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Multivariate Statistical Reduction of Cross Section Uncertainties in Neutron Reactions

Introductive study

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Summary

Nuclear reactions induced by fast neutrons starting from 0.5 MeV up to 25 MeV with emission of alpha particles were investigated. Cross sections and their uncertainties, angular correlations and forward – backward asymmetry effects were evaluated with Talys, SPSS and author's own computer codes. Contribution to the cross section of nuclear reaction mechanisms like direct, compound and pre-equilibrium together with discrete and continuum states of residual nuclei were determined. Theoretical evaluations are compared with existing experimental data and parameters of nuclear potential in incident and emergent channels are obtained. Using cross section and angular correlation data from Talys, forward – backward effects are obtained for different incident neutron energies and target with finite dimensions. Simulated forward – backward asymmetry coefficient is sensible lower than the effect measured in the experiment The difference can be explained by the presence of other emergent channels including alpha particles and not by the presence of so-called non-statistical effects.

Factor analysis data reduction method was tested on neutron induced cross section data sets given by variation of the Wood Saxon (WS) potential parameters in the incident and emergent channels.

The present work was realized in the frame of fast neutron scientific program in FLNP JINR Dubna.

Outline

1. Introduction

2. Theoretical background

3. Computer codes and calculations

4. Results and discussion

5. Conclusions

1. Introduction

Fast neutron reactions - investigated for a long time at LNF facilities

Fundamental research – new data on nuclear reaction mechanisms and structure of nuclei

Applicative researches – precise nuclear data for nuclear fission and fusion reactors; reprocessing of *U* and *Th* for transmutation and energy projects and ADS; Fast Neutron Activation Analysis

Neodymium Nucleus – 5 stable isotopes, $^{142, 143, 145, 146, 148}$Nd (Z=60)
- of interest in many applications – permanent powerful magnets; Samarium – Neodymium dating -> age relationship of rocks and meteorites

^{143}Nd reactions with fast neutrons – alpha channels very low cross section

Investigated process – ^{143}Nd(n,α)^{143}Ce with FN from 0.5 MeV up to 25 MeV

2. Theoretical background

The cross section for (n,α) reaction (Hauser – Feshbach) (HF)

- without fluctuation correction factor

$$\sigma_{n\alpha} = \pi \lambda_n^2 \frac{T_n T_\alpha}{\sum_c T_c}$$

- with fluctuation correction factor

$$\sigma_{n\alpha} = \pi \lambda_n^2 \frac{T_n T_\alpha}{\sum_c T_c} W_{n\alpha}$$

$T =$ **transmission coefficient**

$W_{n\alpha} =$ **width fluctuation correction factor**

Differential cross section

$$\frac{d\sigma}{d\Omega} = \pi \lambda^2 (2l+1) T_l \sum_J \frac{A_J\left(l, j \mid l^{'}, j^{'} \mid \theta\right)}{1 + \sum_{p,q} \frac{T_p\left(E_q^{'}\right)}{T_{l'}\left(E^{'}\right)}}$$

$$A_J\left(l, j \mid l^{'}, j^{'} \mid \theta\right) = \sum_{m,m'} \left|(l, j; 0m \mid l, j; Jm)\right|^2 \left|(l', j'; m'm - m' \mid l', j'; Jm)\right|^2 \left|Y_{l'm'}(\theta, \varphi)\right|^2$$

A contains the dependence on

- quantum numbers in incident and emergent channels (l, j, l’, j’, J, m)
- solid angle (Ω (θ,ϕ))

2. Theoretical background

Quantum mechanical approach used

$$T(l,E) = 1 - \left|U_l(E)\right|^2$$

$W_l^{+}(r) =$ **Ingoing wave function**

$W_l^{-}(r) =$ **Outgoing wave function**

Reflection factor

$$U_l = \left\{ \frac{D_l - R\left[\frac{1}{W_l^-}\frac{dW_l^-}{dr}\right]}{D_l - R\left[\frac{1}{W_l^+}\frac{dW_l^+}{dr}\right]} \frac{W_l^-}{W_l^+} \right\}_{r=R}$$

Solution of Radial Schrodinger Equation

$$W_l(r) \sim W_l^-(r) - U_l W_l^+(r)$$

Logarithmic derivative $D_l = R\left[\frac{1}{W_l}\frac{dW_l}{dr}\right]_{r=R}$

Radial Schrodinger Equation

$$\frac{d^2 W_l(r)}{dr^2} + \frac{2m}{\hbar^2}\left[E_l - V(r) - \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]W_l(r) = 0$$

Antonio Foderaro, The Neutron Interaction Theory, The MIT Press, Cambridge, Massachusetts and London, England, 1971

2. Theoretical background

Quantum mechanical approach used

For neutrons - combination of Neumann (*n*) and Bessel (*j*) functions

$$W_l^+(r) = kr\left[n_l(kr) + ij_l(kr)\right] \qquad W_l^-(r) = kr\left[n_l(kr) - ij_l(kr)\right]$$

For charged particles - combination of Regular (*F*) and Irregular (*G*) Coulomb functions

$$W_l^+(r) = kr\left[F_l(kr) + iG_l(kr)\right] \qquad W_l^-(r) = kr\left[F_l(kr) - iG_l(kr)\right]$$

Widths Fluctuation Correction Factor (WFC)

- Represents a correlation between incident and emergent channels

$$W_{ab} = \left(1 + \frac{2\delta_{ab}}{\nu_a}\right)\int_0^\infty \prod_c \left(1 + \frac{2T_c x}{\nu_c \sum_i T_i}\right)^{-\left(\delta_{ac} + \delta_{bc} + \frac{\nu_c}{2}\right)} dx$$

- At low energies WFC = 1

- Then slowly decreasing with energy

$$\nu_a = 1.78 + \left(T_a^{1.212} - 0.78\right)\cdot e^{-0.228\sum_c T_c}$$

- Mainly three ways of evaluation

- Moldauer expression chosen

P. A. Moldauer, Rev. Mod. Phys., v. 36, p. 1079, 1964

3. Computer codes and calculations

Own computer code

We implemented Hauser – Feshbach (HB) approach

We realized a software in Mathematica able to compute:

- The regular and irregular Coulomb functions for neutral and charged particles and their derivatives

- For Coulomb functions no approximations were used

- The transmission coefficients for neutral and charged particles

- Implementation the quantum mechanical approach

- The cross section is obtained by taking into account the fluctuation factor and other open channels (n, n’, p, γ)

- This software was used for the evaluation of the (n,α) cross section and alpha strength functions

C. Oprea, A, Mihul, A.I. Oprea, (CERN-Proceedings-2019-001):126-130

A.I. Oprea, C. Oprea, C. Parvutoiu, D. Vladoiu, Rom Rep in Phys 63(1):107-114, 2011

3. Computer codes and calculations

TALYS Codes

- free software working under Linux operating system in continue development

- friendly interface

- a large number of models for nuclear structure and nuclear reactions (direct, compound, pre - equilibrium) implemented

- data base on nuclear structure for a large number of nuclei

- allows to evaluate: nuclear structure data, inclusive and exclusive cross sections (XS)

- **Inclusive XS – Ex,** in a binary reaction A(a,b), b will be considered emergent particles from other possible open channels

- **Exclusive XS** – in a binary reaction b will be considered emergent particles from a well defined "b+B" exit channel

- **Talys** will be used in the XS calculations of fast neutron induced reactions with emission of alpha particles

A.J. Koning, S. Hilaire and M.C. Duijvestijn, .TALYS-1.0., Proceedings of the International Conference on Nuclear Data for Science and Technology, April 22-27, 2007, Nice, France, editors O.Bersillon, F.Gunsing, E.Bauge, R.Jacqmin, S.Leray, EDP Sciences, p. 211, 2008

3. Computer codes and calculations - Data Reduction

Factor Analysis - Principal Components

. Let suppose a set of *n* subjects and a number of *m* measurements

An $\{Y_{ij}\}$ matrix can be defined; $i=1,2,..,m$; $j=1,2,..,n$

$$\bar{y}_i = \frac{\sum_{j=1}^{n} y_{ij}}{n}$$

Average values

$$s_i = \sqrt{\frac{\sum_{j=1}^{n} (y_{ij} - \bar{y}_I)^2}{n-1}}$$

Standard deviation – $i = 1,..,m$

$$s_{ik} = \frac{\sum_{j=1}^{n} (y_{ij} - \bar{y}_i)(y_{kj} - \bar{y}_k)}{n-1}$$

Covariance

$$r_{ik} = \frac{\sum_{j=1}^{n} (y_{ij} - \bar{y}_i)(y_{kj} - \bar{y}_k)}{\sqrt{\sum_{j=1}^{n} (y_{ij} - \bar{y}_i)^2 (y_{kj} - \bar{y}_k)^2}} = \frac{s_{ik}}{s_j s_k}$$

Correlation Coefficients

$$z_{ij} = \frac{y_{ij} - \bar{y}_i}{s_i}$$

Elements of standard normalized matrix ***Z***

3. Computer codes and calculations - Data Reduction

Factor Analysis (FA) - Principal Components

$$r_{ik} = \frac{\sum_{j=1}^{n} z_{ij} z_{kj}}{\sqrt{\sum_{j=1}^{n} z_{ij}^2 \sum_{j=1}^{n} z_{kj}^2}} = s_{ik} = \frac{\sum_{j=1}^{n} z_{ij} z_{kj}}{n-1}$$

Correlation coefficients written by ***Z*** matrix elements

$$\frac{ZZ'}{n-1} = R = S$$

Then the following matrix relation is obtained between normalized ***Z***, correlation ***R*** and standard deviations ***S*** matrixes, respectively

Main goal of FA is to describe variables (m) by a series of factors (λ_t) r<m

Factors are obtained by solving eigenvalues problem

$$(\mathbf{R} - \lambda_l \mathbf{I})\boldsymbol{\alpha}_\mathbf{l} = \mathbf{0}$$

Where $\{\alpha\}$ are the matrix elements of ***A*** – factor loading matrix

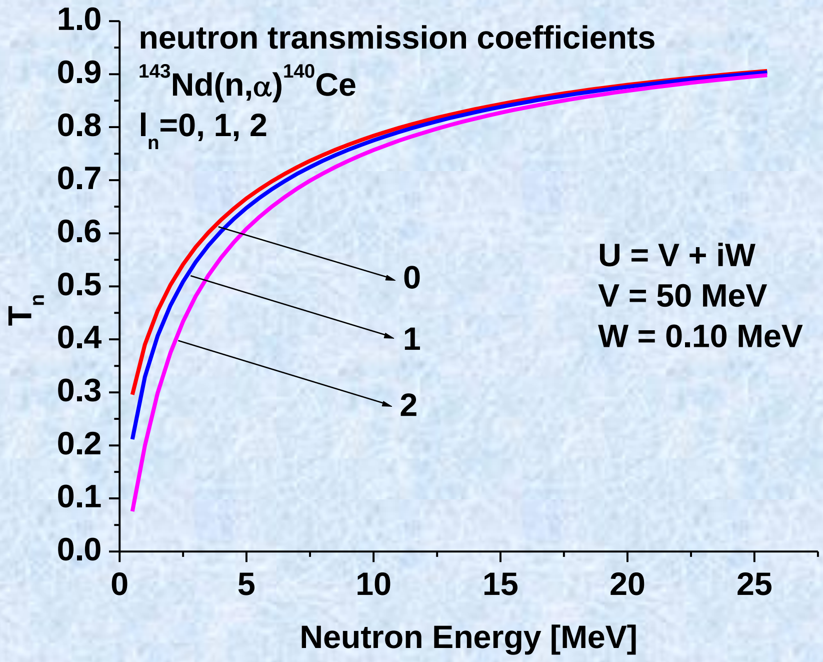
From the matrix relation ***Z = AP*** with ***P*** – factor score matrix

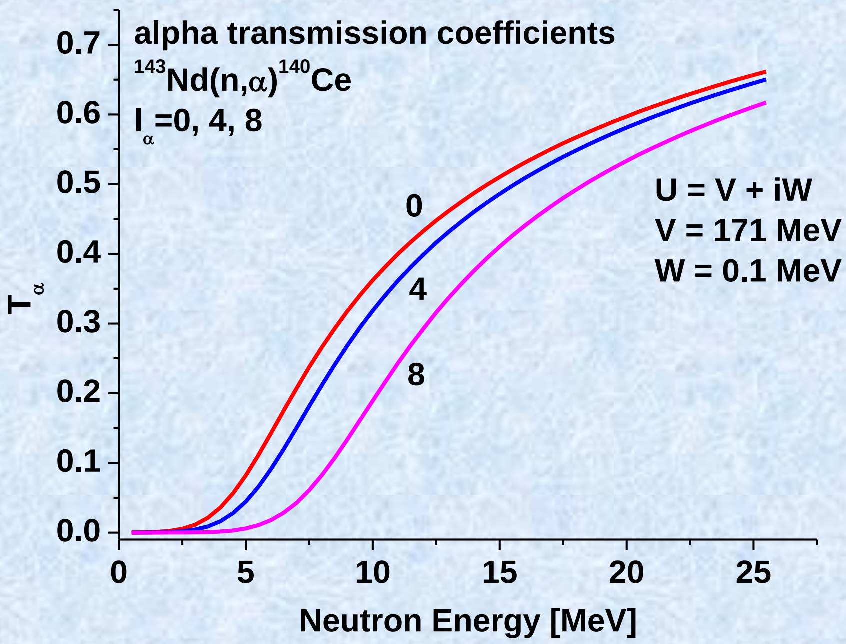
4. Results and discussion. $^{143}Nd(n,\alpha)^{140}Ce$ Transmission coefficients

$^{143}Nd(n,\alpha)^{140}Ce$ ($Q_{n\alpha}$ = 9.72 MeV) neutrons 0.5 to 25 MeV - orbital momentum $l_n = 0,1$

- Spin and parity of ^{143}Nd and ^{140}Ce nuclei, $J^{\Pi} = (7/2)^-$ and 0^+ respectively
- considered γ, p, n, n', α channels;

Neutron energy dependence of neutron and alpha transmission coefficients



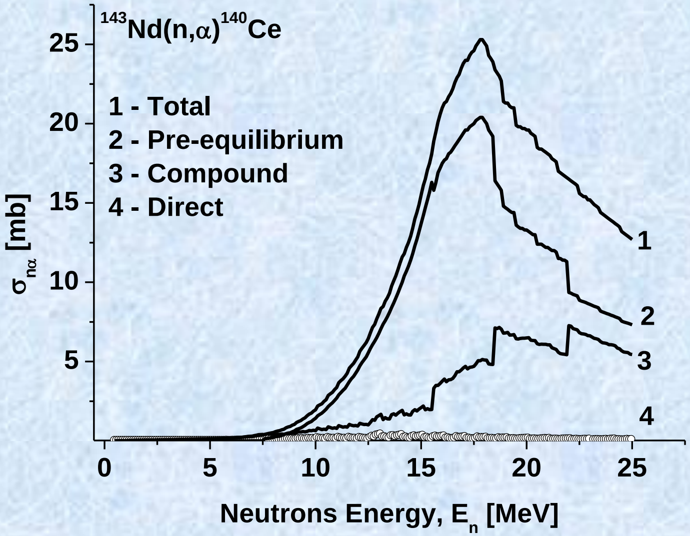


Orbital momentum: neutrons – $l_n = 0,1,2$; alphas – $l_\alpha = 0,4,8$

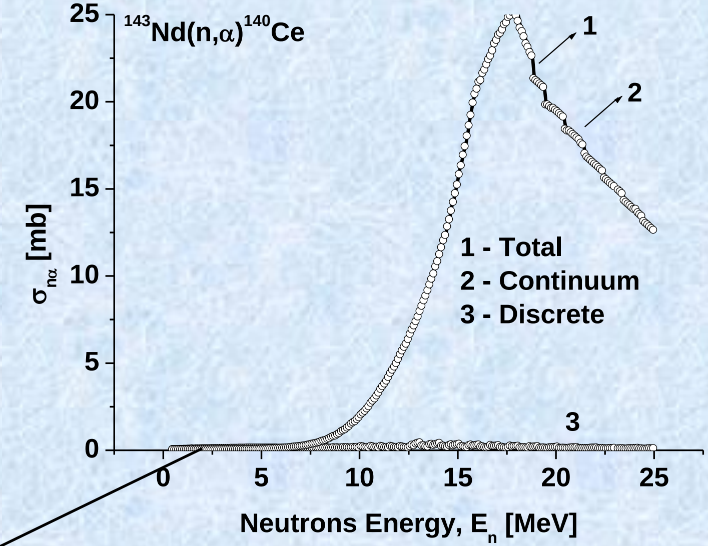
Calculated with our soft based quantum mechanical approach

4. Results and discussion – $^{143}Nd(n,\alpha)^{140}Ce$ – Talys cross sections evaluations

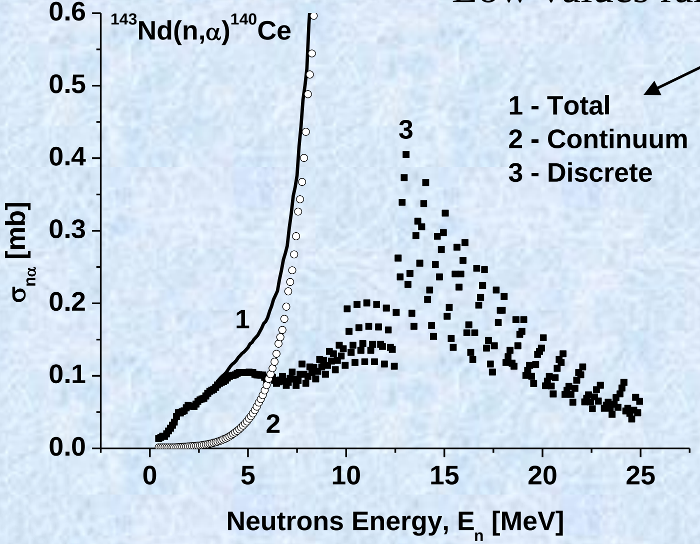
XS – Contribution of reaction mechanisms



Discrete + Continuum states of res nucleus



Low values range



At low energies compound processes + discrete states are dominant.

At higher energies compound pre-equilibrium processes + continuum states becomes dominant

Direct processes can be neglected in alpha channel but not in inelastic and protons ones

30 discrete states on inelastic channels

10 discrete states for γ, proton and alpha channels

4. Results and discussion. Comparison with experimental data ^{143}Nd(n,α) XS

E_n [MeV]	Exp [mb]	Eval /1/ [mb]	Eval /2/ [mb]	Dir. [mb]	Comp [mb]	Discr [mb]	Cont [mb]
4⌖0.23	0.12 ⌖0.01	0.14	0.10934	0.00066	0.10868	0.09574	0.01360
5⌖0.16	0.21 ⌖0.01	0.26	0.17993	0.00338	0.17596	0.14339	0.03595
6⌖0.12	0.31 ⌖0.03	0.37	0.25981	0.05826	0.20154	0.13390	0.12590

Experimental data were obtained at EG-5 Electrostatic Generator from FLNP JINR Dubna and Tandem from Pekin University – Heavy Ions Physics Institute using a double Ionization Chamber

Evaluation /1/ - own soft realized in Mathematica –> Hauser-Feshbach approach + compound processes

- direct and pre-equilibrium processes are not considered.

- Rectangular potential for incident and emergent channels U = V + iW

Evaluation /2/ - Talys -> (Comp + Dir + Pre-eq) and (Discrete and Continuum States of Resid. Nucleus

Compound Processes are dominant. Both Direct and Compound Processes are of pre-equilibrium origin

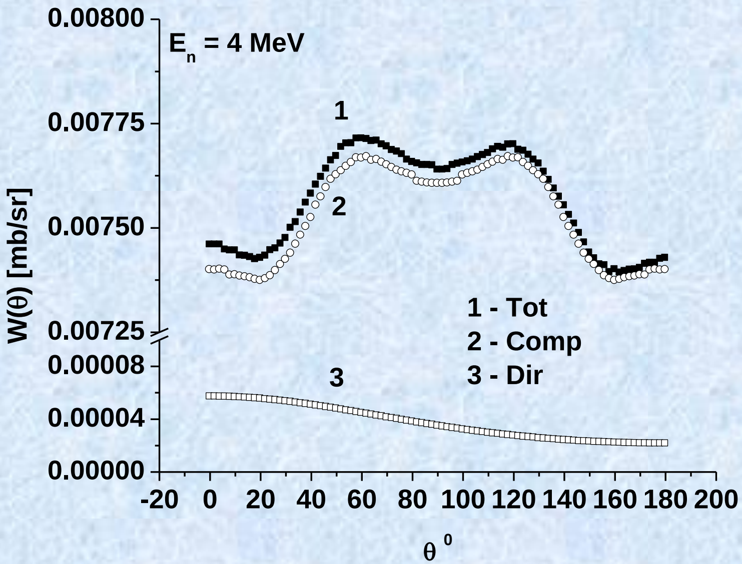
From Table it is possible to observe how continuum states increase their contribution to the XS

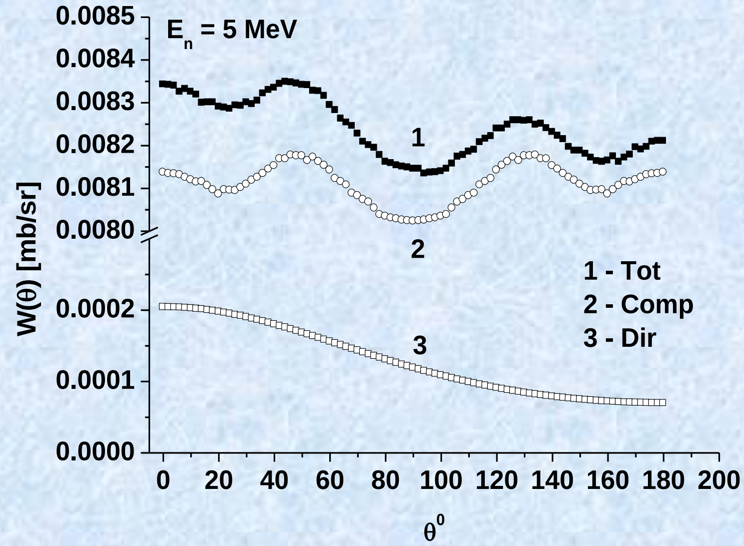
- Wood – Saxon Potential with real and imaginary part with volume, surface and spin-orbit components

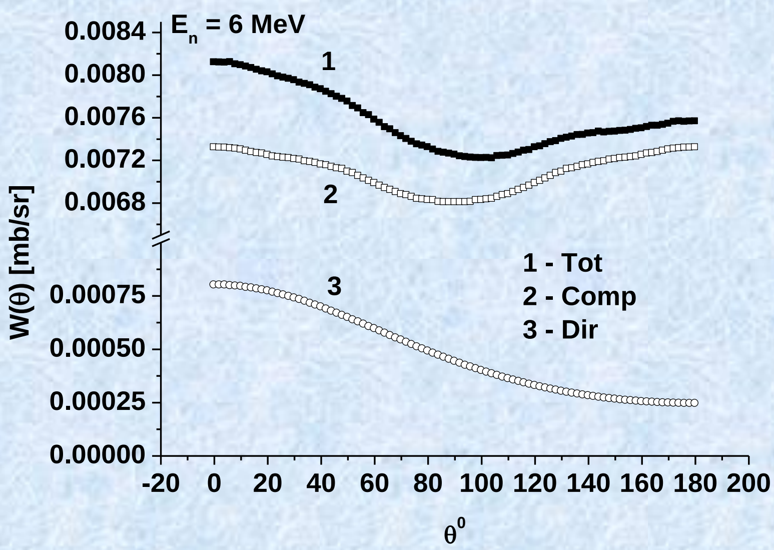
- levels density – Fermi Gas model

Present Table – necessary in the analysis of angular correlations and measured forward – backward (FB) measured effect

4. Results and discussion. Differential XS and FB Effects







Differential cross sections calculated with Talys

Separated the contribution of compound and direct processes

For given energy compound processes are dominant

Direct processes are much lower than compound ones

Importance of direct processes is increasing with the energy

Further calculations

For 4 and 5 MeV diff. XS described numerically

For 6 MeV $W(\theta) = p_0 + p_1\cos(\theta) + p_2\cos^2(\theta)$

4. Results and discussion. A_{FB} Forward – Backward Ratio

E_n [MeV]	$(A_{FB})_{exp}$	$(A_{FB})_{eval}$ g[mg/cm^2]=0	$(A_{FB})_{eval}$ g=0.04077	$(A_{FB})_{eval}$ g=0.4.077	$(A_{FB})_{eval}$ g=4.077
4⌖0.23	1.25⌖0.12	1.0076	-	-	-
5⌖0.16	1.78⌖0.18	1.0142	-	-	-
6⌖0.12	2.50⌖0.25	1.0368	1.0369 ⌖0.0073	1.0351 ⌖0.0059	1.0368 ⌖0.0061

Punctual target A_{FB} ratio

$$A_{FB} = \frac{A_{FW}}{A_{BW}} = \frac{\int_0^{\pi/2} W(\theta)\sin(\theta)d\theta}{\int_{\pi/2}^{\pi} W(\theta)\sin(\theta)d\theta}$$

Finite target with thickness g -> A_{FB} – evaluated by MC simulation

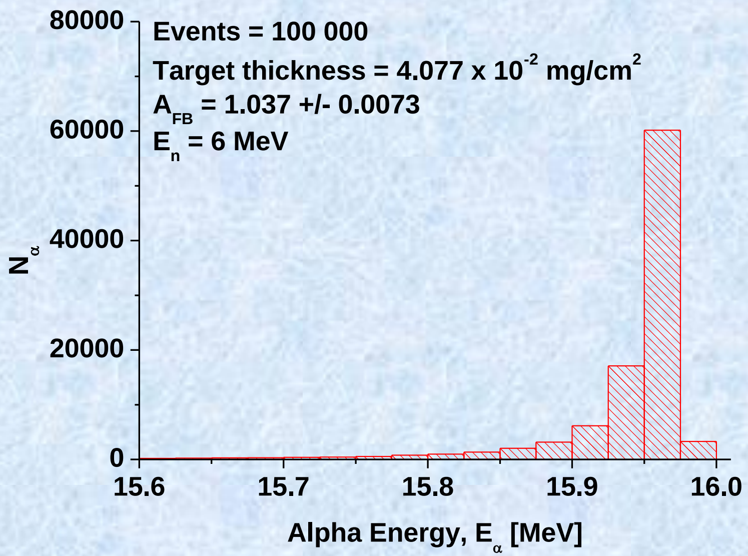
Direct Method

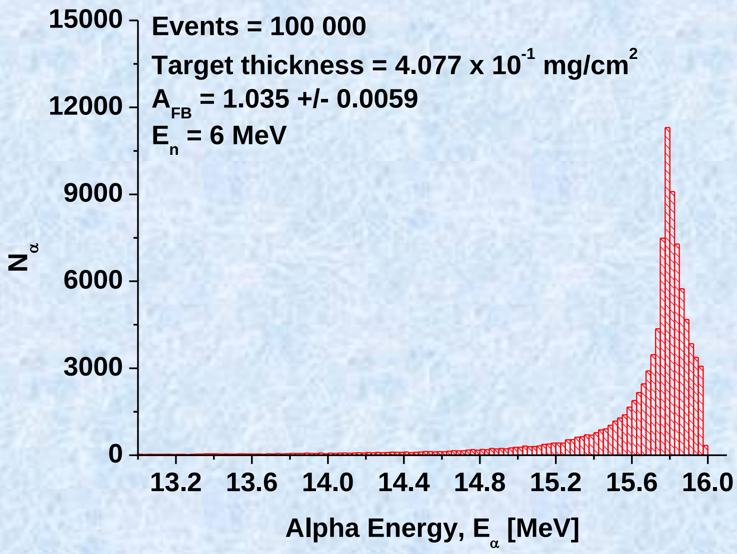
$$\frac{2\pi}{\sigma_{n\alpha}}\int_0^{\theta_c} W(\theta)\sin(\theta)d\theta = r \Rightarrow \theta_c, r \in [0,1), \theta \in [0,\pi)$$

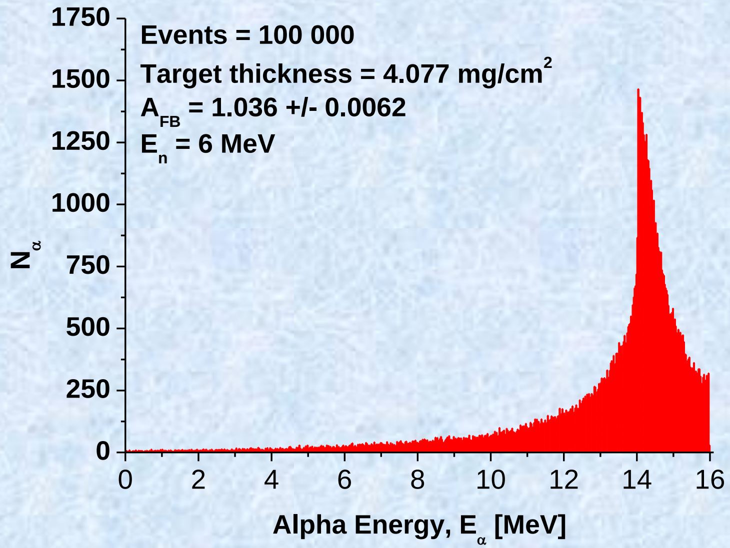
Simple MC modeling of alpha particles going out from a finite ^{143}Nd target with radius 10 cm:

- by direct method angular correlation is obtained -> W(θ)
- using alpha particle energy loss in ^{143}Nd from tables it is determined if particle is escaping from the target
- energy and number of alphas going out from the target are determined
- different thickness g[mg/cm^2] are tested (g = 0, 0.04077, 0.4077, 4.077)
- for A_{FB} measurements it is necessary that g[m]<p_{max}[m] , alpha particles maximum path s
- in our case p_{max} = 9.53E-5 m for E_α 🕓 16 MeV; g[g/cm^2] = 4.077 = g[m] = 6.82E-6 m
- direct component cannot give such a high forward asymmetry -> possible interference with elastic, inelastic and proton channels where direct process is important.

4. Results and discussion. Simulated Alpha Spectra







Events = 100 000

- neutron flux = 1, E_n = 6 MeV

- with the increasing of the thickness number of alpha particles which are not escaping from the target is decreasing and alpha spectra is also enlarging

- for the figures absorbed alpha particles are about 30, 300, 3000 (thick target)

- straggling is not considered

4. Results and discussion. Nuclear Potential Parameters

Wood - Saxon Potential

Volume WS – Real Part | Volume WS – Imaginary | Surfce WS – Imaginary

	V_V [MeV]	r_{vv} [fm]	a_v [fm^{-1}]	W_V [MeV]	r_{wv} [fm]	a_{wv} [fm^{-1}]	W_D [MeV]	r_{wd} [fm]	a_{wd} [fm^{-1}]
N_chann	**49.78**	**1.226**	**0.657**	0.11	1.226	0.657	3.42	1.260	0.521
α_chann	**171.83**	**1.227**	**0.657**	0.330	1.340	0.500	4.00	1.520	0.345

Spin orbit – Real Part | Spin orbit – Imaginary

	V_{so} [MeV]	r_{vso} [fm]	a_{vso} [fm^{-1}]	W_{so} [MeV]	r_{wso} [fm]	a_{wso} [fm^{-1}]
N_chann	6.16	1.186	0.632	0.01	1.062	0.590
α_chann	0	0	0	0	0	0

4. Multivariate Analysis. Initial Data

Parameters of components of WS potential were varied

Modified Parameters

- For incident and emergent channels (neutron and alpha)
- Real and Imaginary part of Volume WS potential (V_V, W_V)
- Imaginary part of Surface WS potential – W_D

Range of variation (0.5 P_0, 2P_0)

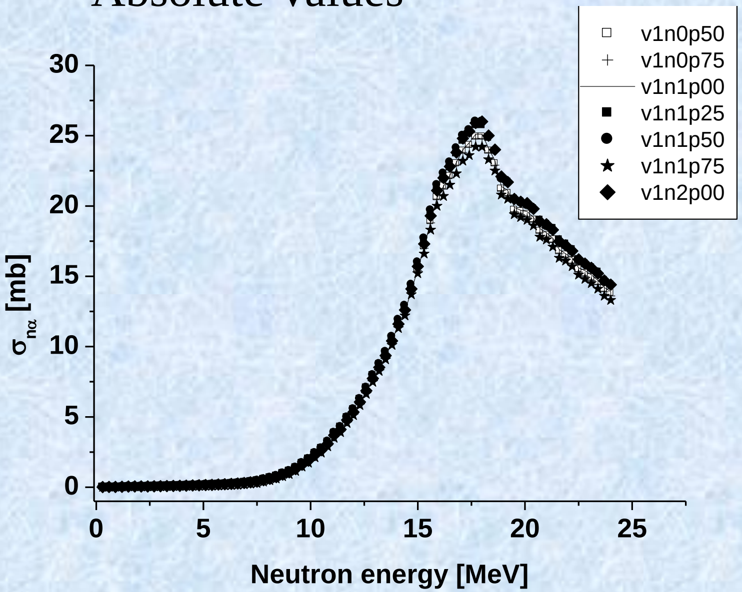
P_0	V_V [MeV]	W_V [MeV]	W_D [MeV]
N_chann	49.78	0.10	4.00
α_chann	171.83	0.11	3.42

4. Multivariate Analysis. Cross Section Data

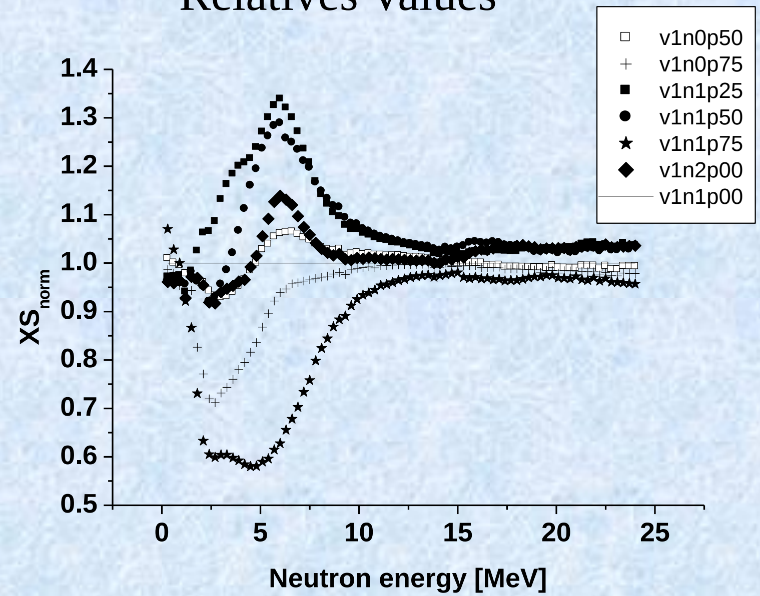
Incident Neutron Channel – Volume Parameters Variations - Real Part

$P_0 = V_{V0} = 50$ MeV, $P = V_V \in \{0.5, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00\} \times V_{V0}$

Absolute Values



Relatives Values



$Xs_{norm} = \sigma_{n\alpha}(P)/\sigma_{n\alpha}(P_0)$

Neutron Volume Real Part V_V values are not affecting very much the absolute values of XS

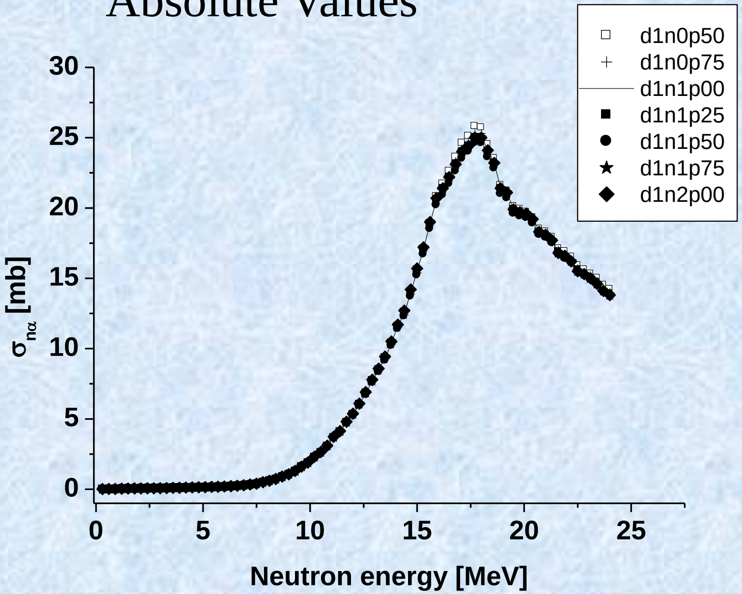
The relative values especially in the low energy part up to 5 - 10 MeV incident energy affected

4. Multivariate Analysis. Cross Section Data

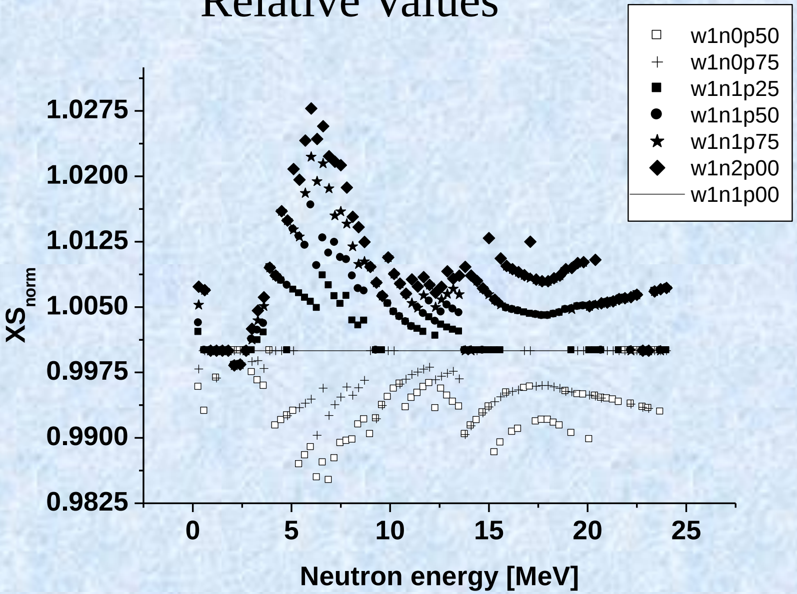
Incident Neutron Channel – Volume Parameters Variations - Imaginary Part

$P_0 = W_{V0} = 0.10$ MeV, $P = W_V$ ∈ {0.5, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00} × W_{V0}

Absolute Values



Relative Values



$Xs_{norm} = \sigma_{n\alpha}(P)/\sigma_{n\alpha}(P_0)$

Neutron Volume Imaginary Part W_V values are not affecting very much the absolute values of XS (few mb)

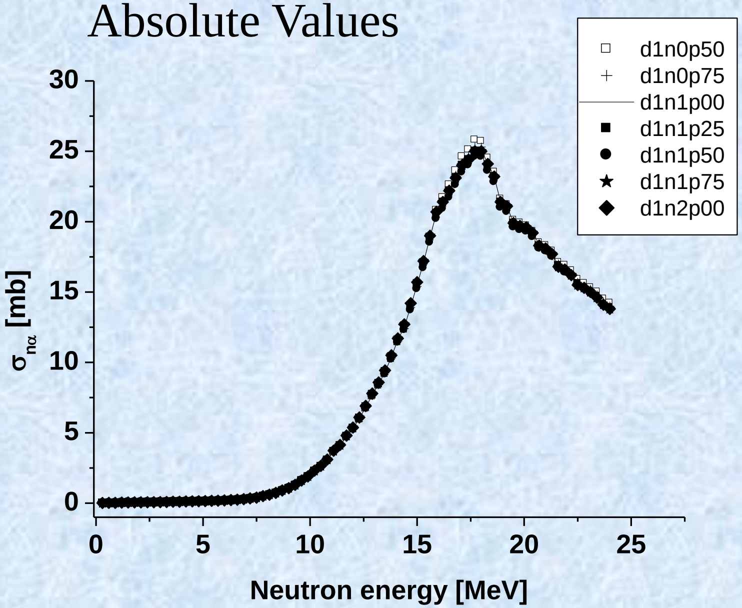
Affected relative values especially in the whole energy interval with about 2 - 3%

4. Multivariate Analysis. Cross Section Data

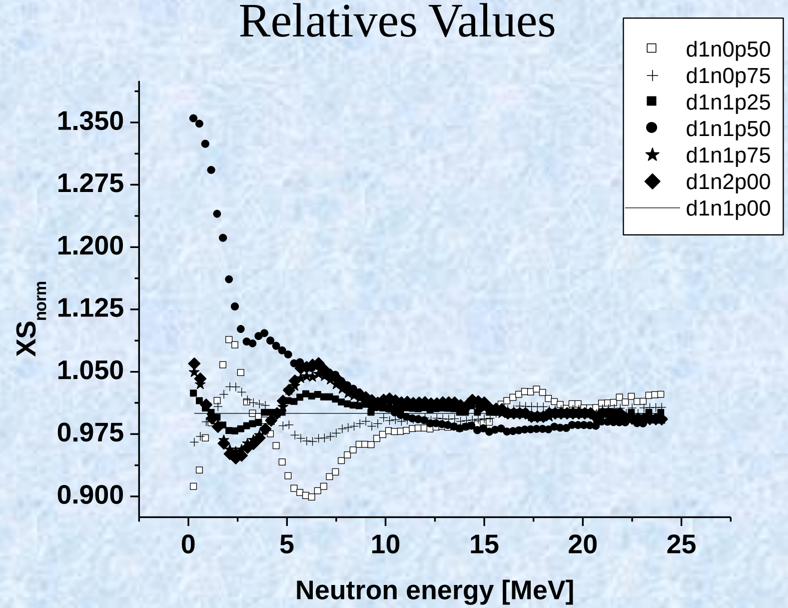
Incident Neutron Channel – Surface Parameters Variation - Imaginary Part

$P_0 = W_{D0} = 4.00$ MeV, $P = W_V \in \{0.5, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00\} \times W_{D0}$

Absolute Values



Relatives Values



$Xs_{norm} = \sigma_{n\alpha}(P)/\sigma_{n\alpha}(P_0)$

Neutron Surface Imaginary Part W_D values are not affecting very much the absolute values of XS (in the range of few mb)

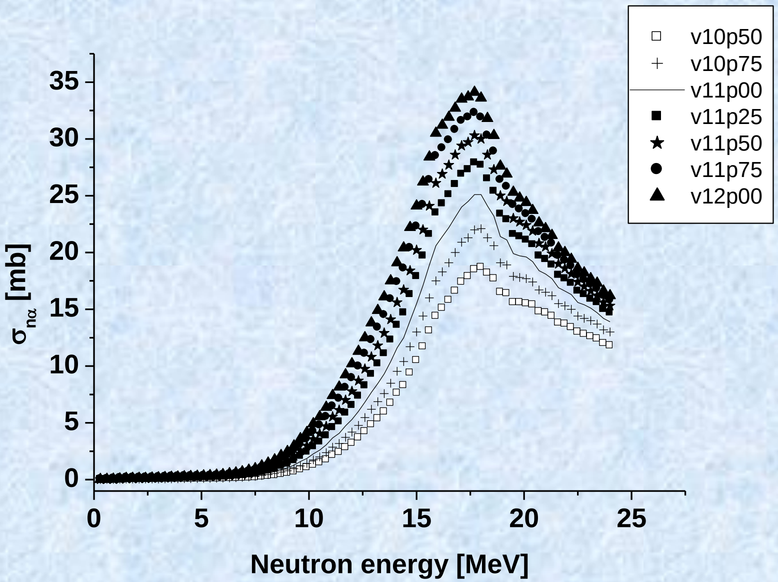
Relative values especially at low energy up to 5 MeV with about 30%

4. Multivariate Analysis. Cross Section Data

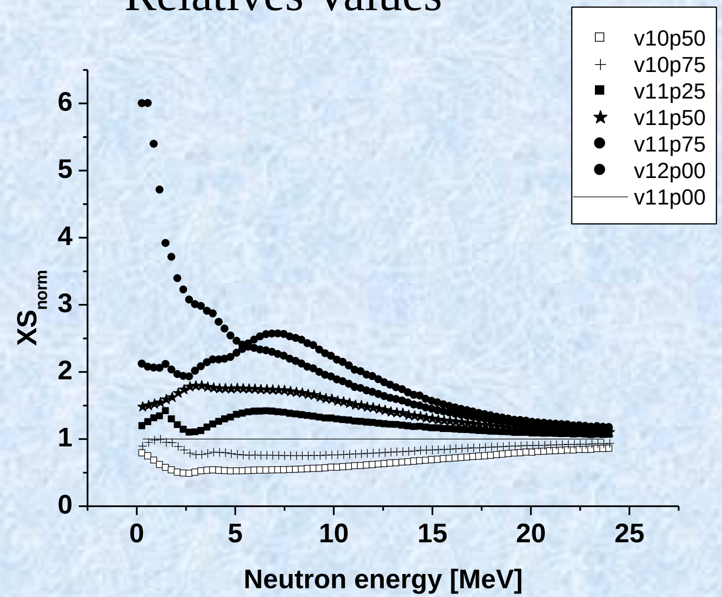
Emergent Alpha Channel – Volume Parameters Variation - Real Part

$P_0 = V_{V0} = 172$ MeV, $P = V_V \in \{0.5, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00\} \times V_{V0}$

Absolute Values



Relatives Values



$Xs_{norm} = \sigma_{n\alpha}(P)/\sigma_{n\alpha}(P_0)$

Alpha Volume Real Part V_V – very sensible in 10 – 20 MeV range

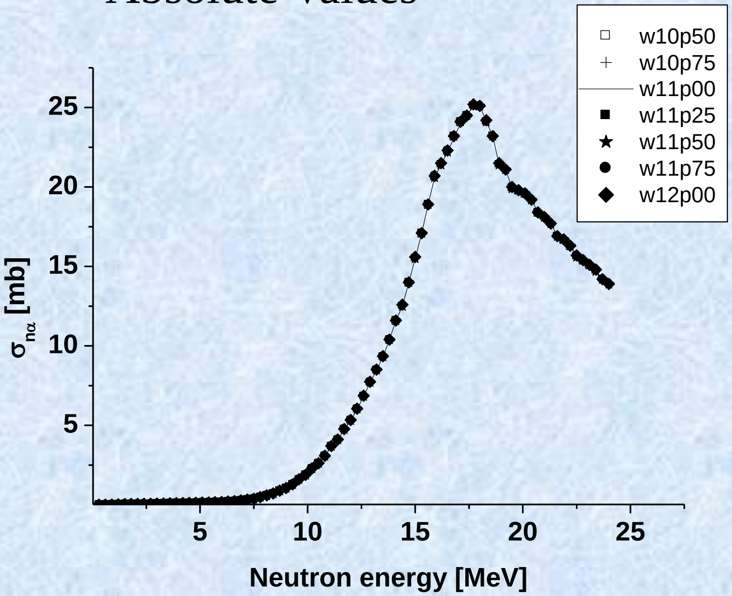
Relative values - at low energy up to 5 MeV – are changing by 3 – 6 times

4. Multivariate Analysis. Cross Section Data

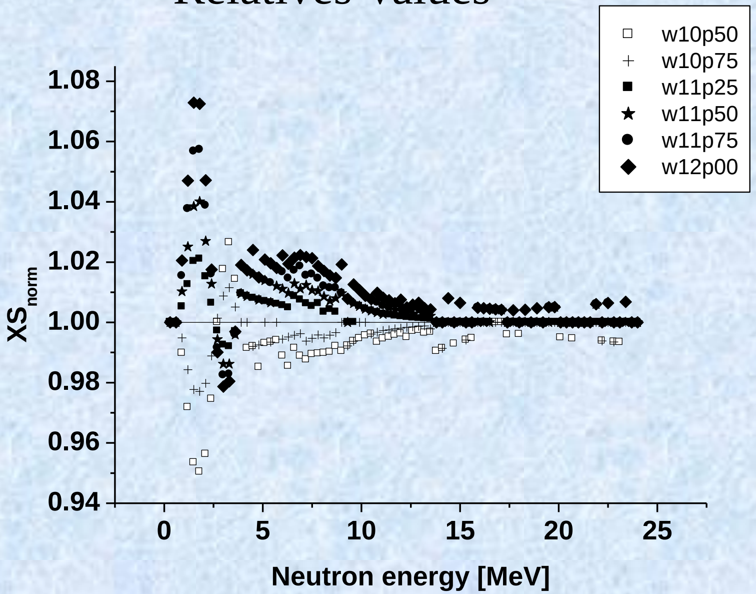
Emergent Alpha Channel – Volume Parameters Variation - Imaginary Part

$P_0 = W_{V0} = 0.11$ MeV, $P = W_V \in \{0.5, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00\} \times W_{V0}$

Absolute Values



Relatives Values



$Xs_{norm} = \sigma_{n\alpha}(P)/\sigma_{n\alpha}(P_0)$

Alpha Volume Imaginary Part W_V – Absolute Values not sensible

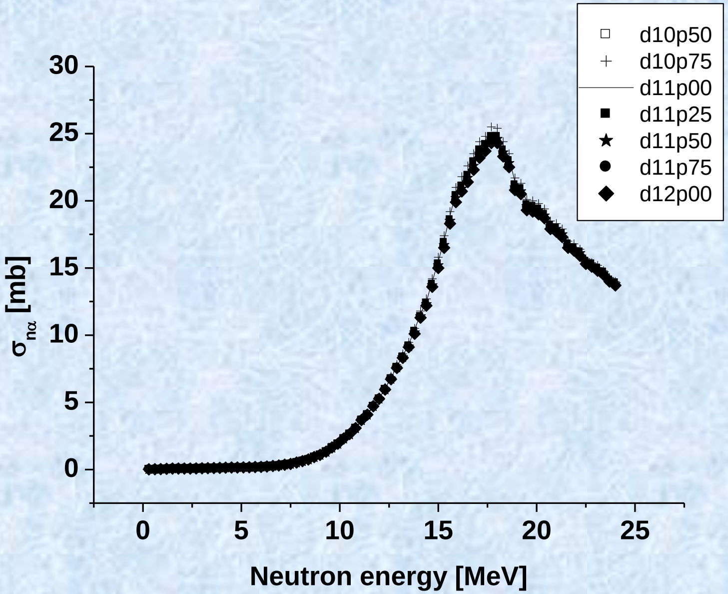
Relative values - at low energy up to 5 MeV – are changing about 8%

4. Multivariate Analysis. Cross Section Data

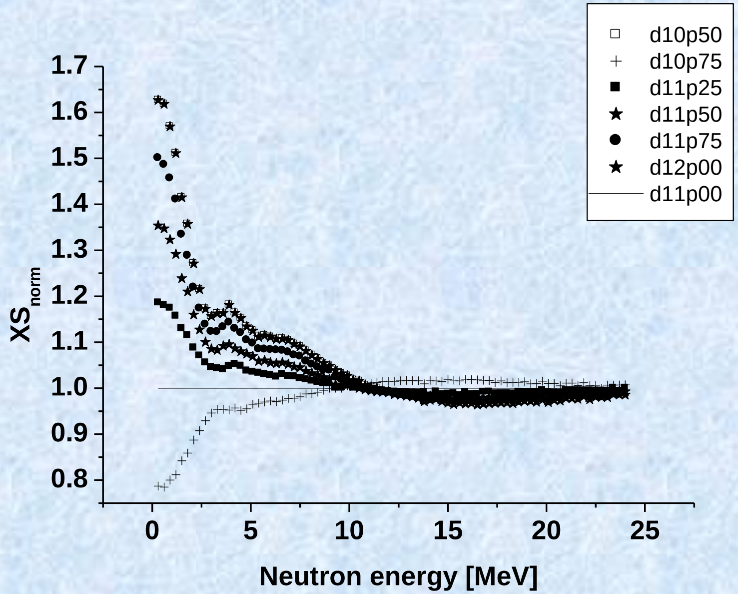
Emergent Alpha Channel – Surface Parameters Variation - Imaginary Part

$P_0 = W_{D0}$ = 3.42 MeV, $P = W_D$ ✇ {0.5, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00} ⯑ W_{D0}

Absolute Values



Relatives Values



$Xs_{norm} = \sigma_{n\alpha}(P)/\sigma_{n\alpha}(P_0)$

Alpha Surface Imaginary Part W_D – Absolute Values not sensible

Relative values - at low energy up to 5 MeV – are changing up to 70%

4. Factor Analysis – Principal Components (Data Reduction)

- Cross sections data were generated with Talys by variation of real and imaginary part of volume components (**V_V, W_V**) and imaginary part of surface component (**W_D**) in the incident and emergent channels (6 parameters, **P**)

- Parameters were varied from 0.50 P_0 up to 2 P_o with a step of 0.25

- Neutrons energy interval is starting from 0.3 MeV up 24 MeV with a step of 0.3 MeV

- Transformation Matrix consisted of m = 42 variables and n = 80 subjects

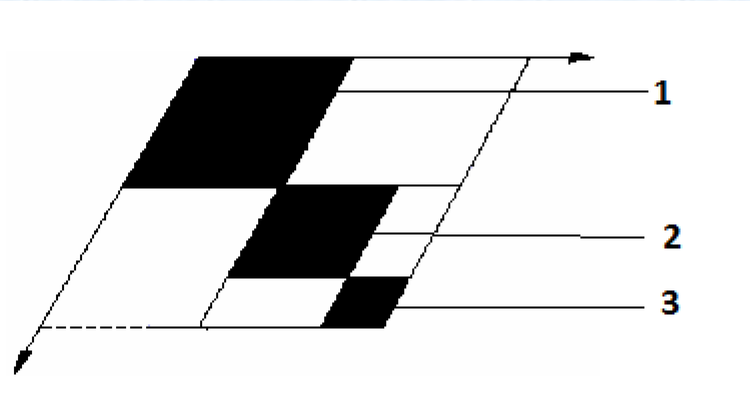
- Cross sections values are given by variations of 6 parameters of volume and surface components of WS potentials

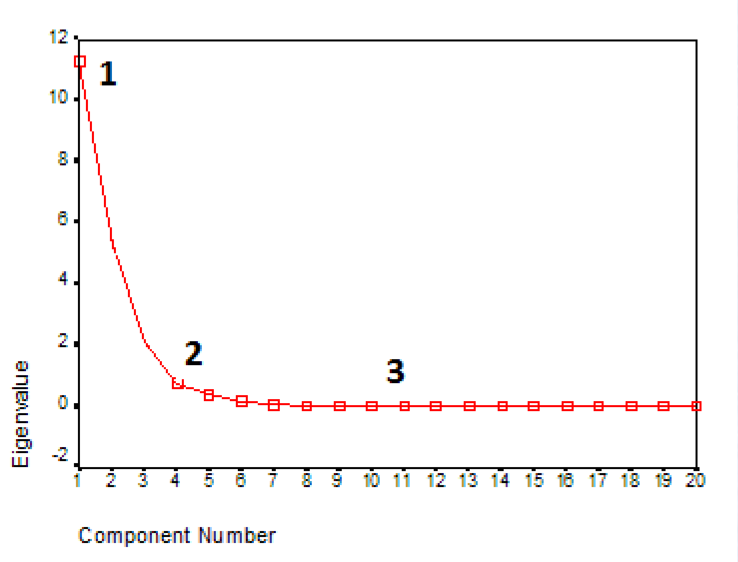
- Solving the eigenvalues problem followed by the main factors extraction by applying Principal Component Analysis

4. Factor Analysis – Principal Components (Data Reduction)

Solving eigenvalues problem

$$(\mathbf{R} - \lambda_l \mathbf{I})\boldsymbol{\alpha}_\mathbf{l} = \mathbf{0}$$





Structure of **R** matrix

- 2 factors are obtained λ_1, λ_2
  - 1 – with a weight larger than 90%
  - 2 – weight about few percents
  - 3 – neglected part (< 1 %)

- The most important factor (1) is given by Real part of Volume WS potential (V_V) of alpha channel
- The second factor (2) most probably comes from Imaginary part of Surface component (W_D) of alpha channel
- Region (3) – influence of other variables statistically can be neglected

Low energy part and Giant Resonance are most sensible for variables changes

5. Conclusions – Cross Sections Analysis

Cross sections

Good description of fast neutrons (0.5 – 25 MeV) CS experimental data using own codes and Talys
Differential cross sections were evaluated
Concurrence of different nuclear reaction mechanism were evidenced

Asymmetry effects

Analyzed A_{FB} ratio. High difference between theory and experiment. Evaluation of direct component, for different configurations and energy cannot give such a high forward asymmetry -> It can be explained by the presence of channel with an important direct component

Future tasks

- New theoretical evaluations based on the new XS and diff. XS measurements in wide neutron energy interval

- To determine for each energy the contribution of nuclear reaction mechanisms based on future experimental data

- Improvements of Monte Carlo simulation

- Theoretical follow up of new nuclear experimental data on strength functions

Present work - proposal for new experiments at FLNP JINR Dubna Facilities

5. Conclusions – Multivariate Analysis

- **Cross sections** data were obtained by variation of parameters of Wood Saxon potential in the incident and emergent channels
- \- Real & Imaginary part of Volume WS potential and Surface Imaginary Part were are the most influencing on XS data
- \- Their values were modified from 2 times lowers and 2 times higher related to the parameters describing the experimental data
- Where obtained absolute and relative XS data
- \- As it is expected in a higher degree XS data are sensible to the Real Part of Volume WS potential
- XS data are not so sensible to the variation of other parameters
- The influence of WS potentials parameters of other open channels, like proton, ^{3}He, or multiparticle emission were not investigated

5. Conclusions – FA – Data Reduction

Principal components analysis was tested on $^{143}Nd(n,\alpha)^{140}Ce$ cross section data for neutrons from 0.3 MeV up to 25 MeV by changing parameters of optical potential in the incident and emergent channels

The multivariate statistical uncertainties reduction was done by SPSS 10.4 by using Talys calculations and database and they were in fair agreement with the standards.

Cross sections FA analysis -> 2 factors with the first factor very well represented

First factor -> is loaded by the Real part of the Volume WS potential from alpha channel

Second factor -> is loaded by the Imaginary part of the Surface WS potential from alpha channel

Third and other factors are mostly irrelevant.

Mainly small differences and high degree of correlations in cross sections energy dependences are reflected in the main featured factors

THANK YOU VERY MUCH FOR YOUR ATTENTION! ☺

Oradea – TRANSYLVANIA - ROMANIA