DE LA RECHERCHE À L'INDUSTRIE

Does one shot Bayesian is equivalent to sucessive update ? Bayesian inference : some matrix linear algebra

C. De Saint Jean

CEA Cadarache, DEN/DER/SPRC/LEPh

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General problem:

Multigroup cross section adjustment of N integral experiments E_i with covariance matrix M_E.

$$M_E = \begin{pmatrix} M_{E1} & C_{12} & \cdots & C_{1N} \\ C_{21} & M_{E2} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ C_{N1} & \cdots & \cdots & M_{EN} \end{pmatrix}$$

Two analysis could be made :

- case 1 $\rightarrow$ analysis of N experiments successively
- case 2 $\rightarrow$ once-trough analysis : all experiments once as a meta-experiment

Are the posterior covariances on multigroup cross section equal for both analysis ?

Analysis of N experiments successively

Step 1 : analyze 1st experiment

$$M_1^{-1} = M_0^{-1} + S_1^T M_{E1}^{-1} S_1$$

where M_1 is the multigroup cross section covariances matrix and S_1 are the sensitivities of the calculated experiment (1st experiment).
Step n : analyze nth experiment with a priory coming from step (n-1) :

$$M_n^{-1} = M_{n-1}^{-1} + S_n^T M_{En}^{-1} S_n$$

where M_n is the multigroup cross section covariances matrix at step n and S_n are the sensitivities of the calculated experiment (nth experiment). A simple recurrence demonstration gives :

$$M_N^{-1} = M_0^{-1} + \sum_i^N S_i^T M_{Ei}^{-1} S_i$$

Once-throught adjustment : all experiments at the same time

Analysis of all experiments at the same time will give a posterior covariances matrices of the following form :

$$M_*^{-1} = M_0^{-1} + S^T M_E^{-1} S$$

Where :

$$S = \begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_N \end{pmatrix}$$

No correlations between experiments($Cij = 0$)

If there are no correlations between experiments ($Cij = 0$), we have :

$$M_E = \begin{pmatrix} M_{E1} & 0 & \cdots & 0 \\ 0 & M_{E2} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & M_{EN} \end{pmatrix}$$

M$_E$ being a block diagonal matrix, one can prove that :

$$M_E^{-1} = \begin{pmatrix} M_{E1}^{-1} & 0 & \cdots & 0 \\ 0 & M_{E2}^{-1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & M_{EN}^{-1} \end{pmatrix}$$

No correlations between experiments($Cij = 0$)

Which gives :

$$M_*^{-1} = M_0^{-1} + \begin{pmatrix} S_1^T & S_2^T & \cdots & S_N^T \end{pmatrix} \begin{pmatrix} M_{E1}^{-1} & 0 & \cdots & 0 \\ 0 & M_{E2}^{-1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & M_{EN}^{-1} \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_N \end{pmatrix}$$

Thus,

$$M_*^{-1} = M_0^{-1} + \sum_i S_i^T M_{Ei}^{-1} S_i$$

No correlations between experiments($Cij = 0$)

If the sensitivities calculated in case 1 are recalculated at each steps (potential effect of changes of cross section of step (n-1) to the calculation of sensitivities at step n) one may end up with slight differences on posterior covariance matrices of Case 1 and Case 2:

$$M_*^{-1} \sim M_N^{-1}$$

On the contrary, if the sensitivities are not recalculated in case 1 at each steps, once-trough calculation and successively analysis are giving the same results for posterior covariance matrices on cross sections :

$$M_*^{-1} = M_N^{-1}$$

Correlations between experiments

On the other hand, if there are correlations between experiments ($Cij \neq 0$), then, we have :

$$M_E^{-1} \neq \begin{pmatrix} M_{E1}^{-1} & 0 & \cdots & 0 \\ 0 & M_{E2}^{-1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & M_{EN}^{-1} \end{pmatrix}$$

There are no reasons why final covariances should be equal.

$$M_*^{-1} \neq M_N^{-1}$$