

DE LA RECHERCHE À L'INDUSTRIE



PRELIMINARY COVARIANCES OBTAINED WITH CONRAD/GROMACS FOR H(H₂O)

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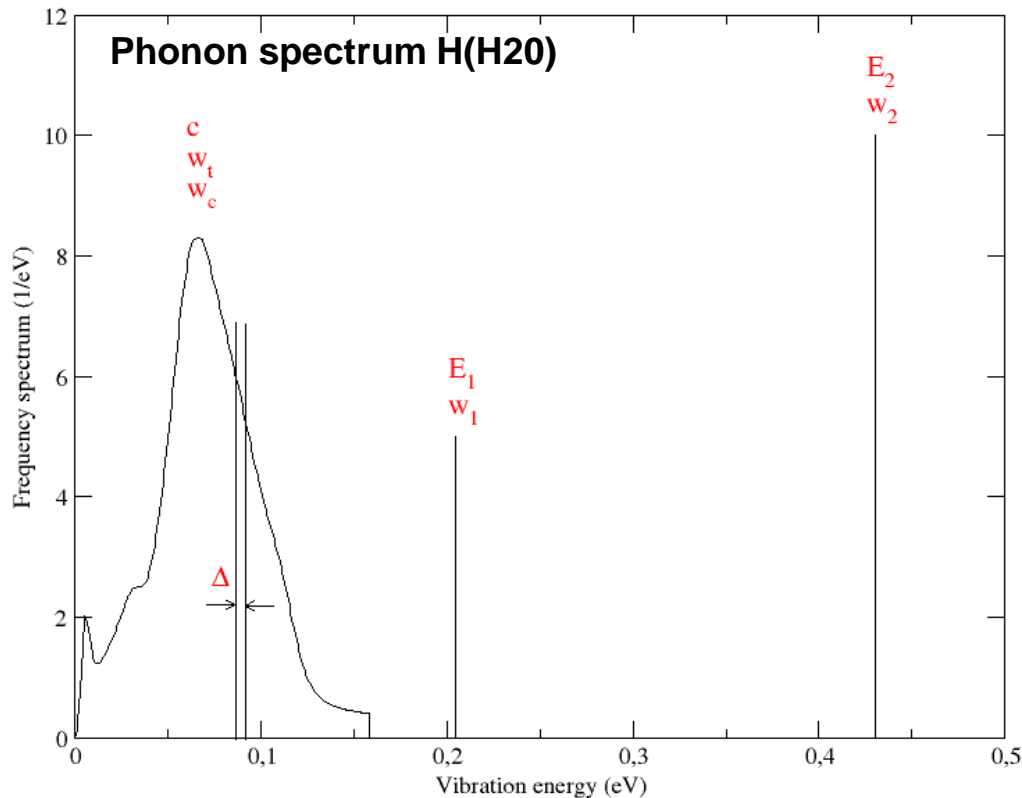
Two main objectives:

- Create covariance matrix between the **LEAPR** parameters for JEFF-3.1.1
- Create covariance matrix between the **GROMACS** parameters for the CAB model

Propagation of the TSL uncertainties to integral calculations by using:

- Direct perturbation of the model parameters
- Random $S(\alpha, \beta)$ tables (TMC calculation)
- Sensitivity calculations

The first step was to determine the covariance matrix between the LEAPR parameters



⇒ 2 main parameters were identified:

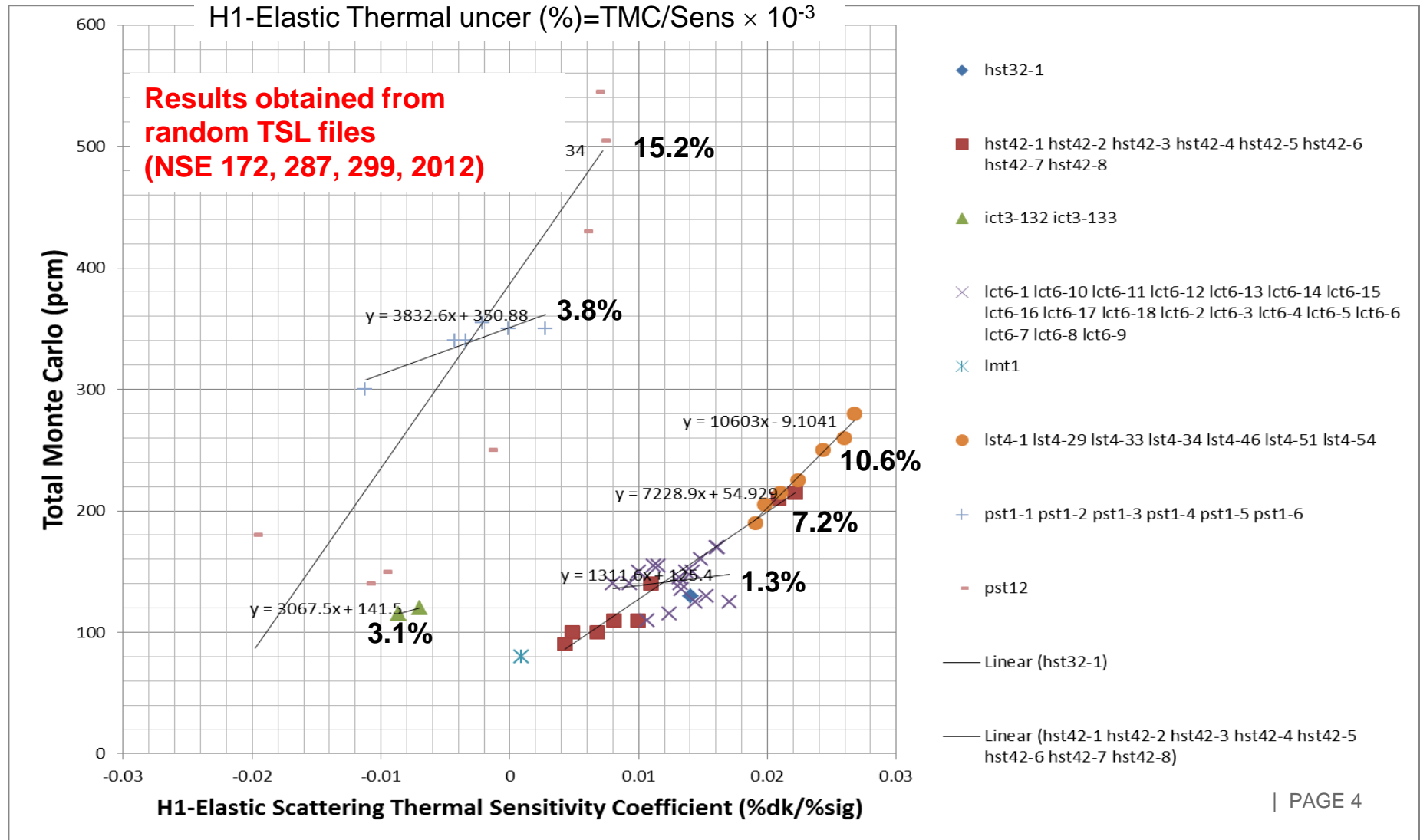
- The energy bin Δ uses to reconstruct the phonon spectrum
- The weight w_t of the translational vibration mode

<https://www.oecd-nea.org/science/wpec/sg42/>

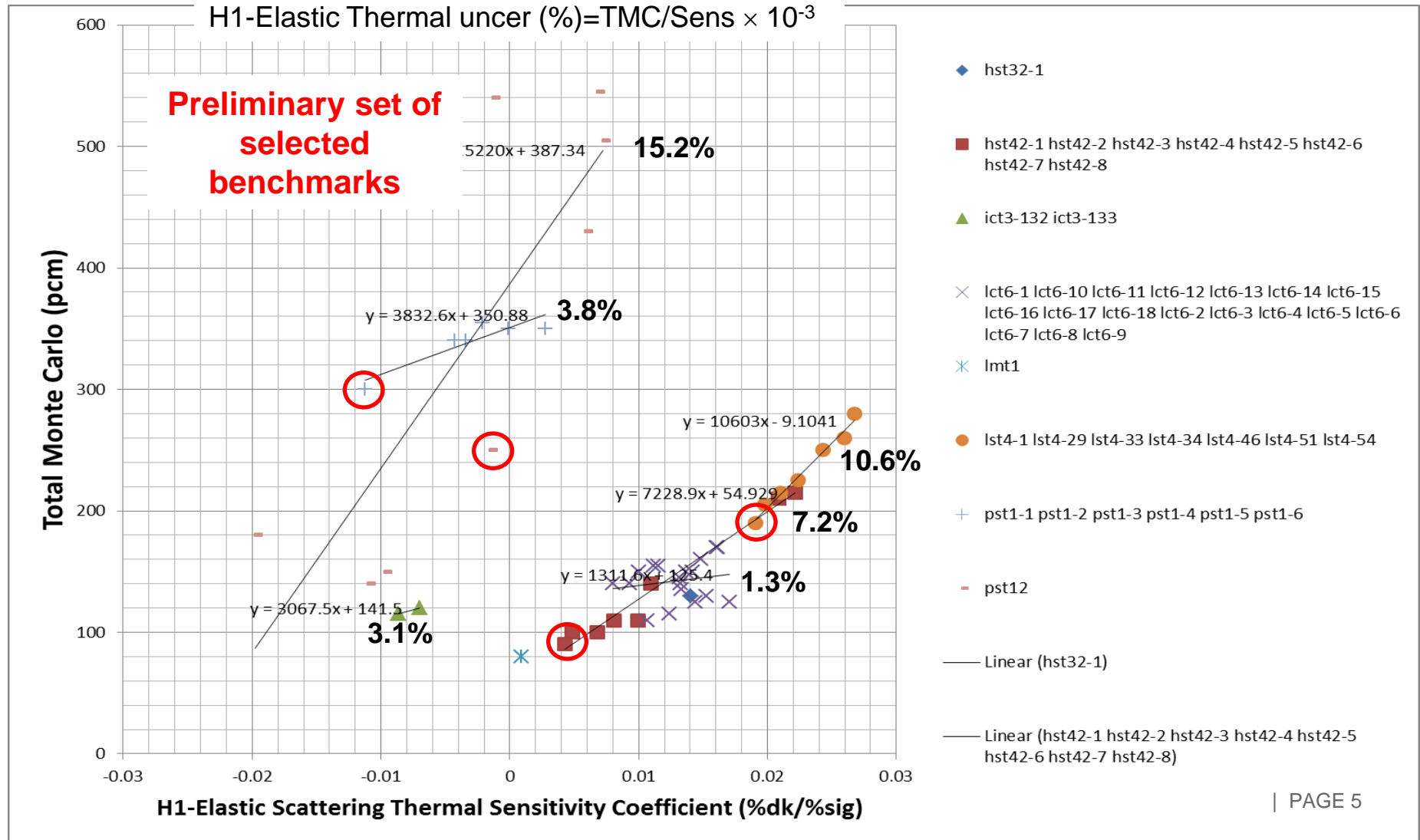
Random TSL files for TMC calculations

1000 H in H₂O files based on JEFF-311 (access to these documents is restricted to SG42 members)

TMC . vs. H1-Elastic Scattering Thermal Sensitivity Coefficient



TMC . vs. H1-Elastic Scattering Thermal Sensitivity Coefficient



Propagation of the TSL uncertainties of JEFF-3.1.1

Random files can be found in the WPEC/SG42 web site
<https://www.oecd-nea.org/science/wpec/sg42/>

Low sensitivity to H(H₂O)

| Benchmarks | EALF (eV) | EAFG (eV) | TMC | | Direct perturbation of the LEAPR parameters |
|------------------------|-----------|-----------|---------|---------|---|
| | | | SERPENT | MCNP | TRIPOLI4 |
| HEU-SOL-THERM-042-001 | 0.0320 | 0.0388 | | 35 pcm | |
| LEU-SOL-THERM-004-001 | 0.0421 | 0.0735 | | 53 pcm | 79 pcm |
| PU-SOL-THERM-012-014 | 0.0783 | 0.1440 | | 132 pcm | |
| PU-SOL-THERM-001-001 | 0.0885 | 0.1660 | 157 pcm | 160 pcm | 145 pcm |
| LEU-COMP-THERM-007-001 | 0.2870 | 0.5910 | | 110 pcm | |
| MISTRAL-2 (MOX) | - | - | | | 199 pcm |

Cf. presentation of Juan Pablo Scotta on MISTRAL-1

→ Excellent agreement between the different codes and methods

THE JOURNAL OF CHEMICAL PHYSICS 135, 224516 (2011)

A flexible model for water based on TIP4P/2005Miguel A. González and José L. F. Abascal^{a)}*Departamento de Química Física, Facultad de Ciencias Químicas, Universidad Complutense de Madrid, 28040 Madrid, Spain*

(Received 29 July 2011; accepted 2 November 2011; published online 14 December 2011)

Water potential
TIP4P/2005f

CAB model for water

Annals of Nuclear Energy 65 (2014) 280–289



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CAB models for water: A new evaluation of the thermal neutron scattering laws for light and heavy water in ENDF-6 format

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NUCLEAR SCIENCE AND ENGINEERING: 172, 164–179 (2012)

Zero Variance Penalty Model for the Generation of Covariance
Matrices in Integral Data Assimilation Problems

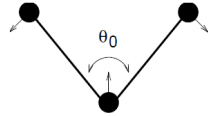
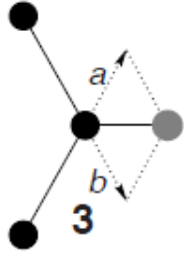
G. Noguere,* P. Archier, and C. De Saint Jean

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and

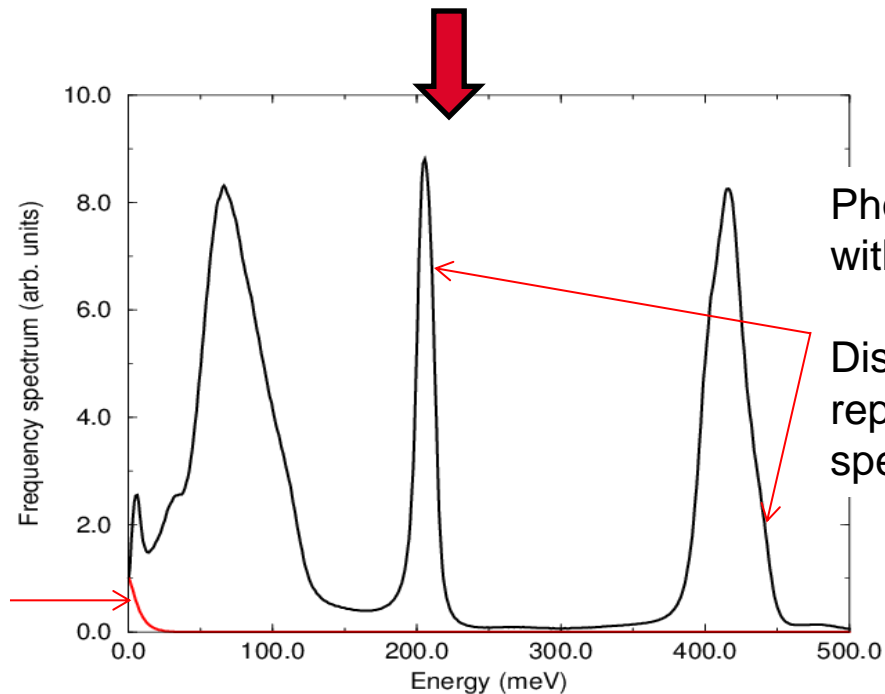
B. Habert

*CEA, DEN, DTN Cadarache
F-13108 Saint Paul les Durance, France*Zero Variance Penalty model
in CONRAD

| Contributions | GROMACS functions | Parameters |
|---------------------------|---|------------------|
| Lennard-Jones potential | $V_{LJ}(r_{ij}) = 4\epsilon_{ij} \left(\left(\frac{\sigma_{ij}}{r_{ij}} \right)^{12} - \left(\frac{\sigma_{ij}}{r_{ij}} \right)^6 \right)$ | sigma epsilon |
| Electrostatic interaction | $V_c(r_{ij}) = f \frac{q_i q_j}{\epsilon_r r_{ij}}$ | e |
| Morse potential | $V_{morse}(r_{ij}) = D_{ij} [1 - \exp(-\beta_{ij}(r_{ij} - b_{ij}))]^2$ | b D Beta |
| Harmonic angle potential | $V_a(\theta_{ijk}) = \frac{1}{2} k_{ijk}^{\theta} (\theta_{ijk} - \theta_{ijk}^0)^2$  | theta0 k |
| « dummy » atom |  | a=b |

Prior values, variances and correlations

| | | | | | | | | | | | | | | | | | | |
|---------|---|-------------|-----|-------------|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|--|--|--|--|
| sigma | = | 3.16440E-01 | +/- | 3.16440E-03 | (1.0%) | 100 | | | | | | | | | | | | |
| epsilon | = | 7.74907E-01 | +/- | 7.74907E-03 | (1.0%) | 0 | 100 | | | | | | | | | | | |
| e | = | 0.55640E+00 | +/- | 0.55640E-02 | (1.0%) | 0 | 0 | 100 | | | | | | | | | | |
| b | = | 0.09419E+00 | +/- | 0.09419E-02 | (1.0%) | 0 | 0 | 0 | 100 | | | | | | | | | |
| D | = | 432.581E+00 | +/- | 432.581E-02 | (1.0%) | 0 | 0 | 0 | 0 | 100 | | | | | | | | |
| beta | = | 22.8700E+00 | +/- | 22.8700E-02 | (1.0%) | 0 | 0 | 0 | 0 | 0 | 100 | | | | | | | |
| theta0 | = | 107.400E+00 | +/- | 107.400E-02 | (1.0%) | 0 | 0 | 0 | 0 | 0 | 0 | 100 | | | | | | |
| k | = | 367.810E+00 | +/- | 367.810E-02 | (1.0%) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100 | | | | | |
| a | = | 0.13288E+00 | +/- | 0.13288E-02 | (1.0%) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100 | | | | |



Phonon spectrum generated with the GROMACS results

Discrete oscillators are replaced by a continuous spectrum

Diffusion constant (translational mode) is deduced from the GROMACS results

Linear least-squares : « best » estimate of the GROMACS parameters are found iteratively by solving the following generic equations

$$\begin{array}{l} \text{iteration 1} \quad (M_x')^{-1} = M_x^{-1} + G^t C^{-1} G \\ \quad \quad \quad X' = X_0 + M_x' G^t C^{-1} (Y - Y_{th}(X_0)) \\ \\ \text{with} \quad \quad M_x = \text{diag}(\text{var}(x_{01}) \dots \text{var}(x_{0n})) \\ \\ \text{and} \quad \quad G_{ij} = \left. \frac{\partial y_{thi}}{\partial x_j} \right|_{X=X_0} \quad \text{for } i = 1, m \text{ and } j = 1, n \end{array}$$

$$\begin{array}{l} \text{iteration 2} \quad (M_x'')^{-1} = M_x^{-1} + G^t C^{-1} G \\ \quad \quad \quad X'' = X' + M_x'' G^t C^{-1} (Y - Y_{th}(X')) \\ \\ \text{with} \quad \quad G_{ij} = \left. \frac{\partial y_{thi}}{\partial x_j} \right|_{X=X'} \quad \text{for } i = 1, m \text{ and } j = 1, n \end{array}$$

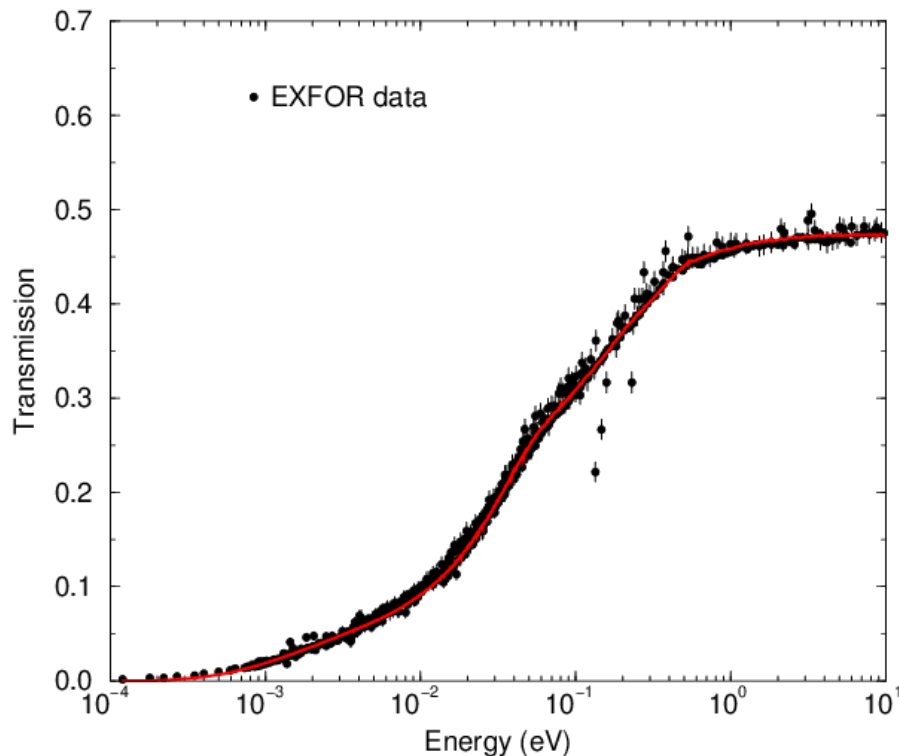
iteration 3 ...

Such a fitting model was not designed to account for uncertainties of “systematic” origin.

This problem was solved in CONRAD by introducing **marginalization techniques**

These methods account for uncertainties of the “**nuisance**” and “**latent**” model parameters

Example : H2O transmission is calculated as follows at a temperature T



$$T_{th}(E) = NT_R(E) + B(E)$$

$$T(E) = \exp\left[-n \sum_i r_i \sigma_{T,i}(E)\right]$$

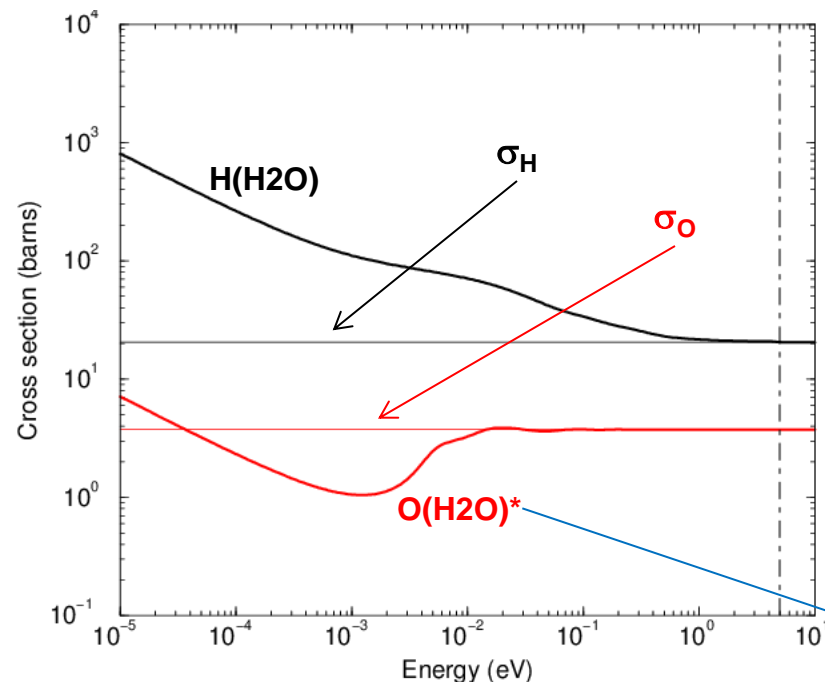
Where N is the normalisation, B a « pseudo background » and n the aerial density

+ resolution function $R_E(E')$ for double-deferential data

⇒ nuisance parameters of interest are $N, B, n, T, R_E(E')$...

Latent variables (as opposed to observable variables which can be fitted on the experimental data) may define redundant parameters or hidden variables that cannot be observed directly. This term reflects the fact that such variables are really there, but they cannot be observed or measured for practical reasons.

Example : contributions of O(H₂O) and free atom cross sections of H and O



uncertainties are
unknown for the
moment

*O(H₂O) from Jose Ignacio Marquez Damian

Latent variables (as opposed to observable variables which can be fitted on the experimental data) may define redundant parameters or hidden variables that cannot be observed directly. This term reflects the fact that such variables are really there, but they cannot be observed or measured for practical reasons.

Example : contributions of O(H₂O) and free atom cross sections of H and O

| References | σ_H | σ_O |
|-----------------------------|-----------------|-----------------|
| IKE model (JEFF-3.1.1) | 20.478 b | 3.761 |
| Atlas of Neutron Resonances | 20.491±0.014 b | 3.761±0.006 b |
| STD2006 (IAEA) | 20.436±0.041 b | |
| JEFFDOC-1488 | 20.474 ±0.012 b | 3.761 ± 0.005 b |



±0.2%



±0.2%

Latent variables (as opposed to observable variables which can be fitted on the experimental data) may define redundant parameters or hidden variables that cannot be observed directly. This term reflects the fact that such variables are really there, but they cannot be observed or measured for practical reasons.

Example : contribution of the translational weight ω_t

In the CAB model, the translational weight is deduced from the measurements performed by Novikov

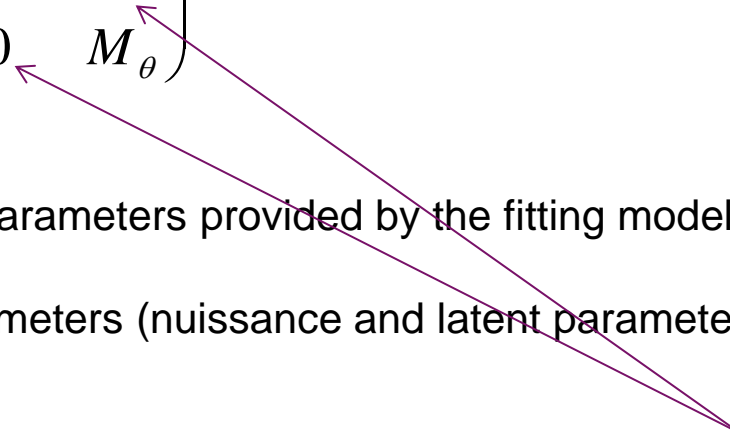
Diffusion masses for light water, from measurements by Novikov et al. (1990).

| T [K] | m_{diff}/m_{H_2O} |
|---------|---------------------|
| 300 | 6.49 |
| 400 | 2.10 |
| 500 | 2.25 |
| 600 | 1.44 |

$$\omega_t = \frac{m_H}{m_{diff}} = \frac{m_H}{m_{H_2O}} \left(\frac{m_{diff}}{m_{H_2O}} \right)^{-1}$$

⇒ a relative uncertainty of **10%** is assumed in the calculations

After the fitting procedure, the covariance matrix between the parameters μ is defined as:

$$\mu = \begin{pmatrix} x \\ \theta \end{pmatrix} \quad \Sigma = \begin{pmatrix} M_x & 0 \\ 0 & M_\theta \end{pmatrix}$$
Two purple arrows originate from the zero elements in the covariance matrix. One arrow points from the top-right zero to the text 'the fitting model' in the definition of M_x . The other arrow points from the bottom-left zero to the text 'nuisance and latent parameters' in the definition of M_θ .

M_x Covariance Matrix between the GROMACS parameters provided by the fitting model

M_θ Covariance matrix between the auxiliary parameters (nuisance and latent parameters)

⇒ under this fitting model, the cross-covariance matrix between x and θ contained only **zeros**

Non-zero elements can be calculated by using « Variance Penalty » terms:

D.W . Muir, « The contribution of individual correlated parameters to the uncertainty of integral quantities », Nucl. Inst. Meth. A 644, 55 (2011)

The model parameters and the full covariance matrix has to be partitioned as:

$$\mu = \begin{pmatrix} x \\ \theta \end{pmatrix} \quad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

By definition, the observable parameters x are the « **passive** » parameters and the latent parameters θ are the « **active** » parameters

An estimation of Σ can be obtained by adding two covariance matrices

$$\Sigma = M_x + H_\theta$$

The « statistical » contribution M_x can be estimated via a fitting model and the « systematic » contribution H_θ can be obtained analytically by introducing **variance penalty terms** $P(z, \theta)$

$P(z, \theta)$ is a measure of the contribution of the uncertainty of the latent and nuisance variables to the variance of a given calculated quantity z . By definition

$$P(z, \theta) = GH_{\theta}G^T$$

The covariance matrix H_{θ} has the following form

$$H_{\theta} = \begin{pmatrix} M_{x,\theta}M_{\theta}^{-1}M_{x,\theta}^T & M_{x,\theta}^T \\ M_{x,\theta} & M_{\theta} \end{pmatrix}$$

For a vector quantity z of general dimension k , the derivative matrix $G=(G_x, G_{\theta})$ of the quantity z to the parameters x and θ is defined as:

$$G_x = \begin{pmatrix} \frac{\partial z_1}{\partial x_1} & \cdots & \frac{\partial z_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial z_k}{\partial x_1} & \cdots & \frac{\partial z_k}{\partial x_n} \end{pmatrix} \quad G_{\theta} = \begin{pmatrix} \frac{\partial z_1}{\partial \theta_1} & \cdots & \frac{\partial z_1}{\partial \theta_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial z_k}{\partial \theta_1} & \cdots & \frac{\partial z_k}{\partial \theta_m} \end{pmatrix}$$

By using the partitioned form of G , we have

$$P(z, \theta) = G_x M_{x,\theta} M_\theta^{-1} M_{x,\theta}^T G_x^T + G_x M_{x,\theta} G_\theta^T + \left(G_x M_{x,\theta} G_\theta^T \right)^T + G_\theta M_\theta G_\theta^T = 0$$

The « zero variance penalty » condition lead to

$$M_{x,\theta} = -(G_x^T G_x)^{-1} G_x^T G_\theta M_\theta$$

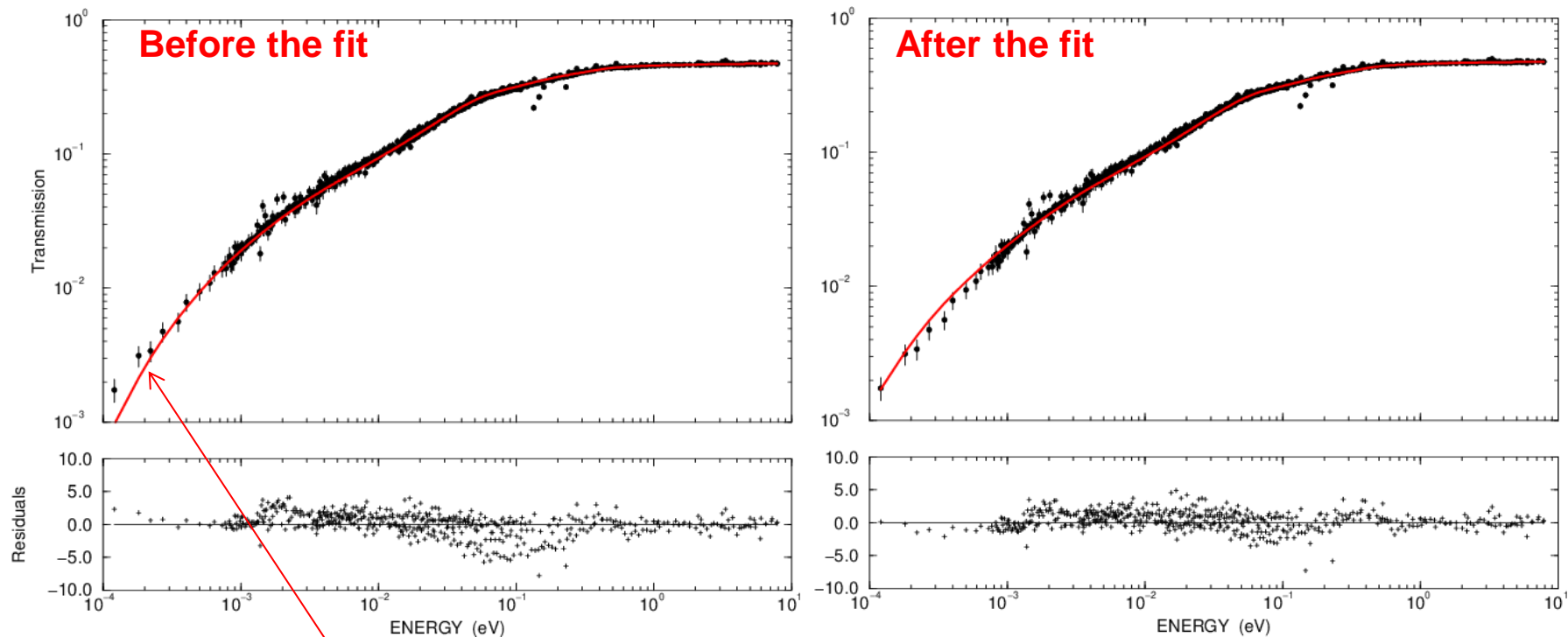
According to the definition of Σ , we have

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \quad \begin{cases} \Sigma_{11} = M_x + (G_x^T G_x)^{-1} G_x^T G_\theta M_\theta G_\theta^T G_x (G_x^T G_x)^{-1} \\ \Sigma_{12} = -(G_x^T G_x)^{-1} G_x^T G_\theta M_\theta \\ \Sigma_{22} = M_\theta \end{cases}$$

Covariance matrix between the parameters:

- GROMACS parameters
- Experimental parameters
- LEAPR parameters

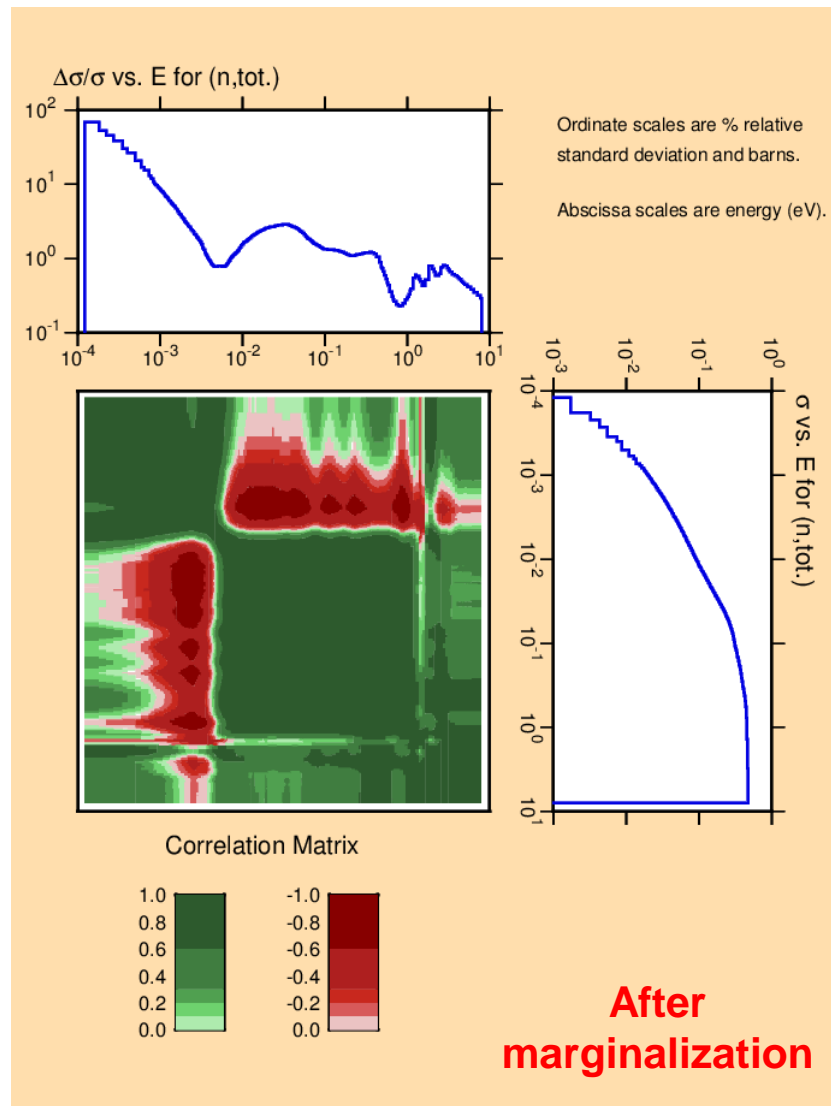
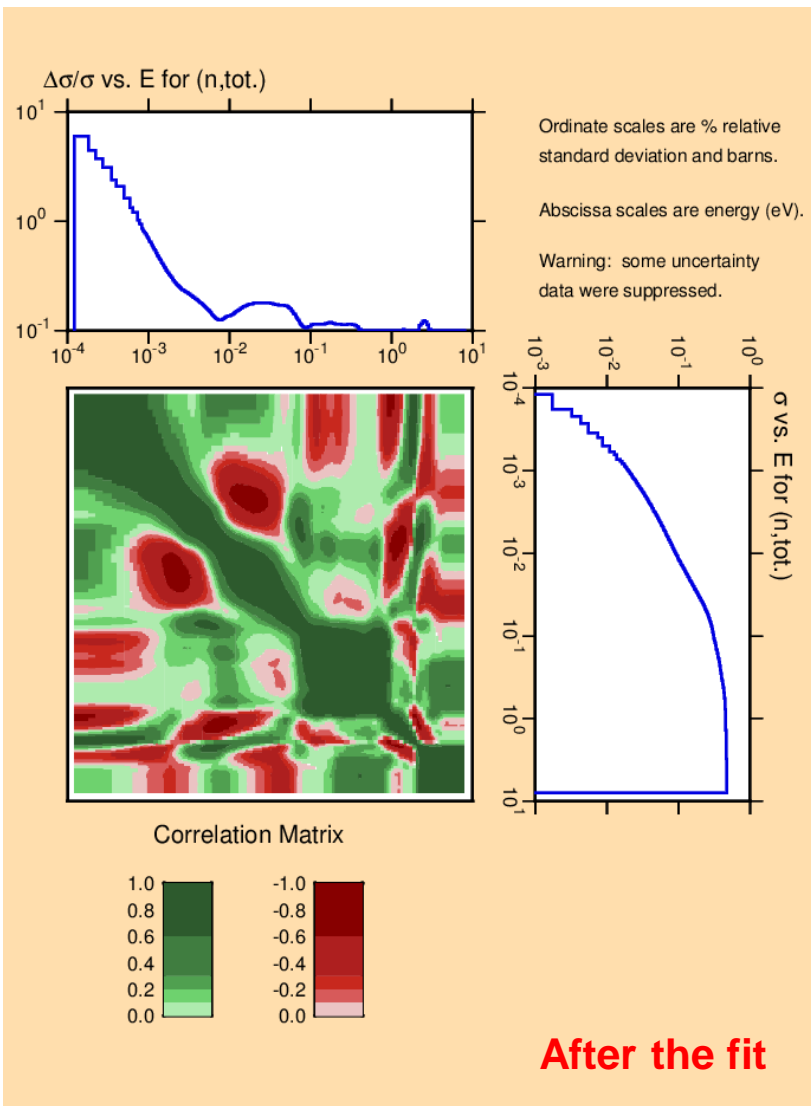
Two-step CONRAD calculation for producing Model Parameter Covariance Matrix for GROMACS



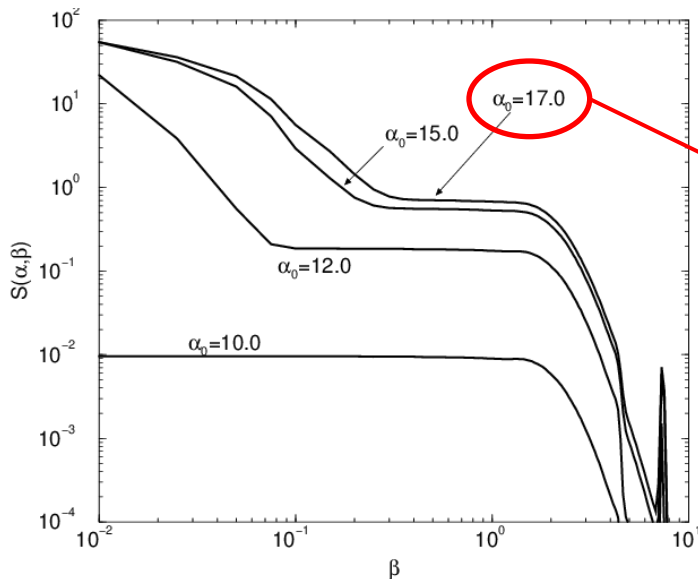
prior model parameters from
CAB MODEL

Prior and posterior model parameters
are nearly equivalent

Relative uncertainties and correlation matrix for H2O (transmission)



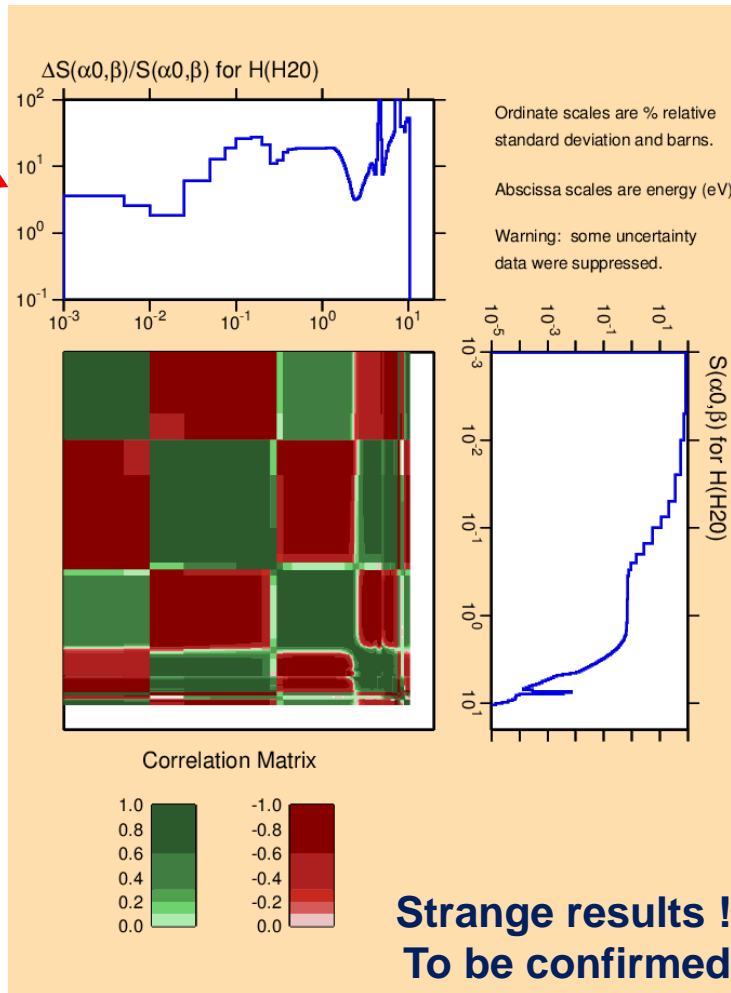
Relative uncertainties and correlation matrix on $S(\alpha, \beta)$



Calculated with the Model
Parameter Covariance Matrix
obtained after marginalization



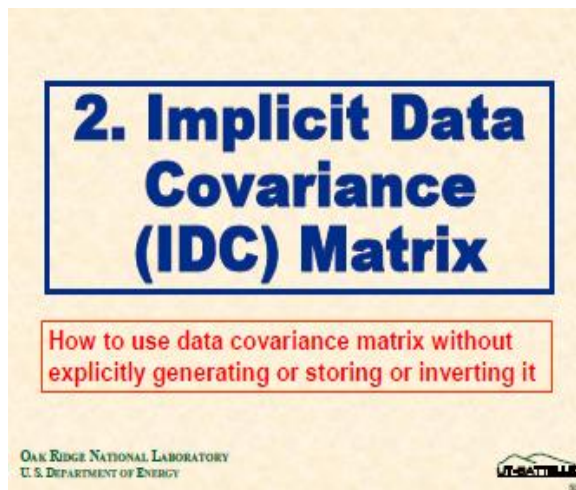
**How to store large covariance
matrix for $S(\alpha, \beta)$?**



**Strange results !
To be confirmed**

IDC : a solution to store large Resonance-Parameter Covariance Matrix

« A concise method for storing and communicating the Experimental Covariance Matrix », N.M. Larson, ORNL/TM-2008/104, 2008



- Available in SAMMY and CONRAD to read AGS covariance file (used at the IRMM)
- Cf. WPEC/SG-36 «Evaluation of experimental data in the resolved resonance region»

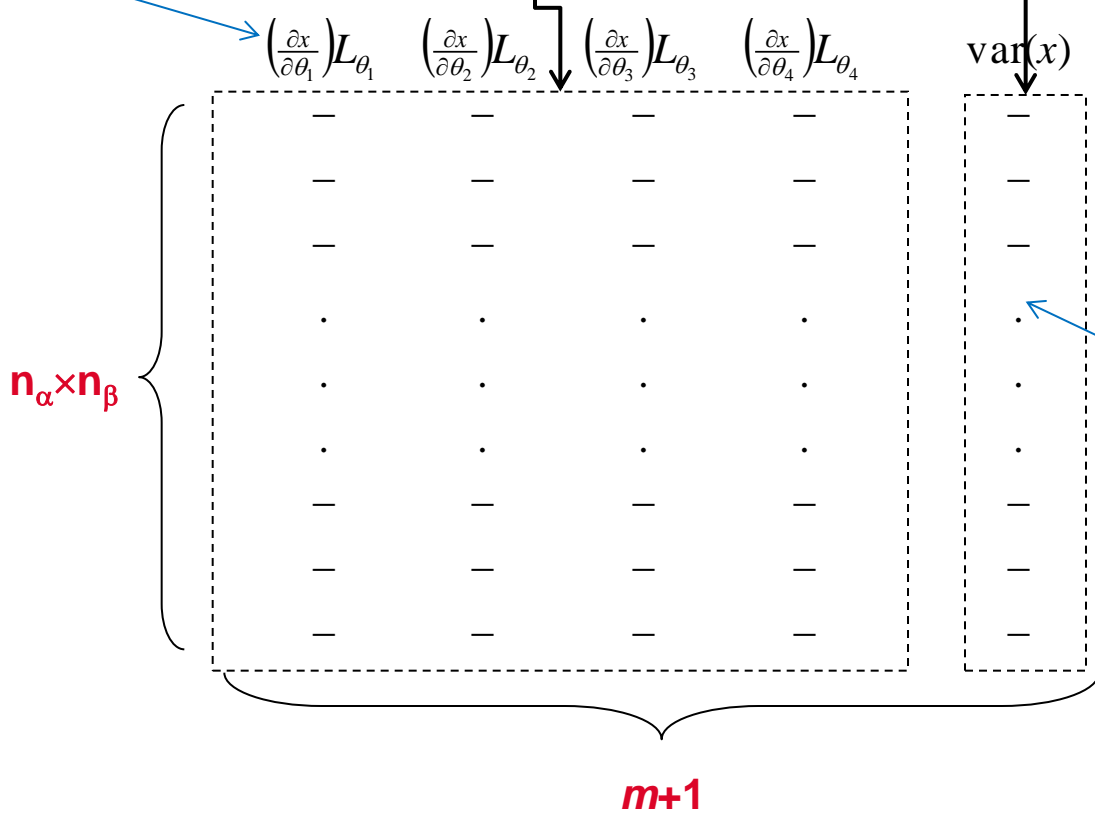
⇒ IDC format can be also used to store covariance matrix for $S(\alpha, \beta)$

IDC format to store large covariance matrix

Derivatives of $S(\alpha, \beta)$ to the nuisance parameters introduced in the marginalization procedure

$$D = S_{\theta} S_{\theta}^T + M$$

diagonal matrix



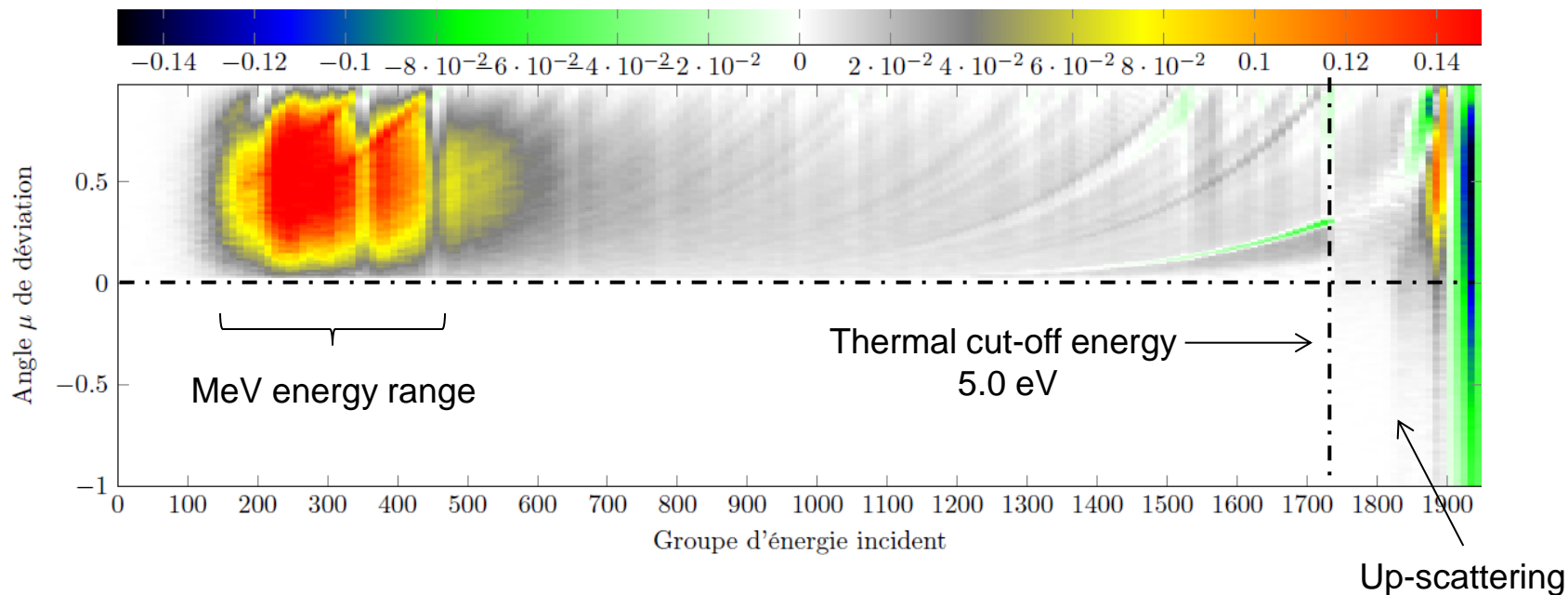
Variance on $S(\alpha, \beta)$ from the fitting procedure
Correlations from the fit are neglected

For a given integral quantity z , the variance is calculated as follow:

$$\text{Var}(z) = SDS^T$$

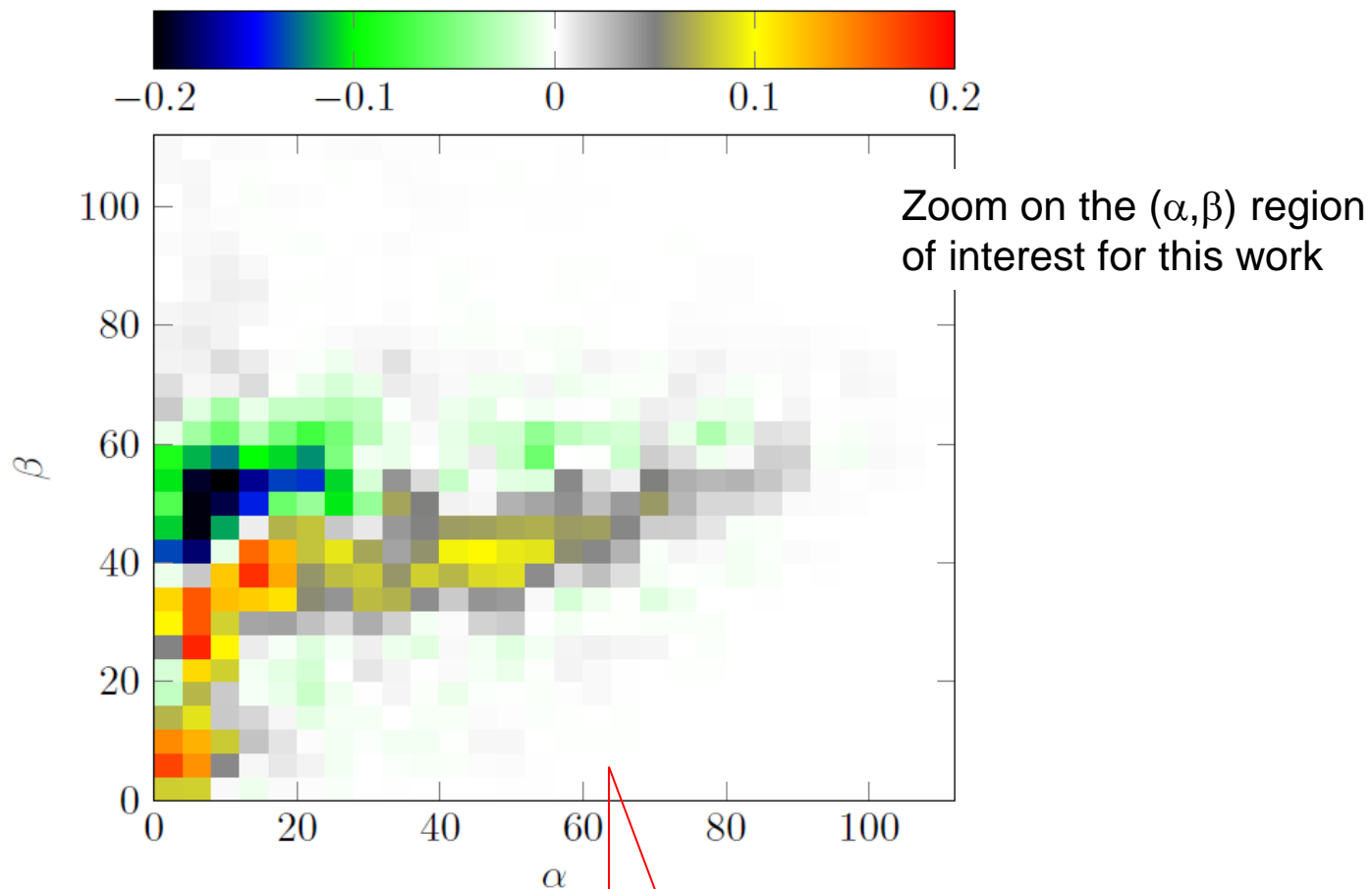
Where D is the covariance matrix for $S(\alpha, \beta)$ and S contains the derivatives of z to $S(\alpha, \beta)$

Exemple: Sensitivity to the elastic scattering of H(H₂O) in **PST001.1** as a function of the incident energy and μ obtained with TRIPOLI4-IFP (PhD thesis G.Truchet, CEA Cadarache)



Propagation of TSL uncertainties to integral calculations

Exemple: Sensitivity to $S(\alpha,\beta)$ of $H(H_2O)$ in **PST001.1** obtained with TRIPOLI4-IFP (PhD thesis G.Truchet, CEA Cadarache)



⇒ Final calculations $\text{Var}(z) = \mathbf{SDS}^T$ not yet performed !

Present results on TSL uncertainties

- ☑ **CONRAD** can be use to produce Model Parameter Covariance Matrix for LEAPR
- ☑ **CONRAD** can be use to produce Model Parameter Covariance Matrix for GROMACS
- ☑ Production of large covariance matrix for $S(\alpha, \beta)$ is possible with the **IDC formalism**
- ☑ **TRIPOLI4-IFP** can be used to propagate TSL uncertainties to integral data

Next steps

- ☐ Include double-differential and mu-bar data in the fitting procedure
- ☐ Generate random files for the CAB model
- ☐ Finalize the propagation of the uncertainties with TRIPOLI4-IFP

Acknowledgment

Special thanks to Jose Ignacio Marquez Damian for his help on using GROMACS and for the fruitful discussions and advices.