ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT

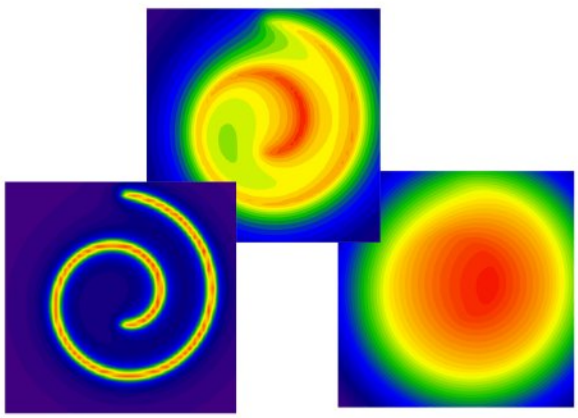
OECD

AEN NEA Agence pour l'énergie nucléaire
Nuclear Energy Agency

TRAINING COURSE

ON

ANALYTICAL BENCHMARKS: CASE STUDIES IN NEUTRON TRANSPORT THEORY



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Course Text: *The Analytical Benchmarks for Nuclear Engineering Applications*, B. Ganapol, OECD © 2008, NEA No. 6292, Organisation for Economic Co-operation and Development

Reference Text: *Nuclear Reactor Theory*, G. Bell and S. Glasstone, Van Norstrand and Reinhold, NY, 1970.

COURSE MOTIVATION

The study of the neutron transport equation is a delicate blend of theoretical mathematics, numerical methods and computational strategies describing the interaction of neutrons and nuclei. Not only do we gain physical insight from the solution to the transport equation, but we also create new mathematics and numerical methods for the solution of transport equations. The solution of the transport equation has been considered by some of the most creative and accomplished scholars of our time, including S. Chandrasekhar, E. Wigner, G. Plazcek, K. Case, N. Weiner and P. Zweifel to name a few. Neutron transport theory, considered an innovative branch of applied mathematics, was an essential element in the development of deterministic numerical methods for the analysis of nuclear systems and should remain so.

However, with the passage of time, ever-increasing computational advances and the success of Monte Carlo computational transport, neutron transport theory is becoming a theory of the past. During the 60's and into the 70's, there was hardly an issue of *Nuclear Science and Engineering* or *Annals of Nuclear Energy* that did not contain at least two articles on the subject. Now, such articles are relatively infrequent at best and are primarily found in applied mathematics journals. While, it was never meant to be the main design tool for commercial power reactor design, neutron transport theory investigations did, and still do, play a role in the nuclear enterprise. In particular, the approximate methods of reactor theory are rooted in the neutron transport equation, which itself comes from the larger realm of statistical mechanics. For example, diffusion theory, which is probably today's most useful tool for reactor design, comes directly from the neutron transport equation by elimination of neutron direction. The generation of multigroup group cross sections in the resonance region of the neutron spectrum is another example of neutron transport theory, but without spatial dependence. There is no question we are in danger of losing a beautiful and elegant theory, which, along with introducing concepts of analytical benchmarking, is a major reason for offering a course on analytical methods in neutron transport theory.

In this course, *Case Studies in Neutron Transport Theory*, we shall concentrate on transforming theoretical solution representations of the neutron transport equation into numerically useable forms. The course will study reactor physics from neutron slowing down to multidimensional multigroup theory and criticality. Though the backdrop is reactor physics, our emphasis will be on analytical manipulations of the transport equation and the numerical realization of its solutions.

COURSE OBJECTIVES

The main objective of this course is to provide a basis for understanding the fundamental concepts of neutron transport theory. This will include recent theoretical as well as numerical advances in analytical benchmarking.

Course attendees will become familiar with

- Analytical forms of the transport equation
- Analytical methods of solutions in various geometries
- Numerical evaluation of analytical representations
- Semi-analytical benchmarking techniques.

Finally, the course material may spark the imagination of those participants who are especially innovative, and who seek a creative outlet.

APPLICATION OF COURSE KNOWLEDGE

The course material will find use in several forms. First, knowledge of analytical solutions increases one's awareness of what is available for prediction. While analytical solutions to idealized transport scenarios do not necessarily apply directly to operating systems, they do provide guidance regarding operation of portions of a system. However, the most widespread use of the course material will be for the generation of standards to which one can compare proposed or legacy algorithms. This will provide operational testing of an algorithm as well as an overall algorithmic assessment. The course will include several demonstrations of the analytical benchmark library that accompanies the text as well as exercise in analytical benchmarking practices.

COURSE PREPARATION

Participants should be familiar with reactor physics and the operation of nuclear systems. In addition, some familiarity with mathematics through vector calculus and linear algebra is helpful. The participant should also be familiar with elementary numerical methods and come with an open mind awaiting new information.

COURSE CONTENT

We consider reactor physics concepts in three distinct physical categories, energy dependence, space dependence and coupled energy and space dependence.

1. Derivation of the Neutron Transport Equation (4 Hours)
After an introduction and brief review of essential reactor physics concepts, we heuristically derive the neutron transport equation. A sketch of the statistical mechanical derivation of the neutron transport equation from the Boltzmann equation follows. Finally, we briefly discuss features of the eight additional forms of the transport equation.

2. Neutron Slowing Down (3 Hours)
Our first transport solution is for neutron slowing down through elastic and inelastic scattering in the absence of material boundaries. The slowing down equation is solved analytically as well as in the multigroup approximation.

3. Monoenergetic Reactor Theory (4 Hours)
We next introduce spatial variation through the monoenergetic transport equation derived from both mathematical and physical considerations. Solutions in an infinite medium and slab result from a numerical Fourier transform inversion and the *Fn* method respectively.

4. Multigroup Transport Theory (3 Hours)
We now consider spatial variation in the multigroup approximation. Fourier transform inversion provides numerical solutions in an infinite medium followed by a Green's function method for a slab.

5. Multidimensional Transport Theory (3 Hours)
The final benchmark is for a 3D multigroup neutron transport in an infinite medium resulting from circular, broken elliptical and spiral sources. The origin of the analytical solution and its novel evaluation are discussed.

6. The future of Analytical Benchmarking (1 Hour)
A discussion of what form future benchmarking activity may take as well as the concept of "benchmarking in place" concludes the course presentation.