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INFLUENCE OF DELAYED NEUTRON IMPORTANCE ON CALCULATED K-EFF FOR THERMAL SYSTEMS

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Where $\bar{\phi}^*d$ and $\bar{\phi}^*p$ are the mean adjoint (importance) of delayed and prompt neutrons averaged over their respective energy spectra and spatially over the whole core.

Rearranging (1),

$$\Delta\rho = \beta eff(1-\bar{\phi}^*p/\bar{\phi}^*d) \qquad (2)$$

ie, the correction is a function of the fractional difference of the importance of prompt and delayed neutrons. If delayed neutrons are more important (have a higher adjoint) than prompt neutrons then the calculation will underpredict k-eff.

2.2 Evaluation of Mean Adjoint from Reactor Physics Lattice Calculations

For a lattice of cells containing fuel pins, the mean adjoint for fission born neutrons can be derived from finite-diference reactor physics calculations.

$$\bar{\phi}^* = \sum_{f=1}^{F} \sum_{x=1}^{X} P_{fx} \sum_{i-1}^{I} \chi_{fxi} \; \phi^*_{xi}/P_{tot} \qquad (3)$$

Where the three summations are over fissile isotope (f), pin cell number (x), and neutron energy group number (i).

P_{tot} is the fission rate integrated over the core.

P_{fx} is the fission rate of isotope f in cell x.

χ_{fxi} is the fractional yield of neutrons in energy group i from fission of isotope f in cell x.

In WIMS, and other codes, the approximation is made that the fission spectrum is independent of fissile isotope and incident neutron energy so that the fractional yields of fission neutrons are constant over the reactor.

$$\bar{\phi}^* = \sum_{x=1}^{X} P_x \sum_{i=1}^{I} \chi_i \; \phi^*_{xi}/P_{tot} \qquad (4)$$

In Section 3 of this report it is shown that in cases where the core is made up of many identical pin cells, the spatial dependence of ϕ* is small so that an approximate value of the mean adjoint can be derived from a reflected single cell calculation, ie

$$\bar{\phi}^* = \sum_{i=1}^{I} \chi_i \; \phi_i^* \qquad (5)$$

By substituting either prompt or delayed emergence spectra (χi) in Equations (4) or (5) the mean adjoint for the two types of fission neutrons can be calculated.

2.3 LWRWIMS/SNAP Calculations

Calculations of the neutron adjoints in DIMPLE assemblies are made routinely (3) as part of the derivation of β-eff, which is a requirement of the Safety Approval for each new DIMPLE core.

Whole core, 2D models are run using LWRWIMS to provide cell by cell neutron production rates and cross-sections in 17 energy groups. The cross-sections are passed to a 2D SNAP calculation which produces 17 group neutron flux adjoints over the core.

$\bar{\emptyset}^*$ can be calculated for prompt neutrons using these adjoints, the LWRWIMS production rates and prompt delayed emergence spectra, from Equations (4) or (5) as appropriate.

3 RELATIVE IMPORTANCE OF PROMPT AND DELAYED NEUTRONS IN VARIOUS BENCHMARK EXPERIMENTS

3.1 DIMPLE Whole Core Calculations

The relative importance of prompt and delayed neutrons has been calculated for several DIMPLE assemblies using the LWRWIMS/SNAP model described in Section 2.3 via Equation 4. The core loading of each assembly is shown in Figures 1 to 5 and the results are summarised in Table 1.

Also shown are the values of β-eff for each core calculated by the same route. Using these values, corrections to account for the assumption of prompt neutron spectra have been derived from Equation 2 are shown in column 4.

For these assemblies the importance of delayed neutrons is greater than prompt neutrons. Calculations which assume only prompt fission neutron spectra would therefore underpredict k-eff. The largest underprediction (0.14%) is for S03B which is a small, high leakage assembly. This might be expected since high energy neutrons are more likely to leak from the core without causing further fissions. In this way the importance of fast neutrons is reduced in high leakage systems.

3.2 DIMPLE Single Cell Calculations

For assemblies containing a lattice of identical pin cells, calculations can be greatly simplified by modelling the core as a single cell reflected on all boundaries. Neutron leakage can then be modelled by imposing radial and axial buckling terms.

Table 2 shows a comparison of relative worth for the DIMPLE assemblies using the whole core model and the single cell model. Agreement is excellent for all cases except S02A for which the core geometry was not a simple repeated pin cell but included borated steel walls to form compartments within the assembly (see Figure 2). A comparison of the neutron adjoints in the whole core calculations and the single cell calculations is made in Figure 6. The comparison shows how the importance of fast neutrons is enhanced relative to thermal neutrons by absorption in the boron walls. This tends to cancel the leakage effects which reduce the worth of fast neutrons.

Calculations for the VALDUC criticality experiments (Section 3.3) which contain no borated wall materials have been based on reflected single cell models.

3.3 VALDUC Criticality Benchmarks

Some of the Criticality Experiments made at the CEA, VALDUC facility in France are currently being analysed using MONK6 as part of an International Benchmarking programme (3). As in WIMS, MONK6 assumes that all neutrons are born with prompt fission energies.

Four of these experiments, covering the range of fuel/moderator ratios within the Benchmark Study have been analysed here to assess the effect of delayed neutron worth on calculated reactivity. The assembly geometries are shown in Figure 7 and details of the experiments are given in Reference 4.

The loadings consist of 4.74% enriched UO_2 fuel pins at various cell pitches ranging from undermoderated to highly over-moderated conditions.

As noted above, this type of assembly can be represented as a reflected cell with critical bucklings imposed. The relative importance of the fission neutrons is then derived from Equation 5.

The results are summarised in Table 3. As for the DIMPLE cases, delayed neutrons are worth more than prompt neutrons. There is therefore a tendency for k-eff to be underpredicted if no account is taken of delayed neutron fission spectra. The largest correction (0.13%) appears for the 1.60cm pitch which is close to the optimum fuel/moderator ratio for this size of fuel pin. At optimum pitch the critical volume is smallest and the leakage (and Buckling) is high. This result is therefore consistent with the DIMPLE cases where the correction is highest for S03B. Again the size of the corrections is small being similar to the experimental uncertainties on measurements of k-eff.

3.4 Bierman Experiments

The current MONK6 Benchmarking Programme also includes analysis of criticality experiments performed at the Battelle PNL Critical Mass Laboratory by Bierman, Clayton and Durst (5, 6). For these experiments clusters of low enriched UO_2 fuel pins with light water moderation were separated by a range of Boral or Borated Steel Plates. In two cases the clusters were separated by a water gap only. The experimental configuration is shown in Figure 8.

The calculations made for the DIMPLE cores have shown that assemblies with absorbing walls which strongly absorb thermal neutrons have a relative importance of prompt to delayed neutrons close to unity. For these experiments then, only the case where no absorbing walls were present, has been modelled to give a 'worst case' value.

The results are presented in Table 4 and show that even in the worst case the effect accounts for only 0.09% in k-eff. This is very low compared to the experimental uncertainties which are between 0.23 and 0.34%.

3.5 Parametric Survey for Light Water Power Reactor Fuel

The current NEA Fuel Burn-up Benchmark (7) is based on a single cell model of PWR fuel for a variety of fuel depletion and cooling times including fresh fuel. Based on the data given for this Benchmark and extending the range of fresh fuel enrichment, the following calculations have been made.

Case	Fuel	Moderator
1	Fresh Fuel, 3.6% enriched, 300K	Full Density 0ppm Boron, 300K
1b	Fresh Fuel, 811K	Density = 0.7295 550ppm Boron, 570K
2	30GWd/t Fuel, 300K	Full Density 0ppm Boron, 300K
2b	30GWd/t Fuel, 811K	Density = 0.7295 0ppm Boron, 570K
3	Fresh Fuel, 2% enriched, 300K	Full Density 0ppm Boron, 300K
4	Fresh Fuel, 7% enriched, 300K	Full Density 0ppm Boron, 300K

In all cases the pin cell pitch was 1.33cms which gives a typical fuel/moderator volume ratio for a PWR core. For Case 1 the calculations were made for a range of bucklings spanning the critical bucklings for all the other configurations.

These calculations cover a range of fuel and moderator conditions through the fuel cycle in a PWR. The results are presented in Table 5 and are summarised in Figure 9. It is seen that the relative importance of prompt and delayed neutrons is related mainly to the leakage from the core.

In a commercial PWR with a core height of about 4 metres and a radius of about 1.6 metres the critical buckling is of the order of 0.002 to 0.003cm^{-2}. On this basis the relative importance of prompt and delayed neutrons would be very close to unity and any error in calculated k-eff would be trivial (<0.02%).

The worst cases are evidently for higher enriched fresh fuel. For UO_2 fuel pin/water geometries the highest critical leakage would be found in high enriched fuel with optimum moderation. This situation is modelled in Case 5 which gives a relative importance of 0.76 for 93% enriched UO_2 fuel with a β-eff of 0.82. The correction would therefore be about 0.2% in k-eff.

At this level the effect of delayed neutron importance on calculated k-eff, is of similar magnitude to the experimental uncertainties in measured k-eff. Typically for critical experiments these are in the range 0.1 to 0.3% (1σ).

3.6 Comparison with Kiefhaber's Results for Fast Systems

Kiefhaber's study covered a range of fast neutron systems with various enrichments. In general the results are similar to the thermal systems covered here. The corrections to k-eff ranged between -0.2 and +0.05%, with the largest underprediction being found for high enriched, high leakage cores.

In low enriched fast systems the effect of fission in U238, at neutron energies above 1MeV, plays an important role in neutron importance. For these systems the prompt neutrons may be more important than the delayed neutrons and k-eff may therefore be slightly overpredicted.

4 CONCLUSIONS

The effect of delayed neutron importance on calculated k-eff has been assessed for a range of UO_2 fuel pin lattices.

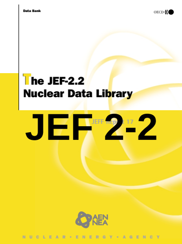
The results have been used to provide correction factors for Reactor Physics or Criticality calculations where the code assumes that all neutrons are born in the fission product spectrum.

Correction factors for Benchmark calculations using LWRWIMS and MONK for Critical Experiments at DIMPLE, VALDUC and the Battelle Laboratories lie in the range 0-0.15% in k-eff. Particularly for storage geometries where the assembly is divided by absorbing plates the corrections are very small ($\simeq$0.01%).

A parametric survey based on irradiated and unirradiated PWR fuel shows that the relative importance of prompt and delayed neutrons is related mainly to core neutron leakage. For a large PWR power reactor core with low leakage, the correction factors are again small at about 0.02% in k-eff.

The worst case identified is for a critical assembly of high enriched fuel at optimum moderation. For this case the correction is 0.2% in k-eff which is of similar order to experimental precisions achieved in critical experiments.

On the basis of the configurations studied, it is concluded that the fission spectrum used in WIMS and MONK is valid for k-eff calculations in light water moderator systems. Even in extreme cases the assumption of prompt neutron energy spectra



for all fission neutrons leads to an error in k-eff which is trivial compared to normal safety margins for criticality assessments.

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Table 1

Corrections to Calculated K-eff for Relative Importance Prompt Neutrons in DIMPLE Critical Core Assemblies

Core	Relative Importance	Beff (%)	Correction (%dk/k)
S01A	0.953	0.81	0.04
S02A	0.913	0.75	0.07
S03B	0.834	0.82	0.14
S06A	0.984	0.78	0.01
S06B	0.980	0.78	0.02

Table 2

Comparison of Whole Core and Reflected Cell Calculation

Core	Relative Importance		Ratio
	Whole Core	Reflected Cell	
S01A	0.953	0.953	1.0003
S02A	0.913	0.895	0.9805
S03B	0.834	0.840	1.0068
S06A	0.984	0.982	0.9988

Table 3

Neutron Importance VALDUC Cell calculations

Neutron Group	Normalised Importance Cell Pitch (cms)				Fission Spectra	
	1.26	1.60	2.10	2.52	Delayed	Prompt
1	0.709	0.590	0.595	0.670	0.00	0.03
2	0.730	0.666	0.696	0.770	0.00	0.12
3	0.787	0.748	0.781	0.842	0.01	0.21
4	0.853	0.837	0.865	0.905	0.04	0.23
5	0.913	0.914	0.933	0.953	0.11	0.18
6	0.960	0.964	0.973	0.982	0.24	0.11
7	1.022	1.022	1.018	1.013	0.25	0.06
8	1.048	1.048	1.039	1.027	0.18	0.03
9	1.071	1.070	1.055	1.038	0.09	0.02
10	1.089	1.088	1.068	1.046	0.07	0.01
11	1.114	1.111	1.086	1.058	0.00	0
Relative Importance	0.865	0.843	0.863	0.900		
Beff (%)	0.84	0.84	0.81	0.77		
Correction (%dk/k)	0.11	0.13	0.11	0.08		

Table 4

Corrections to BIERMAN Wholecore Calculations

Experiment		Relative Importance	Beff (%)	Correction (%dk/k)
No.1	3 Clusters of 17X20 UO2 pins 2.35% Enriched, 2.032cm pit	0.923	0.78	0.06
No.2	3 Clusters of 15X8 UO2 pins 4.31% Enriched, 2.54cm pitch	0.886	0.80	0.09

Table 5

Relative Importance of Delayed and Prompt Neutrons in PWR Reactor Fuel at Stages Through the Cycle

a) Critical Buckling Search

Buckling (cm-2)	K-eff Case 1	Case 2	Case 1b	Case 2b	Case 3	Case 4
0	1.4448	1.1738	1.3095	1.0595	1.2925	1.5528
0.004	1.2528	1.0173	1.0583	0.8568	1.1156	1.3542
0.009	1.0624	0.8623	0.8330	0.6754	0.9409	1.1560
0.015	0.8854	0.7185	0.6440	0.5233	0.7794	0.9707
Interpolated Critical Buckling (cm-2)	0.0108	0.0046	0.0052	0.0011	0.0072	0.0139

b) Relative Importance

Buckling (cm-2)	Relative Importance Case 1	Case 2	Case 1b	Case 2b	Case 3	Case 4
0	1.019	1.031	1.031	1.048	1.025	1.018
0.004	0.957	0.968	0.945	0.96	0.962	0.956
0.009	0.894	0.905	0.864	0.878	0.899	0.894
0.015	0.834	0.844	0.793	0.807	0.837	0.834

c) Interpolated Values at Critical

	Case 1	Case 2	Case 1b	Case 2b	Case 3	Case 4
Buckling (cm-2)	0.0108	0.0046	0.0052	0.0011	0.0072	0.0139
Relative Importance	0.8732	0.9521	0.9361	1.0244	0.9206	0.8436

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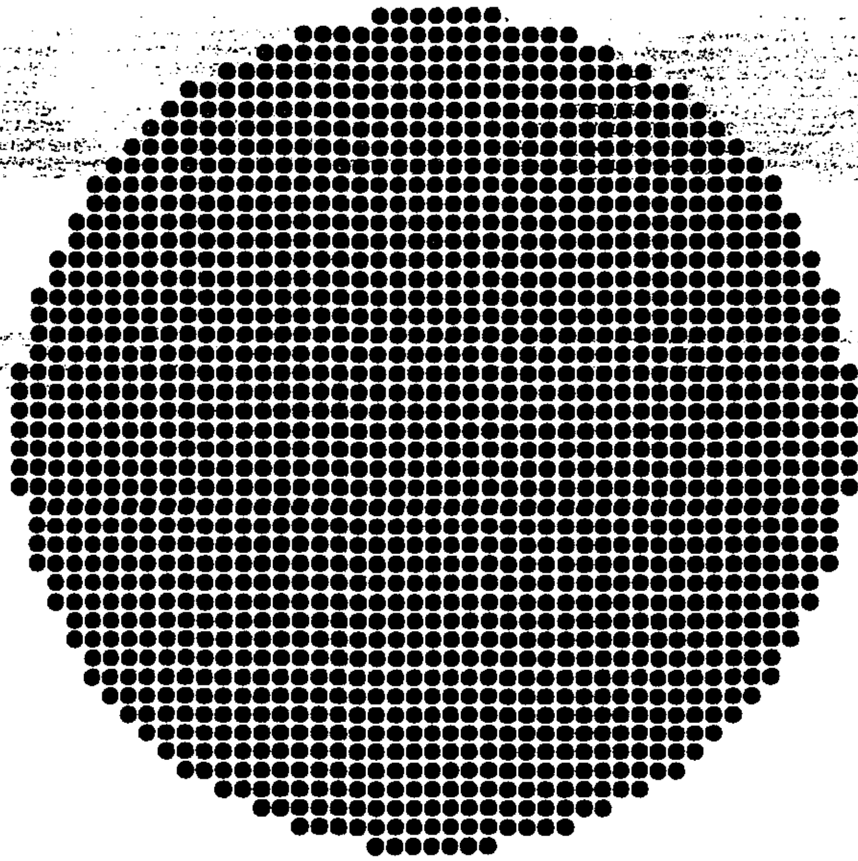
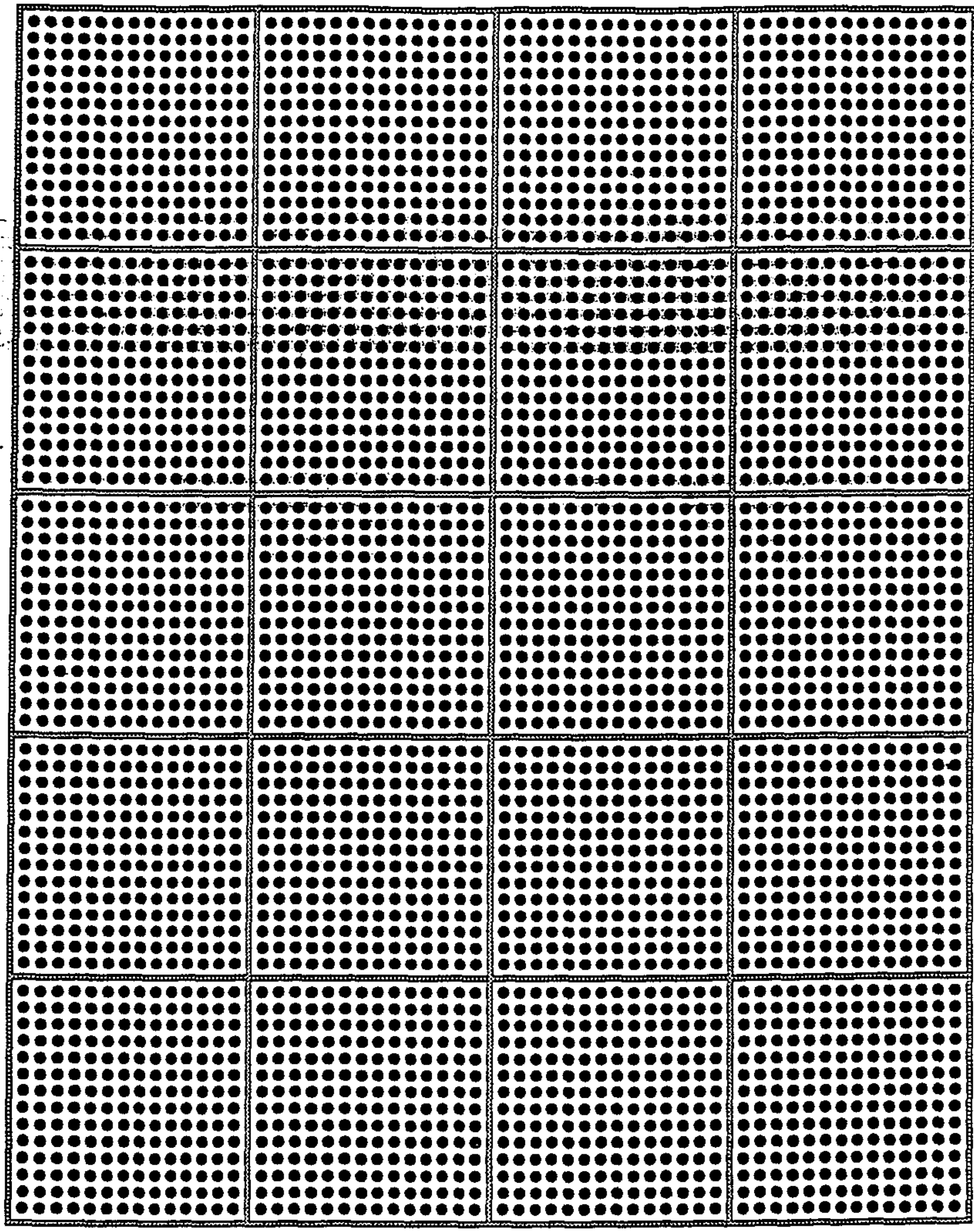


KEY

● 3% UO_2 Fuel Pins

Pitch = 1.32cm

Figure 1 Core Loading Plan For DIMPLE Assembly S01/A



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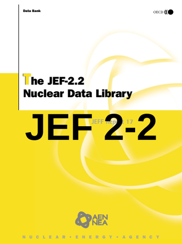
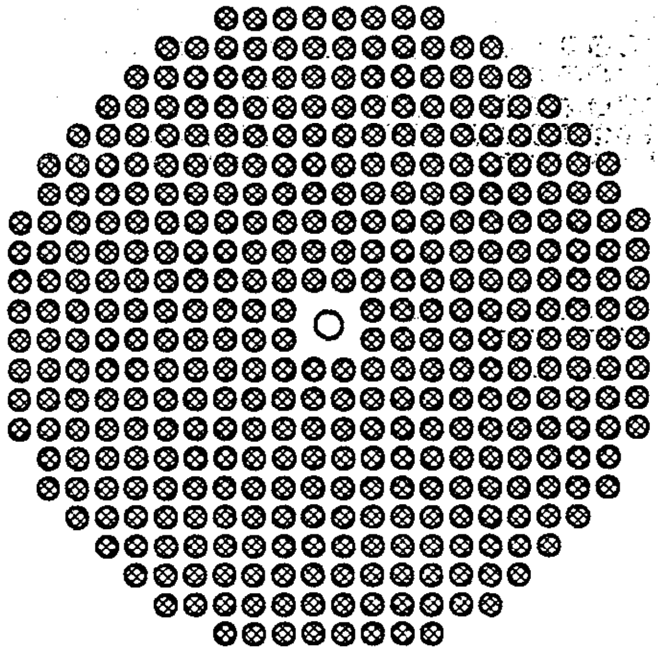
- ● 3% UO_2 Fuel Pins
- Boron Steel Skip Wall
- Pitch = 1.79cm

Figure 2 Core Loading Plan For DIMPLE Assembly S02A





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⊗ 7% UO_2 Fuel Pins

○ Guide Tube

Pitch = 1.32cm

Figure 3 Core Loading Plan For DIMPLE Assembly S03B

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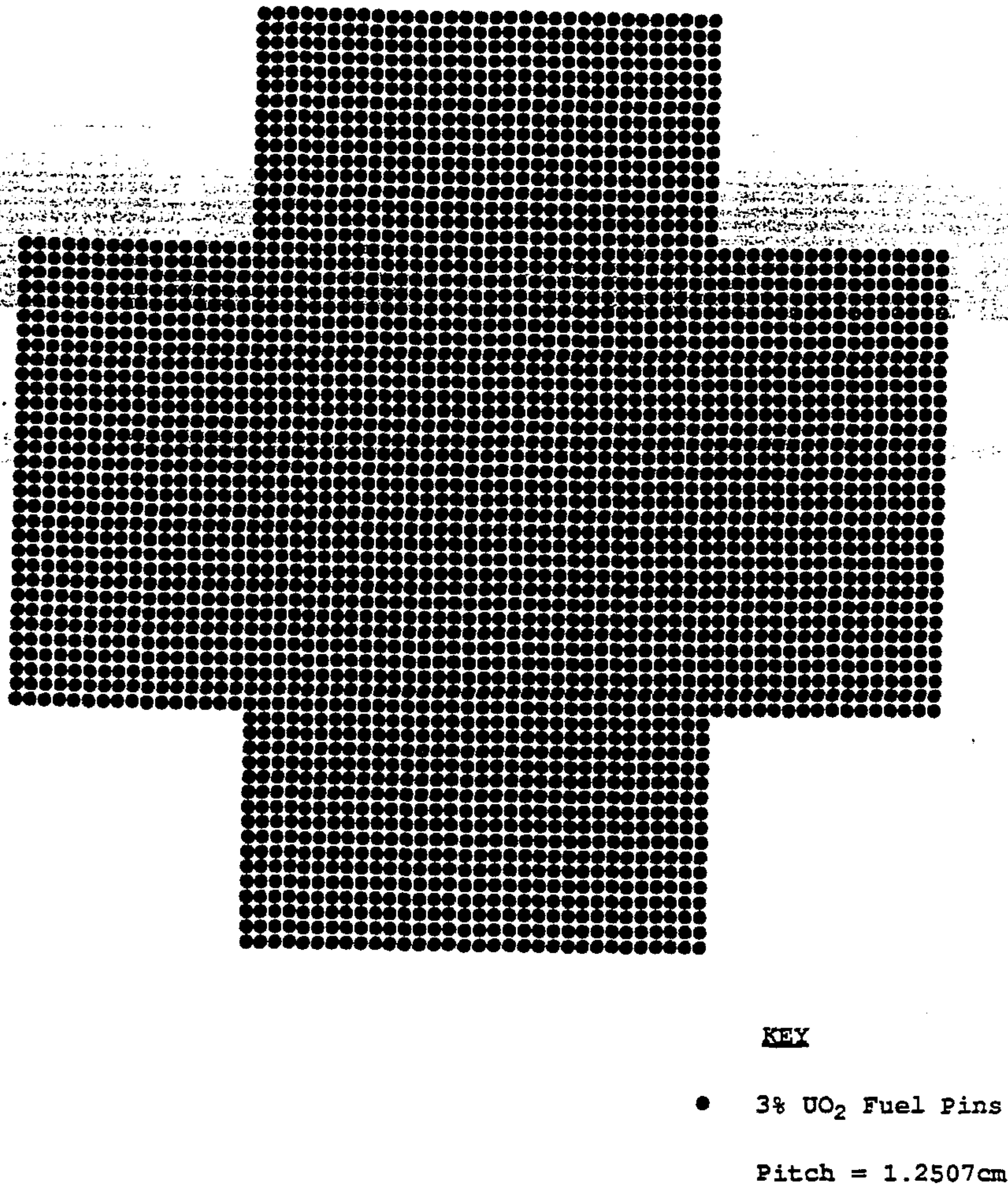
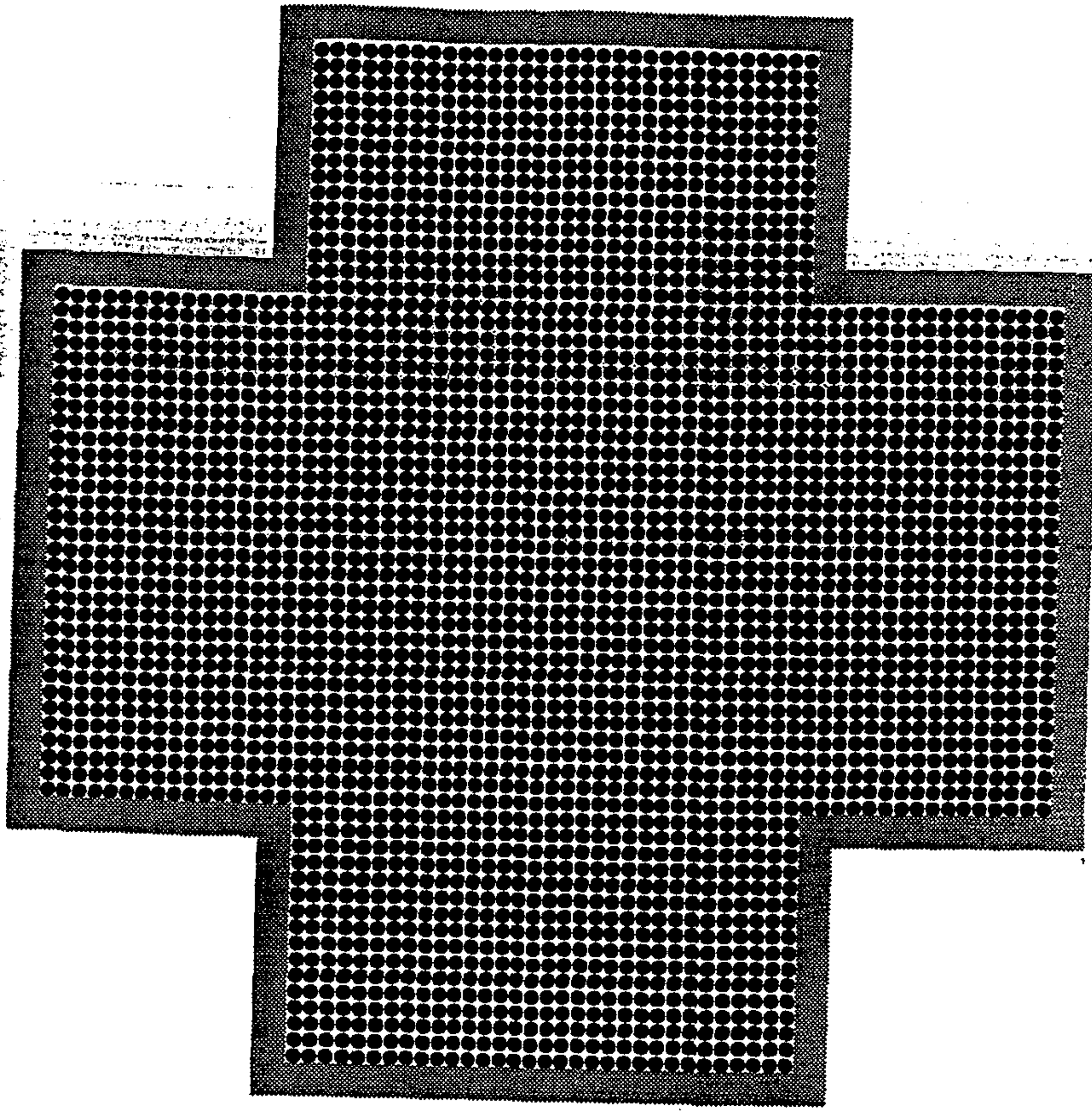


Figure 4 Core Loading Plan For DIMPLE Assembly S06/A



KEY

● 3% UO_2 Fuel Pins

▩ Stainless Steel Baffle

Pitch = 1.2507cm

Figure 5 Core Loading Plan For DIMPLE Assembly S06/B

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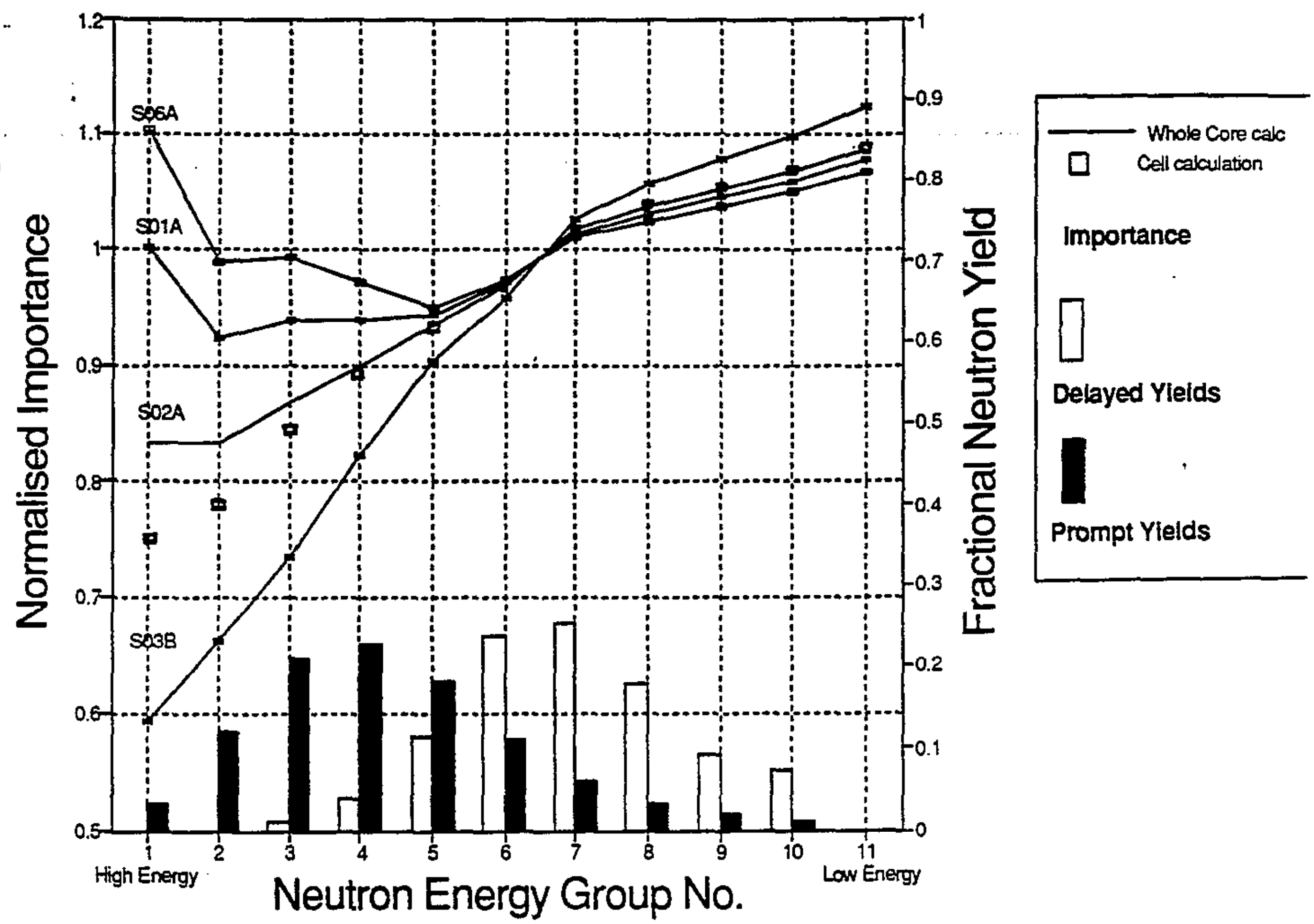
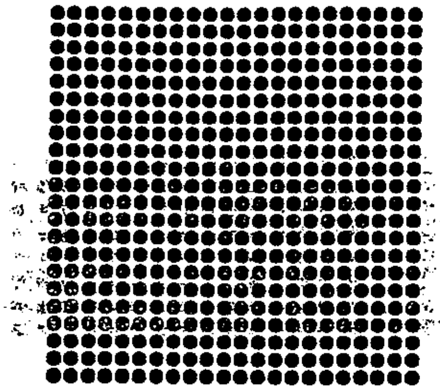
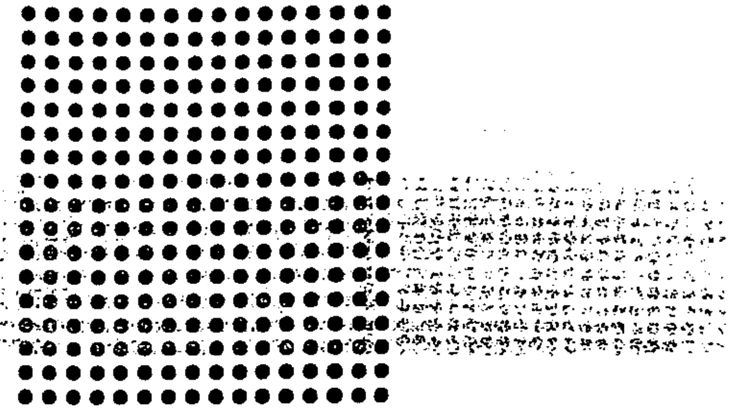


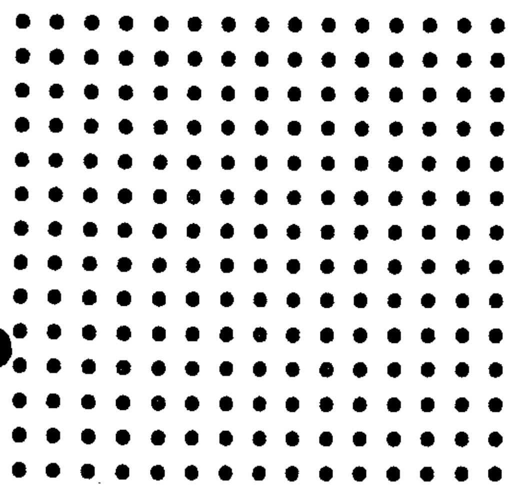
Figure 6 Neutron Importance as Function of Energy In Critical DIMPLE Core Assemblies



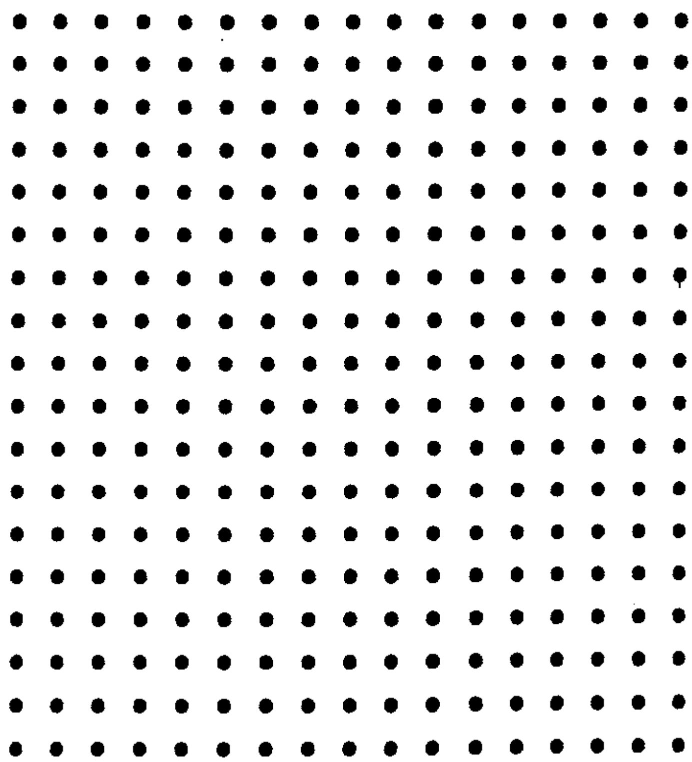
Case 1 22X22pins, Pitch = 1.26cms



Case 2 16X17 pins, Pitch = 1.60cms

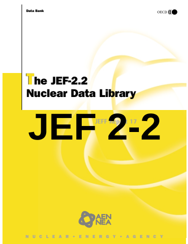


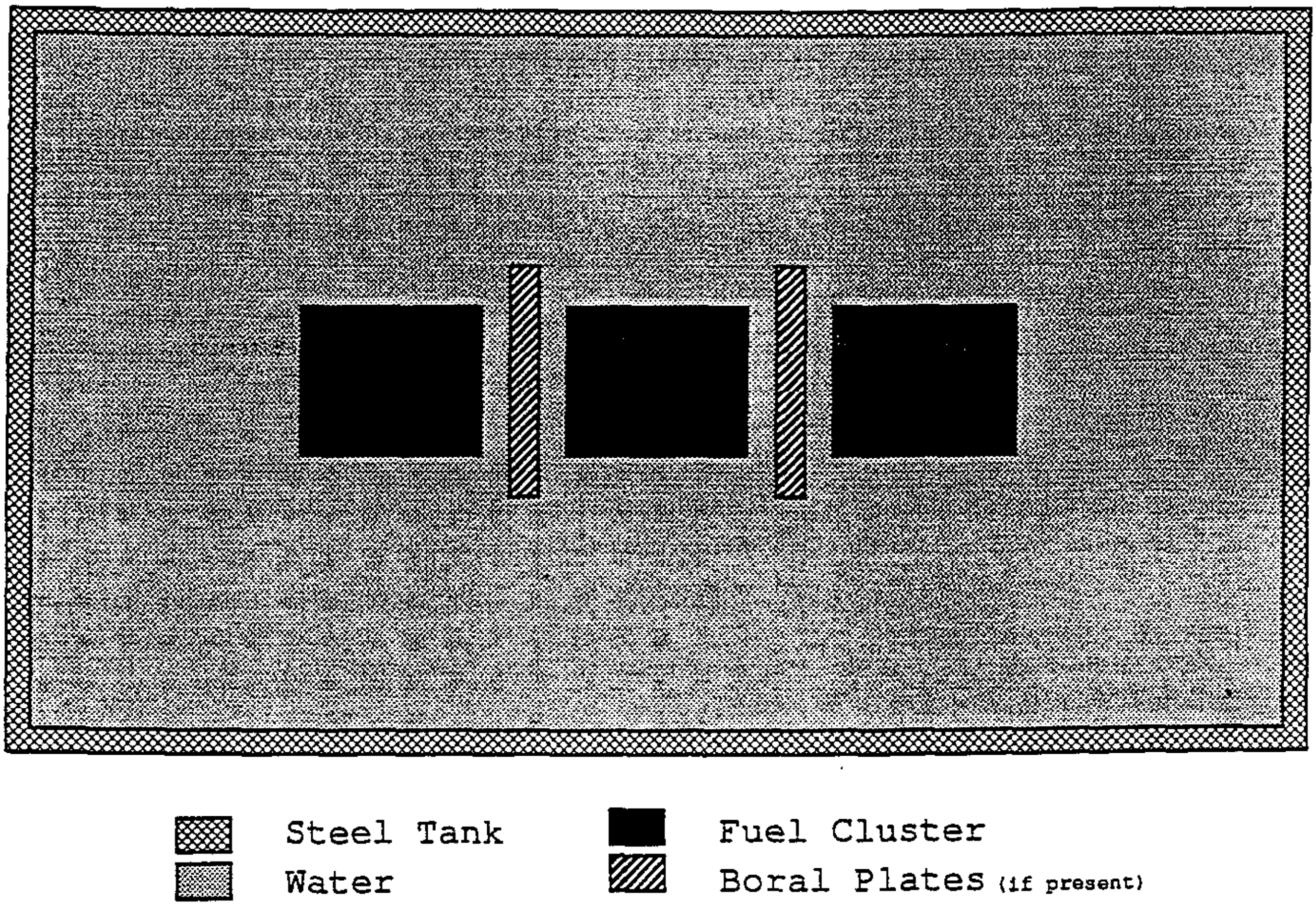
Case 3 15X15 pins, Pitch = 2.10cms



Case 4 17X18 pins, Pitch = 2.52cms

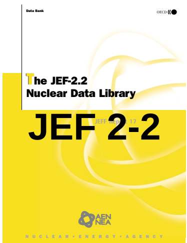
<u>**Figure 7 Core Loading Plan for VALDUC Experiments**</u>

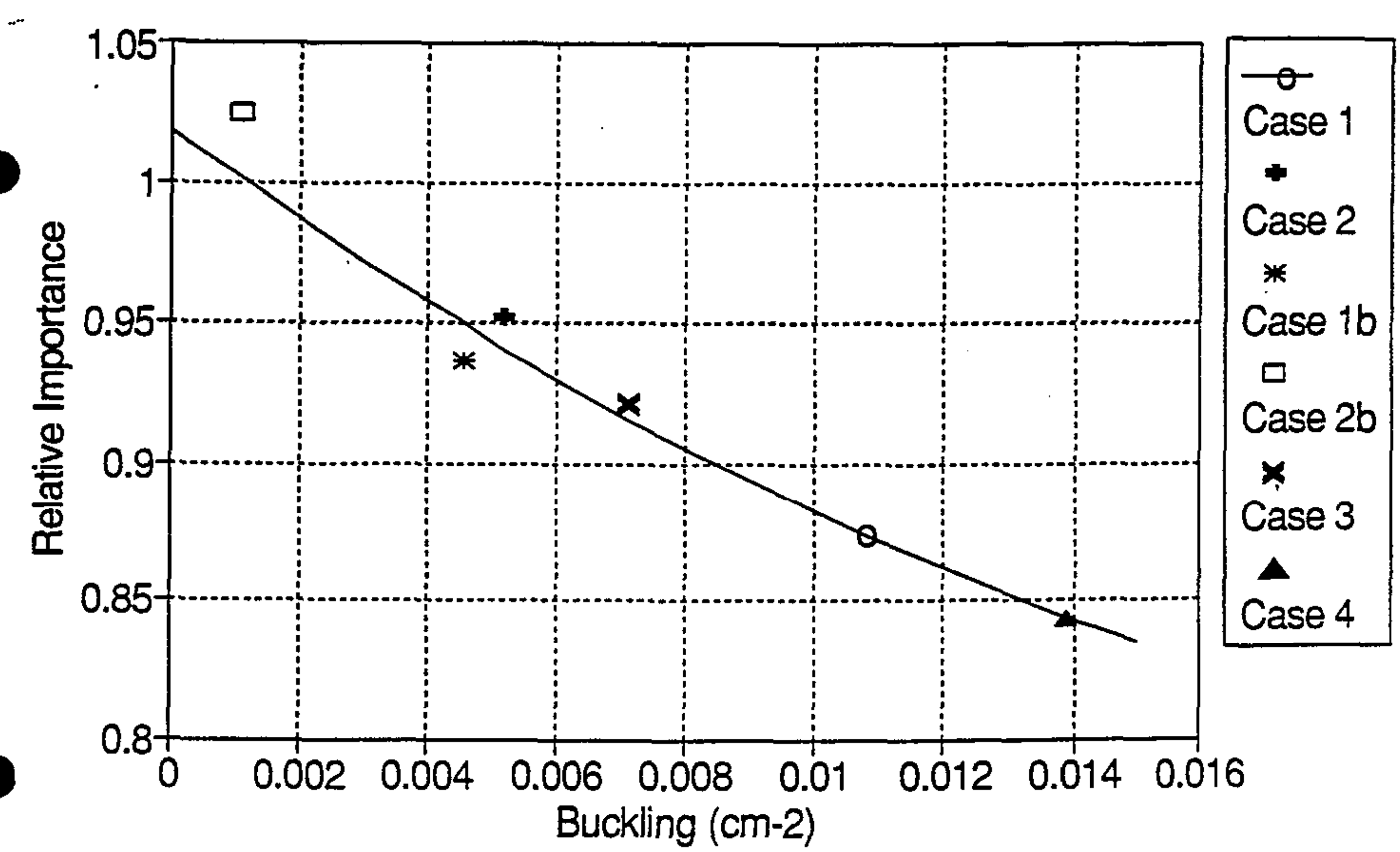




Case 1. No Boral Plates, Fuel Clusters 17X20 2.35%enriched UO2 Pins on 2.032cm pitch
Case 2. No Boral Plates, Fuel Clusters 15X8 4.31%enriched UO2 Pins on 2.54cm pitch

Figure 8 Core Loading Plan For Bierman Experiments





Note. For Cases1b,2,2b,3&4 the plotted data correspond to the critical buckling for each system.
For Case 1 the values have been plotted for a range of bucklings with the critical value shown as an "o"

Figure 9 Relative Importance at Stages Through the Fuel Cycle and Various Fuel Enrichments