FISSION PRODUCT YIELDS

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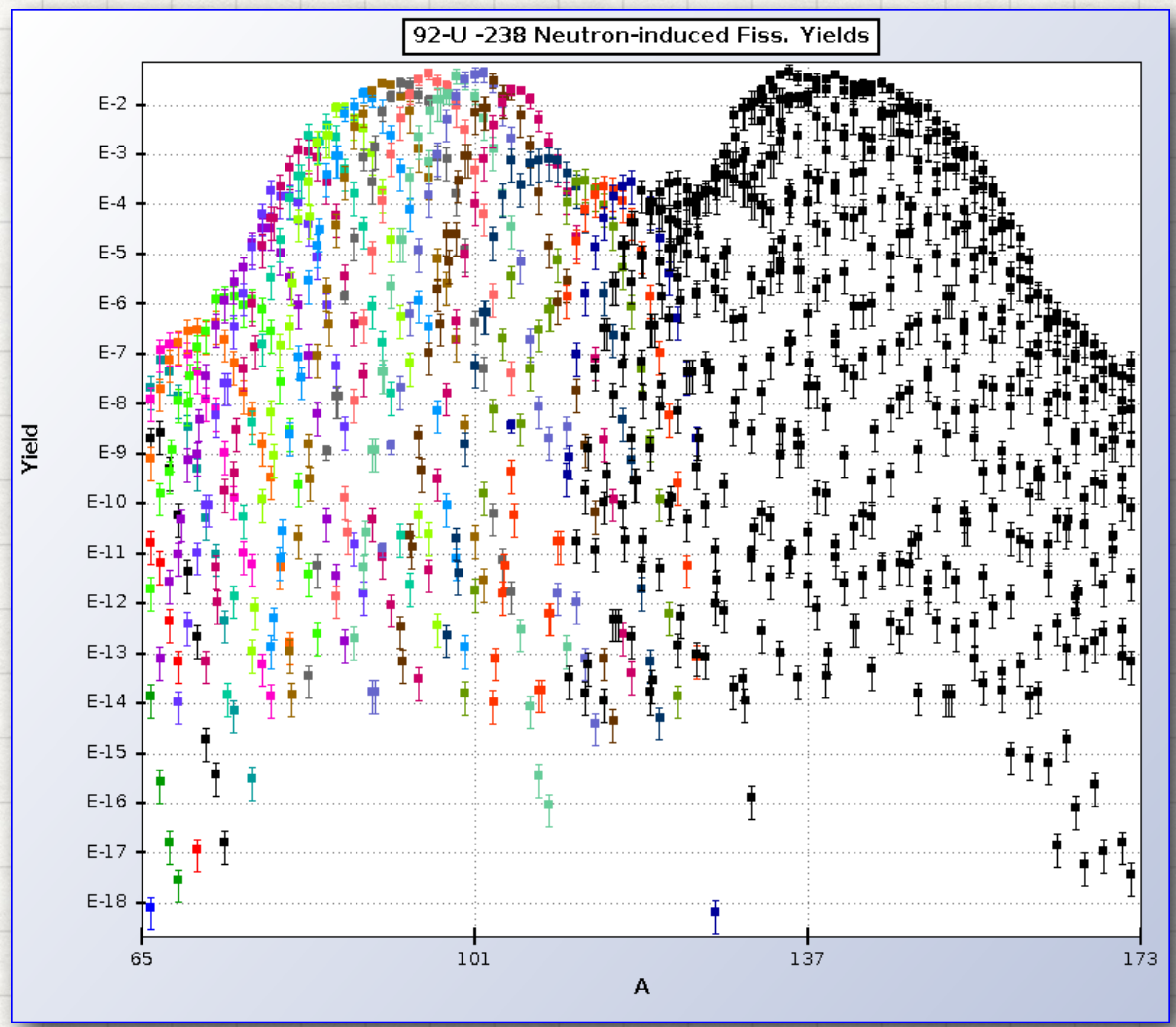
OUTLINE

- **Overview of FPY theory**
- **Some GNDS format requirements are unrealistic**
  - Storing Q matrix as a <matrix> is silly
  - Start and stop times for Q are equally silly
- **Some GNDS format requirements have important and useful implications**
  - Need for (proper) covariances for Q, CY, IY
  - Need for finer energy grids in CY, IY
- **We neglected consideration of ternary fission**



ENDF/B-VII.1 ^{238}U(n,f) independent yields from SIGMA plot

RATE (AKA BATEMAN) EQUATION

The rate equation for particle species i following a fission burst at $t = 0$ sec. is given by

$$\frac{dN_i}{dt} = \sum_j \lambda_j b_{j\to i} N_j(t) - \lambda_i N_i(t) \tag{1}$$

where

$N_i(t)$ is the population of species i at time t in units of (particles)

λ_j is the total rate at which species j in units of (decays/unit time)

$b_{j\to i}$ is the branching ratio for species j to decay into species i (no units)

$\lambda_{j\to i} \equiv \lambda_j b_{j\to i}$ is the rate at which species j decays into species i in units of (decays/unit time)

The species i may be the ground state or an isomer.

Take away message: FPY will be intimately related to decay branching ratios and half-lives

WAYS TO SOLVE THE BATEMAN EQUATION

- Numeric integration (Leap Frog, ...)
- Laplace transform:

$$N_n(t) = \sum_{i=1}^{n} \left[N_i(0) \times \left(\prod_{j=1}^{n-1} \lambda_{j\to n} \right) \times \left(\sum_{j=i}^{n} \frac{\exp(-\lambda_{j\to n} t)}{\prod_{p=i,p\neq j}^{n} (\lambda_{p\to n} - \lambda_{j\to n})} \right) \right]$$

- Use someone else's code (i.e. FISPACT, FISPIN, Origen, DecayNetwork, PyNE, ROOT, CINDER, MONTEBURNS, ...)

Take away message #1: Care needed when rates very close

Take away message #2: Propagating uncertainties from decay data cannot be done "the easy way" with GLSQR. A Monte Carlo approach is probably needed. As far as we know, one has ever done it.

FISSION PRODUCT YIELDS

- Independent fission yield, IFY_i, is the probability that species i is promptly populated after fission event

- The Cumulative fission yield is recursively defined as

$$CFY_i = IFY_i + \Sigma\, B_{ij}\, CFY_j$$

- B_{ij} is the probability of the j^{th} material in the network will decay to the i^{th} material.

- Can be shown that $CFY_i = IFY_i + \Sigma\, A_{ij}\, IFY_j = \Sigma\, Q_{ij}\, IFY_j$, where A_{ij} is the product of decay branching ratios.

- If there is a single fission burst and $N_i(t)$ gives the number of nuclides for the i^{th} material as function of time, then $CFY_i = N_i(t=\infty)$ for stable or very long-lived nuclides, $T_{1/2} > 10^{15}$ years.

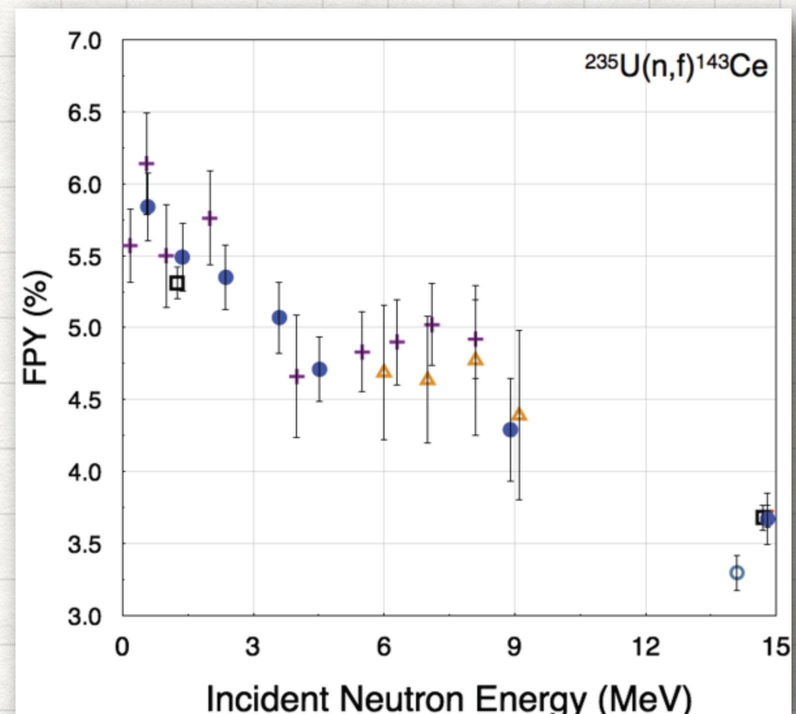
- In an operating nuclear reactor, if a given material has achieved equilibrium, that is $dN_i/dt = 0$, then $N_i(t)$ per fission is proportional to $CFY_i \times T_{1/2i}$

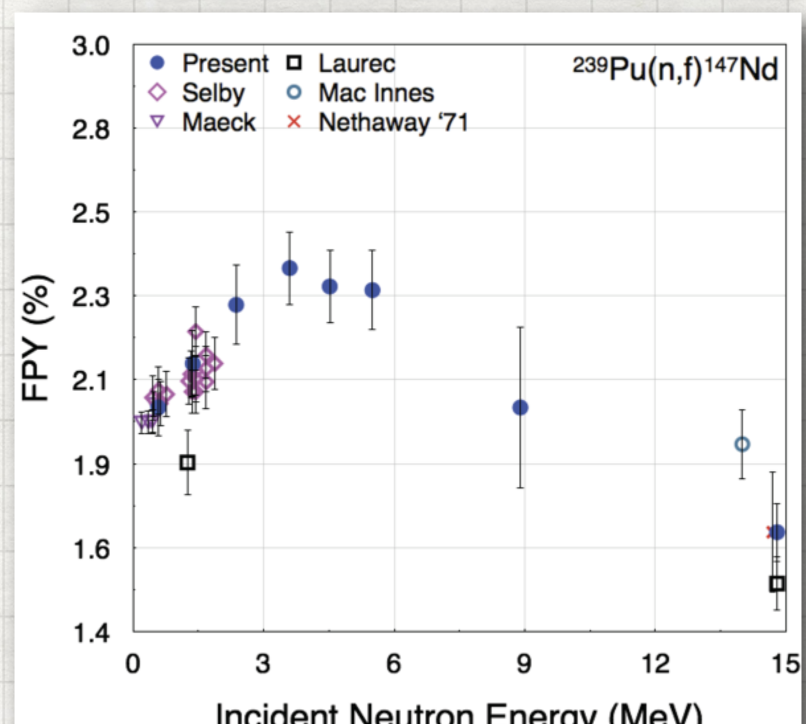
TAKE AWAY MESSAGES

- The Q (or A or B) matrix is computable directly from the (e.g.) ENDF/B decay sublibrary and the list of isotopes in the CFY & IFY tables is all that is needed.
- A precomputed Q matrix & (if computable) Q matrix covariance might be useful for some users

YIELDS HAVE ENERGY DEPENDENCE

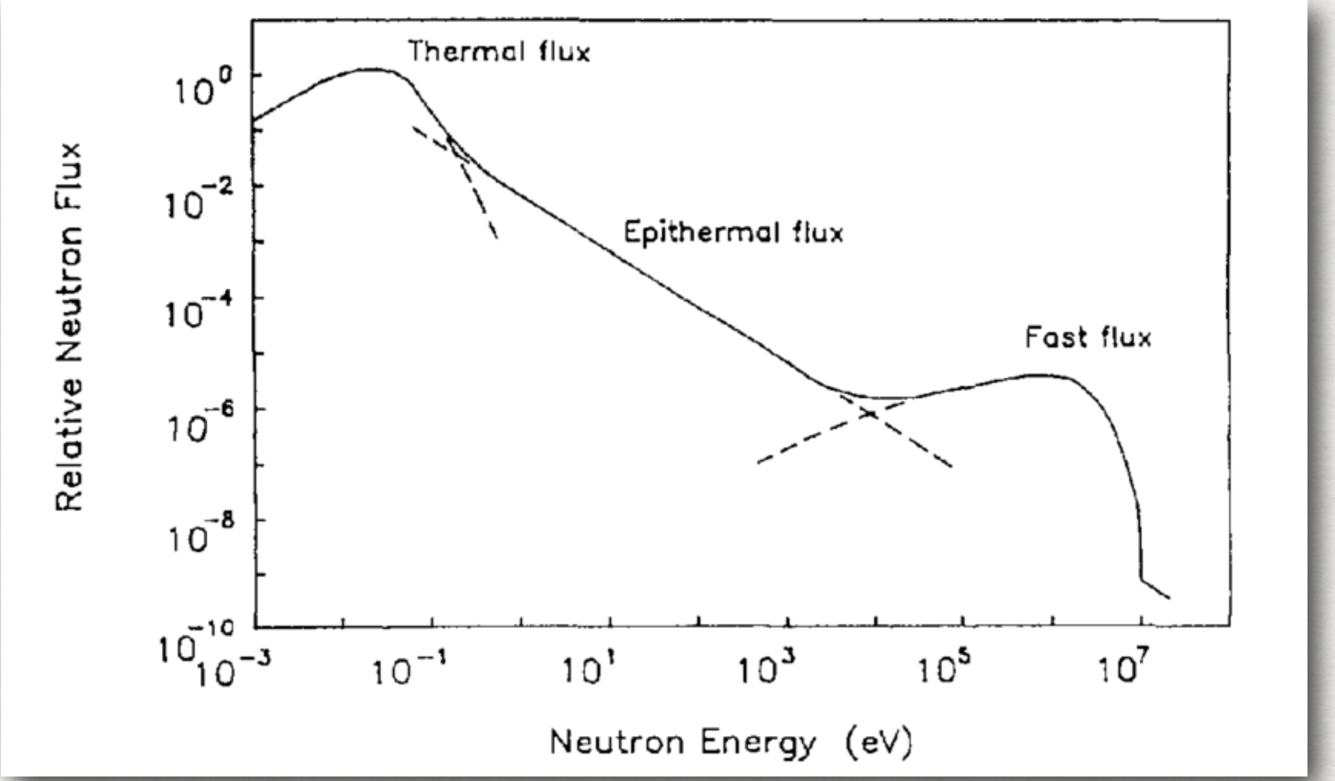
- **England and Rider files have three "fake" energies: thermal, fission spectrum and 14 MeV.**
- Major requirements:
    - Enable $\mathrm{CFY}_i(E)$ and $\mathrm{IFY}_i(E)$
    - Provide covariance between different yields at different energies
    - Provide covariance between CFY and IFY
- **This argues for using XYs1d's for the energy dependence of yields**





M. Gooden et al. Nucl. Data Sheets 131: 319-356 (2016)

PROVIDING THIS ENERGY DEPENDENCE IS TRICKY



- **Thermal and fission spectrum values are spectrum averages tied to high precision data (e.g. Maeck)**
- **Must do GLSQR fit to this data**
  - Energy dependence only known for a few IFY's
  - Energy dependence of CFY "inherited" from IFY
  - Need model of Z,A dependence of IFY to propagate energy dependence to CFY
- **Extra wrinkle: without unified grid, sum rules fail**
- **There are many sum rules we must obey**
  - 2 fragments (unless there's ternary fission, an issue for gas production)
  - baryon number conservation
  - charge conservation
  - …

- **Evaluation methodology will impact form of covariances we can generate**
  - Machine Learning (Random Forest, GLSQR, Mixture Density Networks, …)
  - Hauser-Feshbach-like (CGMF or YAHFC)
  - all of the above?
  - Joint BNL/LANL project should clarify within 1-2 year

Path forward is murky, but are things to consider:

1. **need correlation matrix between yields**
2. **mass or charge yields (Y(A) or Y(Z)) are well known and constrain yields like total cross section constrain partials**