RECOMMENDED VALUES OF THE DELAYED NEUTRON YIELD FOR : U-235 ; U-238 AND Pu 239

E. FORT, V. ZAMMIT-AVERLANT, M. SALVATORES, A. FILIP

SUMMARY :

Today there is no well established theoretical model to predict with the required accuracy the fission delayed neutron yield ν_d. In this field the recommended data result from the rare experimental data analysis or from purely phenomenological or semi-phenomenological models. There is another source of valuable information: the related integral data or β_{eff} data.

In this report we demonstrate ,via a carefull analysis of the experimental methods leading to revisited experimental β_{eff} values and associatated uncertainties, that for the major nuclei the ν_d evaluated data are of acceptable quality.

For U-235,U-238 and Pu-239 we recommend ν_d values for the thermal and the fast reactor ranges which have been obtained from a statistical consistent adjustment to the β_{eff} data.

In the course of this study we show that the energy dependance of ν_d ,suspected from a physics point of view ,probably exists with a different magnitude according to the nucleus. Concerning the major nuclei it is of negligible importance for the applications.

The improvement of the higher Pu isotopes and minor actinides data should be the strong reason to develop the theoretical investigations of the delayed neutron generation mechanism at the same level as the necessary experimental activity.

TABLE of CONTENTS

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I. INTRODUCTION

The delayed neutron yield ν_d is a basic nuclear constant used to calculate the so called β_{eff} parameter. This one characterises, for safety purpose, a given critical medium by a proper reactivity margin which obviously depends on the proportion of delayed neutrons in the total fission neutron emission. In addition, the knowledge of the β_{eff} parameter is essential for the purpose of normalization for the reactivity and for the time characteristics of transients

For the reasons above mentioned, β_{eff} is an important parameter which is considered in the High Priority Request List (HPRL).

J. ROWLANDS [1] on behalf of the Reactor Designers and Physicists took up the recommendations by P.HAMMER [32] and defined target accuracies for β_{eff} .

These are : $\pm$ 3 % for mock-ups
$\pm$ 5 % for Power Reactors

All these uncertainties are expressed in 1 σ unit.

The questions are :

- Are the data and calculationnal methods, today available, of sufficient quality to predict the β_{eff} with the required accuracy ?

- How to improve them if necessary ?

Our answer to these questions will be developed along the following items (on page 3), which all have in common the objective to demonstrate that the integral measurements of β_{eff} are a valuable source of information, complementary to microscopic measurements. To note that a complete demonstration should be addressed in another paper.

Most of the conclusions are based on the thesis work by V. ZAMMIT-AVERLAND [2].

In this paper the following notations have been used :

as variable		as upperscript or underscript	
F	: Fission rate	i	: isotope
I	: Importance	k	: temporal group
ν	: neutron Yield	g	: energy group
α	: relative abundance	f	: fission
λ	: Decay constant	t	: total
Φ	: direct flux	p	: proton
Φ^*	: adjoint flux	d	: delayed
χ	: neutron energy distribution	z	: given zone in the reactor
Σ	: macroscopic cross-section		

Some examples:

$\nu_{d_i}(E)$: energy dependent delayed neutron yield for the isotope i

$F_{U5}^{\exp}(\vec{0})$: Fission rate of U-235 measured at the centre of the reactor core

$\Sigma_{f_i}^{g,z}$: macroscopic fission cross-section of the isotope I ,for the energy group g, in the zone z of the reactor

II. FORMALISMS

KEEPIN in 1956 first defined the β_{eff} parameter in the frame of a general assumption of energy independence of the delayed neutron yields.

The reference expression for β_{eff} is as follows :

$$\beta_{eff} = \frac{\sum_i \sum_{k=1}^{6} \overline{\nu_{d_i}} . \alpha_i^k . \int_V \left\{ \int_0^\infty \sum_{fi}(E,\vec{r}).\Phi(E,\vec{r})dE . \int_0^\infty \chi_{d_i}^k(E').\Phi^*(E',\vec{r})dE' \right\} d\vec{r}}{\sum_i \int_V \left\{ \int_0^\infty \nu_{t_i}(E).\sum_{fi}(E,\vec{r}).\Phi(E,\vec{r})dE . \int_0^\infty \chi_{t_i}(E').\Phi^*(E',\vec{r})dE' \right\} d\vec{r}} \qquad (1)$$

in which i refers to one fissile isotope of the fuel, k to one out of the 6 temporal groups.

The reference to the temporal aspects is justified for sake of consistency with the kinetic parameter calculations used for transient description, but this is not a necessity.

As a matter of fact if one notes that the sum $\sum_{K=1}^{6} \alpha_i^k \chi_{di}^k (E')$ which depends on the emitted neutron energy E' only can be read as $\chi_{di}(E')$ that is the total delayed neutron spectrum at equilibrium (ie, the spectrum after any prompt and/or delayed transient extinction), an expression equivalent to (1) can be derived :

$$\beta_{eff} = \frac{\sum_i \overline{\nu_{d_i}} . \int_V \left\{ \int_0^\infty \sum_{fi}(E,\vec{r}).\Phi(E,\vec{r})dE . \int_0^\infty \chi_{d_i}(E').\Phi^*(E',\vec{r})dE' \right\} d\vec{r}}{\sum_i \int_V \left\{ \int_0^\infty \nu_{t_i}(E).\sum_{fi}(E,\vec{r}).\Phi(E,\vec{r})dE . \int_0^\infty \chi_{t_i}(E').\Phi^*(E',\vec{r})dE' \right\} d\vec{r}} \qquad (2)$$

From this expression one observes that the β_{eff} can be regarded as describing a quasi static situation.

In the recent years ,a possible influence of the incident energy on the delayed neutron yield has been evoked in the energy range where the invariance of ν_d was systematically considered so far.

The energy dependence here mentioned has nothing in common with the factors which also influence : odd-even effect ,contribution from different fission chances ..,...

We refer to the dependence which appears in the first few MeV where the first chance exists solely. It is due to the energy dependence of the direct and cumulative precursor yields [3], [4], which is well admitted although the laws are not well known.

Actually the precursors are located in the vicinity of neutron closed shells so that the neutron binding energy be small compared to Q_{β}-.

For one fissile nucleus :

$$\nu_d(E) = \int_0^{\infty} \bar{n}_d(t,E)\, dt \tag{3}$$

$$\bar{n}_d(t,E) = \sum_{\ell} \lambda_{\ell}\, Pn_{\ell}\, Y_{\ell}(E)\, e^{-\lambda_{\ell} t} + \sum_{H} \lambda_H\, P_{nH}\, Y_H(E)\, e^{-\lambda_H t} \tag{4}$$

$\bar{n}_d(t,E)$ represents the total average number of delayed neutrons emitted at the time t after a scission induced by a neutron of energy E.

In relationship (4) distinction is made between the precursors (produced with a cumulative yield Y and decaying with a time constant λ) belonging to the light peak labelled ℓ and the heavy peak labelled H.

When the incident energy increases the yields of the light peak decrease on average. The behaviour is reverse for the precursors of the heavy peak. The global energy dependence finally results from the competition of these two antagonists components (a competition which limits the amplitude of the variation),in addition to the other effects such as the odd-even effect....

It is worth mentioning that the same reasons which make possible a dependence of $\bar{\nu}_{di}$ will obviously affect the K temporal group abundance's α_i^k and decay constants λ_i^k, but our review is restricted to the consequences on $\bar{\nu}_d$ and β_{eff}.

To take into account the energy dependence of $\bar{\nu}_{di}$ the original KEEPIN's formalism has been modified by A. FILIP [5], keeping the basic meaning of β_{eff} as the ratio of the delayed neutron and the total neutron productions.

$$\beta_{eff} \equiv \sum_i \beta_{eff_i} = \frac{\sum_i \int_V \left\{ \nu_{d_i}(E) \cdot \sum_{f_i}(E,\vec{r}) \cdot \phi(E,\vec{r})\, dE \cdot \int \chi_{d_i}(E') \cdot \phi^*(E',\vec{r})\, dE' \right\} d\vec{r}}{\sum_i \int_V \left\{ \nu_{t_i}(E) \cdot \sum_{f_i}(E,\vec{r}) \cdot \phi(E,\vec{r})\, dE \cdot \int \chi_{t_i}(E') \cdot \phi^*(E',\vec{r})\, dE' \right\} d\vec{r}.} \quad (5)$$

To note that the denominator is the normalization integral in the classical first order eigenvalue perturbation theory.

It has been abundantly demonstrated by [2] that both formalisms and related codes give identical results when used in similar conditions.

In the following we will refer to this modified formalism as the KEEPIN's modified formalism.

The β_{eff} calculation is heavy since it requires a complete core calculation : the β_{eff} value depends on quantities which are functions of energy and space, such as $\varnothing(E,\vec{r})$ the neutron flux, $\varnothing^*(E',\vec{r})$ the importance function, and $\sum_{fi}(E,\vec{r})$ the fission rate of the isotope i.

The calculationnal methods used by V. ZAMMIT-AVERLAND are those of the recent code system ERANOS. They are deterministic methods based on the BOLTZMAN equations solution in the P_N transport approximation. In cell calculation (performed using ECCO code) the neutron showing down is treated in a fine group mesh (1968 gr for the energy interval 19.64 MeV down to 10^{-5} eV) with consideration to the collision anisotropy. Probability tables are used to treat heterogeneous configurations and self shielding effects.

The ERALIB1 multigroup (1968 gr) library associated to ERANOS has been derived from JEF2 by a consistent statistical adjustment on the indications from an integral data base of 355 integral data including most of the media considered in β_{eff} measurement. The spatial calculations are based on a R, Z modelization and are performed using data condensed into a consistent 33 gr scheme.

The performances demonstrated in the reference [5] of this "formulaire" for fast core calculations are such that the main neutronic parameter (critical mass, Bucklings, k^+ and k^∞, spectral indices of different types, ... etc) are calculated with excellent accuracy and negligible biases, that is one of the most important general conditions for microscopic data validation on integral data.

This means that the above quantities $\varnothing, \varnothing^*, \sum_{fi}$ are calculated in a very reliable way for the β_{eff} measurements.

It follows that the values of β_{eff} calculated with the system ERANOS + ERALIB are dependent on the quality of nuclear data for ν_{di} and χ_{di} essentially, since the biases on the other nuclear data have been demonstrated to be small. These nuclear data are in a limited number.

In these conditions the improvement of the non temporal constants (related to delayed neutron emission) by a statistical adjustment procedure is possible also with a limited number of experimental β_{eff} data, provided some additional conditions are satisfied, as shown later on.

The ERANOS + ERALIB1 system is perfectly adapted to fast systems and to a less degree to thermal ones. It is important to indicates that for these ones this has been possible thanks to the performances of the cell code ECCO and also to the thermal integral data included in the adjustment procedure.

To calculate the very fast systems like the LOS ALAMOS bare spheres a special data base SHIVA [7] has been used. In SHIVA the cross sections have been treated in 172 groups with a specific weighting function and have received a P_5 expansion. Because of the very simple geometry the modelization is 3D.

III VALIDATION OF $\bar{\nu}_{di}$ DATA ON β_{eff} DATA

Since the formalism relating β_{eff} values to ν_{di} data is well established and assuming a correct calculation for the sensitivity coefficients , then it becomes obvious that reliable information on ν_{di} can be derived from β_{eff} measurements.

The first practical demonstration of this can be found in the numerous works by FILIP and D'ANGELO [24] and by PANG [32] who have also demonstrated that accurate ν_{di} values can be obtained, especially when the β_{eff} .are measured in « clean » cores.

But another important general condition for ν_{di} data validation is that the integral data base contains as numerous as possible, with sensitivities extended over the whole energy range of interest.

This is the reason why an important effort has been devoted in [2] to :

a) Realize a data base as large as possible.

b) Carefully analyse the various experimental methods in order to revisit the experimental β_{eff} values and the associated uncertainties.

For what concerns the U fuelled mock-ups the integral information extends over a large energy range from thermal to the very fast range (fission spectrum) but excluding the 14 MeV range. For what concerns the Pu fuelled criticals the available information is restricted to the fast and very fast ranges.

In other words, the validation for U isotopes (^{235}U and ^{238}U) will be effective from the thermal energy to a few MeV, excluding the range where there is a competition between the 2^{d} and the 3^{rd} chances fission.

For Pu the information is on the isotope 239 only.

The reliable information will be restricted to the fast and very fast ranges, i.e. from a few KeV to a few MeV.

On the whole, 21 β_{eff} measured data have been considered, that is significantly more than the only known similar work that is the one by A. D'ANGELO [8].

Among them, 11 are related to U isotopes solely, while 2 only are related to Pu solely. The 8 remaining data give information on the isotopes all together.The data have been obtained by the following technics :

- Californium Source : Experiments R2, ZONA 2 performed in MASURCA
  7A, 7B, 9C1, 9C2 performed in SNEAK.

- Covariance : Experiments CRef, PuCSS, RSR, U9, UFe-Ref, UFe-Leak performed in ZPR.

- Frequencies : Experiments XIX-I, XIX-3 performed in FCA
  MISTRAL performed in EOLE
  R2, ZONA 2 performed in MASURCA

- α, Rossi : Experiment R2 performed in MASURCA.

The experiments R2 and ZONA 2 of the so called international programme BERENICE are particularly attractive since 3 different technics have been used. An analysis of the differences in β_{eff} values could be used to try to quantify the systematic errors relevant to each experimental method.

It is a general remark that differences are observed in the values published in the literature for a same experiment and a same technic.

This can be understood if one notes that a part of the experimental β_{eff} value is calculated using calculational methods (and modelizations) and nuclear data bases.

Symbolically ones writes :

$$\beta_{eff} = P_m \times P_c$$

Where P_m and P_c refer to the measured and calculated parts respectively.

Although the differences are not dramatic they justified the careful investigation which has been undertaken in order to understand whether they could be due to a reanalysis of the raw experimental data or to a recalculation of P_C.

In particular, the P_c have been systematically recalculated with our methods and our data bases, at least for a complete consistency with the β_{eff} values calculated in view of a future statistical adjustment.

This way of doing is perfectly justified since P_c depends on the global neutronic characteristic of a core (see the various expressions of P_c) and not on the delayed neutron emission.

These recalculations, resulting in correction to the published values, have been performed for the cores for which the information was available, i.e. for all experiments but SNEAK experiments.

In what follows we describe the details of the corrections which have been made.

Experimental data revisited.

- Method of the Californium Source.

As used for the BERENICE experiments where the Cf source is placed at the core center, the β_{eff} has been obtained by :

$$\beta_{eff} = \frac{S_{Cf} \cdot I_{Cf}}{\Delta\rho \cdot P} = \frac{S_{Cf}}{\Delta\rho \cdot F_{U_5}^{exp}(\bar{0})} \cdot \frac{1}{INR} \cdot \frac{I_{Cf}(\bar{0})}{I_f(\bar{0})}$$

INR: Relative Normalization Integral, that is the integral (dominator of (5)) normalized to the référence center values.

The calculated part is represented here by :

$$P_c = \frac{1}{INR} \cdot \frac{I_{Cf}(\bar{0})}{I_f(\bar{0})}$$

To be noted that a more accurate normalization would have required the measurement of the fission rates of all the fissiles isotopes of significant concentration in the fuel, together with traverses of the reference fission rates and of their importance.

For the SNEAK experiments the calculated part has been reestimated but no difference has been observed.

- Method of Frequencies

This method has been used in the BERENICE, FCA and MISTRAL programs.

- In the BERENICE experiments βeff is obtained by :

$$\beta_{eff}\,2 = \frac{1}{(1+|\rho\$|)^2} \cdot \frac{V_1 V_2}{DSPI} \cdot \frac{1}{F_{U_5}^{\exp}(\vec{0})} \cdot \frac{2.D}{K_{cal}}$$

The calculated part P_c is :

$P_c = \frac{2.D}{K_{Cal}}$; $with\ K_{cal} = \frac{F}{F_{réf}^{cal}(\vec{O})}$; F represents the total fission rate over the whole core.

- In the FCA experiments [9] :

$$\beta_{eff}\,2 = \frac{1}{(1+|\rho\$|)^2} \cdot \frac{V_1 . V_2}{DSPI} \cdot \frac{1}{\sum_i F_i^{\exp}(\vec{0})} \cdot \frac{2.D.\sum_i F_i^{cal}(\vec{0})}{F}$$

The calculated part P_c is represented, here, by :

$$P_C = \frac{2.D.\sum_i F_i^{cal}(\vec{O})}{F}$$

It is has been checked that BERENICE's and FCA's expressions give equivalent results ,given that the contributions of the fissile nuclei other than U-235 are not significant .

- In the MISTRAL experiments [10]

$$\beta_{eff}\,2 = \frac{1}{(1+|\rho\$|)^2} \cdot \frac{V_1 . V_2}{DSPI} \cdot \frac{1}{T} \cdot \frac{m_5}{M_5} \cdot \frac{\sigma_{f_{U235}}^{CF}}{\sigma_{f_{U235}}^{Coeur}} \cdot a \cdot b \cdot D_\nu \cdot \frac{F_{/cell}}{F_{U235}^{CF}}$$

Where :

V_1 an V_2 : are average indications from measurements by 2 fission chambers.

DSPI : Spectral Power Density of Interaction of the 2 measurements chains.

m_5 : mass of U-235 in a fission chamber positioned in the core center.

M_5 : mass of U-235 in the core.

$\sigma_{f_{U235}}^{FC}$; average fission cross section of U-235 in the fission chamber.

a, b : respectively axial and radial form factor.

Dν : part of the so called DIVEN factor D related to the prompt neutron emission.

F_{U5}^{FC} : fission rate in the fission chamber (in the center of the core).

F_{cell} : total fission rate divided by the number of cells.

The « calculated part » is related to the quantity :

$$P_C = \frac{\sigma_{f_{U235}}^{CF}}{\sigma_{f_{U235}}^{coeur}} . a . b . D_\nu . \frac{F_{cell}}{F_{U_{235}}^{CF}}$$

- α-Rossi Method

This method has been used in the frame work of BERENICE program for the R2 experiment.

For this particular type of frequencies technic, β_{eff} is obtained by :

$$\beta_{eff}\,2 = \frac{D}{(1+|\rho\$|)^2 \cdot \frac{2F.S.\Delta t}{N} \cdot \frac{a}{a+C}} = \frac{1}{(1+|\rho\$|)^2 \cdot \frac{S}{N} \cdot \Delta t \cdot \frac{a}{a+C}} \cdot \frac{1}{F_{U_5}^{\exp}(\bar{0})} \cdot \frac{1}{2} \cdot \frac{D}{K_{cal}}$$

Where :

C : counting rate per time unit.

l : prompt decay constant.

N : total counting N = CΔt.

S : total number of « correlated » events.

D : complete DIVEN factor.

The calculated part is represented by the quantity :

$$P_C = \frac{1}{2}\frac{D}{K_{cal}}$$

- Covariance method

This method due to E. BENNET [11] has been used for β_{eff} measurement in ZPR mock-up, also in MASURCA (BERENICE program) (and in FCA for the program XIX, but not analysed by us).

The β_{eff} value is given by :

$$\beta_{eff}\,2 = \frac{3}{2}\frac{D}{\tau \cdot \sigma_{12}^2 \cdot F \cdot \left((1+|\rho\$|)\right)^2} = \frac{1}{\tau \cdot \sigma_{12}^2 \cdot F^{\exp}(\bar{r}_0) \cdot (1+|\rho\$|)^2} \cdot \frac{3}{2} \cdot \frac{D \cdot F^{cal}(\bar{r}_0)}{F}$$

Where :

τ : counting time.

σ_{12}^2 : covariance of the countings by 2 detectors (labelled 1 and 2) during the time δ.

The calculated part P_c is represented by :

$$P_c = \frac{3}{2}\frac{D}{K_{cal}}$$

- For the experiments SHE-8, GODIVA, JEZEBEL what was available was the experimental value of the parameter β_{eff}/Λ, Λ neutron life-time in the core. In these conditions, the β_{eff} was written as :

$$\beta_{eff} = \frac{\beta_{eff}}{\Lambda} \times \Lambda$$ where the calculated P_c is :

$$P_c = \Lambda$$

Special mention should be made of the so called DIVEN factor since it appears in most of the recalculated parts of the β_{eff}.

This dispersion factor is the extension to the reactor neutron spectrum of the original DIVEN factor used to describe the statistical dispersion of the prompt neutrons emitted per one fission induced by monocinetic neutrons.

For a given energy E, the dispersion factor is :

$$D = \overline{\nu(E).(\nu(E)-1)} \,/\, \overline{\nu(E)^2}$$

where the bars denote an arithmetic averaging.

It appears that, for the most important isotopes U-235, U-238, Pu-239, this « microscopic » dispersion factor is constant with energy, at least for $E \leq 500$ KeV.

It follows that the initial definition of the « integral » DIVEN factor :

$$D = \frac{\int_V \int F(E,\vec{r})dEd\vec{r} \int_V \int \nu_p(E).\left(\nu_p(E)-1\right).F(E,\vec{r})\,dE.\left[\int \chi_f(E').\phi^*(E',\vec{r})\,dE'\right]^2 d\vec{r}}{\left[\int_V \int \nu_p(E).F(E,\vec{r})\,dE.\int \chi_f(E').\phi^*\left(E',\vec{r}\right)dE'\,d\vec{r}\right]^2}$$

becomes in an approximation only valid for a fission reactor spectrum :

$$D = \frac{\overline{\nu_p(E).(\nu_p(E)-1)}}{\overline{\nu_p(E)}^2}.\frac{\int_V \int F(E,\vec{r})dEd\vec{r}.\int_V \int \overline{\nu_p(E)}^2.F(E,\vec{r})dE.\left[\int I(E,\vec{r})dE\right]^2 d\vec{r}}{\left[\int_V \int \overline{\nu_p(E)}F(E,\vec{r})dE.\int I(E,\vec{r})dEd\vec{r}\right]^2}$$

This expression of the DIVEN factor can be split into 2 terms :

- one related to the dispersion of the emitted neutrons denoted D_ν,approximately constant with the energy (at least for E< 500 KeV),

- a second one related to the « effectiveness » of these emitted neutrons and expressed as a function of space and energy over the core volume. This term is denoted D_s and is expressed as :

$$D_S = \frac{\int_V \int F(E,\vec{r})dEd\vec{r}.\int_V \int \overline{\nu_p(E)}^2.F(E,\vec{r})dE.\left[\int I(E,\vec{r})dE\right]^2 d\vec{r}}{\left[\int_V \int \overline{\nu_p(E)}F(E,\vec{r})dE.\int I(E,\vec{r})dEd\vec{r}\right]^2}$$

A more general (and exact) expression for D requires that the microscopic dispersion factor D_ν be weighted by an adequate and consistent factor.

We have chosen the one proposed by the FCA team, so that D_ν is expressed as :

$$D_V = \frac{\iiint_V \overline{\nu_p(E).(\nu_p(E)-1)} F(E,\vec{r})dE.\left[\int I(E,\vec{r})dE\right]^2 d\vec{r}}{\iiint_V \overline{\nu_p(E)}^2 F(E,\vec{r})dE.\left[\int I(E,\vec{r})dE\right]^2 d\vec{r}}$$

It is checked that the relationship $D = D_S.D_V$ is respected.

In the effective recalculation of D_ν, the variance $\sigma^2_{\overline{\nu_p(E)}}$ was expressed as a linear function of $\overline{\nu_p}(E)$ as proposed by J. FREHAUT [12].

$$\sigma^2_{\overline{\nu_p}(E)} = a \times \overline{\nu_p}(E) + b$$

This general relationship has been adapted to each nucleus of interest [13] as follows :

For Pu-239 $\sigma^2_{\overline{\nu_p}} = 0.27 \times \nu_p(E) + 0.541$

U-235 $\sigma^2_{\overline{\nu_p}} = 0.224 \times \nu_p(E) + 0.718$

U-238 $\sigma^2_{\overline{\nu_p}} = 0.172 \times \nu_p(E) + 0.872.$

The recalculation of P_c has been performed for the experiments which were sufficiently documented. It follows that the recalculation has been complete or simply partial according to the available information (for example, for the DIVEN factor the calculation concerned D_ν and D_s or was simply limited to D_ν).

The table 2 lists the parameters which have been modified and compares the recalculated and the published experimental values.

Mock-up	Method	βeff (pcm) Published	βeff (pcm) Recalculated	Modified parameters
R2	Californium source	735	755.0	INR - I_{Cf}/I_f
	Frequencies	711	727.6	F/Fref - Dν - Ds
	α-Rossi	728	745.0	F/Fref - Dν - Ds
ZONA2	Californium source	356	359.1	INR - I_{Cf}/I_f
	Frequencies	338	350.0	F/Fref - Dν - Ds
7A	Californium source	395	395.0	--
7B	Californium source	429	429.0	--
9C1	Californium source	748	748.0	--
9C2	Californium source	416	416.0	--
CRef	Covariances	381	383.6	Dν
PuCSS	Covariances	222	223.4	Dν
RSR	Covariances	335	337.3	Dν
U9	Covariances	725	731.4	Dν
UFeRef	Covariances	667	670.8	Dν
UFeLeak	Covariances	672	675.8	Dν
XIX-1	Frequencies	733	734.4	F/Fref - Dν - Ds
XIX-3	Frequencies	252	252.3	F/Fref - Dν - Ds
MISTRAL-1	Frequencies	788	789.7	Dν
SHE-8	Cinetic parameter	696	696.0	Λ
GODIVA	Cinetic parameter	645	603.1	Λ
JEZEBEL	Cinetic parameter	190	143.1	Λ

Table 2 : List of integral data (βeff data) considered in the present analysis, their published and revised values resulting from parameter modifications

Uncertainties revisited

a) Measured parameters

To be fully consistent with the re-evaluation of β_{eff} and to prepare the adjustment process the uncertainties on β_{eff} experimental values have to be re-estimated. This has been made by considering all the parameters including those which have been measured. The parameters are :

- S_{Cf}, $\Delta\rho$, $F_{ref}^{exp}(\vec{O})$, $1+Q$, DSP, S/N, $\frac{a}{a+T}$, Δt for the class of measured parameters.

- INR, D, $I_{Cf} / I_f(\vec{\theta})$, K for the class of calculated parameters.

The uncertainties ε_β on β_{eff} have been calculated using the error propagation law :

$$\varepsilon^2_{\beta_{eff}} = \sum_i S^2_{x_i} \varepsilon_{x_i} + 2 \sum_{i,J} S_{x_i} S_{x_j} Cov(x_i, x_j)$$

The S_{xi} refers to the relative derivative of β_{eff} with respect to the x_i parameters measured with a total uncertainty ε_{x_i} .

The covariance terms can be neglected when the parameters have been measured or calculated in uncorrelated ways.

$$\varepsilon^2_{\beta_{eff}} = \sum_i \left(S^2_{x_i} \varepsilon^2_{x_i} \right)$$

Therefore the general expressions adopted for the total standard deviation depend on the type of technique used. They are as follows :

- Californium Source method

$$\varepsilon_{\beta_{eff}} = \sqrt{\varepsilon_{[S_{Cf}]}^{2} + \varepsilon_{[\Delta\rho]^2}^{2} + \varepsilon_{\left[F_{ref}^{exp}(\bar{0})\right]^2}^{2} + \varepsilon_{\left[I_{Cf}/I_f(\bar{0})\right]}^{2} + \varepsilon_{[INR]^2}^{2}}$$

since

$$\frac{S_{Cf}}{\beta_{eff}} \cdot \frac{\partial \beta_{eff}}{\partial S_{Cf}} = \frac{I_{Cf}/I_f(\bar{0})}{\beta_{eff}} \cdot \frac{\partial \beta_{eff}}{\partial\, I_{Cf}/I_f(\bar{0})} = 1 \qquad \frac{\Delta\rho}{\beta_{eff}} \cdot \frac{\partial \beta_{eff}}{\partial \Delta\rho} = \frac{INR}{\beta_{eff}} \cdot \frac{\partial \beta_{eff}}{\partial INR} = -1$$

- Frequencies technique

$$\varepsilon\beta_{eff} = \frac{1}{2}\sqrt{\varepsilon^2_{\left[(1+|\rho_\$|)^2\right]} + \varepsilon\left[F_{ref}^{exp}(\bar{0})\right]^2 + \varepsilon^2_{[K_{cal}]} + \varepsilon^2_{[D]} + \varepsilon^2_{[DSP]}}$$ in general.

The technique used for the MISTRAL-1 experiment leads to the following relationship :

$$\varepsilon\beta_{eff} = \frac{1}{2}\sqrt{\varepsilon^2_{\left[(1+|\rho_\$|)^2\right]} + \varepsilon[F]^2 + \varepsilon^2_{[D]} + \varepsilon^2_{[DSP]}}$$

- α-Rossi technique

$$\varepsilon\beta_{eff} = \frac{1}{2}\sqrt{\varepsilon^2_{\left[\frac{\alpha}{\alpha+T}\right]} + \varepsilon[S/N]^2 + \varepsilon^2_{[\Delta t]} + \varepsilon^2_{[\rho_\$]} + \varepsilon^2_{[D]} + \varepsilon\left[F_{ref}^{exp}(\bar{0})\right]^2 + \varepsilon[K_{cal}]^2}$$

- Covariance method

$$\varepsilon\beta_{eff} = \frac{1}{2}\sqrt{\varepsilon^2_{[\tau\sigma_{12}]} + \varepsilon^2_{\left[(1+\rho_\$)^2\right]} + \varepsilon^2_{[D]} + \varepsilon^2_{[F]}}$$

The uncertainties on the measured part P_m of the β_{eff} are detailed in the table 3.

Mockup		S_{Cf}	$\Delta\rho$	F	ρ	DSP	S/N	$\tau\sigma^2$	P_m
R2	Cf. Source	1.6	1.5	1.8	--	--	--	--	**2.8**
	Frequencies	--	--	1.8	--	2.4	--	--	**3.0**
	α-Rossi	--	--	1.8	--	--	1.0	--	**2.1**
Zona2	Cf. Source	1.8	1.5	1.8	--	--	--	--	**2.8**
	Frequencies	--	--	1.8	--	2.4	--	--	**3.0**
7A	Cf. Source	1.5	1.0	1.5	--	--	--	--	**2.4**
7B	Cf. Source	1.5	1.0	1.5	--	--	--	--	**2.4**
9C1	Cf. Source	2.0	3.0	1.5	--	--	--	--	**3.9**
9C2	Cf. Source	2.0	3.5	1.5	--	--	--	--	**4.3**
CRef	Covariances	--	--	3.0	1.0	--	--	1.5	**3.5**
PuCSS	Covariances	--	--	3.0	1.0	--	--	1.5	**3.5**
RSR	Covariances	--	--	3.0	1.0	--	--	1.5	**3.5**
U9	Covariances	--	--	3.0	1.0	--	--	1.5	**3.5**
UFeRef	Covariances	--	--	3.0	1.0	--	--	1.5	**3.5**
UFeLeak	Covariances	--	--	3.0	1.0	--	--	1.5	**3.5**
XIX-1	Frequencies	--	--	1.8	1.5	3.0	--	--	**3.8**
XIX-3	Frequencies	--	--	1.8	1.5	3.0	--	--	**3.8**
Mistral-1	Frequencies	--	--	1.8	--	1.4	--	--	**2.3**

Table 3 : Uncertainties on the measured part of β_{eff}

b) Calculated parameters

We recall that the calculated parts P_c of β_{eff} have been obtained with the system ERANOS+ERALIB1. To calculate the uncertainties affecting these quantities we followed the calculationnal scheme adopted in ERANOS based on given energy and spatial meshes to calculate the sensitivity coefficients. We used also the covariance matrices associated to ERALIB1 Library.

To be short, we will only report here the final results, the details of the calculation can be found in the reference [2].

- Relative Integral of Normalization INR

We have :

$$INR = \frac{\sum_i \sum_z \left[\sum_g \nu_i^g \sum_{f_i}^{g,z} \varnothing^{g,z} \cdot \sum_g \chi_i^g \varnothing^{*g,z} \right]}{\sum_g \sum_{f_{ref}}^{g,o} \varnothing^{g,o} \sum_g \chi_f^g \varnothing^{*g,o}}$$

The sensitivity to the prompt neutron yield in the energy group g

$$S_{INR/\nu_i^g} = \frac{\sum_z \left[\nu_i^g \sum_{f_i}^{g,z} \varnothing^{g,z} \sum_g \chi_i^g \varnothing^{*g,z} \right]}{\sum_i \sum_z \left[\sum_g \nu_i^g \sum_{f_i}^{g,z} \varnothing^{g,z} \sum_g \chi_i^g \varnothing^{*g,z} \right]}$$

To note that the indirect effect terms have not been considered.

Concerning the sensitivity coefficients relative to the fission cross section one has to differentiate the reference isotope from the others :

$$S_{INR/\Sigma_{f_{ref}}^{v}} = \frac{\sum_{z}\left[\nu_{ref}^{g}\cdot\sum_{f_{ref}}^{g,z}\varnothing^{g,z}\sum_{g}\chi_{ref}^{g}\varnothing^{*g,z}\right]}{\sum_{i}\sum_{z}\left[\sum_{g}\nu_{i}^{g}\sum_{f_i}^{g,z}\varnothing^{g,z}\sum_{g}\chi_{i}^{g}\varnothing^{*g,z}\right]} - \frac{\sum_{f_{ref}}^{g,o}\varnothing^{g,o}}{\sum_{g}\sum_{f_{ref}}^{g,o}\varnothing^{g,o}}$$

$$S_{INR/\Sigma_i^g} = \frac{\left[\nu_i^g\cdot\sum_{f_i}^{g,z}\varnothing g,z\cdot\sum_{g}\chi_i^g\varnothing^{*}g,z\right]}{\sum_{i}\sum_{z}\left[\sum_{g}\varnothing^{g,z}\sum_{f_i}^{g,z}\varnothing g,z\sum_{g}\chi_i^g\varnothing^{*}g,z\right]}$$

The sensitivity coefficients are normalized in a way which eliminates the imperfections (minimized) resulting from the multigroup scheme and also from neglecting the fissiles isotopes existing in very small quantities.

The uncertainty on INR is given by :

$$\left[S^{+}_{INR/\nu_p,\Sigma_f}\cdot V_{\nu_{p,\Sigma_f}}\cdot S_{INR/\nu_p,\Sigma_f}\right]^{1/2}$$

$S_{INR}/\nu_p,\Sigma_f$ is the sensitivity matrix of INR with respect to ν_p and Σ_f.

V_{ν_p,Σ_f} : covariance matrix relative to ν_p, and Σ_f.

- DIVEN factor

Neutron Diven factor.

Having adopted the following notations :

$$F_i^{gz} = \sum_{f_i}^{gz}\cdot\phi^{gz} \qquad F_i^{z} = \sum_{g}\sum_{fi}^{gz}\cdot\phi^{gz} \qquad F_i = \sum_{z}\sum_{g}\sum_{f_i}^{gz}\cdot\phi^{gz}$$

$$I_i^{gz} = \chi_i^g \,.\, \phi^{*gz} \qquad I_i^z = \sum_g \chi_i^g \,.\, \phi^{*gz} \qquad I_i = \sum_z \sum_g \chi_i^g \,.\, \phi^{*gz}$$

D_V reads as:

$$D_V = \frac{\sum_z \sum_i \left[\sum_g \left({\nu_{p_i}^g}^2 + (a_i - 1)\, \nu_{p_i}^g + b_i \right) . F_i^{gz} . \left[\sum_g I_i^{gz} \right]^2 \right]}{\sum_z \sum_i \left[\sum_g {\nu_{p_i}^g}^2 . F_i^{gz} . \left[\sum_g I_i^{gz} \right]^2 \right]}$$

The various sensitivity coefficients are written as :

$$S_{D_V/\nu_{p_i}^g} = \frac{\sum_z \left[\left(2{\nu_{p_i}^g}^2 + (a_i - 1)\, \nu_{p_i}^g \right) . F_i^{g,z} . \left[I_i^z \right]^2 \right]}{\sum_z \sum_i \left[\sum_g \left[\left({\nu_{p_i}^g}^2 + (a_i - 1)\, \nu_{p_i}^g + b_i \right) . F_i^{g,z} \right] . \left[I_i^z \right]^2 \right]} - 2 \frac{\sum_z \left[\nu_{p_i}^g . F_i^{g,z} . \left[I_i^z \right]^2 \right]}{\sum_z \sum_i \left[\sum_g \left[\nu_{p_i}^g . F_i^{g,z} \right] . \left[I_i^z \right]^2 \right]}$$

$$S_{D_V / \Sigma_{fi}^g} = \frac{\sum_z \left[\left(2{\nu_{p_i}^g}^2 + (a_i - 1)\, \nu_{p_i}^g + b_i \right) . F_i^{g,z} . \left[I_i^z \right]^2 \right]}{\sum_z \sum_i \left[\sum_g \left[\left({\nu_{p_i}^g}^2 + (a_i - 1)\, \nu_{p_i}^g + b_i \right) . F_i^{g,z} \right] . \left[I_i^z \right]^2 \right]} - \frac{\sum_z \left[{\nu_{p_i}^g}^2 . F_i^{g,z} . \left[I_i^z \right]^2 \right]}{\sum_z \sum_i \left[\sum_g \left[{\nu_{p_i}^g}^2 . F_i^{g,z} \right] . \left[I_i^z \right]^2 \right]}$$

$$S_{D_V/a_i} = \frac{\sum_z \left[\sum_g \left[a_i\, \nu_{p_i}^g \,.\, F_i^{g,z} \right] . \left[I_i^z \right]^2 \right]}{\sum_z \sum_i \left[\sum_g \left[\left({\nu_{p_i}^g}^2 + (a_i - 1)\, \nu_{p_i}^g + b_i \right) . F_i^{g,z} \right] . \left[I_i^z \right]^2 \right]}$$

$$S_{D_V/b_i} = \frac{\sum_z \left[\sum_g \left[b_i \,.\, F_i^{g,z} \right] . \left[I_i^z \right]^2 \right]}{\sum_z \sum_i \left[\sum_g \left[\left({\nu_{p_i}^g}^2 + (a_i - 1)\, \nu_{p_i}^g + b_i \right) . F_i^{g,z} \right] . \left[I_i^z \right]^2 \right]}$$

The uncertainty on D_v is :

$$\varepsilon_{D_v} = \sqrt{\varepsilon^2_{D_v/\nu_p,\Sigma_f} + \varepsilon^2_{D_v/a} + \varepsilon^2_{D_v/b}}$$

$$= \left(S^+_{D_v/\nu_p,\Sigma_f} \cdot V_{\nu_p,\Sigma_f} \cdot S_{D_v/\nu_p,\Sigma_f} + S^+_{D_v/a} \cdot V_a \cdot S_{D_v/a} + S^+_{D_v/b} \cdot V_b \cdot S_{D_v/b} \right)^{1/2}$$

- The « spatial » component of the DIVEN factor is written as :

$$D_S = \frac{\sum_i F_i \cdot \sum_z \sum_i \left[\sum_g \left(\nu^g_{p_i} \right)^2 \cdot F_i^{gz} \cdot \left[I_i^z \right]^2 \right]}{\left[\sum_z \sum_i \left[\sum_g \nu^g_{p_i} \cdot F_i^{gz} \cdot I_i^z \right] \right]^2}$$

Hence, the expressions of the sensitivity coefficients are :

$$S_{D_S}/\nu^g_{p_i} = \frac{2 \sum_z \left[\left(\nu^g_{p_i} \right)^2 \cdot F_i^{gz} \cdot \left[I_i^z \right]^2 \right]}{\sum_z \sum_i \left[\sum_g \left(\nu^g_{p_i} \right) \cdot F_i^{gz} \cdot \left[I_i^z \right]^2 \right]} - \frac{2 \sum_z \left[\nu^g_{p_i} \cdot F_i^{gz} \cdot I_i^z \right]}{\sum_z \sum_i \left[\sum_g \nu^g_{p_i} \cdot F_i^{gz} \cdot I_i^z \right]}$$

$$S_{D_S}/\Sigma^g_{f_i} = \frac{\sum_z F_i^z}{\sum_i F_i} + \frac{\left(\nu^g_{p_i} \right)^2 F_i^{gz} \left[I_i^z \right]^2}{\sum_z \sum_i \left[\sum_g \left(\nu^g_{p_i} \right)^2 \cdot F_i^{gz} \cdot \left[I_i^z \right]^2 \right]} \frac{2 \nu^g_{p_i} \cdot F_i^{gz} \cdot I_i^z}{\sum_z \sum_i \left[\sum_g \nu^g_{p_i} \cdot F_i^{gz} \cdot I_i^z \right]}$$

The uncertainty on D_s is obtained by :

- $$\varepsilon_{D_S} = \left(S^+_{D_S/\nu^g_p \cdot \Sigma^g_f} \cdot V_{\nu^g_p \cdot \Sigma^g_f} \cdot S_{D_S/\nu_p,\Sigma^g_f} \right)^{1/2}$$

- k_{cal} facteur

We remind of the expression of K_{cal} :

$$K_{cal} = \frac{F_{ref}^{cal}}{F}, \qquad \text{F representing the total fission rate over the reactor.}$$

The sensitivity coefficients are :

$$S_{K_{cal}/\Sigma_{f_{ref}}^{g,\vec{0}}} = \frac{F_{ref}^{g,\vec{\theta}}}{F^{\vec{0}}} - \frac{F_{ref}^{g,\vec{0}}}{F}$$

$$S_{K_{cal}/\Sigma_{f_i}^{g,z}} = \frac{F_i^{g,z}}{F}$$

$$S_{K_{cal}/\Sigma_{ref}} = \frac{F_{ref}}{F} - 1 \qquad (<0)$$

$$S_{K_{cal}/\Sigma_{fi}} = \frac{F_i}{F}$$

$$\sum_i S_{K_{cal}/\Sigma_{f_i}} = 0$$

The uncertainties on the calculated part of β_{eff} are given in the table 4.

Mockup		INR	D	K_{cal}	P_c
R2	Cf. Source	1.20	--	--	**1.20**
	Frequencies	--	2.24	1.16	**2.52**
	α-Rossi	--	2.24	1.16	**2.52**
Zona2	Cf. Source	1.21	--	--	**1.21**
	Frequencies	--	2.59	1.26	**2.88**
7A	Cf. Source	1.50	--	--	**1.50**
7B	Cf. Source	1.50	--	--	**1.50**
9C1	Cf. Source	1.50	--	--	**1.50**
9C2	Cf. Source	1.50	--	--	**1.50**
CRef	Covariances	--	2.60	--	**2.60**
PuCSS	Covariances	--	3.00	--	**3.00**
RSR	Covariances	--	2.72	--	**2.72**
U9	Covariances	--	2.17	--	**2.17**
UFeRef	Covariances	--	2.39	--	**2.39**
UFeLeak	Covariances	--	2.38	--	**2.38**
XIX-1	Frequencies	--	2.34	1.15	**2.61**
XIX-3	Frequencies	--	2.86	1.17	**3.09**
Mistral-1	Frequencies	--	2.04	--	**2.04**

Table 4 : Uncertainties on the « calculated » part of β_{eff}

Finally the recommended experimental β_{eff} values and the assigned uncertainties are displayed in the table 5.

The data in ( ) refer to values published in the literature.

		βeff (pcm)	Incertitudes (%)	
R2	Cf Source	755.0 (735)	**3.1**	**(2.2)**
	Frequencies	727.6 (711)	**2.0**	**(2.5)**
	α-Rossi	745.0 (728)	**1.6**	**(2.3)**
ZONA2	Cf. Source	359.1 (356)	**3.1**	**(2.0)**
	Frequencies	350.0 (338)	**2.1**	**(2.7)**
7A	Cf. Source	395.0 (355)	**2.8**	**(3.0)**
7B	Cf. Source	429.0 (429)	**2.8**	**(3.0)**
9C1	Cf. Source	748.0 (748)	**4.2**	**(3.2)**
9C2	Cf. Source	416.0 (416)	**4.6**	**(4.5)**
Cref	Covariances	383.6 (381)	**2.2**	**(2.0)**
PuCSS	Covariances	223.4 (222)	**2.3**	**(2.0)**
RSR	Covariances	337.3 (335)	**2.2**	**(2.0)**
U9	Covariances	731.4 (725)	**2.1**	**(2.0)**
UFeRef	Covariances	670.8 (667)	**2.1**	**(2.0)**
UFeLeak	Covariances	675.8 (672)	**2.1**	**(2.0)**
XIX-1	Frequencies	743.4 (733)	**2.3**	**(2.5)**
XIX-3	Frequencies	252.3 (252)	**2.5**	**(2.5)**
MISTRAL-1	Frequencies	789.7 (788)	**1.5**	**(1.5)**
SHE-8	Cinetique	696.0 (696)	**4.6**	**(4.6)**
GODIVA	Cinetique	603.1 (645)	**4.6**	**(4.6)**
JEZEBEL	Cinetique	143.1 (190)	**4.6**	**(4.6)**

Table 5 : Recommended values for β_{eff} and uncertainties

The experimental β_{eff} values depend on the experimental technique and on the « calculated » part.

The difference related to the calculated part are perfectly understood (in that respect, the use of our recent « formulaire » ERANOS+ERALIB1 is certainly an advantage). This is not the case for the differences due to the experimental technique although they are statistically acceptable.

Concerning the uncertainties the differences are modest but will play an important role in the adjustments.

This relatively favourable situation is very often the result of « compensating » effects. This is true, in particular, concerning the DIVEN factor as it can be seen in the table 6.

	Uncertainties on the DIVEN factor					
	D_ν		D_s		D	
	Present	Published	Present	Published	Present	Published
R2	1.92	4.00	1.16	0.5	2.24	4.03
ZONA2	2.28	4.00	1.22	0.5	2.59	4.03
Cref	2.35	2.00	1.11	0.5	2.60	2.06
PuCSS	2.78	2.00	1.11	0.5	3.00	2.06
RSR	2.47	2.00	1.13	0.5	2.72	2.06
U9	1.86	2.00	1.11	0.5	2.17	2.06
UFeRef	2.09	2.00	1.15	0.5	2.39	2.06
UFeLeak	2.08	2.00	1.15	0.5	2.38	2.06
XIX-1	2.08	2.00	1.08	0.5	2.34	2.06
XIX-3	2.62	2.00	1.15	0.5	2.86	2.06
MISTRAL-1	2.04	2.00			2.04	2.00

Table 6 : Uncertainties on D_ν and D_s. Comparison between the published and our proposed values

It appears also that, in most cases, the uncertainties are not calculated by the experimentalists but simply estimated on the basis of previous notorious results (the remarkable work by BENNET is often taken as a reference). We think that the uncertainties because of their important impact on the adjustment have to be carefully estimated, experiment by experiment.

IV IMPROVEMENT OF ν_D DATA BY A STATISTICAL ADJUSTMENT PROCEDURE

General conditions

An adjustment of nuclear parameters on Integral data can be considered as a transfer of information from integral data to microscopic data.

For this transfer to be effective and exact several conditions have to be fulfilled :

1. Integral data base as large as possible populated with independent, clean and informative data having sensitivity profiles extending over the whole energy range of interest.

2. Calculational methods in Neutronics with limited bias.

3. No distortion of information due to the nuclear data treatment.

4. Existence of an efficient theoretical tool to organize the transfer of information.

By precisely specifying the conditions of application of the statistical consistent adjustment method we have demonstrated the existence of the required tool (condition 4).

It is well accepted that the condition 3 is satisfied when using the modern versions of NJOY.

There is a continuous effort to demonstrate the condition related to point 2.

The point 1 could be considered as more questionable, especially for what concerns the aspect of the cleanness of the data (referring to the generally accepted definition of clear data). As a matter of fact there is a dependence of the β_{eff} value on :

a : the experimental method type

The differences observed when using different techniques (See R2 and ZONA 2 experiments) are of the order of magnitude of the uncertainties (and the argument relative to this point can be neglected).

b : the « Calculated » part

$$\beta^i_{eff} = P^i_m . P^i_c$$

$$\text{var}\,\beta^i_{eff} = \text{var}\,P^i_n + \text{var}\,P^i_c \qquad \text{in relative unit.}$$

Apart the only case of α-Rossi measurement on R2 (see tables 3 and 4) the uncertainty on the calculated part of β_{eff} is smaller than the one on the measured part. This is the consequence of the excellent performances of our ERALIB-ERANOS system to calculate in a very confident way the parameters involved in P_c.

This favourable situation justifies an acceptance of the condition 1.

The transfer of information is effective when the modified nuclear data are consistent with the integral data. This is obtained by a consistent statistical adjustment procedure governed by a generalized χ^2 minimization. The χ^2 value is used as an indicator of consistency and allows, with the help of complementary theoretical considerations to identify in an integral data base the data affected by a systematic error.

The power of the method has been demonstrated by the performances of the ERALIB1 library [6]. In the present adjustment a similar method has been used in order to satisfy the condition 4.

Covariance data

In data adjustment, the uncertainty information is an essential parameter to be determined as accurately as possible.

Unfortunately this uncertainty information is very scarce in both nuclear and integral data, especially for what concerns the covariance terms which play an important role .

In order to improve the conclusions of the adjustment we have generated these covariance terms on the basis of experimental data analysis and personal judgement.

Nuclear data

In order to consider the assumed energy dependence of $\nu_d(E)$, the energy range 10^{-5} ev - 20 MeV has been divided in 5 groups which are :

Group number	Boundaries		Range of expected information
Group 1	10^{-5} eV - 10 KeV	:	Thermal and epithermal range (LWR).
Group 2	10 KeV - 500 KeV	:	Fast reactor range + fission spectrum (FBR).
Group 3	500 KeV - 4 MeV	:	Fission spectrum + Fast reactor range (Spheres + FBR).
Group 4	4 MeV - 7 MeV	:	1st and 2nd chance fission competition (Spheres) and related effects.
Group 5	7 MeV - 20 MeV	:	Multichance fission competition-Poor information.

In JEF2, $\bar{\nu}_d(E)$ for U-235 [14] and Pu-239 [15] has been evaluated using LENDL's model [16] and $\bar{\nu}_p(E)$ evaluated data, while the U-238 delayed Yield [17] has been evaluated after analysis of the experimental data. It happens that the evaluation for this last nucleus is very similar to our own calculations using LENDL's model. This is the reason why the 3 nuclei U-235, U-238, Pu-239 have been treated in the same way for the error bar assignment.

The standard deviations derived from LENDL's parameter and $\bar{\nu}_p(E)$ uncertainties are so high (13 % - 22 %) that they have not been considered. The final error bars for the $\bar{\nu}_d$ of the main nuclei have been estimated on the basis of BLACHOT's estimations in the thermal range [18] and have been increased as a function of energy referring to the experimental information when available [see table 6].

For the higher Pu isotopes we referred to the experimental data only.

The covariance terms have been estimated by taking into account the competition between the various chances of fission.

Isotopes	group 1 0 - 10 keV	group 210 keV - 500 keV	group 3 500 keV - 4 MeV	group 4 4 MeV - 7 MeV	group 5 7 MeV - 20 MeV
U235	3 %	3 %	4 %	6 %	7 %
U238	6 %	6 %	7 %	9 %	10 %
Pu239	4 %	4 %	5 %	7 %	8 %
Pu240	10 %	10 %	12 %	14 %	15 %
Pu241	9 %	9 %	10 %	12 %	13 %
Pu242	20 %	20 %	21 %	23 %	24 %

Table 7 : Standard deviations assigned to the reviewed nuclei by energy group

The following correlation matrix (see table 8) has been assigned for all nuclei (This, of course, is an approximation).

Group	1	2	3	4	5
1	1	0.80	0.64	0.30	0.09
2	0.80	1	0.80	0.40	0.12
3	0.64	0.80	1	0.50	0.15
4	0.30	0.40	0.50	1.00	0.30
5	0.09	0.12	0.15	0.30	1.00

Table 8 : Standard deviation correlation matrix for the isotope i
(The same for all isotopes)

The uncertainties on the calculated β_{eff} obtained by propagation of the errors on $\nu_d(E)$, $\nu_p(E)$, $\Sigma_{fi}(E)$ are shown in the table 9. The last column indicates the uncertainty due to $\nu_d(E)$ only, in order to show this component is predominent in the total uncertainty on β_{eff} and give an additional justification of the adopted policy to validate $\nu_d(E)$ on β_{eff} data .

	U235	U238	Pu239	Pu240	Pu241	Pu242	Total	($\nu_d(E)$)
R2	2.30	1.55	--	--	--	--	**3.01**	**2.77**
Zona2	0.07	3.18	1.67	0.38	0.24	0.04	**3.80**	**3.62**
7A	0.24	3.30	1.54	0.13	0.09	0.00	**3.84**	**3.66**
7B	0.34	3.80	1.13	0.09	0.06	0.00	**4.16**	**3.98**
9C1	2.20	1.79	--	--	--	--	**3.07**	**2.83**
9C2	0.38	3.22	1.45	0.13	0.06	0.00	**3.75**	**3.55**
CRef	0.05	3.85	1.41	0.16	0.13	0.01	**4.28**	**4.10**
PuCSS	--	--	3.75	0.12	0.06	0.00	**3.93**	**3.75**
RSR	0.04	2.94	1.98	0.20	0.12	0.01	**3.76**	**3.56**
U9	1.40	3.49	--	--	--	--	**3.95**	**3.76**
UFeRef	2.90	0.02	--	--	--	--	**3.12**	**2.90**
UFeLeak	2.90	0.02	--	--	--	--	**3.13**	**2.90**
XIX-1	2.75	0.31	--	--	--	--	**3.00**	**2.77**
XIX-3	0.30	0.60	3.10	0.26	0.05	0.04	**3.38**	**3.18**
MISTRAL-1	2.63	0.78	--	--	--	--	**2.96**	**2.74**
SHE-8	2.97	0.05	--	--	--	--	**3.17**	**2.97**
GODIVA	3.36	0.16	--	--	--	--	**3.56**	**3.36**
JEZEBEL	--	--	4.28	0.43	0.06	0.00	**4.55**	**4.31**

Table 9 : Uncertainties on β_{eff} calculated from various contributions

Since these errors are greater than the errors on experimental data, an adjustment is relevant and there will be an effective transfer of information (in particular relatively to the uncertainties) from integral to nuclear data.

Integral Data

The final uncertainties on experimental values of β_{eff} have been determined elsewhere (see section II, table 5, page) in order to obtain a complete uncertainty matrix for the Integral data.

We will define here a correlation matrix for them.

It has been established by using the rule that statistic and systematic errors have to be quadratically added to obtain the final total error.

Let's define for the set of β_{eff} data :

- $V^{stat,k}_{\beta^{exp}_{eff}}$ the statistic uncertainty matrix for the parameter k in β^{exp}_{eff}

- $V^{syst,\ell}_{\beta^{exp}_{eff}}$ the systematic uncertainty matrix for the parameter l in β^{exp}_{eff}.

The complete uncertainty matrix is $V_{\beta eff}$

$$V_{\beta^{exp}_{eff}} = \sum_{k} V^{start,k}_{\beta^{exp}_{eff}} + \sum_{\ell} V^{syst,\ell}_{\beta^{exp}_{eff}}$$

Covariance terms exist in the k statistic covariance matrices in particular in those for which the parameter k (any parameter implied in any expression giving β_{eff} or β^2_{eff}) has been measured in the same reactor by the same team using the same detector. This is true, as an example, for the parameter $F_{exp}(\vec{r}_0)$ for the R2 and ZONA 2 experiments performed in MASURCA.

These covariance terms have been neglected according to two arguments :

- The same treatment should be applied to all the experimental parameters but it would be difficult or even impossible to get the original data (counting rates, corrective factors, ...).

- Some of these terms appear in the calculated part P_c for normalization purpose.

On the contrary the systematic covariance matrices correspond to the parameters involved in P_c terms which have all been calculated by us and for which we consider a full correlation ($P_{kk} = 1$).

If σ stands for the standard deviation, the term specifying the correlation between the final uncertainties $\left(\sigma_{\beta^{exp}_{eff,i}}, \sigma_{\beta^{exp}_{eff,j}} \right)$ for two measured β_{eff} values, $\beta^{exp}_{eff,i}$ and $\beta^{exp}_{eff,j}$ is obtained as follows : $\beta^{exp}_{eff,i}$, $\beta^{exp}_{eff,j}$

$$\mathrm{corr}\left(\beta^{exp}_{eff,i}, \beta^{exp}_{eff,j}\right) = \frac{\sum_{\ell} \sigma_i^{\ell} \cdot \sigma_j^{\ell'}}{\sigma_{\beta^{exp}_{eff,i}} \cdot \sigma_{\beta^{exp}_{eff,j}}}$$

The numerical values of the correlation matrix are given the table 10, the ordering corresponding to the following legend :

1 : R2-Cf	2: R2-Fréquence	3 : R2-αRossi	4 : Z2-Cf	5 : Z2-Fréquence	6 : 7A	7 : 7B
8 : 9C1	9 : 9C2	10 : CRef	11 : PuCSS	12 : RSR	13 : U9	14 : UFeRef
15 : UFeLeak	16 : XIX-1	17 : XIX-3	18 : Mistral-1	19 : She-8	20 : Godiva	21 : Jezebel

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	1.0	0.2	0.2	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2		1.0	0.7	0.0	0.6	0.0	0.0	0.0	0.0	0.3	0.3	0.3	0.2	0.2	0.2	0.5	0.6	0.3	0.0	0.0	0.0
3			1.0	0.0	0.5	0.0	0.0	0.0	0.0	0.3	0.4	0.3	0.3	0.3	0.3	0.4	0.5	0.3	0.0	0.0	0.0
4				1.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5					1.0	0.0	0.0	0.0	0.0	0.3	0.3	0.3	0.2	0.3	0.3	0.5	0.6	0.4	0.0	0.0	0.0
6						1.0	0.4	0.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7							1.0	0.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8								1.0	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9									1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10										1.0	0.5	0.5	0.4	0.5	0.5	0.2	0.3	0.4	0.0	0.0	0.0
11											1.0	0.5	0.5	0.5	0.5	0.3	0.3	0.4	0.0	0.0	0.0
12												1.0	0.5	0.5	0.5	0.3	0.3	0.4	0.0	0.0	0.0
13													1.0	0.4	0.4	0.2	0.2	0.3	0.0	0.0	0.0
14														1.0	0.5	0.2	0.3	0.3	0.0	0.0	0.0
15															1.0	0.2	0.3	0.3	0.0	0.0	0.0
16																1.0	0.6	0.3	0.0	0.0	0.0
17																	1.0	0.4	0.0	0.0	0.0
18																		1.0	0.0	0.0	0.0
19																			1.0	0.0	0.0
20																				1.0	0.1
21																					1.0

Table 10 : Correlation matrix for experimental β_{eff} values

- Adjustment results

The adjustment has been performed with the AMERE code [19] a substantially modified French version of the AMARA code [20].

It is worthwhile to recall that in this work only ν_d has been adjusted, the sensitivity calculations of β_{eff} to the delayed spectrum χ_d having been omitted since the coefficients are expected to be very small.

This has been verified in the following way :

β_{eff} were calculated using the same spectrum χ_d (the one of U-238) for all nuclei and compared to the value obtained with the original, specific (different spectra). Differences of the order of 1 pcm were observed for the R2 and ZONA 2 mock-ups. This can be understood by observing that the delayed neutron spectrum is located in an energy region where the reactor adjoint flux ϕ^* is small and slowly varying with the energy as it can be seen on the figure 1 related to the R2 mock-up and taken from the work of V. ZAMMIT [2].

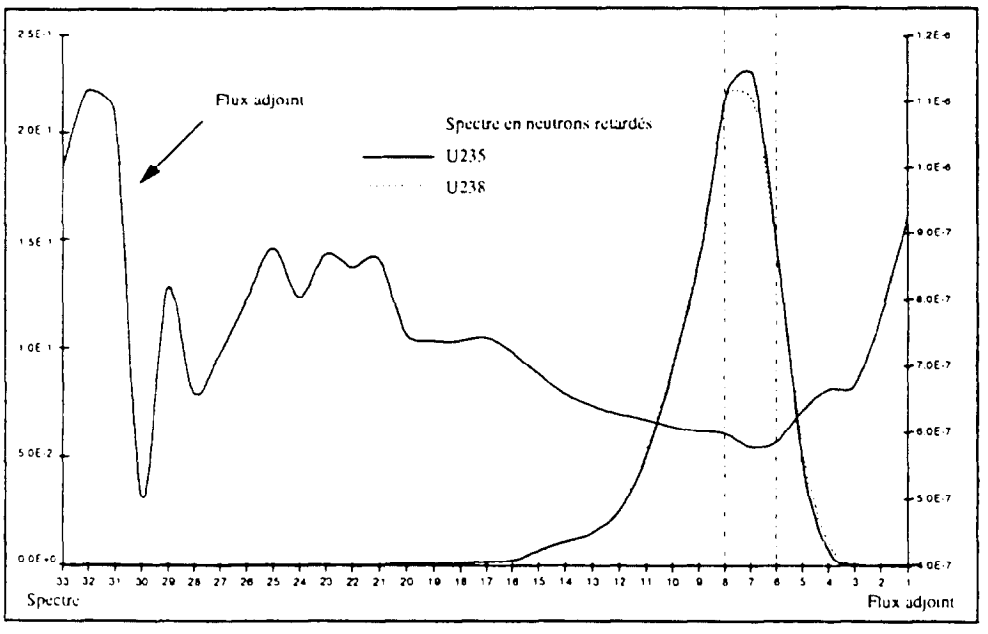

Figure 1 : Position of the delayed neutron spectrum relatively to the expansion of the adjoint flux

The same argument can be used to demonstrate that β_{eff} is not very much sensitive to the flux Φ. This point has been checked by performing calculations with a multigroup library derived from JEF2.2 instead of ERALIB1 Library (that is adjusted). For the same mock-up R2 and ZONA 2 differences of only ~ 3 pcm has been observed.

The present system of 21 integral data could be considered insufficient for the ν_d adjustment of 6 nuclei, unless to consider that the sensitivities to the higher Pu isotopes are very small (due to small content in the fuel) and that the adjustment cannot bring any additional information on these isotopes. Finally 15 nuclear data (3 nuclei * 5 energy groups) are adjusted on 21 integral data so that we can conclude that the adjustment is meaningful for U-235, U-238 and Pu-239.

Before any adjustment the differences between experimental (E) and calculated (C) (with the modified KEEPIN formalism) data are given in the table 11, when using JEF2.2 data for ν_d.

Mock-up		C (pcm)	E (pcm)	(E-C)/C (%)
R2	Source Cf	741.2 ± 3.0 %	755.0 ± 3.1 %	**1.86 ± 4.3 %**
R2	Frequencies	741.2 ± 3.0 %	727.6 ± 2.0 %	**- 1.84 ± 3.6 %**
R2	α-Rossi	741.2 ± 3.0 %	745.0 ± 1.6 %	**0.51 ± 3.4 %**
Zona2	Source Cf.	348.7 ± 3.8 %	359.1 ± 3.1 %	**2.98 ± 4.9 %**
Zona2	Frequencies	348.7 ± 3.8 %	350.0 ± 2.1 %	**0.39 ± 4.3 %**
7A	Cf. Source	387.5 ± 3.8 %	395.0 ± 2.8 %	**1.94 ± 4.8 %**
7B	Cf. Source	437.6 ± 4.2 %	429.0 ± 2.8 %	**- 1.98 ± 5.0 %**
9C1	Cf. Source	748.4 ± 3.1 %	748.0 ± 4.2 %	**- 0.06 ± 5.2 %**
9C2	Cf. Source	399.1 ± 3.8 %	416.0 ± 4.6 %	**4.23 ± 5.9 %**
CRef	Covariances	380.8 ± 4.3 %	383.6 ± 2.2 %	**0.72 ± 4.8 %**
PuCSS	Covariances	221.8 ± 3.9 %	223.4 ± 2.3 %	**0.72 ± 4.6 %**
RSR	Covariances	328.6 ± 3.8 %	337.3 ± 2.2 %	**2.65 ± 4.4 %**
U9	Covariances	725.5 ± 4.0 %	731.4 ± 2.1 %	**0.81 ± 4.5 %**
UFeRef	Covariances	674.4 ± 3.1 %	670.8 ± 2.1 %	**- 0.53 ± 3.8 %**
UFeLeak	Covariances	674.3 ± 3.1 %	675.8 ± 2.1 %	**0.22 ± 3.8 %**
XIX-1	Frequencies	763.3 ± 3.0 %	743.4 ± 3.1 %	**- 2.61 ± 3.8 %**
XIX-3	Frequencies	253.6 ± 3.4 %	252.3 ± 3.5 %	**- 0.50 ± 4.2 %**
Mistral-1	Frequencies	808.2 ± 3.0 %	789.7 ± 1.5 %	**- 2.29 ± 3.3 %**
SHE-8	Cinetique	694.2 ± 3.2 %	696.0 ± 4.6 %	**0.26 ± 5.6 %**
Godiva	Cinetique	602.1 ± 3.6 %	603.1 ± 4.6 %	**0.17 ± 5.8 %**
Jezebel	Cinetique	139.0 ± 4.6 %	143.1 ± 4.6 %	**2.95 ± 6.5 %**

Table 11 : (E-C)/C values obtained using JEF2.2 data for ν_d

The differences are small, always smaller than 3 % (except for C2 experiment) that is the limit given in the HPRL. This means that the JEF2 data are of sufficient quality.

Similar calculations have been performed in the same conditions using the ENDFB-VI ν_d data.

The corresponding values (E-C)/C are in table 12.

Mock-up		C (pcm)	E (pcm)	(E-C)/C (%)
R2	Source Cf	729.1	755.0 ± 3.1 %	**3.55**
R2	Frequencies	729.1	727.6 ± 2.0 %	**- 0.21**
R2	α-Rossi	729.1	745.0 ± 1.6 %	**2.18**
Zona2	Source Cf.	337.0	359.1 ± 3.1 %	**6.54**
Zona2	Frequencies	337.0	350.0 ± 2.1 %	**3.84**
7A	Cf. Source	387.5	395.0 ± 2.8 %	**1.94**
7B	Cf. Source	419.1	429.0 ± 2.8 %	**2.36**
9C1	Cf. Source	733.5	748.0 ± 4.2 %	**1.98**
9C2	Cf. Source	384.0	416.0 ± 4.6 %	**8.34**
CRef	Covariances	365.5	383.6 ± 2.2 %	**4.95**
PuCSS	Covariances	220.3	223.4 ± 2.3 %	**1.41**
RSR	Covariances	318.3	337.3 ± 2.2 %	**5.98**
U9	Covariances	694.7	731.4 ± 2.1 %	**5.29**
UFeRef	Covariances	678.3	670.8 ± 2.1 %	**- 1.10**
UFeLeak	Covariances	678.2	675.8 ± 2.1 %	**- 0.35**
XIX-1	Frequencies	767.9	743.4 ± 3.1 %	**- 3.19**
XIX-3	Frequencies	241.2	252.3 ± 3.5 %	**0.42**
Mistral-1	Frequencies	806.9	789.7 ± 1.5 %	**- 2.14**
SHE-8	Cinetique	700.7	696.0 ± 4.6 %	**- 0.67**
Godiva	Cinetique	597.9	603.1 ± 4.6 %	**0.87**
Jezebel	Cinetique	159.2	143.1 ± 4.6 %	**- 10.11**

Table 12 : (E-C)/C values obtained using ENDFB-VI data for ν_d

Compared to those obtained with JEF2.2 the results obtained with ENDFB-VI indicate a general underestimation by a rather significant amount, and they exhibit a much larger dispersion.

This is essentially due to the ν_d data for U-238 which are much lower, although the ν_d data for the other nuclei in both files are very similar in the energy range of major interest (except for the spheres) as it can be seen in the figures 2, 3 and 4.

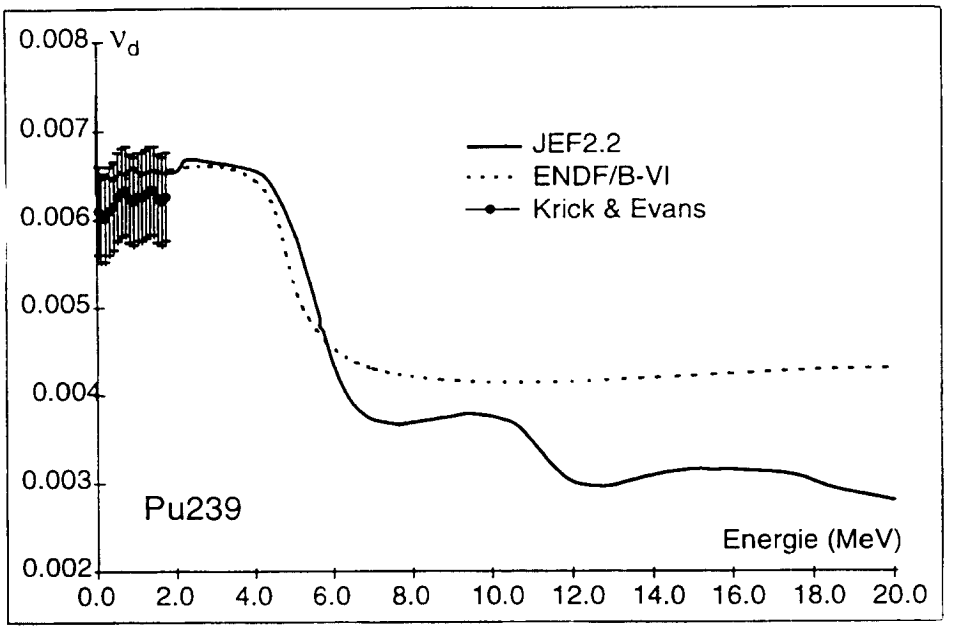

Figure 2 : ν_d data in JEF2.2 and ENDFB-VI for Pu-239

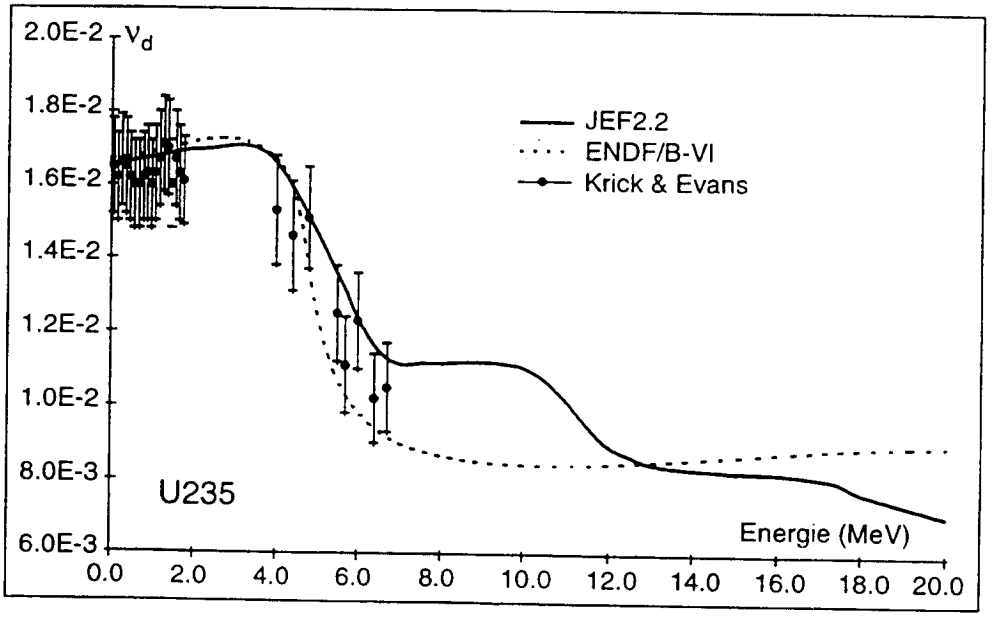

Figure 3 : ν_d data in JEF2.2 and ENDFB-VI for U-235

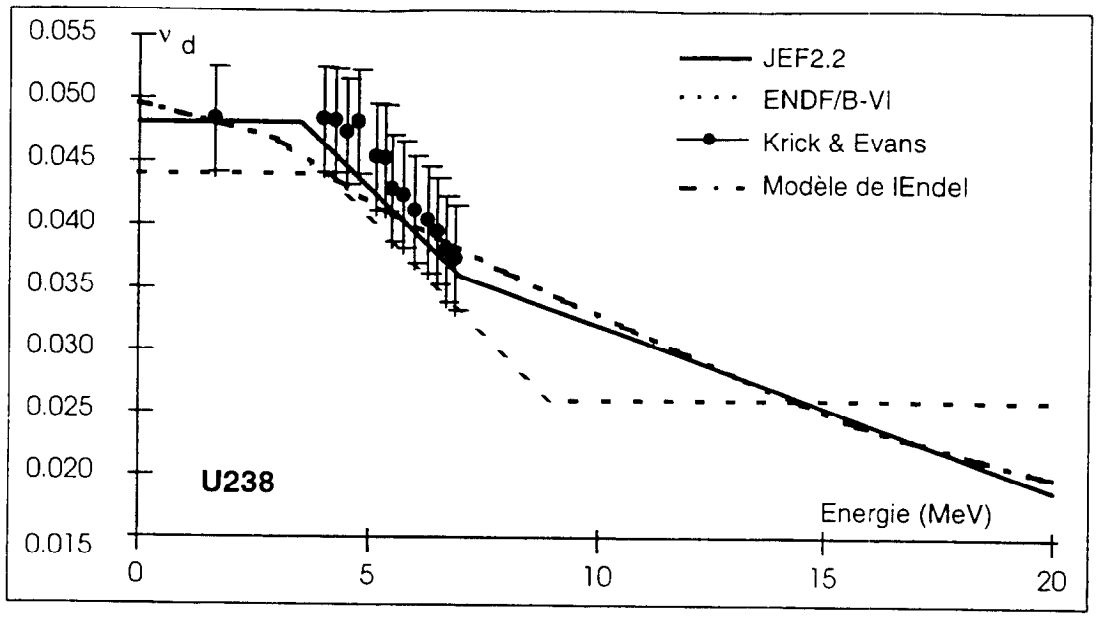

Figure 4 : ν_d data in JEF2.2 and ENDFB-VI for U-238

In a general way the β_{eff} calculations performed with JEF2.2 compare advantageously to the results obtained, in the course of ages, by d'ANGELO and co-workers using different systems of codes and libraries and different ν_d data sets, namely KEEPIN [21] and [22], ENDFB-V [23] and JEF1 [24] as shown in the table 13 below.

Mockup		Ce travail	[18]	[19]	[20]	[21]
R2	Cf. source	1.9 ± 4.3 %	- 3.5 ± 2.2 %	--	--	--
R2	Frequencies	- 1.8 ± 3.6 %	- 4.4 ± 2.5 %	--	--	--
R2	α-Rossi	0.5 ± 3.4 %	- 2.1 ± 2.2 %	--	--	--
Zona2	Cf. source	3.0 ± 4.9 %	1.7 ± 2.0 %	--	--	--
Zona2	Frequencies	0.4 ± 4.3 %	- 2.1 ± 2.7 %			
7A	Cf. Source	1.9 ± 4.8 %	2.9 ± 3.0 %	29 ± 3.0 %	3.1 ± 3.0 %	4.8
7B	Cf. Source	- 2.0 ± 5.0 %	1.0 ± 2.0 %	1.0 ± 2.0 %	1.0 ± 2.0 %	2.4
9C1	Cf. Source	- 0.1 ± 5.2 %	0.0 ± 3.2 %	1.3 ± 3.2 %	1.0 ± 3.2 %	1.4
9C2	Cf. Source	4.2 ± 5.9 %	5.1 ± 4.5 %	7.6 ± 4.5 %	7.5 ± 4.5 %	6.7
Cref	Covariances	0.7 ± 4.8 %	2.4 ± 2.0 %	2.4 ± 2.0 %	1.0 ± 2.0 %	4.1
PuCSS	Covariances	0.7 ± 4.6 %	4.3 ± 2.0 %	4.3 ± 2.0 %	- 7.4 ± 2.0 %	1.8
RSR	Covariances	2.7 ± 4.4 %	2.4 ± 2.0 %	2.4 ± 2.0 %	2.0 ± 2.0 %	4.4
U9	Covariances	0.8 ± 4.5 %	1.4 ± 2.0 %	1.4 ± 2.0 %	- 2.0 ± 2.0 %	- 0.1
UFeRef	Covariances	- 0.5 ± 3.8 %	- 4.4 ± 2.0 %	- 4.4 ± 2.0 %	- 5.7 ± 2.0 %	- 2.9
UFeLeak	Covariances	0.2 ± 3.8 %	3.7 ± 20 %	3.7 ± 20 %	6.5 ± 2.0 %	- 2.2

Table 13 : Comparison of β_{eff} calculated to experimental values using different ν_d data sets

This means that the data in JEF2.2 are satisfactory at least for the energy range of prime importance (E < 4 MeV) for application. They can still be improved by a statistical adjustment.

The adjustment procedure is controlled by a generalized χ^2 minimization. Adjusted data with minimized biases are obtained when the χ^2 (after adjustment) value lies within a confidence interval. This one is defined as a function of a (1-2α) chosen value for the probability for χ^2 to be a correct estimation of the mean value of a KHI2 distribution (approximated by a Gaussian) with 1 as a mean value and $\sqrt{\frac{2}{N}}$ as standard deviation .

N is the degree of freedom of the system and equals the number of integral data.

If $\alpha = 1{,}35.10^{-3}$, then

$$\mathrm{Prob}\left(1-3\sqrt{\frac{2}{N}} < \chi^2 < 1+3\sqrt{\frac{2}{N}}\right) = 0.9973.$$

In the present case, we have :

	$1-3\sqrt{2/N}$	χ_r^2	$1+3\sqrt{2/N}$
Before adjustment	0.074	0.477	1.926
After adjustment	0.074	0.407	1.926

The reduced χ_r^2 value, significantly lower than 1 (before and after adjustment) tends to indicate that the uncertainties on β_{eff} are probably overestimated.

The tables 14 and 15 respectively give the relative modification on ν_d and the impact on the β_{eff} values.

	Corrections (%)					Uncertainties (%)				
	Group 1	Group 2	Group 3	Group 4	Group 5	Group 1	Group 2	Group 3	Group 4	Group 5
U235	- 1.49	- 0.35	- 0.07	0.14	0.07	0.8	0.8	1.4	3.4	4.4
U238	0.85	1.06	1.55	0.82	0.11	3.1	2.5	1.5	4.4	6.2
Pu239	0.74	1.28	1.89	1.60	0.62	1.4	1.2	1.7	4.0	5.0
Pu240	0.81	1.01	1.32	0.90	0.32	6.3	6.3	6.9	8.3	8.9
Pu241	0.40	0.48	0.79	0.30	0.10	5.7	5.7	6.4	7.7	8.3
Pu242	0.08	0.10	0.13	0.08	0.02	12.8	12.8	13.4	14.7	15.3

Group 1 : 0 - 10 KeV
Group 2 : 2 : 10 KeV - 500 KeV
Group 3 : 500 KeV - 4 MeV
Group 4 : 4 MeV - 7 MeV
Group 5 : 7 MeV - 20 MeV

Table 14 : Corrections on ν_d due to the adjustment

Mock-up		β_{eff} (pcm)		(E-C)/C (%)	
Name	Methods				
R2	Californium	741.2 ± 3.0 %	741.2 ± 1.3 %	1.9 ± 4.3 %	1.9 ± 3.3 %
R2	Frequencies	741.2 ± 3.0 %	741.2 ± 1.3 %	- 1.8 ± 3.6 %	- 1.8 ± 2.4 %
R2	α Rossi	741.2 ± 3.0 %	741.2 ± 1.3 %	0.5 ± 3.4 %	0.5 ± 2.1 %
ZONA2	Californium	348.7 ± 3.8 %	353.1 ± 1.4 %	3.0 ± 4.9 %	1.7 ± 3.4 %
ZONA2	Frequencies	348.7 ± 3.8 %	353.2 ± 1.4 %	0.4 ± 4.3 %	- 0.9 ± 2.5 %
7A	Californium	387.5 ± 3.8 %	392.1 ± 1.4 %	1.9 ± 4.8 %	0.7 ± 3.1 %
7B	Californium	437.6 ± 4.2 %	442.8 ± 1.4 %	- 2.0 ± 5.0 %	- 3.1 ± 3.1 %
9C1	Californium	748.4 ± 3.1 %	749.2 ± 1.3 %	- 0.1 ± 5.2 %	- 0.2 ± 4.4 %
9C2	Californium	399.1 ± 3.8 %	403.6 ± 1.4 %	4.2 ± 6.0 %	3.1 ± 4.8 %
CRef	Covariances	380.8 ± 4.3 %	385.8 ± 1.4 %	0.7 ± 4.8 %	- 0.6 ± 2.6 %
PuCSS	Covariances	221.8 ± 3.9 %	224.3 ± 1.5 %	0.7 ± 4.5 %	- 0.4 ± 2.8 %
RSR	Covariances	328.6 ± 3.8 %	332.9 ± 1.4 %	2.7 ± 4.4 %	1.3 ± 2.6 %
U9	Covariances	725.5 ± 4.0 %	729.7 ± 1.4 %	0.8 ± 4.5 %	0.2 ± 2.5 %
UFeRef	Covariances	674.4 ± 3.1 %	670.1 ± 1.3 %	- 0.5 ± 3.7 %	0.1 ± 2.5 %
UFeLeak	Covariances	674.3 ± 3.1 %	670.0 ± 1.3 %	- 0.5 ± 3.7 %	0.9 ± 2.5 %
XIX-1	Frequencies	763.3 ± 3.0 %	757.5 ± 1.3 %	- 2.6 ± 4.3 %	- 1.9 ± 2.7 %
XIX-3	Frequencies	253.6 ± 3.4 %	256.6 ± 1.4 %	- 0.5 ± 4.9 %	- 1.7 ± 2.8 %
MISTRAL-1	Frequencies	808.2 ± 3.0 %	798.9 ± 1.3 %	- 2.3 ± 3.4 %	- 1.2 ± 2.0 %
SHE-8	Cinetique	694.2 ± 3.2 %	684.3 ± 1.4 %	0.3 ± 5.6 %	1.7 ± 4.8 %
GODIVA	Cinetique	602.1 ± 3.6 %	601.4 ± 1.5 %	0.2 ± 5.8 %	0.3 ± 4.9 %
JEZEBEL	Cinetique	139.0 ± 4.6 %	141.3 ± 2.0 %	3.0 ± 6.5 %	1.3 ± 5.0 %

Table 15 : Impact on the β_{eff} values resulting from ν_d adjustment

As already said the adjustments on the higher Pu isotopes are non significant.

The finalized ν_d multigroup values for the 3 major isotopes are given in the table 16.

		Group 1 0 - 10 KeV	Group 2 10 - 500 KeV	Group 3 0.4 - 4 MeV	Group 4 4 - 7 MeV	Group 5 7 - 20 MeV
U235	Before	1.654E-02 ± 3.0 %	1.656E-02 ± 3.0 %	1.681E-02 ± 4.0 %	1.539E-02 ± 6.0 %	1.127E-02 ± 7.0 %
	After	1.629E-02 ± 0.8 %	1.650E-02 ± 0.8 %	1.680E-02 ± 1.4 %	1.541E-02 ± 3.4 %	1.128E-02 ± 4.4 %
U238	Before	4.810E-02 ± 6.0 %	4.810E-02 ± 6.0 %	4.809E-02 ± 7.0 %	4.438E-02 ± 9.0 %	3.3567E-02 ± 10.0 %
	After	4.851E-02 ± 3.1 %	4.861E-02 ± 2.5 %	4.884E-02 ± 1.5 %	4.474E-02 ± 4.4 %	3.571E-02 ± 6.2 %
Pu239	Before	6.471E-03 ± 4.0 %	6.414E-03 ± 4.0 %	6.579E-03 ± 5.0 %	6.085E-03 ± 7.0 %	3.797E-03 ± 8.0 %
	After	6.519E-03 ± 1.4 %	6.496E-03 ± 1.2 %	6.703E-03 ± 1.7 %	6.182E-03 ± 4.0 %	3.821E-03 ± 5.0 %

Table 16 : Evolution of ν_d and standard deviation values as a result of the adjustment

These results call for some comments :

- With respect to JEF2.2 there is no dramatic change due to the adjustment, all the modifications being less than the standard deviations.

- The adjustment tends to increase the slope in $\nu_d(E)$ when existing (U-235 and Pu-239) or to introduce a positive slope when there is none in the evaluation.

Concerning U-238 this conclusion tends to weaken the performances of LENDL's model.

Before a definite conclusion some work is to be made about the consistency of the adjustments on ν_p on one side (calculations are performed with ERALIB1) and on ν_d on the other side.

To finalize this adjustment work, the corrections of the table 13 have to be unfolded in order to produce adjusted pointwise $\nu_d(E)$ curves.

The number of integral data and the consistency of the obtained results enable us to produce recommended values in the format adopted in the past, i.e., ν_d values for the thermal reactor and the fast reactor ranges.

These recommended values have been obtained from the multigroup adjusted data of the table 16 after convolution by classical LWR or FBR spectra.

These values are :

Thermal reactor range :

U235 : 1.642 ± 0.018 (1.1 %)
U238 : 4.839 ± 0.126 (2.6 %)
Pu239 : 0.654 ± 0.010 (1.6 %)

Fast reactor range :

U235 : 1.653 ± 0.017 (1.0 %)
U238 : 4.855 ± 0.112 (2.3 %)
Pu239 : 0.654 ± 0.009 (1.4 %)

V.CONCLUSION

• V.1 Conclusion relative to the present status

This validation work has demonstrated that most of the major files exhibit ν_d data for the 3 most important nuclei (U-235, U-238, Pu-239) of sufficient quality for realistic β_{eff} values to be produced.

The hypothesis recently raised, of an energy dependence of ν_d in the first chance fission range has been confirmed. This energy dependence is of very modest practical importance and even doesn't really appear in the recommended average ν_d values, essentially because of compensations existing in the folding by the reactor neutron spectra (small increase below 4MeV compensated by the sharp decrease above 4MeV).

Referring to the few cases studied in the framework of the JEF2 file validation the so called LENDEL's model seems to be a tool of sufficient quality (although improvements are needed for correct predictions above 5-6 MeV) to produce reliable $\upsilon_d(E)$ data.

The present validation brought poor information on the higher Pu isotopes. For the data evaluators it is suggested, for a first check ,to recalculate the ν_d data for these isotopes with LENDEL's model assuming that reliable $\nu_p(E)$ data are available. For the data users it is reminded that the ν_d(E) data (for the higher Pu isotopes) used in this validation work have been taken from ENDF-B VI (JEF2.2= ENDF-B VI).

• V.2 Suggestions for the future

- Relative to LENDEL's model

This model corresponds to an average description of the macroscopic physical effects. It is probably not adequate to describe fine microscopic effects, such as fluctuations at resonance energies which might be of practical importance.

Fluctuations at resonance energies have been observed in the prompt fission neutron yield $\nu_p(E)$ for Pu-239 and, to a smaller extent, for Pu-241 and U-235.

For Pu-239 these fluctuations have been calculated as functions of spin and (n,γf) effects [25]. These reasons have not been contradicted by recent results obtained at GEEL by DEMATTE ,HAMBSCH and BAX [33] who analyzed the fluctuations in terms of a (n,γf)

effect and of a small $\overline{TKE}$ effect spin dependent ($\overline{TKE_{J^\pi=1^+}}$ - $\overline{TKE_{J^\pi+0^+}}$ =0.068 ± 0.054 MeV). In contrast to Pu-239, the fluctuations observed for the $\nu_p(E)$ of U-235 are due to fluctuations in the fission fragment yields correlated to fluctuations in $\overline{TKE}$ according to HAMBSH and al. [26] .

Concerning U-235 ,OHSAWA and OYAMA [27] have calculated fluctuations in the delayed fission neutron yield $\upsilon_d(E)$ by identifying the precursors in the fluctuations of the fragments yields (after neutron emission) and by using the Pn data from MANN [28] and from WAHL [29].

In these conditions, the fluctuations appear as dips (See figure 5) in contradiction with what has been obtained in JEF2 for Pu-239.or U-235 by (mis)using the LENDEL'S model. The positive correlation between the fluctuations in $\nu_p(E)$ and in $\nu_d(E)$,as predicted by OHSAWA and OYAMA ,is more consistent with a Physics understanding but should receive an experimental confirmation.

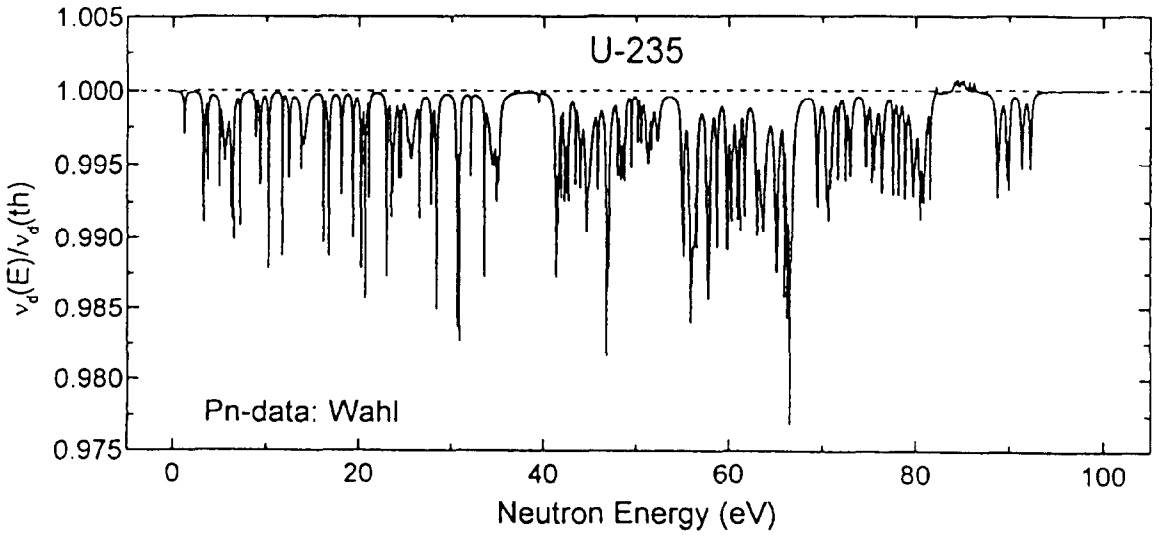

Figure 5 : Relative variation of $\frac{\nu_d(E)}{\nu_d(thermal)}$ as suggested by the reference [27]

Concerning some minor actinides (Np-237, Am-241, Am-243, Cm-245) for which the interest is more related to scientific knowledge than really needed for classical reactor design, a calculation, again using LENDEL'S model, could be a good first approach justified

by a recently improved knowledge of $\nu_p(E)$ [30]. As an example, the ν_d(E) data of Np-237 [4] obtained after a reevaluation of $\nu_p(E)$ have been a posteriori confirmed by accurate (2%) experimental data obtained by V.PIKSAIKIN [31] (see figure 6).

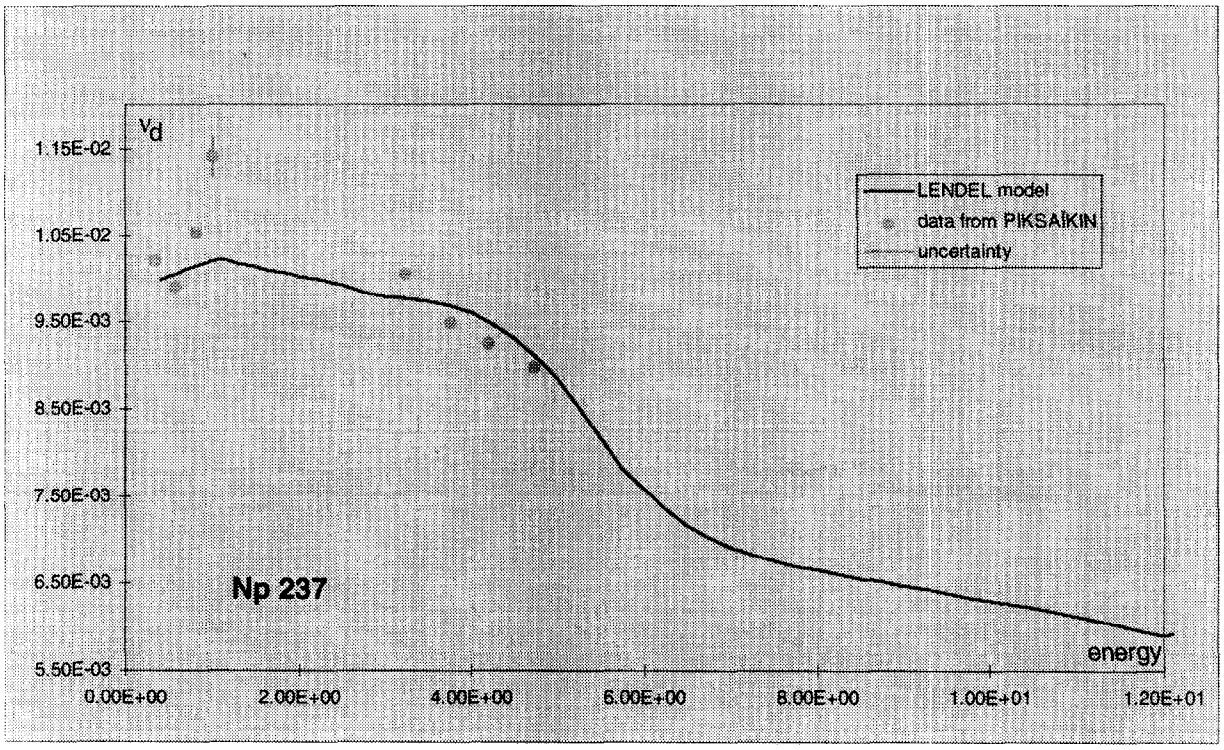

Figure 6 : $\nu_d(E)$ data calculated [4] with the LENDEL's formalism compared with the recent experimental data by PIKSAIKIN [31] .

In the present version of this model the effects related to the various fission chances are averaged and mixed altogether. An improvement in the same context of the semi-empirical approach would be to treat separately the effects of each fissionning system and to derive the final global $\nu_d(E)$ by a summation weighted by the various fission probabilities :

$$\nu_d(E) = \alpha_1(E)\nu_d^1(E) + \alpha_2(E)\nu_d^2(E) + ...$$

$\alpha_1(E) =$ 1st chance fission probability

$\alpha_2(E) = 2^{nd}$ chance fission probability

- Relative to more sophisticated models

A consistent treatment of $\nu_p(E)$ and $\nu_d(E)$ certainly requires more basic approaches than LENDEL's. A modelization of the scission mechanism to derive realistic and accurate values

for fragment mass yields is the only way. This is a long term and quite challenging task since this modelization must be effective over a large energy range, that would be a breakthrough with respect to the few existing models. Nevertheless, some work along this line, have been started in OBNINSK [3]. Such initiatives have to be encouraged, together with experimental work devoted either at microscopic ($\nu_d(E)$) or at integral aspects (β_{eff}).

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