



# *Uncertainty and sensitivity methods in support to Level 2 PSA*

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WORKSHOP ON EVALUATION OF UNCERTAINTIES IN RELATION  
TO SEVERE ACCIDENTS  
AND LEVEL 2 PROBABILISTIC SAFETY ANALYSIS  
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- In this talk, mainly comments on the suitability of uncertainty and sensitivity methods for Level 2 PSA

**The content is the point of view of authors, and all comments are welcomed !**

Part of this work is partly on-going in the framework of the WP 5.2 of



(See paper B. Chaumont Wednesday)

- 1 task: reviewing possible (non usual) methods for uncertainty and sensitivity analysis in support to L2 PSA
  - (non exhaustive) list of references for a description of methods
    - NEA/CSNI/R(94)20
    - NEA/CSNI/R(97)35 *(with examples)*
    - NEA/CSNI/R(99)10 (mainly Session III) et NEA/CSNI/R(99)22
    - Proceedings of the *International Workshop On Level 2 PSA and Severe Accident Management* (OCDE/CSNI/WGRISK, Köln, March 2004) *(with examples)*, including:
      - N. Devictor N. et al. Advances in methods for uncertainty and sensitivity analysis. *Proceedings of the workshop "Level 2 PSA and Severe Accident Management"*, OCDE/AEN/CSNI/WGRISK, Köln, March 2004 *(with examples)*
- + bibliography in these documents.

- In the framework of the study of the influence of uncertainties on the results of severe accidents computer codes (and specially for Best Estimate codes), and then on results of Level 2 PSA (responses, hierarchy of important inputs...)
- Why taken account uncertainty ?
  - A lot of sources of uncertainty
  - To show explicitly and traceably their impact  $\Rightarrow$  decision process that could be robust against uncertainties
- Some applications of treatment of uncertainty by probabilistic methods
  - For a best understanding of a phenomenon
    - To evaluate the most influential input variables. To steer R&D.
  - For an improvement of a modelling or a code
    - Calibration, Qualification...
  - In a risk decision-making process
    - Hierarchy of contributors  $\Rightarrow$  interest for actions to reduce uncertainty or to define a mitigation mean (for example a SAM measure)
    - Confidence intervals or probabilistic density functions or margins...
- In any analysis, we must keep in mind the choice in modelling and the assumptions.
  - Case : a variable has a big influence on the response variability, but we have a low confidence on his value...

Three main sources of uncertainty:

– Parameter uncertainty

→ physical variables and model parameter

- Statistical and mathematical tools exist

– Model uncertainty

Due to the incomplete knowledge in the phenomena that can occur during a severe accident

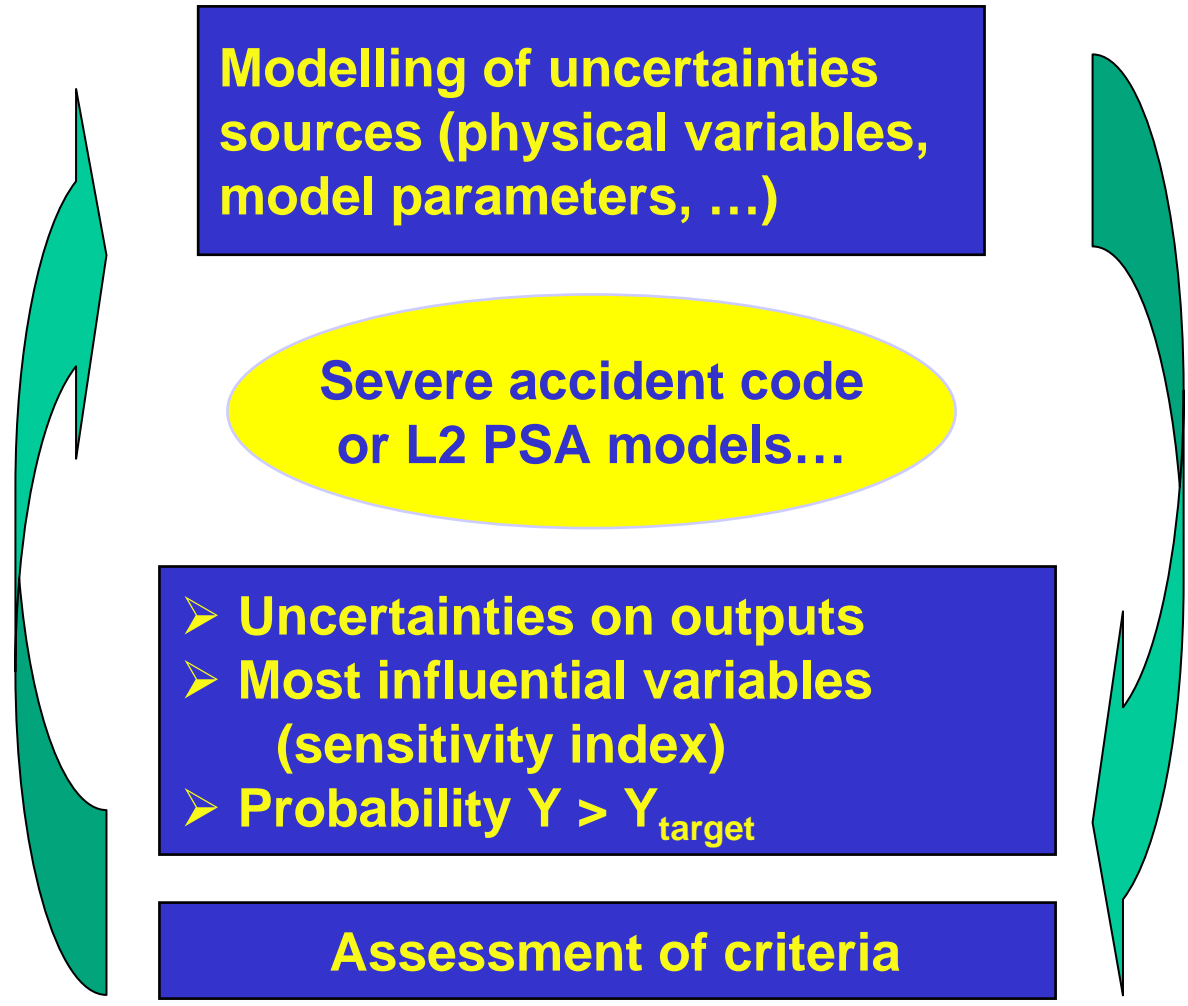
- Usually studied by parametric study

– Scenario uncertainty

- Related to the completeness of the analysis and whether there area any fault sequences that have not been included in the analysis
- Usually reduced by carrying out a peer review of the analysis

Uncertainty sources may be divided, according to their origin, into:

- Stochastic uncertainty
- Epistemic uncertainty



Main objective of input parameter uncertainty Characterization: to characterize as well as possible our state of knowledge about the system.

- Stochastic uncertainty
  - Classical statistical techniques (maximum likelihood, method of moments, bootstrap, ...)
- Epistemic uncertainty
  - Bayesian estimation (use of generic and specific data to build up prior distribution function and likelihood function)

$$\pi(\theta | \mathbf{x}, H) = \pi(\theta | H) L(\mathbf{x} | \theta, H)$$

- Expert Judgment (use of structured protocols to get and combine expert opinions)

*Comments: at the present time, works for building a coherent mathematical theory for uncertainty that involves different paradigm are on-going (see for example the Dempster-Shafer theory or theory of evidence); these works seems promising, but not at the present time mature from an industrial point of view.*

The practical approach can be summarized by three cases:

- **Case 1.** If a lot of experience feedback data is available, the frequential statistics is generally used. The objectivist or frequential interpretation associates the probability with the observed frequency of an event. In this interpretation, the confidence interval of a parameter,  $p$ , has the property that the actual value of  $p$  is within the interval with a confidence level  $\alpha$ ; this confidence interval is calculated based on measurements.
- **Case 2.** If data is not as abundant, expert opinion may be used to obtain modeling hypotheses. The Bayesian analysis is used to correct a priori values established based on expert opinion as a function of observed events. The subjectivist (or Bayesian) interpretation understands probability as a degree of belief in a hypothesis. In this interpretation, the confidence interval is based on a probability distribution representing the analyst's degree of confidence in the possible values of the parameter and reflecting his/her knowledge of the parameter.
- **Case 3.** If no data is available on a parameter, its probabilistic representation may be obtained from a model and from the knowledge of the uncertainties on the input parameter of this model. The data to be gathered thus concerns the input parameters. The quality of the probabilistic analysis is a function of the credibility of statistics concerning these input parameters and that of the model. The following cases can be discerned:
  - A *structural reliability-type approach* if the sought value is a probability,
  - An *uncertainty propagation-type approach* if a statistic around the most probable value is considered.

- “Case 1”, where a large enough sample is available, i.e. the sample allows “characterization of the relevant distribution with a known and adequate precision”, begs the following questions:
  - Question 1 : Is the selected distribution type relevant and justifiable? From the various statistical models available, what would be the optimal distribution choice?
  - Question 2 : Would altering the distribution (all other things being equal) entail a significant difference in the results of the application?
  - Question 3 : How can uncertainty associated with sample representativeness be taken into consideration (sample size, quality, etc)?
  
- Justification could be more difficult for Cases 2 and 3 (*more expert judgment*).

In industrial studies, the characteristics of databases must be taken into consideration because the implementation of an adjustment can be difficult; for example the following frequently encountered scenarios:

1. the sample size is small and therefore asymptotic results need to be handled cautiously as well as approximations of more or less valid moments of an order greater than two;
2. sample data values are measured with an uncertainty;
3. sample homogeneity is not verified (mixture of samples taken from different populations, overlaying of phenomena, etc.);

If the area of interest is a distribution tail, it should be noted that the statistical theory and above all associated tools are less developed.

- In a PSA level 2 it is necessary to estimate as accurately as possible all relevant output variables.
  - Full characterization of a random variable → its probability density function.
  - Could be summarized by some numeric statistics like the mean, the standard deviation and **order statistics** (see *paper E. Chojnacki*), among others.
  - Additionally, there are several graphics that provide visual information about the shapes of the aforementioned functions.
- PDF (probability density function)
  - Once the **pdf** of an output is determined, all statistics of interest can be directly and easily obtained, including the assessment that the probability of an output exceeds a threshold.
  - From a theoretical point of view, the best method to build the pdf is Monte-Carlo methods (*suitable for static and dynamic models and for probabilistic models with continuous or discrete variables*).
  - Monte-Carlo methods, variance reduction techniques, moments methods, classical statistics could be easily included in a Level 2 PSA software.
  - **The main drawback** : a large number of calculations. To avoid this problem, it can be interesting to build a **response surface** or **surrogate model** or **simplified model** which approximates the complex physical phenomena.
- When new information are available and some *pdf* of inputs change, statistical results about outputs could be (sometimes) obtained from old results without new computations by using techniques like “distribution sensitivity techniques” for example.

Distribution sensitivity analysis addresses the changes in the output distribution induced by changes in the distribution of the inputs.

- The weighting method (provides update of the mean)

$$\hat{\mu}_2 = \frac{1}{n} \sum_{i=1}^{i=n} \frac{f_2(\mathbf{x}_i)}{f_1(\mathbf{x}_i)} y(\mathbf{x}_i)$$

- The rejection method (provides update of the cdf)
- The extended rejection method (provides update of the cdf)

$$\hat{\mu}_{2r} = \frac{\sum_{i=1}^n Y(\mathbf{x}_i) \frac{f_2(\mathbf{x}_i)}{f_1(\mathbf{x}_i)}}{\sum_{i=1}^n \frac{f_2(\mathbf{x}_i)}{f_1(\mathbf{x}_i)}}$$

$$P(Y(\mathbf{x}_i)) = \frac{\frac{f_2(\mathbf{x}_i)}{f_1(\mathbf{x}_i)}}{\sum_{i=1}^n \frac{f_2(\mathbf{x}_i)}{f_1(\mathbf{x}_i)}}$$

**Sensitivity is the study of the influence of the inputs on the outputs.**

- Main problem: Under such a generic definition there may be many different interpretations.
  - Sensitivity considered as output variable response to an increment in some of the inputs (non-probabilistic interpretation).
  - Sensitivity considered as correlation between inputs and outputs
  - Sensitivity considered as monotonic relation between inputs and outputs
  - Sensitivity considered as more complex polynomial relation
  - Sensitivity considered as specific relation between different parts of range of definition of different variables
  - Sensitivity considered as fractional contribution to the output variance (variance based techniques)
  - Sensitivity considered as output distribution changes as a result of input distribution changes (distribution sensitivity techniques)

$$y = f(x_1, \dots, x_p) \quad (\text{where } y \text{ could be a probability})$$

- 1<sup>st</sup> Question : what is the impact of a variation of the **value** of an input variable on the **value** of the response Y ?

- Gradient, differential analysis
- Often deterministic approach
- *Interest for the prevention of “cliff edge effects”*

- 2<sup>nd</sup> Question : what is the part of the **variance** of Y that comes from the **variance** of  $X_i$  (or a set  $\{X_i\}$ ) ?

$$\frac{V(E[Y|X_i])}{V(Y)}$$

- Usual sensitivity indices
  - Pearson’s correlation coefficient, Spearman’s correlation coefficient, Coefficients from a linear regression, PRCC...
- In the case of **non linear** or **non monotonous** : Sobol’s method or FAST
  - *with very time consuming code (→ use of response surface),*
  - *problems with correlated uncertainties.*
    - All these indices are defined under the assumptions that the variables inputs are statistically independent.

SA techniques may be used pursuing different objectives, all of them related to getting knowledge about the behavior of the system studied, in other words, related to getting information about the input-output relation.

- guidance as to where to improve the state of knowledge in order to reduce the output uncertainties most effectively,
  - to steer research and development efforts,
  - better understand the modeling
  - to obtain a good confidence in the results.
- They may be divided into numerical and graphical techniques, and may in many cases be used simultaneously, using the same data set.
  - Many different SA techniques:
    - Regression based techniques
    - Non parametric statistics used to identify relations between regions of input parameters and output variables
    - Analysis of variance (ANOVA) based techniques
    - Design of experiments.
    - Distribution sensitivity techniques.
  - All these techniques are “complementary” and could be used according the characteristics of the physical models and the statistics of interest.
  - All these methods assume that the random input variables are statistically independent. New results dependent random variables are available (see references).

- Classical sensitivity indexes like PCC, PRCC SRC...
  - could easily be implemented in software, or the database defined by the sets of inputs and computed outputs could be easily post-processed in statistical software.
  - But underlying assumptions should always kept in mind when the results are analyzed.
- FAST and Sobol
  - define the points for which calculations are required → a coupling between L2 PSA software and statistical software is then necessary.
  - For these two last methods, a lot of calculations could be required according a given precision, then the use of *surrogate models* could be necessary.
  - We does not think it is possible to apply such methods to a full L2 PSA. But they could be of big interest if they are applied to sub-models or parts of L2 PSA.
- Sensitivity analysis permits to distinguish the influence of aleatory and epistemic uncertainties.
  - This difference is important for the decision-maker, because the way for mitigating their impact are different. For example case of the most influential uncertainties are epistemic and uncertainty of the output is “unacceptable”, an input uncertainty could be better known (and then its influence reduced) with new experimental results.
  - In case of the most influential uncertainties are aleatory and uncertainty of the output is “unacceptable”, in the most cases it is necessary to modify the design of the structure or to define an additional barrier for example.

- To assess such probability has an interest in some decision-making process, by for examples:
  1. to assess the probability that a sequence or a set of sequences exceed a fixed value;
  2. to assess the confidence interval on that probability,
  3. to assess the most influential variables on that probability.

- 2 main families of methods
  - Monte-Carlo simulations and variance reduction techniques
  - FORM/SORM

We think it is very difficult to introduce **FORM/SORM** method for “intensive” use in a Level 2 PSA, because the robustness of the scheme should be verified before. And feedback experience shows that efficient optimization methods are sensitive to be used.



- But they could be of big interest if they are applied to sub-models or parts of L2 PSA (see paper in proceedings of PSA'05).

Simulations	FORM/SORM
<b>Results</b>	<b>Results</b>
Failure probability Error on the estimation  Probability distribution of the response	Failure probability Most influential variables (on the probability) Efficiency (depends on the number of random variables)
<b>Assumptions</b>	<b>Assumptions</b>
No assumptions on the random variables (discrete, continuous, dependency, etc.) No assumptions on the limit state function	Continuous random variables  Continuous limit state function (more suitable for optimization step)
<b>Drawbacks</b>	<b>Drawbacks</b>
Computation costs (depends on the probability level)	No error on the estimation Global minimum is required, but it is necessary to obtain all the minima of the optimization problem.

- Interest for a response surface (or meta-model or surrogated model):
  - Good capability in **approximation** (study on the training sample) ;
  - Good capability in **prediction** ;
  - **Low CPU time for a calculation.**
- Data needed in a Response Surface Method (RSM) :
  - a training sample **D** of points  $(\mathbf{x}(i), z(i))$ , where  $P(\mathbf{X}, Z)$  the probability law of the random vector  $(\mathbf{X}, Z)$  (unknown in practice) ;
  - a family **F** of function  $f(\mathbf{x}, \mathbf{c})$ , where **c** is either a parameter vector or a index vector that identifies the different elements of **F**.

- The best function in the family F is then the function  $f_0$  that minimized a risk function :

$$R(f) = \int L(z, f(\mathbf{x}, \mathbf{c})) dP(\mathbf{x}, y)$$

- In practice, often use of an empirical risk function :

$$R_E(f) = \frac{1}{N} \sum_{i=1}^N [z(i) - f(\mathbf{x}(i), \mathbf{c})]^2$$

- Polynomial models
- Generalized Linear Models (GLM)
  - Regression models (assumption : continuous function).
  - Other possibility : discriminant function (logit, probit models).
  - Qualitative and quantitative inputs.
- Thin plate spline
  - Regression models (assumption : continuous function).
- PLS (Partial Least Squares)
  - Regression models (assumption : continuous function).
  - Qualitative and quantitative inputs.
- Neural networks
  - Regression models (assumption : continuous function).
  - Other possibility : discriminant function (logit, probit models).
- A simplified « physical » model (3D → 1D, ...)

- The characteristic « good approximation » is subjective and depends on the use of the response surface.
  - What is the future use of the built response surface ?
  - What are the constraints that are forced by the use ?
  - How to define the validity domain of a response surface ?
    - Calibration, modelling, prediction, probability computation...
    - Specific criteria in the decision making process
      - Conservatism / A bound on the remainder / Better accuracy in a interest area (distribution tail...).
- How defines the expected accuracy ?
  - Ratio “residual deviance / null deviance” ?
  - Calibration : representativeness of the most influential parameters,
  - Prediction : robustness : bias/variance compromise,
  - The quality of the response surface should be compatible with the accuracy of the studied code.

## Statistics

(often under assumptions like Gauss-Markov assumptions...)

- Variance analysis
- Estimator of the variance  $\sigma^2$
- $R^2$  statistics
- Confidence area  $1-\delta$  for coefficients  $\mathbf{c}$
- ...

**Prediction** : test base (bias), cross validation

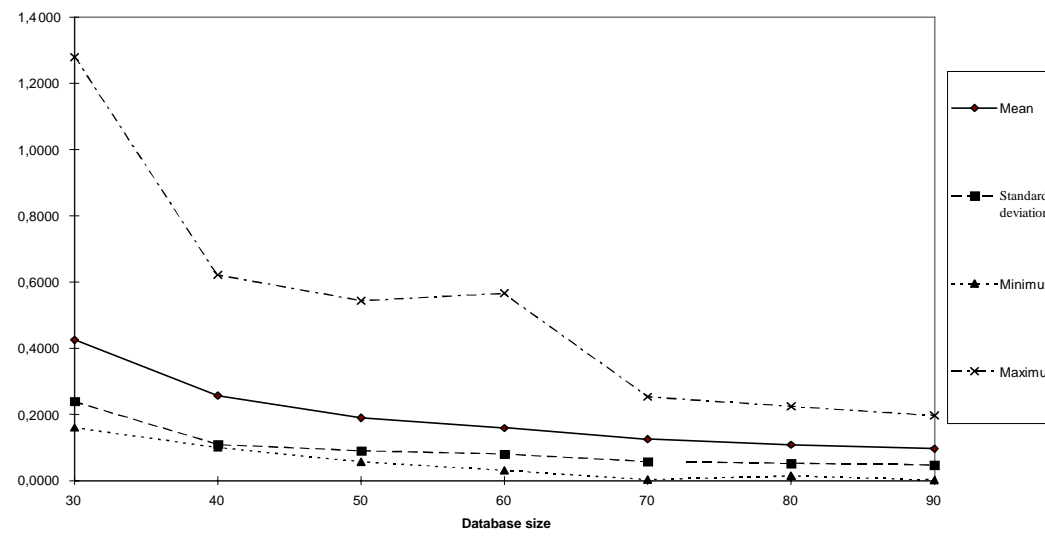
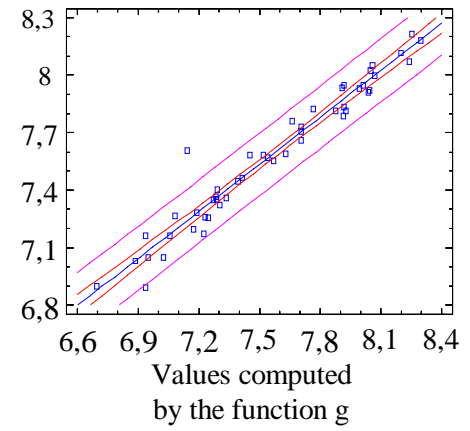
## Bootstrap method

to improve the estimation of the bias between learning and generalization error, to estimate the sensitivity of the trained model  $f$  in relation to available data.

## Comparison of results

- Pdf of the output, Confidence interval...

Values computed by a polynomial response surface



- Dealing with uncertainties in Level 2 PSA and severe accidents requires using a set of statistical techniques needed to assess input uncertainty, to propagate uncertainties in an efficient way, to characterize appropriately output uncertainty and to get information from computer code runs through the intelligent use of sensitivity analysis techniques.
- In the on-going study, we have trying to give information about the interest, the capacities and the suitability of these methods and techniques for the purpose of Level 2 PSA and physic codes used in support.
  - These methods could be not suitable, from a theoretical point of view:
    - the phenomena that are modelled by the computer code are discontinuous in the variation range of influent parameters;
    - input variables are statistically dependent.
  - It could be noticed that a lot of these methods are computing time consuming, and seems more suitable for the analyse of submodels or for focusing about a question.
- The state of the art of Level 2 PSA shows that different strategies have been applied for the writing of Level 2 PSA and the uncertainty studies. One of the next steps of our work could be to compare the underlying assumptions of these strategies, and to propose recommendations for uncertainty and sensitivity analysis.
- Practical interest of these “new” methods should be confirmed, by applications on « real » problems are planned in SARnet.



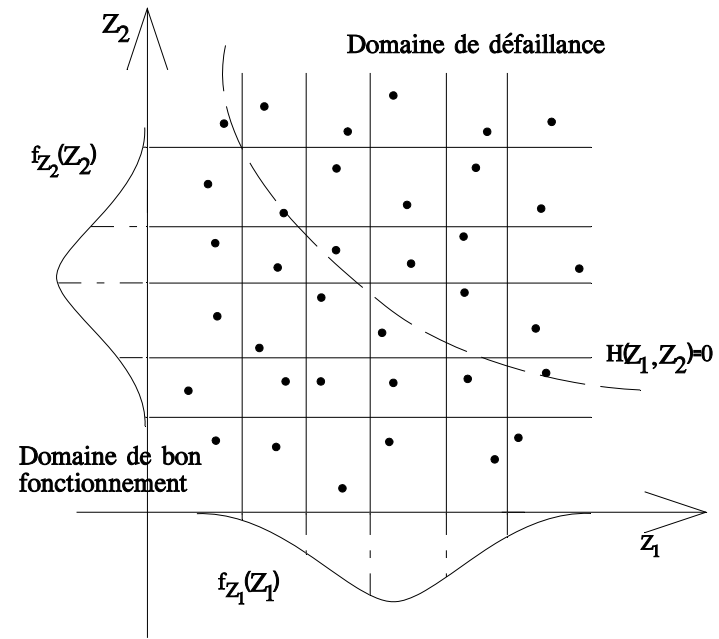
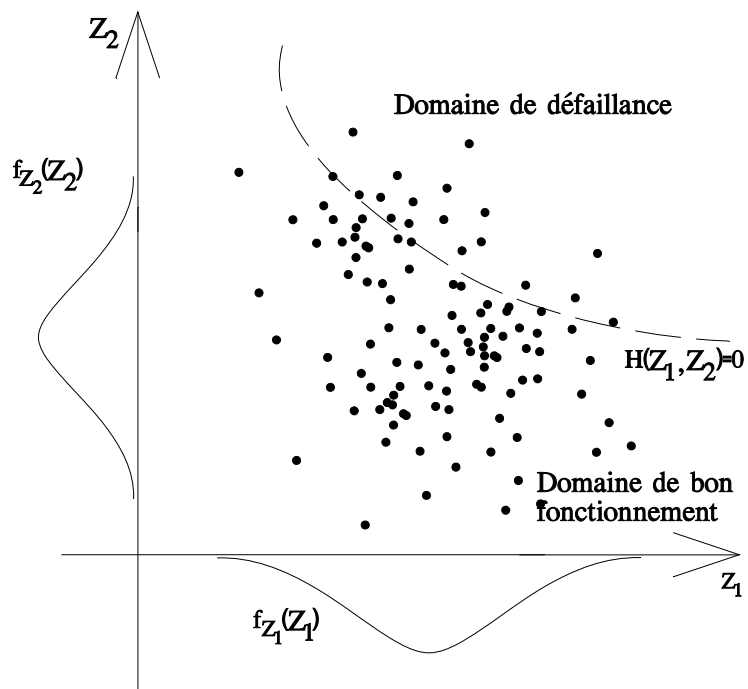
- Probability distribution
  - Simulation + fit + statistical tests (*asymptotical*)
- First statistical moments
  - Statistics on a sample (convergence, Bootstrap)
  - Approximation of the standard deviation

$$s[f(x_1, \dots, x_n)] = \left[ \sum_i \left( \frac{\partial f}{\partial x_i} \right)^2 s^2(x_i) + 2 \sum_{i=1}^n \sum_{j=i+1}^n \left( \frac{\partial f}{\partial x_i} \right) \left( \frac{\partial f}{\partial x_j} \right) \text{cov}(x_i, x_j) \right]^{1/2}$$

- Confidence interval
  - From the density function
  - Wilks formula

$$P\{P(m \leq Y \leq M) \geq \alpha\} \geq \beta$$

$$1 - \alpha^N - N(1 - \alpha)\alpha^{N-1} \geq \beta$$



–Variance reduction methods:  
*conditional MC, stratified MC,*  
*Hypercube Latin*

–More suitable for the computation  
 of a probability : *importance*  
*sampling, directional simulation*

–Practical problem with very time  
 consuming code → Response surface



- Probabilistic transformation  $Z \longrightarrow U$   
 ( $U_i$  is  $N(0,1)$ -distributed and are independents)
- In  $U$ -space, a new failure surface  $G(U)=H(T(Z))=0$
- Design point and Hasofer-Lind index  $U^*$

$$\beta_{HL} = \min_{G(u)=0} \sqrt{u^t u}$$

- FORM approximation

$$P_F \approx \Phi(-\beta_{HL})$$

- SORM approximation (Breitung)

$$P_F \approx \Phi(-\beta_{HL}) \prod_{i=1}^n (1 - \beta_{HL} \kappa_i)^{-1/2}$$

- Sensitivity factors

$$\alpha_i = \frac{u_i}{\beta_{HL}}$$

