2}^{N} \omega_E \{(S_E - S_1) M_\sigma (S_i - S_1)^T + V(\Delta E_1) + V(\Delta E_i)\} \; (i = 2, 3, ..N)$$

The Product of Exponentials Bias Factor Method

$$f_{PE} = \frac{E_{PE}}{C_{PE}} = \frac{\prod_{i=1}^{N} E_i^{F_i}}{\prod_{i=1}^{N} C_i^{F_i}}.$$

$$R_R = R_C \cdot f_{PE} = R_C \cdot \frac{\prod_{i=1}^{N} E_i^{F_i}}{\prod_{i=1}^{N} C_i^{F_i}}$$

$$V(f_R) = \left(S_R - \sum_{i=1}^{N} \omega_i S_i\right) M_\sigma \left(S_R - \sum_{i=1}^{N} \omega_i S_i\right)^T + V\left(\sum_{i=1}^{N} \omega_i \Delta E_i\right)$$

Variance is minimized with respect to the weights (first derivative equal to zero).

$$\sum_{j=1}^{N} \omega_j \{S_i M_\sigma S_j^T + V(\Delta E_j)\} - S_R M_\sigma S_i^T = 0 \quad (i = 1, 2 \ldots N)$$

The Best Representativity Method

$$S_k = \sum_{i=1}^{N} \omega_i S_i \qquad S_R^T M_\sigma S_R = S_k^T M_\sigma S_k$$

$$(S_R - S_k)^T M_\sigma (S_R - S_k) = minimum$$

$$T_E \lambda \overrightarrow{\omega} = \overrightarrow{P_R}$$

Where T_E the matrix of rank N containing all the products $S_i^T M_\sigma S_j$ among the i and j experiments, λ is the Lagrange multiplier value, $\overrightarrow{\omega}$ is the vector containing the weights ω_i, and $\overrightarrow{P_R}$ is the vector containing the products $S_R^T M_\sigma S_i$ for all experiment i.

$$(\Delta Z/Z)^2 = \sum_{i=1}^{N} \omega_i^2 (f_i - 1)^2 + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_i \, \omega_j Corr_{ij} \left\{ (f_i - 1)^2 \left(f_j - 1\right)^2 \right\}^{\frac{1}{2}}$$

$$f_R = \frac{1}{1 + \Delta Z/Z}$$

The Extended Bias Factor Method

This method by Gandini (1985) is the oldest of the methods that uses more than one experiment, and it uses the adjustment methodology in order to provide the same results but using the bias factors without needs of calculating adjusted cross sections and the related adjusted covariance matrix. We had to reformulate and recast the equations in order to fix problems with the original formulation and define the weights.

$$f_R - 1 = \overrightarrow{P_R}\; G^{-1}(F - U) \qquad \vec{\omega} = \overrightarrow{P_R}\; G^{-1}$$

$$\omega_0 = 1 - \left(\overrightarrow{P_R}\, G^{-1} U\right) = 1 - \vec{\omega} U \qquad f_R = \omega_0 + \vec{\omega}\overline{F}$$

$$R'^2 = R^2 - \sum_{i=1}^{N} \omega_i S_R^T M_\sigma S_i$$

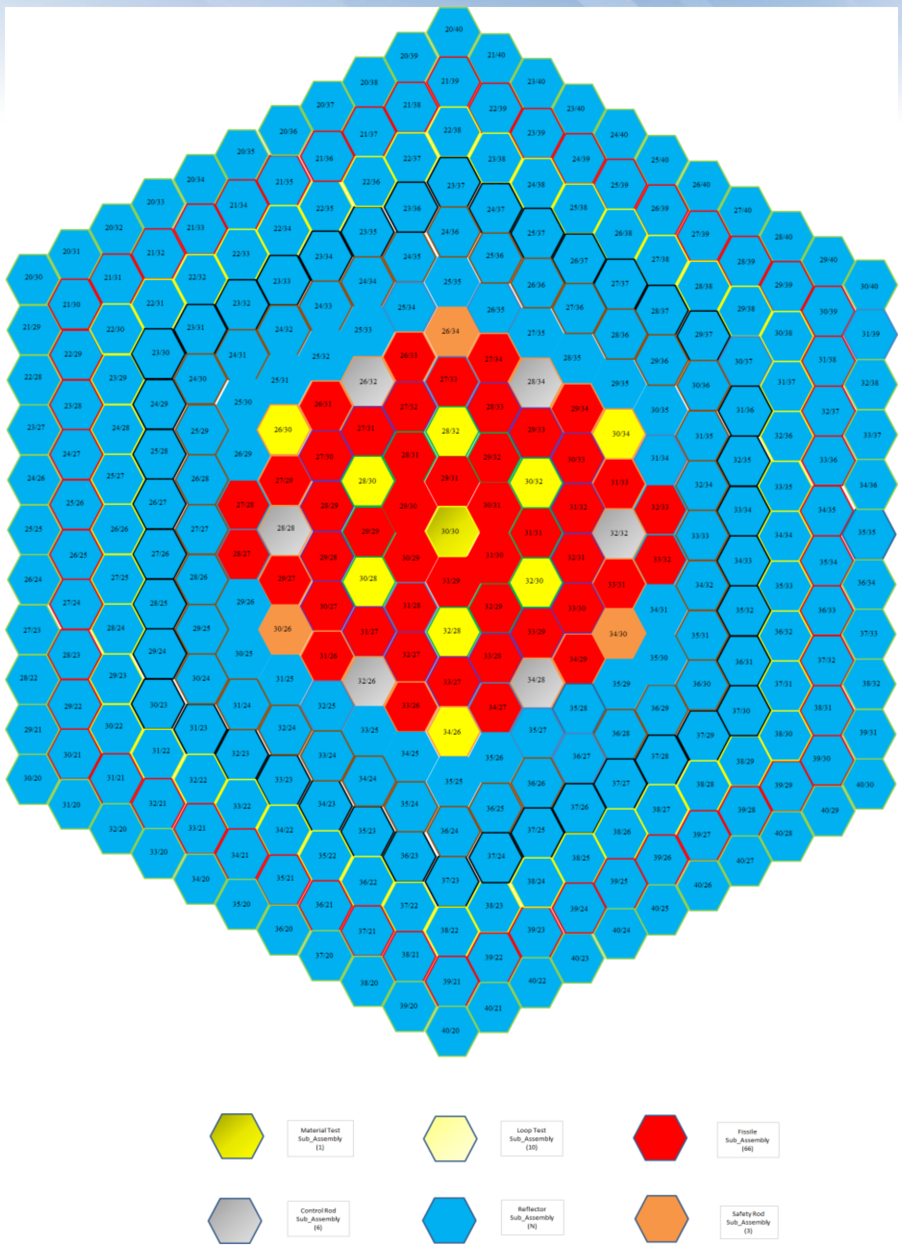
$$Q_i = \frac{\omega_i S_R^T M_\sigma S_i}{\sum_{i=1}^{N} \omega_i S_R^T M_\sigma S_i}$$

This is the relative measure of how much each individual experiment contributes to reduce the reactor uncertainty related to nuclear data. In this way, it is possible to establish the hierarchy among the used experiments.

Application to a Fast Test Reactor

- The application is the critical mass (K_{eff}) of a typical fast test reactor with metallic fuel and enriched U and Pu.
- A threshold of 0.75 was used for the representativity factor in order to select the relevant experiments.
- Four uncorrelated experiments were selected:
  - ZPPR-15 A
  - CIRANO 2B
  - FFTF start up configuration
  - ZPR3-56B
- No correlation among experiments exist, and calculational uncertainty was put equal to zero (with Monte Carlo is in the pcm range).
- Besides bias factor methods, also adjustments was performed using the same experiments in order to compare results.

FTR


Material Test Sub_Assembly (1)
Loop Test Sub_Assembly (10)
Fissile Sub_Assembly (66)
Control Rod Sub_Assembly (6)
Reflector Sub_Assembly (N)
Safety Rod Sub_Assembly (3)

ZPPR-15

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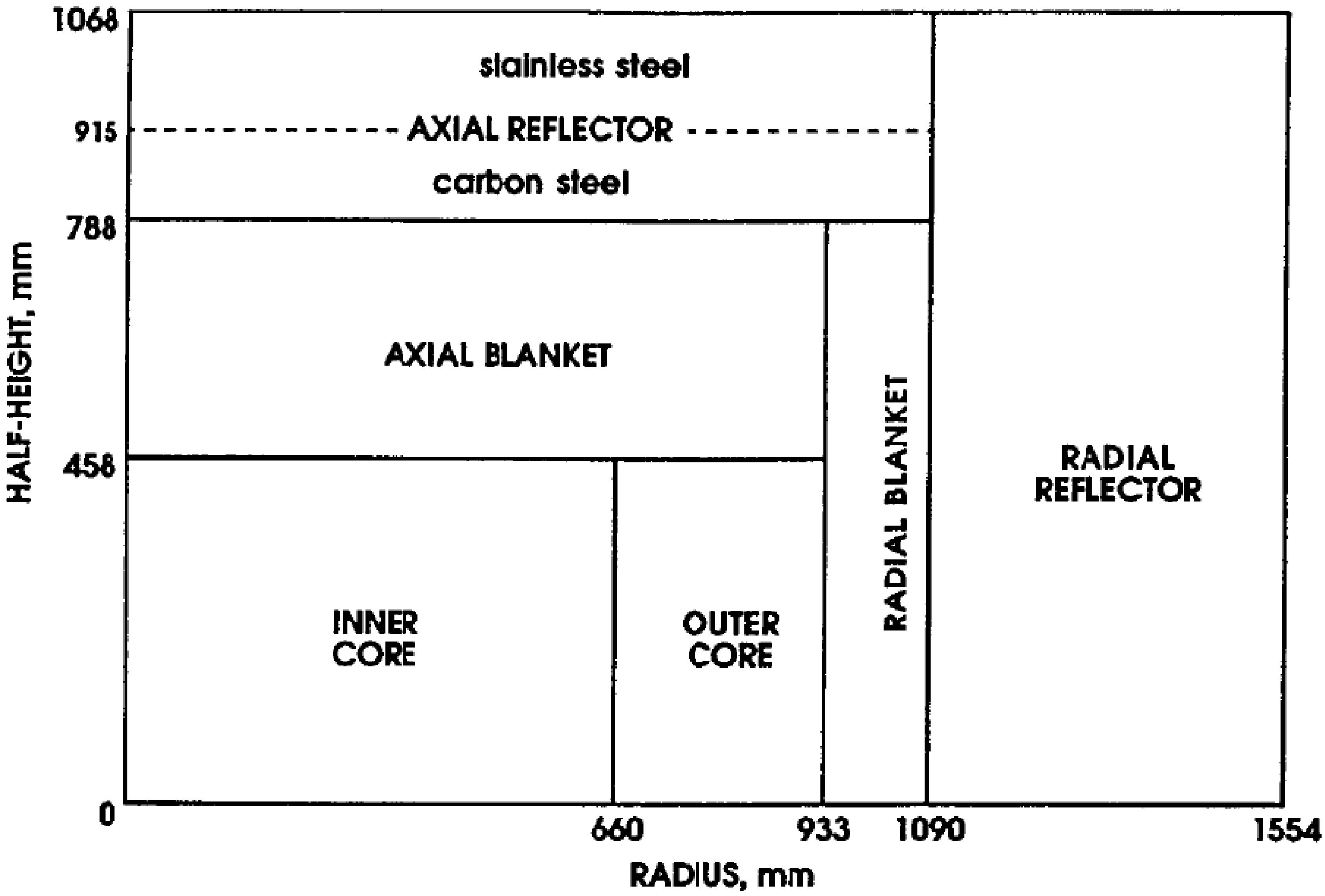
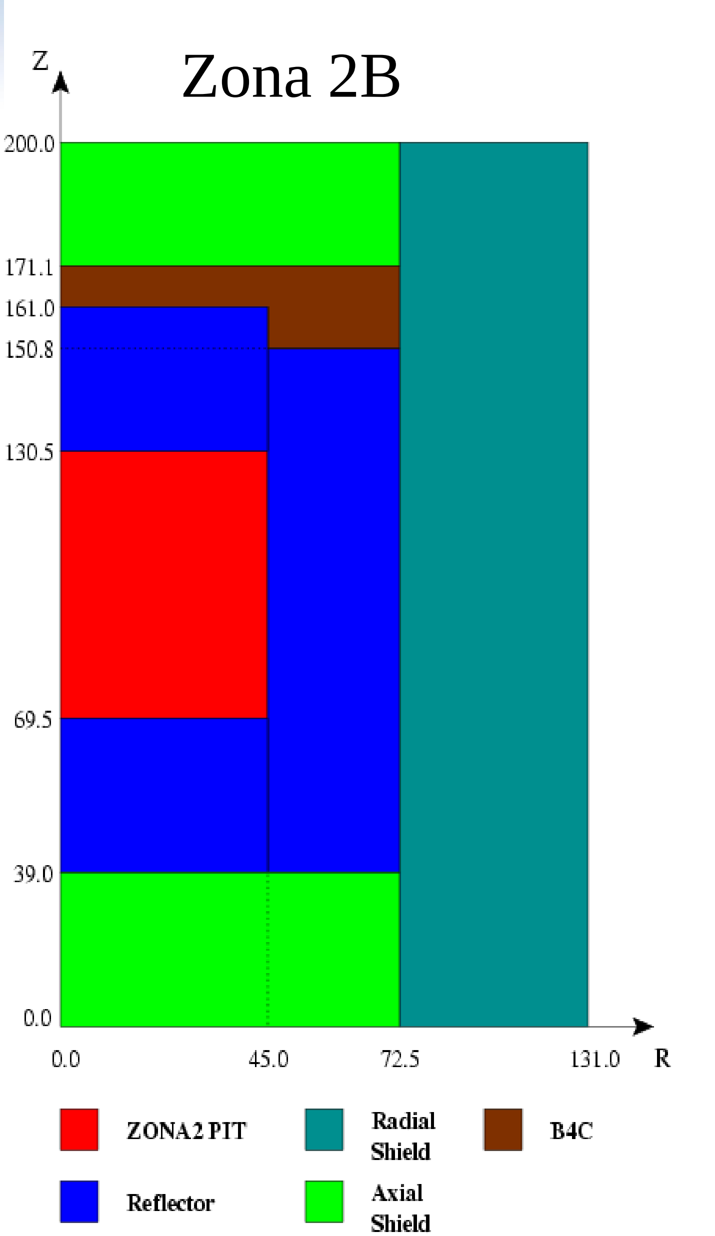
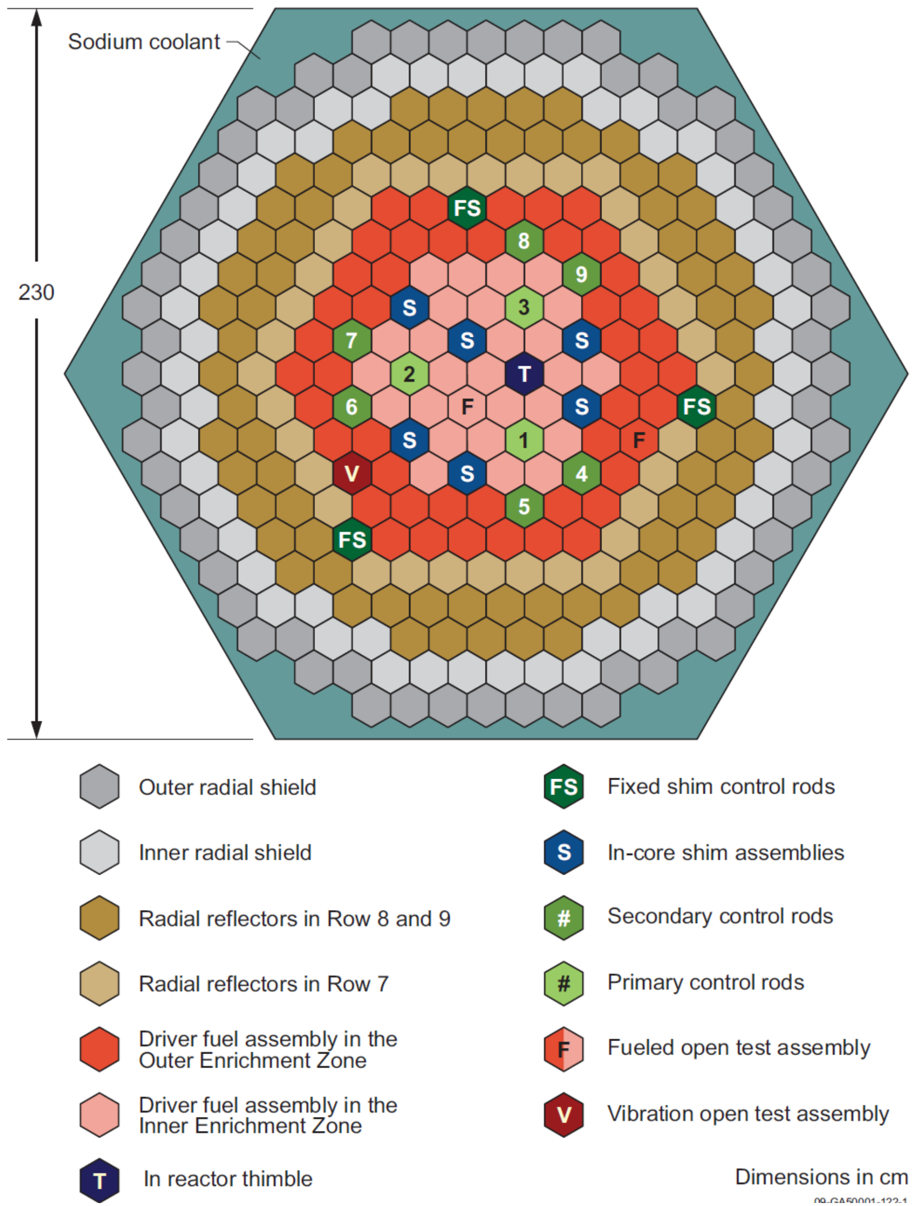


CIRANO


Zona 2B
Z
200.0
171.1
161.0
150.8
130.5
69.5
39.0
0.0
0.0
45.0
72.5
131.0
R
ZONA2 PIT
Radial Shield
B4C
Reflector
Axial Shield

FFTF First Critical Core



ZPR3-56B

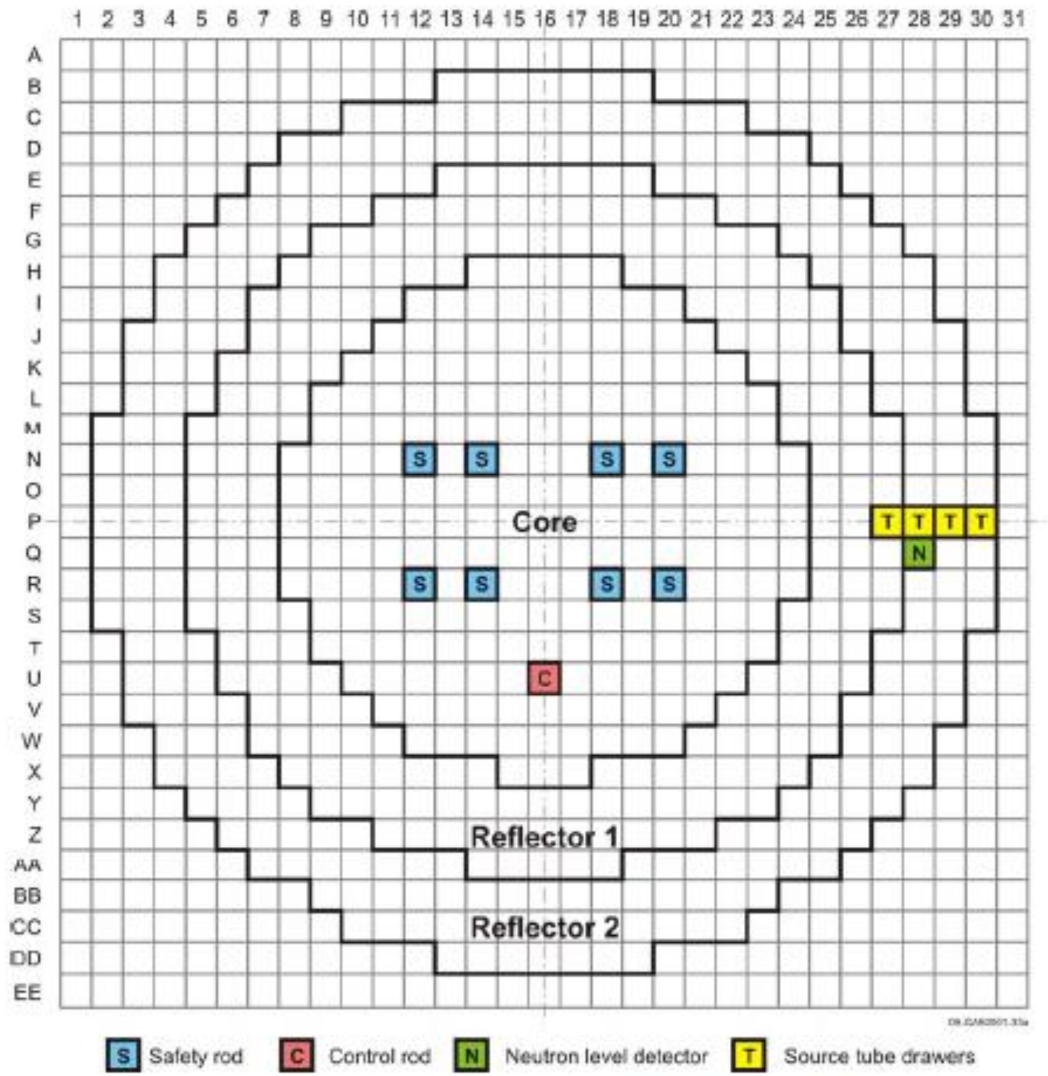


Figure 4. ZPR-3/56B Loading 17 Core Layout – Half 1 (Stationary Half).

Characteristics of the Experiments

Experiment	ZPPR-15 A	CIRANO 2B	FFTF	Estimated ZPR3-56B
C/E	0.99822	1.00644	1.00380	1.00312
Bias Factor	1.0018	0.9936	0.9962	0.9969
Nucl. Data Uncer. (pcm)	994	846	690	759
Exper. Uncert. (pcm)	89	200	211	150
Represent. Factor	0.7818	0.9485	0.8617	0.8364
Reduced Uncert.[a)] (pcm)	536	272	436	471
Total Uncert. (pcm)	543	338	484	494

Table II. List of K_{eff} experiments, bias factors, and uncertainty with respect to the FTR. Starting FTR Uncertainty: 859 pcm.

a) Using Standard Bias Factor Methodology

3 Experiments Weights (no exper. unc.)

Method	ZPPR-15 A	FFTF	ZPR3-56B
RWBF	0.2629	0.3969	0.3403
GBF	0.0941	0.8554	0.0504
PEBF [a)]	0.0658	0.9833	0.0043
BRBF [b)]	0.0658	0.9833	0.0043
EBF [c)]	0.0658	0.9833	0.0043

Table VIII. Bias Factor weights for different methods. 3 experiments used. No experimental uncertainty taken into account.

a) Exponents of the product

b) $\lambda\omega$.

c) Value of the extra term ω_0=-0.0533

3 Experiments Weights (with exper. unc.)

Method	ZPPR-15 A	FFTF	ZPR3-56B
RWBF	0.3796	0.3031	0.3174
GBF	0.1665	0.4552	0.3783
PEBF [a)]	0.1809	0.4108	0.3665
BRBF [b)]	0.0658	0.9833	0.0043
EBF [c)]	0.1809	0.4108	0.3665

Table IX. Bias Factor weights for different methods. 3 experiments used. Experimental uncertainty taken into account.

a) Exponents of the product

b) $\lambda\omega$.

c) Value of the extra term ω_0=0.0418

3 Experiments Results

Method	FTR Bias Factor	Combined Represent. Factor	FTR Nucl. Data Unc. pcm	Exper. Unc. pcm	FTR Total Unc. pcm
RWBF	0.9985	0.9312	313	154	349
GBF	0.9974	0.8574	442	173	475
PEBF	0.9976	0.8566	443	167	474
BRBF	0.9954	0.8624	435	227	490
EBF	0.9976	0.8480	435	133	455
ADJ	0.9976	0.8480	435	133	455

Table X. FTR K_{eff} Bias Factor and uncertainty for different methods. 3 experiments used. Experimental uncertainty taken into account.

4 Experiments Weights

Method	ZPPR-15 A	CIRANO 2B	FFTF	ZPR3-56B
RWBF	0.2684	0.3426	0.1841	0.2049
GBF	0.1799	0.7945	0.1099	-0.0843
PEBF [a)]	0.2067	0.8039	0.0238	-0.1116
BRBF [b)]	0.2563	1.0984	-0.1599	-0.3060
EBF [c)]	0.2067	0.8039	0.0238	-0.1116

Table XII. Bias Factor weights for different methods. 4 experiments used. Experimental uncertainty taken into account.

a) Exponents of the product

b) $\lambda\omega$.

c) Value of the extra term ω_0=0.1112

4 Experiments Results

Method	FTR Bias Factor	Combined Represent. Factor	FTR Nucl. Data Unc. pcm	Exper. Unc. pcm	FTR Total Unc. pcm
RWBF	0.9969	0.9023	370	169	407
GBF	0.9951	0.9508	266	190	327
PEBF	0.9955	0.9530	260	180	316
BRBF	0.9953	0.9600	240	182	302
EBF	0.9955	0.9600	240	191	307
ADJ	0.9955	0.9600	240	191	307

Table XIII. FTR K_{eff} Bias Factor and uncertainty for different methods. 4 experiments used. Experimental uncertainty taken into account.

4 Experiments Relative Contribution

Experiment	Q_i (%)
ZPPR-15 A	21.4
CIRANO 2B	86.1
FFTF	1.9
ZPR3-56B	-9.5

Table XIV. Relative contribution of each experiment to the total uncertainty reduction for the Extended Bias Factor Method.

CONCLUSIONS

- **Bias factor methods have been compared in a practical case, the critical mas of typical fast test reactor, using four different experiments and compared against cross section adjustments using the same experiments.**
- **Among the considered methods the GBF and BRBF have shortcomings, while the PEBF and the EBF provide identical results except for the experimental uncertainty component. However, only the EBF reproduces the results obtained by an adjustment that uses the same experiments. This is done without producing adjusted cross sections and associated adjusted covariance matrix**
- **The standard bias factor method is preferable in presence of significantly high representativity factors (e.g. 0.95).**
- **In bias factor methods is crucial to use experiments that capture the physics of the integral parameter under consideration. For the FTR is very important to capture the reflector effect.**