Workshop on Evaluation of Uncertainties In Relation To Severe Accidents and Level 2 Probabilistic Safety Analysis

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Session II: Methods for Uncertainty Assessment

The use of Monte-Carlo simulations and order statistics for uncertainty analysis of a LBLOCA transient (LOFT L25)

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Contents

Uncertainty Analysis : Why ?

-To demonstrate that the NPPs are designed to respond safely at numerous postulated accidents, computer codes are used.

-The models of these computer codes are an approximation of the real physical behaviour occurring during an accident. Moreover, the data used to run these codes (inputs or models data) are known with a certain accuracy. Therefore the code predictions are not exact values.

-To deal with these uncertainties, safety demonstration can follow two different ways. The first way is to use conservative codes. These codes contain deliberate pessimisms and unphysical assumptions. It is then argued that the overall predictions are worse than the reality. The second way is to use best estimate codes with best-estimate data to evaluate best-estimate predictions.

-If best-estimate codes are used, then it is required to take into account the uncertainties (cf. NRC regulatory guide).

Uncertainty Analysis : Why ?

The use of best-estimate codes instead of conservative codes is motivated by both economical and safety reasons :

-the economical reasons :
It is expected that the use of best-estimate codes will allow to relax unnecessary technical specifications and operating limits set up by conservative codes.

-the safety reasons :
Due to the presence of numerous counter-reactions, it is difficult to prove the conservatism of conservative codes. Moreover the use of best-estimate codes allows to improve accident management procedures thanks to a better understanding of accident progress.

The aim of the uncertainty analysis is to provide '*reasonable*' uncertainty margins for the code results taking into account the uncertainties of inputs and models data. '*Reasonable*' means conservative but not over-conservative..

Types of uncertainty propagation methods

- **Deterministic methods :**
  The investigation of the variation domain relies on the ability of the analyst and therefore on the uncertainty margins.

- **Probabilistic methods :**
  The investigation of the variation domain is based on probability theory. However, the analyst must provide PDFs for each uncertain parameter and their possible correlations

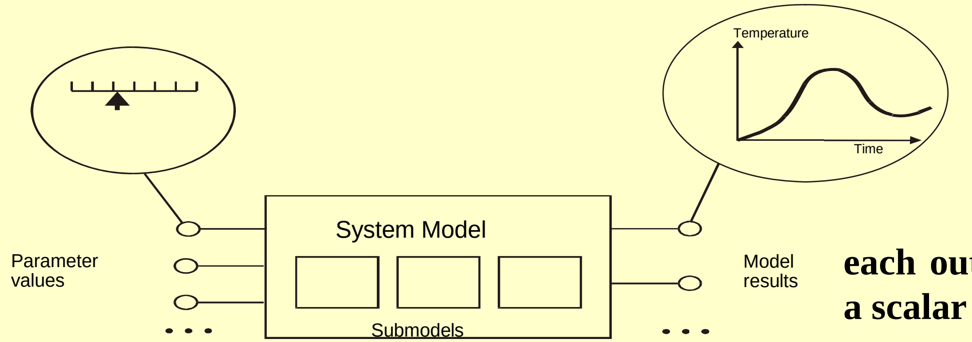
- **Possibilistic (or fuzzy) methods :**
  These methods are a generalization of interval calculation.

- **Hybrid methods :**
  Hybrid methods are a combination of probabilistic and possibilistic methods.

Principle of probabilistic uncertainty methods

Best-estimate calculation without uncertainties

each input X_i is a scalar value

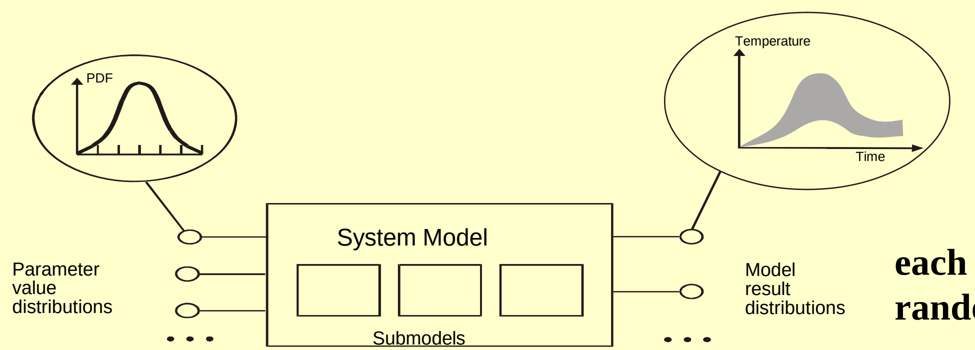


each output Y is a scalar value

$$Y = \text{computer code}\ (X_1, \ldots, X_n)$$

Best-estimate calculation with uncertainties

each input X_i is a random variable



each output Y is a random variable

$$Y = \text{computer code}\ (X_1, \ldots, X_n)$$

Probabilistic methods : Principle and advantages

Principle : to weight the likelihood of parameters values in order to quantify the likelihood of output values

Method : a specific PDF is attributed to each uncertain parameter with their possible intercorrelations, in order to evaluate the PDFs or CDFs associated to output values

Advantage : The knowledge on input values (i.e. their likelihood) is directly converted into knowledge on output values without any additional assumptions or expert opinions. The likelihood of output values is a mathematical consequence of the joint PDF of uncertain parameters through the computer code.

Drawbacks : How to select appropriate PDFs for uncertain parameters ?

How to calculate the PDFs of code responses ?

(Generally, it is analytically impossible because we have tens or hundreds of uncertain parameters with large ranges of variation , and moreover the code response is only implicitly defined))

Probabilistic methods : Principle and advantages

Monte-Carlo simulation allows to estimate any usual statistics

Mean, variance, percentiles… can be derived from the average of observed values

Data : X random variable $f_X(x)$ **its density and G any real function**

Statistical estimators : $E(G(X)) = \int G(x) f_X(x) dx$

Large numbers law : $\frac{1}{N}\sum_{1}^{N} G(x_i) \xrightarrow{N \longrightarrow \infty} E(G(X))$

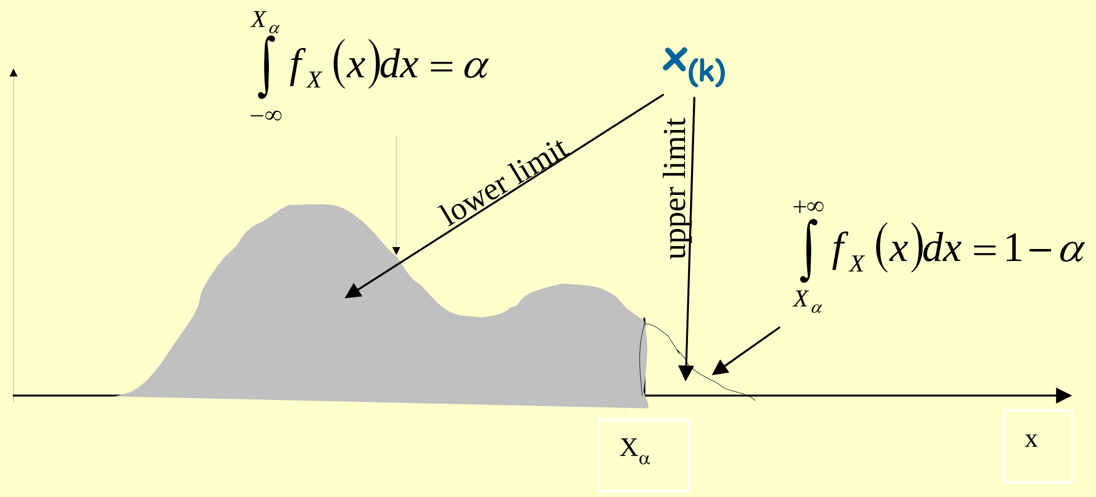
Examples :

If	**G(x) = x**	**then E(G(X)) : mean**
	G(x) = x^2	**E(G(X)) : variance**
	G(x)=1 if $x \leq x_0$ **and 0 else**	**E(G(X)) : CDF in** x_0

Use of order statistics

Order statistics are statistics using sorted sample values : $x_{(1)}<x_{(2)}<\ldots<x_{(n)}$
Order statistics are a way to derive direct and robust estimations of percentiles without additional assumptions such as response surfaces or fit tests

Definition : **α-fractile or percentile denoted X_α** : deterministic value which divides the PDF into 2 parts such as :



Probability $(X \leq X_\alpha)$ = confidence $(X \leq X_\alpha) = \alpha$

α is a measure of *reasonable* feature of safety margins

Probability $(x_{(k)} \leq X_\alpha)$ = confidence $(x_{(k)} \leq X_\alpha) = \beta$

β is a measure of *confidence* that $x_{(k)}$ is lower than X_α

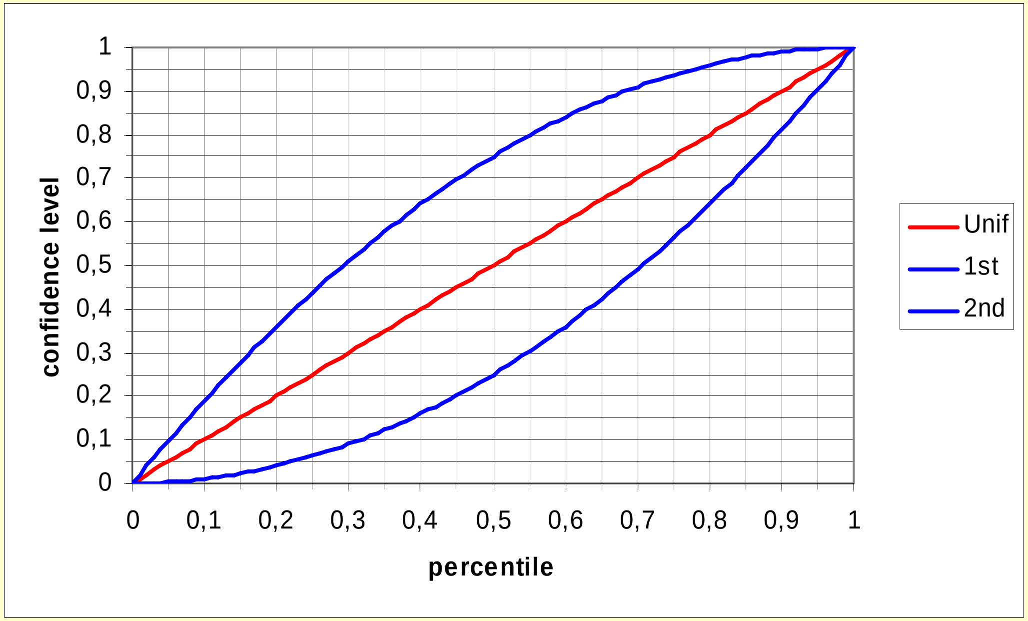
Use of order statistics

Property : the probability that the k^{th} sorted value out of a sample of size n is lower or upper than a given percentile does not depend on the law of the sample ; it is given by the beta law $\beta(k,n-k+1)$.

$$Proba(x_{(k)} \leq X_{\alpha}) = F_{\beta(k,n-k+1)}(\alpha)$$

where $\beta_{(k,n-k+1)}(x) = n!/[(k-1)!(n-k)!]\, x^{k-1}(1-x)^{n-k}$

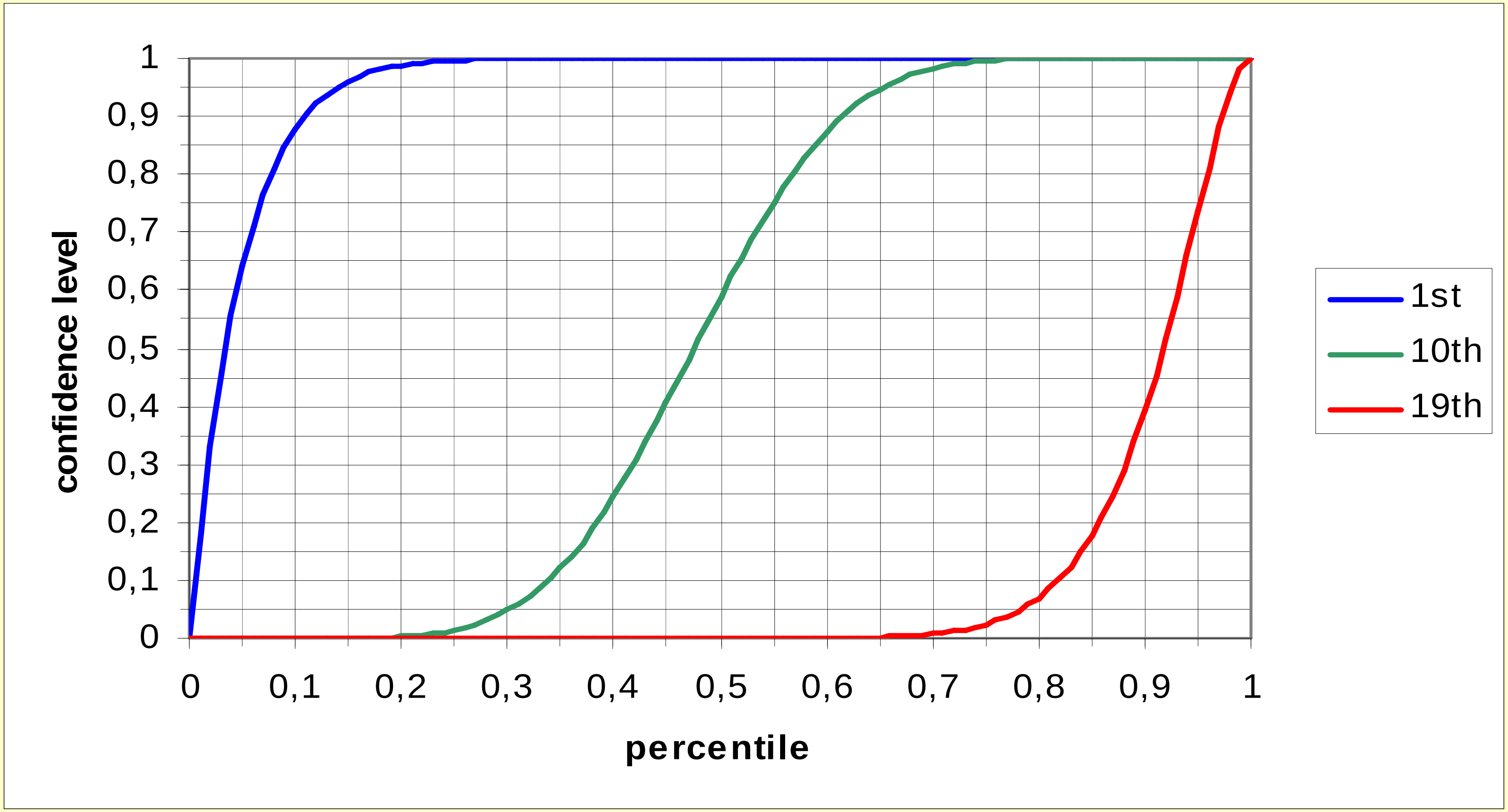
Example of a sample of size 1 and of size 2



Use of order statistics

How to use the 1st, 10th, 19th draw out of 20 to estimate lower, likely, upper values of percentiles ?

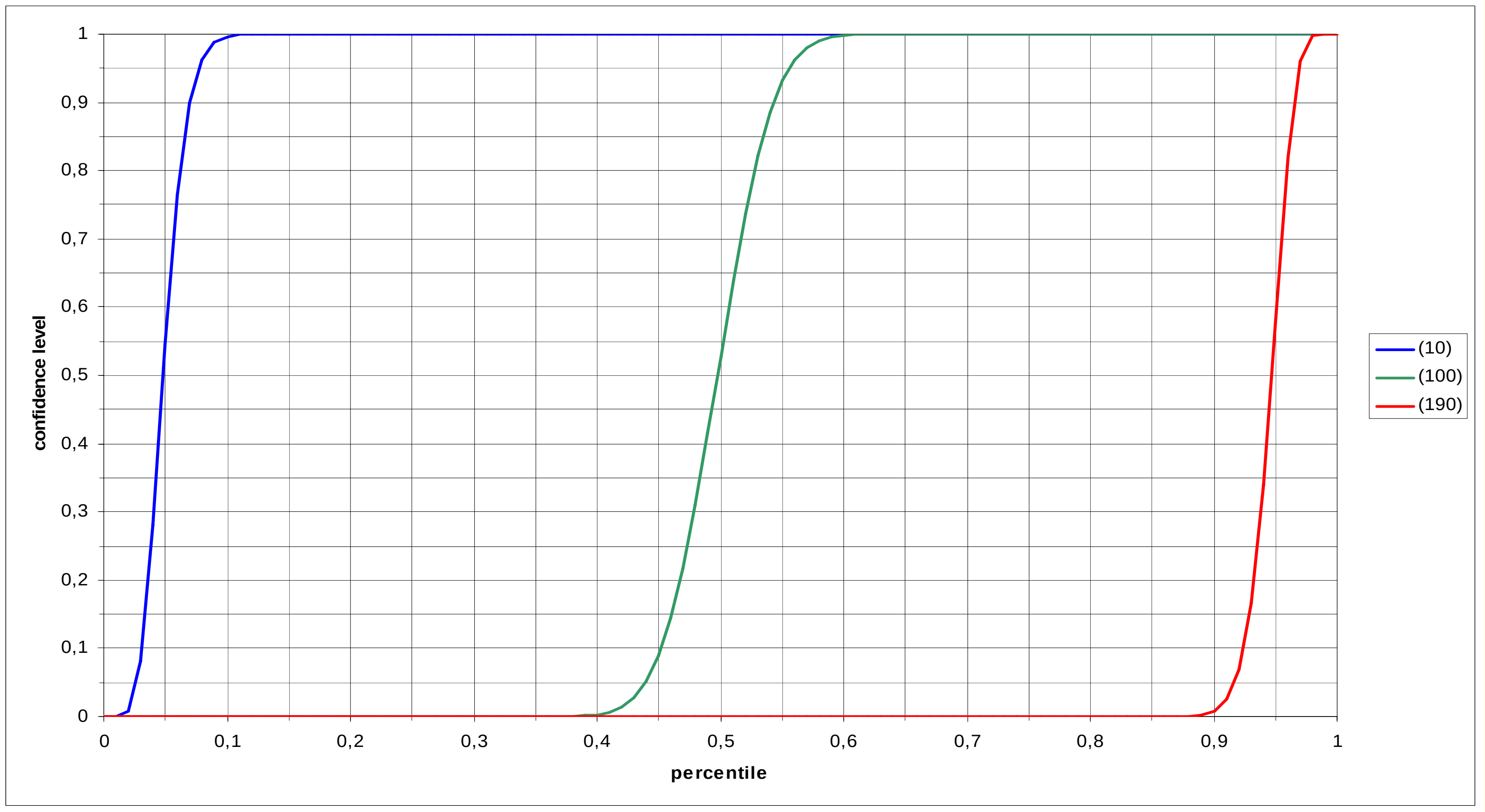
Example of a sample of size 20



Use of order statistics

How to use the 10th, 100th, 190th draw out of 200 to estimate lower, likely, upper values of percentiles ?

Example of a sample of size 200



Use of order statistics

How to use order statistics to derive the minimum sample size n which allows to derive an upper limit of the percentile α at the confidence level β ?

$$k=n \Rightarrow \beta_{(n,1)}(x) = n\, x^{n-1} \Rightarrow \alpha^n \leq 1-\beta \quad \Leftrightarrow \quad n \geq \ln(1-\beta)/\ln(\alpha)$$

Wilks' formula widely used in MC applications to limit the sample size

Table : minimum sample size
(to get an upper limit of a percentile α at the confidence level β.)

α \ β	0.80	0.90	0.95	0.99
0.80	8	16	32	161
0.90	11	22	45	230
0.95	14	29	59	299
0.99	21	44	90	459

Use of order statistics

How to evaluate the sample size effect on the accuracy of estimated percentiles ?

Example : 95% confidence interval from a 200-sample around the percentile 95%

Probability($x_{(184)} > X_{95\%}$) = 2.4%

and Probability($x_{(196)} < X_{95\%}$) = 2.6% .

$\Rightarrow$ Probability($x_{(184)} < X_{95\%} < x_{(196)}$) = 95% .

The difference between $X_{(196)}$ and $X_{(184)}$ represents the accuracy obtained on the percentile 95% from this limited sample

Use of order statistics

Conclusion

Advantage :

✓easy to perform

✓no selection of variables ⇒ unlimited number of uncertain parameters

✓direct upper and lower estimation of percentiles taking into account the limited sample size

✓no response surfaces (generally very difficult to obtain)

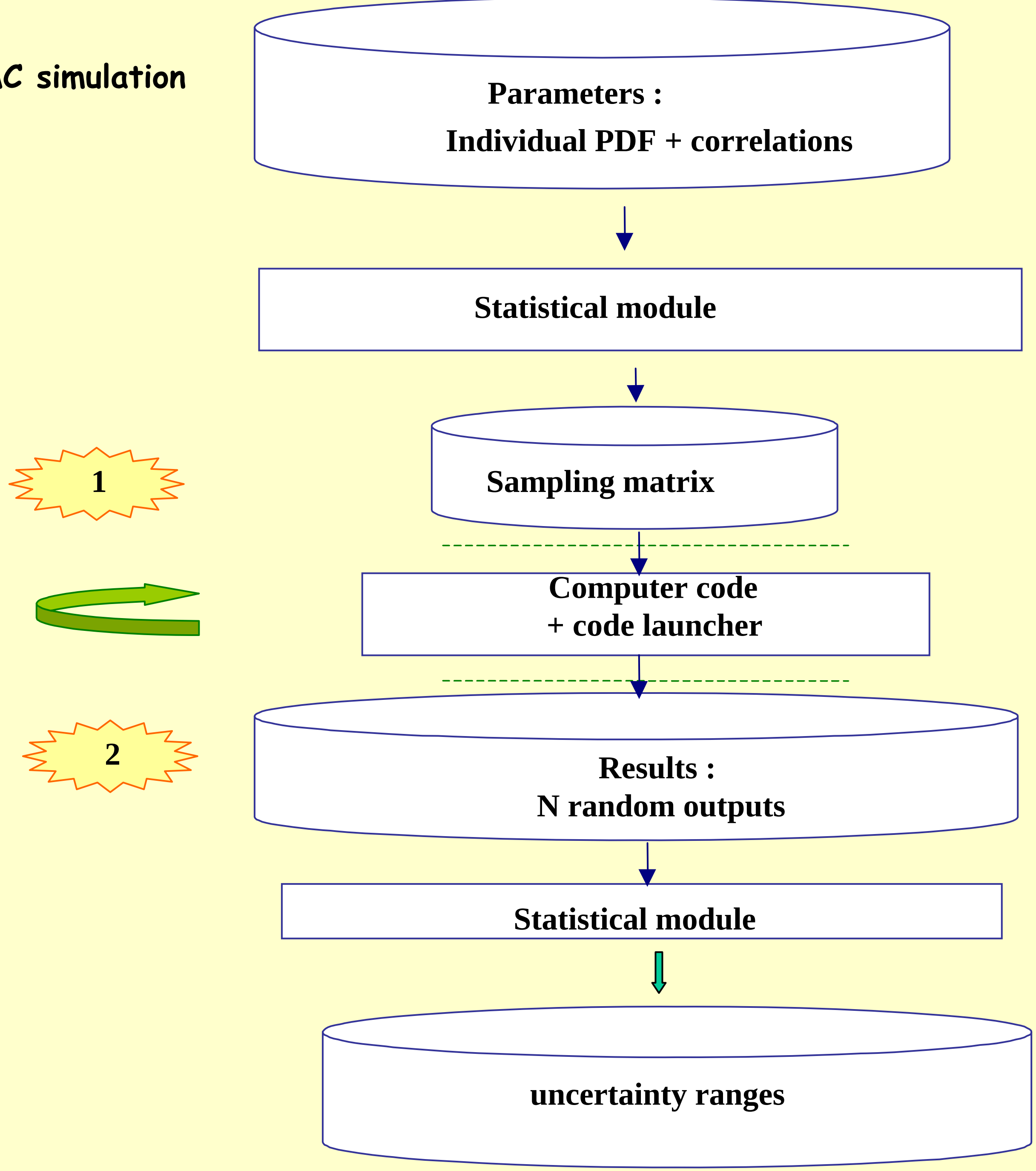
✓no fit tests (not very reliable, specially for the tails of distribution)

Drawbacks :

✓need to know PDFs parameters and their dependencies (inherent in probabilistic methods)

✓may require a large number of code calculations (inherent in SRS methods)

Flowchart of MC simulation



Description of LOFT L2-5 :

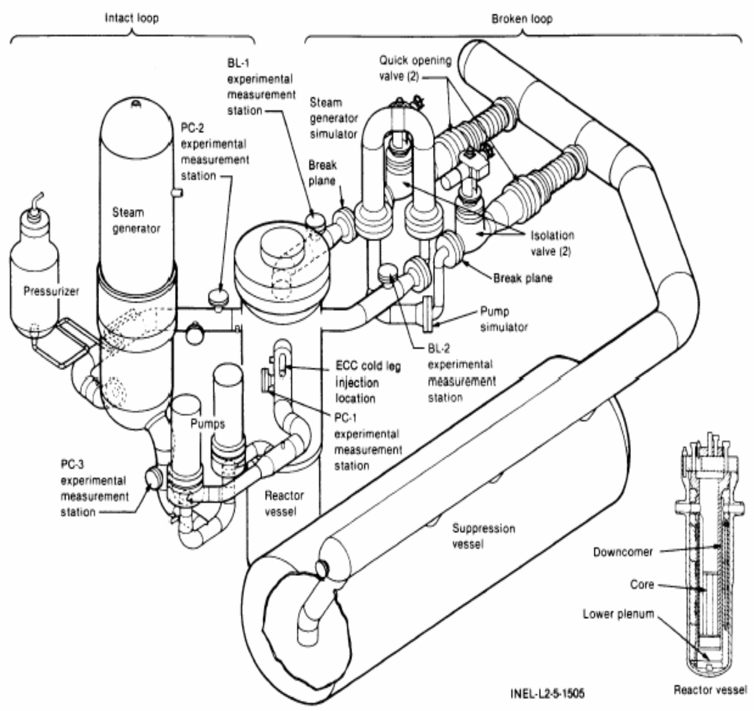


Figure A-1. Axonometric projection of LOFT system.

The BEMUSE program is divided in two steps. The first step consists to perform an uncertainty analysis on an experimental test and the second step on a NPP. Each of these two steps is made up of three phases :

- **First step (Phases 1, 2 and 3): an uncertainty analysis of LOFT L2-5**

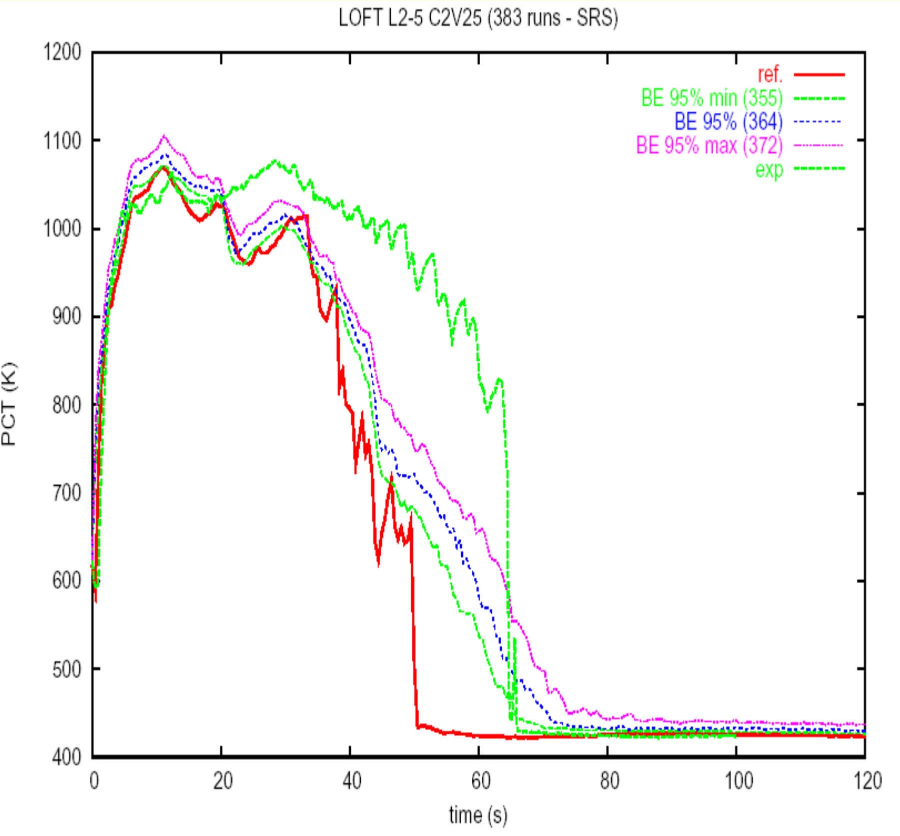
- Phase 1 : a priori presentation of the uncertainty evaluation methodology to be used by the participants ,
- Phase 2 : re-analysis of the ISP-13 exercise, post-test analysis of the LOFT L2-5 test calculation,
- Phase 3 : uncertainty evaluation of the L2-5 test calculations, first conclusions on the methods and suggestions for improvement.

- **Second step (Phases 4, 5 and 6): performing this analysis for a NPP-LB.**

- Phase 4 : best-estimate analysis of a NPP-LBLOCA,
- Phase 5 : sensitivity studies and uncertainty evaluation for the NPP-LB (with and without methodology improvements resulting from phase 3),
- Phase 6 : status report on the area, classification of the methods, conclusions and recommendations.

BEMUSE program: IRSN results

Preliminary results :



First assumption: Uncertain parameters modelled by uniform laws:

-experimental value out of the uncertainty band after 20s

-Reference value ~ percentile 95%

⬆

Non-symmetrical uncertainty ranges: for example [0.15 ; 6.5] (Film boiling transfer coefficient)

Second assumption: Uncertain parameters modelled by piecewise uniform laws:

For example: { 50% in [0.15 ; 1]
50% in [1 ; 6.5]

BEMUSE program: IRSN results

BE value, [BE 5%min,BE 5%max] and [BE 95%min,BE 95%max] (confidence level: 95%)

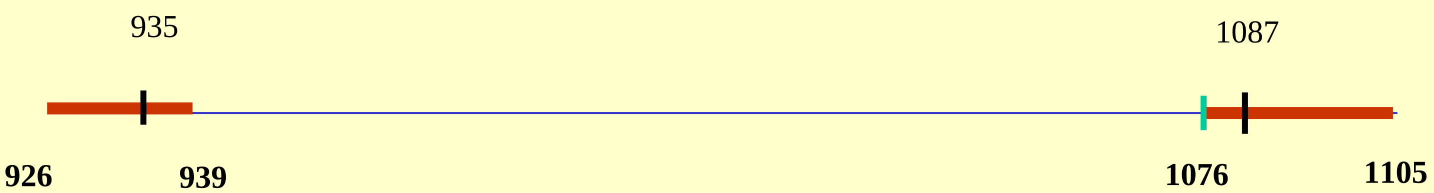


Figure 1: 383 runs with 27 uncertain parameters (uniform law)

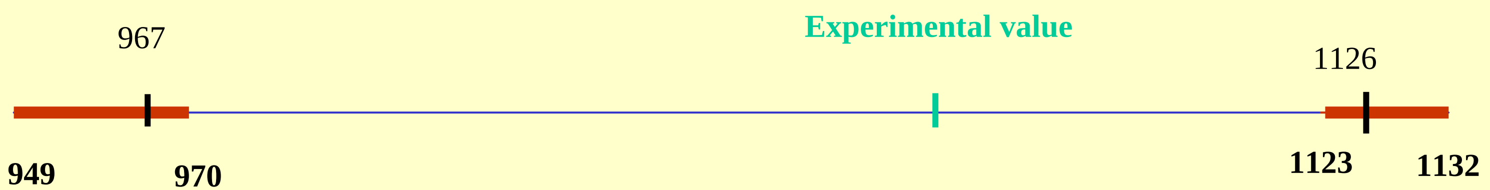


Figure 2: 490 runs with 27 uncertain parameters (piecewise unifom law)

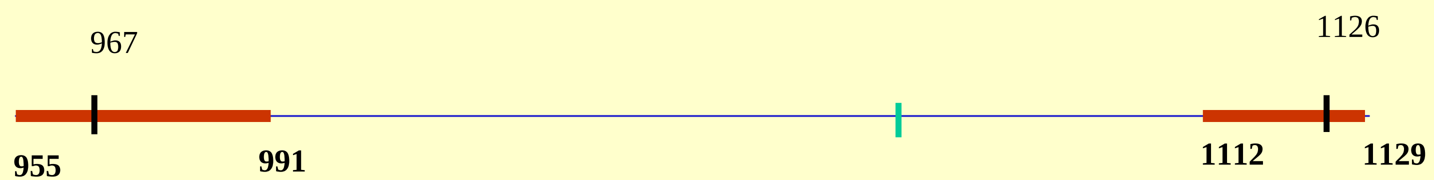


Figure 3: 200 runs with the 3 most influencial uncertain parameters
(piecewise uniform law)

Conclusions :

The use of order statistics in Monte-Carlo simulations provides :

- extremely simple and powerful way to evaluate uncertainty margins
  - *uncertainty range of the peak clad temperature ~150K,*

- confidence intervals around these estimations can also be derived. That allows to quantify the sample size effect :
  - *accuracy of percentile 95%~±10K*

- robust results (in comparison with other statistical techniques) :
  - no assumptions on the number of uncertain parameters and on their PDFs, due to an approximation by a response surface.

However, the quality of results depends obviously on the ability of the computer code to model the physics, on the exhaustiveness of uncertain parameters and on the choice of PDFs to model the input uncertainties