Workshop on Evaluation of Uncertainties
In Relation To Severe Accidents and
Level 2 Probabilistic Safety Analysis

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Session II: Methods for Uncertainty Assessment

Influence of mathematical modelling of knowledge : application to the transfer of radionuclides in the environment

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Influence of mathematical modelling of knowledge : application to the transfer of radionuclides in the environment

Contents

Introduction : limits of MC-methods

Monte-Carlo methods need a lot of knowledge.

1°) Choice of the input PDFs

Example of uniform distribution

Ignorance does not mean equiprobability

2°) Knowledge of possible dependencies between uncertain parameters

Example of independence assumption

No dependence information does not mean stochastic independence

Such assumptions may lead to an artificial reduction of uncertainty margins and thus can deteriorate the relevance of the decision making.

Current practices to mitigate difficulties encountered in MC simulations

- Use of deterministic penalizing values
- Use of penalizing PDFs or penalizing dependencies
- Double MC techniques
- Use of fuzzy theory
- Hybrid theory : a combination of fuzzy and probability theories

Fuzzy modelling : an extension of interval calculation

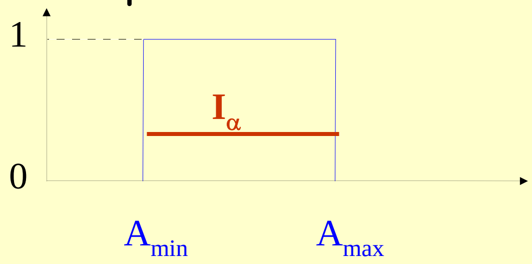
<u>Definition 1</u> : a fuzzy set is an extension of a classical set, with a membership function instead of a characteristic function

<u>Definition 2</u> : an α-cut is the set of points with a membership $\geq \alpha$.

<u>Definition 3</u> : a fuzzy number is a fuzzy set for which the α-cuts are nested intervals I_α : $I_1 \subset I_\alpha \subset I_0$

Example 1 : a flat fuzzy number *A*

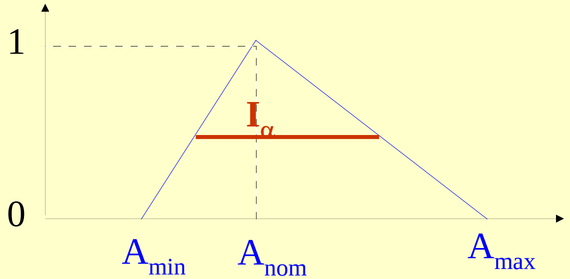
membership function



$$I_\alpha = [A_{min} , A_{max}] \qquad \forall \alpha \in [0 , 1]$$

Example 2 : a triangular fuzzy number A

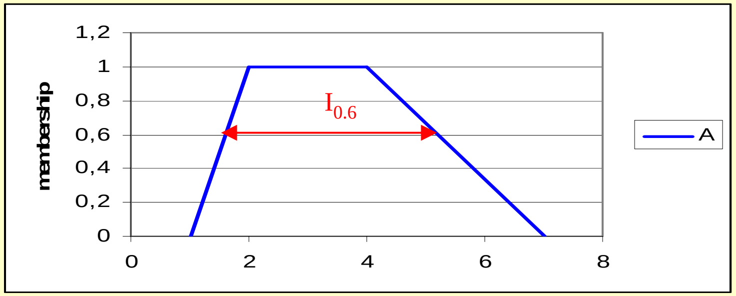
membership function



$$I_\alpha = [A_{min} + \alpha\,(A_{nom} - A_{min})\,,\ A_{max} - \alpha\,(A_{max} - A_{nom})\,]$$

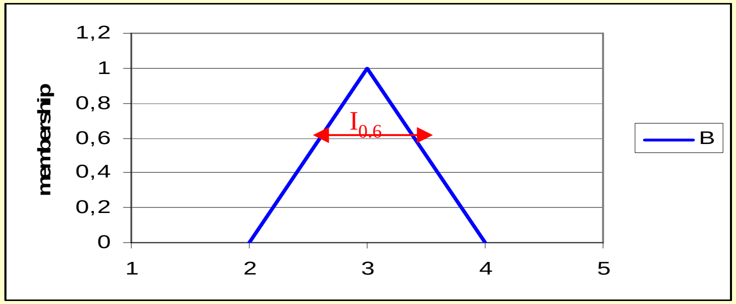
Fuzzy modelling : an extension of interval calculation

Trapezoidal fuzzy number A = (1, 2, 4, 7).



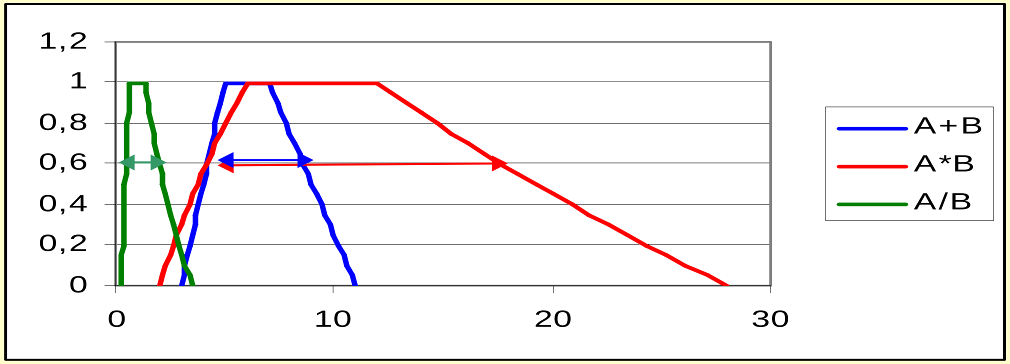
Interval A = [1 , 7]

Triangular fuzzy number B = (2, 3, 4).



Interval B = [2 , 4]

Results : A+B, A*B, A/B



A/B = [0.25 , 3.5]   A+B = [3 , 11]   A*B = [2 , 28]

The fuzzy calculation is an interval calculation performed for each α-cut I_α

Links between possibility theory and probability theory

A possibility measure as a probability measure is a way to measure the confidence associated to an event.

By definition, a possibility measure must satisfy the three following axioms :

$\pi(\emptyset)=0$,
$\pi(E)= 1$ where E is the whole set,
$\pi(A \cup B)=\max(\pi(A), \pi(B))$ for any subsets A and B.

Let us remind the Kolmogorov axioms of a probability measure P :

$P(\emptyset)=0$,
$P(E)= 1$ where E is the whole set,
$P(A \cup B)= P(A) + P(B)$ for any subsets A and B such as $A \cap B= \emptyset$.

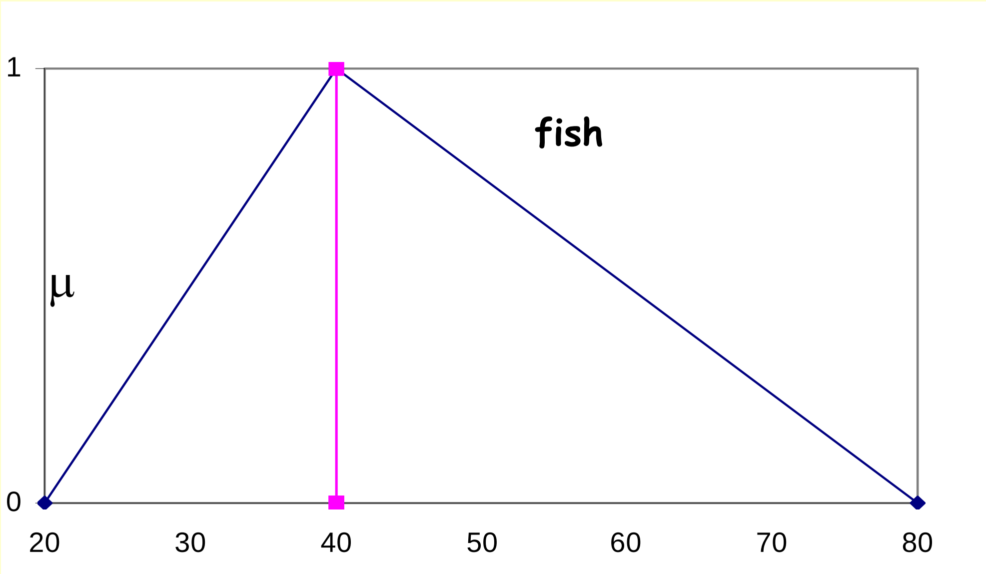
N.B 1 $\pi(A)=1$ and $\pi(\text{non } A)= 1$ means that no information on the occurrence of the event A is known.

N.B 2 $\pi(A)=0$ means that the event A is impossible, therefore $\pi(\text{non } A)= 1$.

Links between fuzzy numbers and possibility theory

Example : Quantity of ingested fish Q

membership



$\mu(30)=0.5$
$\mu(40)=1.0$
$\mu(50)=0.75$
$\mu(60)=0.5$
$\mu(70)=0.25$
$\mu(80)=0.$

A membership function measures the membership of an element to a fuzzy set.

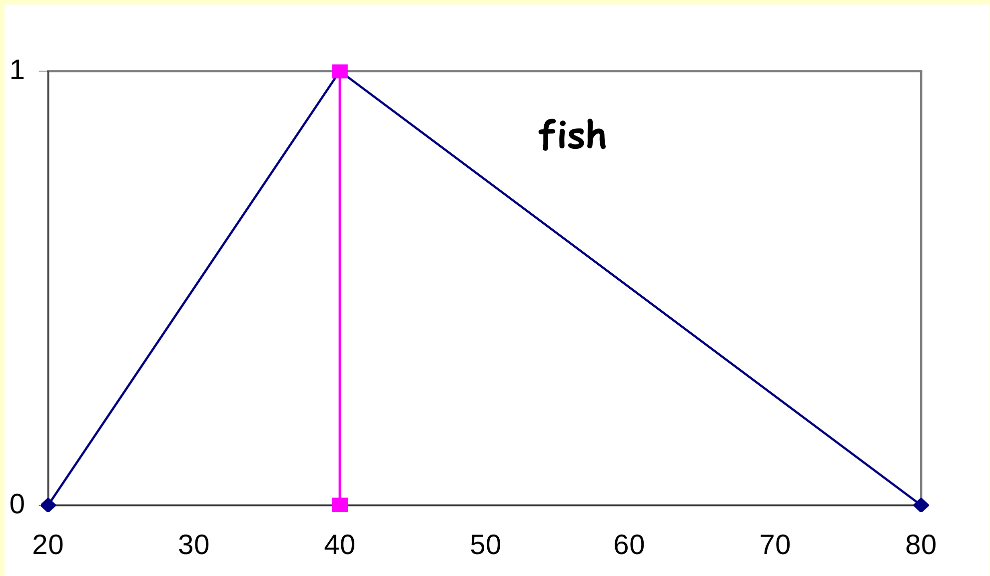
A membership function does not allow to measure the confidence associated to an event.

A membership function allows to define a possibility measure from which it is possible to measure the confidence associated to an event.

Links between fuzzy numbers and possibility theory

Example : Quantity of ingested fish Q

Possibility π



$\pi(\{40\}) = 1.0$

$\pi([20,30]\ )= 0.5$

$\pi([70,80]\ )= 0.25$

$\pi([70,80] \cup [20,30])=0.5$

A fuzzy number defines a possibility distribution:

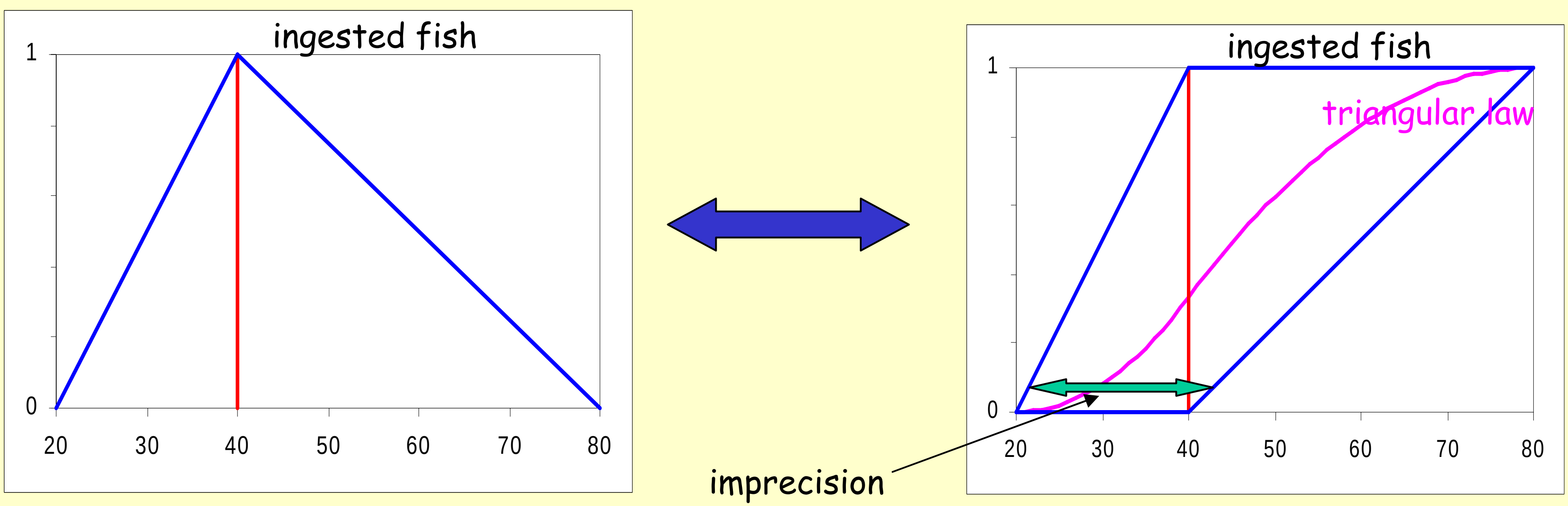
$$\pi(E)= \pi(E \text{ knowing } Q)= \max(\mu(x)) \quad x \in E$$

Links between possibility theory and probability theory

A possibility distribution is similar to a family of PDFs

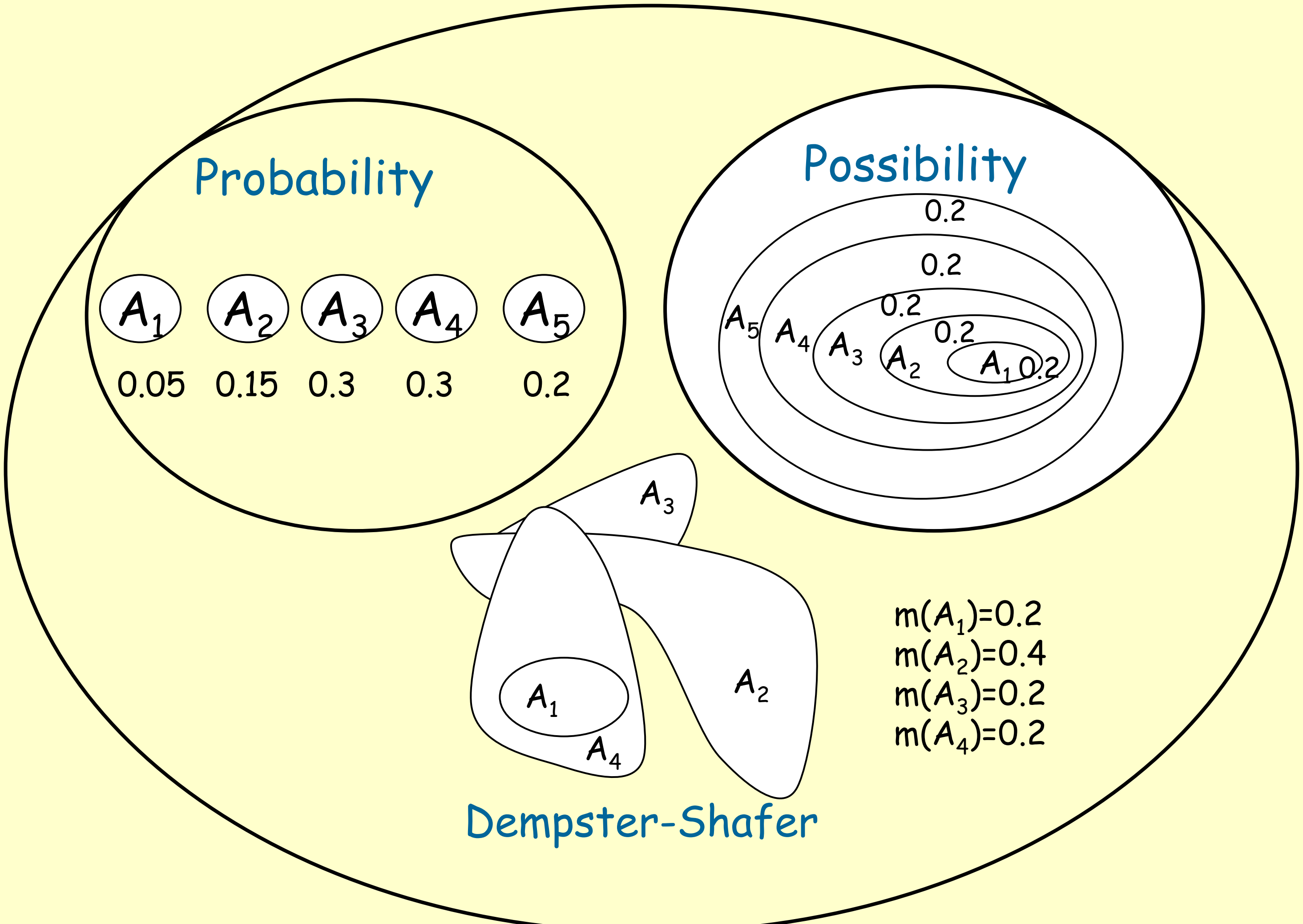
Example : triangular possibility

possibility distribution **Equivalent set of PDFs**



A triangular distribution of possibility contents all the probabilities with the same mode and support.

Dempster-Shafer theory: an unified framework for possibility and probability theories



Dempster-Shafer theory: an unified framework for possibility and probability theories

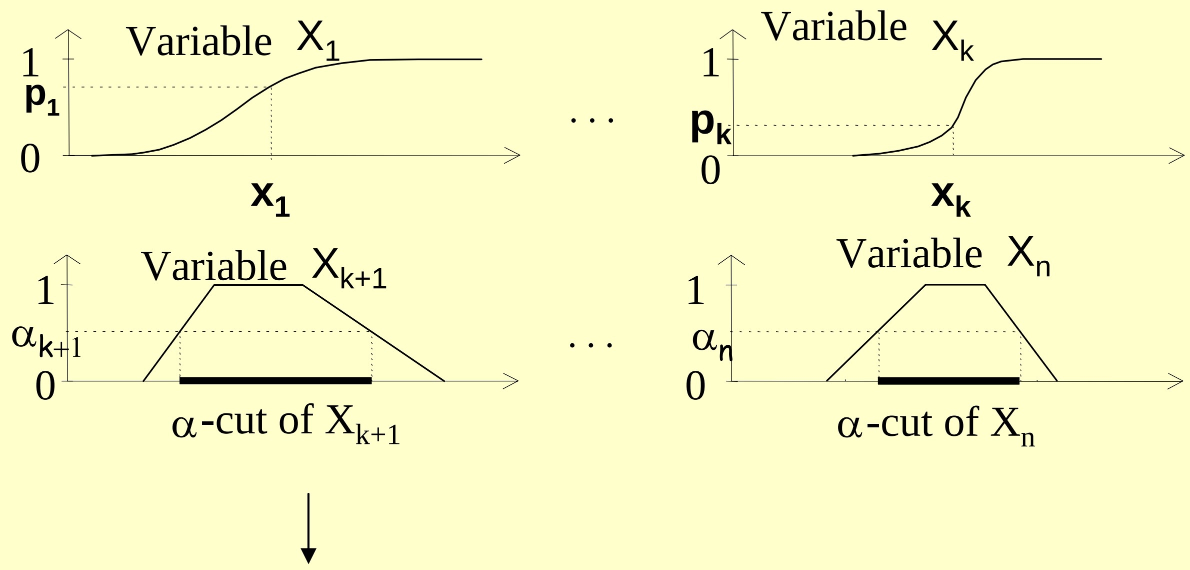
Uncertainty propagation : the 'Dempster-Shafer' method

Principle = extended MC simulations

Model = $M(X_1,\ldots,X_k,X_{k+1},\ldots,X_n)$; $X_1,\ldots,X_k$ probabilities and $X_{k+1},\ldots,X_n$ possibilities

If variability: enough knowledge available
→ PDF

If imprecision: not enough knowledge available
→ family of PDFs encoded by a possibility distribution

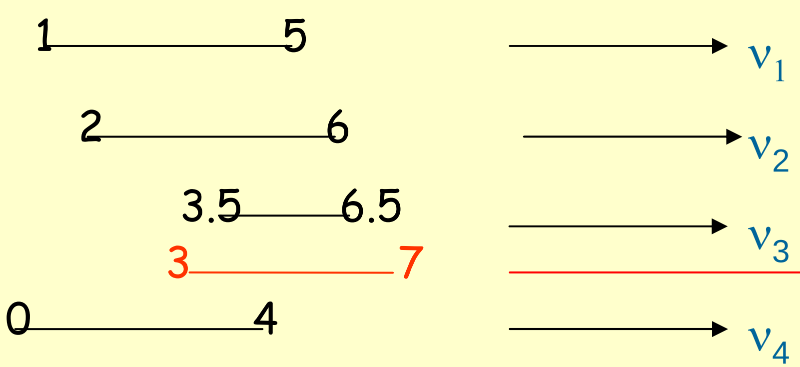


Result = sample of random intervals

Dempster-Shafer theory: an unified framework for possibility and probability theories

How to evaluate the uncertainty of an event? For example x ∈ [3,7]

Example of results:



$$\underbrace{\nu_3}_{\text{lower probability}} \le \mathrm{Proba}(x \in [3, 7]) \le \underbrace{\nu_1+\nu_2+\nu_3+\nu_4}_{\text{upper probability}}$$

In MC simulations all the $\nu_i = 1/N$

Application to a simple example

A simple radionuclide transfer model from maize to man through the consumption of milk

Absorbed dose $D = A * Q * F * M$

A : the maize activity is a random variable, known from experimental measurement

Q : the quantity of eaten maize :
very likely between 10 and 14 kg/day and cannot be out of the interval [4 , 35],

F : the transfer factor from maize to milk
in the interval [0.001 , 0.005] with 0.003 day/litre for the most likely value,

M : the quantity of ingested milk
in the interval [70 , 280] with 140 litre/year for the most likely value.

The knowledge related to the parameters Q, F, M is not enough to define a specific PDF. The set of PDFs checking these conditions can be easily encoded by the mean of possibility distributions.

Application to a simple example

Modelling of dependencies between uncertainty 'sources'

Stochastic independency and epistemic independency

Stochastic independency.
The stochastic independency between two uncertain variables means that there is slight likely to have simultaneously extreme values between random variables and leads to a compensating effect between uncertainty sources.

Epistemic independency
The epistemic independency assumes that the information related to the two uncertain variables have the same reliability. With this assumption, the uncertainties may cumulate themselves .

Property :

Epistemic independency is similar to an ignorance of the stochastic dependency.
Indeed, the use of the epistemic independency as in the interval calculation, leads to cumulate uncertainties when no information is available about compensating effects. At the opposite, the use of stochastic independence assumptions, when it is possible, limits the over-conservatism of standard interval calculations.

Application to a simple example

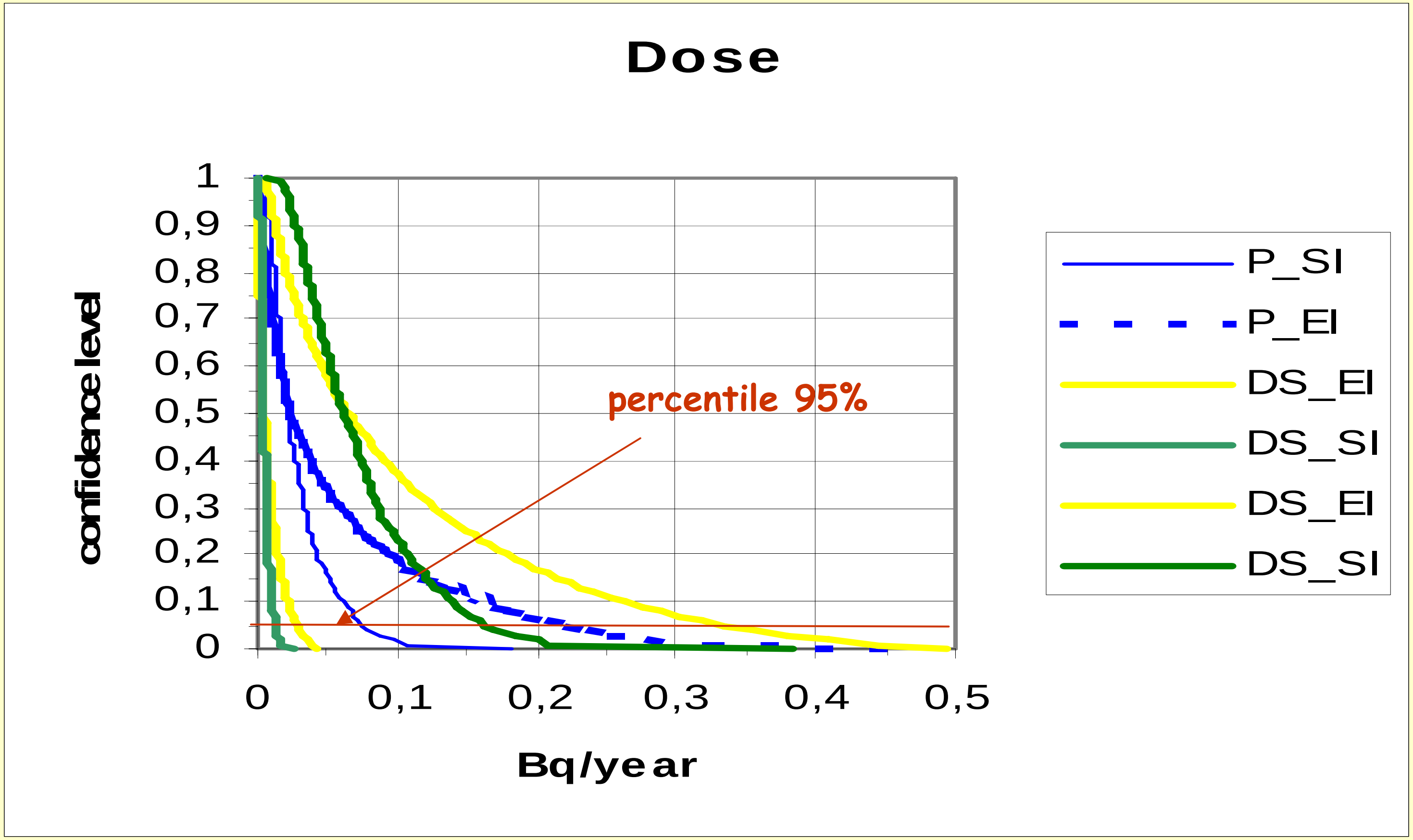
Modelling of uncertainty 'sources'

parameter	Probabilistic methods : P_SI and P_EI	Dempster-Shafer methods DS_SI and DS_EI
maize activity A	random variable : lognormal distribution $m=-5.76$, $\sigma=0.58$	random variable : lognormal distribution $m=-5.76$, $\sigma=0.58$
quantity of maize Q	random variable : trapezoidal distribution (4, 10, 14, 35)	fuzzy variable : trapezoidal distribution (4, 10, 14, 35)
transfer factor F	random variable : triangular distribution (0.001, 0.003, 0.005)	fuzzy variable : triangular distribution (0.001, 0.003, 0.005)
quantity of milk M	random variable : triangular distribution (70, 140, 280)	fuzzy variable : triangular distribution (70, 140, 280)

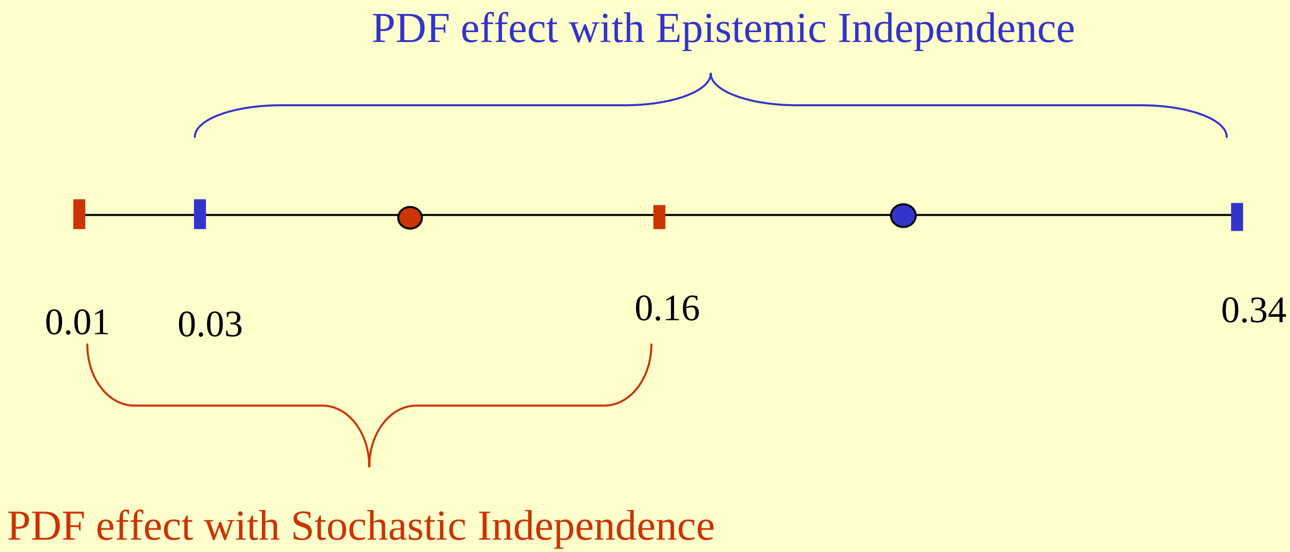
_SI for Stochastic Independence assumption and
_EI for Epistemic Independence assumption

Application to a simple example

Result : CCDF of the activity



Application to a simple example



Percentile 95% derived from MC simulations

Conclusion :

These figure shows the importance of the assumptions related to the choice of marginal distributions (a factor ~10 on the percentile 95%) and their dependencies (a factor ~3 on the percentile 95%) on the uncertainty margins.

Conclusion

The 'Dempster-Shafer' uncertainty methodology

- has the same advantages of MC simulations :

very easy to perform , unlimited number of uncertain parameters, independent of models complexity

+ possibility to relax, when not enough knowledge is available, PDFs and dependencies assumptions, allowing to derive reliable results.

- No confusion between epistemic uncertainties (family of PDFs) and stochastic uncertainties (PDF).