DETERMINATION OF REACTIVITY BY A REVISED ROD-DROP TECHNIQUE IN THE MUSE-4 PROGRAMME – COMPARISON WITH DYNAMIC MEASUREMENTS

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Abstract

The MUSE-4 experimental programme in the zero power fast facility MASURCA at CEA Cadarache aims at studying accelerator-driven-systems (ADS). One of the main objectives is the qualification of measuring and monitoring sub-critical reactivities. Rossi- and Feynman-α and pulsed neutron source techniques based on the use of the GENEPI deuteron accelerator are investigated to assess the prompt neutron decay constant α and to deduce the reactivity.

Reactivity reference levels are determined using a revised rod drop technique and static source multiplication methods for comparison with values extracted from dynamic measurements. This article focuses on the determination of the inherent source and the reactivity sensitivity to kinetic constants and counting rate in the rod drop technique using the point kinetic model.

Introduction

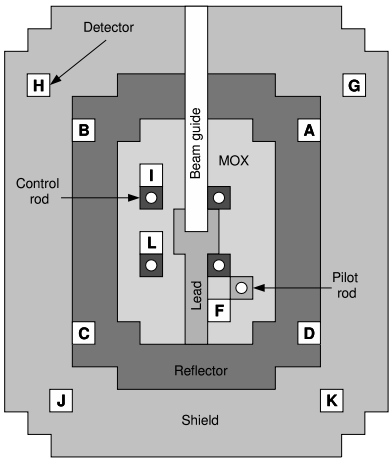
The MUSE-4 programme (multiplication with an external source) aims at qualifying codes for future ADS systems but also at developing reactivity measurement techniques in highly sub-critical cores (k≈0.97) without critical reference. A pulsed neutron source technique using the GENEPI deuteron accelerator and Rossi- and Feynman-α techniques are investigated to measure the prompt decay constant α and to yield reactivity. In order to qualify those techniques the MUSE-4 programme starts from a critical configuration REF followed by subsequent reactivity levels: SC0 at k≈0.995 and SC2 at k≈0.97. Reference reactivity values are inferred from classical rod drop and source multiplication methods. The determination of inherent source and uncertainties due to counting rate and kinetic constants in the inverse point kinetic equation commonly used for rod drop experiments are addressed to get an accurate reference value of reactivity.

Experimental set-up

For the MUSE-4 experiments, MASURCA is loaded with MOX fuel. A deuterium or tritium target surrounded by a lead buffer to simulate a spallation target is placed in the centre of the core. Source strength is up to 40 000 neutrons per pulse for the D-D reaction with a resolution of 1μs and a maximum frequency of 5 kHz. Detectors (^{235}U fission chambers) are located in the fuel, reflector and shield region (see Figure 1).

Up to now the series of experiments have been made in REF (critical) and SC0 (k≈0.995) configurations using the deuterium target. Feynman- and Rossi-α analyses have been performed in both configurations using a specially designed time marking acquisition system NIKO. [1] Pulsed neutron source technique have been investigated in the SC0 configuration using different reactivity levels via control and pilot rod position adjustments (see Figure 1). A reference reactivity level is obtained using rod drop of the pilot rod in the REF configuration. Reactivity values of the SC0 configuration are derived from the reference reactivity using the source multiplication technique.

Figure 1. **Schematic view of the MUSE-4 core**



Rod drop technique – Methodology

Pilot rod drop experiments are analysed using inverse kinetic equations in the point model. This model seems convenient for all detectors because the measured reactivity is known to be less than 50¢ [2] and because the regions of MASURCA are well coupled. [3] In addition, the inversion of the kinetic equations is used in its reduced form where reactivity is expressed in dollars:

$$\rho_{\$}(t)=1+\Lambda^{*}\frac{\dot{c}(t)}{c(t)}-\Lambda^{*}\frac{\varepsilon\hat{Q}(t)}{c(t)}-\sum_{j}\alpha_{j}\frac{c(0)}{c(t)}e^{-\lambda_{j}t}-\sum_{j}\lambda_{j}\alpha_{j}\int_{0}^{t}\frac{c(u)}{c(t)}e^{-\lambda_{j}(t-u)}du \qquad (1)$$

This form takes advantage of the lower sensitivity of the effective relative yields $\alpha_j=\beta_j/\beta$ to reactivity level compared with absolute yields β_j. [4] And the reduced generation time $\Lambda^{*}=\Lambda/\beta$ is a physical parameter that is free from fission-integral normalisation. [5]

Determination of reactivity using a discretised form of Equation (1) is submitted to the adjustment of the source term $\Lambda^{*}\varepsilon\hat{Q}/c(t)$ that is not negligible due to the intrinsic source of the MOX-fuelled core. Reactivity uncertainties come from counting rate and kinetic parameters.

Determination of the reactivity and the source term

The source $Q=\varepsilon\hat{Q}$ of Equation (1) is expressed as the effective rate of neutrons detected due to the intrinsic source. This parameter, which is core and detector dependent through the efficiency ε, is generally unknown. To determine $\rho_{\$}$ is to estimate both variables (Q, $\rho_{\$}$). The additional criteria used to solve Equation (1) is always the constancy of reactivity within a selected time domain after the drop. The problem is the determination of the adequate time domain where reactivity is constant. From the point model without counting rate fluctuations, any time domain after the drop is adequate, but this is not the case for experiments.

In the literature, [6-8] many estimators are extracted from different ways of satisfying the constant reactivity criterion. A means to select the appropriate estimator is to look for their accordance and their robustness to the time domain selection. The most simple method to make reactivity constant is an iterative approach on the source value Q that yields a zero-valued slope for a linear fit of reactivity on the appropriate time domain. Using either weights coming from statistical uncertainties on counts or not we derived two couples of estimators: $(\hat{Q}^{W},\hat{\rho}_{\$}^{W})$ and $(\hat{Q},\hat{\rho}_{\$})$. Another approach used by Hoogenboom *et al.* [6] is to recast Equation (1):

$$c=\frac{g}{1-\rho_{\$}}+\frac{\Lambda^{*}Q}{1-\rho_{\$}} \qquad (2)$$

where g is the delayed neutron source and time-dependence is dropped for reactivity (constant hypothesis), g and counting rate. A linear fit of function $c(g)$ on the same appropriate time domain gives estimators. The fit can be either weighted or made using the so-called grouping method (robust fit). It gives two more estimators: $(\hat{Q}^{L},\hat{\rho}_{\$}^{L})$ and $(\hat{Q}^{GM},\hat{\rho}_{\$}^{GM})$.

Comparison of those estimators are presented in Table 1 for monitor C in the reflector (see Figure 1). The parameter is the beginning time of the fitting domain. The ending time is taken constant in the post-drop stabilised state. This time corresponds to an approximate stable counting rate and is not well defined. But all estimators are relatively insensitive to this parameter. Time τ stands for the arrival time of the pilot rod. In order to ease comparisons the sources Q are normalised to the pre-drop (constant) counting rate.

Table 1. **Reactivity (¢) and source strength (s^{-2}) estimators sensitivity to initial fitting bound**

Initial times	$\hat{Q}^{GM}$	$\hat{Q}^{L}$	$\hat{Q}$	$\hat{Q}^{W}$	$\hat{\rho}^{GM}$	$\hat{\rho}^{L}$	$\hat{\rho}$	$\hat{\rho}^{W}$
τ	120.3	119.1	120.4	121.4	37.0	36.9	37.1	37.3
$\tau + 25$	120.9	119.7	120.9	122.0	37.1	37.0	37.3	37.5
$\tau + 100$	123.5	121.8	123.2	124.0	37.9	37.5	37.9	38.1
$\tau + 125$	123.3	121.7	123.6	124.3	37.8	37.5	38.0	38.2

A similar trend to increase reactivity and source with the initial fitting bound is observed for all estimators. Up to 125 seconds after the drop, reactivity and source deviations are less than 3%. After 125 seconds, fluctuations are observed in the determination of reactivity for all the estimators. The total dispersion between operators is less than 2% for cases from τ to τ+125s.

All estimators for reactivity are consistent and deviations with initial fitting time are more or less the same. We choose to keep the simplest estimator $\hat{\rho}_{\$}$ obtained by minimisation of the reactivity slope from the time of drop τ to the new stabilised state.

Causes of the drift in reactivity are not a point kinetic effect because a simulated counting rate from this theory yields a constant value after the drop of the rod. Besides, from comparison of different detector responses, the amplitude and duration of the drifts are changing. It has to be considered as a spatial/energetic effect that is outside our initial model hypothesis. It induces systematic uncertainties. These are derived by (a) determining the maximum range of beginning time for fit, (b) verifying that the resultant set of reactivity estimates is not significantly different from a Gaussian distribution (95% level confident) and (c) by taking half the maximum dispersion as the estimated uncertainty. It yields a ±1.1% uncertainty for the monitor C.

Uncertainties in reactivity due to counting rate and kinetic constants

Uncertainties of the estimator $\hat{\rho}_{\$}=\overline{\hat{\rho}_{\$}^{n}}$ which is the mean of the time dependent reactivity over the chosen area of fit are investigated through (a) re-sampling techniques and (b) propagation calculations. Both techniques permit a derivation of the uncertainties in reactivity coming from the counting rate and/or the kinetic constants.

Re-sampling techniques

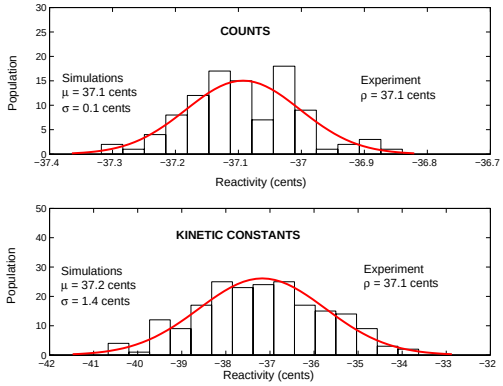
Taking $\overline{p_i}$ as the mean i-th parameter and d_i its associated centred probability distribution we generate for each parameter a random number issued from d_i. Repeating the process we get N sets of parameters $\{p_i\}_{k=1...N}$. Using for parameters the kinetic constants α_j and λ_j that are normally distributed and/or the discretised counting rate c_n that follows a Poissonian law, we obtain via Equation (1) a distribution of reactivity $\hat{\rho}_{\$}$. The standard deviation of this reactivity set is the uncertainty due to either counting rate or kinetic constants or both.

It is noteworthy that uncertainties in Λ^* do not play any role in the reactivity uncertainty. Considering Equation (1), the derivative term is negligible and the source term $\Lambda^* \varepsilon \quad \hat{Q}/c(t)$ is fitted to *cst/c(t)*. Λ^* has effect on the source determination but not on reactivity expressed in dollars.

Up to now we have considered no correlation between kinetic constants in this procedure except the normalisation $\Sigma_j \alpha_j = 1$. Uncertainties due to kinetic constants and counting rate are kept separated because the former is treated as systematic and the latter as statistical. Effective kinetic constants pre-calculated from the Tuttle evaluation [9] by the ERANOS code system using 33 energy groups and R-Z geometry [10] is used in Equation (1). Associated "effective" uncertainties are not derived by ERANOS. To get an estimate of these variances we used those of ^{239}Pu and ^{238}U which are the two main components of the ERANOS β_{eff} calculation for the MUSE-4 core.

Distributions and final uncertainties for the monitor C are illustrated in Figure 2 for counting rate and kinetic constants with variance issued from Tuttle's ^{239}Pu evaluations.

Figure 2. **Distribution of simulated reactivity obtained by re-sampling technique**



Taking into account the inherent fluctuations of the uncertainties obtained by re-sampling, we can consider a ±4% systematic uncertainty for the kinetic constants (for all monitors). In all cases, statistical uncertainties derived from fluctuation in counting rate are less than ±1%. This is in agreement with a former study made at CEA for the MASURCA reactor. [11]

Uncertainty propagation calculations

Taking advantage of the simplicity of the discretisation of Equation (1) and the estimator $\hat{\rho}_\$$ we can derive analytical formulas for partial derivates $\delta\hat{\rho}_\$ / \delta p_i$ where p_i are counting rate and kinetic constants (see previous paragraph). Assuming (a) no correlation between discretised counting rate and kinetic constants and (b) no uncertainties in the source term (previously derived from the previous paragraph) we get from propagation theory:

$$Var(\hat{\rho}_\$) = Vk^t \, Mk \, Vk + Vc^t \, Mc \, Vc \qquad (3)$$

where Mk is the covariance matrix of parameters λ_j, α_j, Mc is the diagonal matrix of counting rate variances (no correlations) and Vk and Vc are the vectors of partial derivatives of $\hat{\rho}_\$$ vs. the kinetic parameters and the discrete counting rate.

Again, both type of uncertainties are kept separated. Uncertainty due to counting rate is in agreement with the one found from re-sampling techniques that is less than ±1%. Systematic uncertainties due to kinetic constants can be addressed using a real correlation matrix to take these correlation effects into account.

Indeed as pointed out by Loaiza and Haskin, [12] correlation between α_j and λ_j can have noticeable effects on uncertainties yielded from these kinetic constants. In order to have an idea of the impact we constructed the covariance matrix *Mk* using either the correlation matrix reported from the GODIVA experiments [12] or a matrix that respects the normalisation $\Sigma_j \alpha_j = 1$. Variances of α_j and λ_j still come from the Tuttle evaluation for ^{239}Pu and ^{238}U but also for ^{235}U because the GODIVA experiments contained 93% ^{235}U enriched fuel.

The results presented in the Table 2 are insensitive to detectors and can be assimilated as systematic uncertainties.

Table 2. **Uncertainties in reactivity due to propagation of kinetic constant uncertainties**

Correlation matrix	**Tuttle's standard deviations – Isotope**	$\sigma_{\hat{\rho}_\$}$
No correlation	^{239}Pu	3.7%
GODIVA	^{239}Pu	7.7%
No correlation	^{238}U	4.1%
GODIVA	^{238}U	6.8%
No correlation	^{235}U	5.2%
GODIVA	^{235}U	6.3%

Clear increases of uncertainties using the GODIVA correlation matrix are seen. For ^{235}U, which is the most adapted isotope to the correlation matrix from the GODIVA experiment, the increase is lowered. That is to say it is reasonable to think that the adapted correlation matrix would give an approximate 1% increase over the case without correlation except normalisation of the α_j 's. Prevailing isotopes of the MUSE-4 core are ^{239}Pu and ^{238}U. The selected conservative systematic uncertainty used for kinetic constants is ±5%.

Experimental results

Rod drop measurement of the pilot rod in the REF configuration resulted in a reference value for core, reflector and shield monitors. Source multiplication using correction factors for large reactivity determination yields the reference reactivity levels in the SC0 configuration. Comparison from coupled Rossi- and Feynman-α / pulsed neutron source method is then achieved to qualify those techniques for reactivity monitoring.

Rod drop and source multiplication reactivities

Pilot rod drop results are presented in the Table 3 for all detectors. Derived uncertainties due to counting rate are presented from the re-sampling σ_{CR}^{ps} and the propagation techniques σ_{CR}^{pc}. Systematic uncertainty for the source adjustment procedure is reported as σ_Q and a ±5% uncertainty (also systematic) due to kinetic constant is added to obtain the total uncertainty σ_{tot}. The statistical uncertainties are negligible compared with systematic values and the total uncertainty is about ±5 to 6%.

Table 3. **Uncertainties in reactivity due to propagation of kinetic constant uncertainties**

Region	**Monitor**	$\hat{\rho}_{\$}$ (¢)	σ_{CR}^{rs} (%)	σ_{CR}^{pc} (%)	σ_{Q} (%)	σ_{tot} (%)
Core	F	37.5	0.5	0.2	1.4	6.4
	I	37.2	0.4	0.2	1.4	6.4
Reflector	A	37.0	0.4	0.2	0.7	5.7
	B	37.2	0.8	0.2	4.0	9.0
	C	37.1	0.3	0.1	1.1	5.1
	D	37.4	0.2	0.1	0.7	5.7
Shield	G	37.0	0.1	0.1	0.7	5.7
Mean		37.2	0.2	0.1	0.7	6.4

Source multiplication theory was used on monitors F, I, C and D to obtain the reactivity level of the SC0 configuration with the pilot up (PRU), down (PRD) and the case of pilot rod down and 1 control rod down (CRD). Spatial/energetic correction factors (MSM factors) have been applied for the CRD configuration and yield an additional ±3% statistical uncertainty.

Results are presented in Table 4 where uncertainties are coming from the reference reactivity level (statistical + systematic), counting rate and correction factors uncertainties.

Table 4. **Sub-critical reactivity levels (USD) in SC0**

Region	**Monitors**	**Configurations**		
		PRU (by SM)	PRD (by SM)	CRD (by SM)
Core	F	1.32 (6.4%)	1.76 (6.4%)	12.7 (9.5%)
	I	1.38 (7.2%)	1.79 (7.3%)	12.3 (8.1%)
Reflector	C	1.34 (5.6%)	1.73 (5.6%)	12.6 (8.1%)
	D	1.31 (6.1%)	1.73 (6.2%)	12.9 (8.7%)
$\overline{\rho}$ / $\overline{\sigma}$		1.34 (5.9%)	1.75 (5.9%)	12.6 (6.1%)

Mean values are taken as the reference reactivity levels for the SC0 configurations: USD 1.34 (5.9%) for PRU, USD 1.75 (5.9%) for PRD and USD 12.6 (6.1%) for CRD. Those overall uncertainties are evaluated by $\sigma_{tot}^2 = \sigma_{stat}^2 / n + \overline{\sigma_{syst}}^2$ where $\overline{\sigma_{syst}}$ is the mean of the systematic uncertainty for each detector.

Comparisons with dynamic measurements

Prompt period α-values obtained by Feynman- and Rossi-α techniques in the REF configuration with the pilot rod down and by pulsed neutron source measurements in the SC0 configurations for the 3 positions of rods PRU, PRD and CRD are recalled from Reference [1] in Table 5. Derived uncertainties are expressed as half the maximum dispersion of values over significant ranges assuming Gaussian law.

Table 5. **α-values (s-1) derived from dynamic measurements**

Rossi	**Feynman**	**Pulsed neutron source**		
REF	REF	SC0/PRU	SC0/PRD	SC0/CRD
8 017 (8.4%)	8 244 (8.3%)	13 018 (3.5%)	15 578 (2.5%)	63 664 (8.6%)

In order to analyse the capability of sub-critical monitoring by a pulsed neutron source method, we proceed as follow:

- Use prompt period α_R and α_F of Rossi- and Feynman-α measurements and the associated reference reactivity level obtained by rod drop (≈ 37¢) to yield two reduced generation times: Λ^*_R and Λ^*_F.
- Use Λ^*_R, Λ^*_F and the generation time evaluated by ERANOS calculations Λ^*_{calc} with the pulsed neutron source prompt periods in configurations PRU, PRD and CRD to yield associated reactivities.

This permits a maximum reduction of the use of calculated constants. Indeed, in the first point, the reactivity obtained via Equation (1) is very insensitive to Λ^*_{calc} because the derivative term $\Lambda^* \dot{c}/c$ is negligible and the source term is adjusted by fit. Only relative neutron yields interfere. However we make the hypothesis that Λ^* is constant from REF to all SC0 configurations.

Reduced prompt generation times and reactivity for PRU, PRD and CRD configurations are synthesised within Table 6 and compared to rod drop/source multiplication reactivities.

Table 6. **Comparison of reactivities in the SC0 configuration**

Method	Λ^* (μs)	ρ_{PRU} (USD)	ρ_{PRD} (USD)	ρ_{CRD} (USD)
Rossi	171	1.23 (10.8%)	1.66 (10.5%)	9.88 (13.4%)
Feynman	166	1.16 (10.8%)	1.59 (10.5%)	9.59 (13.3%)
Calculation	158	1.05	1.46	9.03
Rod drop + SM	X	1.34 (5.9%)	1.75 (5.9%)	12.6 (6.1%)

Uncertainties in brackets are issued from square sum of relative standard deviations because we assume no correlation between the rod drop, Rossi- and Feynman-α and pulsed neutron source experiments. From PRU and PRD configurations, experimental reactivities derived from either Rossi or Feynman methods are in agreement to 1 σ with rod drop + SM values. This is not the case with the CRD configuration where a disagreement of nearly 25% is observed. The point model used in the pulsed neutron source could be deficient and/or the generation time increased. The deduced pilot rod worth from $\delta = \rho_{PRD}$-ρ_{PRU} is 43¢ for both Feynman and Rossi methods. Although only indicative, this value is in agreement with the worth of the pilot rod in the REF configuration. Calculations systematically underestimate reactivities and with the assumed measurement uncertainties they are not in agreement to 1 σ with rod drop + SM values in either configuration. Under-estimations of the generation time seem probable.

Conclusions

From rod drop measurement on pilot rod in the MUSE-4 programme, a ±4% systematic uncertainty due to non correlated α_j , λ_j and an additional ±1% due to correlation has been emphasised. An additional restriction to the point model has been derived even for ≈40¢ sub-criticality. It leads to an uncertainty lower than ±2% which is detector dependent. The overall uncertainties permit a derivation via source multiplication method of reactivities as low as USD 12.6 with a ±6% uncertainty.

These reactivity references qualify the monitoring of reactivity by pulse neutron source method coupled with noise measurements around USD 1.5 with a 5% to 10% confidence. Reactivities issued from the use of calculated reduced generation time shows an approximate 20% underestimation in the same configurations. Finally around USD 12.5 (k≈0.96) pulsed neutron source in the point kinetic model underestimate reactivity about nearly 25%.

Starting from a critical reference configuration, rod drop coupled with source multiplication techniques stay can accurately determine sub-critical reactivity levels in the range of use of future ADS. Capability to monitor reactivity using pulsed neutron source and Rossi- and Feynman-α techniques has been clearly shown in MUSE-4 for sub-critical levels up to USD 2. First results obtained for lower levels (k≈0.96) will be detailed by the end of 2002 in the SC2 configuration (k≈0.97) of the MUSE-4 programme. This will certainly extend the monitoring capabilities of pulsed neutron source and noise measurement techniques to reactivity levels for future ADS.

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