Of Rocket Science, Finance, and Nuclear Data: REWIND (Ranking Experiments by Weighting for Improved Nuclear Data)

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Introduction

- **This work is still under development, and therefore there are not yet final results but still many questions to be answered.**
- **Last time a new methodology was proposed, PIA (Progressive Incremental Adjustment); however, the proposed hierarchy for the progressive use of experiments was based on expert judgment more than a scientific sound basis.**
- **This work tries to establish a methodology for ranking experiments by looking at the potential gain they can produce in an adjustment.**
- **This work interconnects the calculation of rocket trajectories, money investment strategies, and nuclear data assimilation.**

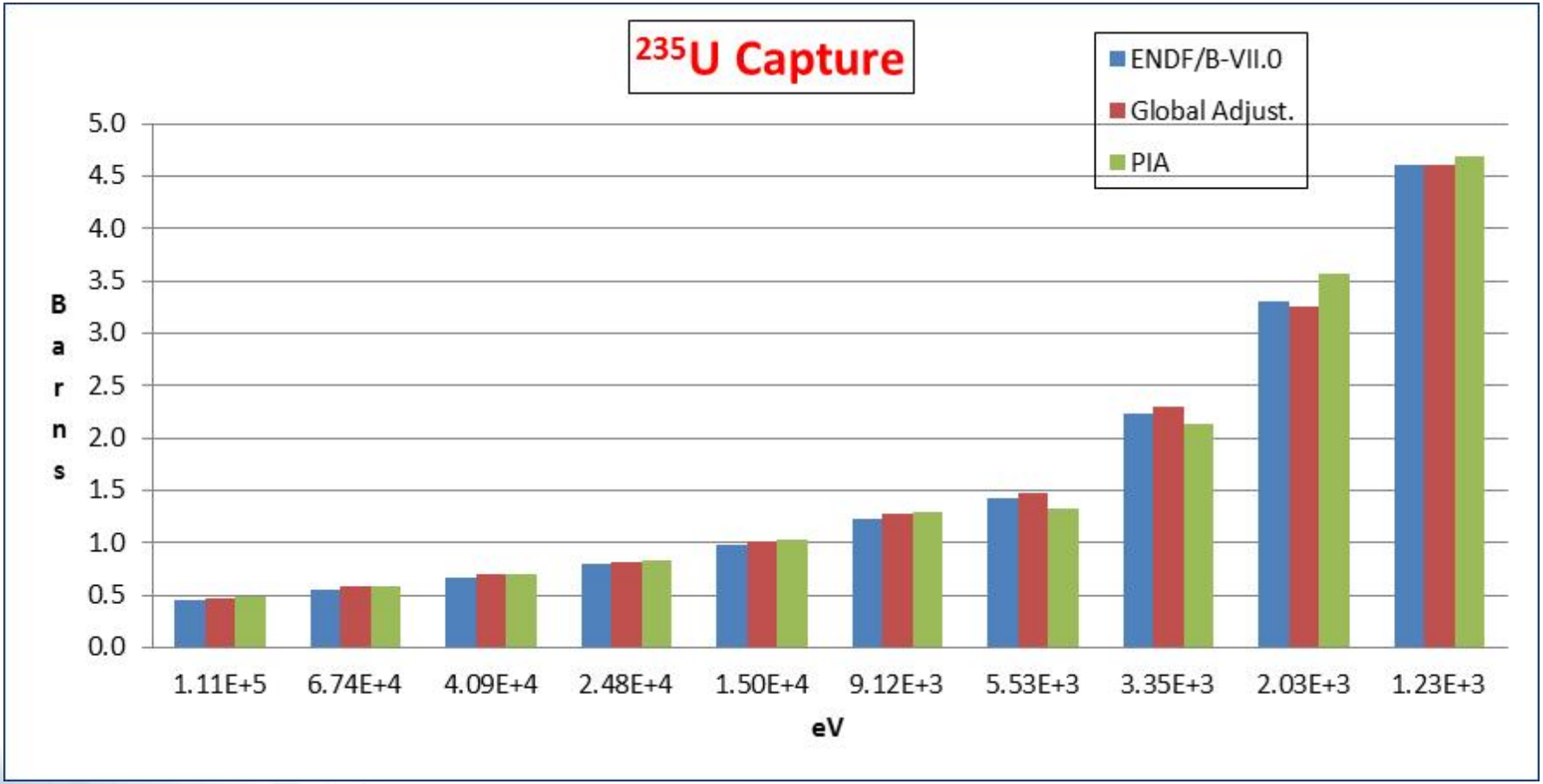
PIA (Progressive Incremental Adjustment)

- The proposed approach for avoiding compensations and give more reliable feedback to evaluator is to perform a progressive incremental adjustment.
- In PIA the starting point is giving priority to the utilization of experiments of elemental type (those sensitive to a specific cross section), following a definite hierarchy on which type of experiment to use (see next slide).
- Once the adjustment is performed, both the new adjusted data and the new covariance matrix are kept. This will limit the range of variability of the adjusted cross sections.
- In the final steps integral experiments that are sensitive to a large variety of cross sections (global type like critical mass) are added.

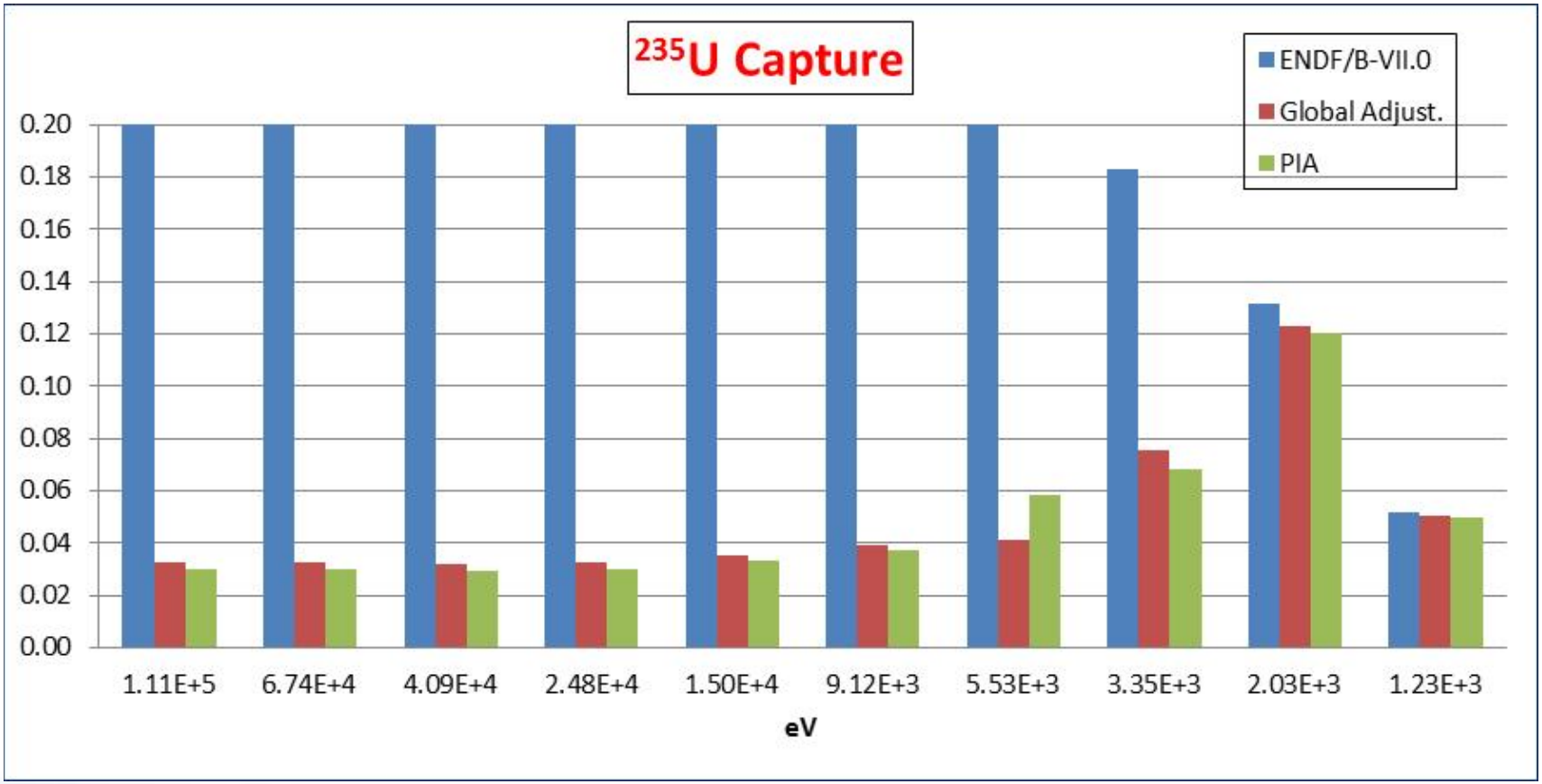
PIA Experiment Hierarchy

- **For actinides:**
  1. **Fission spectral indices: sensitive to fission cross sections (but also to inelastic and fission spectrum, in the case of threshold fission cross sections)**
  2. **Irradiation experiments: sensitive to capture cross sections (and second order to fission) and (n,2n)**
  3. **Sample oscillation experiments and other experiment sensitive to inelastic (e. g. transmission, flat/steep adjoint as in STEK and SEG)**
  4. **Critical masses**
  5. **Reactivity variations (both reactivity coefficients and associated to fissile isotope variations in the same core geometry)**
- **For structural materials:**
  1. **Propagation experiments (inelastic and elastic)**
  2. **Sample oscillations (add capture)**
  3. **Critical masses**
  4. **Reactivity variations (e.g. k-infinity type experiments, sodium void, control rods)**

σ change comparison PIA against Global



Standard deviation change comparison PIA against Global



Main Results

- A new adjustment strategy has been proposed, PIA (Progressive Incremental Adjustment), where priority is given to elemental type experiments in order to cope with the problem of compensations.
- An exercise applied to the previously presented ENDF/B-VII.0 (Global) adjustment has shown that, if we trust the elemental experiments, compensations occur in integral type of experiments (e. g. critical masses).
- Moreover, PIA indicates some significant impact on both central values and standard deviations.
- The new covariance matrix obtained by PIA produces significantly reduced uncertainty on target reactors.

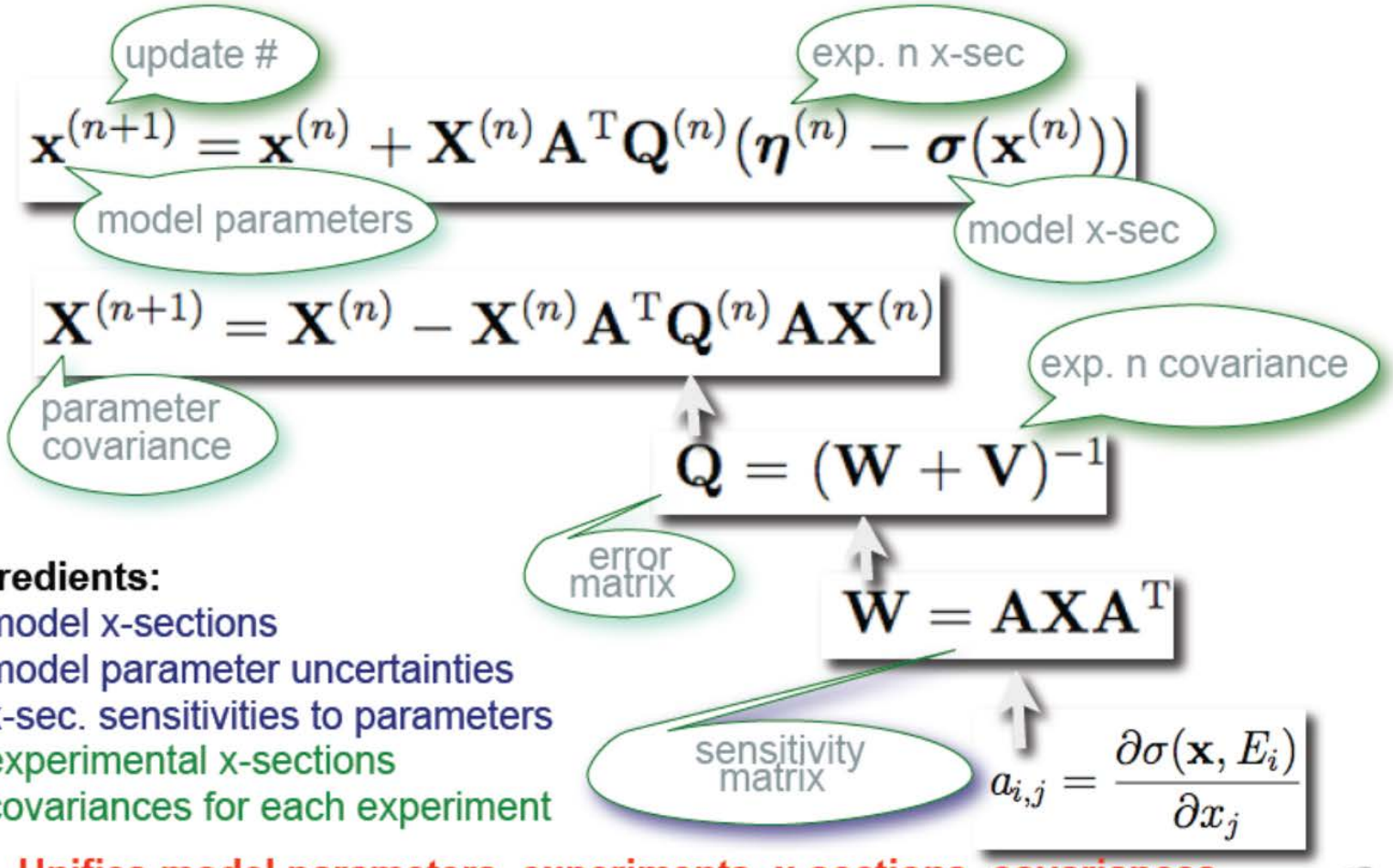
Kalman Filtering and Rocket Trajectories

- Rudolf E. Kálmán proposed the formulation of the filtering technique, also known as linear quadratic estimation (LQE), in 1960 (there are claims that others proposed earlier, 1958, the same formulation).
- The formulation is practically identical to the GLSM that we currently apply for our data adjustment, but applied only to one measurement.
- Kalman filtering is commonly applied in his original form, which is progressive, by nuclear data evaluators.
- It was during a visit by Kalman to NASA that he saw the applicability of his ideas to the problem of trajectory estimation for the Apollo program, leading to its incorporation in the Apollo navigation computer.
- At the time analogue computers were used, relying on a model with parameters that were progressively improved by measuring the actual rocket position.

From M. Herman BNL

Fast neutron region

Kalman: Bayesian, Generalized Least Squares approach

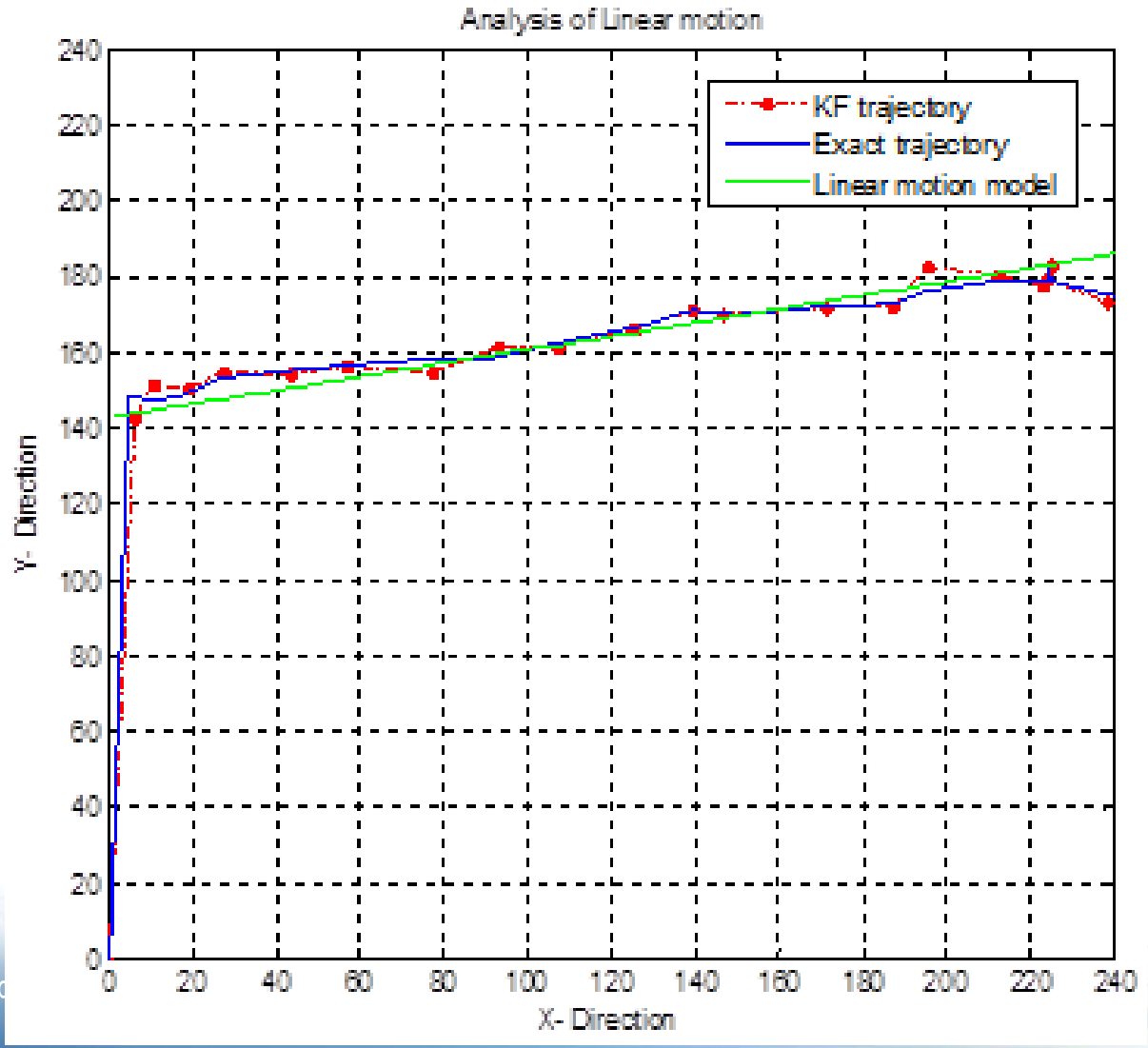


Ingredients:

- model x-sections
- model parameter uncertainties
- x-sec. sensitivities to parameters
- experimental x-sections
- covariances for each experiment

Unifies model parameters, experiments, x-sections, covariances

Trajectory Correction with Kalman Filtering



Progressive Adjustment Issue

- The Kalman filtering technique is naturally progressive because it is applied to a time series (including measurements), and, therefore, time dictates the progressive adjustment.
- It is not clear to me how the nuclear data evaluators prioritize the measurements to be used in a successive way in the Kalman filtering.
- As the PIA exercise has shown, the succession of experiments to be used in the progressive adjustment will impact significantly the final results both in terms of central values and a posteriori covariance.
- I have done an attempt to rank experiments by using a technique I have developed for optimizing portfolios of investment assets.

MPT (Modern Portfolio Theory)

- **MPT defines an investment strategy that seeks to find the optimal portfolio of assets that, for a given return, minimizes the standard deviation or conversely maximizes the return for a given standard deviation. These pairs create a set called the efficient frontier, from which an optimal portfolio is selected given the amount of risk the investor is willing to assume.**
- **In MPT the standard deviation of a portfolio characterize the volatility, and in turn its risk. More volatile (higher standard deviation) is the portfolio, riskier is the portfolio.**
- **MPT formalizes the concept of diversification, which is habitually employed in asset selection to reduce the volatility of a portfolio. This can be achieved by selecting assets that are not correlated, or better yet negatively correlated, which results in a portfolio with a lower volatility than any of the individual assets selected.**

Sharpe Ratio Optimization

- We construct a portfolio with *N* assets, and for each asset *i* there is a return r_i^y, where *y* indicates the period of the return (e. g. year). The return of the portfolio is the weighted sum of the individual asset of the returns (sum of the weights equal to one).
- Given the time distribution of the returns the standard deviation of each return can be calculated using the classical statistical formulation:

$$SD_i = \sqrt{\frac{\sum_{j=1}^{NY}(r_i^j - r_i^\mu)^2}{NY-1}}$$

- The portfolio correlation matrix C_p is a symmetric matrix containing 1 on the diagonal and on the off-diagonal in row *i* column *k* cor_{ik}, calculated using again the classical formula:

$$cor_{ik} = \frac{\sum_{j=1}^{NY}((r_i^j - r_i^\mu)(r_k^j - r_k^\mu)}{(NY-1)SD_iSD_k}$$

Sharpe Ratio Optimization

Using the correlation matrix the portfolio covariance matrix D_p can be calculated in a matrix form as:

$$D_p = SD\ C_p SD^T$$

where SD is a square matrix containing all SD_i on the diagonal and zero everywhere else. Finally, the portfolio standard deviation SD_p can be calculated in matrix form as:

$$SD_p = wD_p w^T$$

where w and w^T are column and row vectors containing the asset weights, respectively.

We have proposed a new quantity, Cor_p, to characterize the internal correlation of the portfolio. We start from the complete correlated and uncorrelated portfolio standard deviations:

$$SD_p^{cc} = \sum_{i=1}^{N} w_i SD_i \qquad SD_p^{cu} = \sqrt{\sum_{i=1}^{N} (w_i SD_i)^2}$$

Sharpe Ratio Optimization

Then we can calculate, approximately the internal portfolio correlation as:

$$Cor_p = \frac{SD_p - SD_p^{cu}}{SD_p^{cc}}$$

the smaller Cor_p is, the more diversified the portfolio.

Let us now introduce the risk adjusted performance measure. This measure, the Sharpe Ratio, is often mentioned as "Reward to Variability". It is a way of ranking portfolios inside the efficient frontier described in MPT by return per unit of risk assumed. It is defined as:

$$SR_p = \frac{r_p^a - r_{fr}^a}{SD_p}$$

Where r_p^a is the mean annual portfolio return, r_{fr}^a is the mean annual risk-free interest rate (generally associated with cash). Note that this is very similar to the information ratio (mean over the standard deviation of a series of measurements).

In order to find the optimal asset weights, the nonlinear optimization process maximizes the Sharpe Ratio with the constraint on the sum of the weights being equal to one.

Application to Integral Experiments: REWIND

Let's consider the set of integral experiments we have as a "portfolio" of assets and calculate the optimal weights that maximize the portfolio "Sharpe Ratio". The asset (experiment) return will be different following the application for which the adjustment is intended.

First, let's define some attributes of the portfolio. The covariance of the portfolio is calculated as:

$$D_p = SD\ C_p SD^T$$

Where SD is the experiment standard deviation and C_p is the correlation among experiments and are calculated using the usual formulation with sensitivity coefficients and cross section covariance.

$$C_{E'E} = \frac{(S_{E'} M_\sigma S_E)}{[(S_{E'} M_\sigma S_{E'})(S_E M_\sigma S_E)]^{1/2}}$$

The standard portfolio standard deviation is then:

$$SD_p = wD_p w^T$$

While the internal portfolio correlation is defined as:

$$Cor_p = \frac{SD_p - SD_p^{cu}}{SD_p^{cc}}$$

Application to Integral Experiments: REWIND

Now let's define the Sharpe Ratio for the case we want to find the optimal experiment weights for improving the information we want on a set of isotopes like those of CIELO. In this case the return of each asset (experiment) is the potential gain an experiment can produce by reducing the uncertainty obtained by the usual sandwich formula limited to the isotopes under consideration. However, to this we have to subtract the experimental uncertainty U_i (both from measurement and calculation). Similarly, the portfolio standard deviations is calculated using only the sensitivity coefficients and covariance data of the isotopes under considerations.

$$SR_p = \frac{\sum_{i=1}^{N} w_i (SD_i - U_i)}{SD_p}$$

Note that the Sharpe Ratio for each experiment, defined in this way, is very similar to the Ishikawa factor. In fact the Sharpe Ratio equal to zero corresponds to the Ishikawa factor equal to 1. Positive Sharpe ratio is what we want from an experiment (corresponding to the Ishikawa factor greater than one). The optimization process maximizes this portfolio Sharpe Ratio in order to find the optimal weights, and, therefore we will obtain a ranking of the experiments. Subsequently we can use this ranking to apply a progressive adjustment like in PIA. Note that the optimization process will reward experiments that are not correlated.

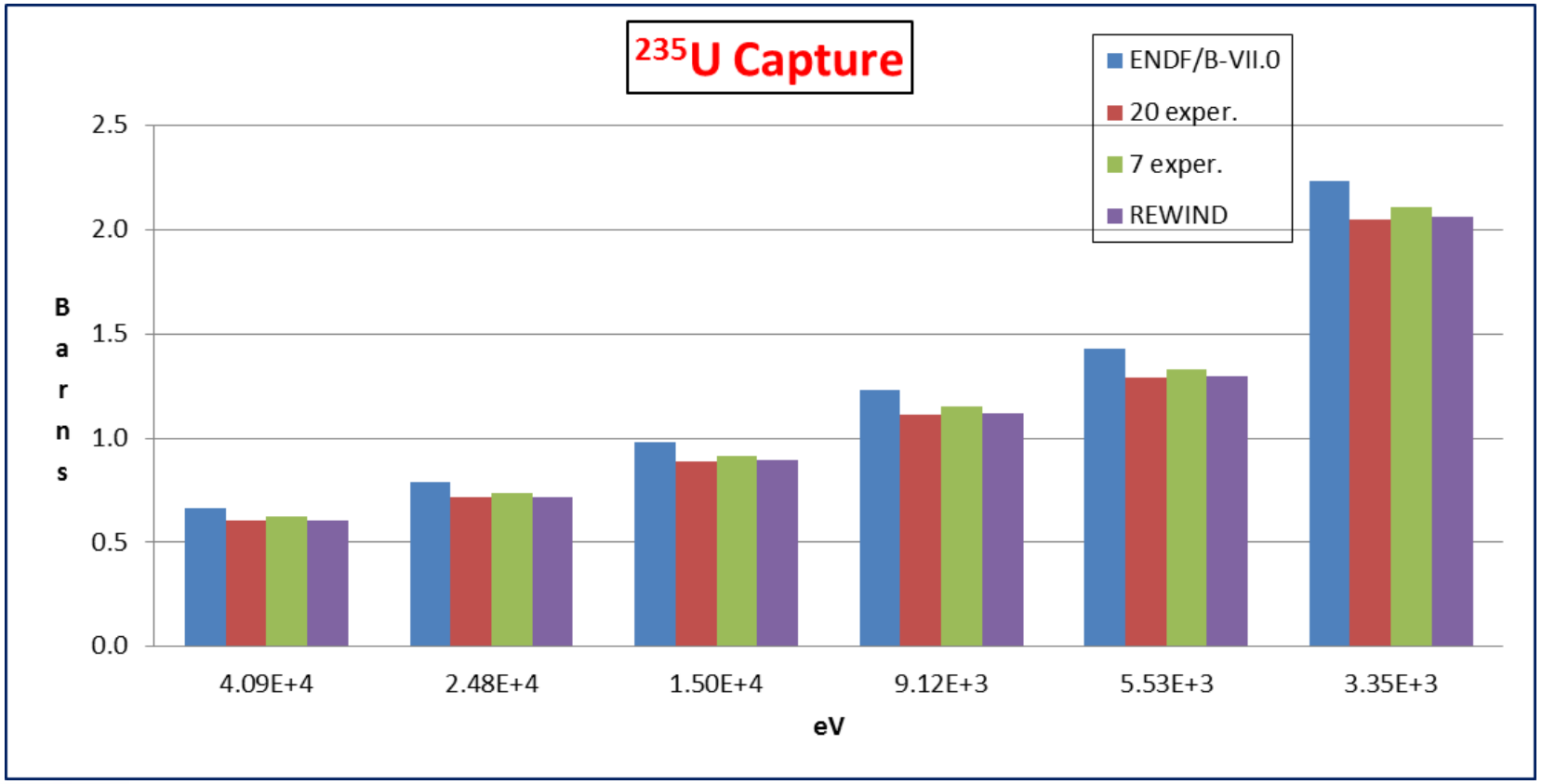
If, instead, the adjustment is targeting a specific reactor design, the experiment return (gain) will be defined in the Sharpe Ratio as the reduction of uncertainty obtained using the representativity factor. We will not see this application in the presentation.

REWIND applied to SG33 set of experiments and 5 Isotopes: ^{23}Na, ^{56}Fe, ^{235}U, ^{238}U, ^{239}Pu

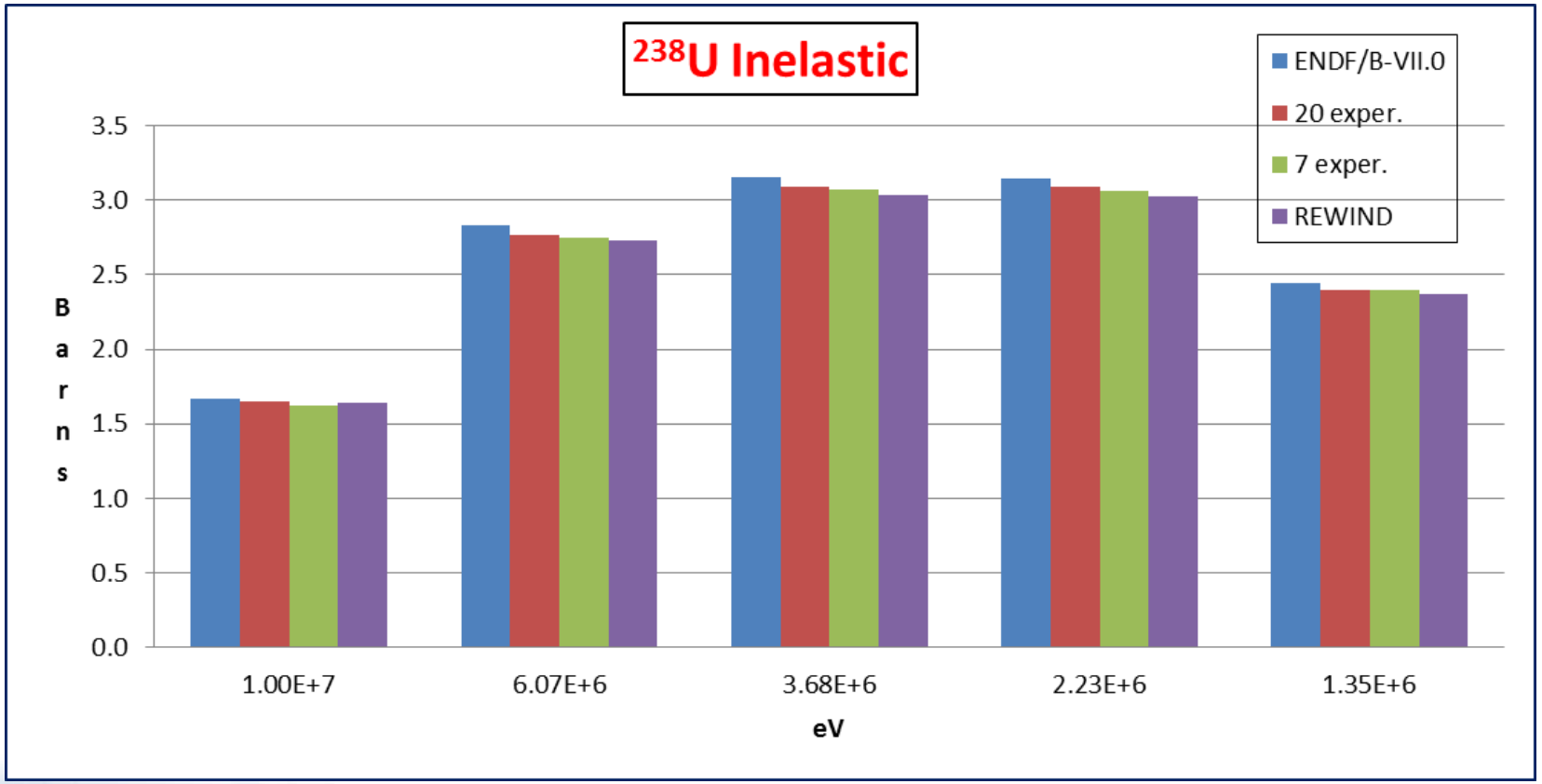
Experiment	Optimal weight %	Rank	Exp. Return %	Sharpe Ratio	Ishikawa Factor	Uncert. before adjust. %	Uncert. after adjust. %
JEZ_Pu239 KEFF	27.8	2	0.45	0.69	1.50	0.30	0.15
JEZ_Pu239 F28/F25	3.4	6	2.26	0.61	1.18	1.68	0.90
JEZ_Pu239 F37/F25	5.0	5	0.91	0.39	0.71	1.02	0.64
JEZ_Pu239 F49/F25	0.0	8	-0.13	-0.15	0.85	0.80	0.53
JEZ_Pu240 KEFF	0.0	8	0.29	0.59	2.44	0.49	0.18
FLATTOP KEFF	38.1	1	0.56	0.65	0.92	0.28	0.16
FLATTOP F28/F25	0.0	8	1.22	0.40	0.84	1.56	0.84
FLATTOP F37/F25	0.0	8	0.60	0.30	0.69	0.98	0.63
ZPR6/7 KEFF	0.0	8	0.76	0.77	1.84	0.42	0.12
ZPR6/7 F28/F25	0.0	8	2.97	0.46	0.63	2.19	1.41
ZPR6/7 F49/F25	0.0	8	-1.70	-2.07	0.29	0.72	0.57
ZPR6/7 C28/F25	0.0	8	-1.17	-0.78	0.47	1.26	0.90
ZPR6/7 PU40 KEFF	0.0	8	0.77	0.78	1.92	0.42	0.12
ZPPR9 KEFF	7.5	4	1.10	0.90	3.83	0.45	0.11
ZPPR9 F28/F25	3.3	7	5.10	0.64	0.81	2.37	1.53
ZPPR9 F49/F25	0.0	8	-1.26	-1.47	0.34	0.72	0.56
ZPPR9 C28/F25	0.0	8	-0.45	-0.29	0.64	1.27	0.90
ZPPR9 STEP3	0.0	8	-0.18	-0.02	0.70	5.44	3.93
ZPPR9 STEP5	0.0	8	2.26	0.23	0.91	6.87	4.88
JOYO KEFF	15.0	3	0.70	0.79	1.67	0.30	0.14

Experiment Portfolio Internal Correlation: -0.02

REWIND applied to SG33 set of experiments σ



REWIND applied to SG33 set of experiments σ



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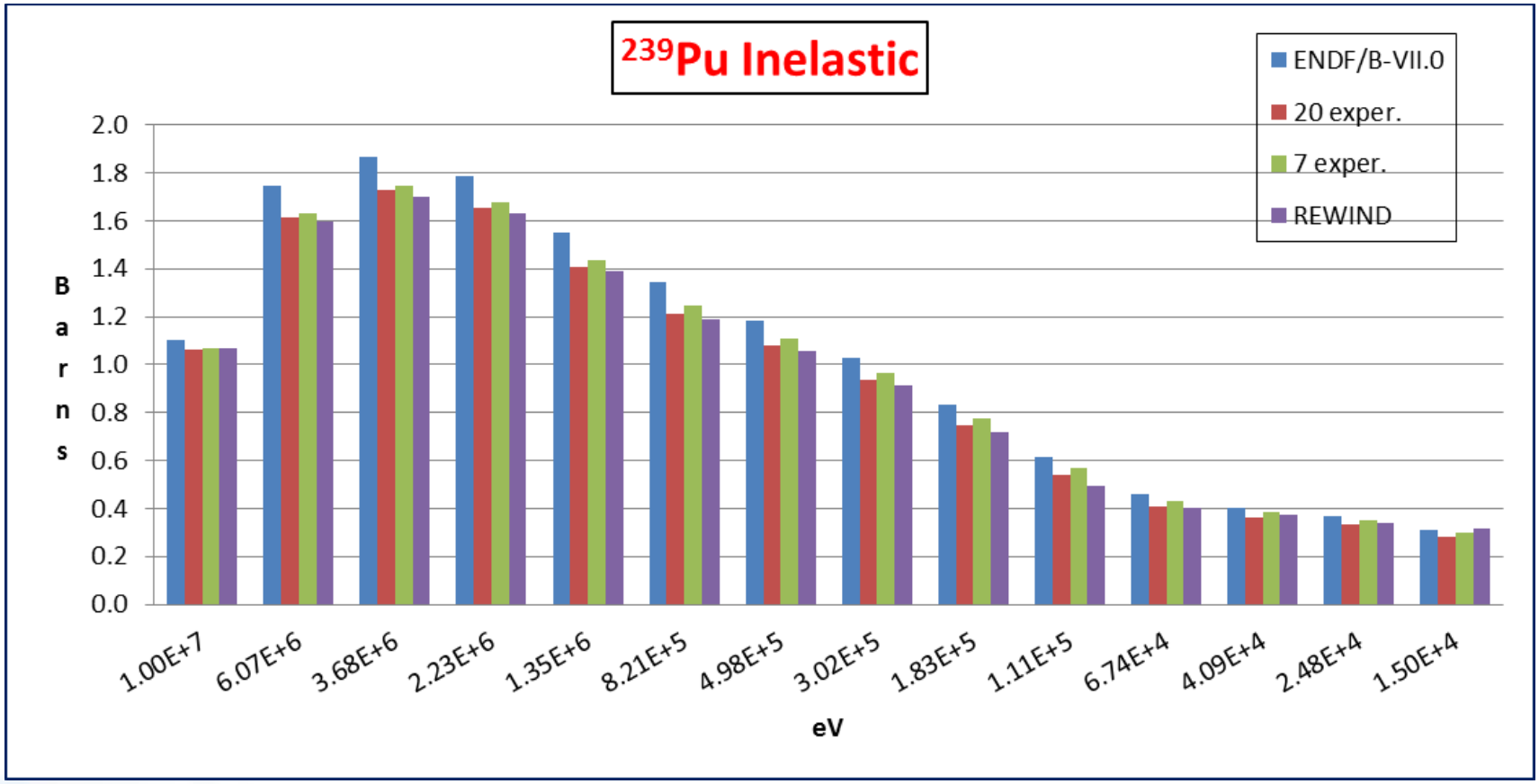
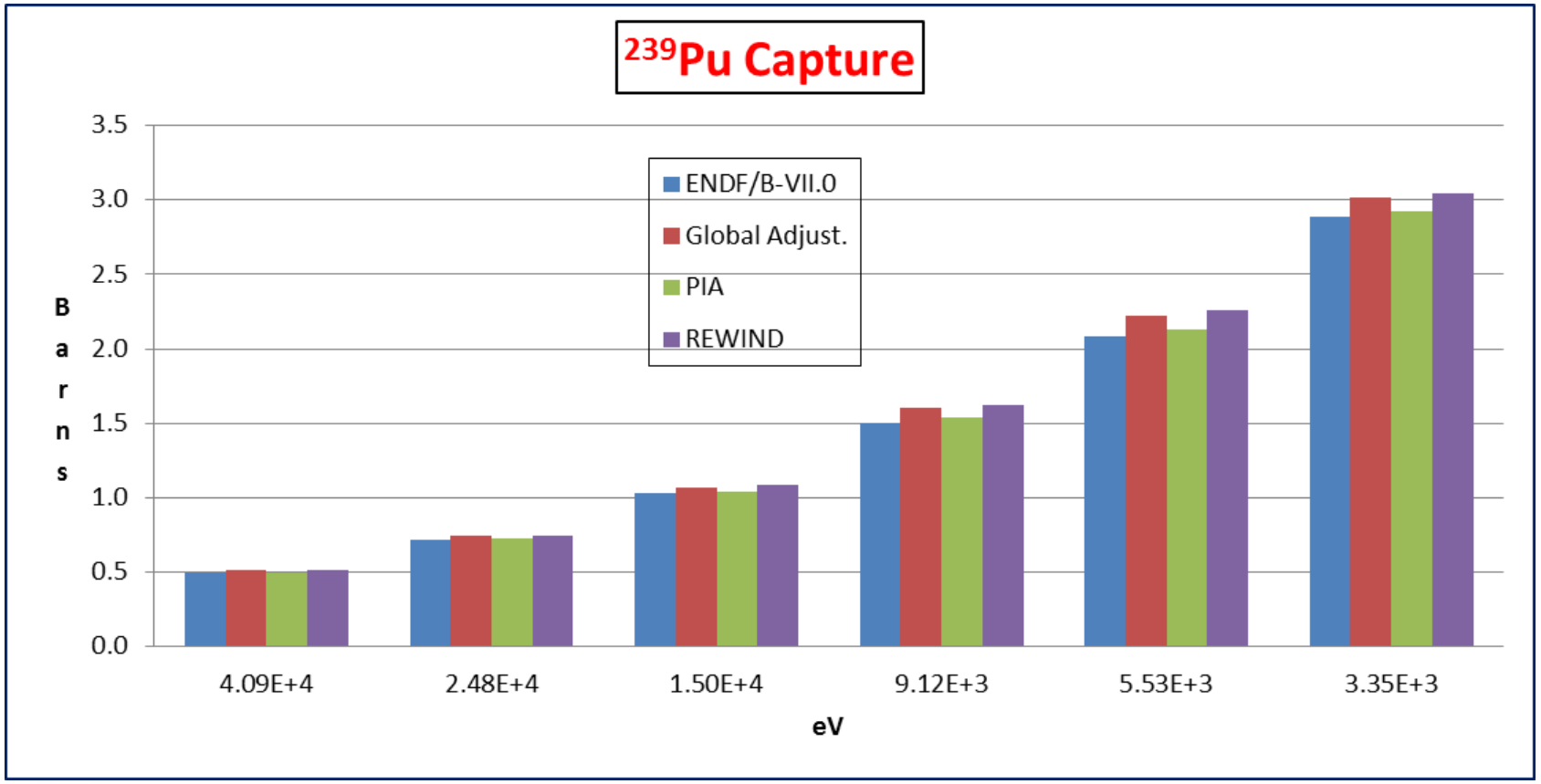
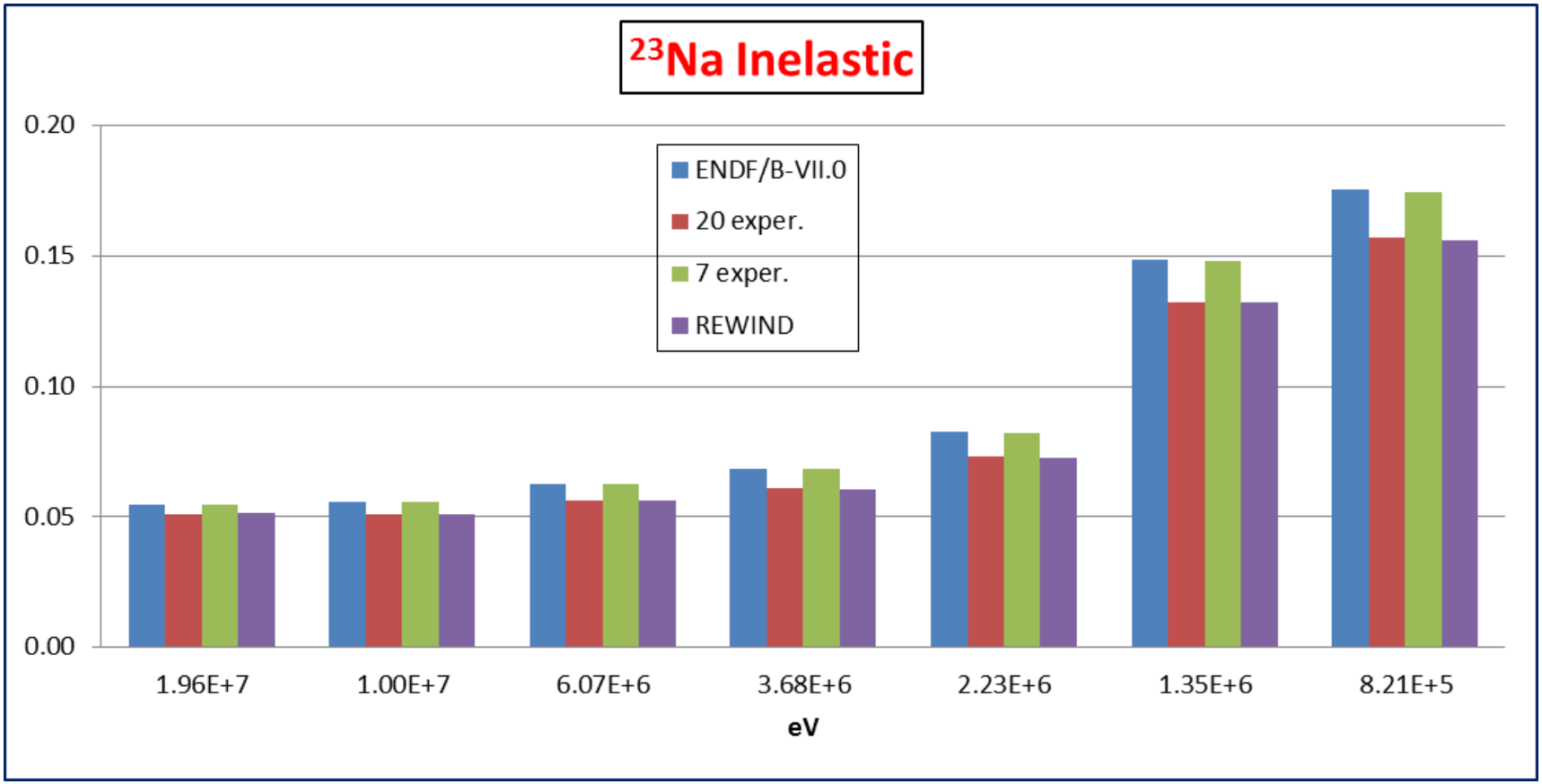


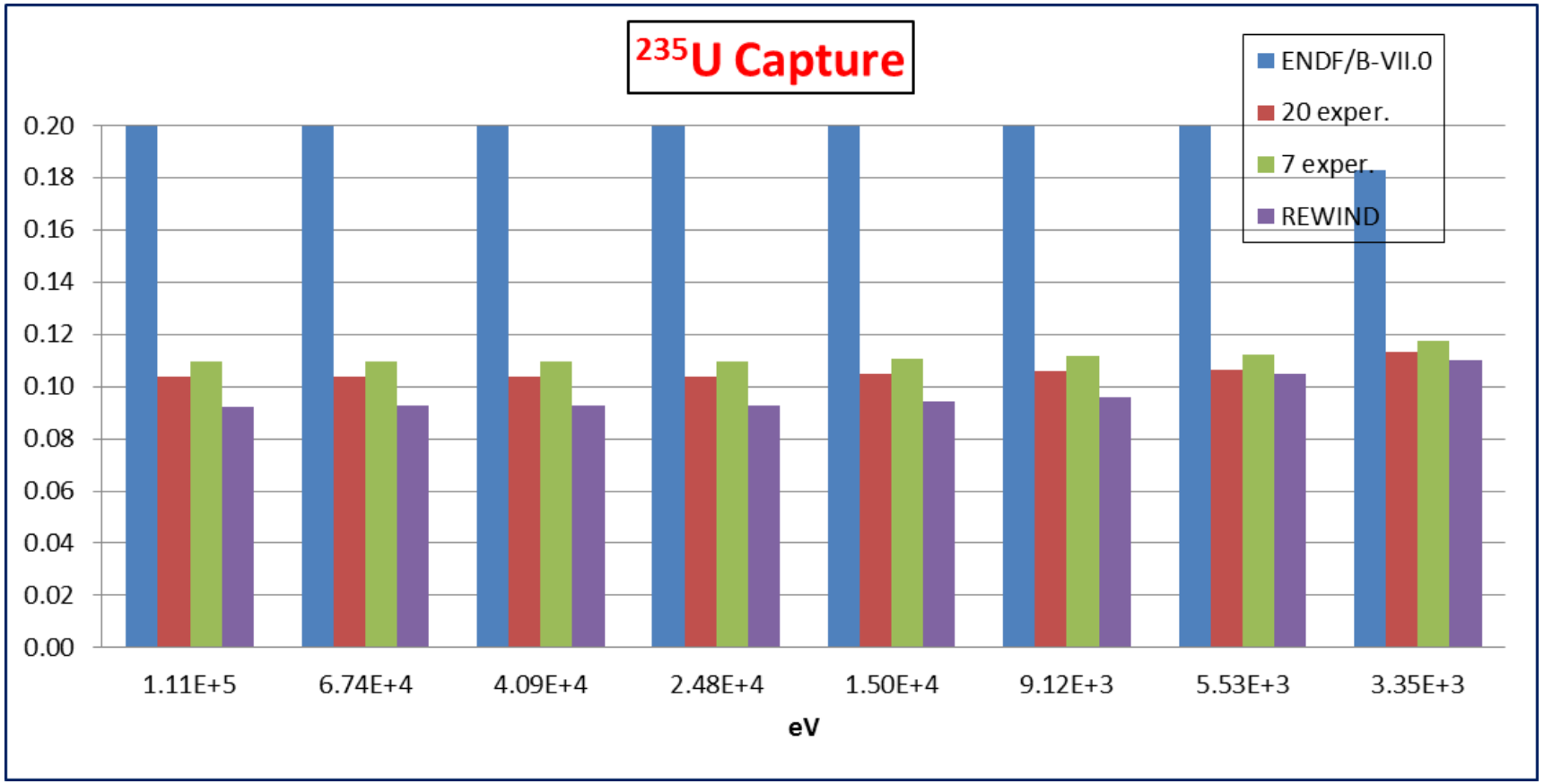
REWIND applied to SG33 set of experiments σ



REWIND applied to SG33 set of experiments Standard Deviation



REWIND applied to SG33 set of experiments Standard Deviation



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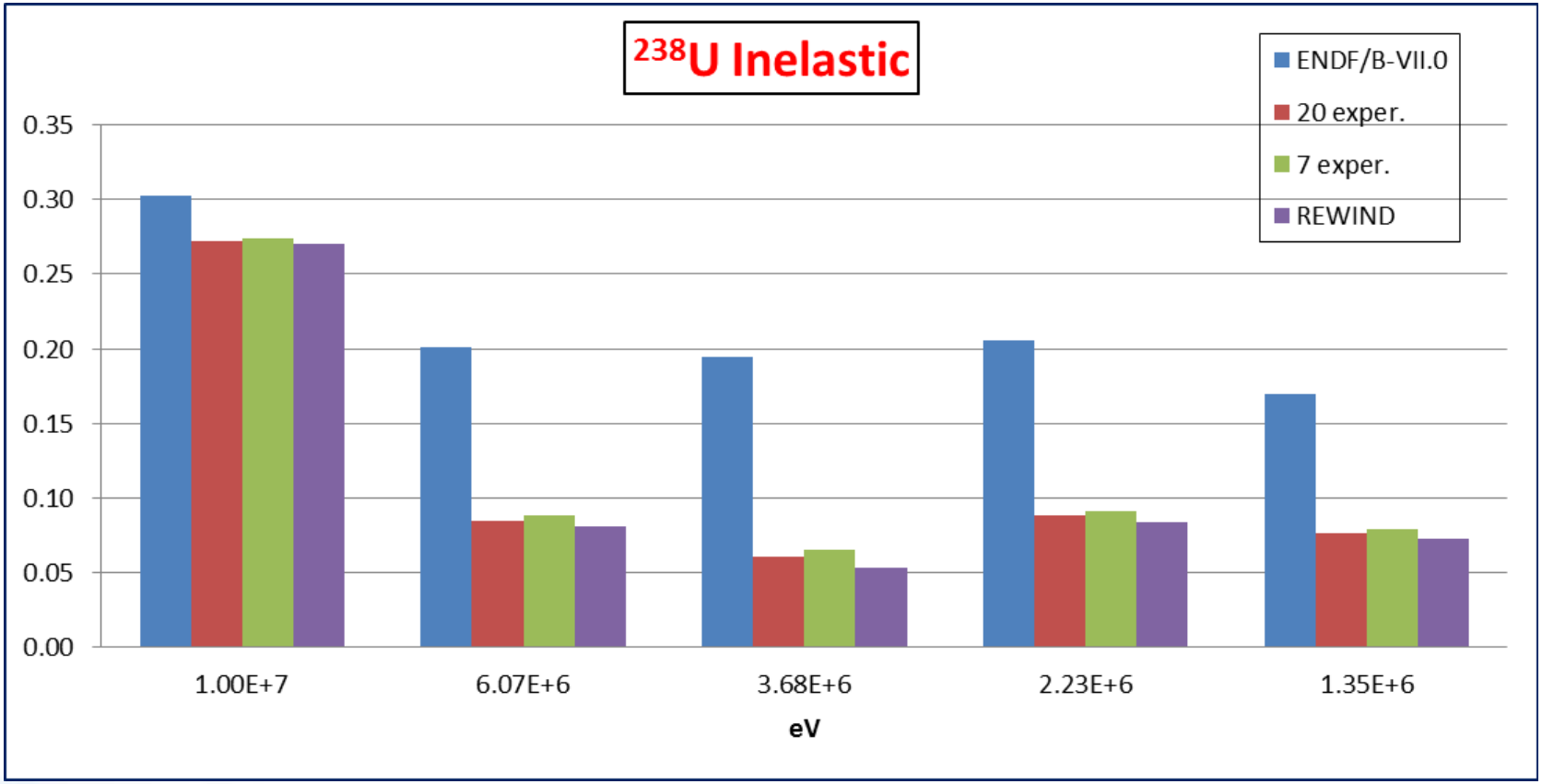

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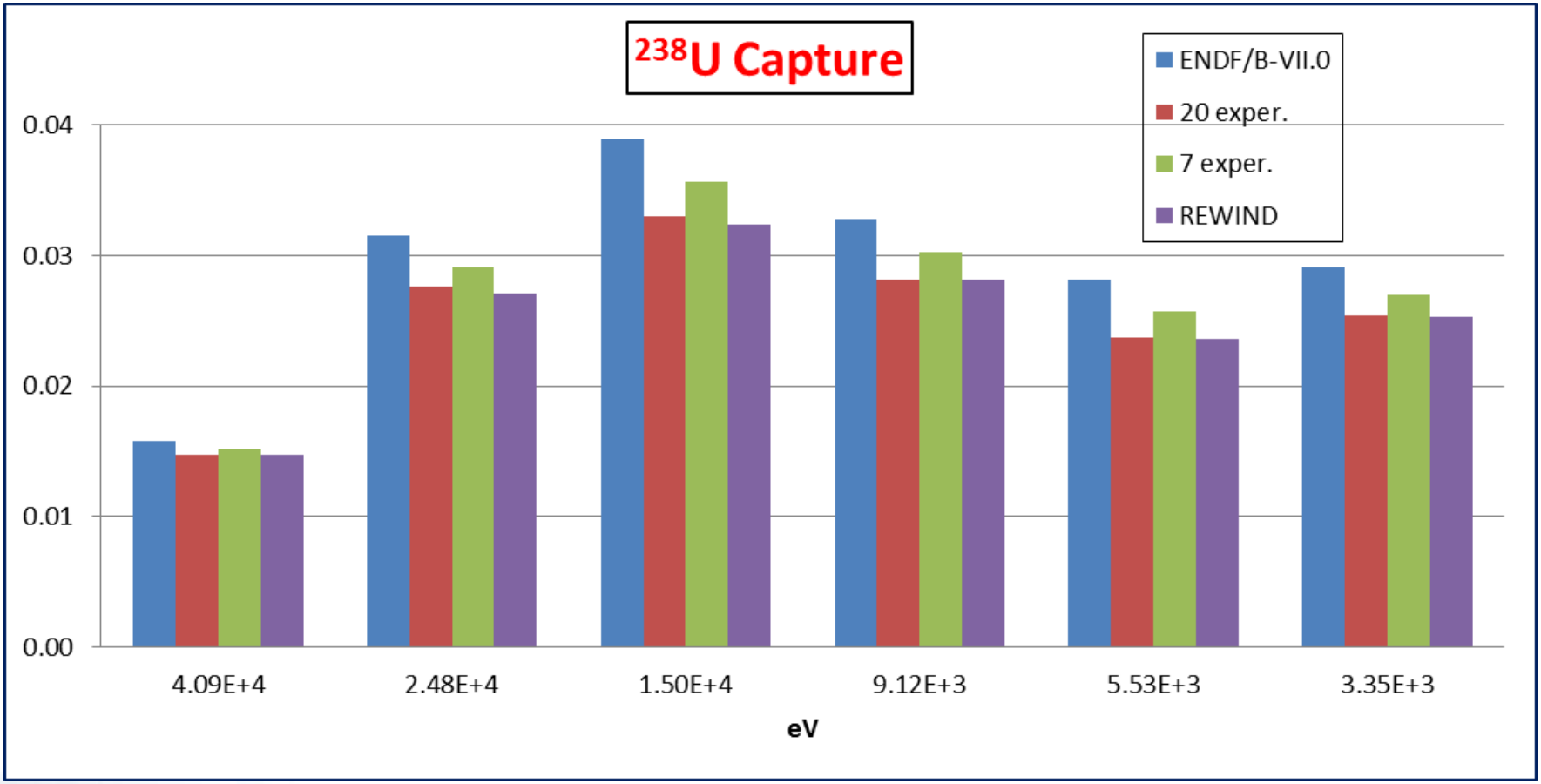

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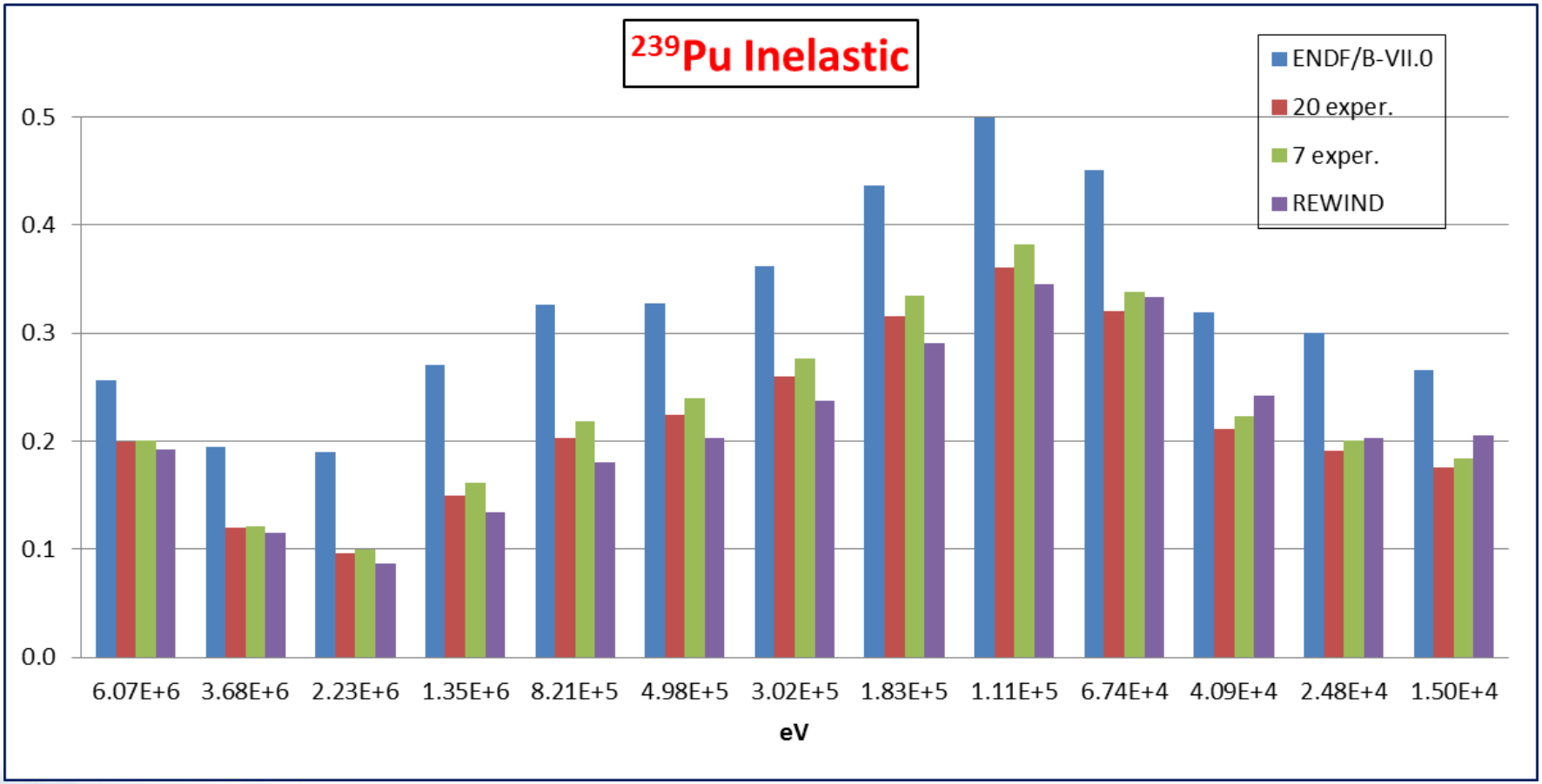

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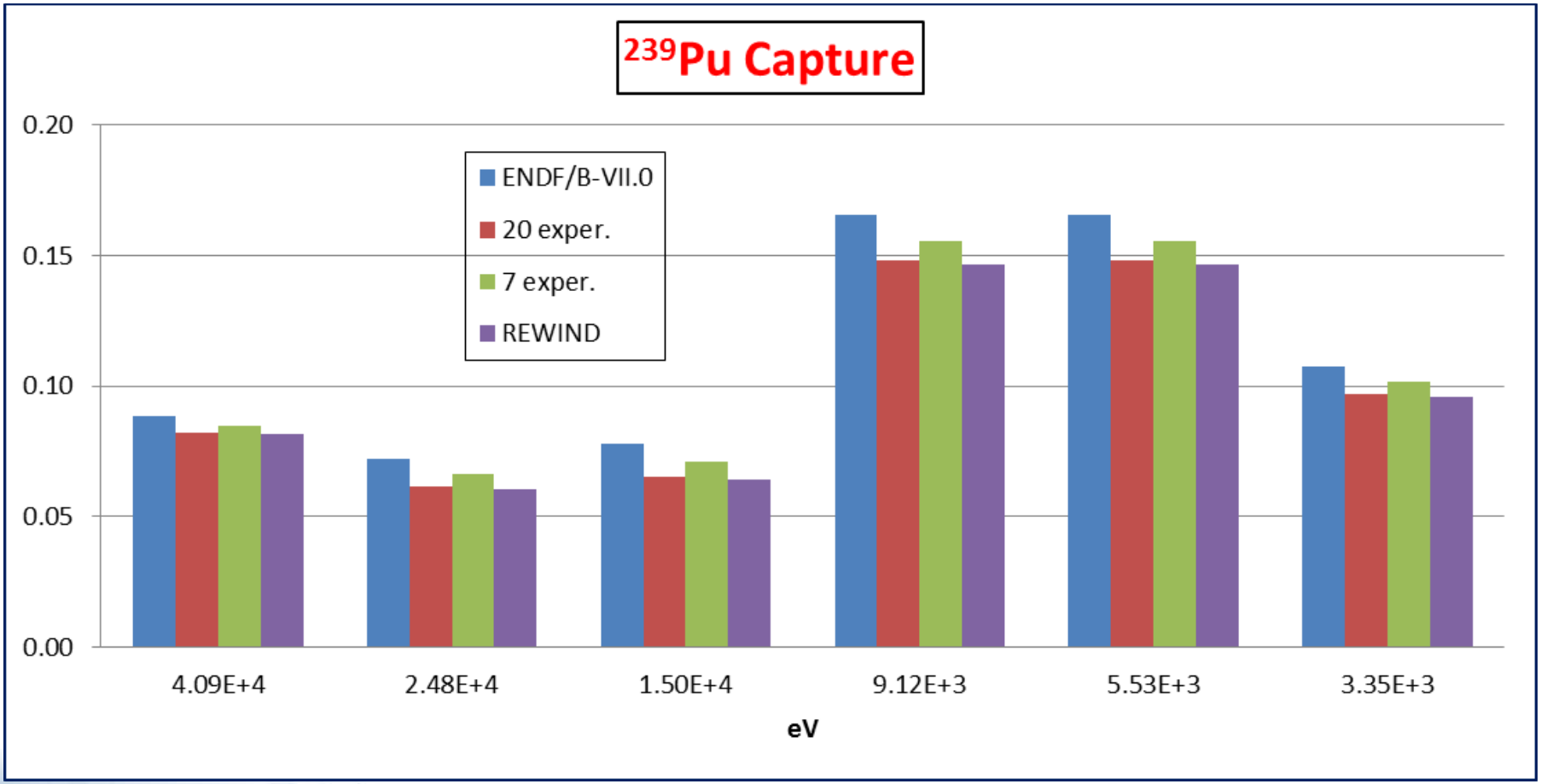

Conclusions

- A new methodology, REWIND, based on optimizing the use of a set of experiments using their potential gain in the adjustment aimed at different goals, has been developed for ranking experiments in a PIA strategy. The PIA strategy follows the original Kalman filtering approach.
- However, the ranking favors more global experiments than the elemental type one, and, therefore, it is not clear if compensations are avoided (the original aim of PIA).
- The REWIND approach is general and specific more appropriate functionals can be considered in the optimization step that give priority to the elemental experiments.
- Another development would be to directly incorporate the REWIND weights in the adjustment formulation, so that the progressive adjustment procedure would be significantly simplified.