

**Formal Handling of the Level 2 Uncertainty Sources and Their Combination with the Level 1 PSA Uncertainties**

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**Abstract**

As an essential part of the Level 2 PSA, a probabilistic treatment of complex phenomenological accident pathways inevitably leads to two sources of uncertainty: (a) an incomplete modeling of these accident pathways and their subsequent impacts on the Level 2 risk, and (b) an expert-to-expert variation in their probabilistic estimates. While the former type of uncertainty is epistemic in nature from the viewpoint that we deal with an uncertainty addressed in the deterministic events, the latter type is a random/aleatory uncertainty. The impacts of the preceding sources of uncertainty on the Level 2 risk measures are different for each other, thus leading to a different conclusion in the decision-making process. An important aspect of the foregoing distinction of uncertainty is that it plays an essential role in identifying what sources of uncertainty impact more on the Level 2 risk and what sources among them should be focused on for a greater reduction of the overall Level 2 uncertainty. Another aspect of the importance is closely related to its role in combining the Level 1 and Level 2 uncertainties. A primary objective of this paper is to explore the aforementioned sources of the Level 2 uncertainty and provide the corresponding approaches for handling them formally. An additional purpose is to provide an approach for combining consistently aleatory uncertainties addressed in the Level 1 PSA with the Level 2 epistemic uncertainties, so that the Level 1 and 2 uncertainties are finally represented as an integrated measure.

**Key Words:** Level 2 PSA, Uncertainty Sources, Aleatory and Epistemic Uncertainties, Combination of Level 1 and 2 Uncertainties, Formal Approaches

**1. Introduction**

As a logical model for the Level 2 probabilistic accident progression analysis, the containment event tree (CET) (or accident progression event tree (APET) in some applications) looks at the severe accident as a series of snapshots in time from the initial conditions of phenomenological accident progressions to a potential containment failure. For this, the CET/APET embraces a definition of initial conditions, criteria for selecting top events, and a probabilistic quantification of the top event branch probabilities. Then, the CET/APET top event asks the basic questions that arise in the course of a severe accident progression, many of which are closely related to the occurrence or nonoccurrence

of potential severe accident pathways within the containment and the induced containment failure. Lots of phenomenological subevents with a different possibility for a given top event are modeled as the corresponding branch points, which are uniquely determined by the prior conditions of the event tree. If the same prior conditions are given exactly for a given event, the following accident pathway is always fixed to a specified one. This means that there is no uncertainty in the choice of a CET branch event if and only if the prior physical conditions involved in a severe accident are completely known at the time of an estimation of the given top event. The problem is that a limited knowledge about these prior conditions gives rise to a different possibility of the subsequent accident pathway and it is not easy to clearly specify which condition for a given accident pathway is the correct one. This is the main reason why we need a probabilistic analysis for the Level 2 deterministic accident pathways.

While the above concept of the Level 2 PSA appears to be currently standardized in a formalized fashion [1-5], the way the sources of its uncertainty are treated has not been so clear as yet [2, 5-6]. This is mainly because the Level 2 PSA deals with probabilistically phenomenological accident pathways which are deterministic in nature; there are no probabilistic models characterizing them. The foregoing situation inevitably leads to two sources of uncertainty: (a) an incomplete modeling of these accident pathways (each incomplete with respect to various aspects of the problem) and their subsequent impacts on the Level 2 risk (i.e., phenomenological and modeling uncertainty), and (b) an expert-to-expert variation in their probabilistic estimates (so called ‘judgmental uncertainty’). While the former type of uncertainty is epistemic in nature from the viewpoint that we deal with an uncertainty addressed in the deterministic events, the latter type is a random/aleatory uncertainty. Consequently, the impacts of the preceding sources of uncertainty on the Level 2 risk measures are different for each other, thus leading to a different conclusion in the decision-making process. This means that some phenomena could have very different models for their estimation and magnitudes applied by different experts by which accordingly the subsequent accident pathways could be modeled differently. An important aspect of the foregoing distinction of uncertainty is that it plays an essential role in identifying why there exist uncertainties in the Level 2 PSA, what kinds of uncertainty is involved there, what sources of uncertainty impact more on the Level 2 risk, and what sources among them should be focused on for a greater reduction of an overall uncertainty. If one knows the reasons, one has a better chance of finding the right methods for reducing them. Additional aspect of the importance is closely related to its role in combining both the Level 1 and Level 2 uncertainties.

A primary objective of this paper is to explore the aforementioned sources of the Level 2 uncertainty and provide the corresponding approaches for handling them formally. An additional purpose of this paper is to provide an approach for combining consistently aleatory uncertainties addressed in the Level 1 PSA with the Level 2 epistemic uncertainties, so that the Level 1 and 2 uncertainties are finally represented as an integrated measure. For this, the underlying portions and sources of the Level 2 uncertainty that would often be encountered in its implementation process are explored in the former part of this paper. The latter part provides the respective approaches for handling them formally from various points of view. Many portions of this paper are based on the previous works of authors [6-12].

## **2. Potential Sources of Uncertainties in the Level 2 PSA**

Uncertainties may be addressed in every step and element of the Level 2 PSA, which make the Level 2 analysis results greater or less unsubstantiated. Before tackling a formal quantification of the uncertainties involved in the Level 2 PSA, it is a natural step to understand why uncertainties arise in the Level 2 PSA, what the underlying source of uncertainty is, and which uncertainties are explicitly accounted for and which ones are not in the analysis.

### **2.1 Two Types of Level 2 Events**

In the Level 2 PSA, there exist two types of events: one is the status of the containment systems whose modeling is made through a plant damage state (PDS) and the other is phenomenological events whose modeling is made through the CET/APET top events. As manipulated in the Level 1 system event trees, an availability of a containment system is associated with a random/aleatory event whose success or failure probability is regarded as a property of the event and whose uncertainty is expressed as a distribution on the probability. Whereas, the Level 2 phenomenological events employed in the CET/APET model are mainly related to the occurrence of specific physical accident pathways which are deterministic in nature, many of which are estimated under the lack of direct data. The underlying probability is fundamentally an expression of an analyst's subjective confidence about the occurrence possibility of the phenomenological events, whose true value is subjected to either an occurrence or nonoccurrence, but not both. Then, the CET/APET branch event probability is more suitably regarded as a measure of uncertainty about the occurrence of the various potential pathways denoted by the corresponding top events. This is the main reason why the CET/APET events are quantified separately with each PDS, for a consistent and practical treatment of the system and phenomenological event probabilities that are different in nature in the implementation process of the Level 2 PSA.

The foregoing separation is very helpful in understanding the nature of the uncertainties and selecting an appropriate measure of uncertainty in practical situations. In turn, it leads to a clearer insight into 'what might actually happen and with what probability' (for PDS system-related events)' or 'how well we know a given problem and how much our knowledge about it might change with additional information' (for CET/APET phenomenological- and modeling-related events). An additionally important aspect is a proper propagation of different uncertainties addressed in the CET/APET evaluation process so that a consistent decision-making is made for the Level 2 risk [13-14]. If both uncertainties are already mixed up in the course of the analysis, a separation for the effect of either type in the resulting overall uncertainty is very difficult. For a practical convenience in estimating both aleatory and epistemic portions of uncertainty, Table 1 gives their summarized feature.

Table 1 Characterization of Random/Aleatory and Epistemic Portions of Uncertainty

Items	Aleatory Portion	Epistemic Portion
Classification criteria	Level of modeling details Knowledge about the laws governing the occurrence of an event Degree of the sensitivity to the initial conditions or environment	
Terminologies	Random/Stochastic, Irreducible, Observable, Inherent Uncertainty	State-of-knowledge, Reducible, Unobservable, Cognitive Uncertainty
Uncertainty sources	Randomness/variability of an event, Circumstance variability	Inaccurate knowledge of a fixed quantity Alternative representations of a true but unknown value
Measure	Probability model of random variability or circumstance variability among unspecified values in the population	Subjective probability model of a fixed but unknown quantity or different distribution or model assumptions
Uncertainty management	We don't know and understand the underlying reasons and behaviors governing randomness, so it is not practically reducible	As we know more about the underlying problem, it can be effectively reduced
Analysis purpose	Answer the question on what might actually happen and with what probability	Answer the question on how well we know and how much our knowledge about it might change with additional information

## 2.2 Epistemic Sources of the Level 2 Uncertainty

As mentioned before, a logical structure of the CET/APET is a means for expressing a type of modeling uncertainty (or experts' subjective opinions) about a PDS-specific accident pathway. This is basically due to a previous assumption by which the accident progression addressed in a Level 2 PSA is uniquely determined by the prior conditions and thus if the same conditions are given, the resultant accident progression is always fixed at one. Problem is that a limited knowledge about the prior conditions gives rise to different accident progression possibilities that are characterized with subjective probabilities for a given CET/APET top event branch point. Whenever possible, the branch probability can be obtained by the overlap of a probability distribution for the occurrence criteria of the branch point and probability distributions about the physical parameters that would be used to determine the relative magnitude of the branch event.

Uncertainties addressed in the foregoing two parameters (i.e., one for the occurrence criteria of event and the other for a parameter controlling the relative magnitude of the event) are characterized as a type of phenomenological uncertainty in the Level 2 PSA, which are estimated by a type of severe accident analysis. Also, such a kind of phenomenological uncertainty is considered as a special case of a modeling uncertainty. Although the both uncertainties are closely related in the CET/APET analysis, it may be more instructive to manipulate distinctively an uncertainty over the phenomena characterizing a given accident pathway (i.e., phenomenological uncertainty or uncertainty in physical parameters) and an uncertainty over a logical structure of the accident pathway models (i.e., modeling uncertainty or uncertainty in the CET/APET structure). By considering "phenomenology" and "modeling" separately, one can separate the question of how well our CET/APET model represents an accident process from the question as to how well we understand the underlying phenomena for the accident process [7,9,13].

### 2.3 Random/Aleatory Sources of the Level 2 Uncertainty

As mentioned above, a formal separation between the random/aleatory and epistemic portions of an uncertainty greatly depends on the level of decomposition and qualification for the events in question. If an event is not clearly defined at the fundamental levels, the potential for a stochastic portion of the probability is inevitable even for phenomenological events. This is the same even for the Level 2 PSA, most of whose events have been characterized as subjective/epistemic uncertainties, and there are three representative cases where the random/aleatory uncertainty might be addressed in the Level 2 PSA.

#### *Case 1: Sequence-to-sequence variability of phenomenological factors*

If any accident pathway in the CET/APET were executed many times for the same PDS and if the outcome of a top event varies in such an execution, there would be a sequence-to-sequence variability [15]. Whereas, as long as the PDSs are properly defined and all the CET/APET top events represent physical processes governed by the physical laws rather than a random feature, there is no variability in a CET/APET sequence for a given PDS. If such a type of variability were observed in a practical experiment, it would be due to a subtle detail in a physical process that is not adequately understood. The corresponding redefinition of either the PDS or the CET/APET model should be followed in that case. Not knowing what the subtle details could be and lacking any evidence for their existence, they are properly treated as an element of uncertainty in the outcome of each CET/APET top event.

#### *Case 2: Variability of phenomenological factors due to a limited resolution of initiating events*

A basic assumption for this case is that if a phenomenon might occur in the core damage accidents leading to a containment failure, its occurrence probability becomes a fraction of the event among all

the Level 1 core damage sequences that result in the phenomenon [16]. This probability describes a random/stochastic process of a given phenomenon, therefore a measure of a physical property of the containment system being studied. When the concept of a PDS is utilized as an initial condition of the Level 2 PSA, however, the details of different accident sequences ending in the same PDS do not have to be retained for the containment model; i.e., a specific sequence often loses its information once it is assigned to its PDS. Probabilistically, this means that there is no variability in the containment response for different plant failure sequences within the same PDS.

### *Case 3: Variability of phenomenological factors due to a limited resolution of prior conditions*

In most cases, the description of the factors leading to a containment phenomenon will be of a limited resolution and the accident sequences with it will differ in many details. Even when an initial accident condition is specified in a PDS, a question for the containment peak pressure resulting from the phenomenon often neglects the existence of subsequences or phenomena that are unspecified in various ways in the definition of PDS. This is mainly due that the aforementioned question says nothing about some factors making them a stochastic process like the initial melt temperature at the time of core melt, and/or the question refers to sequences with the melt exhibiting any technically feasible values [13]. Consequently, a population of values for the initial melt temperature indicate that there is an uncertainty as to which value to use in the estimation of the peak pressure to the given PDS condition, i.e., a random/stochastic variability of the peak pressure within the population of the initial melt temperatures.

There are two reasons why the aforementioned random/stochastic portions of uncertainty are no longer explicitly taken into account in the Level 2 analysis. The first is that in many cases the contribution of a random/aleatory portion of an uncertainty is not so much in the Level 2 PSA, when compared to the corresponding epistemic portion of an uncertainty. The second reason is that a clear separation of the random/aleatory and epistemic portions of an uncertainty is not easy due to the complexity of the severe accident phenomenologies. In fact, the ability to estimate uncertainties of physical phenomena at the level of detail at which they enter the total analysis requires a considerable knowledge of the accident sequences and the underlying phenomena. Finally, their incorporation into the Level 2 CET/APET analysis makes it difficult to quantify explicitly the impact of each uncertainty on the Level 2 risk measure.

## **3. Formal Treatment of the Level 2 Uncertainty Sources**

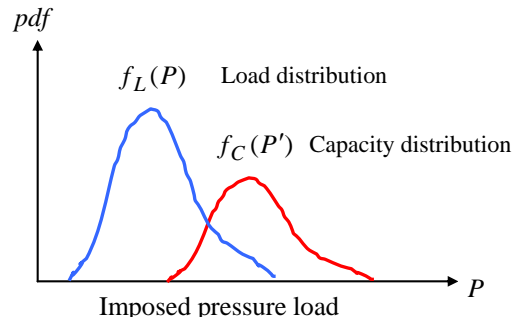
Once we find out what kinds of uncertainty are addressed in the Level 2 PSA, the next step is to identify their impact on the Level 2 risk. Because it is not easy to handle all the sources of uncertainty explored in the previous sections, we have to determine which sources of uncertainty must be explicitly handled in the quantification process and which sources we do not need to do so. From the viewpoint that the epistemic portions of an uncertainty are much more dominant in the Level 2 PSA, this section is focused on how to propagate them statistically to obtain the corresponding uncertainty in the Level 2 risk measure, and how to handle the expert-to-expert variations in the estimates of an uncertainty.

### 3.1 Characterization of the Level 2 Uncertainty Sources

As mentioned before, the Level 2 CET/APET branch probability is interpreted as a measure of an uncertainty about the occurrence or nonoccurrence of the branch event, whose estimation could be made from a transition of the Level 2 accident analysis results into the corresponding branch event probability or an inductive process of the analyst's confidence on the occurrence of the event

### 3.1.1 Modeling and Phenomenological Uncertainty Sources

If possible, the occurrence probabilities of the Level 2 CET/APET branch events and the related phenomena must be determined with the probability distributions for two decision parameters (one for physical impacts imposed on the specified branch event and another for the physical criteria on the occurrence of the event). However, the main difficulty in estimating them is that there is no directly applicable database or statistical model with which to estimate these quantities. These deficiencies allow different experts to arrive at different conclusions about the branch probability, and therefore large uncertainties may exist in the realistic prediction of the Level 2 risk results. To minimize potential uncertainties resulting from the lack of phenomenological data and subjectivism in making quantitative estimates of the branch probabilities, experiments and code analyses are primarily utilized to simulate the expected behavior of an accident phenomenon to be considered in the CET/APET.



$f_L(P)$  = probability distribution (*pdf*) for the imposed load  $P$

$f_C(m_i | P)$  = capacity distribution (*pdf*) of a given failure mode  $m_i$

$m_i$  = failure mode (e.g., leak, rupture, or catastrophic rupture)

$$\text{Mean failure probability, } p_{fail}(m_i) = \int_0^{\infty} f_L(\tau) \int_0^{\tau} f_C(m_i | \tau') d\tau' d\tau$$

$$\text{Failure probability for a given } P, p_{fail}(m_i | P) = \int_0^P f_L(\tau) \int_0^{\tau} f_C(m_i | \tau') d\tau' d\tau$$

Fig.1 Determination of the Level 2 CET branch probability (split fraction)

As a typical example utilizing the phenomenological information, the failure probability of the containment due to a high-pressure melt ejection is estimated by two probability distributions characterizing it: the first is what pressure is generated from the high-pressure melt ejection and the second is how is the pressure strong enough to fail the containment in a specific mode. As shown in Fig.1, the two probability distributions (one for containment peak pressure and another for containment failure pressure) are regarded as a representation of the analysts' uncertainty for the containment pressure resulting from a high-pressure melt ejection for a specific core melt accident sequence and for the containment failure for a given pressure, respectively. Then their combination with a stress strength interference method [3,5,11] gives a single measure (or mean probability) of the analyst's belief that the branch event will occur in a core melt accident as a result of a physical challenge in it. Table 2 summarizes four approaches for determining the containment failure probabilities when the containment peak and failure pressures are subjected to either point values or probability distributions.

For less well-understood accident sequences, on the other hand, the branch probabilities obtained in such a way cannot be rigorously substantiated because the judgment process is not a clearly defined process. Whereas, it may well be the right means for many practical applications when no other alternative means exist [3,10,17]. If information is very limited in estimating the CET/APET branch

probability, however, an expert might rarely ascertain a unique belief to his estimate for the branch probability, thus leading to an interval probability with a different degree of a possibility in his mind. This does not mean the expert is uncertain of the belief in the sense of a probability; rather it means the value of the belief is uncertain. In other words, the interval in a probability is used to give a single representative estimate of the qualitative probability (e.g., a nominal value or mid range) that will be used as the corresponding branch probability, but does not have any statistical meaning in a strict sense because there is no reason for assigning an additional degree-of-belief about the degree-of-belief which is already an expression of an uncertainty. Of course, an assignment of the interval in the belief of a probability depends on the degree of the confidence of the different probability values and the importance of the decision to be made. Distinctive qualitative terms have been proposed for the transition of the analyst's confidence into the subjective probabilities [17].

Table 2 Different Combinations of the Containment Peak and Failure Pressures

Cases	Peak pressure ( $P_{peak}$ )	Failure pressure ( $P_{fail}$ )	Containment failure probability ( $p_{cf}$ )
Case 1	Point estimate	Point estimate	If $P_{peak} > P_{fail}$ , $p_{cf} = 1.0$ If $P_{peak} < P_{fail}$ , $p_{cf} = 0.0$
Case 2	Uncertainty distribution	Uncertainty distribution	The convolution of the two uncertainty distributions results in $0.0 < p_{cf} < 1.0$ .
Case 3	Point estimate	Uncertainty distribution	The cumulative failure probability for a given pressure, results in $0.0 < p_{cf} < 1.0$ .
Case 4	Uncertainty distribution	Point estimate	(a) Obtain point values ( $P_{peak,i}$ , $i = 1$ to $n$ ) from a given peak pressure distribution; (b) The application of Case 1 to each pressure value results in $p_{cf} = 1.0$ or $0.0$ ; (c) The arithmetic average of all $p_{cf}$ gives $0.0 < p_{cf} < 1.0$ .

### 3.1.2 Expert-to-Expert Variation in the Uncertainty Estimates

When a judgmental process is concerned with the estimation of the CET branch probabilities, on the other hand, there are two distinct sample spaces of probability judgments. The first is the usual sample space over which probabilities are estimated as the space of event conditions, and the second is a new sample space over which the dispersion of opinions are measured as the space of the experts' opinions. While the former asserts the probability of an event by an individual (i.e., personal probability), the latter suggests the different opinions of experts on a chance of the individual probability. Moreover, the latter case can be statistically treated, based on the sample space of heterogeneous opinions among the experts [18-19]. These additional uncertainties addressed in the judgmental process of an individual expert are characterized as expert-to-variability in the subjective estimate, which is different from the uncertainty of the occurrence of the CET branch point as a physical event. The latter type of uncertainty involved inherently in the judgmental process gives additional information on the uncertainty to the Level 2 PSA decision-making process. Because these two types of uncertainty are considered equally important, it is necessary to manipulate them explicitly in the Level 2 PSA.

## 3.2 Propagation of Characterized Uncertainties

### 3.2.1 Modeling and Phenomenological Uncertainties

The CET/APET branches can be modeled with either a direct assignment of a branch probability or the use of physical parameters by which the probabilities of the subsequent branch events can be evaluated. A typical example of the latter case is a determination of probabilities for each containment failure mode from various physical parameters (e.g., pressure, temperature, and oxidation) obtained for the prior conditions. In the latter case, a propagation of uncertainties is a greater or lesser difference from the former case.

#### (1) Statistical Propagation of Modeling Uncertainties

In the Level 2 CET/APET analysis, only one accident pathway is physically possible for a specific PDS, but we do not know which one is correct. This feature of CET/APET events has an important implication when their probabilities are propagated in the form of an uncertainty through the model. For this, let's reset the branch probability in the frequency format of a probability, e.g., a deterministic value of one correct accident pathway equal to one and the other pathways equal to zero. This is possible only when we can eliminate all the subjective uncertainties surrounding the occurrence of the branch events. Then, the branch events in question would be either 0 or 1 in the sense of an occurrence frequency. Accordingly, the frequency format of a CET branch probability is a kind of double-delta function that expresses the probability that the frequency is either one or zero, and the frequency between zero and one has a zero probability. Consequently, the branch probability itself is interpreted as a mean value of the corresponding branch frequencies (or a weighted average of uncertainty for the frequency of one and uncertainty for the frequency of zero).

All the CET branch probabilities are similar in that they originate from the uncertainties in the branch events. The above approach makes it possible to statistically propagate uncertainties of the branch events given in the form of subjective probabilities to determine the uncertainty distributions for the frequency of the Level 2 end states. In summary, the uncertainty propagation of the CET branch probabilities is based on the two aforementioned propositions: (a) The CET branch probability is a complete statement of knowledge as the branch probability itself expresses the uncertainty that the event frequency is either one or zero, and (b) This state of knowledge can be interpreted as a double-delta distribution for the occurrence frequency of a specified branch event. An example event tree [7], Fig.2 is considered to assess these circumstances, which is described by binary-type branch models.

In the event tree, most of the branch points relate to the physical condition of the containment during severe accidents. The branch points here relate to questions phrased in terms of exceeding thresholds that mark a distinct change in the physical progression of the accident. There is, however, in this case no probabilistic model by which the relative frequencies of the possible outcomes can be predicted; instead a set of deterministic models with a given set of input conditions, predict the accident progression. In this case, only one outcome is physically possible but one does not know which one is the correct model. Thus, we set a value of one event branch equal to one (and the other branches equal to zero) and an expression of an uncertainty for the subjective branch probabilities is assigned to each branch model. The probabilities may be quantified using the aforementioned stress strength interference method whenever possible. When no other alternative exists, however, engineering judgments are only used to quantify the branch model uncertainties. All the event tree branch probabilities are similar in that they originate from uncertainties in branch models.

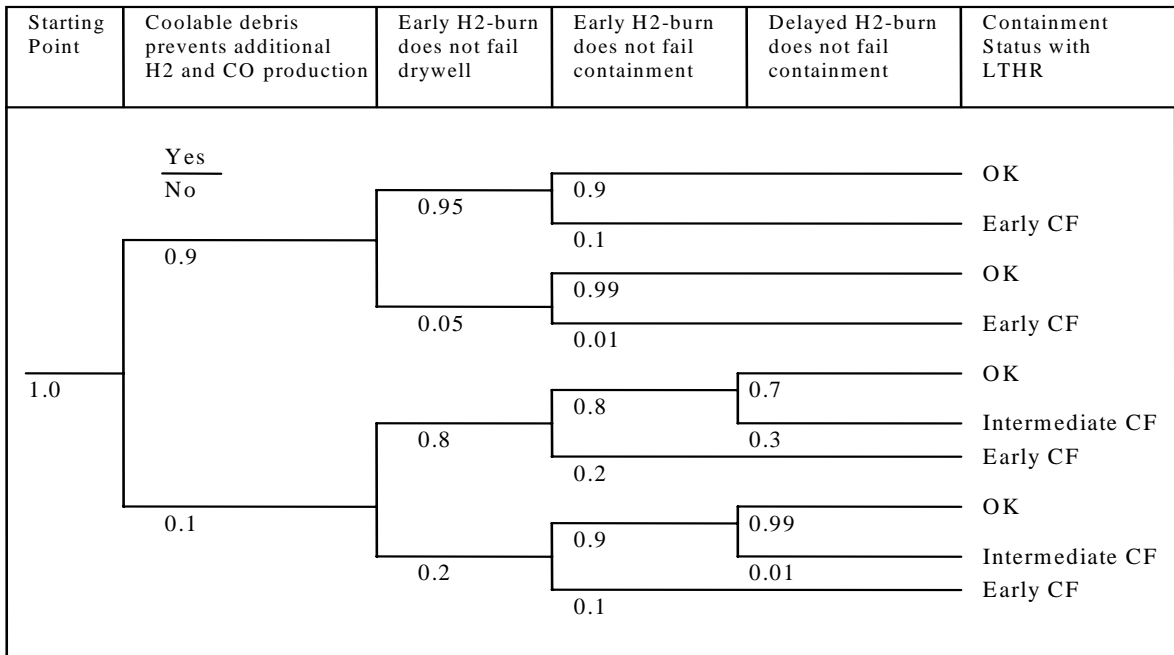


Fig.2 An Example CET for a Sample Calculation

Table 3 Model Uncertainty Expression of the Branch Probability

Input model 1	Top Event 1	Uncertainty
Model element 1	Branch 1 (1,0)	0.9
Model element 2	Branch 2 (0,1)	0.1
Composite probability distribution (0.9,01)		

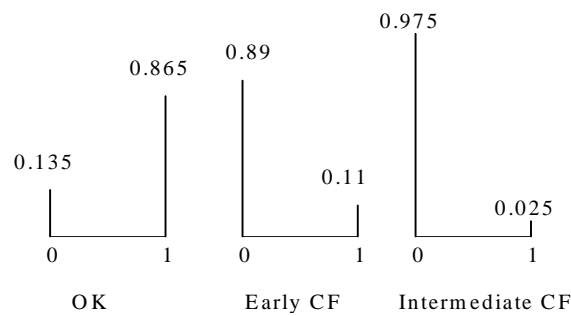


Fig.3 Uncertainty Distributions for the CET End Points

In order to obtain a measure of an uncertainty about the CET end points from Fig.2, first, different branch models or accident pathways have to be sampled with an appropriately chosen statistical procedure (such as Monte Carlo approach). In that case, the relative weights of each branch point sample (i.e., one-zero sample in the frequency format of probability) are sampled, based on the formulated branch probabilities. Table 3 represents the probabilities for each branch model, given in the form of modeling uncertainties. A statistical propagation of the different branch models through the event tree results in a type of probability distribution in the discrete form of a frequency, and their mean frequencies can be then obtained by aggregating the CET end state frequencies. Fig.3 shows the probability distributions for the CET end state frequencies which were obtained through a Latin

Hypercube Sampling (LHS) (i.e., a Stratified Monte Carlo Sampling) approach with 200 simulations, with the corresponding mean frequencies of OK (0.865), Early CF (0.11), and Intermediate CF (0.025).

### (1) Statistical Propagation of Phenomenological Uncertainties

When the branch probabilities are explicitly assigned to the CET/APET model, the CET/APET uncertainty analysis can be made through the aforementioned uncertainty propagation of the branch probabilities. Whereas, when a pressure load itself imposed to the containment is explicitly used for some intended purposes or a convenience of the CET/APET evaluation, an additional step is required to determine statistically the subsequent containment failure modes. As shown in Fig.1, the currently available approach for the uncertainty analysis is to utilize “containment failure probability” curves and conditional probability curves for the underlying containment failure modes. These probability curves can be obtained from the use of individual probability distributions for each failure mode which can be obtained from the structural analysis [11] or the expert elicitation process as implemented in the NUREG-1150 Surry APET analysis [17]. Once the probability distributions for the containment failure and conditional probability distributions for each failure mode are obtained, whether the containment fails or not at a given pressure could be determined in the following distinct ways:

- The question of the failure mode is dealt with entirely on the basis of a conditional probability. The conditional probabilities for each failure mode are a breakdown of the probability between the containment failure modes (such as leak, rupture, and catastrophic rupture), given that a failure occurs [11,17]. Depending on the severe accident progressions, the containment failure may be subjected to a leak mode only or both leak and rupture modes.
- The load and failure pressures are statistically sampled from their own probability distributions through a sampling procedure. If the load pressure is less than the containment failure pressure, the containment does not fail for a given pair of pressure samples, the definition is binary (i.e., deterministic) as given in case 1 of Table 2.
- If the load pressure is greater than or equal to the containment failure pressure, the containment fails. If the containment fails, then containment failure for a given pressure is treated as if the load is applied statistically. For this, the random number between zero and one is used to determine the resultant failure mode. If the random number is less than the leak conditional probability, the failure mode is a leak. If the random number is greater than the leak conditional probability but less than the sum of the leak and rupture conditional probabilities, the failure mode is a rupture. If the random number is greater than the sum of the leak and rupture conditional probabilities, the failure mode is a catastrophic rupture. The foregoing approach is applicable even when the containment failure is subjected to a leak mode only or both leak and rupture modes.

The foregoing approach makes it possible to consider the accident sequence-to-sequence dependency of the phenomenological inputs employed in the CET/APET. For instance, an accident sequence of the CET/APET may be subjected to some degrees of dependency with other sequences, resulting in a similar trend of the pressure loads. In that case, the dependency among those sequences can be quantitatively treated by the use of correlation coefficients in the stage of a statistical sampling for an uncertainty analysis [6,11,17].

### 3.2.2 Statistical Propagation of Expert-to-Expert Variations

When experts are involved in the estimation process of a specific branch probability, the branch probabilities are allowed to vary as each expert may have his own opinion concerning the branch model assumptions with some degree of validity. Table 4 shows a mathematical expression of an

expert-to-expert variation of a branch probability when experts provide their own probability estimates. The foregoing situation leads to an assessment for the impact of a model-to-model variation of each branch event or expert-to-expert variation of each branch probability on the Level 2 risk results. For the PSA application, the probabilistic formulation of a model uncertainty has been made with the probability (or relative weight) of each model over alternate models, whose values may be equally weighted by the experts. In that case, the probability is regarded as an expression of each analyst's degree of belief in that model as being the most appropriate [7, 18-21]. This formulation of a judgmental uncertainty about the branch probability is fundamentally based on an assumption that each of the underlying branch models can be treated as a probability variable in the framework of an uncertainty analysis, with varying degrees of probability estimates or probability distributions.

Table 4 Expert-to-Expert Variability to the Specified Set of Branch Probabilities

Source of Variability	Form of Variability	Mathematical Expression
Experts' Different Weights to Estimated Branch Probability Set	Discrete Weights	<p>Discrete Sets of Branch Probabilities</p> $\sum_i p_{ij} = \sum_j w_j = 1$ <p><math>p_{ij}</math>: Probabilities of <math>i</math>-th branch in <math>j</math>-th branch set,  <math>w_j</math>: experts' weight assigned to <math>j</math>-th branch set</p>

Table 5 Expert-to-Expert Variations in the Branch Probability Distributions

Branch Model	Top Event 1	Top Event 2	Top Event 3	Top Event 4
1 <sup>st</sup> branch	(0.90, 0.10) <sup>a</sup> 0.8 <sup>b</sup> (0.80, 0.20) 0.1 (0.75, 0.25) 0.1	(0.95, 0.05) 0.5 (0.90, 0.10) 0.3 (0.85, 0.15) 0.2	(0.90, 0.10) 0.7 (0.85, 0.15) 0.2 (0.75, 0.25) 0.1	
2 <sup>nd</sup> branch		(0.80, 0.20) (0.80, 0.20) (0.90, 0.10)	(0.99, 0.01) (0.95, 0.05) (0.85, 0.15)	
3 <sup>rd</sup> branch			(0.80, 0.20) (0.80, 0.20) (0.90, 0.10)	(0.70, 0.30) 0.5 (0.80, 0.20) 0.3 (0.75, 0.25) 0.2
4 <sup>th</sup> branch			(0.90, 0.10) (0.80, 0.20) (0.75, 0.25)	(0.99, 0.01) (0.90, 0.10) (0.85, 0.15)

Note: <sup>a</sup>uncertainty distributions given by each expert, <sup>b</sup>degree of belief each expert, given by subjective probabilities (same belief is used for each top event, regardless of dependence cases)

An appropriately chosen statistical procedure for quantifying an overall impact of the experts' different estimates in the branch probabilities produces an envelope of experts' different viewpoints for the Level 2 risk results (e.g., a mean frequency of a specified CET/APET end point). In order to illustrate the foregoing situation [7], three experts are considered to give the uncertainty in each branch probability distribution of the example event tree Fig.2, and the degree of belief on the branch probability distribution asserted by each expert is given by a subjective probability. Table 5 shows the assumed expert-to-expert variation on each branch probability distribution of the event tree. With 200 LHS simulations, the present formal approach for quantifying a modeling uncertainty then produces the envelope of uncertainty distributions on the final results, given in Table 6.

On the other hand, the impact of such an expert-to-expert variation for the uncertainties of CET/APET end points can be assessed through an additional type of sensitivity analysis such as a

‘distributional sensitivity analysis’ [7]. This type of sensitivity analysis is applicable as long as the branch probability is regarded as a type of an uncertainty distribution. According to its definition, the discrete distribution given by the first expert is replaced with its base branch model. Then each base model is propagated through the CET/APET model, one by one, while the remaining branch models have their original uncertainties. Then, the expert-to-expert variation with the greatest impact on the uncertainty of the CET/APET end points can be determined by the relative magnitudes of the dedicated importance measures such as a ‘variation’ or ‘standard deviation (s.d)’.

Table 6 Output Uncertainties due to Expert-to-Expert Variation

End States Output Variable	Uncertainty in mean frequency				
	5 %	50 %	95 %	mean	s.d
OK	0.7515	0.8724	0.8869	0.8524	0.039
Early CF	0.0959	0.1068	0.2234	0.1248	0.037
Intermediate CF	0.013	0.0194	0.046	0.0228	0.01

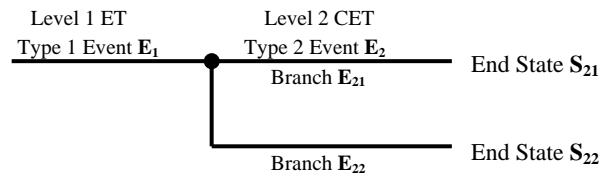
#### 4. Formal Link of the Level 1 and 2 Uncertainties

When a point estimation for the Level 1 accident sequences and the Level 2 accident pathways is made, the distinction between the Level 1 random/stochastic events and the Level 2 deterministic events is only conceptual and does not make the results different for the frequencies of the CET end points. However, when a full propagation of the Level 1 and Level 2 uncertainties is required for an integrated uncertainty distribution about the Level 2 risk results, a strict representation of the uncertainty distributions for the Level 1 events and for the Level 2 events is required.

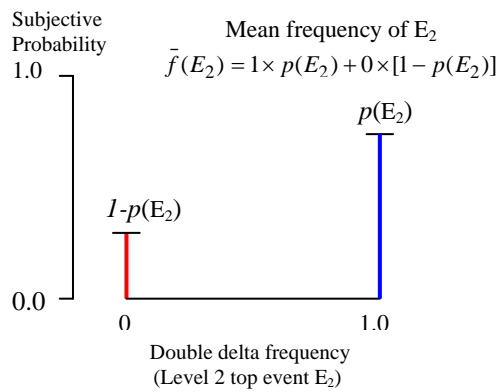
In order to explain explicitly the aforementioned situation of the Level 2 uncertainty analysis, let’s take a typical example that propagates the uncertainties addressed in the Level 1 ET (or Type 1) event probability into the uncertainties addressed in the Level 2 CET/APET branch (or Type 1) event. As mentioned before, the former type of probability is due to a random/stochastic nature of the Level 1 event (i.e., relative probability); the latter type of probability is characterized as a subjective probability. Fig. 4(a) shows an example event tree linking a Level 1 ET end sequence (or PDS) and a Level 2 CET top event. Fig. 4(b) gives an expression of the CET branch probability in the format of a frequency (i.e., an expression of uncertainty distribution for a subjective probability). Then, Fig. 4(c) and Fig. 4(d) explain how an uncertainty distribution of the Level 1 (or Type 1) event ( $E_1$ ) and the uncertainties involved in both the events of the Level 2 CET ( $E_{21}$  and  $E_{22}$ ) (i.e., Type 2 events) are propagated through the CET model to obtain the uncertainties of the two CET end states  $S_{21}$  and  $S_{22}$ , respectively.

Fig. 4(c) and Fig. 4(d) give a conceptual framework for a formal integration of uncertainty distributions for the foregoing types of events. Here, the uncertainty distribution for Type 1 event  $E_1$  is defined as a probability density function (PDF) of the frequency  $f_{E_1}$  (or relative probability  $p_{E_1}$ ), which is a random/stochastic one in nature. Also, the uncertainty distribution for Type 2 event  $E_2$  is defined as a double-delta form of a PDF in the frequency level of the subjective probabilities for the deterministic branch event  $E_{21}$  or  $E_{22}$ . Based on the frequency format of a subjective probability, the frequency of Type 2 event  $E_2$  is characterized as either zero or one. Then, the formal integration of the uncertainties addressed in both the Type 1 and Type 2 events is made in the frequency (or relative probability) levels, with probability densities  $\rho_1$  for Type 1 event  $E_1$  and  $p(E_2)$  for Type 2 event  $E_2$ . In real applications, the impact of the probability densities is reflected in the stage of a statistical sampling for an uncertainty analysis. Table 7 also gives a formal framework for propagating

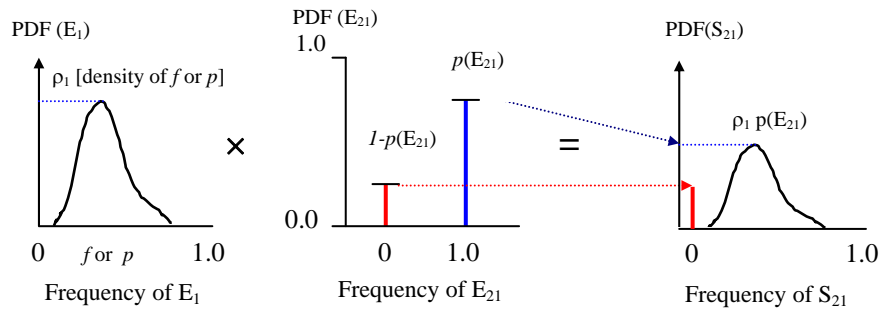
statistically the Level 1 uncertainties to the Level 2 PSA through the present integration model in the frequency level [12].



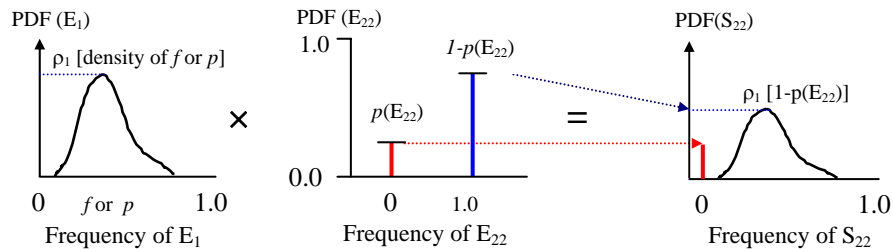
(a) A Combined Model of Level 1 System ET and Level 2 CET



(b) Frequency Format of the Level 2 Top Event Probability (Modeling or Phenomenological Uncertainty)



(c) Propagation of  $E_1$  and  $E_{21}$  Uncertainties into the End State  $S_{21}$



(d) Propagation of  $E_1$  and  $E_{22}$  Uncertainties into the End State  $S_{22}$

Fig.4 Propagation of Type 1 and Type 2 Uncertainties through the Level 2 CET Model

Table 7 Statistical Combinations of the Level 1 Uncertainties with the Level 2 Events

Type of Event and Frequency		Frequencies of the CET/APET end states
$E_1$ : Level 1 Event	$E_2$ : Level 2 Event	
Relative frequency $0 < P_{E_1} < 1$ U-distribution (PDF) $f_1 \equiv f(P_{E_1})$	Relative frequency (2-branches) $E_2 \equiv \begin{cases} E_{21} = 1 \\ E_{22} = 0 \end{cases}$ U-distribution (DUD) $f_2 \equiv \begin{cases} P_{E_2}, & \text{if } E_2 = 1 \\ 1 - P_{E_2}, & \text{if } E_2 = 0 \end{cases}$	
Point estimate $P_1 = E[P_{E_1}]$	Point estimate $P_2 \equiv \begin{cases} 1 \times P_{E_2} + 0 \times (1 - P_{E_2}), & \text{if } E_2 = 1 \\ 0 \times P_{E_2} + 1 \times (1 - P_{E_2}), & \text{if } E_2 = 0 \end{cases}$	Point estimate $P_{stc} = P_1 \times P_2$
Point estimate $P_1 = E[P_{E_1}]$	Uncertainty ( $j$ -th sample) $P_2^j \equiv \begin{cases} 1, & \text{if } E_2 = 1, \text{ weighted by } P_{E_2} \\ 1, & \text{if } E_2 = 0, \text{ weighted by } 1 - P_{E_2} \end{cases}$	Level 2 uncertainty only $P_{stc}^j = P_1 \times P_2^j$
Uncertainty ( $i$ -th sample) $P_1^i = P_{E_1}^i$	Point estimate $P_2 \equiv \begin{cases} 1 \times P_{E_2} + 0 \times (1 - P_{E_2}), & \text{if } E_2 = 1 \\ 0 \times P_{E_2} + 1 \times (1 - P_{E_2}), & \text{if } E_2 = 0 \end{cases}$	Level 1 uncertainty only $P_{stc}^i = P_1^i \times P_2$
Uncertainty ( $i$ -th sample) $P_1^i = P_{E_1}^i$	Uncertainty ( $j$ -th sample) $P_2^j \equiv \begin{cases} 1, & \text{if } E_2 = 1, \text{ weighted by } P_{E_2} \\ 1, & \text{if } E_2 = 0, \text{ weighted by } 1 - P_{E_2} \end{cases}$	Combined uncertainties of Level 1 and 2 Events $P_{stc}^k = P_1^i \times P_2^j$

- (1)  $E_1$  = Level 1 random and stochastic event,  $E_2$  = Level 2 deterministic event,  $P_{E_1}$  = probability of  $E_1$  (relative frequency),  $P_{E_2}$  = probability of  $E_2$  (subjective probability),  $E[P_{E_1}]$  = Expectation of  $P_{E_1}$ ,  $f_1, f_2$  = uncertainty distributions for  $E_1$  and  $E_2$ , respectively,  $P_{stc}$  = probability for source term category (STC), DUD= double-delta uncertainty distribution.
- (2) Type 1 Event ( $E_1$ ): Random and stochastic event, Different outcomes occur at random (e.g., flipping a two-sided coin, always true or occurring event), Probability used to describe relative frequency of outcomes
- (3) Type 2 Event ( $E_2$ ): Deterministic event or parameter value, A single and true, but uncertain outcome (One-sided coin, true or false, occurrence or nonoccurrence), Probability used to describe uncertainty or state-of-knowledge about the outcome.
- (4) Results of the Level 2 PSA accident progression and containment analyses through the CET are containment failure modes characterized as STCs and their frequencies and, in some studies, the conditional probabilities of their occurrence.

## 5. Concluding Remarks

Through the former part of this paper, we have systematically clarified potential sources of a subjective/epistemic and a random/aleatory uncertainty that would often be addressed in the Level 2 PSA and provided some insights into their implications on the Level 2 risk quantification process. As a result of the former part, a clearer answer has been given for the question on why uncertainties arise in the Level 2 PSA, which kind of uncertainty sources exists, and which uncertainties are explicitly accounted for in real applications and which ones are not. Major findings drawn from the former part are summarized as follows,

- If the underlying events are not clearly defined at the fundamental levels, the potentials for the stochastic portion of the probability are inevitable even for phenomenological events although epistemic uncertainty for a modeling or phenomenological event tends to become much more important.

- Even when the stochastic portions of an uncertainty are involved in the phenomenological events, there are two reasons why they are no longer taken into account in the CET/APET analysis. The first reason is that in many cases the contribution of the stochastic portion to the CET branch probability is not so much, when compared to the phenomenological impact assessed for a specific PDS. The second reason is that a clear separation between the stochastic and phenomenological portions is not easy due to the complexity of the severe accident phenomenology. In addition, the possibility of the stochastic variability in the Level 2 events is greatly reduced with a systematic description of the Level 2 initial conditions.

Based on the foregoing clarification of different uncertainty sources, a formal guidance for handling two representative sources of the Level 2 uncertainty (modeling/phenomenological sources of uncertainty and its judgmental source) has been provided through the latter part of this paper, particularly with respect to their characterization, propagation, interpretation, and impact on the PSA Level 2 decision-making process. Finally, an answer has been given for the question on how to propagate consistently two different types of uncertainties characterizing the Level 1 ET and Level 2 CET/APET events into an integrated uncertainty distribution about the Level 2 risk results.

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