NEACRP-L-279

THE NODAL DISCRETE ORDINATES METHOD AND ITS APPLICATION TO LWR LATTICE PROBLEMS

M. R. Wagner and B. Müller

Kraftwerk Union AG, Erlangen
Fed. Rep. of Germany

Summary of an invited paper to be presented at the ANS Topical Meeting on Reactor Physics and Shielding, Sept. 17-19, 1984, Chicago, Ill. USA

THE NODAL DISCRETE ORDINATES METHOD AND ITS APPLICATION TO LWR LATTICE PROBLEMS

M. R. Wagner and B. Müller
Kraftwerk Union AG, Erlangen
Fed. Rep. of Germany

1. INTRODUCTION

The transverse nodal diffusion theory methods /1,2/ for global coarse mesh reactor calculations have reached a high level of sophistication and computational efficiency. The success of these methods has prompted the development of related nodal transport theory methods /3-6/. Most of these apply the nodal transverse integration procedure to generate higher order differencing schemes /4-6/ for the discrete ordinates form of the two-dimensional (x,y)-geometry transport equation which then allow the use of a much coarser mesh for the same accuracy.

The nodal discrete ordinates method (NDOM) /3/ described in this paper is different in several respects. It uses an approach which is much more akin to the nodal diffusion methods than other nodal transport schemes. It has been incorporated as an option, parallel to the diffusion theory nodal expansion method (NEM) /7/, in the framework of the same nodal code MULTIMEDIUM without requiring major changes in the overall iteration strategy or in the input and edit segments of the program.

Section 2 of this paper gives a brief outline of the theory and the basic approximations of NDOM. Results of test calculations for typical LWR lattice problems are presented in section 3. Section 4 then describes the program MULTIMEDIUM and its function in the nodal reactor analysis system /8/ developed at KWU. Finally, a few examples are given for the application of MULTIMEDIUM-NDOM to more general reactor physics problems.

3. BRIEF OUTLINE OF THE THEORY

All transverse nodal methods have in common that the N-dimensional diffusion or transport equation is formally integrated over N-1 spatial dimensions of the computational node before any approximations are introduced. This is done for each spatial direction separately so that, for instance, the three-dimensional diffusion equation is replaced by three coupled one-dimensional diffusion equations for each node. This is the key idea which is believed to be the reason for the success of the transverse nodal methods.

It is easy to solve the three almost identical 1-D equations for varying boundary conditions at the node interfaces with higher order methods. This allows to compute accurately the coupling of large nodes to its neighbors.

The nodal discrete ordinates method /3/ is unique in so far as the two-dimensional (x,y) transport equation (with isotropic scattering)

$$\mu \frac{\partial\phi(x,y)}{\partial x} + \eta \frac{\partial\phi(x,y)}{\partial y} + \Sigma_t \phi(x,y) = S(x,y) \qquad (1)$$

is formally integrated over both the transverse spatial and angular variables. As a result the 2-D transport equation is converted to two coupled one-dimensional slab type equations

$$\mu \frac{\partial\varphi(u,\mu)}{\partial u} + \Sigma_t \varphi(u,\mu) - \Sigma_s \emptyset(u) = Q(u) - L^v(u,\mu), \qquad (2)$$

$$u = (x,y),$$

for the "double" transverse integrated angular flux $\varphi(u,\mu)$. In the simplest version of NDOM the transverse leakage term $L^v(u,\mu)$ is replaced by its spatial average and assumed not to depend on the angular variable μ. The flat, isotropic leakage is then given by

$$L^v(u,\mu) = \frac{1}{\Delta v} \left[J^{+v}(\Delta v) - J^{-v}(\Delta v) + J^{-v}(o) - J^{+v}(o)\right], \qquad (3)$$

where $J^{\pm v}$ are the average partial currents on the transverse faces of the node. The partial currents and the angular moments describing the angular distribution of the face-averaged in- and outgoing surface fluxes are the actual variables of NDOM.

Eqs. (2) are replaced by their conventional discrete ordinate form and solved simultaneously for directions x and y by a direct inversion of the streaming-collision operator (left side of Eq. (2)) in a finite difference approximation. This obviates the need for iterating on and storing of the scattering source. In complete analogy to NEM a new set of outgoing currents and out-moments on all faces of the node and for all energy groups is computed for a fixed set of in-currents and in-moments. This is done for all nodes of the problem and the iteration continues until convergence is achieved.

3. NUMERICAL TESTS FOR LWR LATTICE PROBLEMS

The LWR lattice problems discussed in this section are typical for the type of application for which NDOM was primarily developed. Problem A, Fig. 1, is a supercell of four quarters of PWR fuel assemblies with one quarter containing 2 gadolinium bearing fuel rods (~ 4^{w}/o Gd_2O_3). The actual design problem has 90-degree rotational symmetry. However, for this test case reflective boundary conditions were used since the 2-D S_N reference code TWOTRAN-2 /9/ does not allow rotational symmetry. Problem B is a similar four quarter assembly problem with mixed oxide fuel in one quarter and a control rod inserted in one of the U- assemblies. For both problems reference solutions were computed with TWOTRAN-2. The results of the coarse mesh (1 node/pin) nodal diffusion theory (NEM) and nodal transport (NDOM,S_4) calculations are compared in Table 1. The NDOM calculation reproduces the reference pin powers within 0.6 %. Compared to the diffusion theory result the maximum errors are reduced by a factor of 6, respectively a factor of 10, with only a small increase in computing time.

The third test case, Problem C, Fig. 3, is the BWR bundle benchmark problem from the Benchmark Problem Book ANL 7416, Identification No. 13. The original problem is unrodded. To get a more demanding test case a corresponding problem with a cruciform control rod was also defined *). Results for both cases are summarized in Table 2. The maximum error of the 1 node per pin calculations with MULTIMEDIUM-NDOM is 0.7 %. Note, that for the controlled bundle the fission density distribution varies by a factor of 3.7 across the assembly.

The above problems A, B and C are mathematical benchmarks with pin cell homogenized cross sections. Test calculations have also been done for the heterogeneous BWR Lattice Problems NEACRP-1 and -4 /10/ with cylindrical pins. Results /11/ will be given in the full paper. The NEACRP benchmark problems provide also a useful test of the Consistent-Pin-Homogenization (CPH) procedure /11/ developed at KWU for determining equivalent homogenized pin cell cross sections for use in MULTIMEDIUM. The CPH method is a combination of the SPH-Method as proposed by Kavenoky /12/ and the surface current coupling concept desribed by Jonsson et. al. /13/.

---

*) The problem specification and results of the TWOTRAN reference calculation may be requested from the author.

4. ADDITIONAL REMARKS

The nodal discrete ordinates method has been incorporated in the 10-group assembly transport/burnup program MULTIMEDIUM /3/ as the primary flux module. A new, highly efficient "nodal equivalent acceleration" (NEA) method allows to solve large two-dimensional configurations involving many assemblies in transport theory approximation with computing times of the same order as corresponding (NEM) diffusion theory calculations. The NEA method was initially developed for the acceleration of the iterative solution of large systems of nodal diffusion theory equations /19/ and has recently been generalized for nodal transport calculations /2/. Microscopic, CPH-homogenized pin cell cross section libraries for use in MULTIMEDIUM are computed by the pin cell spectrum depletion program FASER-3 /14/ as a function of local variables (pin burnup, fuel and moderator temperature, moderator density and soluble boron concentration). MULTIMEDIUM fulfils important functions in the KWU nodal reactor analysis code system /8/. The program MULTIMEDIUM is used for assembly homogenization and dehomogenization according to the principles described in the companion paper /8/, as well as for the verification /2/ of the new techniques for reconstructing the pin power distributions of the heterogeneous assemblies from results of converged nodal coarse mesh reactor calculations /8/.

MULTIMEDIUM/FASER-3 has been used for 10-group core calculations of the TRX critical experiments /15/. Table 3 compares the results for TRX-1 with experiment and with some recently reported data from other TRX analyses /16,17/.

The nodal discrete ordinates method has also been applied to some more general reactor physics problems. An example is the ZPPR-7A benchmark problem, a model of a heterogeneous LMFBR. It was shown /2/ that, for this problem, NDOM is competitive with other nodal transport methods. Another example is the LWR pool reactor benchmark problem EIR-3B in an intercomparison of NDOM with various other transport and diffusion theory methods /18/.

The discussion of NDOM would be incomplete without mentioning the limitations of the present version of the method. These limitations are a consequence of neglecting the angular dependence of the transverse leakage term in Eq. (2). Fortunately, the resulting deviation from the exact solution of the transport equation is quite small in most applications, except for deep penetration problems with a small scattering ratio c. However, preliminary investigations indicate that it is possible to find suitable angular approximations of the leakage term $L^{v}(u,\mu)$ in Eq. (2) which improve the asymptotic convergence of NDOM and still allow to retain the computational efficiency of the method.

5. REFERENCES

1 J. J. Dorning: "Modern Coarse-Mesh Methods - A Development of the '70's", Proc. ANS Topical Mtg. Williamsburg, Virginia, Vol. 1, p.3-1 (April 1979)

2 M. R. Wagner, K. Koebke: "Progress in Nodal Reactor Analysis", Proc. ANS Topical Mtg., Salt Lake City, Vol. 2, p. 941 (March 1983) Reprinted in Atomkernenergie/Kerntechnik, 43 (1983)

3 M. R. Wagner: "A Nodal Discrete Ordinates Method for the Numerical Solution of the Multidimensional Transport Equation", Proc. ANS Topical Mtg., Williamsburg, Virginia, Vol. 2, p. 4-117 (April 1979)

4 R. D. Lawrence, J. J. Dorning: "A Discrete Nodal Integral Transport Theory Method for Multidimensional Reactor Physics and Shielding Calculations", Proc. ANS Topical Mtg., Sun Valley, Idaho, p. 840 (Sept. 1980)

5 W. F. Walters, R. D. O'Dell: "Nodal Methods for Discrete-Ordinates Transport Problems in (x,y) Geometry", Proc. ANS/ ENS Intl. Topical Mtg., Munich, FRG, Vol. 1, p. 115(April 1981)

6 R. E. Pevey, H. L. Dodds, Jr.: Trans. Am. Nucl. Soc., 39 (1981) 751

7 H. Finnemann, F. Bennewitz, M. R. Wagner: Atomkernenergie, 30 (1977) 123

8 K. Koebke, M. R. Wagner, H.-J. Winter, "Principles and Application of Advanced Nodal Reactor Analysis Methods", Proceedings of this Conference

9 K. D. Lathrop, F. W. Brinkley, "TWOTRAN-2, An Interfaced, Exportable Version of the TWOTRAN Code for Two-Dimensional Transport", LA 4848-MS (July 1973)

10 M. J. Halsall, "Review of International Solutions to NEACRP Benchmark BWR Lattice Cell Problems", AEEW-R1052 (Dec. 1977)

11 B. Müller, M. R. Wagner: "Improved Transport Theory Approximations for Heterogeneous Assemblies With Strong Local Absorbers", Tagungsbericht Jahrestagung Kerntechnik 1981, p.55, Düsseldorf, FRG (March 1981)

12 A. Kavenoky, "The SPH Homogenization Method", IAEA-TECDOC 231, p. 181, IAEA Technical Comm. Mtg., Lugano, Switzerland (Nov. 1978)

13 A. Jonsson, R. Goldstein, A. Mockel, Atomkernenergie, 24 (1974) 79

14 J. Fite, K. Koebke, B. Müller, M. R. Wagner, "Entwicklung von neutronenphysikalischen Auslegungsmethoden für die Plutonium-Rückführung in Druckwasserreaktoren", BMFT-FB-K82-007 (Sept. 1982)

15 B. Müller, M. R. Wagner, Internal KWU-Report (July 1982)

16 E. Schmidt, P. Rose, "Progress in SAM-CE Monte Carlo Capability for Validating Nuclear Data and Reactor Design Codes", EPRI NP Report, to be published

17 M. W. Crump, C. Durston, A. Jonsson, U. N. Singh, "Analysis of Selected Critical Experiments Using ENDF/B-IV and ENDF/B-V Data", EPRI NP-2855, p. 23-1 (Febr. 1983)

18 R. D. O'Dell, J. Stepanek, M. R. Wagner: "Intercomparison of the Finite Difference and Nodal Discrete Ordinates and Surface Flux Transport Method for a LWR Pool Reactor Benchmark Problem in X,Y-Geometry", Proc. ANS Topical Mtg., Salt Lake City, Utah, Vol. 2, p. 757 (March 1983)

19 K. Koebke, "Advances in Homogenization and Dehomogenization", Proc. ANS/ENS Intl. Topical Mtg., Munich, FRG, Vol. 2, p. 59 (April 1981)

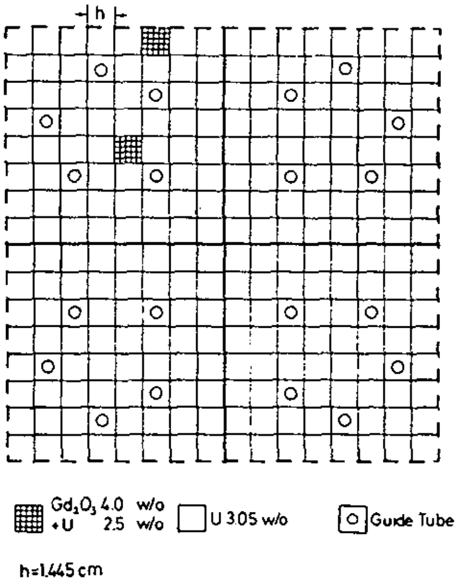

Fig. 1 Problem A
U/Gd Supercell,
Unrodded

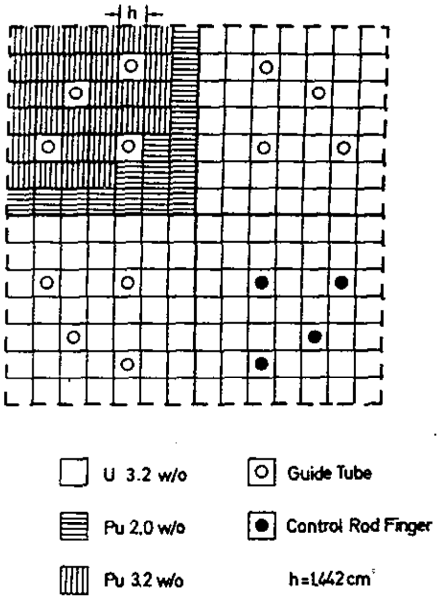

Fig. 2 Problem B
U/Pu Supercell,
Rodded

Table 1 Summary of Results for PWR Test Problems

	Program Mesh/Pin	Angular Approx.	k-eff	Power Density Max.Rel.Error(%) Pin	Power Density Max.Rel.Error(%) Assbly	CP-Time (sec)
Problem A	TWOTRAN-2 4x4	S_4	1.22485	Reference		297
	MULTIMEDIUM 1x1	NEM, P1	1.22418	4.8	-.41	3.4
		NDOM, S_4	1.22495	.43	.03	5.7
Problem B	TWOTRAN-2 4x4	S_4	1.12855	Reference		271
	MULTIMEDIUM 1x1	NEM, P1	1.12387	-3.5	-1.8	3.6
		NDOM, S_4	1.12845	-0.57	-0.1	4.9

CYBER 176

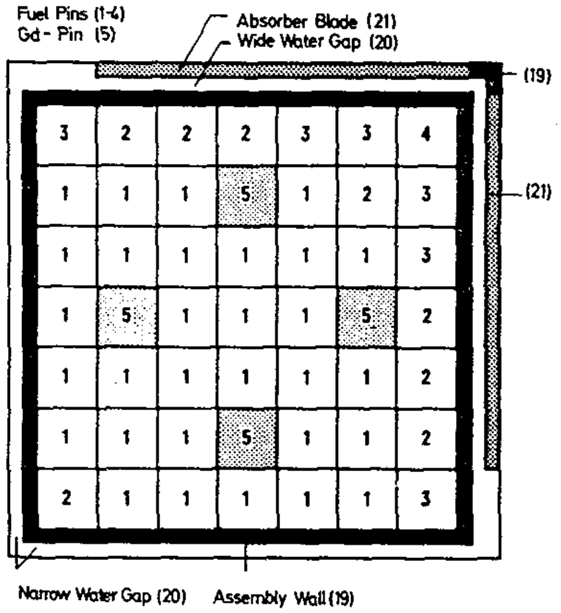

Fig. 3 BWR Bundle, Benchmark Problem Id.13, ANL-7416 with Cruciform Control Rod

Table 2 Summary of Results for BWR Assembly Problem

	Program	Approxim. Mesh/pin	Eigenvalue	Max/Standard Deviation of Pin Powers in %	CP-Time (sec) CYBER 176
BWR Bundle Uncontrolled	TWOTRAN-II	S_8 ,4*4	1.08727	Reference	275*)
	DOT-III	S_8 ,4*4	1.08714	0.3 / 0.1	-
	MM-NDOM	NS_4,1*1	1.08717	0.7 / 0.3	4.5
Controlled	TWOTRAN-II	S_8 ,4*4	0.85106	Reference	296
	MM-NDOM	NS_4,1*1	0.85189	0.6 / 0.4	4.4

*) IBM 360/195

Table 3 Comparison of Measured and Calculated Parameters for TRX-1 Critical Experiment

	Experiment	BNL/EPRI 1983	BNL/EPRI 1983	CE/EPRI 1983	KWU 1982
k_{eff}	1.0	.984	.994	.9944	.9877
ρ_{28}	1.320 ± .021	1.390	1.370	1.364	1.317
δ_{25}	.0987 ± .0010	.0999	.1004	.1012	.1006
δ_{28}	.0946 ± .0041	.0994	.1001	.0984	.1002
C*	.797 ± .008	.807	.8017	.799	.790
Library Programs		ENDF/B-IV SAM/CE	ENDF/B-V SAM/CE	ENDF/B-V DIT/ANISN	KWU Production Library FASER-3/ MULTIMEDIUM
Reference		/16/	/16/	/17/	/15/

92110009