Uncertainties Analysis
Surrogate Models

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  on the use of *Surrogate Models*
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  Sensitivity Analysis by ANOVA

  → link with Form/Sorm methods

  analysis for tail distribution and based on surrogate models (linear or quadratic approximation)

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2) and the basis functions issued from
   $\rightarrow$ Polynomial Chaos (Ghanem, Antoniadis, Le Maître)

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- Actual Research in our Laboratory (PhD)
  → if the Surrogate Model is not *reliable* enough, DoCE (Design of Computer Experiment) using Bagging techniques (Bootstrap Aggregating, Breiman 1996)

Example

- Sensitivity Analysis (Sobol) on Homma and Saltelli model

$$
\begin{aligned}
f(x) &= sin(x_1) + asin^2(x_2) + bx_3^4 sin(x_1) \ \texttt{with} \ a = 7, b = 0.1 \\
\texttt{pdf}(x_i) &= \begin{cases} 1/(2\pi) \ \texttt{if} \ -\pi \leq x \leq \pi \\ 0 \ \texttt{if not} \end{cases}
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- Neural Network Surrogate Model

$\longrightarrow$ Multi Layer Perceptron $3 \times 20 \times 1$

$\longrightarrow$ trained from 500 simulations LHS

Results

- Estimation of Sobol global coefficients

Tarantola method (1997)

Variables	Exact Value	Neural Network	Real Function $f(x)$ 1000 LHS of 500 simulations
x_1	0.56	0.55	$\texttt{mean} = 0.55, \sigma = 0.15$
x_2	0.44	0.44	$\texttt{mean} = 0.45, \sigma = 0.11$
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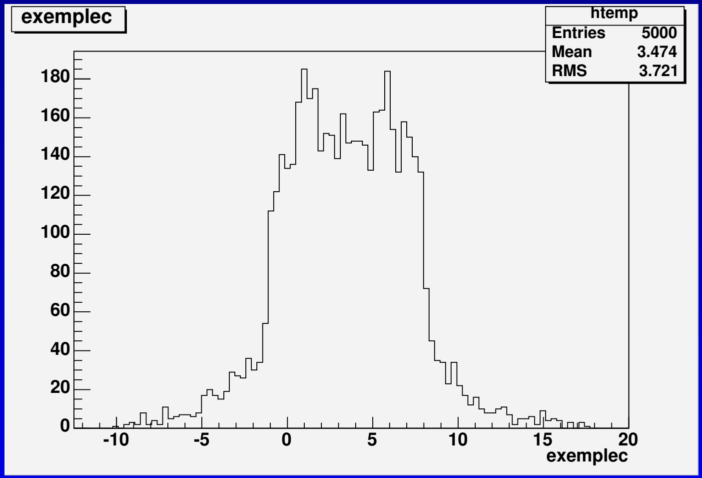

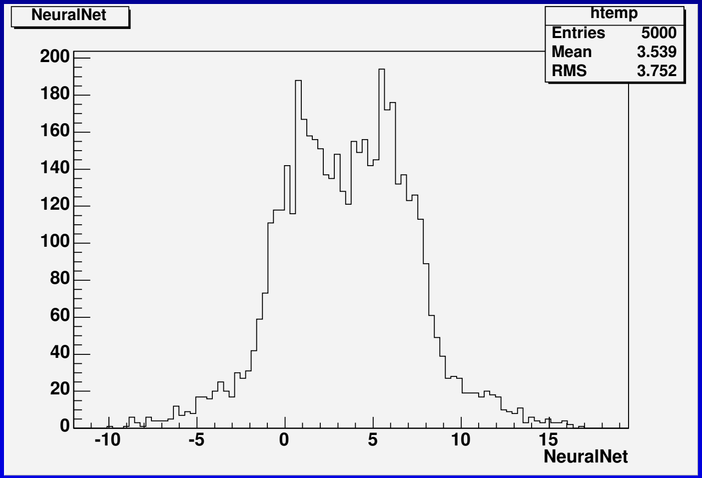

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- Estimation of Sensibility Coefficients (Sobol)
  $\longrightarrow$ after the realization of all LHS Experiments