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# APPENDIX A

## How to Determine Error Matrix of Integral Data

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## I. Introduction

In the core design of fast breeder reactors, it is significant to improve the prediction accuracy of nuclear characteristics from the viewpoint of both reducing cost and insuring reliability of plant. To utilize the past critical experimental data and power reactor operational experience to the reactor design work, the most powerful method is to adjust a cross-section set based on the Bayesian theory and least-square technique (for example, Ref.1), where all related information including C/E (Calculation/Experiment) values, experimental and analytical errors, sensitivity coefficients of various experimental cores and parameters, and cross-section covariance, is synthesized with physical consistency. Based on the Bayes theorem, i.e., the conditional probability estimation method, we can maximize the posterior probability that a cross-section set, $\boldsymbol{T}$, is true, under the condition that the information of integral experiments, $\boldsymbol{R}$, is obtained as below:

$$\boldsymbol{J(T) = (T-To)^t M^{-1}(T-To) + [Re-Rc(T)]^t [Ve+Vm]^{-1}[Re-Rc(T)]} \quad \text{- - - (1)}$$

where,

$\boldsymbol{J(T)}$: The error function targeted for the combined set of differential and integral data,
$\boldsymbol{To}$ : A prior cross-section set before adjustment,
$\boldsymbol{M}$ : Covariance of the prior cross-section set $\boldsymbol{To}$ before adjustment,
$\boldsymbol{Ve}$ : Experimental error matrix of an integral experiment set,
$\boldsymbol{Vm}$ : Analytical modeling error matrix of the integral experiment set,
$\boldsymbol{Re}$ : Measured values of the integral experiment set, and,
$\boldsymbol{Rc(T)}$ : Analytical values of the integral experiment set obtained with the cross-section set $\boldsymbol{T}$.

To minimize the error function J(T), $\mathbf{d}\boldsymbol{J(T)}/\mathbf{d}\boldsymbol{T} = \mathbf{0}$. - - - (2)

After analytical derivations, the posterior cross-section set, $\boldsymbol{T'}$, and its covariance, $\boldsymbol{M'}$, after adjustment are obtained as follows:

$$\boldsymbol{T' = To + MG^t[GMG^t+Ve+Vm]^{-1}[Re-Rc(To)]} \quad \text{- - - (3)}$$

$$\boldsymbol{M' = M - MG^t[GMG^t+Ve+Vm]^{-1}GM} \quad \text{- - - (4)}$$

where,

$\boldsymbol{G}$ **:** Sensitivity coefficients of a cross-section, $\boldsymbol{t}$, to an integral parameter, $\boldsymbol{R}$, that is, $(\mathrm{d}\boldsymbol{R}/\boldsymbol{R})/(\mathrm{d}\boldsymbol{t}/\boldsymbol{t})$.

As seen in these equations, it is need to prepare the error matrices of integral parameters for both experiment and analytical modeling, $\boldsymbol{Ve}$ and $\boldsymbol{Vm}$, to perform the cross-section adjustment procedure. The objective of this section is to establish the reasonable method to quantitatively determine the

integral error matrices which include both diagonal terms (standard deviations) and non-diagonal terms (correlation factors). We call this "Integral covariance estimation method based on the perfect correlation between common error components", or "Error component correlation method" in short.

## II. Experimental Error Matrix

The experimental error values of an integral parameter are usually given by the experimenters with the error components. However, the correlations between plural integral parameters are scarcely found in the experiment report, therefore, we have to estimate the correlation factors from the experimental information here. The Error component correlation method adopts the following three steps:

### (Stage 1) Classification of Error Components to either Common or Independent

First, we list up all-related components of the experimental errors for "Data A" and "Data B" with quantitative values reported, and distinguish each of them into "Common error (i.e., the correlation factor is 1.0) between Data A and B", or "Independent error (i.e., the correlation factor is 0.0)"[1]. If an error component is judged as a mixture of common and independent errors, that is, the correlation factor is not considered as either 1.0 or 0.0, the error component must be divided into more detailed subcomponents until the error component becomes either common or independent error. We admit this classification requirement is extremely hard for the experimenters who evaluate the error components in their report, but nowadays, this kind of strictness is essential to keep the experimental quantities, and the recent experimental database like the NEA/IRPhE handbook begins to attain such detailed experimental error evaluation by the continuous efforts of the authors and reviewers.

### (Stage 2) Summing up of Common and Independent Errors

Next, we sum up the common and independent errors respectively, by the statistical method, that is, the "square, sum and root" means to obtain each standard deviation, $\sigma_{Total}$, the diagonal term of matrix. The statistical treatment is justified by the assumption that all error components are already divided until there are no correlations between any error items in the measurement of an integral parameter. The total errors of Data A and B, that is, the diagonal term of error matrix, ***Ve***, are the summation of common and independent errors as below:

Standard deviation of Data A: $$\sigma_{Total,A} = \sqrt{\sigma^2_{Common.A} + \sigma^2_{Independent,A}} \quad \text{- - - (5)}$$

Standard deviation of Data B: $$\sigma_{Total,B} = \sqrt{\sigma^2_{Common.B} + \sigma^2_{Independent,B}} \quad \text{- - - (6)}$$

where,

$\sigma_{Common}$ : Summing up of all Common error components, and,

$\sigma_{Independent}$ : Summing up of all Independent error components.

### (Stage 3) Evaluation of Correlation Factor

Finally, the correlation factor, non-diagonal term, of Data A and B is derived as the ratio of common errors to the total errors as Eq.7. The "Stage 1 to 3" procedures must be repeated for all matrix

[1] The words "Common" and "Independent" adopted here are usually refered as "Systematic" and "Statistic", respectedly, in many experimental reporting literatures. However, the author prefers using the former ones, since they would express that the intention of this classification is to evaluate their correlation factor for a spcific pair of data in a large matrix, more clearly than the latter words which seem rather vague about the definition range.

elements to generate a full experimental error matrix as the input of adjustment exercise. Note that, for example, the correlation factors between several sodium void reactivity measurements would be changed depending on the combination of void steps, even in the same experimental core.

$$\text{Correlation Factor of Data A and Data B: } \rho_{A,B} = \frac{\sum_i \sigma_{Common,A,i} \times \sigma_{Common,B,i}}{\sigma_{Total,A} \times \sigma_{Total,B}} \quad \text{- - - (7)}$$

where,

Suffix *i*: Common error components between Data A and Data B.

## III. Examples to Evaluate Experimental Error Matrix

Typical examples to estimate the experimental error matrix are shown for the sodium void reactivity (SVR) measurement and the reaction rate ratio (RRR) measurement in the ZPPR-9 core.

### III.1. Sodium Void Reactivity Measurement

Figure A.1 summarizes the evaluation procedure for the SVR measurement in the ZPPR-9 core. The upper part of Fig.A.1 shows the measured void step in the ZPPR-9 experiment (Ref.2). Hereafter, we treat the error values and their correlation between Step 3 and Step 5 of the SVR measurement in ZPPR-9 as an example. Step 3 is a central void case in the core where neutron non-leakage term is dominant for the reactivity change by sodium voiding, on the other hand, Step 5 is an axially whole-core void case where the non-leakage term of the reactivity is largely cancelled by the leakage term. The net reactivity of both steps is almost the same with the value around +30 cents, though the mass of the removed sodium to simulate sodium voiding is quite different by more than factor 2, that is, 31kg for Step 3 and 78kg for Step 5, respectively. The left part of Fig.A.1 is the result of the experimental error evaluation (Ref.2) following the IRPhE evaluation guidance (Ref.3), where the error sources are classified to the three categories, (1) measurement technique, (2) geometry, and (3) composition.

(Stage 1) The detailed explanation of the error evaluation for the SVR measurement can be found in "Section 2.4: Evaluation of Reactivity Data" of Ref.2, and we could judge the quality of error analysis and the classification level of the experimental error components fulfilled the requirement of our error matrix evaluation, that is, the correlation factor of each error component between two measurement must be 1.0 or 0.0. The followings are brief comments for the important error components in Fig.A.1.

λ i and β i/ β: To obtain the cent-unit reactivity by solving the inverse kinetics equation from the flux change measured, the family-wise decay constant (λ i ) values of the delayed neutron precursors and the family-wise delayed neutron fraction ratio (β i/ β ) were needed as the input data[2]. This error component greatly contributes to the total error with the common characteristics between two measurements, since ANL experimenters used the same λ i and β i/ β values through the measurement.

Temperature adjustment: The correction of temperature difference was needed between two measurements. According to the ANL document, the temperature difference is usually 2 degree-C at maximum, and the uncertainty of the temperature coefficient would be 10%. The resulted error values are quite large with the independent characteristics, since the temperature change between two measurements could be considered as random.

[2] Note this is not concerned with the conversion factor (βeff) of the reactivity from cent-unit to delta-k, which is needed to compare the reactivity value of an experiment with that of calculation.

Material mass-induced error: The assumed mass uncertainties were derived from some ANL documents, that is, 0.079% for plutonium, 0.15% for uranium, etc. The mass uncertainties were converted to the reactivity-unit errors using the sensitivity coefficients of each element to each void step, therefore, the error values were slightly changed in Step 3 and 5. This mass-induced error can be considered as a common component, since the shape of sensitivity coefficients for two measurements are found quite similar.

(Stage 2) The summing up results of the error values for Step 3 and 5 are shown in the right table of Fig.A.1. The Total error for Step 3 is 1.92%, and for Step 5, 1.90%. The contributions of common and independent errors are quite comparable.

(Stage 3) Finally, the correlation factor between Step 3 and 5 is shown at the bottom of Fig.A.1. The value is 0.41, which might be physically plausible from the quantitative evaluation of the common and independent error components.

**III.2. Reaction Rate Ratio Measurement**

Figure A.2 shows the error matrix evaluation process for the RRR measurement in the ZPPR-9 core. Here, we treat the foil activation method for the RRR measurement adopted in the ZPPR facility. Thin metallic activation foils were used to measure reaction rates in ZPPR-9 at the upper-left positions of Fig.A.2. Uranium and plutonium metal foils were placed between plates in various drawers in the assembly as shown in the upper-right figure, irradiated and then removed from the drawers. Capture and fission rates in the irradiated foils were determined by counting gamma rays emitted by capture or fission products.

(Stage 1) The error evaluation for the RRR measurement in ZPPR-9 is described in detail in "Section 2.7: Evaluation of Reaction Rate Distributions" of Ref.2. The left part of Fig.A.2 shows the result of the experimental error evaluation.

Error caused by mapping foil activity measurement: This error component consist of (1) the counting statistics, (2) the positioning of a sample above a gamma-ray counter, (3) the foil mass and (4) the discrete channel boundary in peak integration, whose characteristics are all statistical.

Error caused by detector calibration: The absolute calibration of each reaction is necessary to measure the RRR value, which was made by gamma–ray counting of $^{239}$Pu, $^{235}$U and $^{238}$U foils and deposits in back-to-back fission chambers. Note that the error induced by the detector calibration has the systematic characteristics to determine the absolute value of a kind of RRR, such as by averaging the F49/F25 values in the whole core, however, it has the statistic feature when we consider the correlation between two kinds of RRRs, such as between F49/F25 and C28/F25.

Composition-induced error: Since the error caused by the foil composition is included in the mapping foil error, the composition column in Fig.A.2 is related to the chemical analysis error of the core fuel and other core materials, and possesses the common characteristics between two RRRs. The composition error to the reactivity was converted with the sensitivity coefficients like the SVR case, but the magnitude was found to be negligible, compared with other common error mentioned below.

(Stage 2) In the ZPPR experiment, the activation foils of $^{239}$Pu, $^{235}$U and $^{238}$U were irradiated in the same run, and at the same foil folders in a drawer. This means that the common error of two RRRs such as F49/F25 and C28/F25 must include the contribution from the common denominator, F25 in this case. The summing up results of the error values for F49/F25 and C28/F25 are shown in the right table of Fig.A.2. The total error for F49/F25 is 2.0%, and for C28/F25, 1.9%.

(Stage 3) The correlation factors between F49/F25 and C28/F25 becomes 0.32 as shown at the bottom of Fig.A.2.

### III.3. Full Experimental Error Matrix in SG33 Exercise

Appling the above-mentioned methodology, the full matrix of the experimental error for the 20 experiments treated in the SG33 exercise was summarized in Fig.A.3. Additional comments for this table are below:

(1) The correlation factors of the RRRs in Jezebel-Pu239, Flattop and ZPR6-7 are borrowed from those of ZPPR-9, since the denominator of the RRRs, F25, is common in these experiments, and there is scarce information for the former three experiments to evaluate the common and independent components of the RRRs. The F37/F25 ratio is assumed to possess similar characteristics with F28/F25 which has a threshold feature against neutron energy.
(2) From the fuel composition tables of Ref.2 and 4, the plutonium fuel plates used in ZPR6-7, ZPR6-7 Pu240 and ZPPR-9 experiments were found as identical ones. This means at least that the criticality of these three cores must be correlated through the composition errors. In Fig.A.3, the evaluated correlation factors with the sensitivity coefficients of core compositions are added[3]. The correlations among other parameters of these three cores are neglected here, since the effects of common core material to other parameters are usually small compared with that to the criticality.

## IV. Analytical Modeling Error Matrix

The evaluation methodology of the analytical modeling error, ***Vm***, depends on the analytical method adopted to obtain the calculation value of an integral experiment. Here, we consider three kinds of analytical methods, that is, (1) Continuous-energy Monte Carlo method based on the as-built experimental geometry and compositions (MC method, hereafter), (2) Deterministic analytical method based on the combination of the standard calculation and the corrections by the most-detailed analytical models (Deterministic method), and (3) Combination of the deterministic analytical calculation based on the simplified geometry and the correction by the Monte Carlo calculation with as-built geometry (Combined method).

### IV.1. MC Method

The standard deviation (diagonal term) of the analytical modeling error should be supplied from the statistical error value evaluated by the adopted MC code. However, there is one caution to adopt the statistical error evaluated by the MC code. According Ref.5 and 6, the existing MC codes cannot count on the effect of correlation among the fission source over successive MC cycles to evaluate the statistical error. This ignorance will result in the underestimation bias of the real statistical error, by the range of the factor 1.4 to 3.1 (Ref.5) depending on the target calculation systems, or by 69% for a small-size reactor core and by 80% for a medium-size one (Ref.6). Here we recommend to multiply the statistical error evaluated with a MC code by a factor of 2, in order to prepare the analytical modeling error as the input data of the cross-section adjustment.
The correlation factors of the analytical modeling error with the MC method are basically 0.0 from the chaotic nature of the Monte Carlo methodology. This would be also valid even for the correlation between two reaction rates at the same position in a core, since the detailed energy structure of two cross-sections are not identical, therefore, the energy-integrated reaction rate would be independent

[3] The common errors induced by composition errors are only considered here, since there is no information on other common error components among these.

from each other. Only one exception is the correlation between two reaction rate ratios (RRRs) which have the same denominator such as F49/F25 and C28/F25. If the calculation results of two RRRs are obtained by one MC code run, these RRRs have the correlation of +0.5 if the statistical errors of each RRR are the same. Fig.A.4 shows an example of the analytical modeling Error matrix ***Vm*** applied in SG33 exercise, where all calculations are based on the MC method.

### IV.2. Deterministic Method

There is no established methodology to evaluate the analytical modeling error matrix for the deterministic analysis yet. Here we submit a possible estimation method which is based on a kind of "sensitivity consideration" to the detailed degree of a physical modeling. The basic assumption is that the error value of analytical modeling would be large, if the analytical result significantly changes when the degree of physical modeling are replaced from simple one to detailed, such as from the diffusion theory to the transport theory. In other words, the error would be large if the sensitivity to the analytical modeling is large. Figure A.5 summarizes the actual procedure to estimate the analytical modeling error matrix:

(1) First, the correction items of an integral parameter, the criticality for two experimental cores, ZPPR-9 and JOYO Mk-I in Fig.A.5, are listed up with their calculated values. The explanation of each correction is found in Section 4 of Ref.8.
(2) For each correction item, a certain percent[4] of the "smaller" correction value is assumed as the common error between two cores. The concept of this treatment for the transport-theory correction is illustrated in Fig.A.6. The physical mechanism of transport correction comes from the approximated treatment of neutron-flux gradient, which is common between ZPPR-9 and JOYO. However, the sensitivity of the transport effect to JOYO is larger than that to ZPPR-9, since JOYO is a smaller core with steep flux gradient. We assume here the overlapped part of the transport correction would be common between these two cores, and the un-overlapped part might be an independent feature of JOYO only. If the sign of correction values for two cores are opposite, which is the case of the ultra-fine energy effect in Fig.A.5, this correction item would be judged as independent because the physical mechanism must be different between each other.
(3) The percentage value of the correction should be estimated by other engineering judgments, and we recommend a value of 30% from the past adjustment experiences where the chi-square balancing was surveyed among the set of the C/E-1 values, the integral experimental errors, the cross-section covariances and the analytical modeling errors.
(4) The following procedures are similar with those of the experimental error matrix evaluation. The Total error values of two cores are calculated by summing up both the common errors and the independent errors as Eq.5 and 6. The correlation factor is evaluated by the ratio of the common errors to the total errors as Eq.7.

Figure A.5 shows the example of keff correlations between a large core, ZPPR-9, and a small core, JOYO. The correlations of them would be expected to be quite weak, and the estimated result was 0.14. On the other hand, Figure A.7 is an example of the strong correlation case. The integral data evaluated are the criticality of ZPPR-10A, a 600MWe-class FBR core, and that of ZPPR-10C, an 800MWe-class FBR core. The drawer structures and arrangements of both cores are almost identical except small difference of core sizes, therefore, the correlation of them would be expected to be strong, and the estimated factor was 0.82. We consider these results would be coincident with our physical intuition.

[4] The "a certain percent" means that value of the analytical modeling error is assumed to be proportional to the value of correction by the detailed models.

### IV.3. Combined Method

The MC method could be generally regarded as the most desirable analysis tool to obtain the best-estimated calculation values and their error values for the complicated as-built geometry. However, the current computer power is still not enough to calculate small reactivity changes or local reaction rates by the MC method with realistic computing time. Further, the input data of the MC method tends to be huge if the core compositions are very complicated, such as the burnup conditions of a power reactor. On the other hands, the Deterministic method has the advantage that the computing time is relatively short, and it is rather practical to treat such complicated compositions, although the complicated as-built geometry is difficult to directly simulate by the Deterministic method. To make use of the merits of both methods, there is a possibility to combine the MC method with the Deterministic method to obtain the best-estimated calculation values. The idea is to make the calculation result of the Deterministic method equivalent to that of the MC method by the correction. The final values of the Combined method, $R_{Combined}(Best\ estimated)$, would be expressed below:

$$R_{Combined}(Best\ estimated) = R_{Det}(Simplified\ geometry,\ As\text{-}built\ composition) + \{R_{MC}(As\text{-}built\ geomery,\ Simplified\ composition) - R_{Det}(Simplified\ geometry,\ Simplified\ composition)\} \quad \text{- - - (8)}$$

where,

$R_{Det}$: Calculated value by the Deterministic method, and,
$R_{MC}$: Calculated value by the MC method.

In this case, the standard deviation of the Combined method could be considered as the same of the MC method, under the condition that the correction values, the second term of the right hand in Eq.8, are obtained through well-organized simulation models from the analytical modeling error viewpoint, in other words, if the analytical errors caused by two calculations with the Determination method are cancelled in Eq.8. The correlation factors of the Combined method would be also the same with those of the MC method, under such condition.

## V. Concluding Remarks

In order to make the cross-section adjustment, the preparation of the integral experimental and analytical modeling errors is inevitable with the form of matrix, that is, the combination of standard deviations and correlation factors. We proposed the methodology and procedure to evaluate/estimate both matrices here, and hope these will be used in the actual adjustment exercise and improved to more sophisticated way in future.

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Fig. A.1 (Example 1) Sodium Void Reactivity Measurement in ZPPR-9

Table 1.13 Results of Zone Sodium-voiding Measurements in ZPPR-9 (Ref.; "ZPPR-11 Monthly Report for February 1980", ZPR-TM-361, Argonne National Laboratory (Feb. 1980).) (Ref. 2)

| Step No. | Total Zone Size, Drawers | Zone Depth, mm | Total Na Mass [a], kg | Reactivity Change, [b], cent | | Reactivity Adjustment [c], cent |
|---|---|---|---|---|---|---|
| | | | | Cumulative +/- σm (σt) | Step +/- σm (σt) | |
| 1 | 9 | 203.2 | 2.90 | 3.03+/-0.05 (0.10) | 3.03+/-0.05 (0.10) | -0.04 |
| 2 | 37 | 203.2 | 11.94 | 11.56+/-0.04 (0.19) | 8.53+/-0.06 (0.17) | -1.36 |
| 3 | 97 | 203.2 | 31.30 | 29.39+/-0.02 (0.36) | 17.83+/-0.04 (0.32) | 1.22 |
| 4 | 97 | 406.4 | 62.60 | 37.26+/-0.01 (0.43) | 7.87+/-0.02 (0.10) | 0.84 |
| 5 | 97 | 508.0 | 77.88 | 31.68+/-0.02 (0.36) | -5.58+/-0.04 (0.15) | 0.13 |
| 6 | 97 | 685.8 | 105.11 | 24.44+/-0.03 (0.29) | -7.24+/-0.04 (0.15) | -0.82 |
| Off-center Zones [d] | | | | | | |
| 1 | 25 (x axis) | 203.2 | 8.07 | 0.93+/-0.06 (0.12) | | |
| 2 | 25 (y axis) | 203.2 | 8.07 | 0.20+/-0.06 (0.12) | | |

a: A random uncertainty of 1% is assigned to any mass or mass difference.
b: Counting statistics only are included in σm. The value of σt includes uncertainties in the reactivity adjustment and a 1.1% uncertainty in the detector calibration.
c: This uncertainty adjustment accounts for differences in experimental conditions between the reference and the particular step. When comparing the reactivity between steps, an uncertainty is assigned based on the magnitude of the adjustment.
d: Outer core zones in Table 1.25.

Table 2.12 Summary of Uncertainties in the Zone Sodium Void Measurement in ZPPR-9

| Source of Uncertainty | | | | Uncertainty: cents | Uncertainty: % of measured reactivity* | Common error | Independent error |
|---|---|---|---|---|---|---|---|
| Measurement technique | MSM method | Rod drop method | Counting statistics | | +/-(0.2)** | | 0.2 % for Step3 and Step5 |
| | | | $\lambda i$ and $\beta i/\beta$ | | +/-1.0 | 1.0 % for Step3 and Step5 | |
| | | $\frac{R_1 \cdot \varepsilon_2}{R_2 \cdot \varepsilon_1}$ | | | +/-0.2 | | 0.2 % for both step |
| | | $\frac{\beta_{eff,1}}{\beta_{eff,2}}$ | | negligible | | | 0.0 % for both step |
| | | $\frac{S_{eff,2}}{S_{eff,1}}$ | | | +/-0.5 | | 0.5 % for both step |
| | Adjustment | Interface gap | | +/-0.03 | | | Step3: 0.10 %, Step5: 0.09 % |
| | | Temperature | | +/-0.27 | | | |
| | | Pu decay | | +/-0.0015 | | | 0.9 % for both step |
| Geometry | | Interface gap (included in adjustment of measurement technique) | | | -- | 0.00 % for both step | |
| Composition | Assumed deviation of material mass | Pu mass | | | Depend on measured void zones (see Table 2.10(1)) | | -- |
| | | U mass | | | | | |
| | | Stainless steel weight | | | | Step3: 0.72 %, Step5: 0.67 % | |
| | | Sodium mass | | | | | |
| | | O mass | | | | | 1.0 % for both step |
| | | C mass | | | | | |
| | | $^{239}$Pu isotope ratio | | | | | |
| | | $^{235}$U isotope ratio | | | | | |
| | Removed sodium mass | | | | +/-1.0 | 0.16 % for both step | |
| | Difference of stainless steel weight between the sodium-filled plates and the empty plates | | | | +/-0.16 | | |
| | | | | | | (Sub total - Common) Step3: 1.24 %, Step5: 1.21 % | (Sub total - Independent) Step3: 1.46 %, Step5: 1.46 % |
| | | | | | | (Total error) Step3: 1.92 %, Step5: 1.90 % | |

*: Every value in this column depends on the individual measurement case and is a relative uncertainty.
**: generalized uncertainty, refer the actual uncertainties presented in Table 1.13 and Table 1.14)

$$\rho\ (between\ Step3\ \ and\ \ Step5) = \frac{\sum_i \sigma_{Common}(Step3,i) \times \sigma_{Common}(Step5,i)}{\sigma_{Total}(Step3) \times \sigma_{Total}(Step5)} = 0.41$$

Fig. A.2 (Example 2) Reaction Rate Ratio Measurement in ZPPR-9

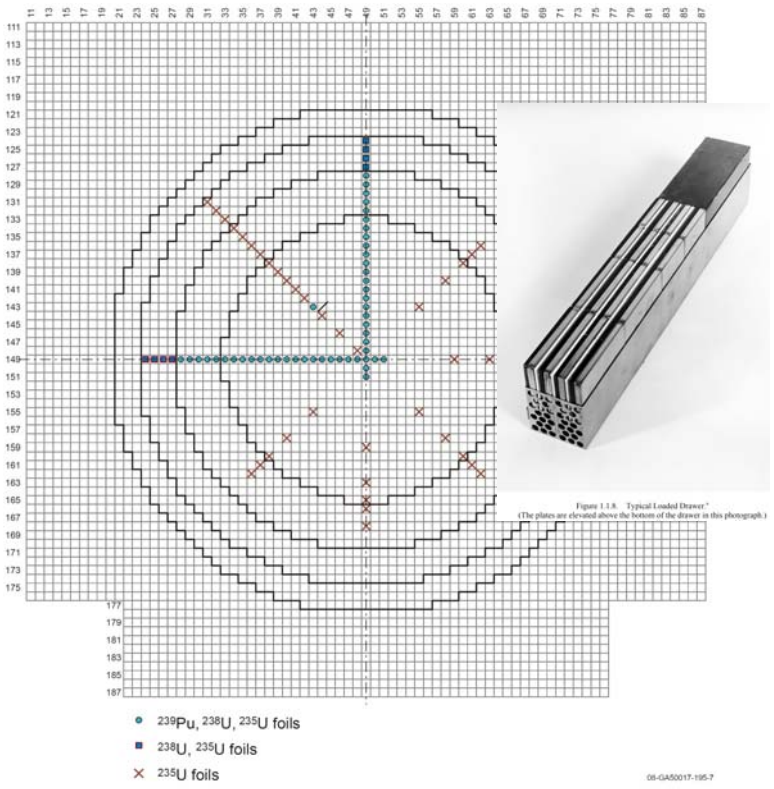


Figure 1.31. Measured Radial Reaction Rate Positions in ZPPR-9.
(Ref. D. N. Olsen, "ZPPR Assembly 9 – Detailed Work Plan No. 3", ZPR-I-306, Argonne National Laboratory (Jun. 1978).)

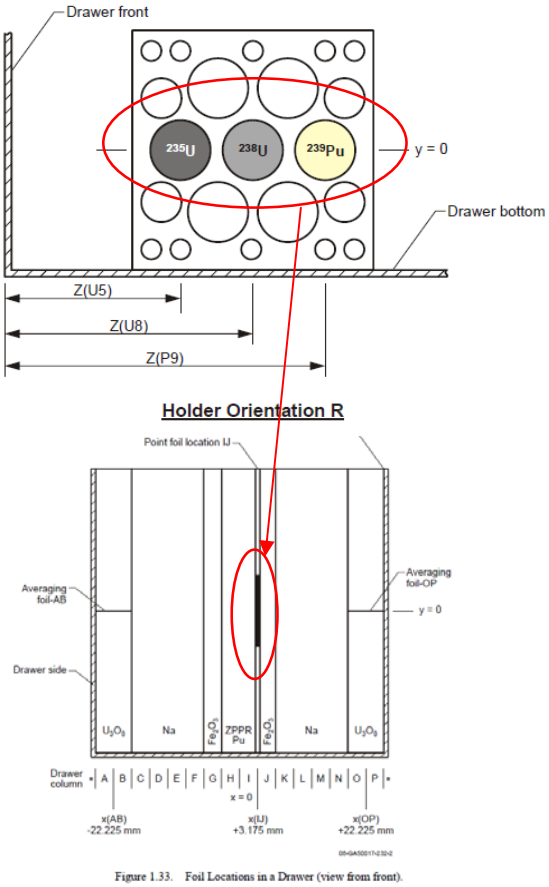


Figure 1.33. Foil Locations in a Drawer (view from front).

Table 2.22 Uncertainties Assigned to the Detector Calibration (Ref. 2)

| Typical uncertainty (% of measured reaction rate) | | | | | | |
|---|---|---|---|---|---|---|
| Reaction Rate | | | | Reaction Rate Ratio | | |
| $^{239}$Pu(n, f) | $^{235}$U(n, f) | $^{238}$U (n, f) | $^{238}$U(n,$\gamma$) | $^{235}$U(n, f)/ $^{239}$Pu(n, f) | $^{238}$U (n, f)/ $^{239}$Pu(n, f) | $^{238}$U(n,$\gamma$)/ $^{239}$Pu(n, f) |
| 1.5 | 1.3 | 1.9 | 1.0 | 1.0 | 1.8 | 1.2 |

able 2.26 Combined Uncertainties of Mapping Foil Data

| | Typical uncertainty (% of measured reaction rate) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $^{239}$Pu fission | | $^{235}$U fission | | $^{238}$U fission | | $^{238}$U capture | |
| | Core | Radial blanket | Core | Radial blanket | Core | Radial blanket | Core | Radial blanket |
| Measurement technique | 1.3* | | 1.1* | | 1.7* | | 1.0* | |
| Geometry | negligible | -- | negligible | 1.0 | negligible | 0.1 | negligible | 0.9 |
| Composition | 0.18 | --- | 0.17 | 0.08 | 0.22 | 0.27 | 0.18 | 0.06 |
| Total | 1.3 | 1.3 | 1.1 | 1.5 | 1.7 | 1.7 | 1.0 | 1.3 |

*: see Table 2.19.

Table 2.27 Combined Uncertainties of Reaction Rate Ratio (in core region)

| | | | Typical uncertainty (% of measured reaction rate ratio) | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | F25/F49 | | F28/F49 | | C28/F49 | |
| | | | F25 | F49 | F28 | F49 | C28 | F49 |
| Measurement technique | Mapping foil | | 1.1 | 1.3 | 1.7 | 1.3 | 1.0 | 1.3 |
| | | Sub-total | 1.7 | | 2.1 | | 1.6 | |
| | Detector calibration | | 1.0* | | 1.8* | | 1.2* | |
| Geometry | | | negligible | | negligible | | negligible | |
| Composition | | | 0.06 | | 0.22 | | 0.05 | |
| Total | | | 2.0 | | 2.8 | | 2.0 | |

*: see Table 2.22.

**In ZPPR-9, the reaction rates were measured in the same run, and at the same foil place.**

**Common error of two reaction rate ratios (e.g. F49/F25 & C28/F25) come from the error of the common reaction rate (F25).**

| Reaction Ratio | | F28/F25 | F49/F25 | C28/F25 |
|---|---|---|---|---|
| Total Error | | 2.7% | 2.0% | 1.9% |
| Correlation factor | F28/F25 | 1.0 | | |
| | F49/F25 | 0.23 | 1.0 | |
| | C28/F25 | 0.23 | 0.32 | 1.0 |

$$\rho\ (between\ F49/F25\ and\ C28/F25) \approx \frac{1.1\% \times 1.1\%}{\sigma_{Total}(F49/F25) \times \sigma_{Total}(C28/F25)} = 0.32$$

Fig. A.3 Experimental Error Matrix ***Ve*** applied in SG33 Exercise

| No. | Core | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Jezebel - Pu239 | keff | 0.2 | | | | | | | | | | | | | | | | | | | |
| 2 | | F28/F25 | 0 | 1.1 | | | | | | | | | | | | | | | | | | |
| 3 | | F49/F25 | 0 | 0.23 | 0.9 | | | | | | | | | | | | | | | | | |
| 4 | | F37/F25 | 0 | 0.23 | 0.32 | 1.4 | | | | | | | | | | | | | | | | |
| 5 | Jezebel - Pu240 | keff | 0 | 0 | 0 | 0 | 0.2 | | | | | | | | | | | | | | | |
| 6 | Flattop | keff | 0 | 0 | 0 | 0 | 0 | 0.3 | | | | | | | | | | | | | | |
| 7 | | F28/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 1.1 | | | | | | | | | | | | | |
| 8 | | F37/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0.23 | 1.4 | | | | | | | | | | | | |
| 9 | ZPR6-7 | keff | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.23 | | | | | | | | | | | |
| 10 | | F28/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3.0 | | | | | | | | | | |
| 11 | | F49/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.23 | 2.1 | | | | | | | | | |
| 12 | | C28/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.23 | 0.32 | 2.4 | | | | | | | | |
| 13 | ZPR6-7 Pu240 | keff | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.13 | 0 | 0 | 0 | 0.22 | | | | | | | |
| 14 | ZPPR-9 | keff | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.31 | 0 | 0 | 0 | 0.30 | 0.117 | | | | | | |
| 15 | | F28/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.7 | | | | | |
| 16 | | F49/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.23 | 2.0 | | | | |
| 17 | | C28/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.23 | 0.32 | 1.9 | | | |
| 18 | | Central Na void | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.9 | | |
| 19 | | Large Na void | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.41 | 1.9 | |
| 20 | Joyo | keff | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.18 |

* Diagonal term: Error value (1 sigma, %)
** Non-diagonal term : Correlation factor (between -1 and +1)

Fig. A.4 An example of Analytical Modeling Error Matrix ***Vm*** applied in SG33 Exercise
(All calculations are based on Continuous-energy Monte Carlo method.)

| No. | Core | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Jezebel - Pu239 | keff | 0.03 | | | | | | | | | | | | | | | | | | | |
| 2 | | F28/F25 | 0 | 0.9 | | | | | | | | | | | | | | | | | | |
| 3 | | F49/F25 | 0 | 0.5 | 0.8 | | | | | | | | | | | | | | | | | |
| 4 | | F37/F25 | 0 | 0.5 | 0.5 | 0.8 | | | | | | | | | | | | | | | | |
| 5 | Jezebel - Pu240 | keff | 0 | 0 | 0 | 0 | 0.03 | | | | | | | | | | | | | | | |
| 6 | Flattop | keff | 0 | 0 | 0 | 0 | 0 | 0.03 | | | | | | | | | | | | | | |
| 7 | | F28/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8 | | | | | | | | | | | | | |
| 8 | | F37/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0.7 | | | | | | | | | | | | |
| 9 | ZPR6-7 | keff | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.03 | | | | | | | | | | | |
| 10 | | F28/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.2 | | | | | | | | | | |
| 11 | | F49/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 1.4 | | | | | | | | | |
| 12 | | C28/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0.5 | 1.2 | | | | | | | | |
| 13 | ZPR6-7 Pu240 | keff | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.03 | | | | | | | |
| 14 | ZPPR-9 | keff | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.02 | | | | | | |
| 15 | | F28/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.1 | | | | | |
| 16 | | F49/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 1.2 | | | | |
| 17 | | C28/F25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0.5 | 1.4 | | | |
| 18 | | Central Na void | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5.3 | | |
| 19 | | Large Na void | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5.0 | |
| 20 | Joyo | keff | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.03 |

* Diagonal term: Error value (1 sigma, %)
** Non-diagonal term : Correlation factor (between -1 and +1)

Fig. A.5 (Example 3) Weak Correlation Case of Analytical Modeling Error

| Analytical Item \ Experiment | | Keff of ZPPR-9 (: A) | | | Keff of JOYO Mk-I (: B) | | |
|---|---|---|---|---|---|---|---|
| | | Value (Ref.2) | Common Error | Independent Error | Value (Ref.5) | Common | Independent Error |
| keff by basic method | | 0.99372 | *multiplied by 0.3* | | 0.98060 | | |
| Correction by detailed model (unit: pcm) | Transport theory | +248 | ±74 | 0 | +1760 | ±74 | ±523 |
| | Mesh-size effect | -93 | ±28 | 0 | -210 | ±28 | ±56 |
| | Ultra-fine energy effect | +103 | 0 | ±31 | -50 | 0 | ±15 |
| | Multi-drawer effect | +47 | 0 | ±14 | 0 | 0 | 0 |
| | Cell-asymmetry effect | -52 | 0 | ±16 | 0 | 0 | 0 |
| Total | | 0.99625 | ±79 | ±37 | 0.99560 | ±79 | ±526 |
| | | | ±88 pcm | | | ±532 pcm | |

*assumed as common*

*assumed as independent*

$$\rho\ (between\ ZPPR-9\ and\ JOYO) = \frac{\sum_i \sigma_{Common}(ZPPR-9)\times\sigma_{Common}(JOYO)}{\sigma_{Total}(ZPPR-9)\times\sigma_{Total}(JOYO)} = 0.14$$

Fig. A.6 Illustration of Common-error Concept in Analytical Modeling Error Estimation

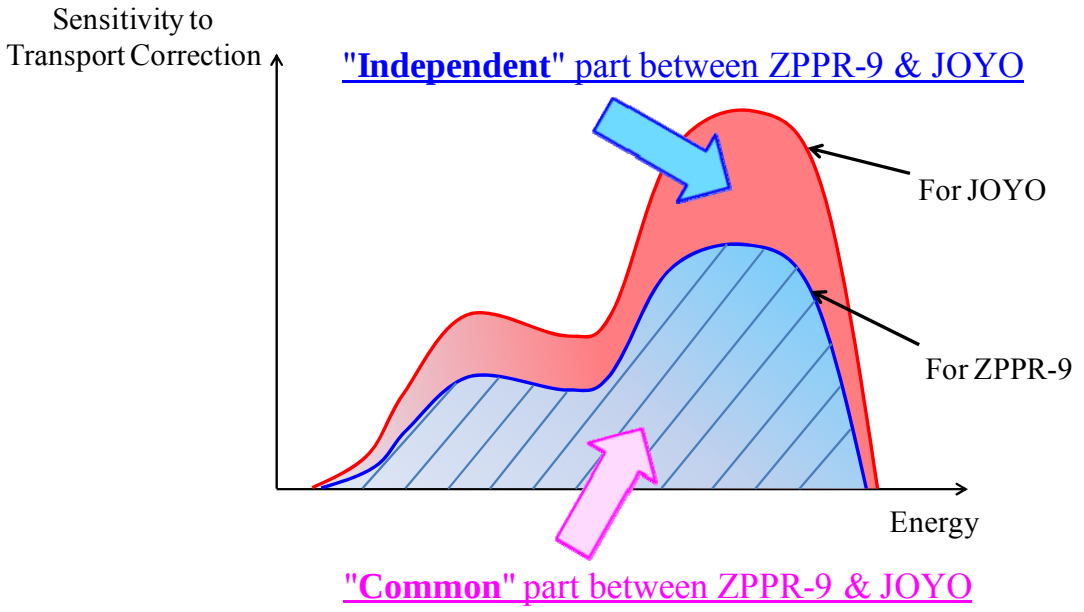

Fig. A.7 (Example 4) Strong Correlation Case of Analytical Modeling Error

| Experiment / Analytical Item | | Keff of ZPPR-10A (: A) (600 MWe-class FBR) | | | Keff of ZPPR-10C (: B) (800 MWe-class FBR) | | |
|---|---|---|---|---|---|---|---|
| | | Value (Ref.6) | Common Error | Independent Error | Value (Ref.7) | Common | Independent Error |
| keff by basic method | | 0.9913 | | | 0.9916 | *multiplied by 0.3* | |
| Correction by detailed model (unit: pcm) | Transport theory | +530 | ±129 | ±93 | +430 | ±129 | 0 |
| | Mesh-size effect | -130 | ±33 | ±21 | -110 | ±33 | 0 |
| | Ultra-fine energy effect | +150 | ±42 | ±16 | +140 | ±42 | 0 |
| | Multi-drawer effect | +40 | ±12 | 0 | +40 | ±12 | 0 |
| | Cell-asymmetry effect | -60 | ±15 | ±10 | -50 | ±15 | 0 |
| Total | | 0.9966 | ±141 | ±97 | 0.9961 | ±141 | 0 |
| | | | ±171 pcm | | | ±141 pcm | |

*assumed as common*

$$\rho\ (between\ ZPPR-10A\ and\ ZPPR-10C) = \frac{\sum_i \sigma_{Common}(ZPPR-10A) \times \sigma_{Common}(ZPPR-10C)}{\sigma_{Total}(ZPPR-10A) \times \sigma_{Total}(ZPPR-10C)} = 0.82$$